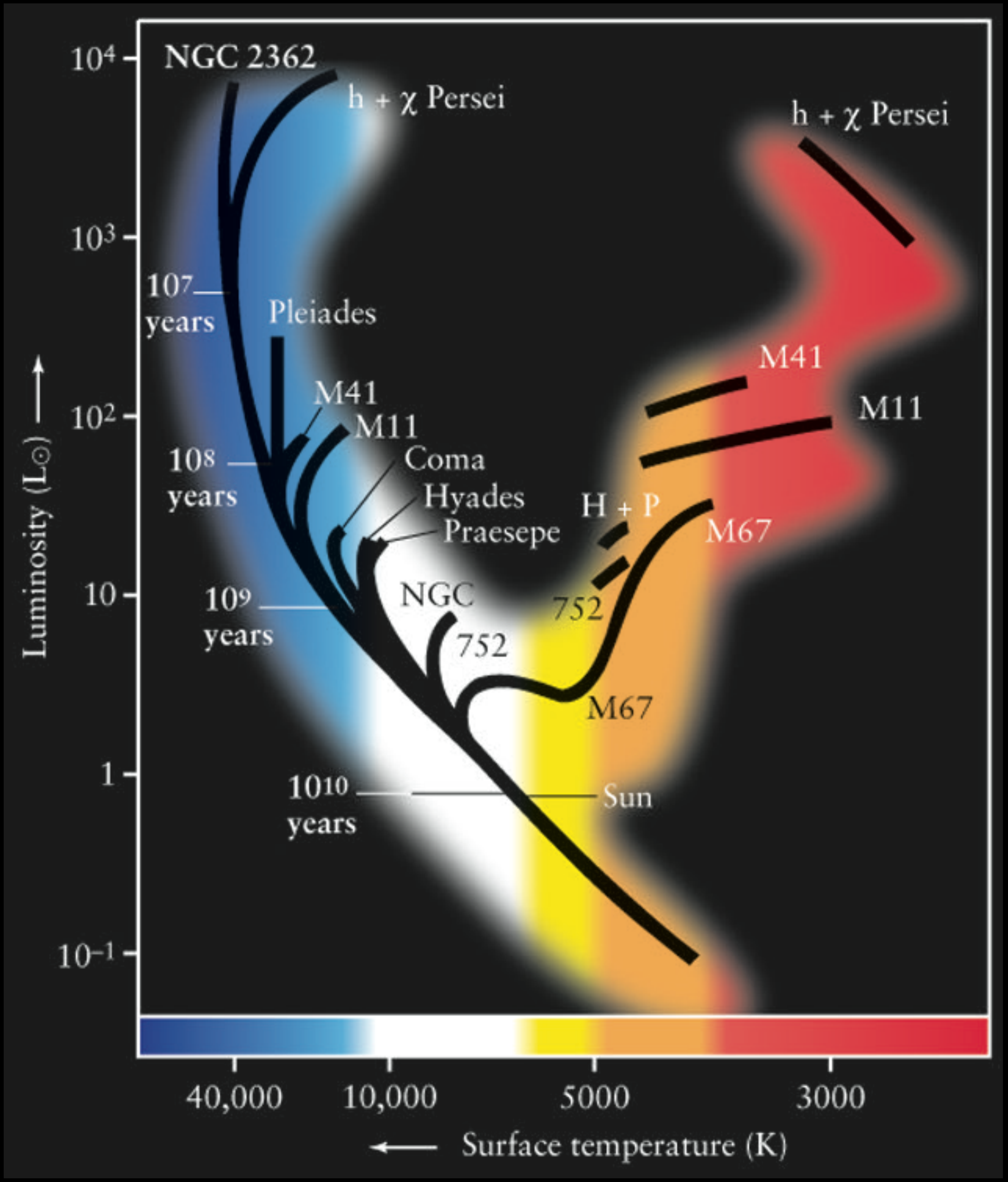
Chap 3: The Evolution of Low-Mass Stars

Chap 3: Key Concepts

- Observations
  - Nothing last forever, even stars
  - H-R diagram of star clusters
- Numerical Models
  - Equations of stellar structure and evolution
  - Stellar evolutionary tracks
- Fine-Tune Models
  - Isochrones (equal-age lines)
  - Fitting cluster H-R diagrams
  - Cluster age estimates
- Model Inferences
  - Main stages and rough lifetimes
  - Changes in the interiors of the stars: e- degenerate core + fusion shells
- Mass-Transfer Binaries
  - Roche Lobe, Lagrange Points
  - Novae, Type Ia SNe, Blue Stragglers

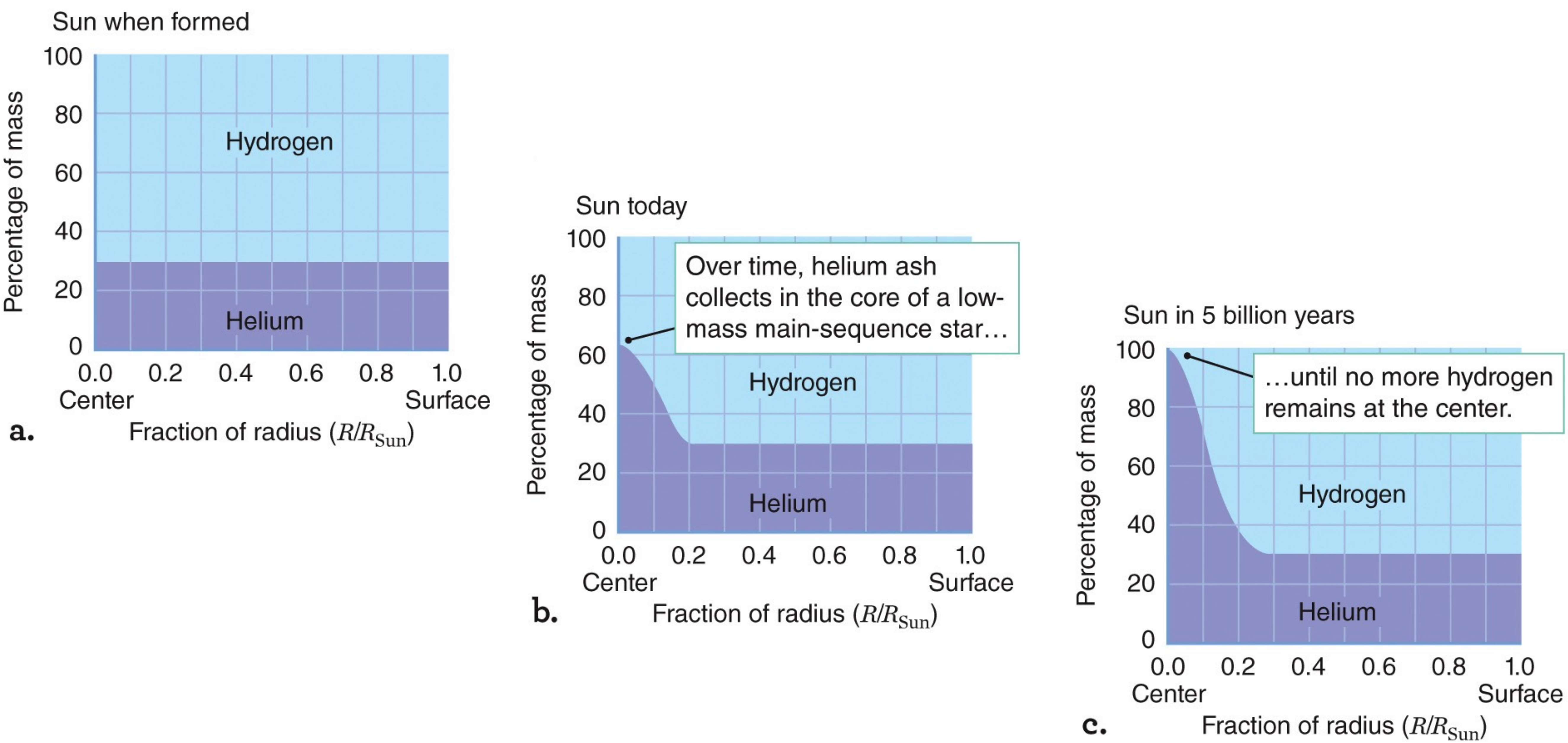


Why do we think stars must evolve?

Logical deduction from our understanding of the Sun

Changes on the Main Sequence due to Fuel Exhaustion

- The chemical composition inside a star changes over time as hydrogen is fused into helium.
- The Sun started with 70 percent hydrogen by mass, but now contains only 35 percent hydrogen in the core.
- What will happen when the hydrogen is exhausted in the core?



Main-Sequence Lifetime of a Star Depends on Its Initial Mass

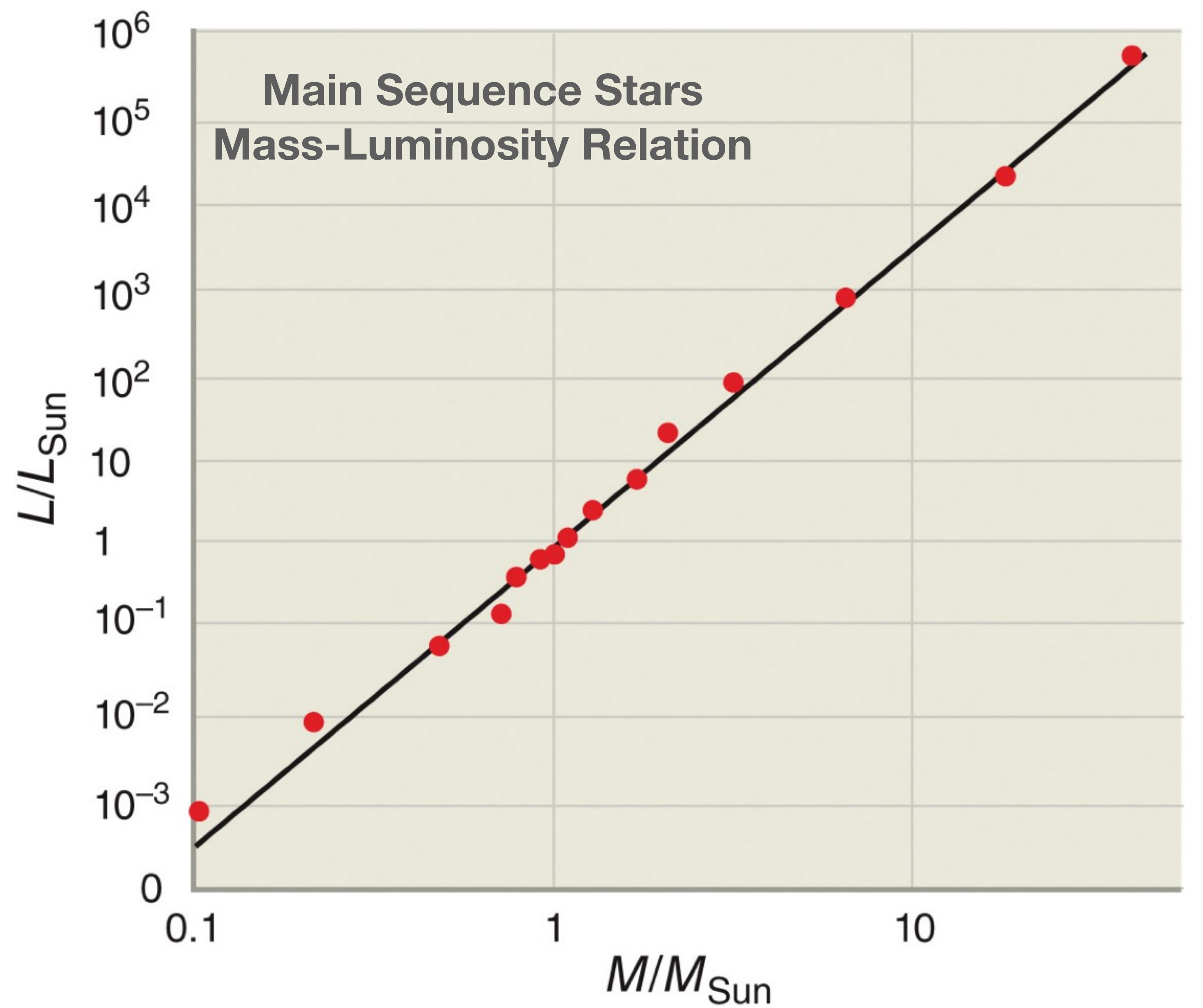
- The main-sequence lifetime of a star is the amount of time that it spends fusing hydrogen as its primary source of energy.

$$\text{Lifetime of star} = \frac{\text{Amount of fuel } (\propto \text{ mass of star})}{\text{Rate fuel is used } (\propto \text{ luminosity of star})}$$

- Stars with high masses have shorter lifetimes.
- Higher-mass stars have more fuel, but they use it more quickly:

$$L \propto M^{3.5}$$

=> MS lifetime ~ $M^{-2.5}$



Calculating the Main-Sequence Lifetime

- In our homework, we have used the proportionalities to get:

$$\text{Lifetime}_{\text{MS}} = 10^{10} \times \frac{M_{\text{MS}}/M_{\text{Sun}}}{\left(M_{\text{MS}}/M_{\text{Sun}}\right)^{3.5}}\ \text{yr} = 10^{10} \times \left(\frac{M_{\text{MS}}}{M_{\text{Sun}}}\right)^{-2.5} \text{yr}$$

- Let's compare the lifetime of a K5 star with the Sun. A K5 star has a mass of 0.67 M_{Sun}:

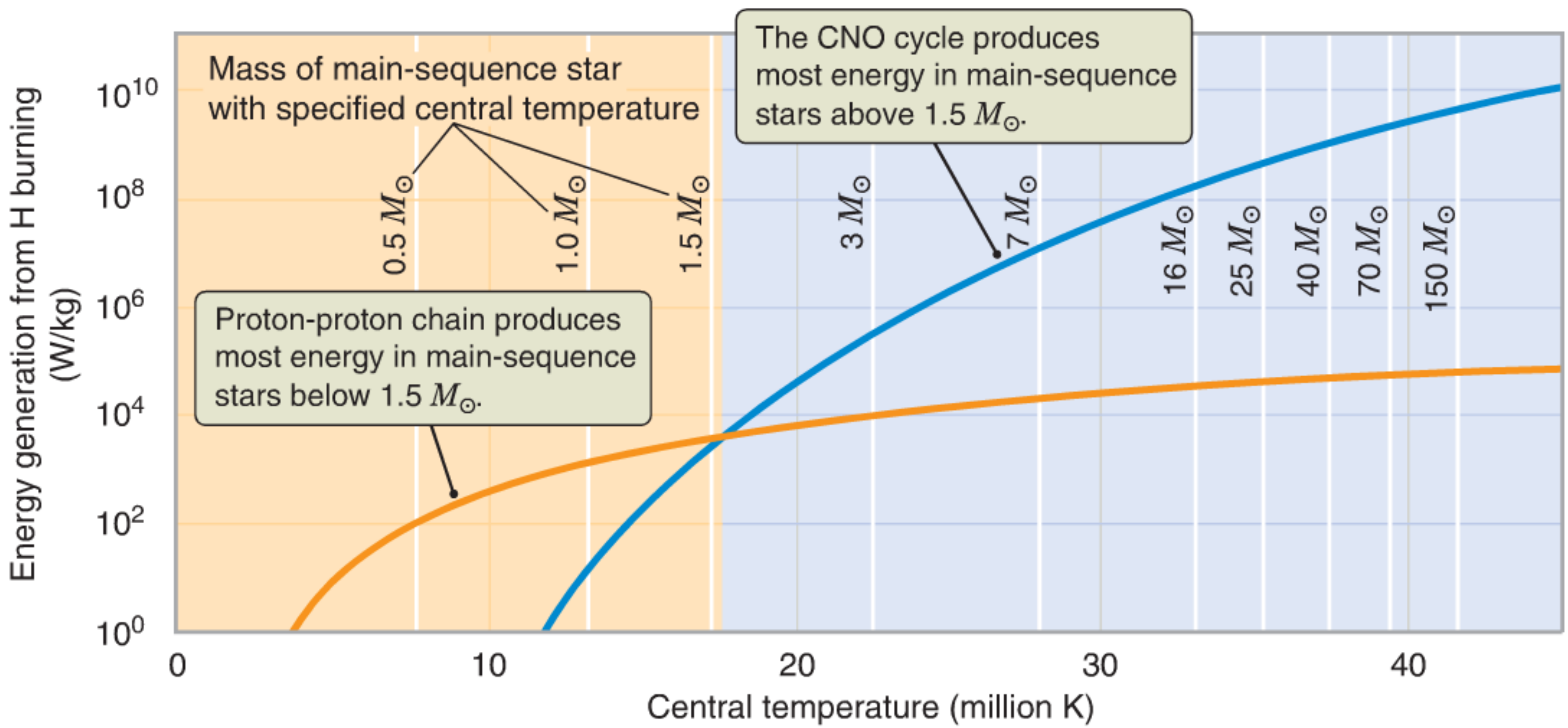
$$\text{Lifetime}_{\text{K5}} = 10^{10} \times (0.67)^{-2.5}\ \text{yr} = 2.7 \times 10^{10} \text{yr}$$

- A **K5** star has a lifetime of **27 billion years**, which is 2.7 times longer than the Sun's lifetime!

- A **O5** main-sequence star has a mass of 60 M_{Sun}, its lifetime is only **360,000 years**! BTW, homo sapiens first evolved in Africa about 300,000 years ago.

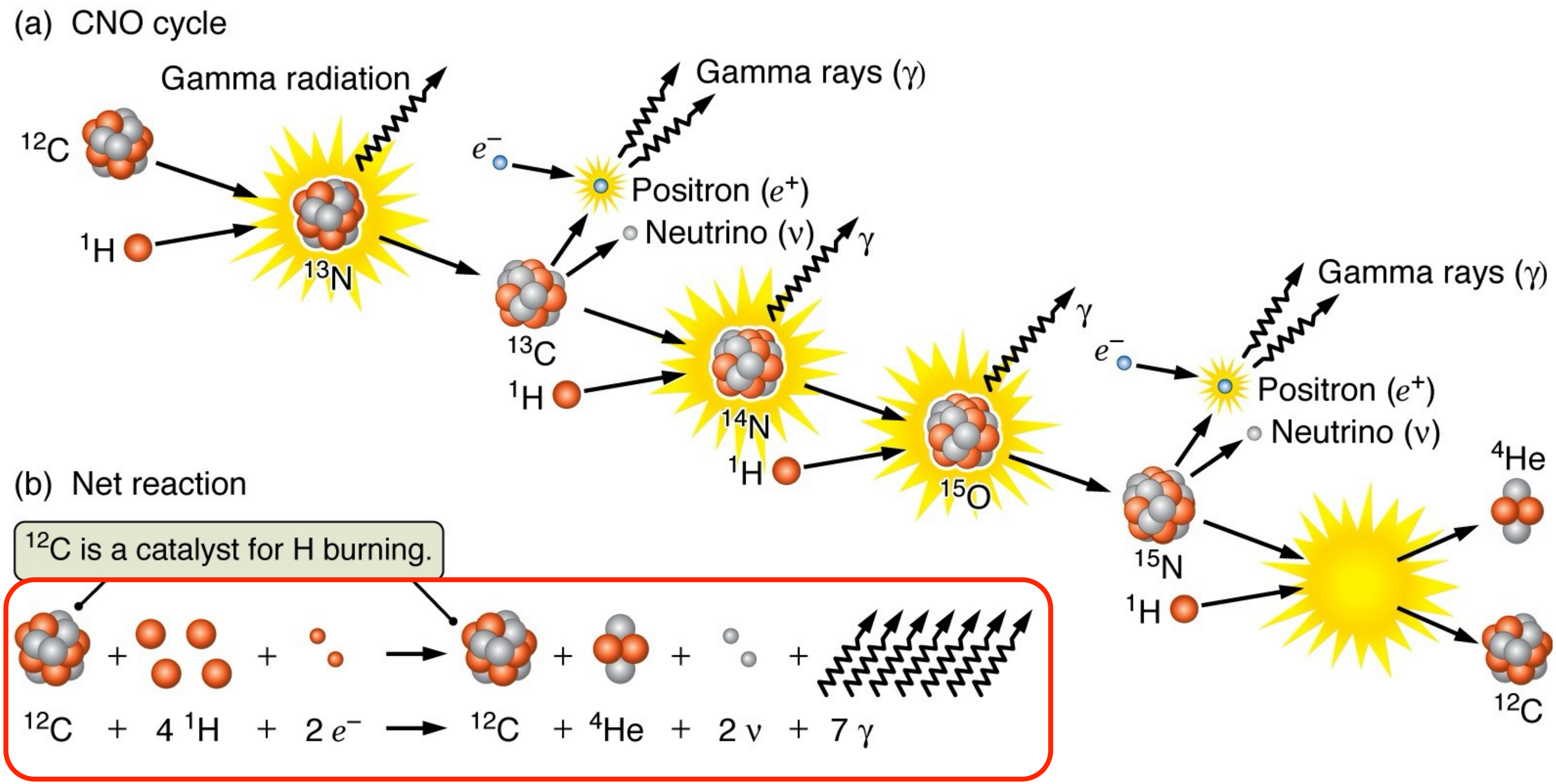
Massive MS stars have higher core temperature but lower core pressure

- Core temperature can be estimated using the **virial theorem**:
$$kT_c \approx GM\mu m_H/R$$
- Core pressure can be estimated from a **force balance**:
$$4\pi R^2 P_c \approx GM^2/R^2 \Rightarrow P_c \approx GM^2/(4\pi R^4)$$
- Main sequence stars show a **mass-radius relation** of:
$$R \propto M^{0.7}$$
- Therefore, $T_c \propto M^{0.3}$ and $P_c \propto M^{-0.8}$



Net reaction of the CNO cycle

- In high-mass stars and the midlife Sun, hydrogen burning proceeds in the CNO cycle instead of the pp chain, due to higher core temperatures.
- The net result is the same as the pp chain: 4 H -> 1 He



Stars Must Change and The Rate of Change Depends on Initial Mass

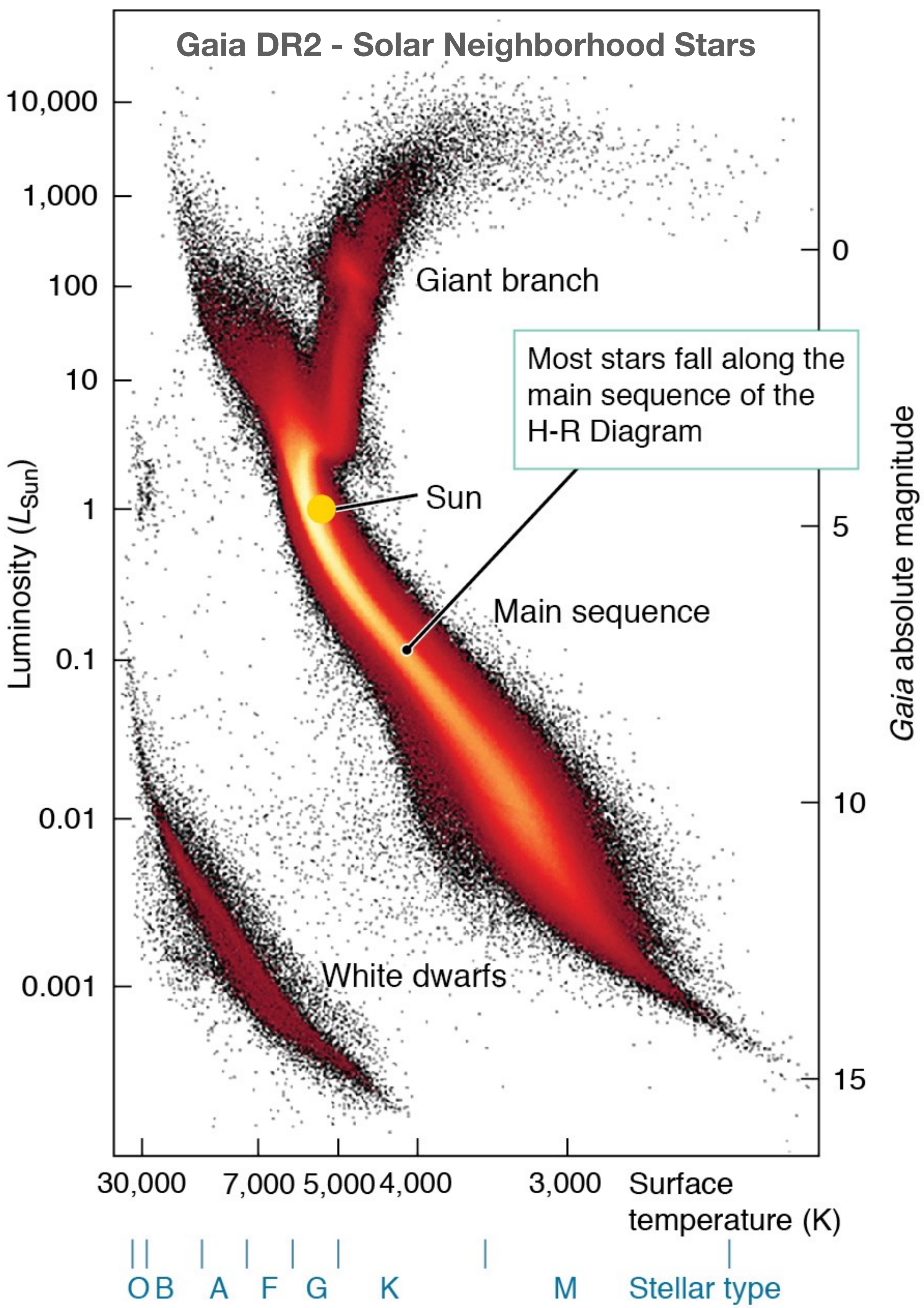
- A star's life depends on mass and composition because the rates and types of fusion depend on the star's mass.
- Stars of different masses evolve differently. There are three categories of stars:
  - **low-mass stars** (Mass < 3 M_{Sun})
  - **intermediate-mass stars** (Mass between 3 M_{Sun} and 8 M_{Sun})
  - **high-mass stars** (Mass > 8 M_{Sun})
- Virial theorem: core temperature increases with stellar mass
  - $2K = -U \Rightarrow \frac{3MkT_c}{\mu m_H} = \frac{3GM^2}{5R} \Rightarrow T_c \propto M/R$
  - For Main-Sequence Stars, $R \propto M^{0.7}$, therefore $T_c \propto M^{0.3}$

Name	High-mass stars	Medium-mass stars	Low-mass stars	Very low-mass stars	Brown dwarfs
Spectral type	O, B	B	A, F, G, K	M	M, L, T, Y
Minimum mass	8 M_{Sun}	3 M_{Sun}	0.5 M_{Sun}	0.08 M_{Sun}	~0.01 M_{Sun} (~13 $M_{Jupiter}$)

How do we know stars evolve?

H-R diagram of star clusters

HR Diagram: Main Sequence, Giant Branch, White Dwarfs, & Luminosity Classes

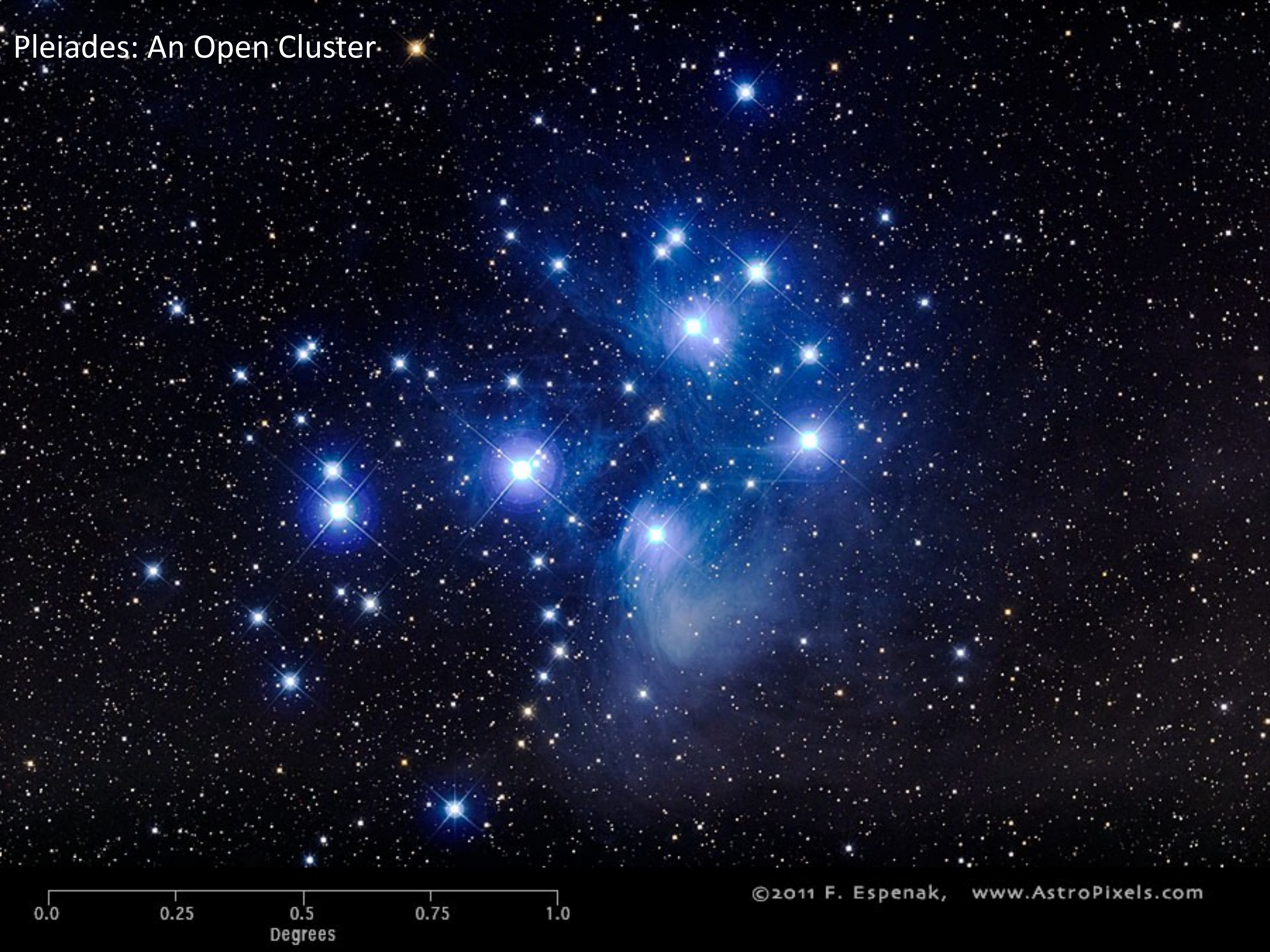


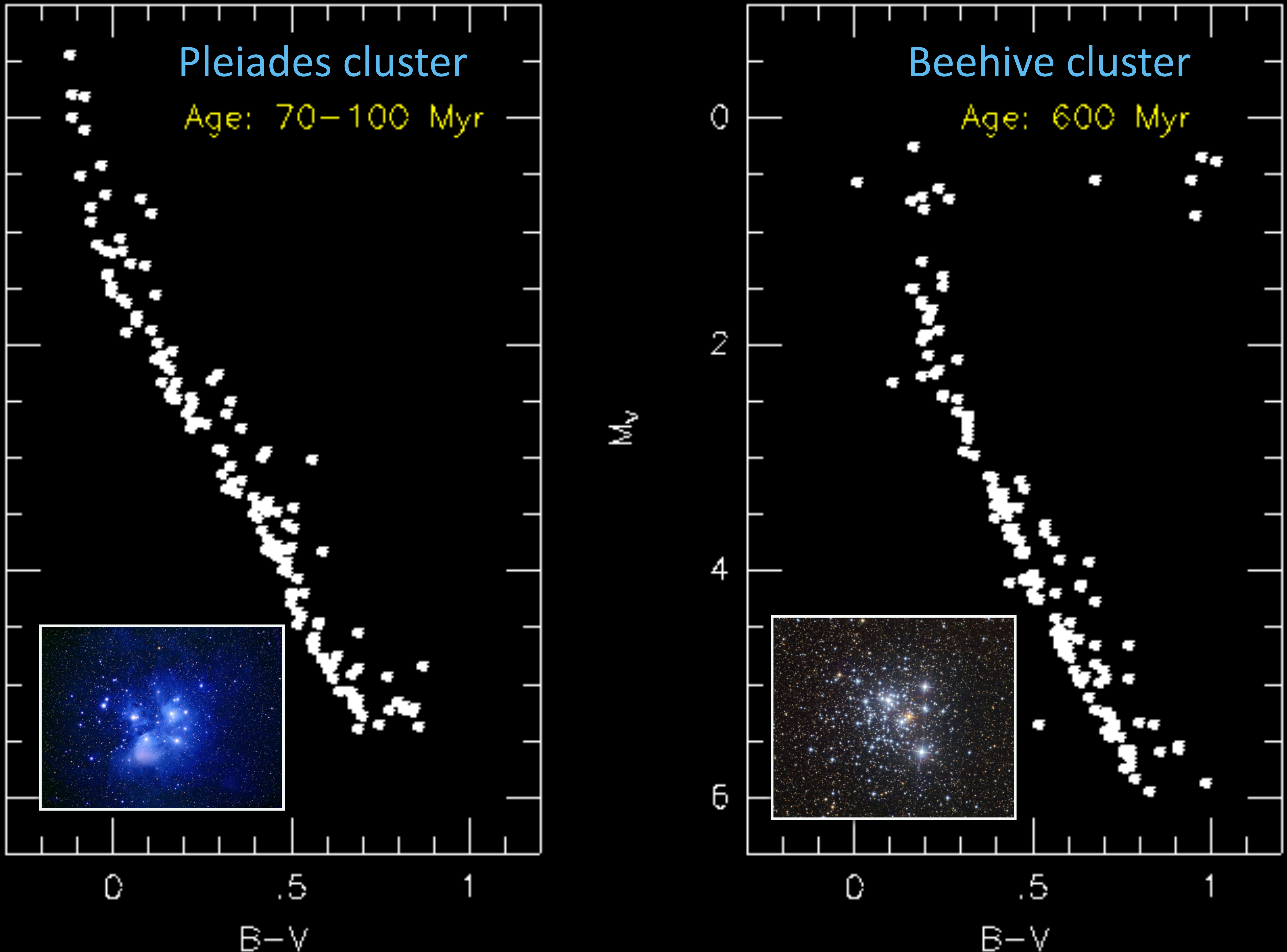
- Most stars, incl. the Sun, are found on the **main sequence,** which runs from luminous/hot stars in upper left corner to low-luminosity/cool stars in lower right corner
- It covers a temperature range of **~10**, and a radius range of **~100**, but a luminosity range of **~10^9**
- The **Giant branch** is connected to the main sequence but branches off to the lower temperature side. That is where the **red giant stars** live
- A separate branch parallel to the main sequence to the lower left, these stars have low luminosities but hot temperatures; this is where the **White Dwarfs** live
- **Spectral classification is 2D**: (1) temperature (OBAFGKM), and (2) luminosity (Ia, Ib, II, III, IV, V)

Star clusters are ideal laboratories to study stellar evolution

- Star clusters are bound groups of stars, all made at **the same time**.
- Each star evolves at a rate set by its mass.
- High-mass stars evolve more quickly along their evolutionary tracks.

Pleiades: An Open Cluster




Pleiades cluster
Age: 70−100 Myr
Beehive cluster
Age: 600 Myr
0
2
4
6
M_V
0
.5
1
B−V
0
.5
1
B−V

H-R diagram of open clusters in the MW galaxy

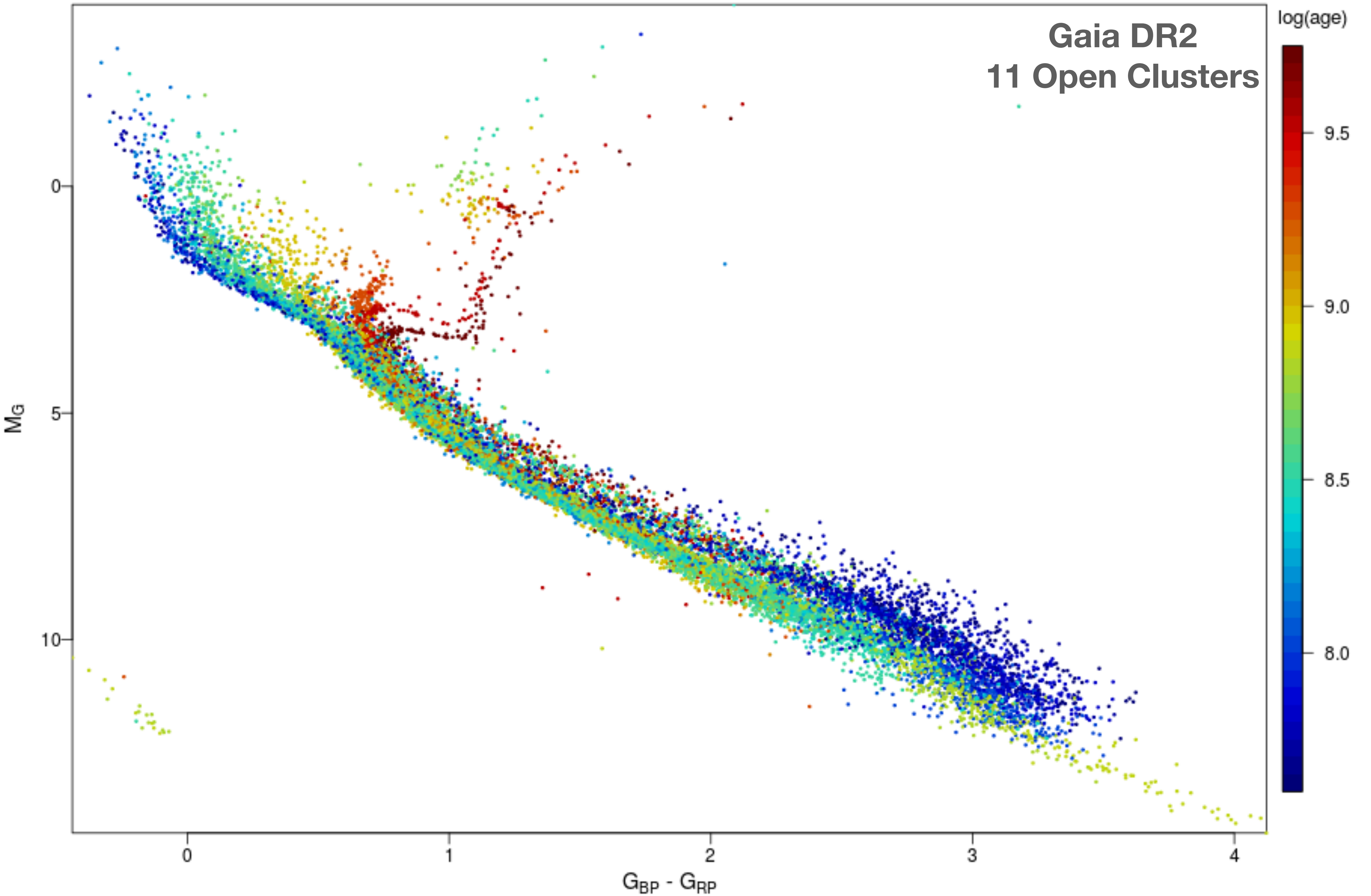


Table 2. Overview of reference values used in constructing the composite HRD for open clusters (Fig. 2).

Cluster	DM	log(age)	[Fe/H]	$E(B-V)$	Memb
Hyades	3.389	8.90	0.13	0.001	480
Coma Ber	4.669	8.81	0.00	0.000	127
Pleiades	5.667	8.04	−0.01	0.045	1059
IC 2391	5.908	7.70	−0.01	0.030	254
IC 2602	5.914	7.60	−0.02	0.031	391
α Per	6.214	7.85	0.14	0.090	598
Praesepe	6.350	8.85	0.16	0.027	771
NGC 2451A	6.433	7.78	−0.08	0.000	311
Blanco 1	6.876	8.06	0.03	0.010	361
NGC 6475	7.234	8.54	0.02	0.049	874
NGC 7092	7.390	8.54	0.00	0.010	248

H-R diagram of globular clusters in the MW galaxy

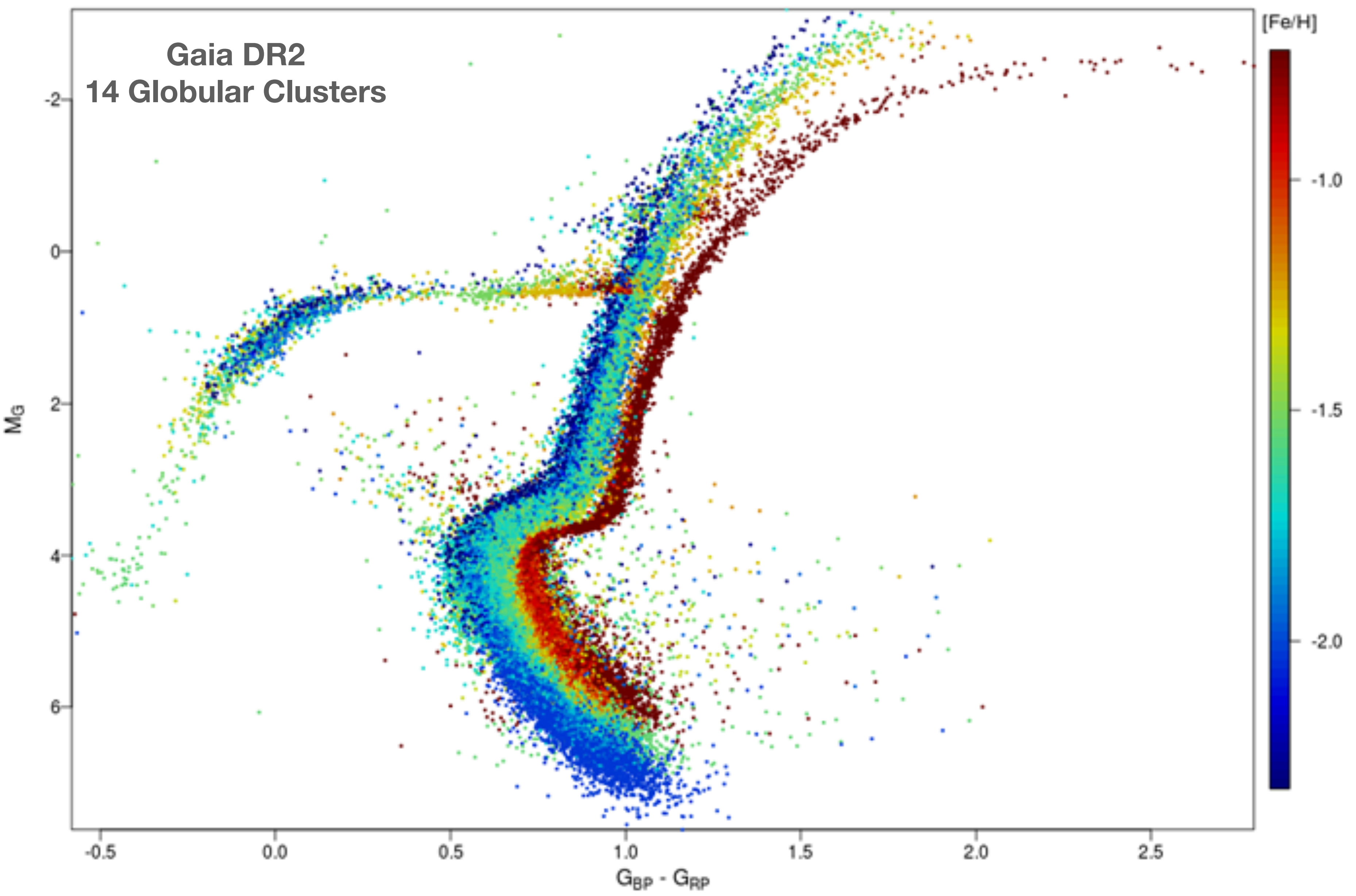
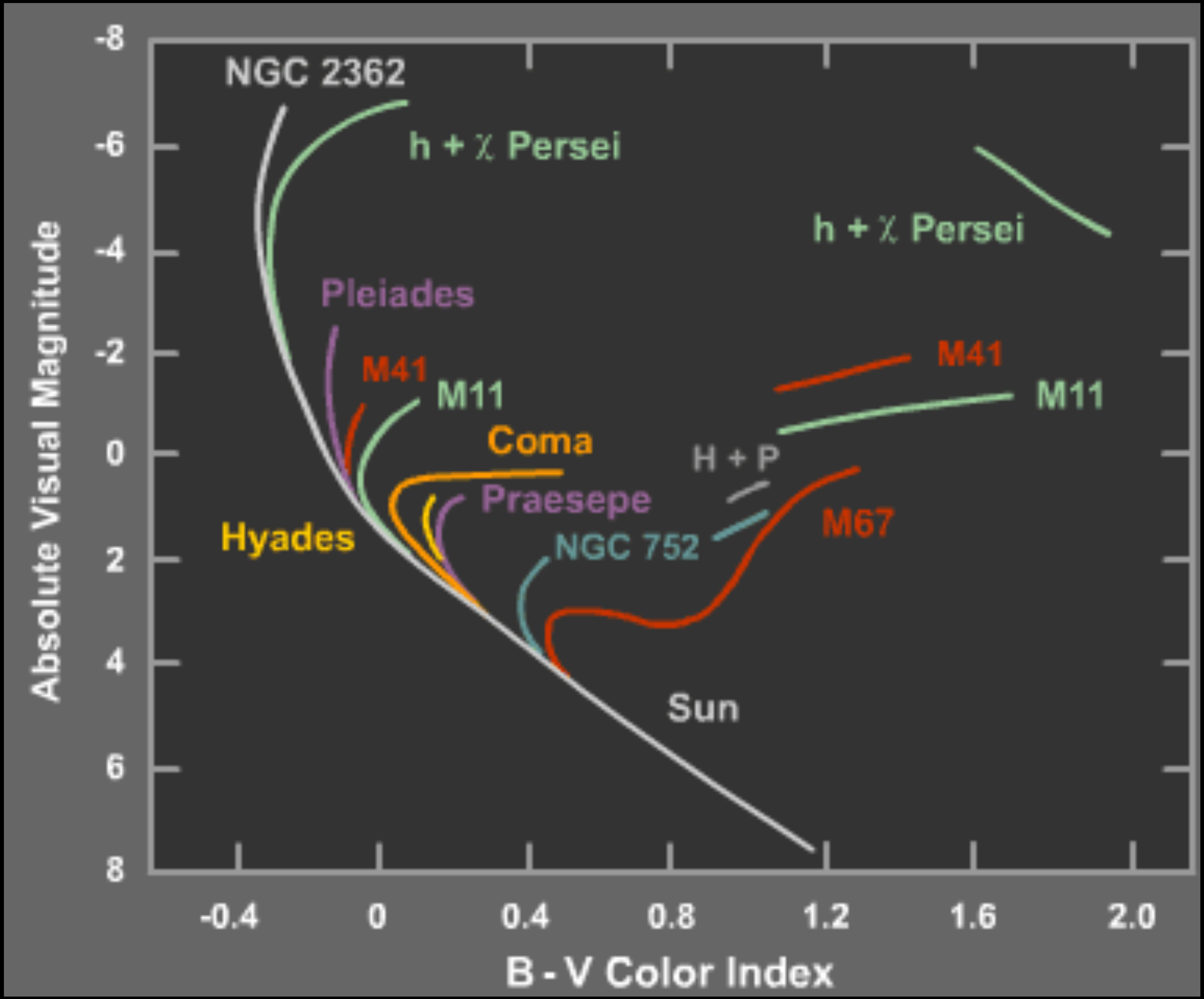


Table 3. Reference data for 14 globular clusters used in the construction of the combined HRD (Fig. 3).

NGC	DM	Age (Gyr)	[Fe/H]	$E(B-V)$	Memb
104	13.266	12.75[a]	−0.72	0.04	21580
288	14.747	12.50[a]	−1.31	0.03	1953
362	14.672	11.50[a]	−1.26	0.05	1737
1851	15.414	13.30[c]	−1.18	0.02	744
5272	15.043	12.60[b]	−1.50	0.01	9086
5904	14.375	12.25[a]	−1.29	0.03	3476
6205	14.256	13.00[a]	−1.53	0.02	10311
6218	13.406	13.25[a]	−1.37	0.19	3127
6341	14.595	13.25[a]	−2.31	0.02	1432
6397	11.920	13.50[a]	−2.02	0.18	10055
6656	12.526	12.86[c]	−1.70	0.35	9542
6752	13.010	12.50[a]	−1.54	0.04	10779
6809	13.662	13.50[a]	−1.94	0.08	8073
7099	14.542	13.25[a]	−2.27	0.03	1016

Simplified H-R diagram of open clusters in the MW galaxy

Can we build a single model to explain all of the star clusters?
If so, what parameter makes different clusters look different on the HRD?



How do we model stellar evolution?

Computational code of stellar evolution

Stellar Structure Models - Basic Equations

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$ HYDROSTATIC EQUILIBRIUM

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$ MASS CONSERVATION

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$ ENERGY EQUATION

$$\left.\frac{dT}{dr}\right|_{rad} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$ RADIATIVE TRANSPORT

$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$ ADIABATIC CONVECTION

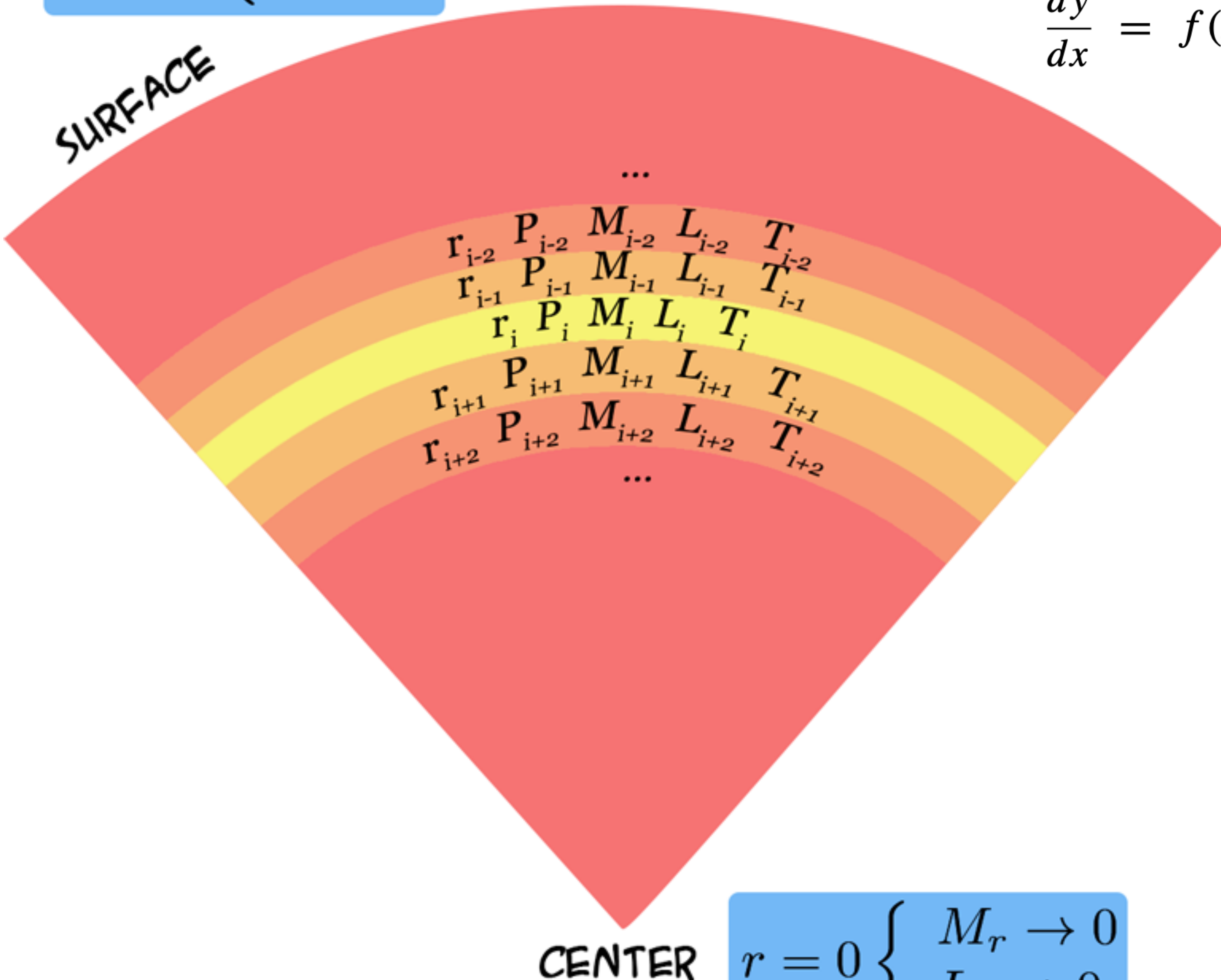
Luis Martínez

$$r = R^* \begin{cases} T \to 0 \\ P \to 0 \\ \rho \to 0 \end{cases}$$

Euler Method: a numerical procedure to solve differential equations

$$y(x_k + \Delta x) \approx y_k + \Delta x \cdot f(x_k, y_k)$$

$$\frac{dy}{dx} = f(x, y)$$



$$r = 0 \begin{cases} M_r \to 0 \\ L_r \to 0 \end{cases}$$

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Modules for Experiments in Stellar Astrophysics (MESA)

https://docs.mesastar.org/en/release-r22.11.1/index.html

http://user.astro.wisc.edu/~townsend/static.php?ref=mesa-web-submit

Motivation

Stellar evolution calculations (i.e., stellar evolution tracks and detailed information about the evolution of internal and global properties) are a basic tool that enable a broad range of research in astrophysics. Areas that critically depend on high-fidelity and modern stellar evolution include asteroseismology, nuclear astrophysics, stellar populations, chemical evolution and population synthesis, astrobiology, binary stars, variable stars, supernovae, novae, compact objects, tidal disruption events, stellar hydrodynamics, and stellar activity.

New observational capabilities are emerging in these fields that place a high demand on exploration of stellar dependencies on mass, metallicity and age. So, even though one dimensional stellar evolution is a mature discipline, we continue to ask new questions of stars. Some important aspects of stars are truly three-dimensional, such as convection, rotation, and magnetism. These aspects remain in the realm of research frontiers with evolving understanding and insights, quite often profound. However, much remains to be gained scientifically (and pedagogically) by accurate one-dimensional calculations, and this is the focus of MESA.

MESA-Web Calculation Submission

To submit a *MESA-Web* calculation, simply enter your email address in the *Email Address* field at the bottom of the form below, and then click the *Submit* button.

The default parameters have been chosen to evolve a 1 $M_\odot$ model from pre main-sequence to white dwarf in less than 2 hours of wall time. To obtain more-Detailed information about each parameter, click on the name of the parameter to visit the corresponding entry on the the *MESA-Web* Input page.

After a calculation completes, you will receive an email with link to a Zip archive that contains the output from *MESA-Web* (note that the link expire after one day). For information on the contents of this archive, see the *MESA-Web* Output page.

Initial Properties

Mass: 1.0 $M_\odot$

Metallicity: 0.02

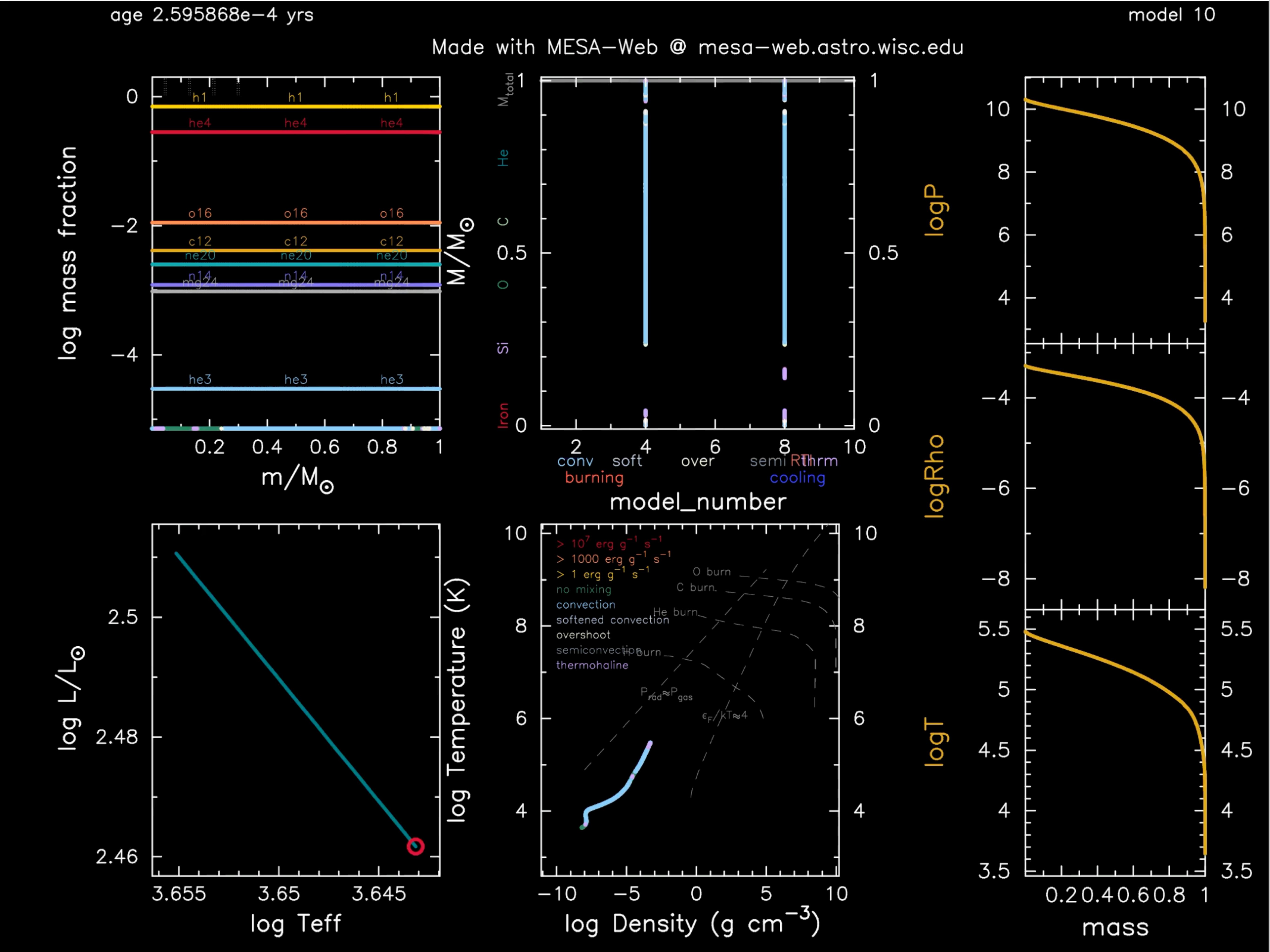
Rotation Rate ($\Omega_{ZAMS}/\Omega_{crit}$): 0.0

Nuclear Reactions

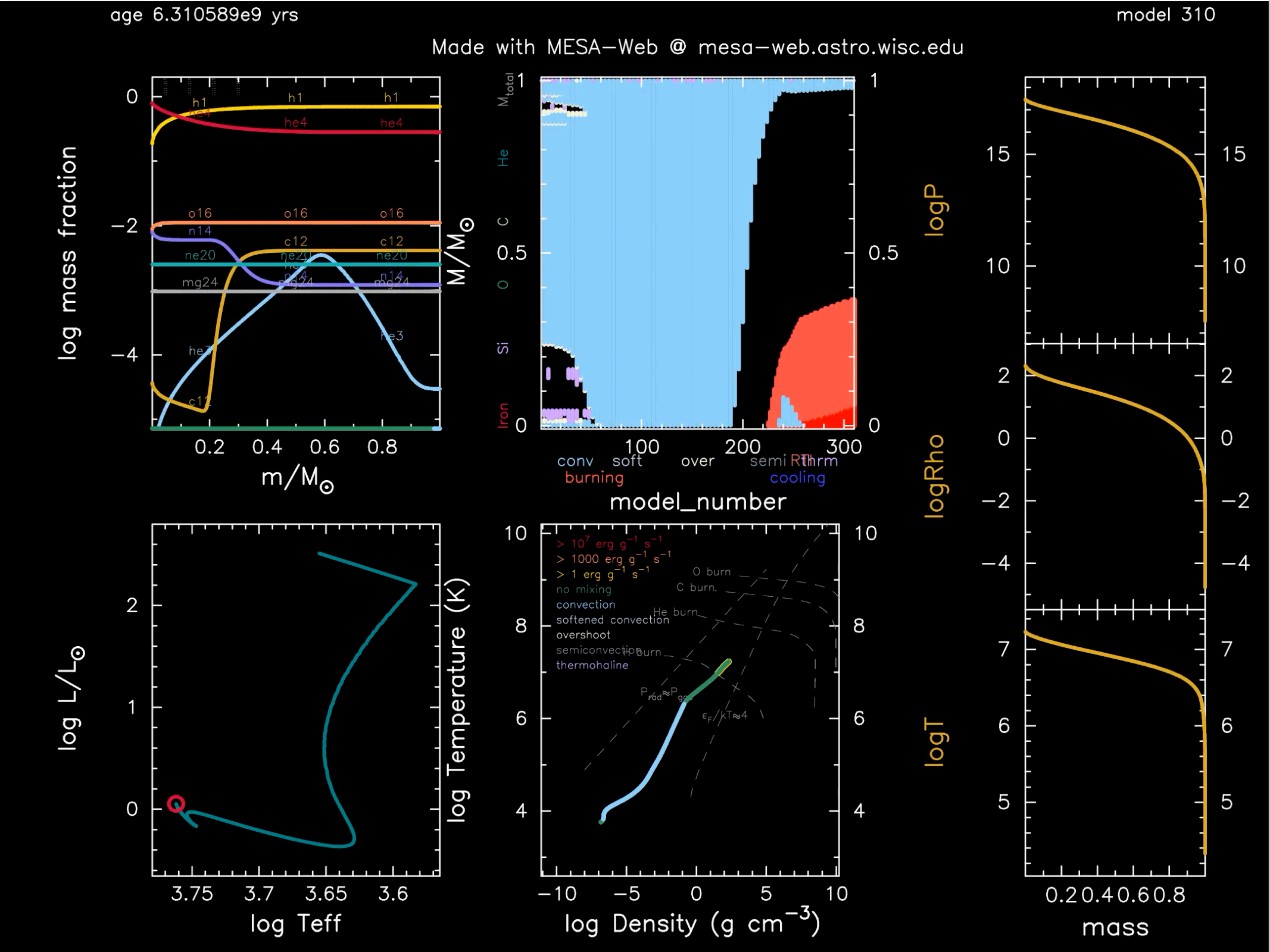
Network: basic

Custom Nuclear Reaction Rate: none

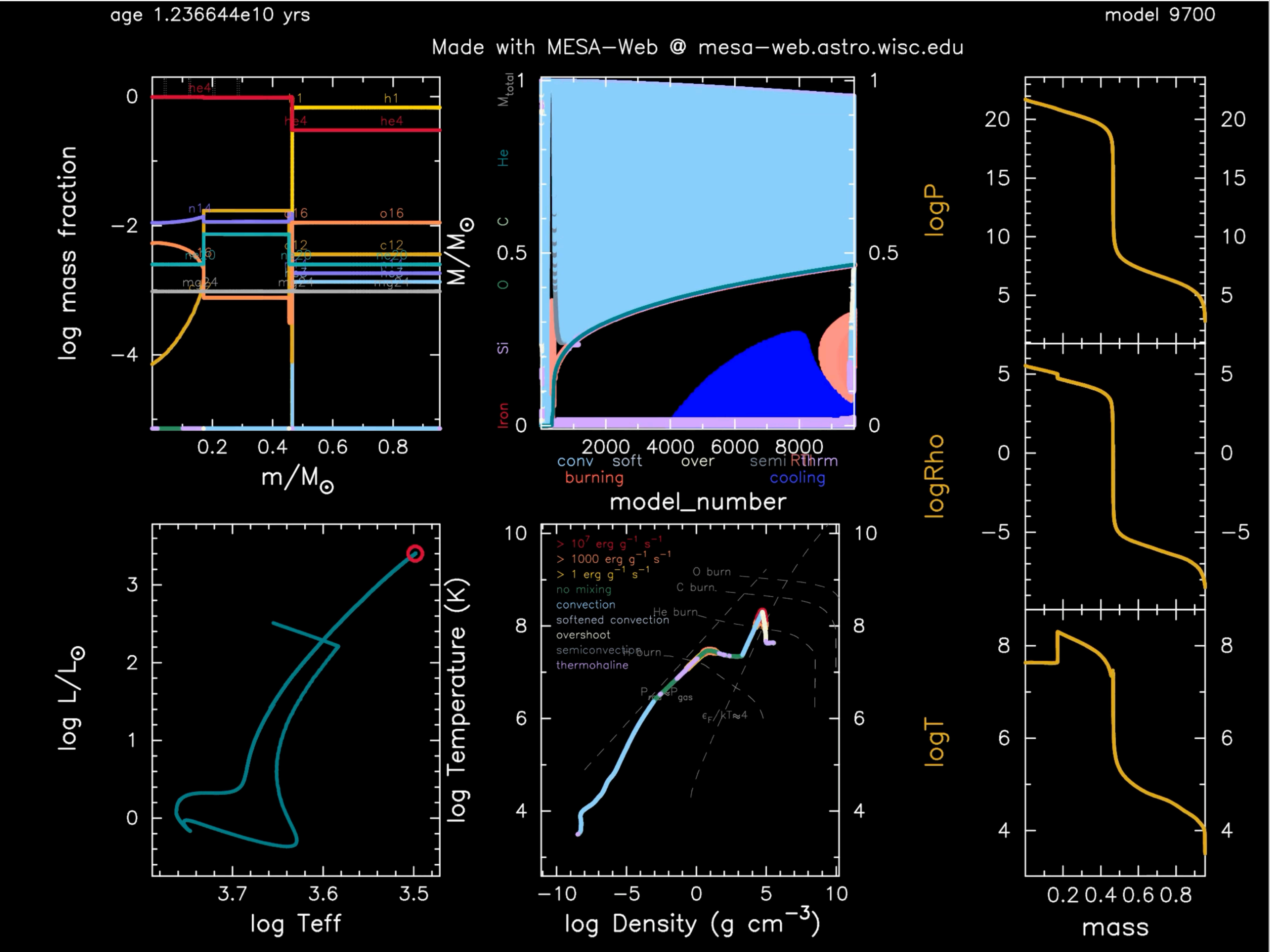
Upload Rate: Choose File no file selected



age 2.595868e-4 yrs
model 10
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
h1
he4
o16
c12
ne20
n14
mg24
he3
m/M⊙
M/M⊙
Mtotal
He
C
O
Si
Iron
conv
soft
over
semi
RTI
thrm
burning
cooling
model_number
logP
logRho
logT
mass
log L/L⊙
log Teff
log Temperature (K)
log Density (g cm−3)
> 10^7 erg g−1 s−1
> 1000 erg g−1 s−1
> 1 erg g−1 s−1
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
Prad≈Pgas
εF/kT≈4



age 6.310589e9 yrs
model 310
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
m/M☉
h1
he4
o16
n14
c12
ne20
mg24
he3
M/M☉
Mtotal
He
C
O
Si
Iron
model_number
conv
soft
over
semi
thrm
burning
cooling
logP
logRho
logT
mass
log L/L☉
log Teff
log Temperature (K)
log Density (g cm⁻³)
> 10⁷ erg g⁻¹ s⁻¹
> 1000 erg g⁻¹ s⁻¹
> 1 erg g⁻¹ s⁻¹
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
Prad≈Pgas
εF/kT≈4



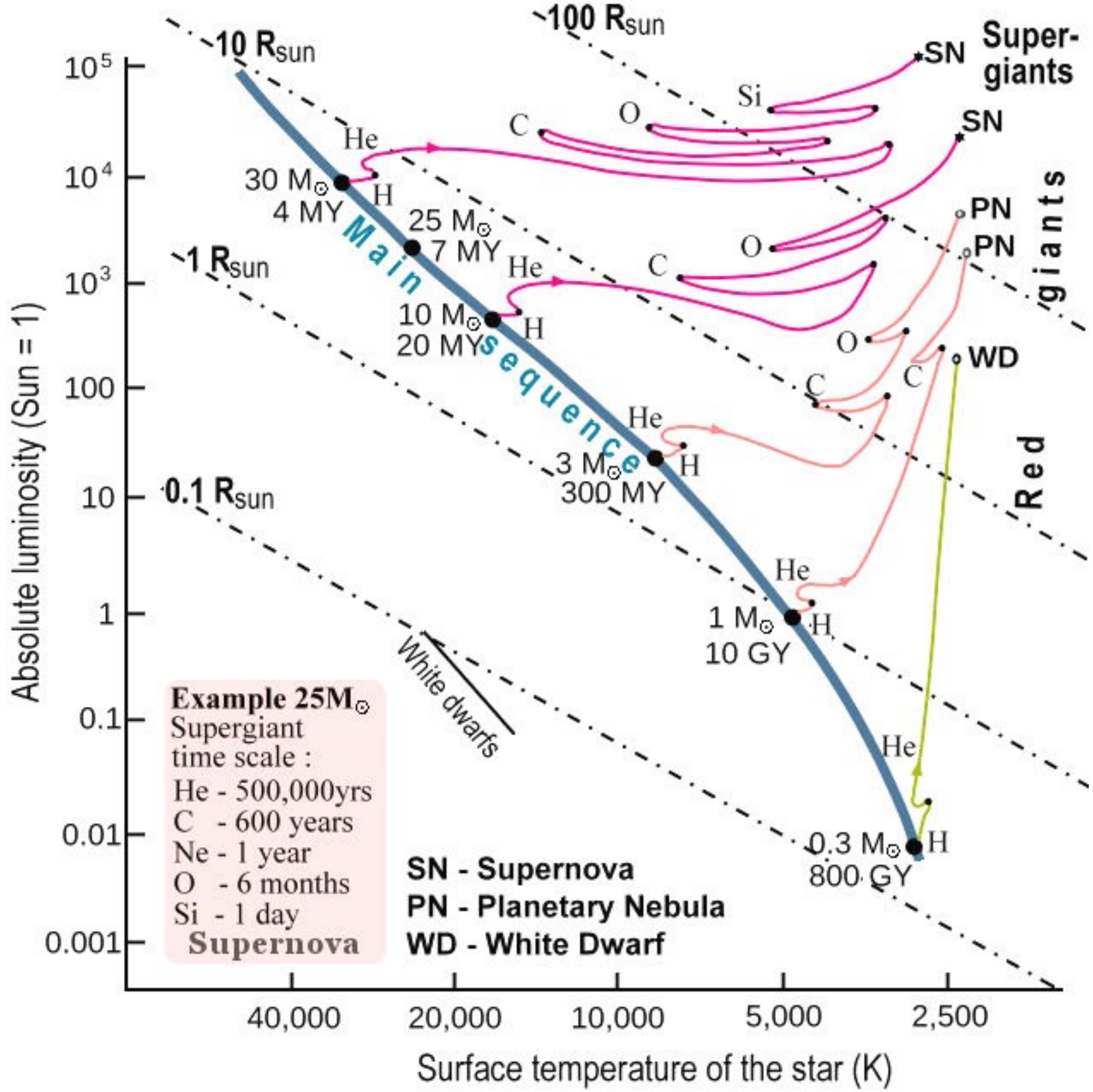
age 1.236644e10 yrs
model 9700
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
m/M⊙
h1
he4
o16
c12
ne20
n14
mg24
M/M⊙
model_number
conv soft over semi Rth thrm
burning cooling
logP
logRho
logT
mass
log L/L⊙
log Teff
log Temperature (K)
log Density (g cm−3)
> 10^7 erg g−1 s−1
> 1000 erg g−1 s−1
> 1 erg g−1 s−1
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
Prad≈Pgas
εF/kT≈4

How do we check stellar evolution models?

Model Isochrones vs. Cluster H-R Diagrams

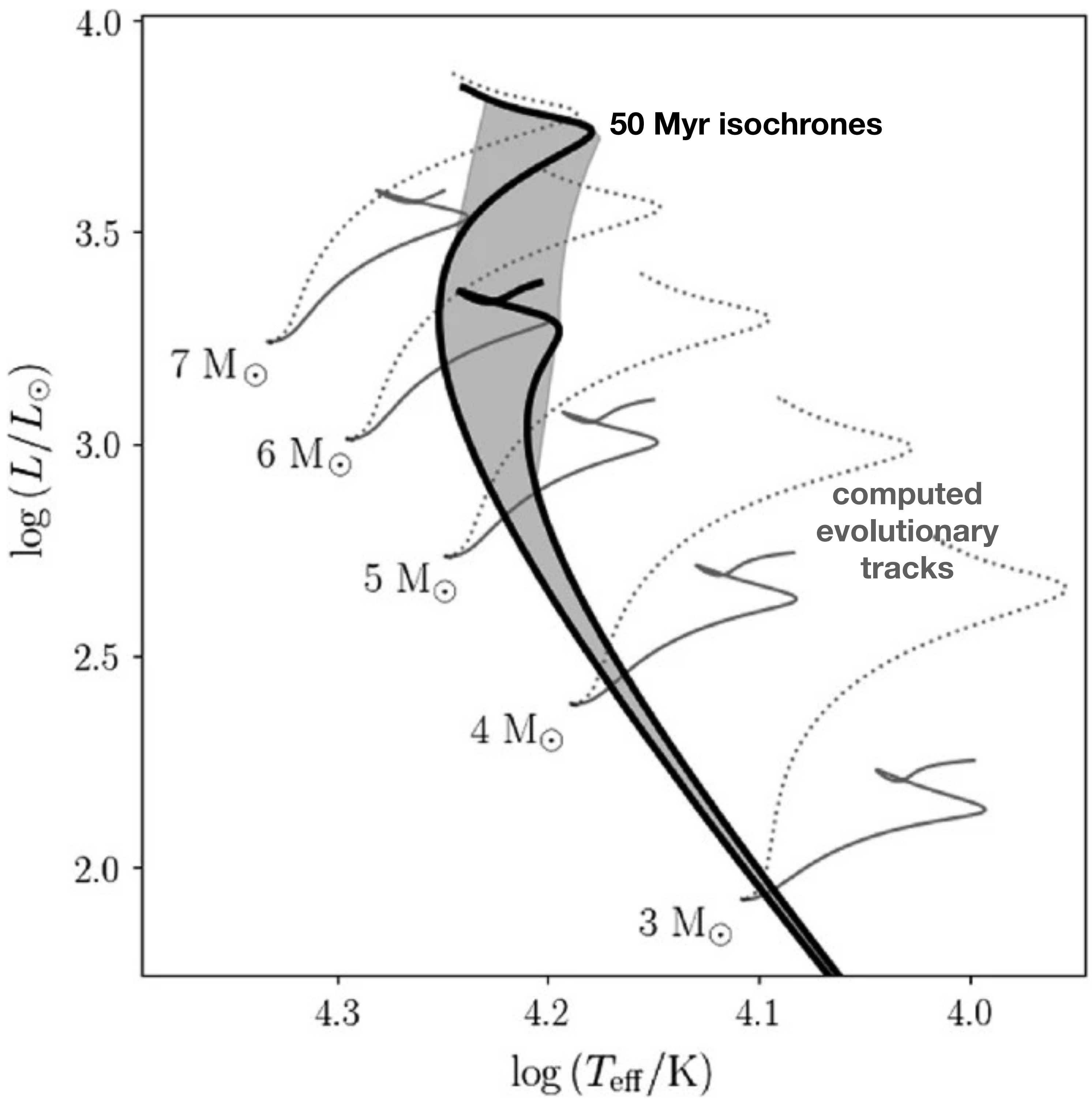
Computed evolutionary tracks of stars with different *initial* masses

- An **evolutionary track** is a computed trajectory of **a single star** on the H-R diagram as it ages over time.

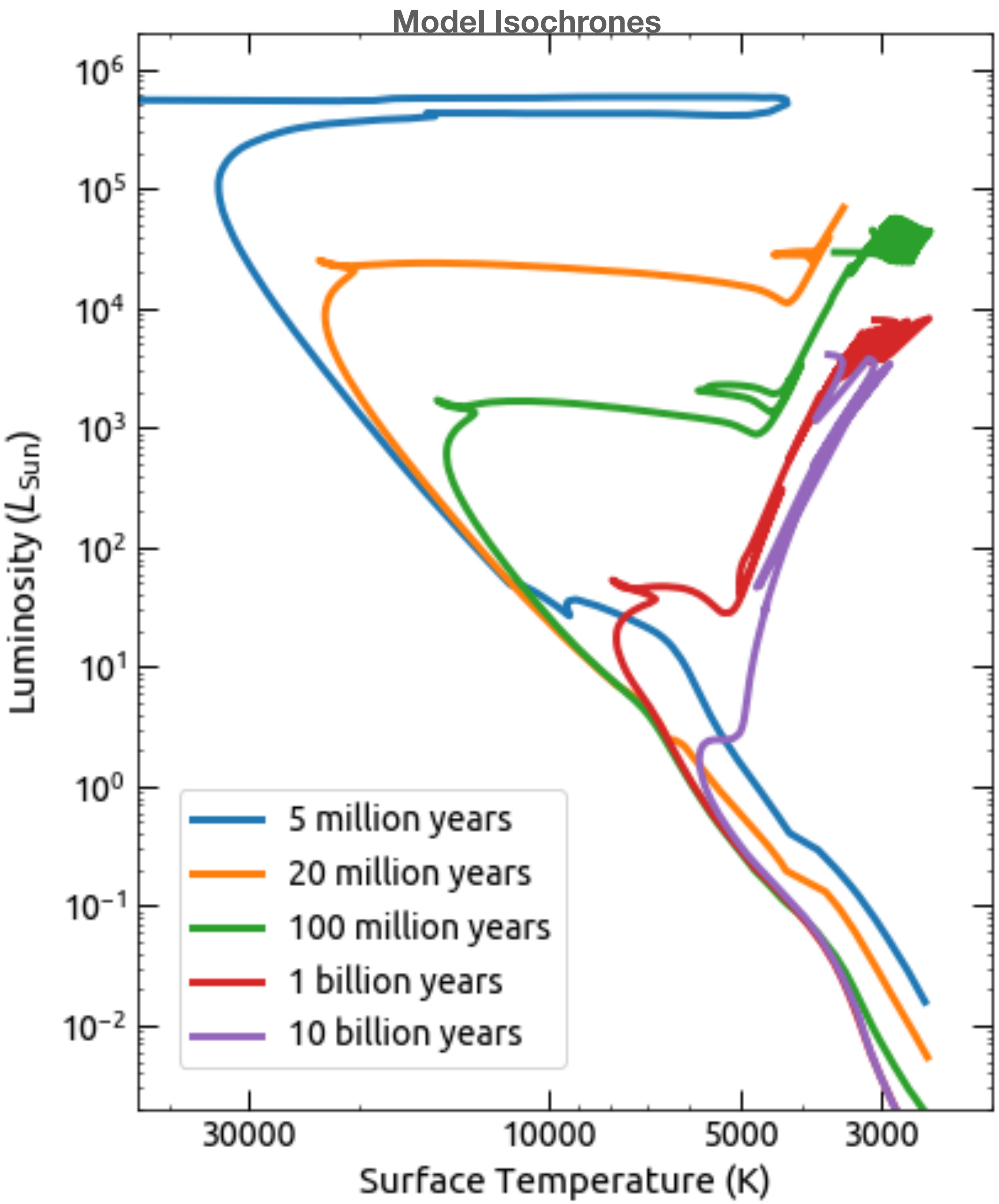


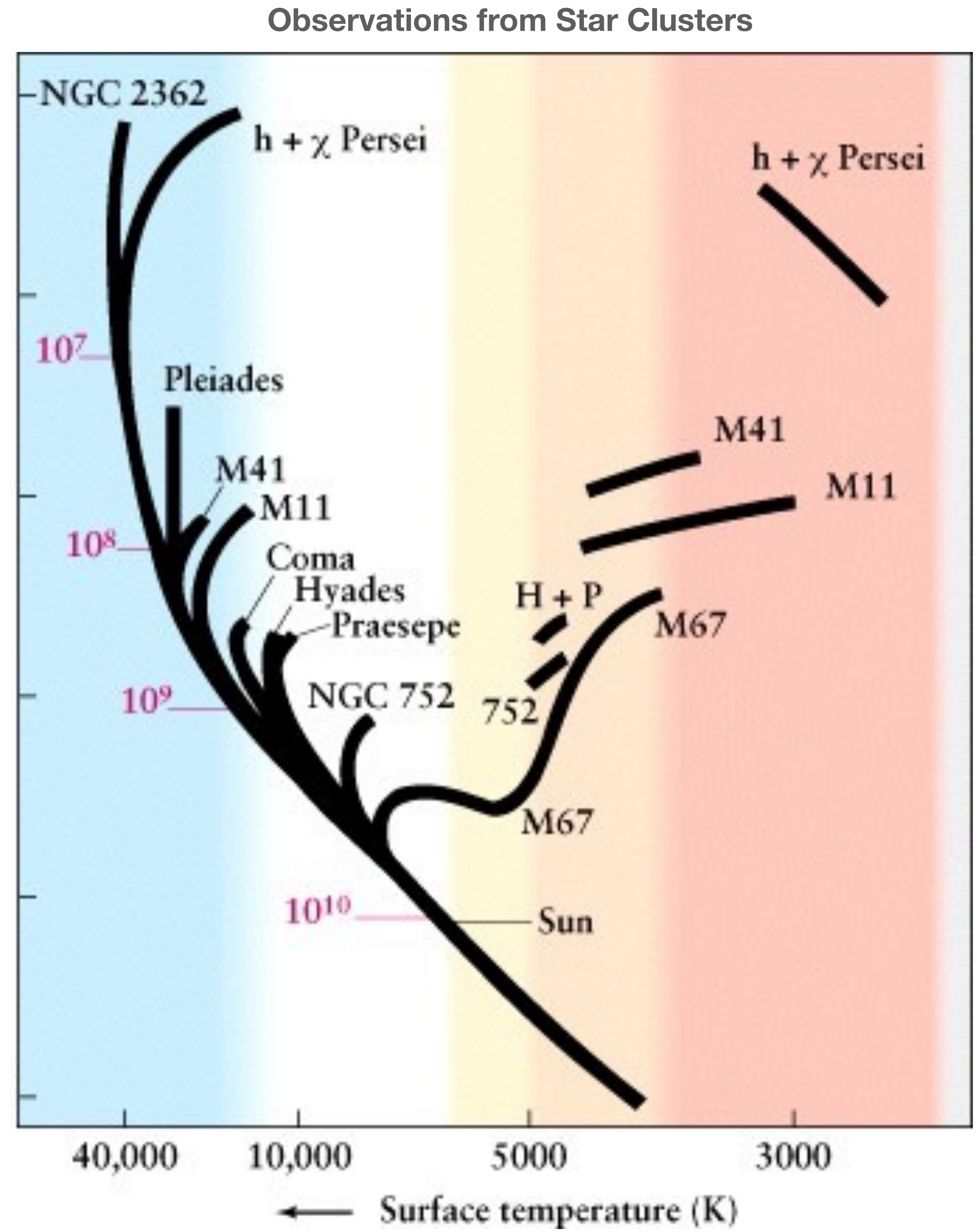
Building isochrones from evolutionary tracks

- An **isochrone** (iso = equal, chrone = time) is a line drawn on the H-R diagram connecting **stars of the same age**



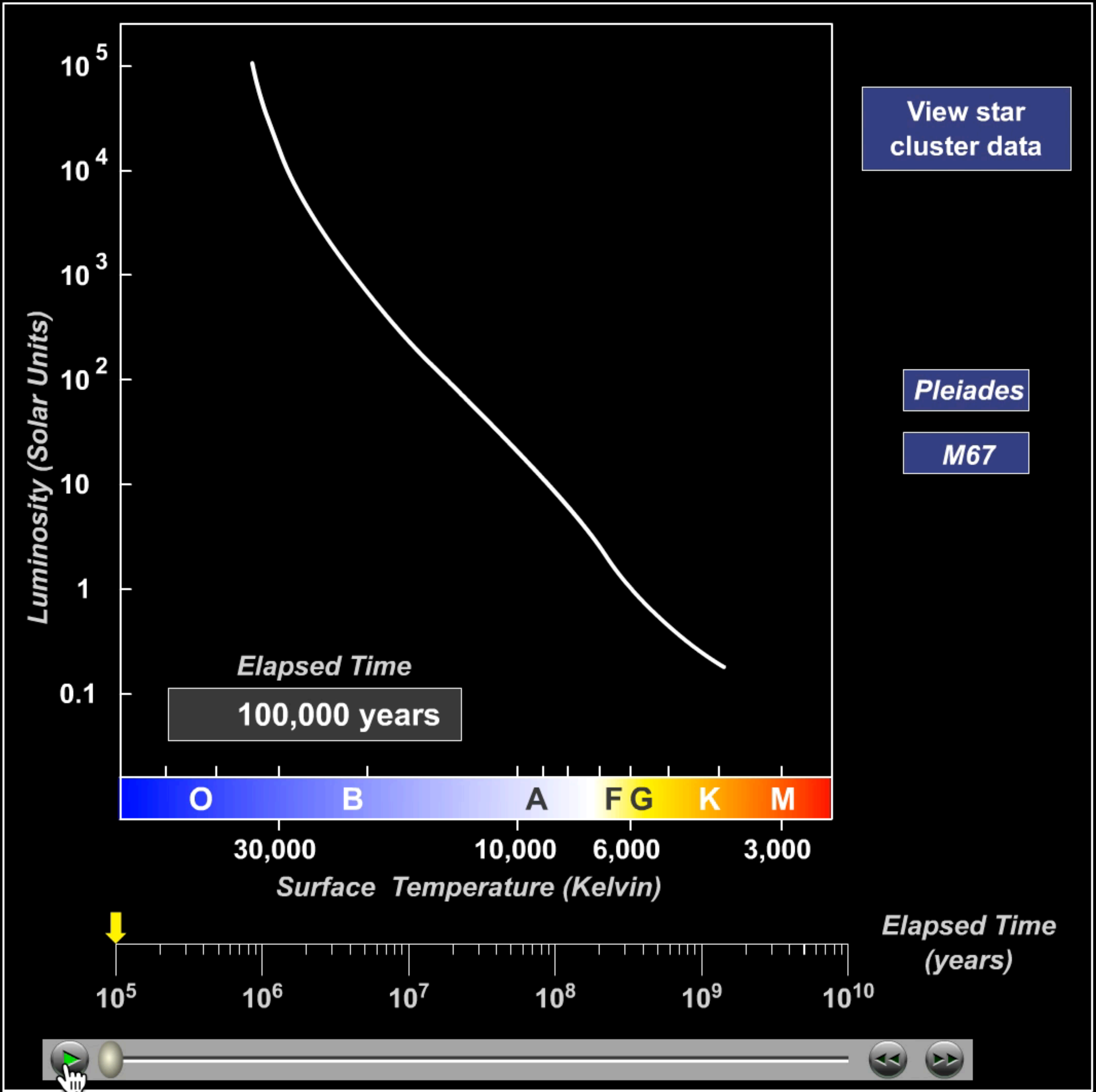
Isochrones change with age: this is how we explain the various different observed H-R diagrams of star clusters





How can you tell if a curve on a HR diagram is an isochrone instead of an evolutionary track?

Using the H-R Diagram to Determine the Age of a Star Cluster



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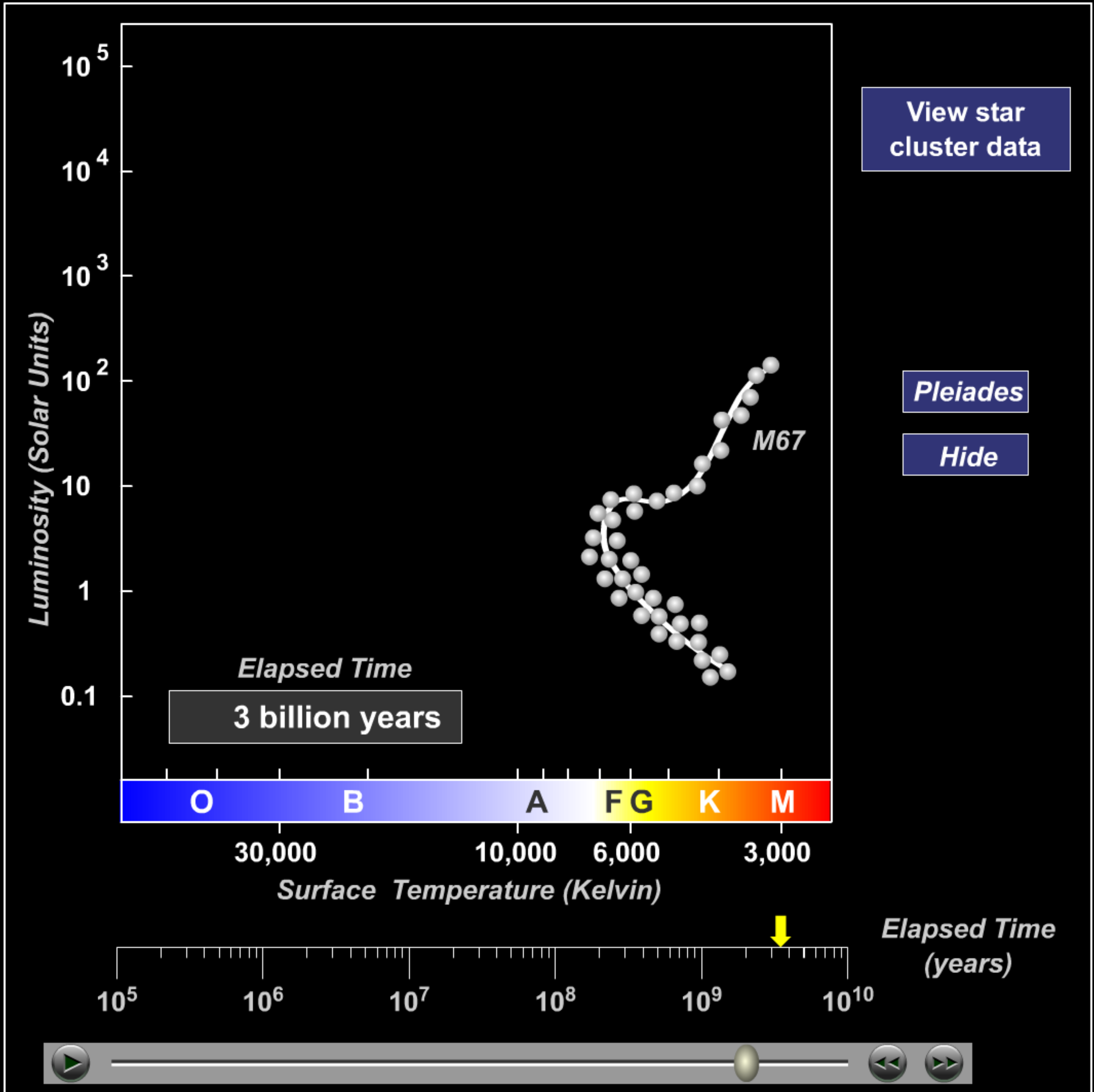
Using the H-R Diagram to Determine the Age of a Star Cluster



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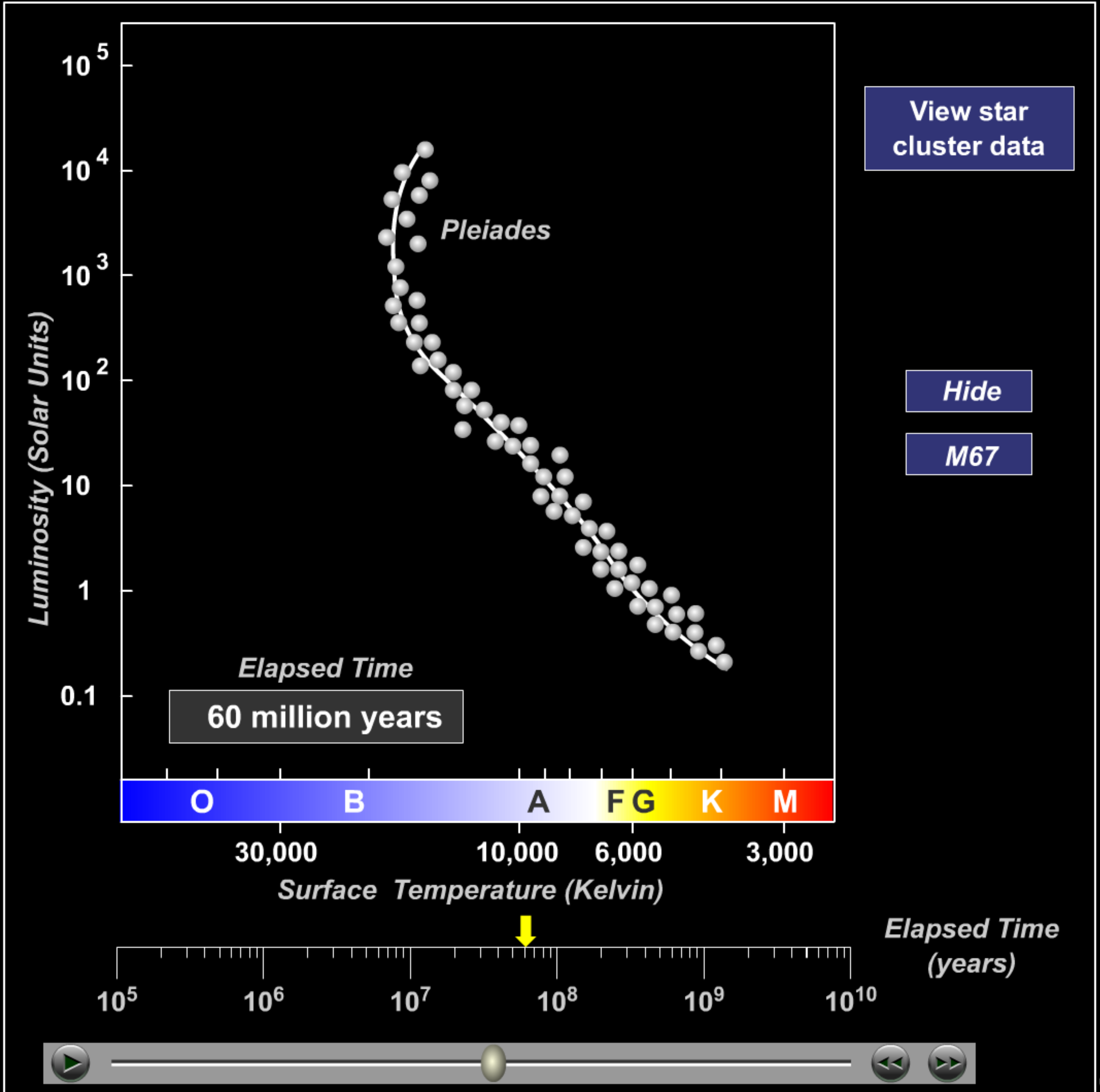
Using the H-R Diagram to Determine the Age of a Star Cluster



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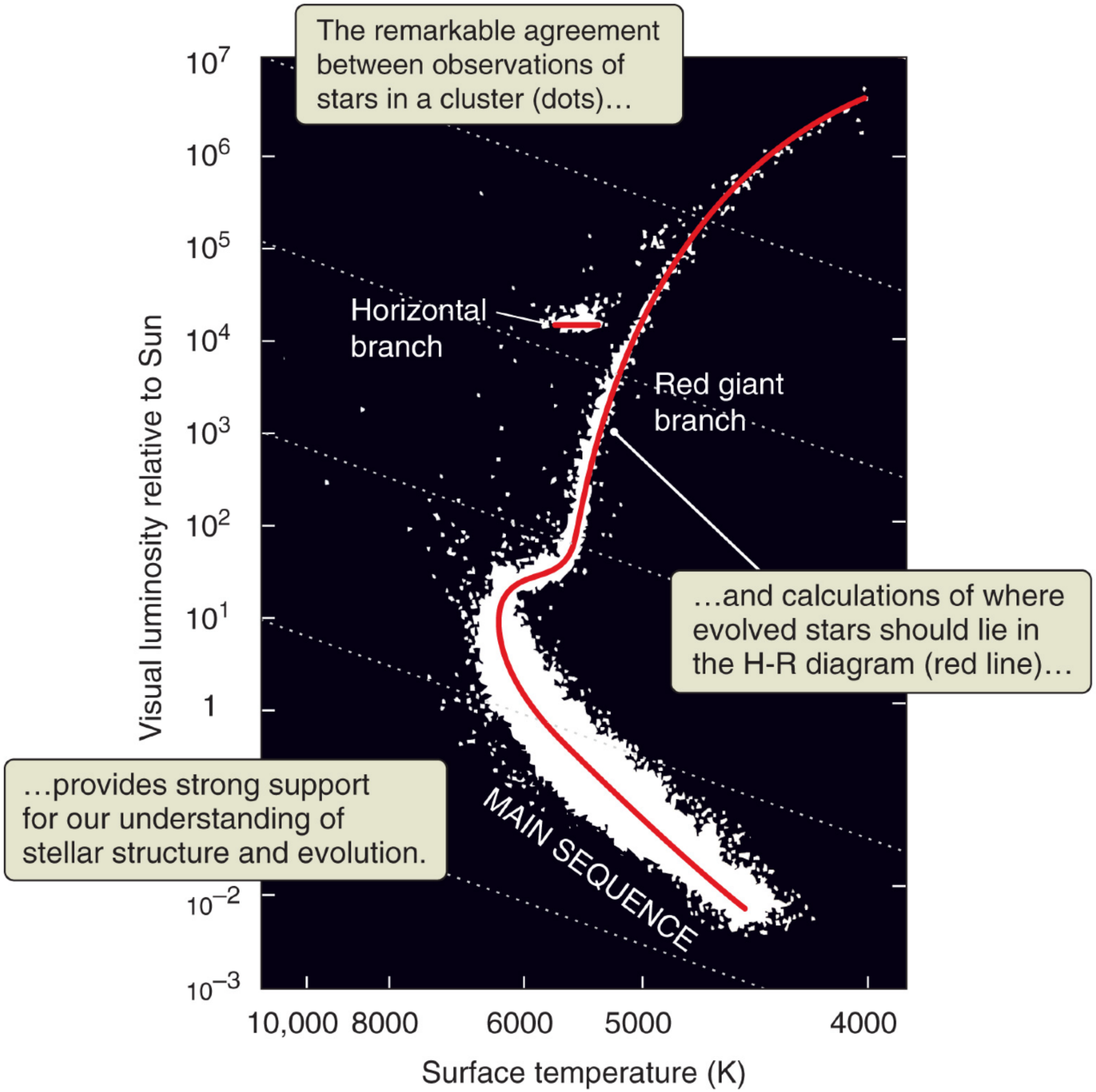
Summary: Model Isochrones vs. Cluster Data



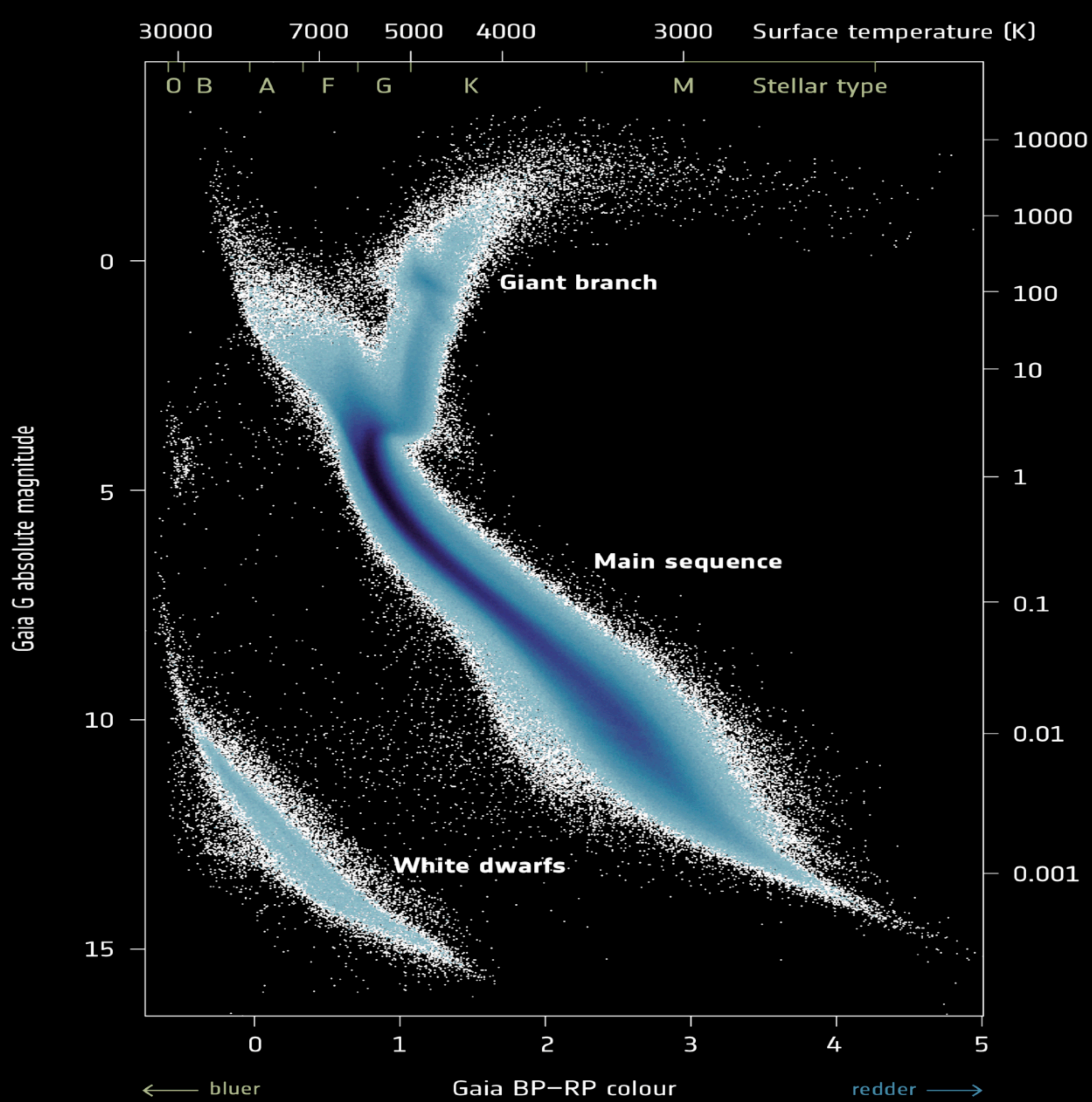
By comparing the distribution of cluster stars on the HR diagram and model isochrones, we can (1) fine-tune stellar evolution models, and (2) estimate age and chemical composition of clusters

Introducing complex stellar populations

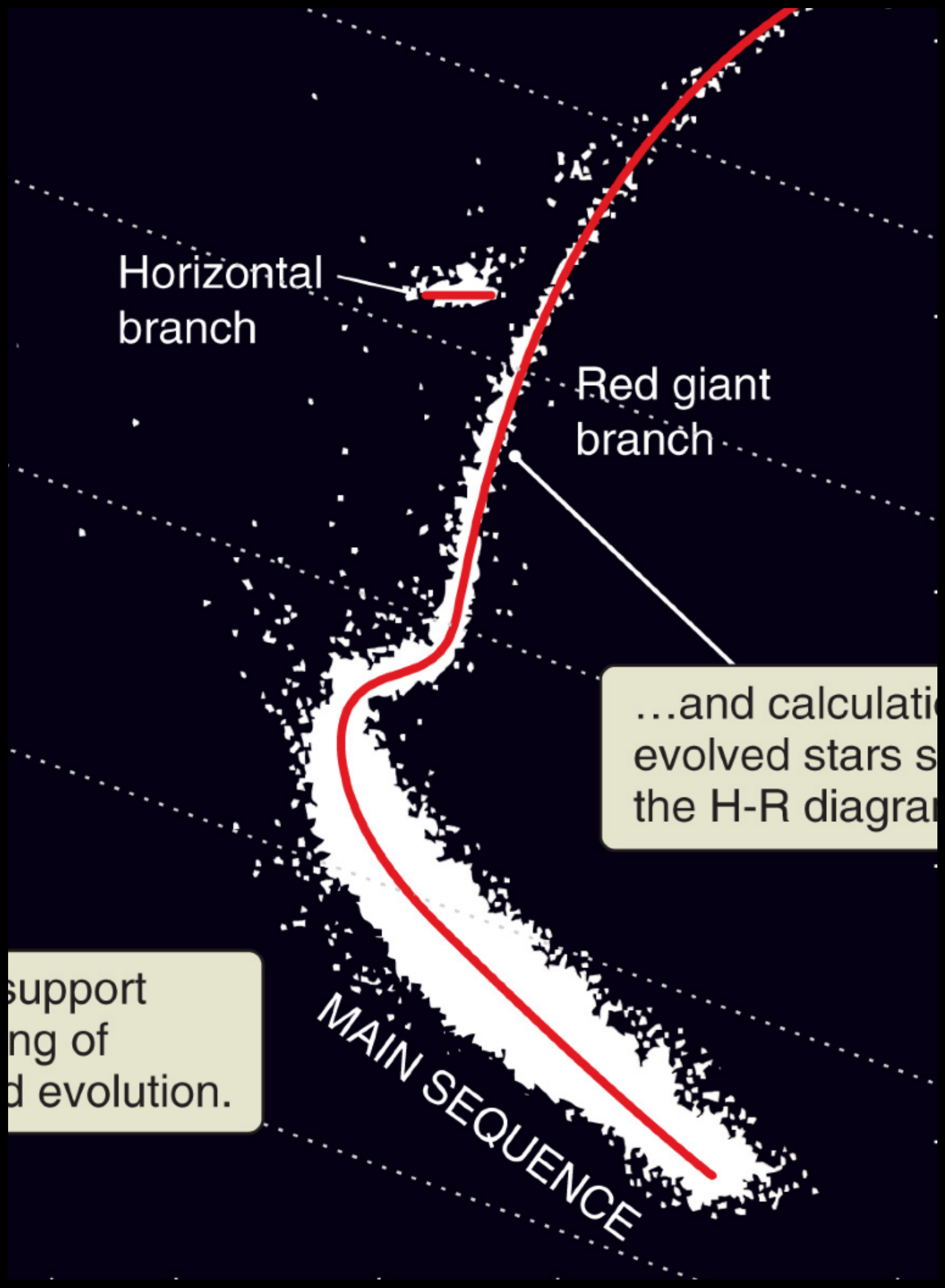
Solar neighborhood stars' H-R diagram

What does the HR diagram of Solar Neighbors tell us?

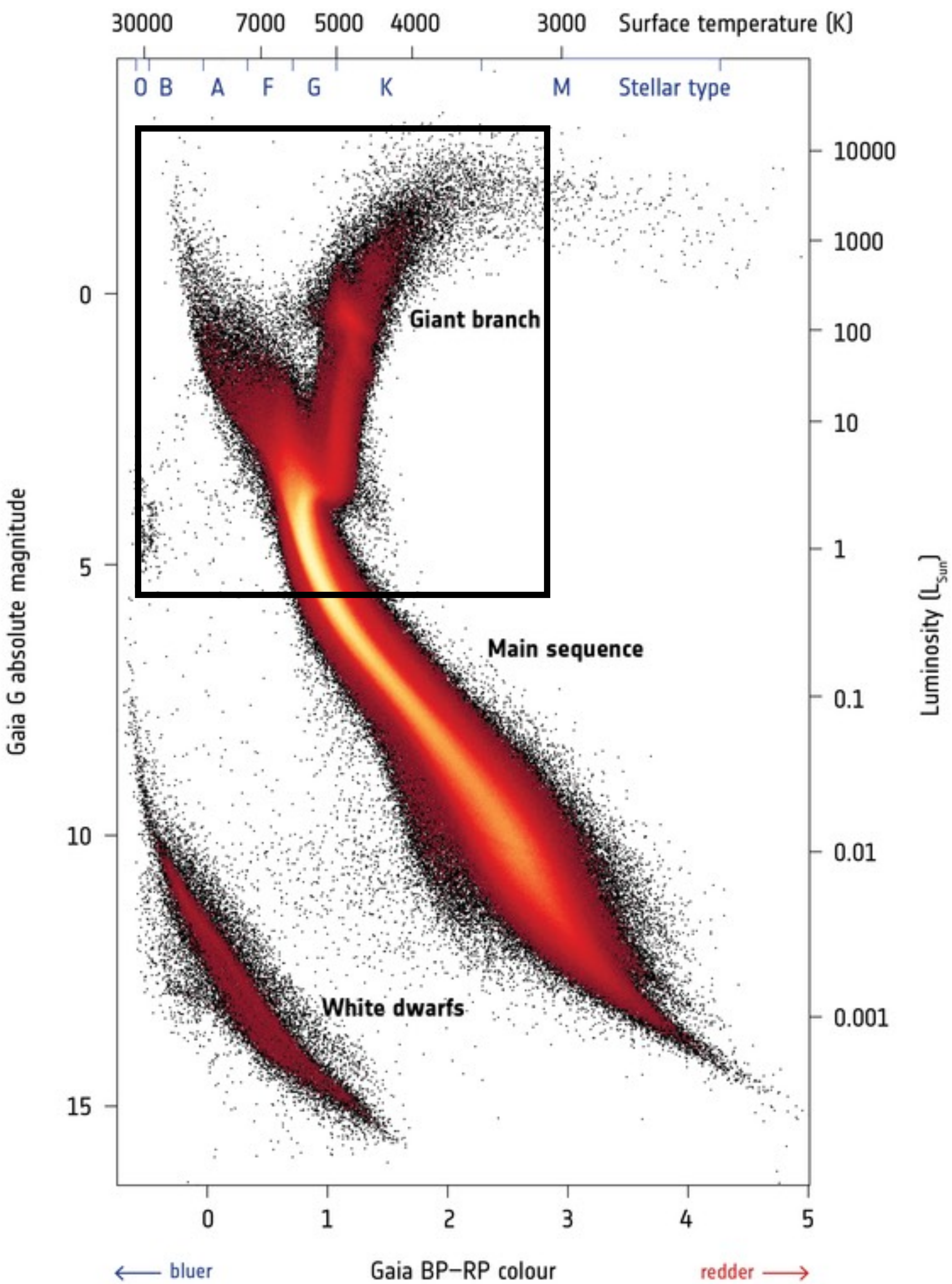
Solar neighborhood HR diagram



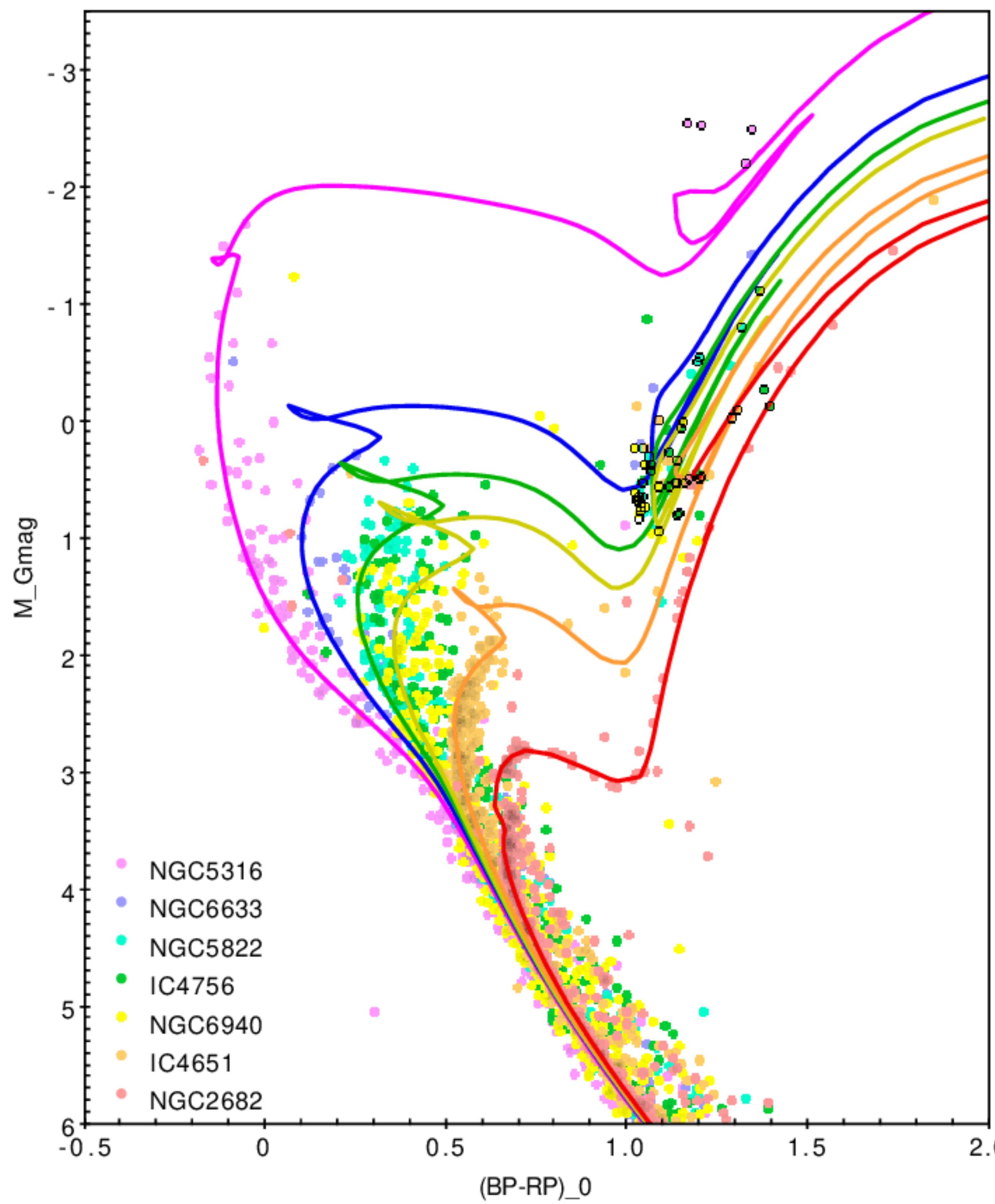
Cluster HR diagram



The Solar neighborhood stars are a mixed stellar population, and its HR diagram can be understood as a combination of multiple isochrones

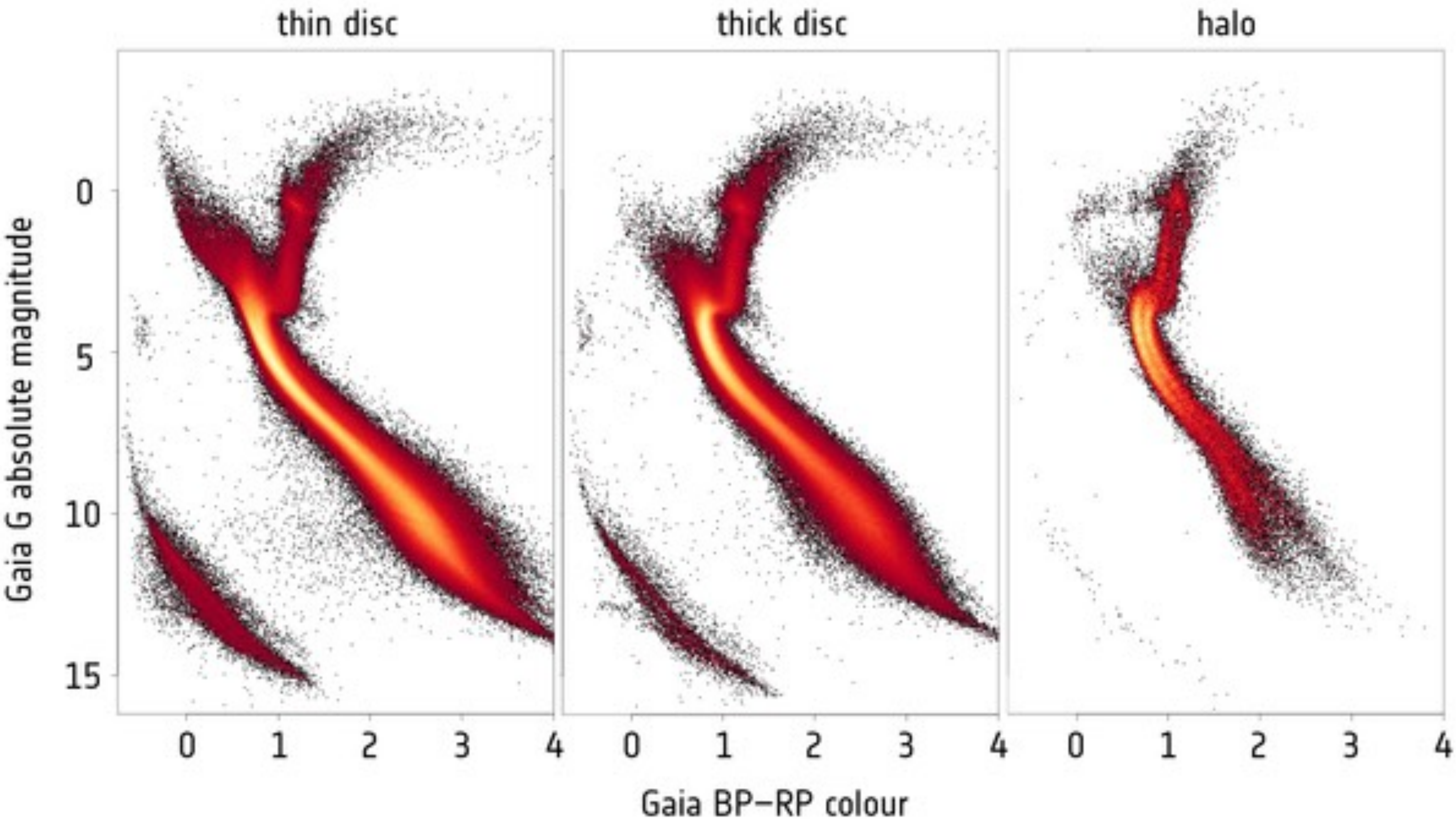


Isochrone fits of seven clusters of different ages

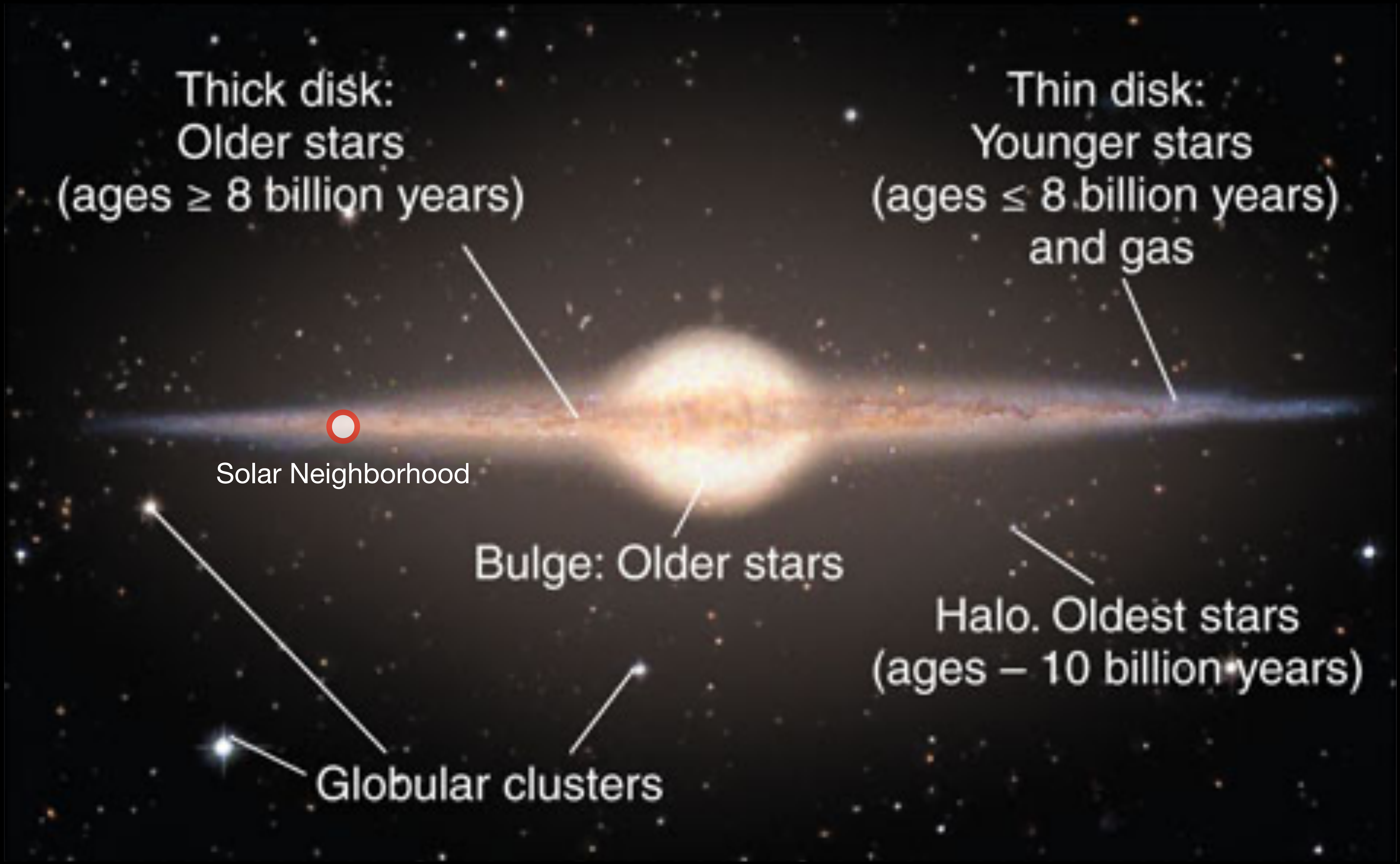


Solar neighborhood stars tell us the formation history of the Galaxy

Solar neighborhood stars separated based on kinematics



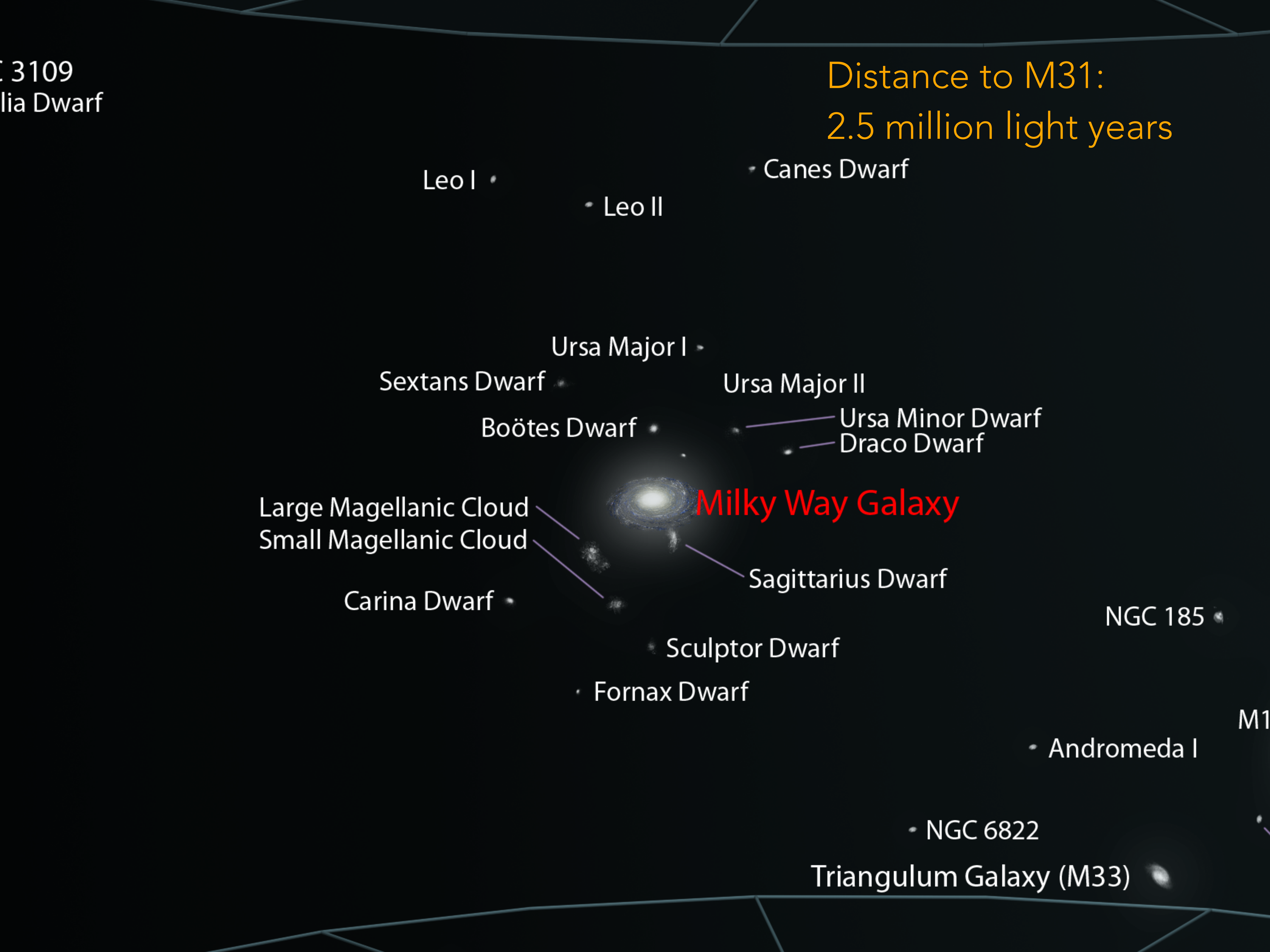
The study of Solar neighbors' HR diagram made us realize that ``Rome wasn't built in a day''



Note: a significant fraction of stars in the Milky Way actually formed in other galaxies

Small galaxies get shredded by large galaxies

Inside the Milky Way's Halo: Tidally Stripped Stars from Dwarf Galaxies



C 3109
lia Dwarf
Distance to M31:
2.5 million light years
Canes Dwarf
Leo I
Leo II
Ursa Major I
Sextans Dwarf
Ursa Major II
Ursa Minor Dwarf
Boötes Dwarf
Draco Dwarf
Milky Way Galaxy
Large Magellanic Cloud
Small Magellanic Cloud
Sagittarius Dwarf
Carina Dwarf
NGC 185
Sculptor Dwarf
Fornax Dwarf
M1
Andromeda I
NGC 6822
Triangulum Galaxy (M33)

The Milky Way and its neighbour Andromeda
are destined to merge within the next 5 billion years

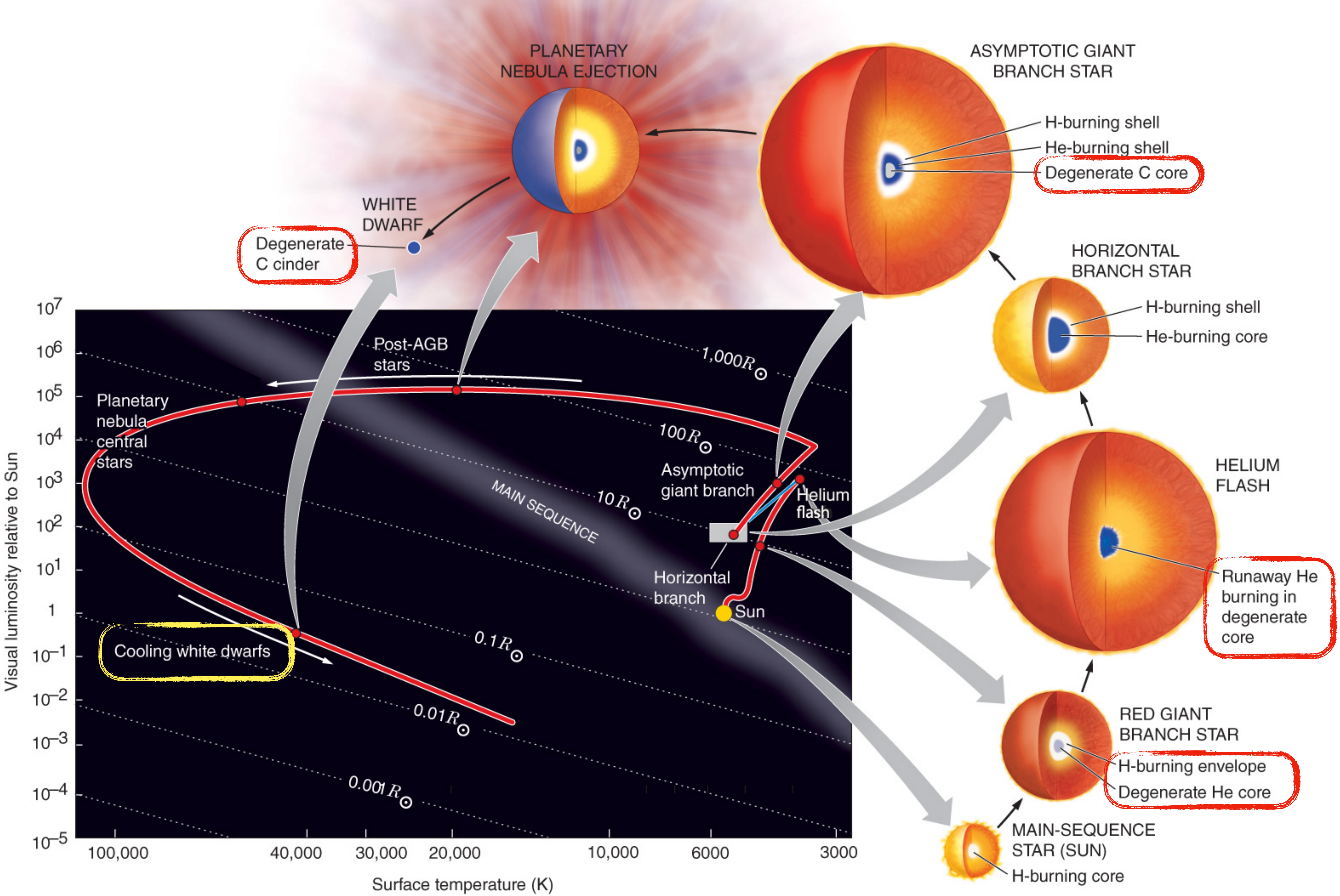
...or so.

This simulation shows what might happen
to the gas (shades of blue) and newly formed stars (red)
when the two galaxies come together.

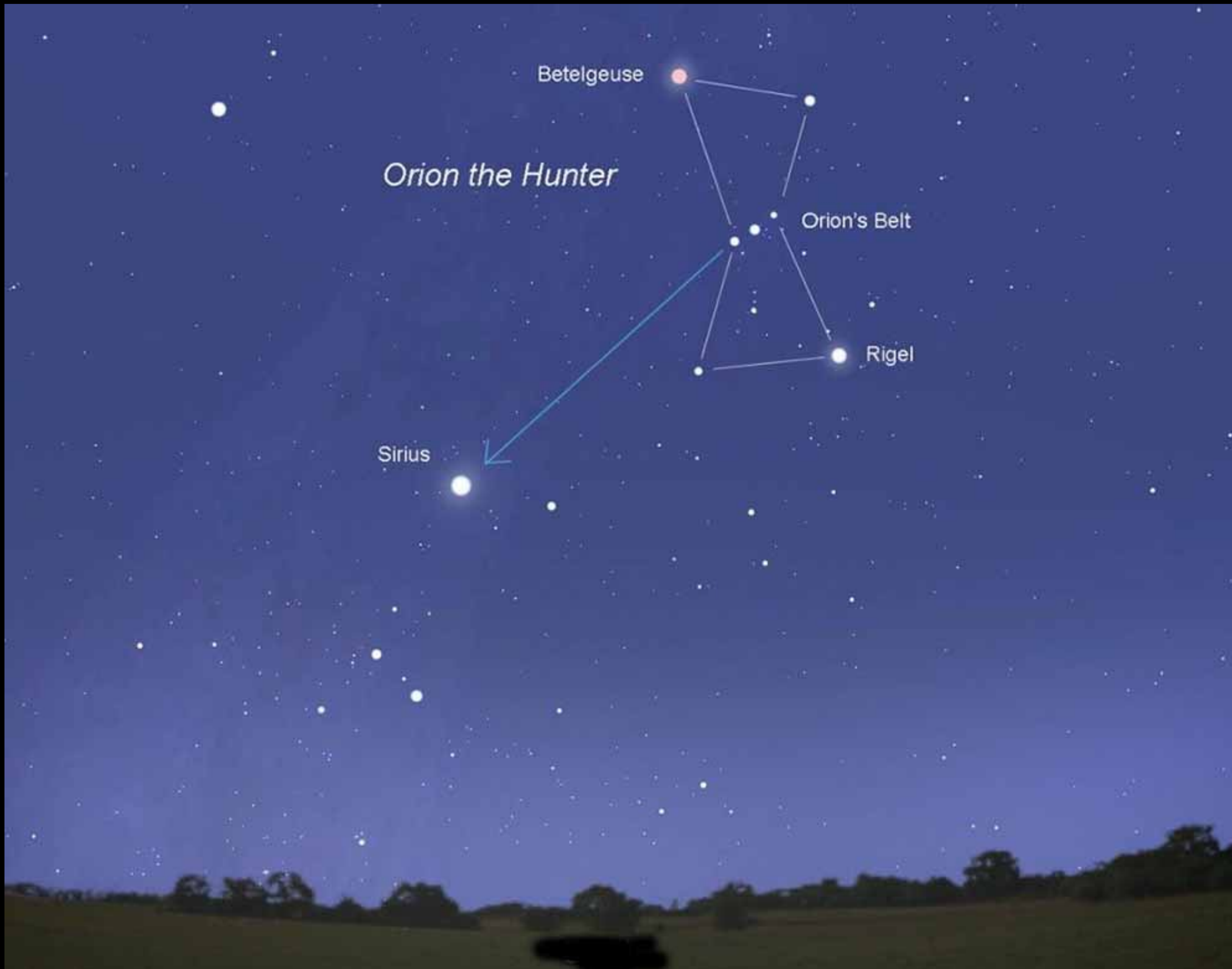
A key prediction of the model is a degenerate core buried at the center of a post-MS star

What is degeneracy?

The four stages in low-mass star evolution where the core is degenerate

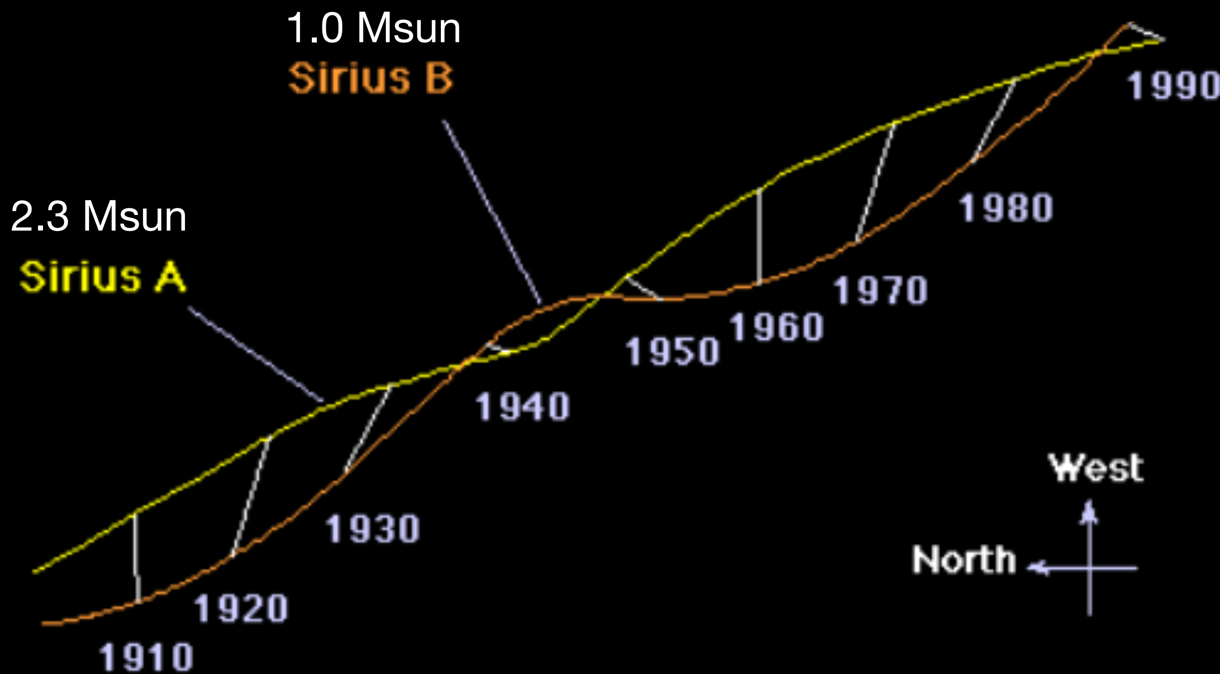


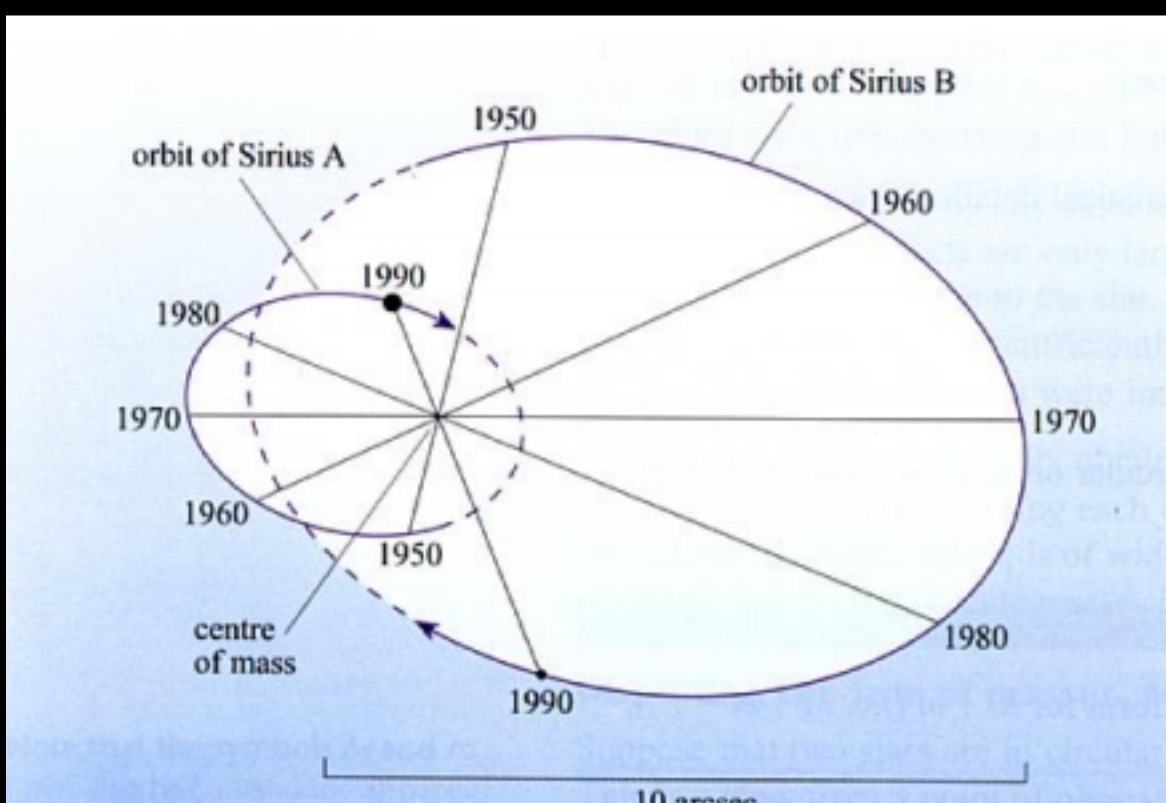
The story of degenerate gas started with the study of Sirius, the brightest star in the night sky



Sirius B - the weird companion of the Dog Star

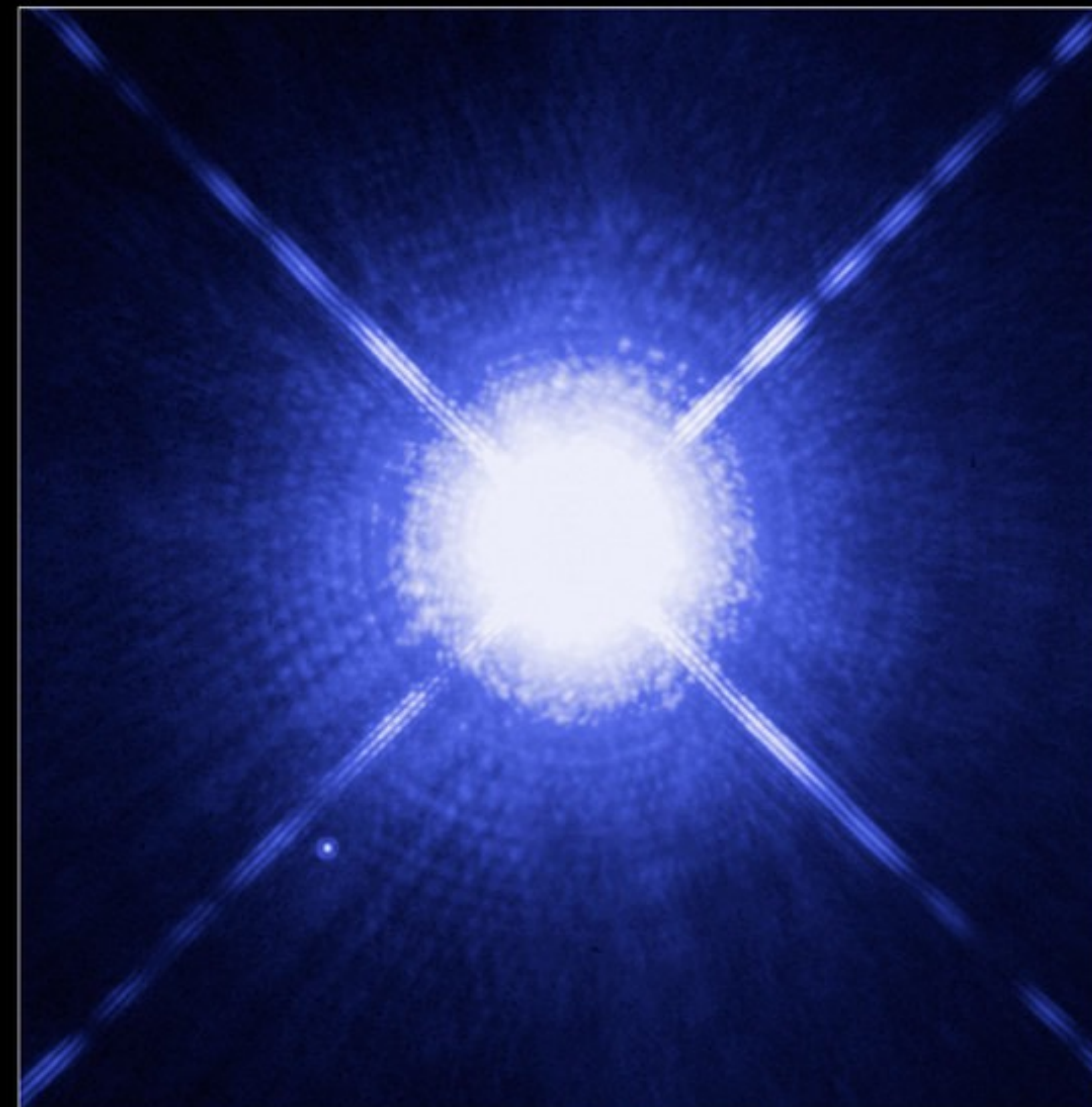
50 year orbit of the binary first inferred by Bessel in 1844
Bessel also measured the first stellar parallax (61 Cygni)





From the orbital motion, it was estimated that A has 2.3 solar mass and B has 1.0 solar mass

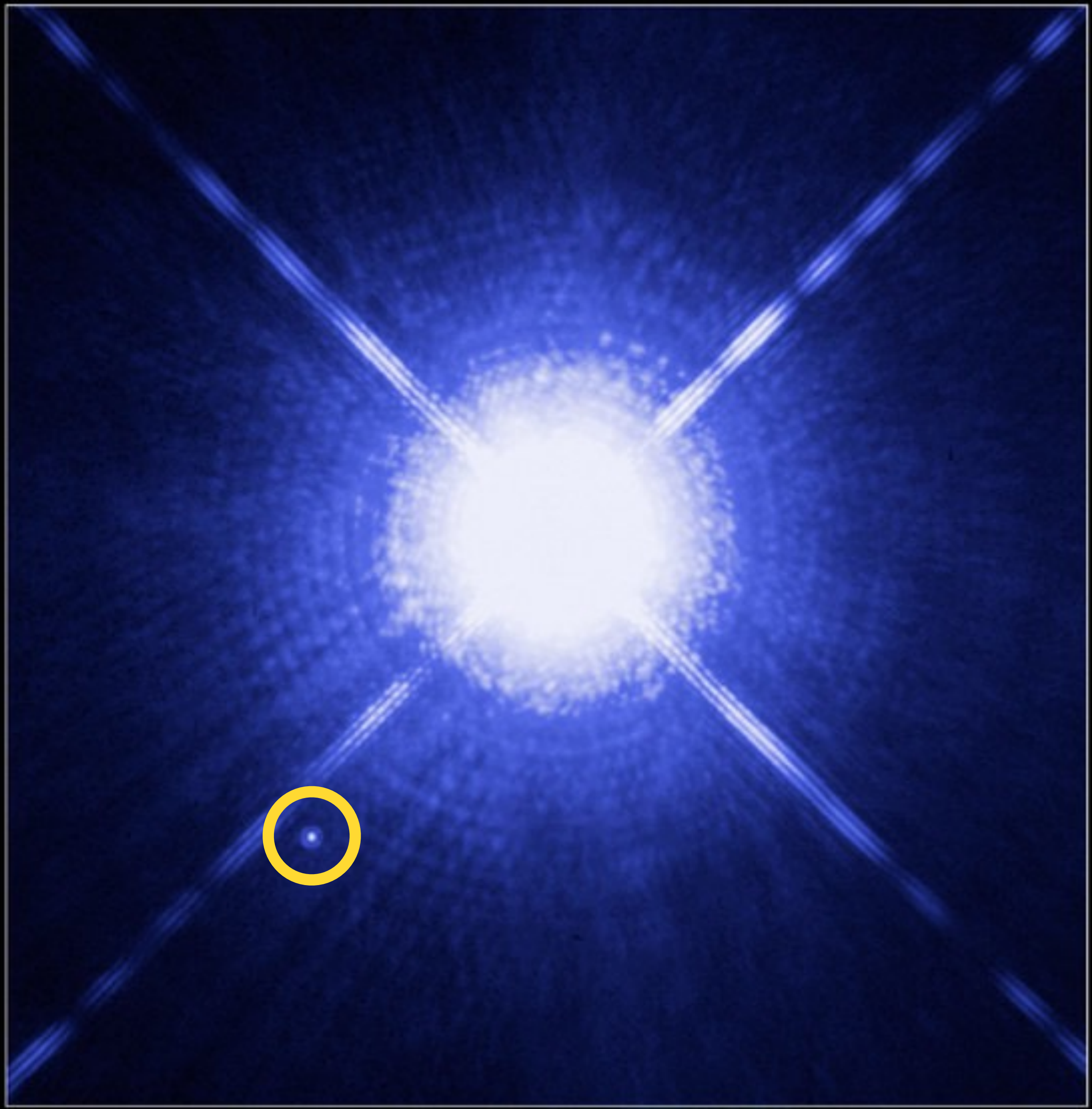
Sirius B is much fainter than Sirius A, is it surprising that if I tell you that it's much hotter (27000 K vs. 9900 K)?



Sirius A and Sirius B
Hubble Space Telescope • WFPC2

NASA, ESA, H. Bond (STScI), and M. Barstow (University of Leicester) STScI-PRC05-36a

Sirius B - the weird companion of the Dog Star



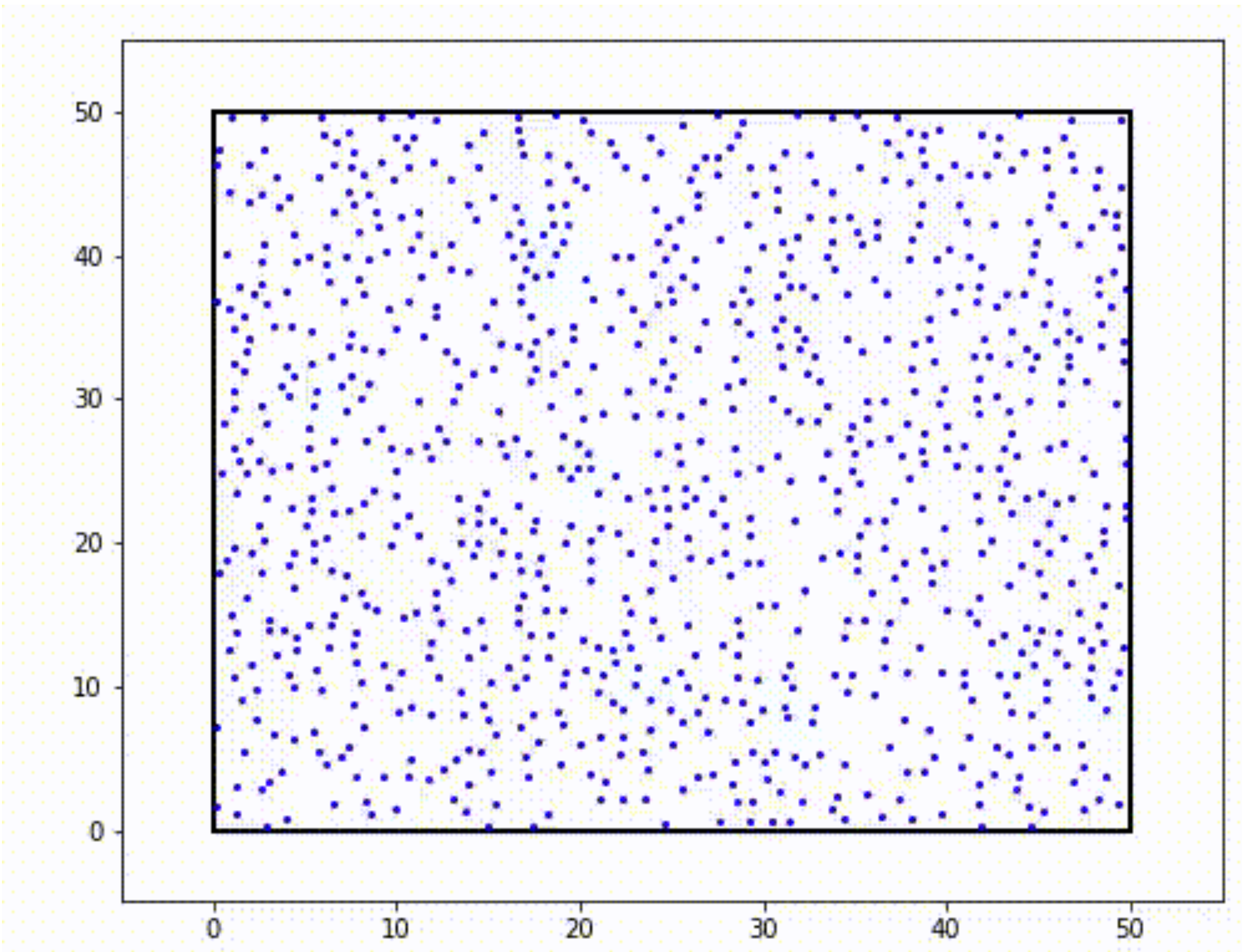
Sirius A and Sirius B
Hubble Space Telescope • WFPC2

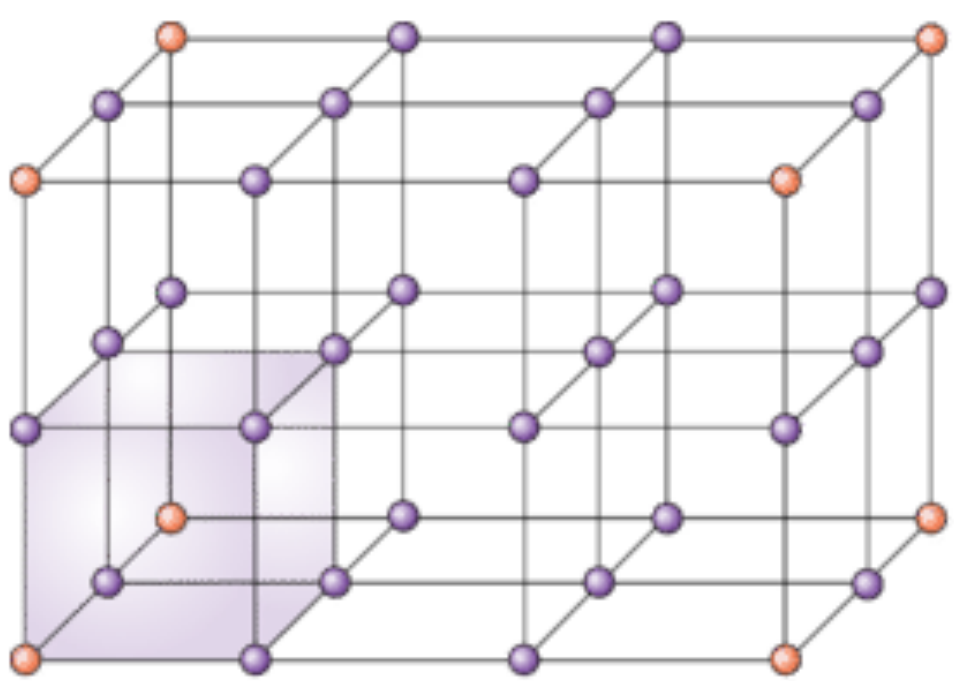
NASA, ESA, H. Bond (STScI), and M. Barstow (University of Leicester) STScI-PRC05-36a

- Inferred properties of Sirius B:
  - 1 Solar Mass
  - 0.03 Solar Luminosity
  - 27,000 K surface temperature
  - 5500 km radius (Earth-size)
- Sirius B represent a class of objects called **White Dwarfs (WDs)**

- The physical conditions of WDs are extreme:
  - extreme density ($\rho \approx$ 3e9 kg/m^3) ($n_e \sim$ 1e36 /m^3)
  - extreme surface gravity (HW)
  - extreme pressure at the center:
  - $P_c \propto GM^2/R^4$

Given a number density, how to estimate the average distance between particles?

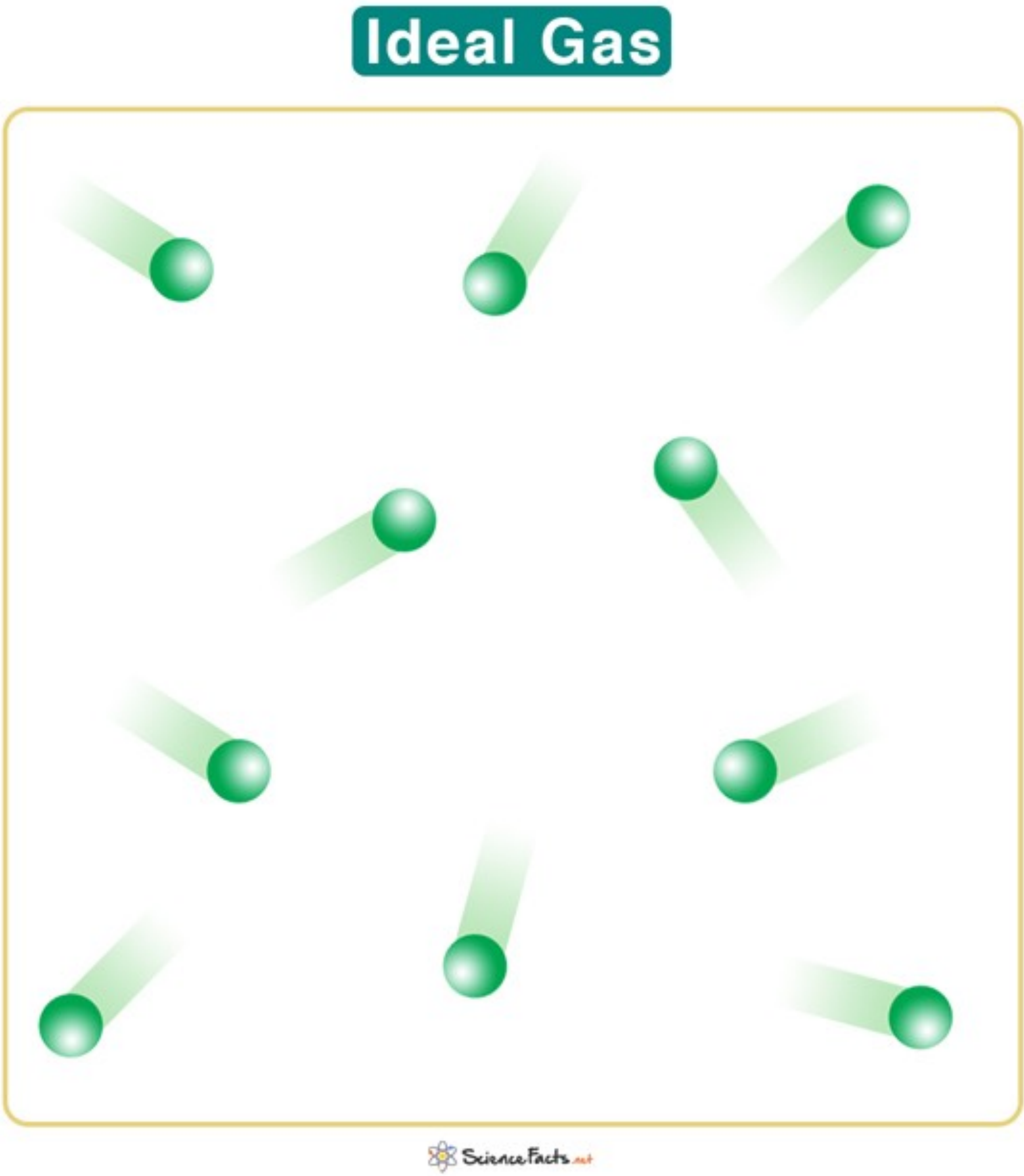




- Extremely high density of WDs:
  - mass density $\rho \approx$ 3e9 kg/m^3
  - number density of electrons: $n_e \sim$ 1e36 /m^3
- What is the average distance between electrons? How does it compare with the size of an atom (~0.1 nm = 1e-10 m)?
  - average distance: $\delta x = n_e^{-1/3} = 10^{-12} m$ (<< 0.1 nm, size of atom)
  - Quantum Mechanics must be important in white dwarfs.

Ideal Gas Pressure

- the pressure from ideal gas is 2/3 the kinetic energy density
  - Gven $P = nkT$ and $K = mv^2/2 = p^2/(2m) = 3kT/2$
  - we have: $P = \frac{2}{3} n \frac{p^2}{2m}$



Deriving degenerate pressure using uncertainty principle

- **Quantum mechanics** start to become important when e- and ions are packed in a very smaller volume
- **Pauli exclusion principle** (1925): no more than one fermion (leptons and baryons) can occupy the same quantum state. So the uncertainty of the fermion's position cannot be larger than their actual separation:

$$\Delta x \lesssim n^{-1/3}$$

- **Heisenberg's uncertainty principle** (1927):

$$\Delta x \Delta p \approx h/2\pi$$

  the smaller the uncertainty in position, the larger the uncertainty in momentum.

- Combining the two and approximating $p \approx \min(\Delta p)$ (ground state):

$$p \approx hn^{1/3}/2\pi$$

- Just like ideal gas, the pressure from degenerate gas is 2/3 the kinetic energy density:

$$P_{\text{degen}} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

Understanding degenerate pressure using Fermi-Dirac Distribution

- Fermions (half-integer spins; e.g., e-, p+, n) follow **Fermi-Dirac distribution**, where the probability of a particle having an energy between *E* and *E+dE* is:

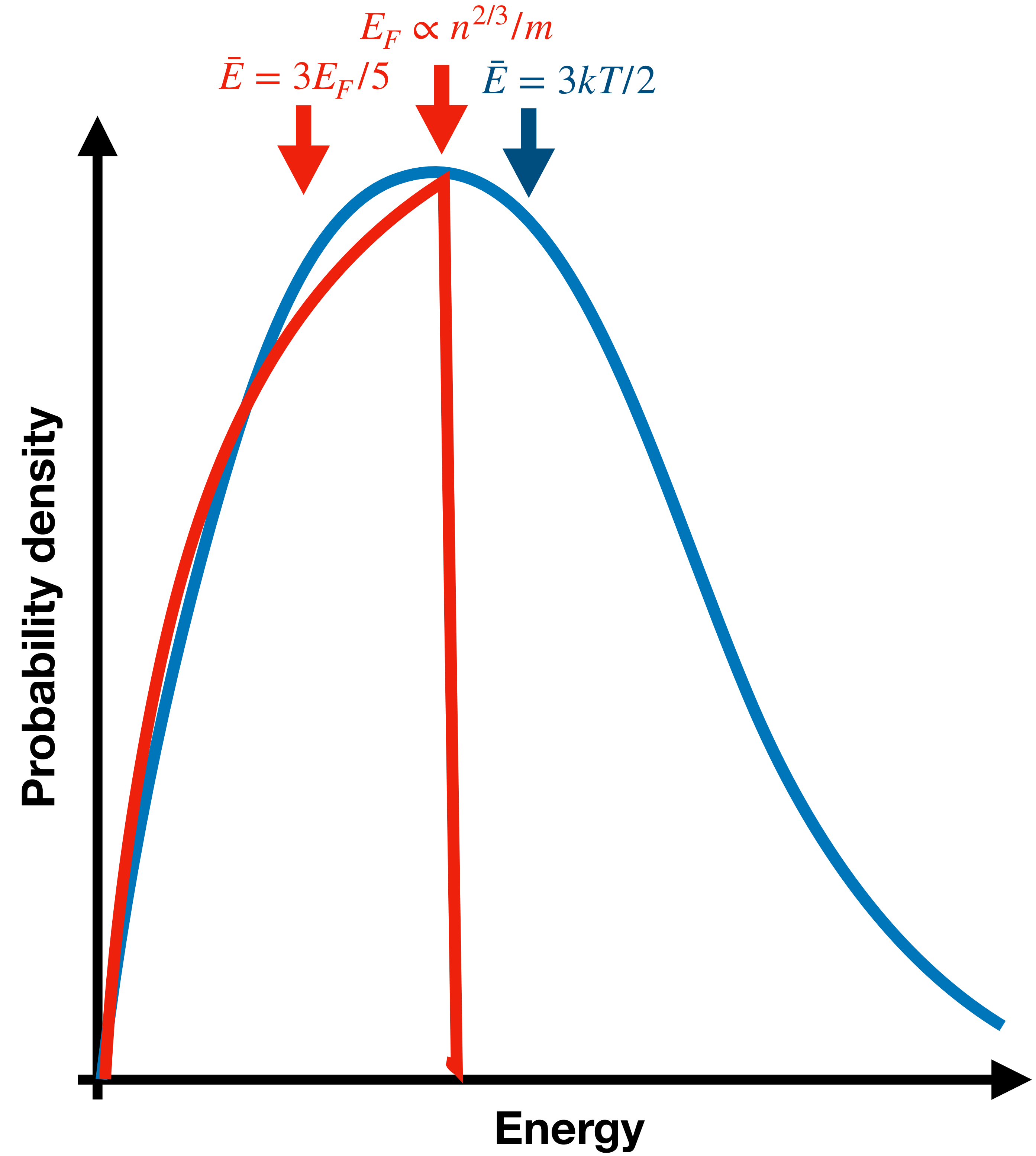
$$f_E dE \propto \frac{E^{1/2} dE}{e^{(E-E_F)/kT} + 1}$$

where $E_F \propto n^{2/3}/m$

- when $kT >> E_F$, it is indistinguishable from the classic Maxwell-Boltzmann distribution: $f_E dE \propto \frac{E^{1/2} dE}{e^{E/kT}}$

  The *mean kinetic energy* is 3kT/2, as a result, pressure depends on both density and temperature

- when $kT << E_F$, all energy states greater than the Fermi energy become unoccupied.
  The *mean kinetic energy* is $3E_F/5$, as a result, pressure depends only on density.



The Condition for Degeneracy: Pressure Comparison

- The pressure from non-relativistic degenerate gas is:

$$P_{\text{degen}} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

- The pressure from ideal gas is:

$$P_{\text{ideal}} = \frac{2}{3} n \left(\frac{3}{2} kT \right) = nkT$$

- The condition for degeneracy is simply:

$$P_{\text{degen}} > P_{\text{ideal}}$$

which can be simplified as:

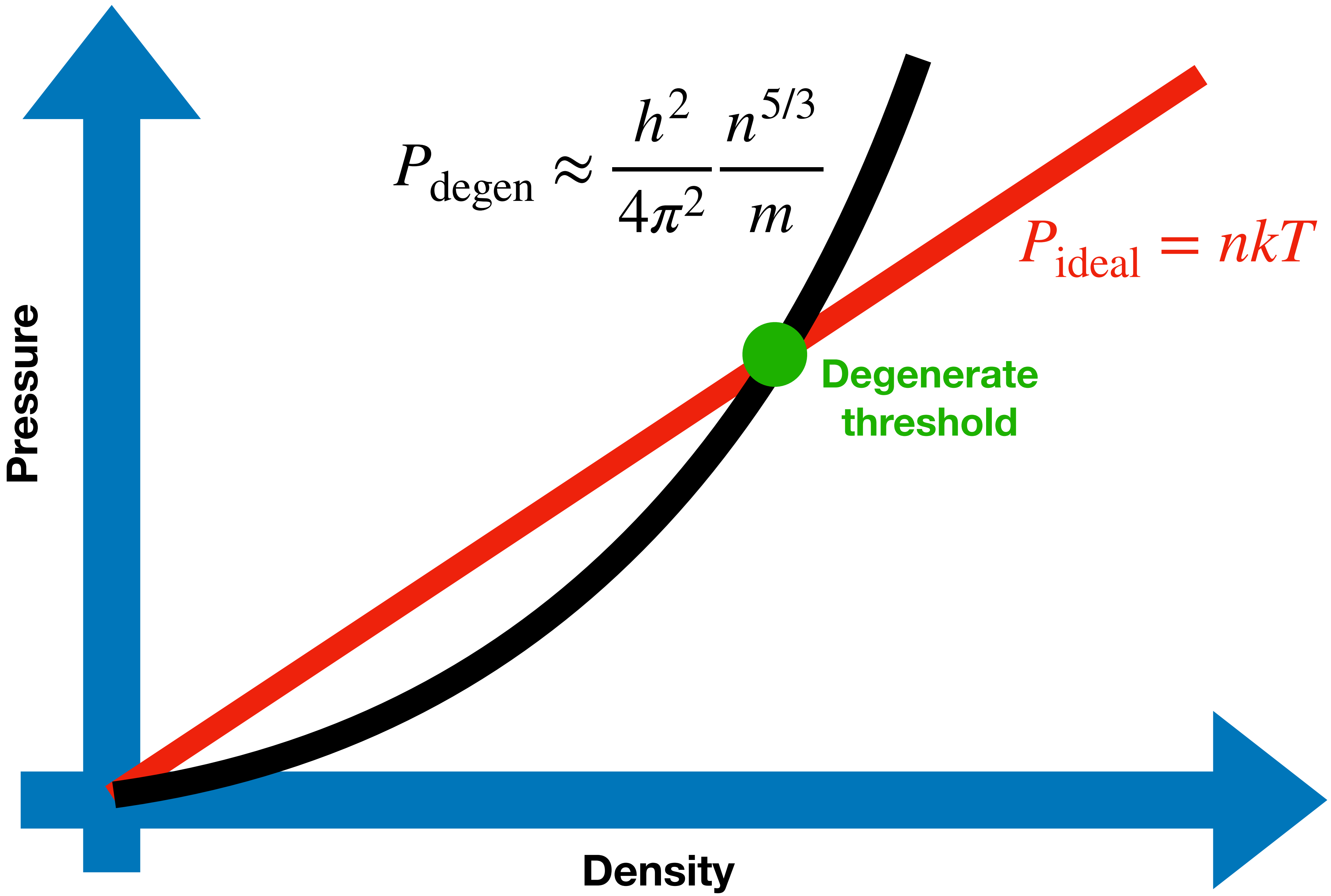
$$\frac{h^2}{4\pi^2} \frac{n^{2/3}}{m} > kT$$

or equivalently (expressed using Fermi energy):

$$kT \lesssim 0.2 \; E_{\text{Fermi}}$$

What is degenerate pressure? How to understand it intuitively?

- The contraction of non-fusing cores packs a large amount of mass into a small volume. Each electron finds its position well constrained, which leads to large momentum and kinetic energy due to the **uncertainty principle**. **Pressure**, as the **density** of **kinetic energy**, increases rapidly as a result of increased **(1) number density** and **(2) kinetic energy per particle**. This pressure due to quantum mechanics is called **degenerate pressure**.



What is degenerate pressure? How to understand it intuitively?

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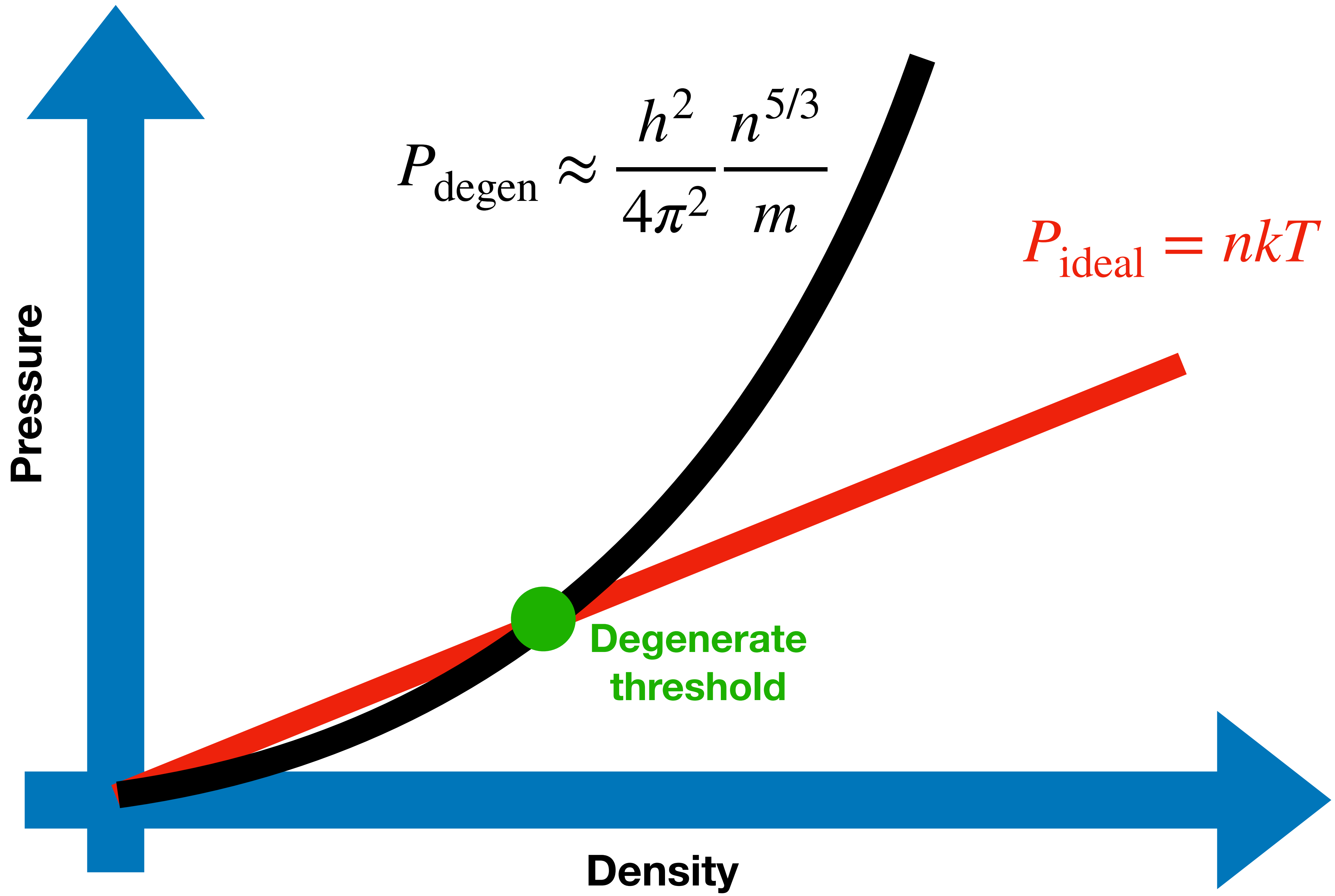


The Condition for Degeneracy: e- vs. ions

- In a previous slide, we derived the condition for degeneracy:

$$\frac{h^2}{4\pi^2}\frac{n^{2/3}}{m} > kT$$

- The above condition can be satisfied when either:
  - **the temperature is very low**, or
  - **the density is very high**
- In the non-fusing core of a star, the density is extremely high, reaching degeneracy condition.
- *Now think about this: If ions and electrons share the same temperature in the core, which component will reach degeneracy first?*

Mass-Radius Relation
&
the Chandrasekhar Limit

In the previous lecture, we derived degenerate pressure using Heisenberg's uncertainty principle, let's apply it to white dwarfs

- **Quantum mechanics** start to become important when e- and ions are packed in a very smaller volume
- **Pauli exclusion principle** (1925): no more than one fermion (leptons and baryons) can occupy the same quantum state. So the uncertainty of the fermion's position cannot be larger than their actual separation:
$$\Delta x \lesssim n^{-1/3}$$
- **Heisenberg's uncertainty principle** (1927):
$$\Delta x \Delta p \approx h/2\pi$$
the smaller the uncertainty in position, the larger the uncertainty in momentum.
- Combining the two and approximating $p \approx \min(\Delta p)$ (ground state):
$$p \approx h n^{1/3}/2\pi$$
- Just like ideal gas, the pressure from degenerate gas is 2/3 the kinetic energy density:
$$P_{\rm degen} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

Understanding the Mass-Size relation of white dwarfs with degenerate pressure and hydrostatic equilibrium

- The pressure from non-relativistic degenerate gas is:

$$P_{\rm degen} = \frac{h^2}{4\pi^2}\frac{n^{5/3}}{m} \propto \left(\frac{M}{R^3 \mu m_H}\right)^{5/3}$$

- Hydrostatic equilibrium provides an estimate of the central pressure:

$$P_{\rm c} \propto \frac{GM^2}{R^4}$$

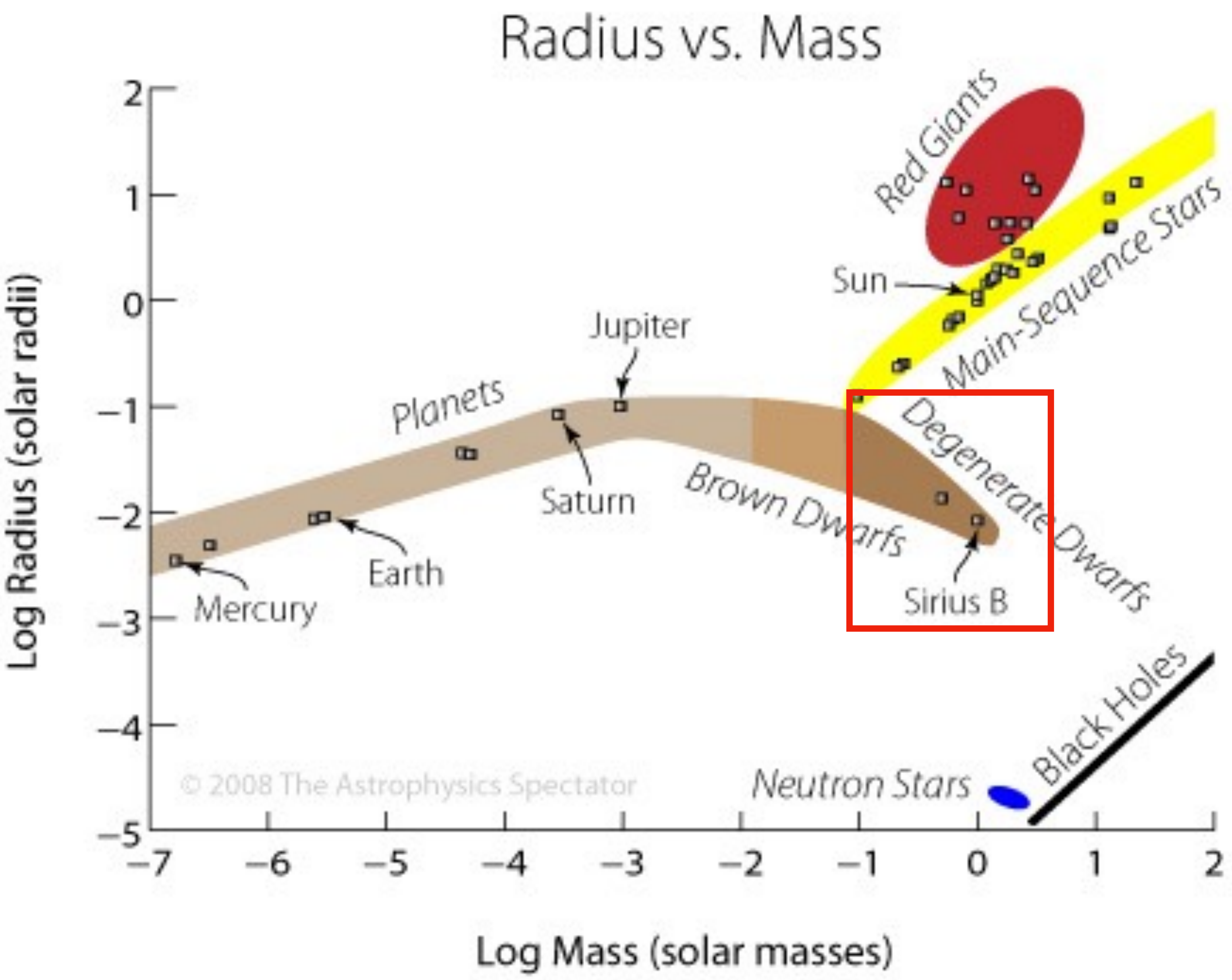
- If degenerate gas provide the central pressure, we can equate the two and solve for the Mass-Radius relation:

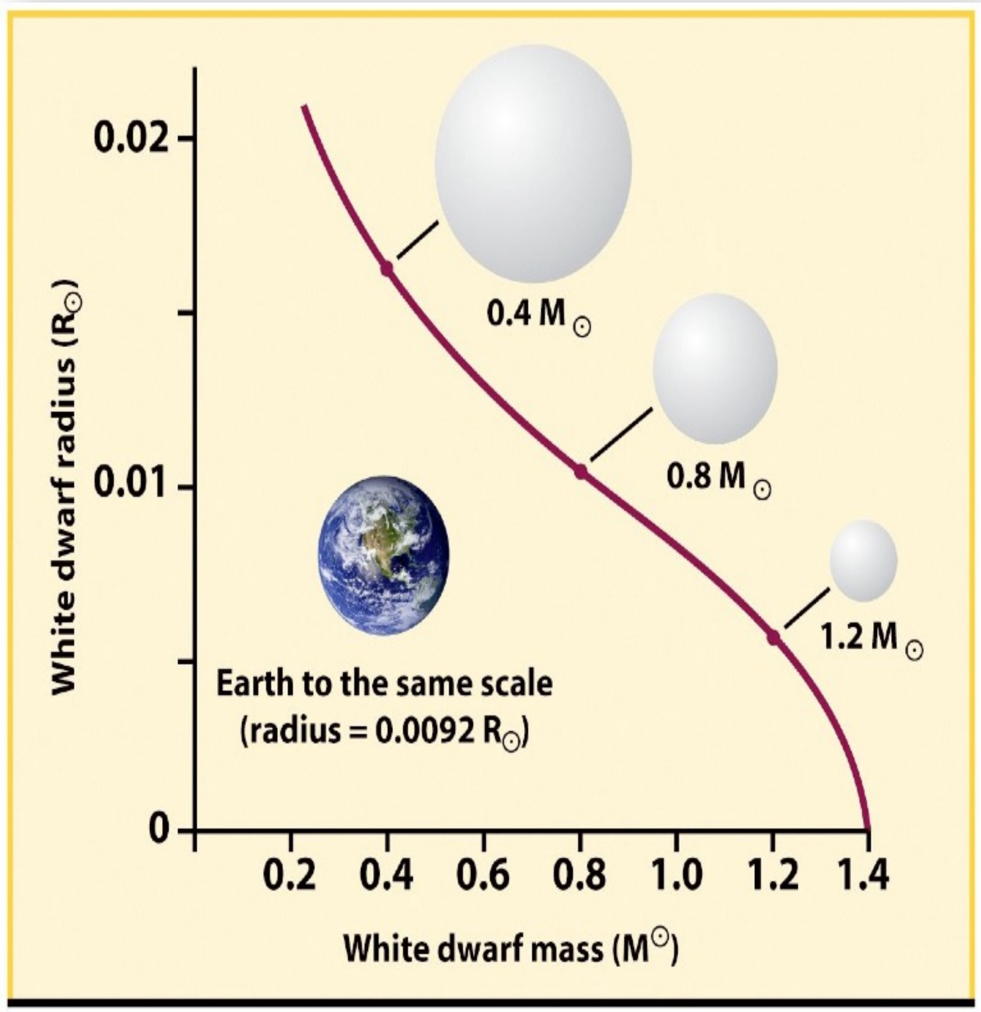
$$\frac{M^{5/3}}{R^5} \propto \frac{M^2}{R^4} \Rightarrow R \propto M^{-1/3}$$

- This is in contrast to main sequence stars, where **$R \sim M^{0.8}$**

Mass-Size Relation of White Dwarfs

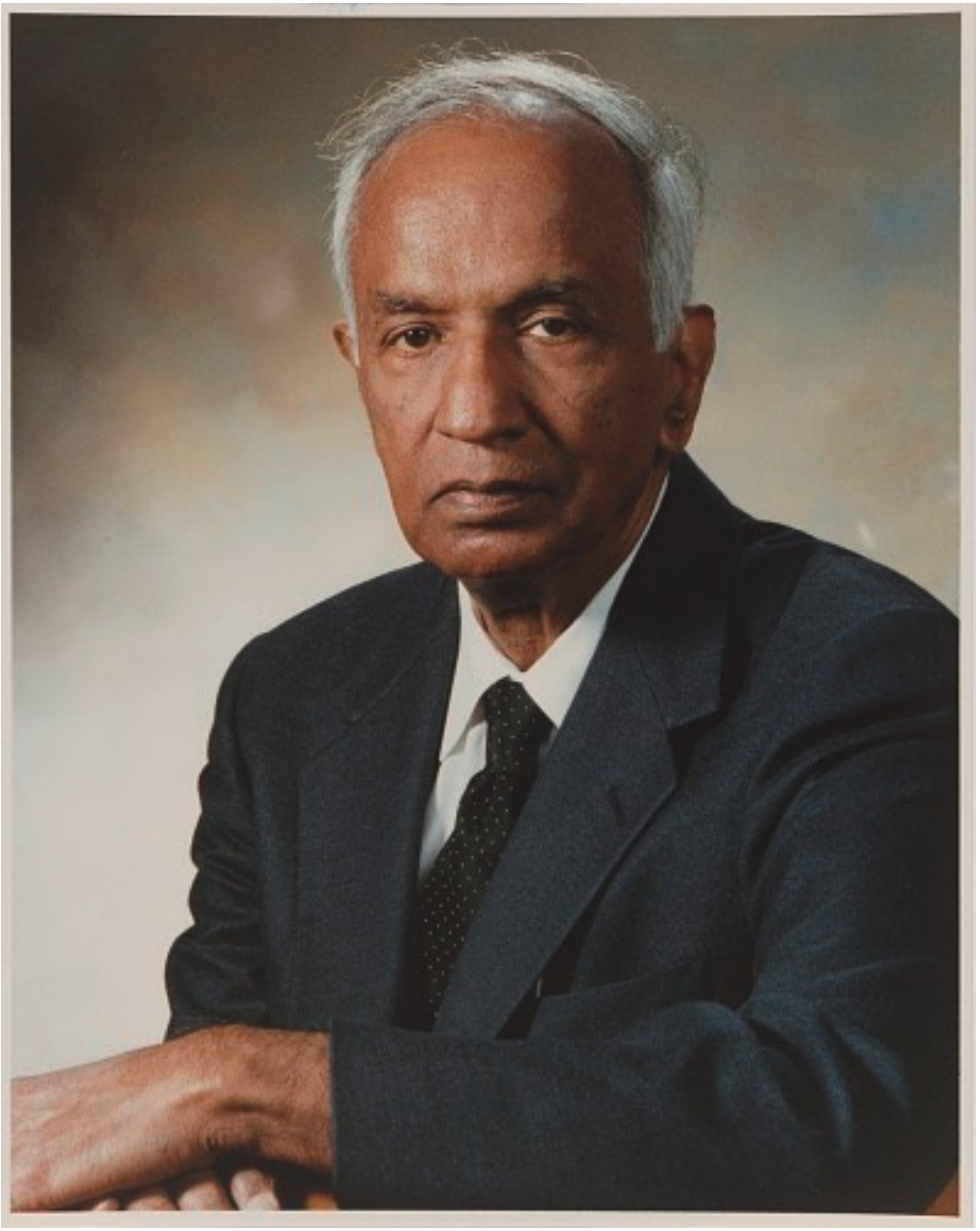
- White dwarfs show a mass-size relation of $\mathbf{R \sim M^{-1/3}}$, which is in contrast to that of main sequence stars, where $\mathbf{R \sim M^{0.8}}$





Subrahmanyan Chandrasekhar (1910-1995)

Subrahmanyan Chandrasekhar (born October 19, 1910, Lahore, India [now in Pakistan]—died August 21, 1995, Chicago, Illinois, U.S.) was an Indian-born American astrophysicist who, with William A. Fowler, won the 1983 Nobel Prize for Physics for key discoveries that led to the currently accepted theory on the later evolutionary stages of massive stars.



If density keeps rising, the electrons will become relativistic!

- Degeneracy pressure is not infinitely powerful, eventually it breaks when degenerate particles become **relativistic (*v* -> *c*)**
- For relativistic particles: $E^2 = p^2c^2 + (mc^2)^2 \approx p^2c^2$
- So the energy density is: $u = n(pc) = nc\dfrac{h}{2\pi\delta x} \approx hcn^{4/3}/2\pi$
- The pressure from relativistic degenerate gas is:

$$P_{\text{degen}} = \frac{1}{3}u = \frac{hc}{6\pi}n^{4/3} \propto \left(\frac{M}{R^3\mu m_H}\right)^{4/3}$$

- Hydrostatic equilibrium provides an estimate of the central pressure:

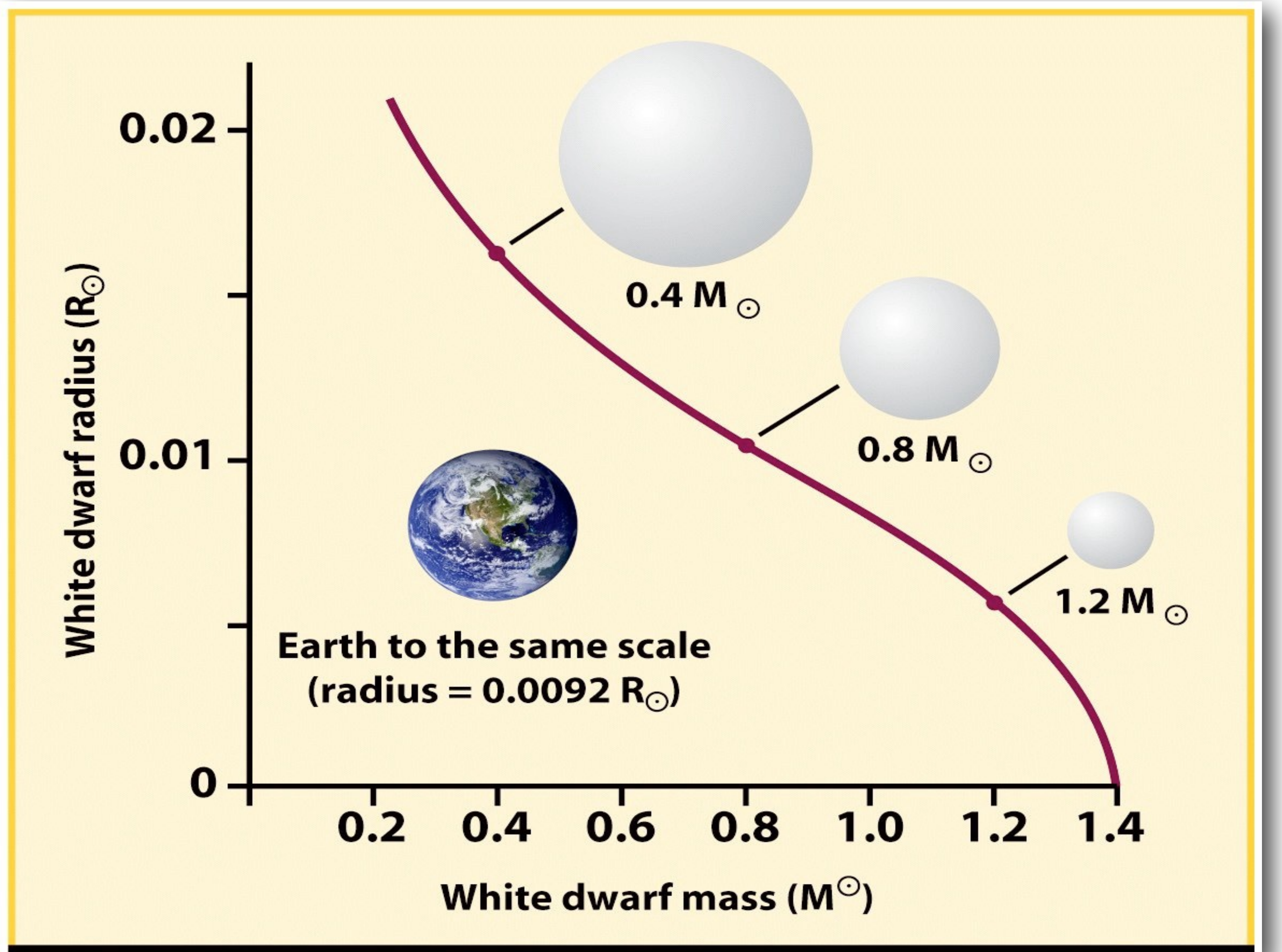
$$P_{\text{c}} \propto G\frac{M^2}{R^4}$$

- If degenerate gas provide the central pressure, we can equate the two, which leads **NOT** to a Mass-Radius relation, but **a mass of the star**:

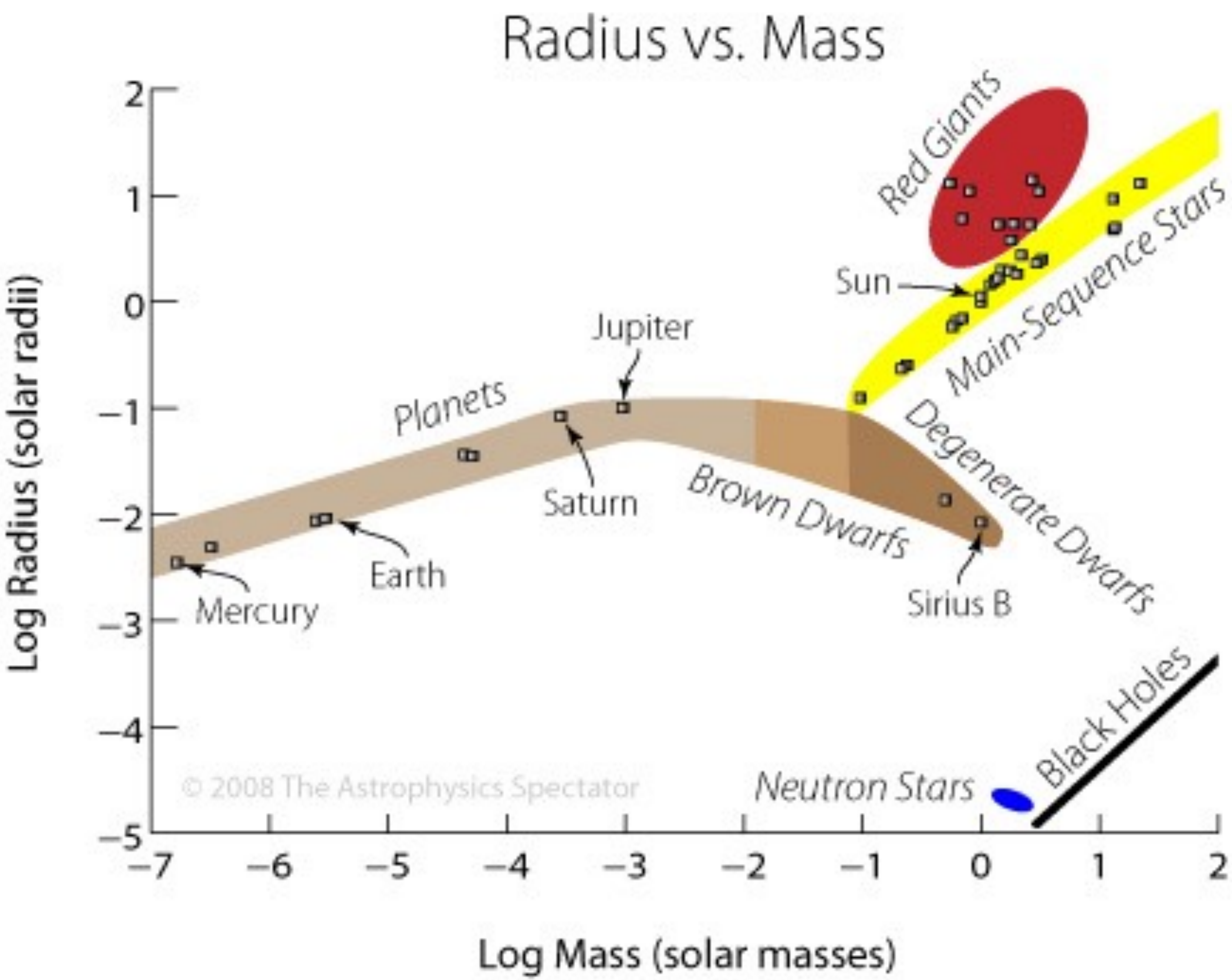
$$\frac{M^{4/3}}{R^4} \propto \frac{M^2}{R^4} \Rightarrow M = \left(\frac{h^3c^3}{8\pi^3G^3m_p^4}\right)^{1/2}$$

The Chandrasekhar Limit of White Dwarfs

- Degeneracy pressure is not infinitely powerful, eventually it breaks when degenerate particles become **relativistic ($p \approx hn^{1/3}/2\pi \rightarrow mc \Rightarrow P \propto n^{4/3}$)**, and that places a limit on the **maximum mass** of the white dwarf, the **1.4 Solar Mass (the Chandrasekhar limit)**
- The collapse of a WD at 1.4 Solar Mass results in **uncontrollable** Carbon fusion (**type Ia SNe**)

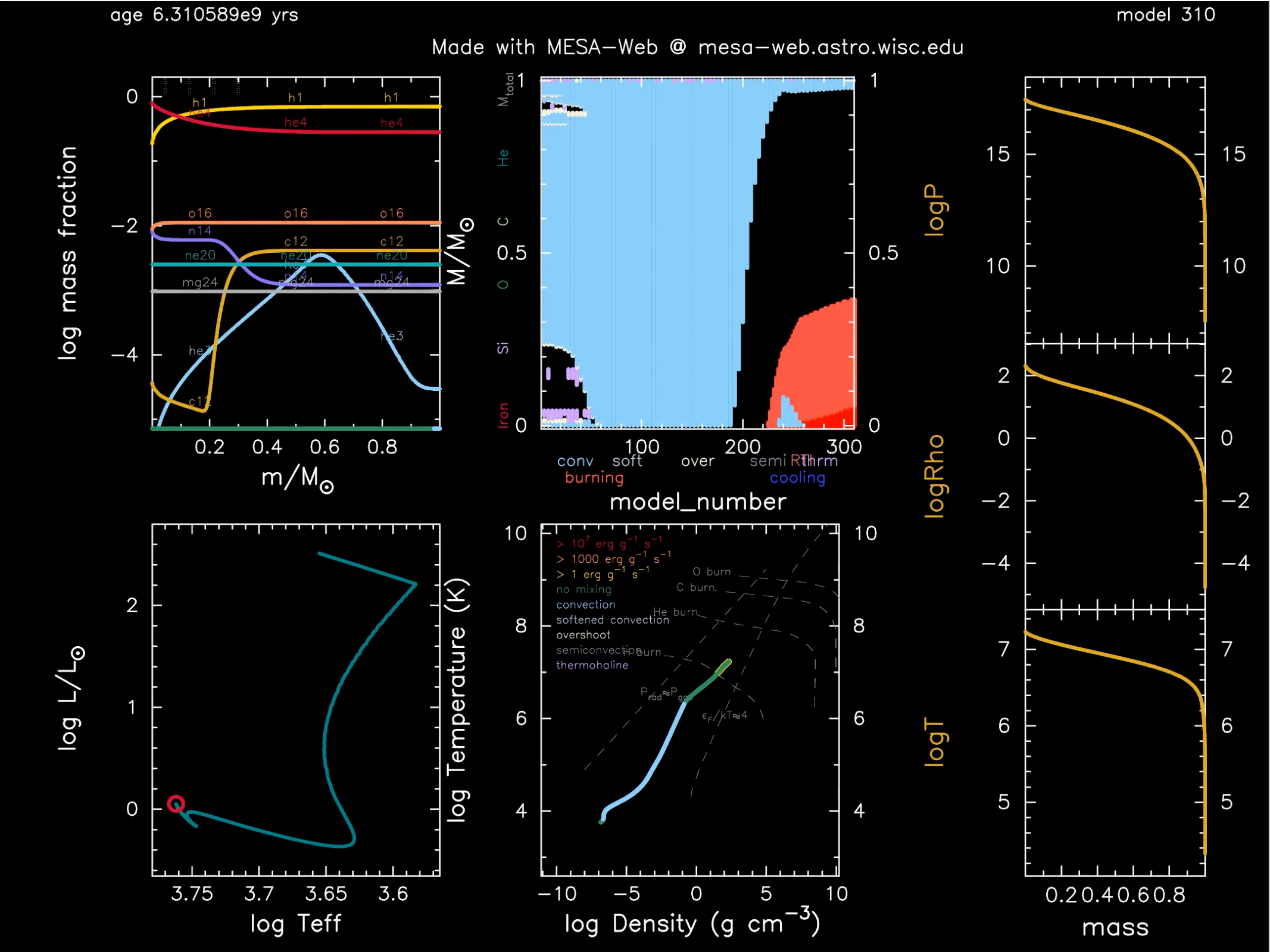


Beyond White Dwarfs: Neutron Stars and Black Holes

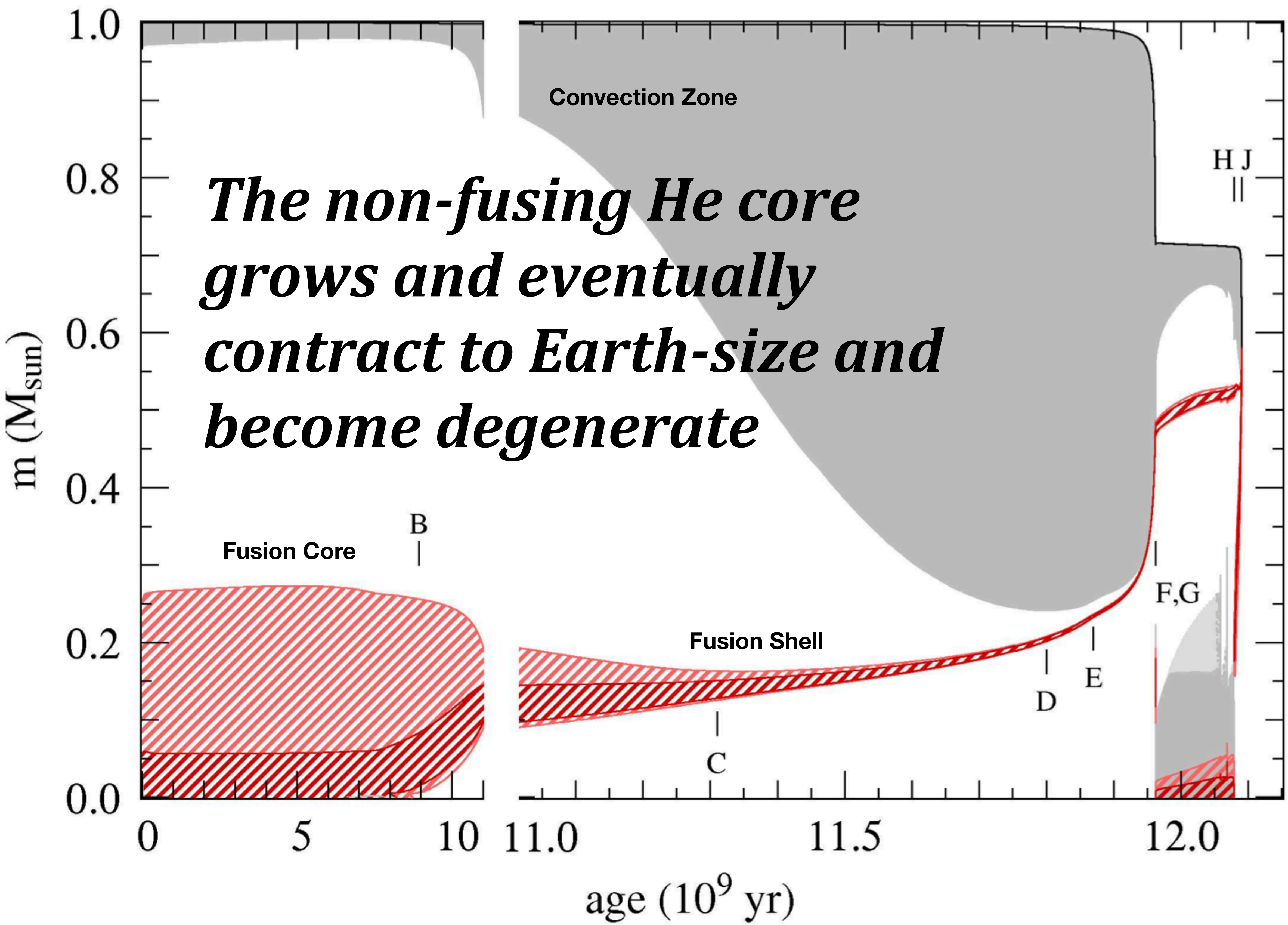


The Onset of Post-Main Sequence Evolution:

developing a degenerate He core


age 6.310589e9 yrs
model 310
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
m/M⊙
h1
he4
o16
n14
c12
ne20
mg24
he3
M/M⊙
model_number
conv
soft
over
semi
burning
cooling
Mtotal
He
C
O
Si
Iron
logP
logRho
logT
mass
log L/L⊙
log Teff
log Temperature (K)
log Density (g cm^-3)
> 10^7 erg g^-1 s^-1
> 1000 erg g^-1 s^-1
> 1 erg g^-1 s^-1
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn

Kippenhahn Diagram of a Star with an Initial Mass of 1.0 Solar Mass



Schonberg-Chandrasekhar Limit

- In 1942, **Schonberg** and **Chandrasekhar** found that when the non-fusing Helium core reaches ~8% of the total mass of the star, the core will contract because under isothermal condition, its pressure can no longer support the envelope:

$$\left(\frac{M_{\text{core}}}{M}\right)_{SC} = 0.37\left(\frac{\mu_{\text{env}}}{\mu_{\text{core}}}\right)^2$$

- The mass-ratio limit above is derived in the following way:
  - Virial theorem (self-gravitating) with external pressure at the core-envelop boundary to calculate the maximum pressure that the core can support:

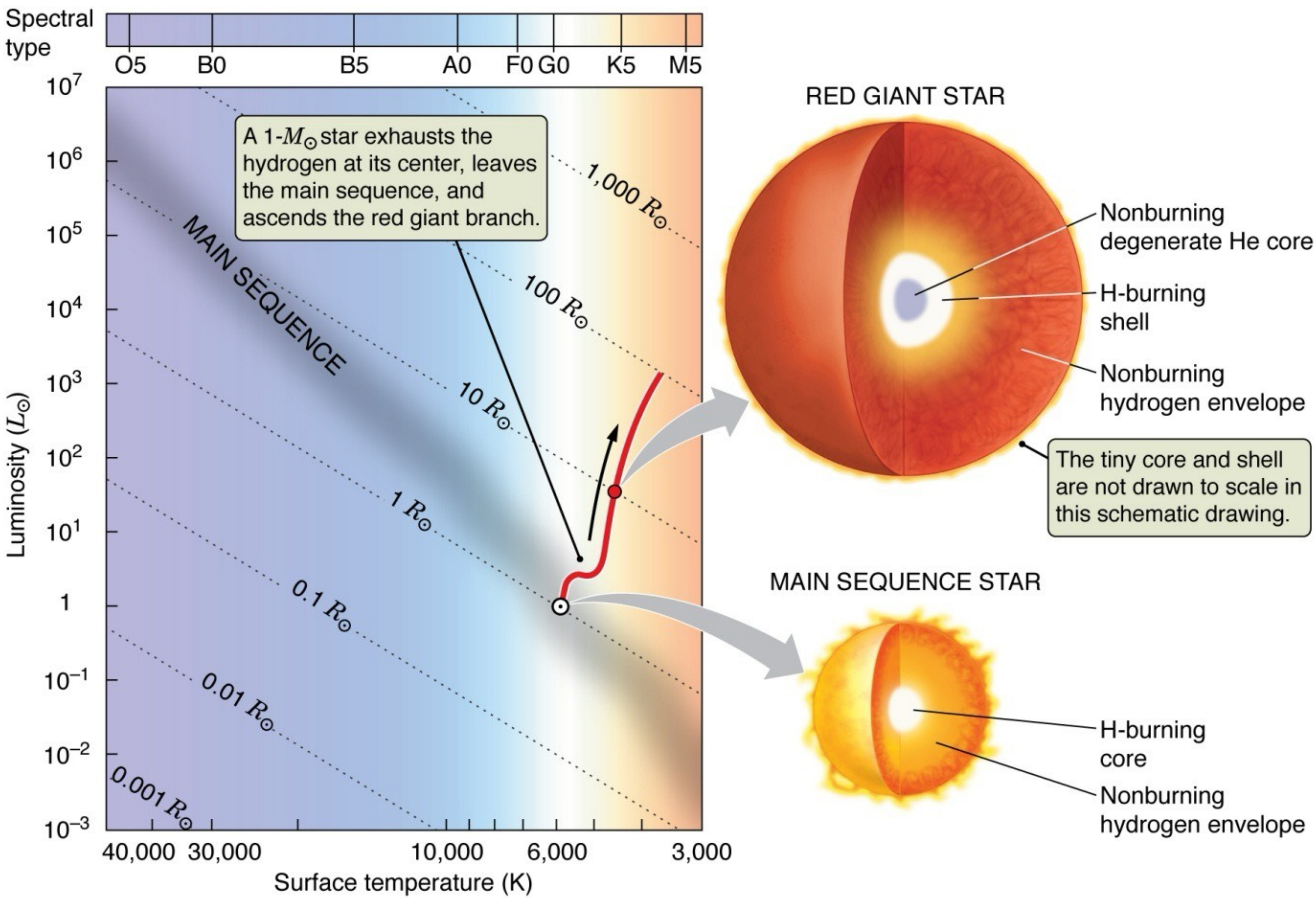
$$P_{\text{core,max}} = \frac{A}{G^3 M_{\text{core}}^2}\left(\frac{kT_{\text{core}}}{\mu_{\text{core}} m_H}\right)^4$$

  - Hydrostatic equilibrium to calculate the actual pressure from the envelope plus the ideal gas law:

$$P_{\text{env}} \approx \frac{G}{4\pi R^4}(M^2 - M_{\text{core}}^2) \approx \frac{B}{G^3 M^2}\left(\frac{kT_{\text{boundary}}}{\mu_{\text{env}} m_H}\right)^4$$

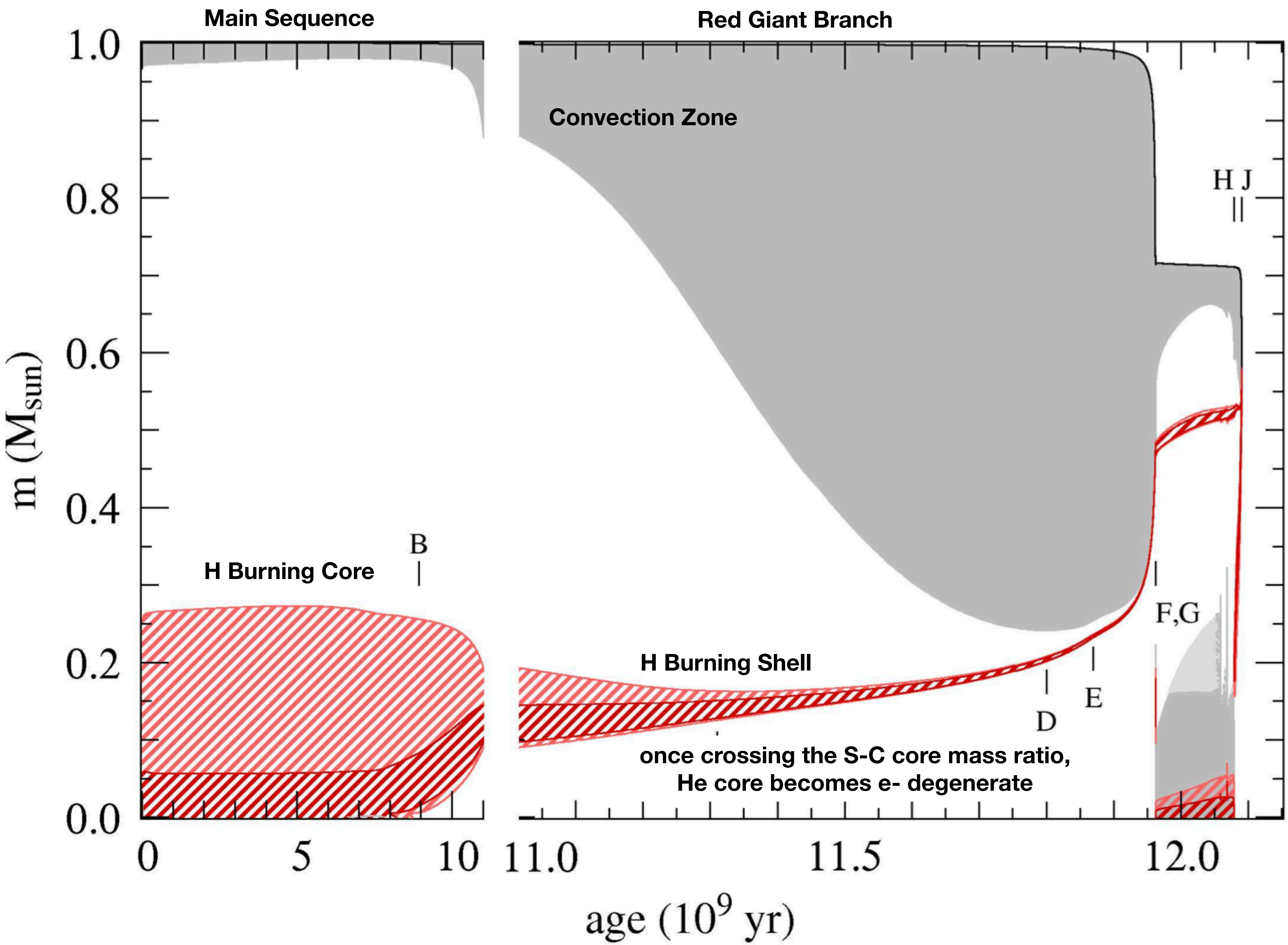
  - Given that at the boundary, the core and the envelope have the same temperature, the condition for collapse ($P_{\text{env}} > P_{\text{core,max}}$) gives us a maximum core-mass-total-mass ratio that is *inversely* proportional to the square of the ratio of the mean molecular mass.

Violation of the SC limit causes the core to contract and become degenerate

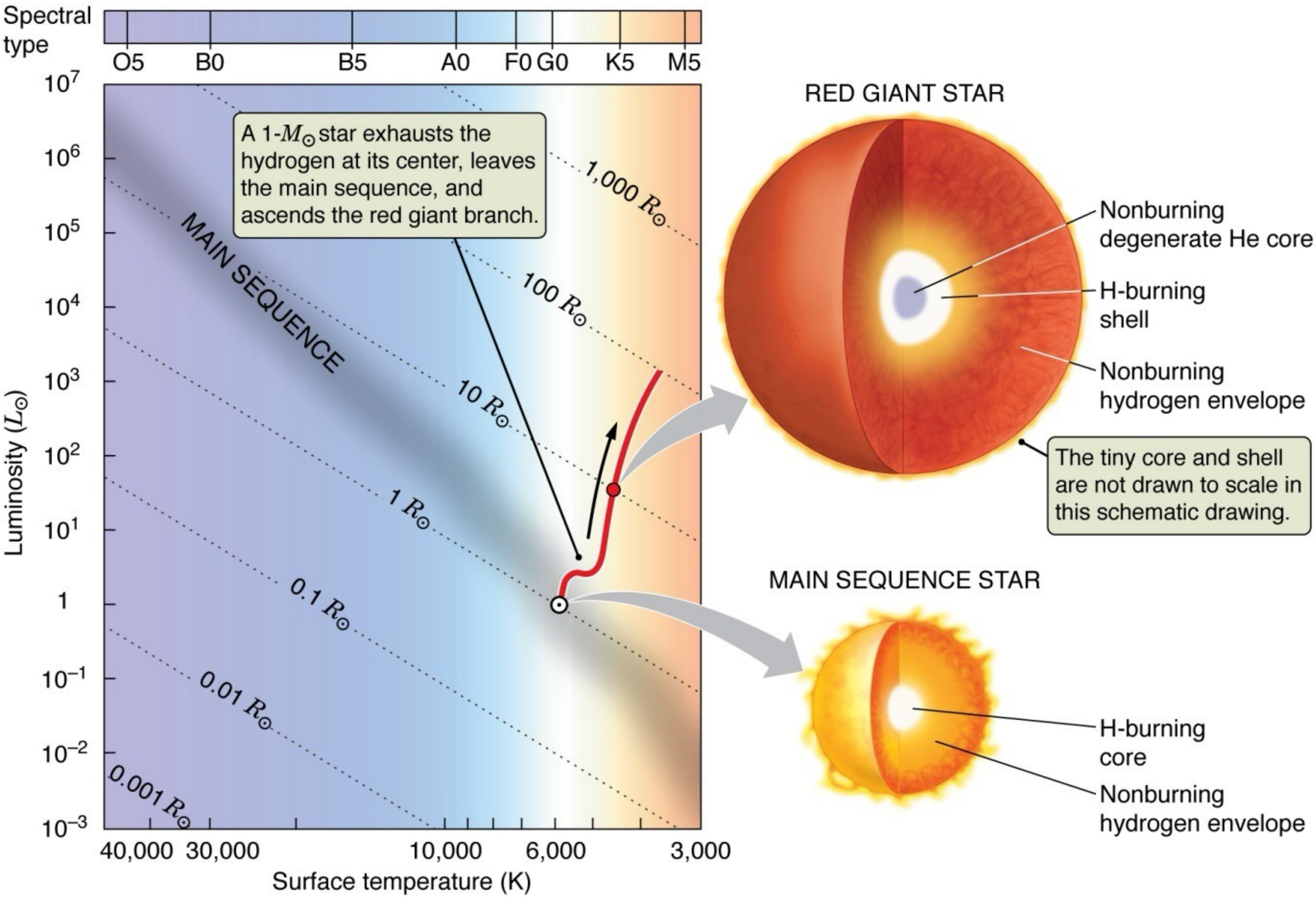


Red Giant Branch

Kippenhahn Diagram of a Star with an Initial Mass of 1.0 Solar Mass

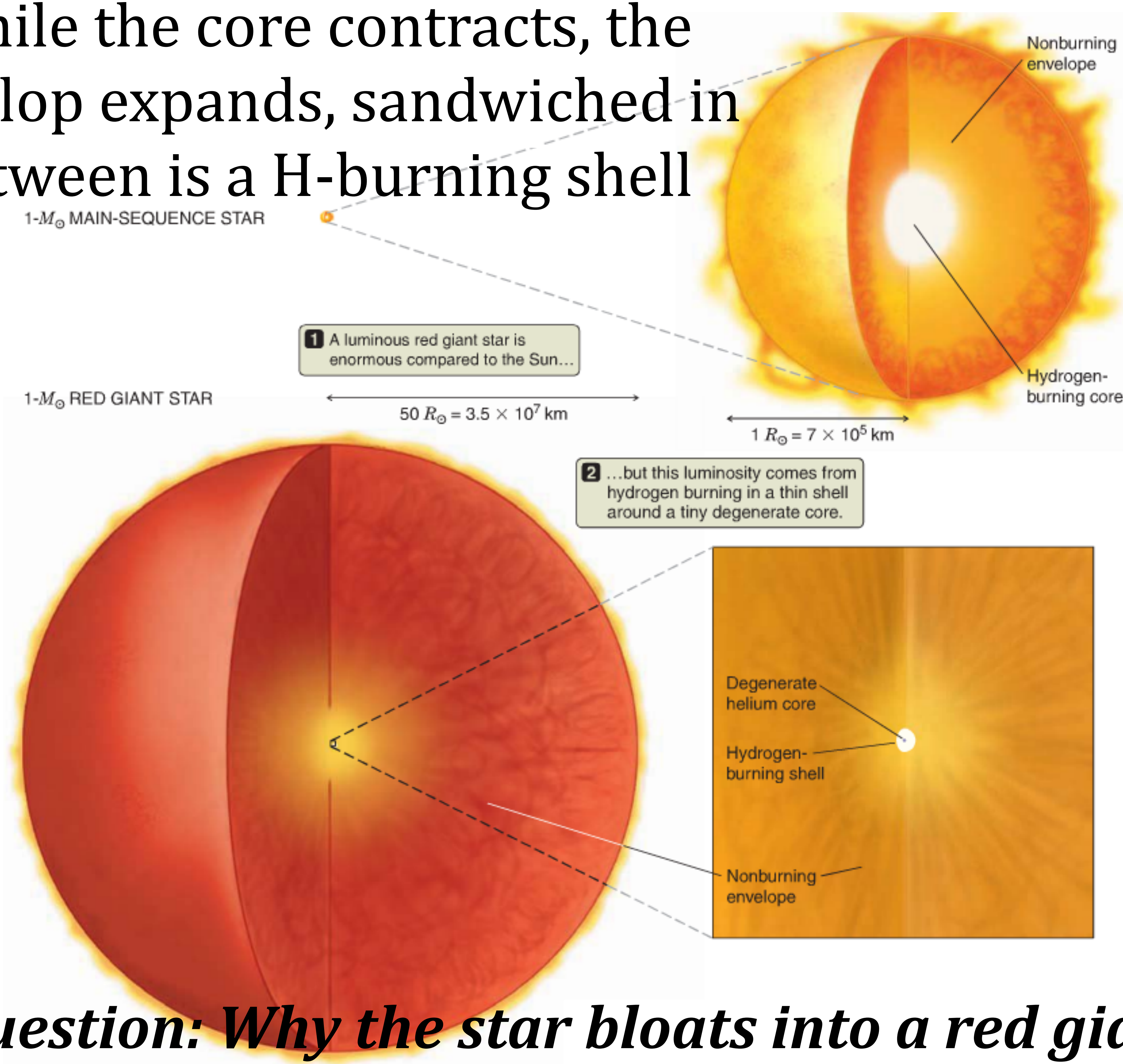


The Red Giant Branch:
H-burning shell + degenerate He core



Sunrise in 7 Billion Years

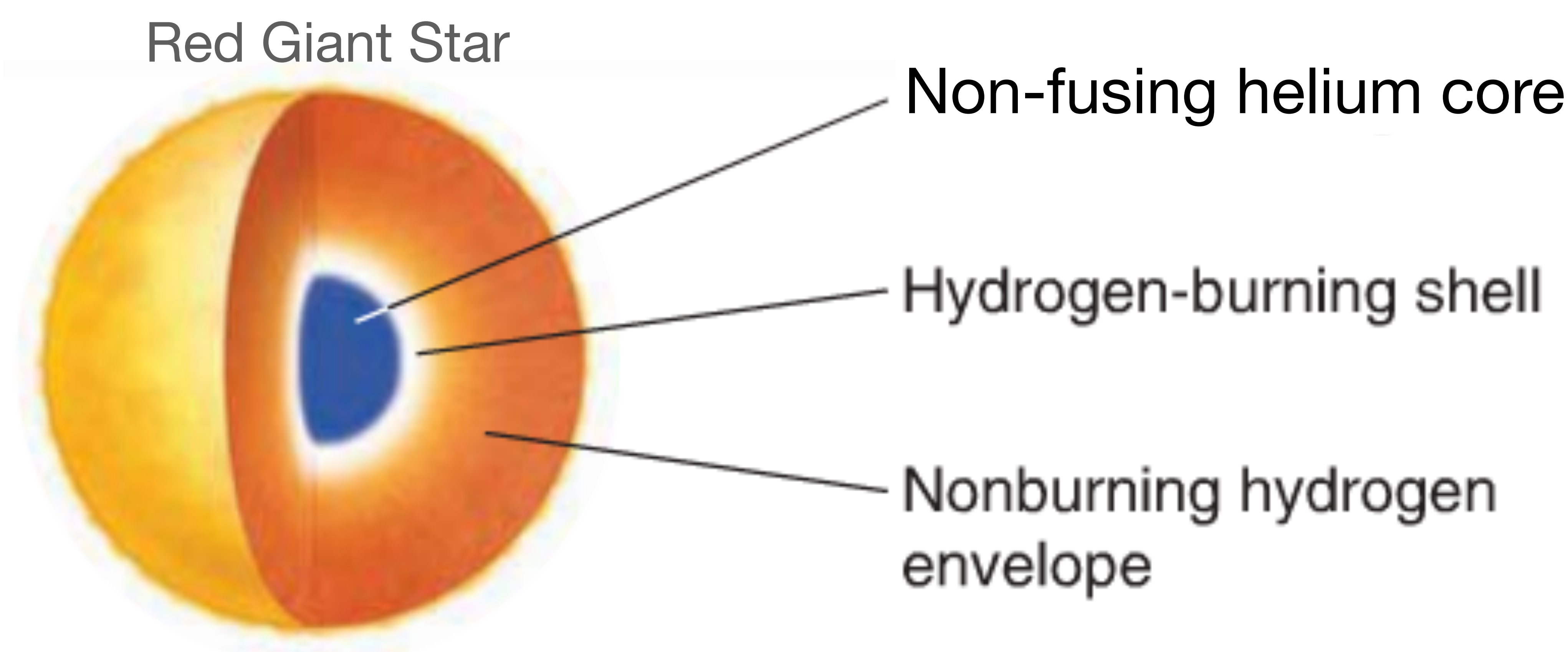
While the core contracts, the envelop expands, sandwiched in between is a H-burning shell



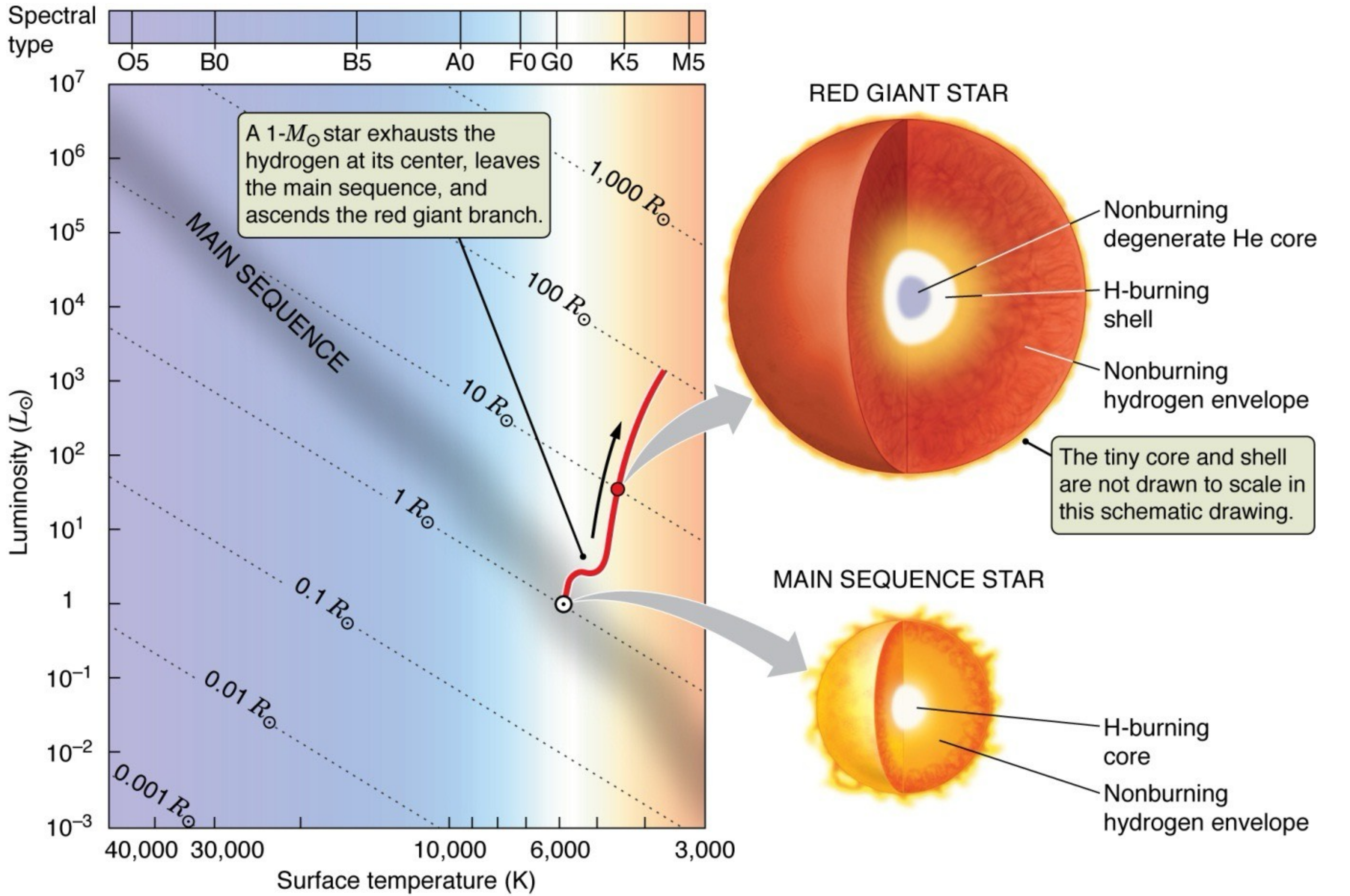
Question: Why the star bloats into a red giant?

The mirror principle of a fusion shell: when one side contracts, the other side expands

- While the **gravitational thermostat** works well to control the **burning core**'s temperature, it cannot control the temperature of a **burning shell**
- When fusion stops in the **core**, it **contracts**. Gravitational potential energy heats up the core and it conducts its heat to the surrounding shell.
- As the **shell's temperature rises**, its fusion reaction rate increases rapidly
- To avoid a thermonuclear runaway, the shell must decrease its temperature by dumping its energy to the **non-burning envelope**, causing the star to expand to a giant.



How Temperature stays roughly constant while Luminosity increases thousands of times?



The H— thermostat controls the atmosphere temperature of red giant stars

Surface temperature decreases

H- increases

Atmosphere becomes more opaque

Surface temperature increases

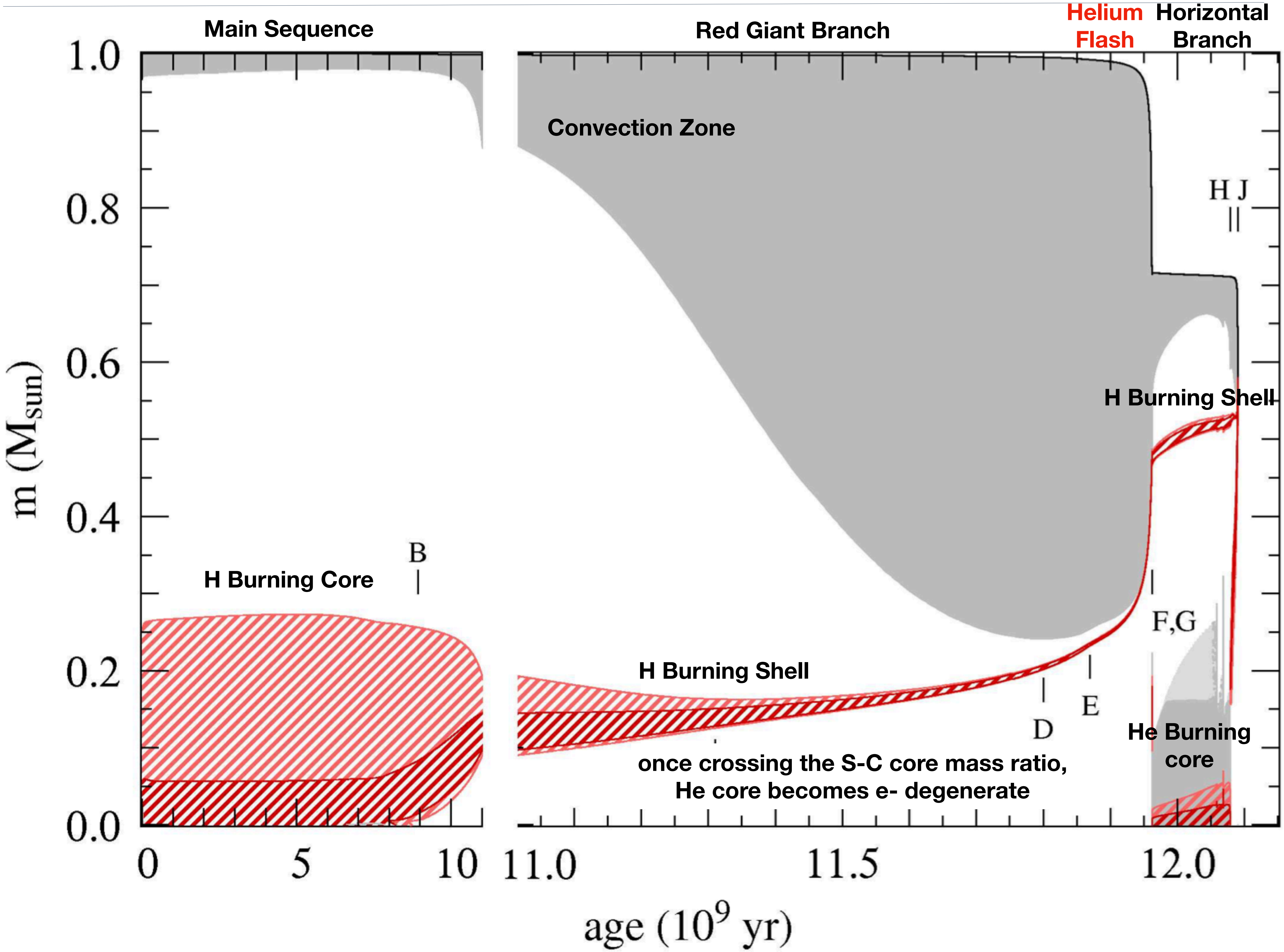
H- decreases

Atmosphere becomes more transparent

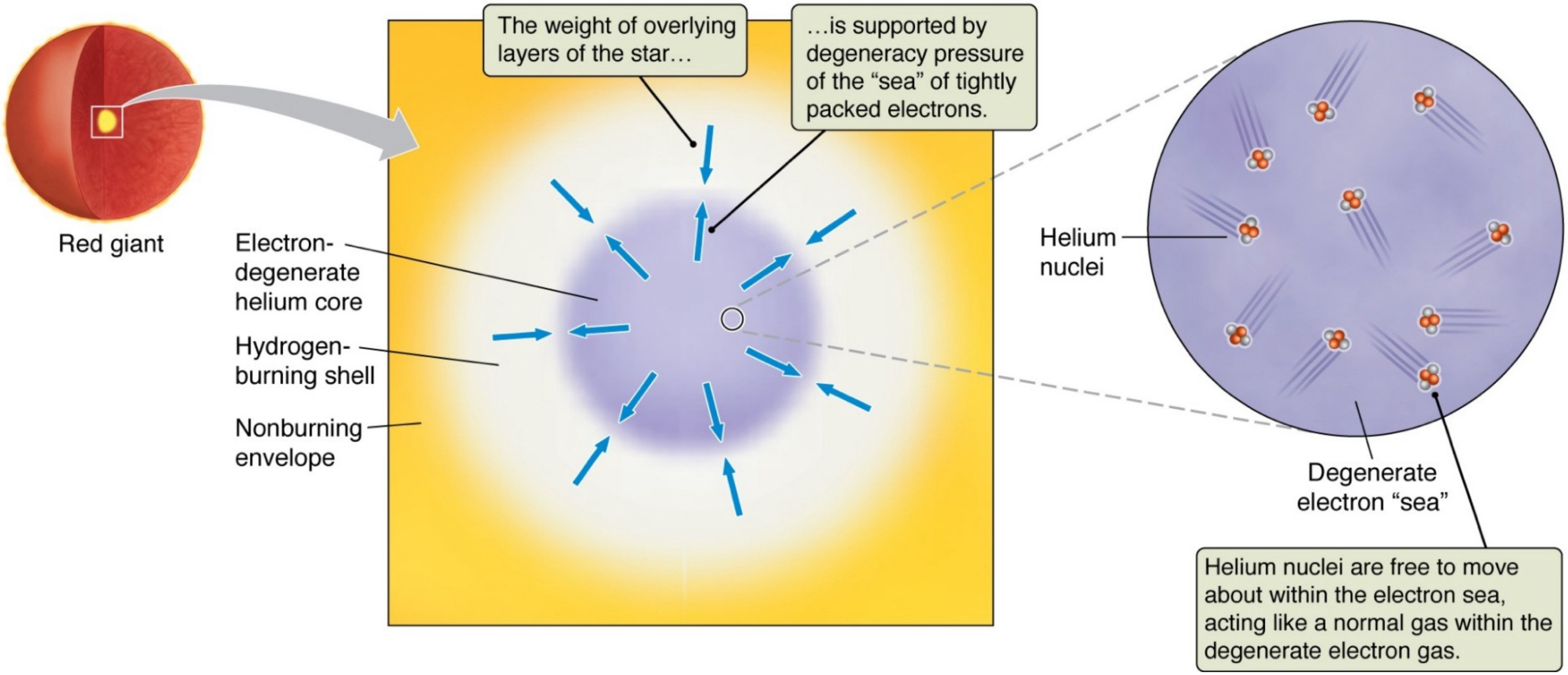
Surface temperature decreases

Helium Flash:
a nuclear explosion inside a star

Kippenhahn Diagram of a Star with an Initial Mass of 1.0 Solar Mass



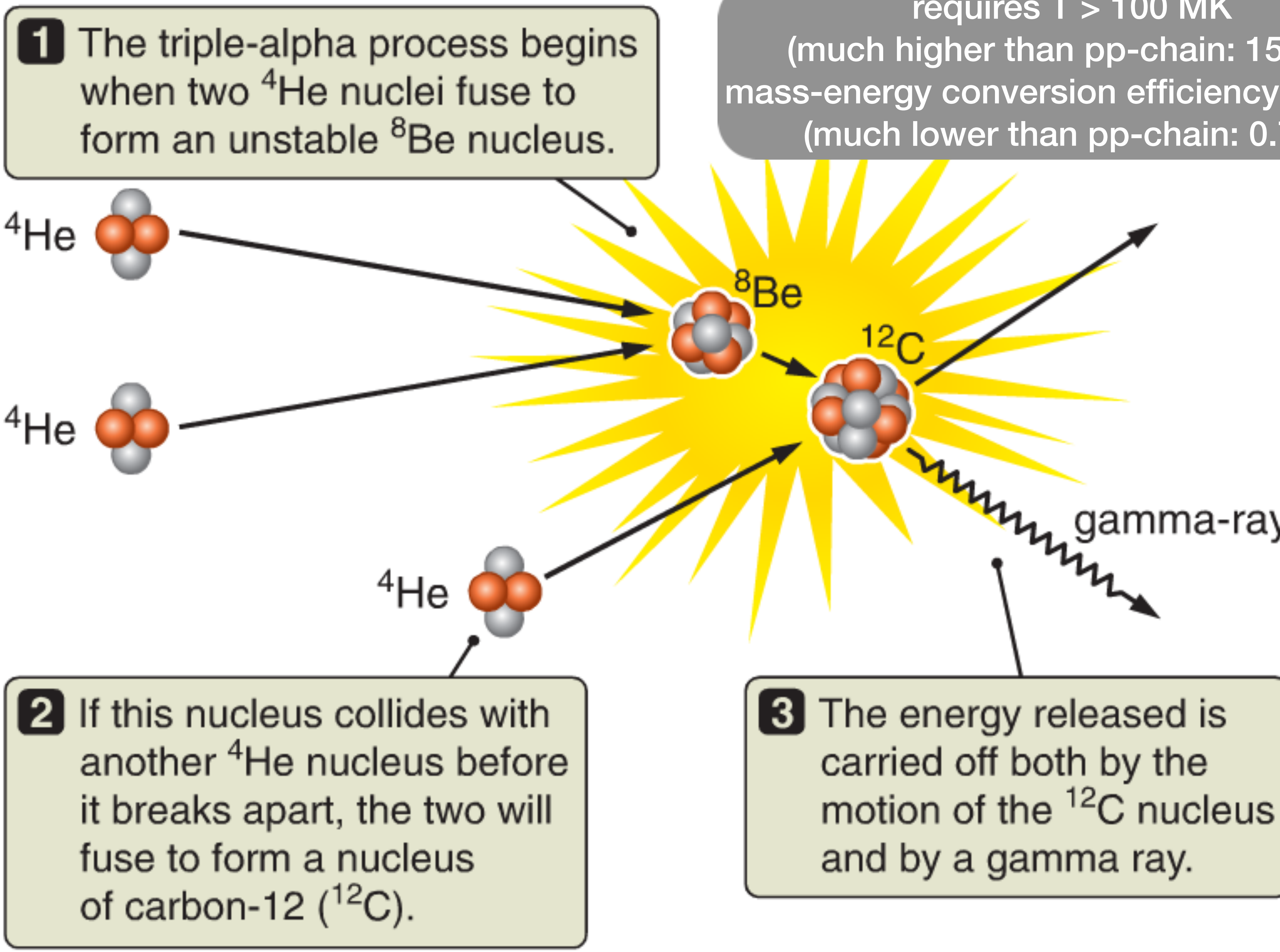
Helium Flash: The End of the Red Giant Phase



- Hydrogen shell-burning adds Helium to the core, making it heavier.
- Core pressure doesn't increase past the electron-degenerate limit, but **ion temperature is allowed to increase without bound**.
- …**until T reaches ~ 100 million K, at which time Helium can fuse.**

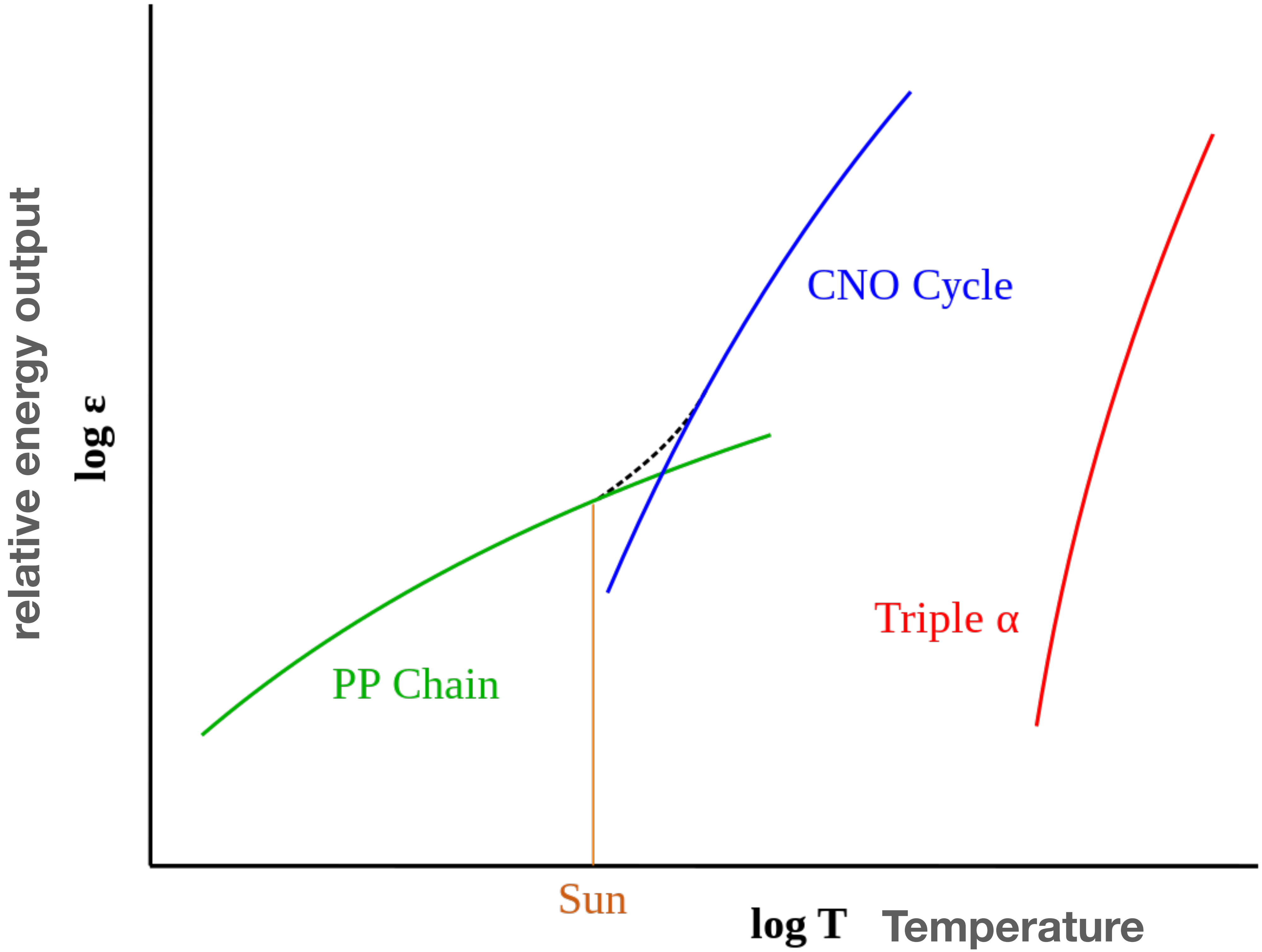
Helium burning – the triple-alpha process

requires T > 100 MK
(much higher than pp-chain: 15 MK)
mass-energy conversion efficiency: 0.065%
(much lower than pp-chain: 0.7%)



Fusion Reaction Rate Strongly Depends on Temperature

- For PP chain, reaction rate ~ T^4, for CNO cycle, rate ~ T^{20}, Triple Alpha cycle, rate ~ T^{40}



Helium Flash: The Star Experiences a Nuclear Explosion at its Center!

THINKSTOCK

Question: Why the onset of He burning in a degenerate core makes a bomb?
Why the nuclear fusion cannot be controlled as in the core of the Sun?

When the gravitational thermostat is out of order

non-degenerate core

H fusion rate increases

T & P increases

Core expands,
work against Gravity

T decreases

H fusion rate decreases

Steady H Burning

electron-degenerate core

He fusion rate increases

$T(He^{2+})$ increases,
P(e-) do not change

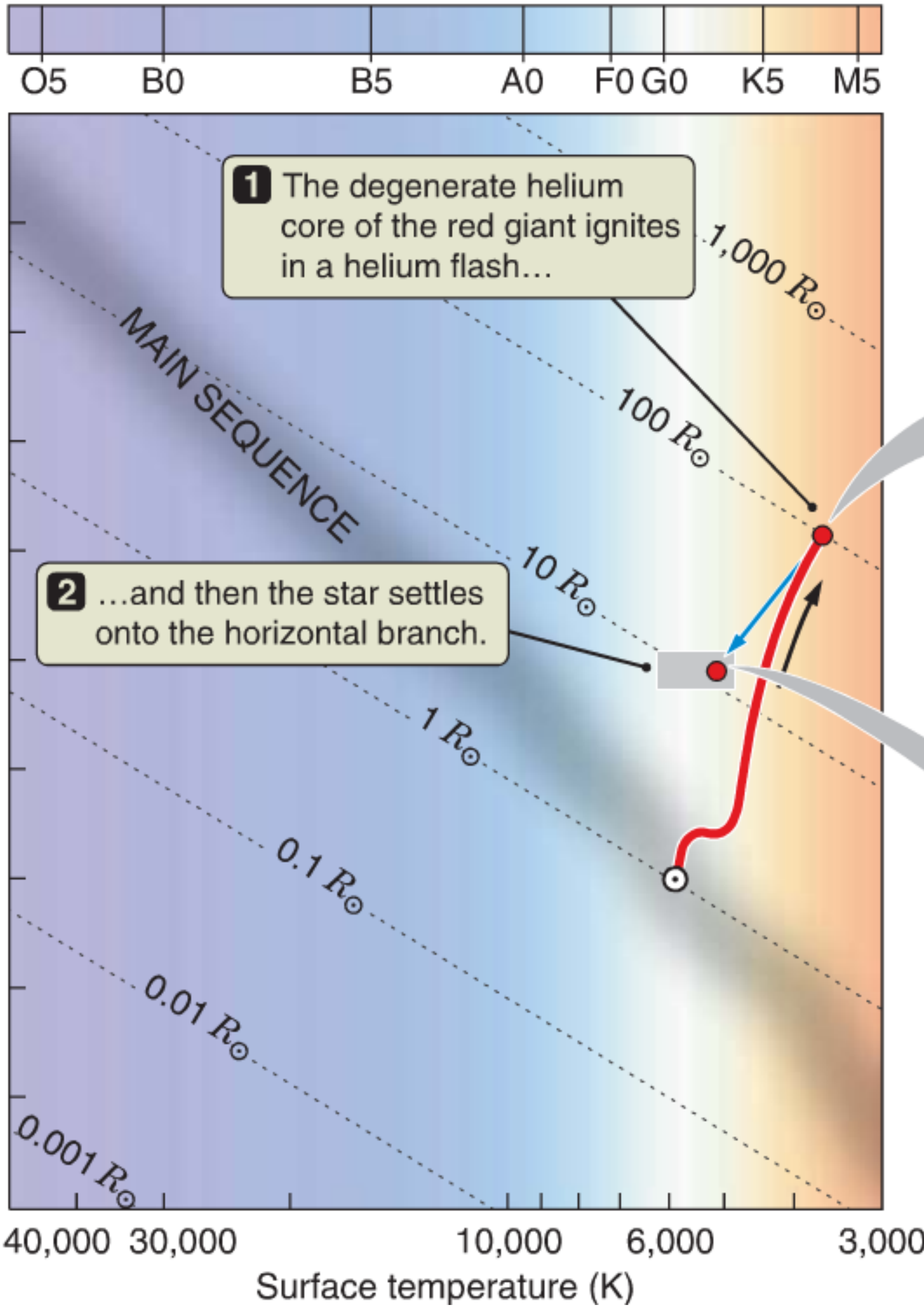
Core doesn't expand

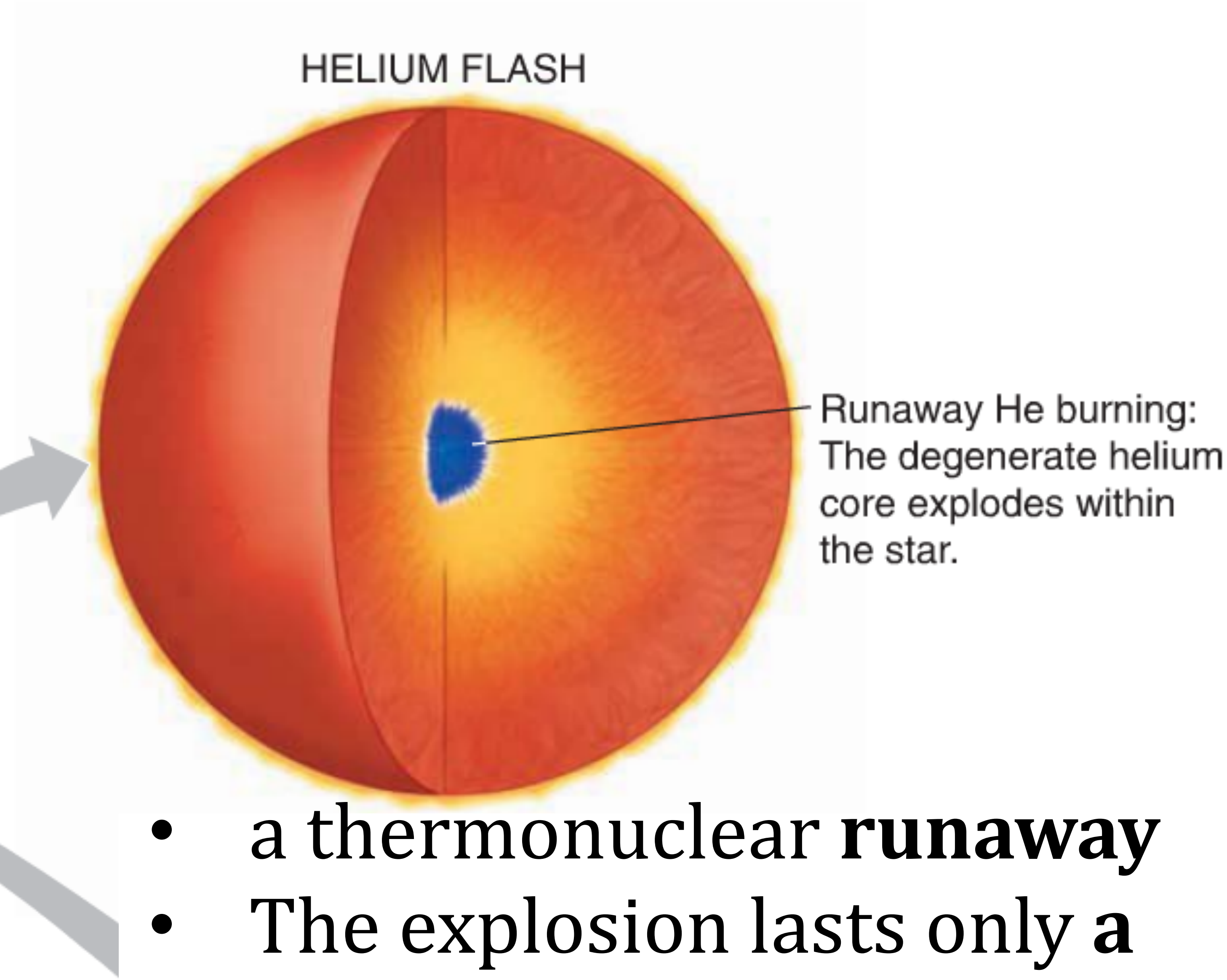
$T(He^{2+})$ still high

He fusion rate increases

He Flash

Helium flash - a thermonuclear runaway





- a thermonuclear **runaway**
- The explosion lasts only **a few hours** thanks to the **heavy envelop**, which helped the star to **regain stability while the core loses degeneracy**

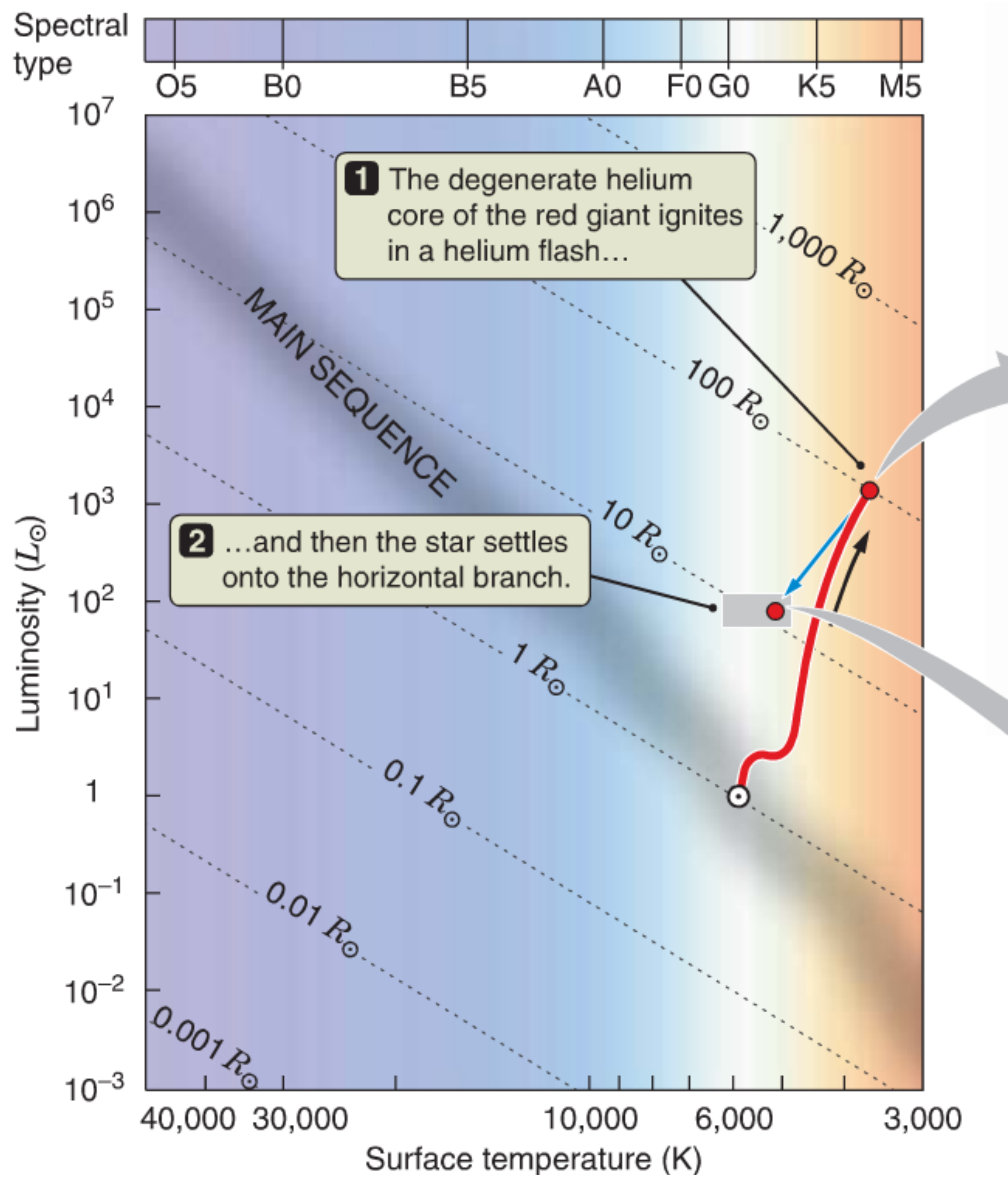
Horizontal Branch

a new main sequence

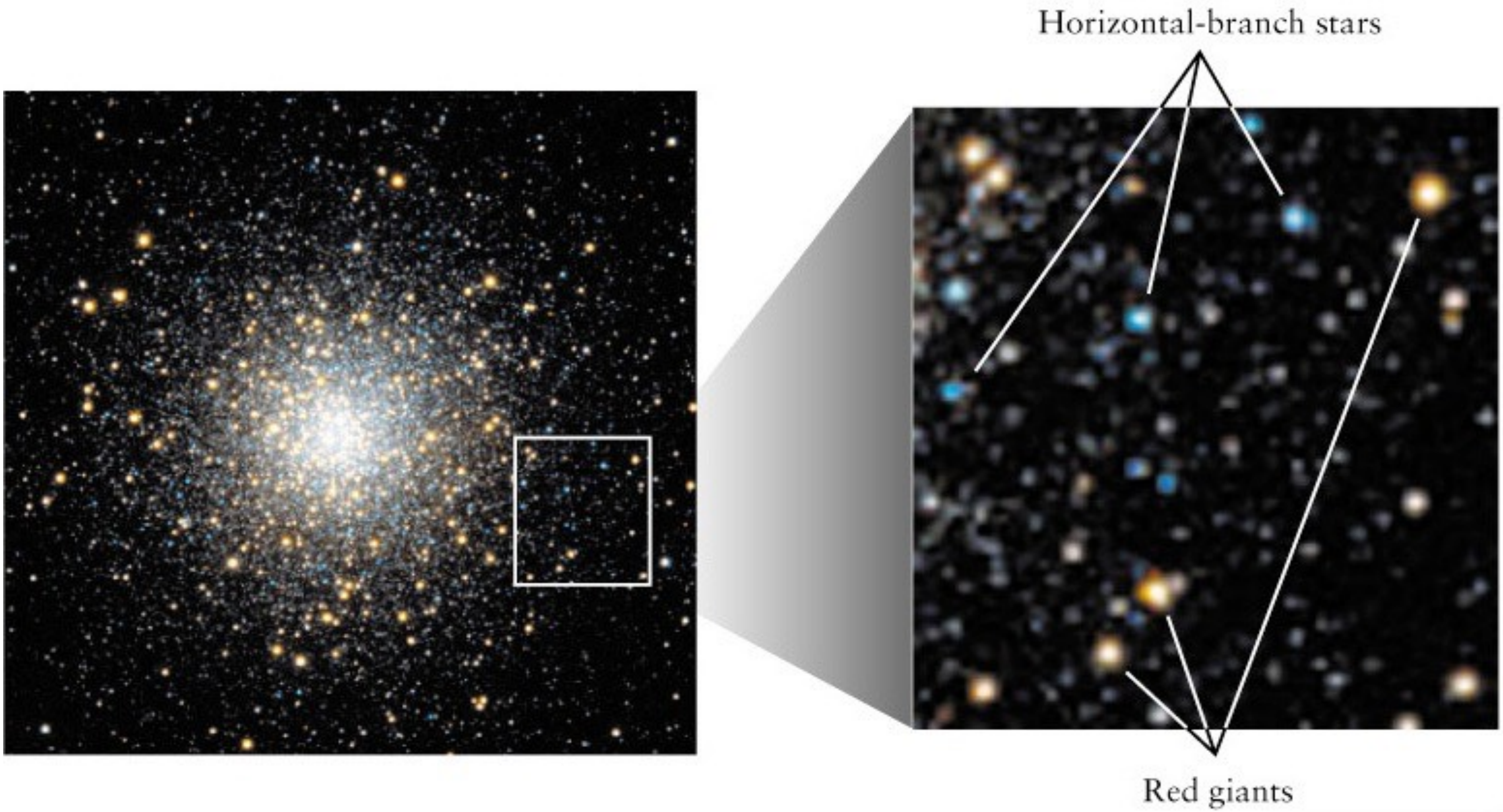
After Helium flash

- Settles onto ***horizontal branch***
- Stable Helium core burning, similar to a main sequence star
- H burning continues in shell surrounding the He burning core

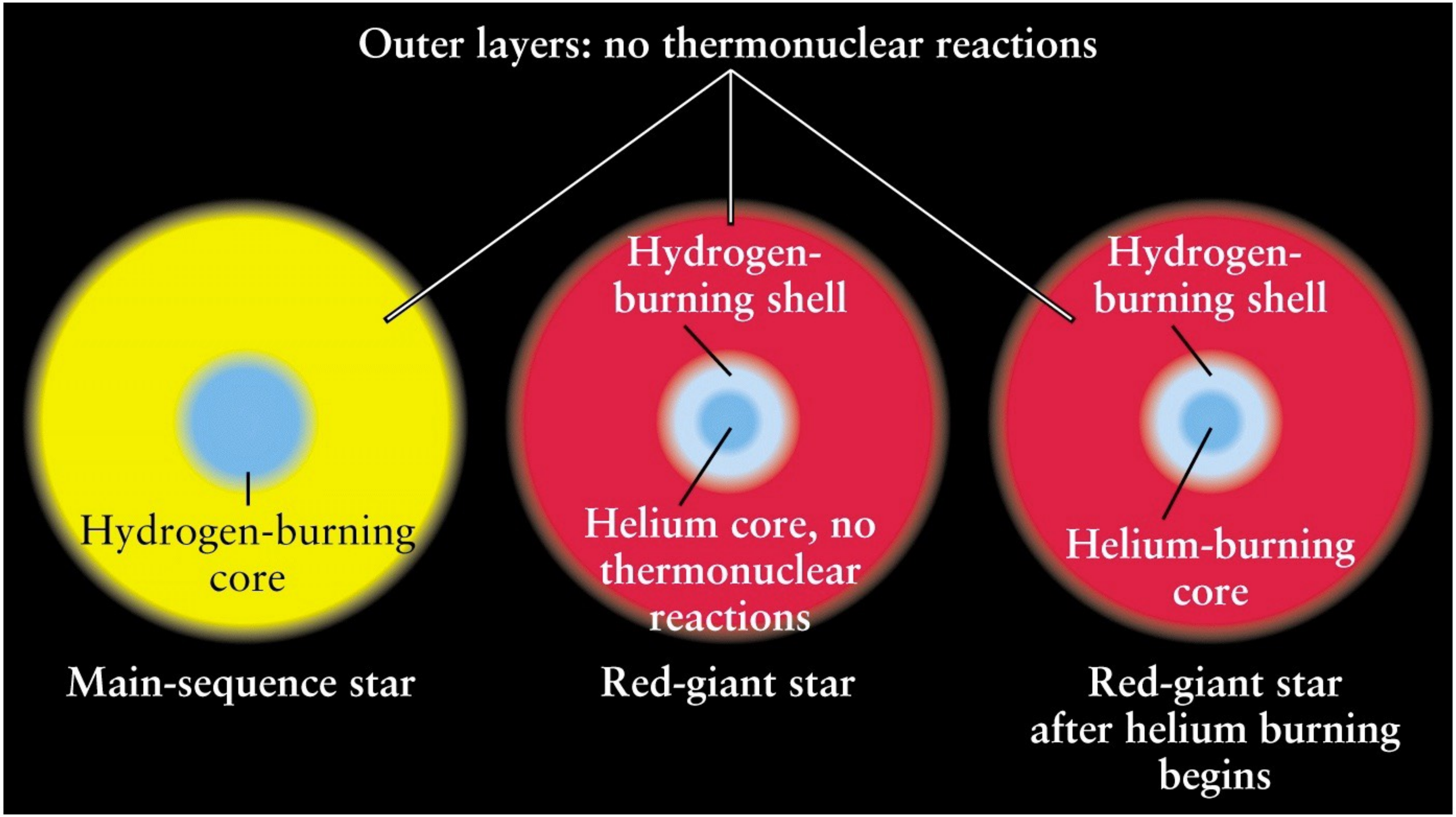
- Main Sequence – 10 Gyr
- Red Giant – 1 Gyr
- He flash - a few hours
- He flash to HB – 100 Kyrs
- HB - 100 Myrs, a new “MS”



Horizontal branch stars are hotter (bluer) than RGs, and are evolving horizontally to left on HR diagram



Main Sequence → Before He flash: Red Giant → After He flash: Horizontal Branch



Post-**MS** evolution:

- non-fusing Helium core, H-burning shell
- Helium-core contract and become **e- degenerate**
- **Red giant** phase (H^- controls surface T as L increases)
- uncontrolled Helium-burning in the e- degenerate core (thermonuclear runaway, **Helium flash**)
- core expands and become non-degenerate, allowing steady Helium burning (**Horizontal branch**)

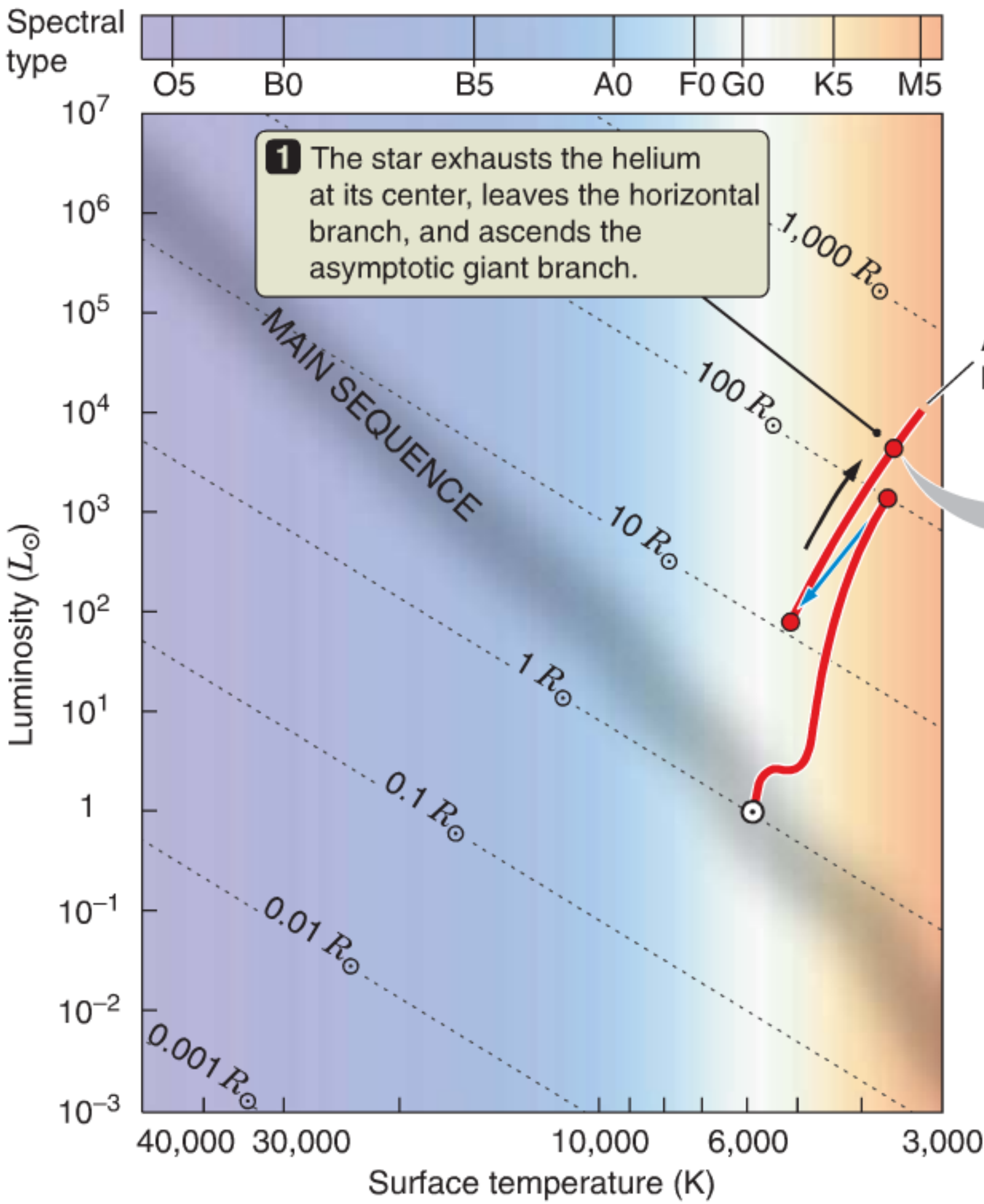
Eventually, Helium will be exhausted at the center of a Horizontal Branch star, and a non-fusing Carbon core will form under a Helium-burning shell?

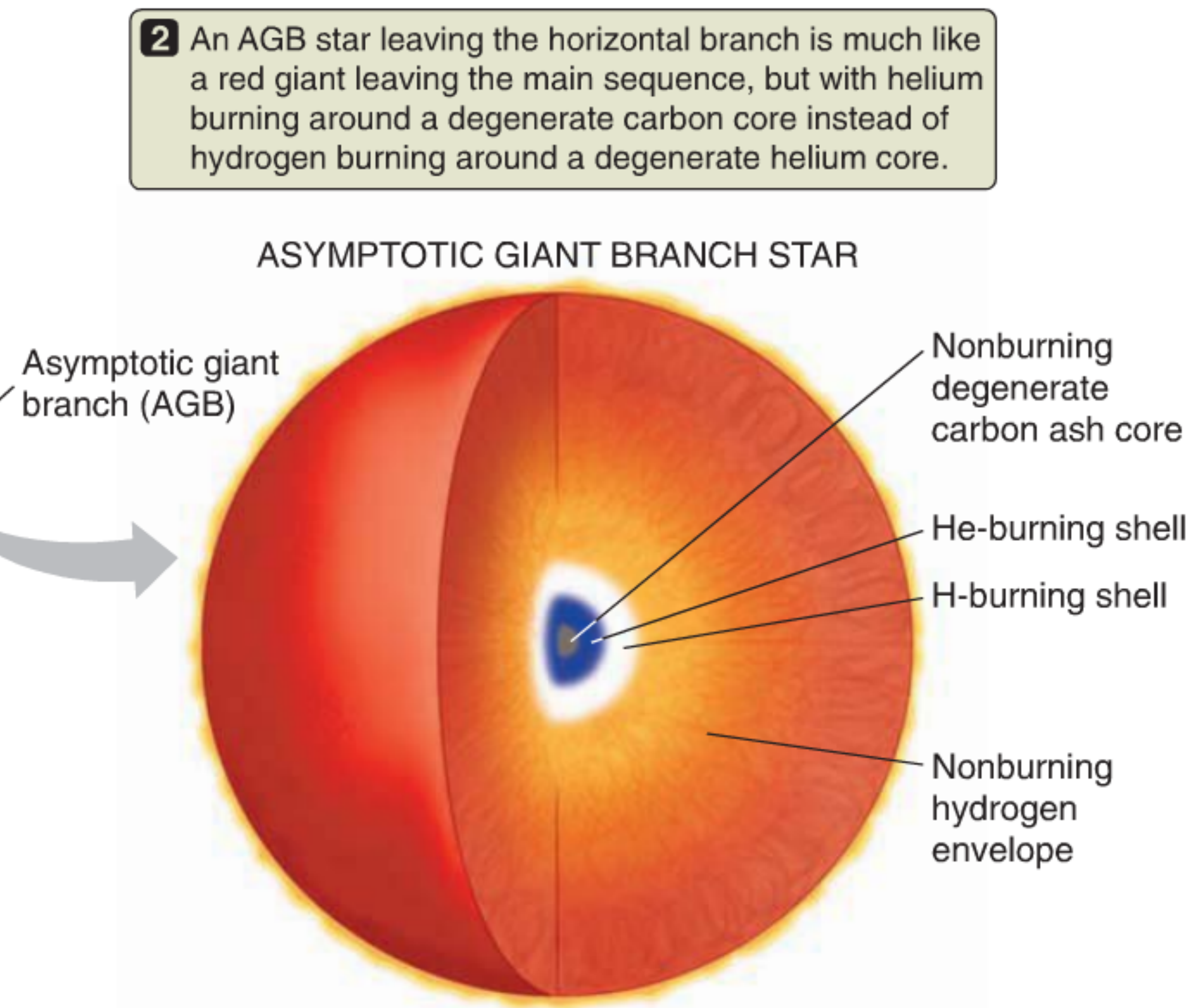
This marks the end of the horizontal branch phase. Based on what you learned about the post-MS evolution, **deduce the post-HB evolution of the star**.

Post-Horizontal Branch Evolution

The Asymptotic Giant Branch (AGB),
Post-AGB phase: Planetary Nebula, White Dwarf

When Helium is depleted in the core, the star evolves into the Asymptotic Giant Branch (AGB)

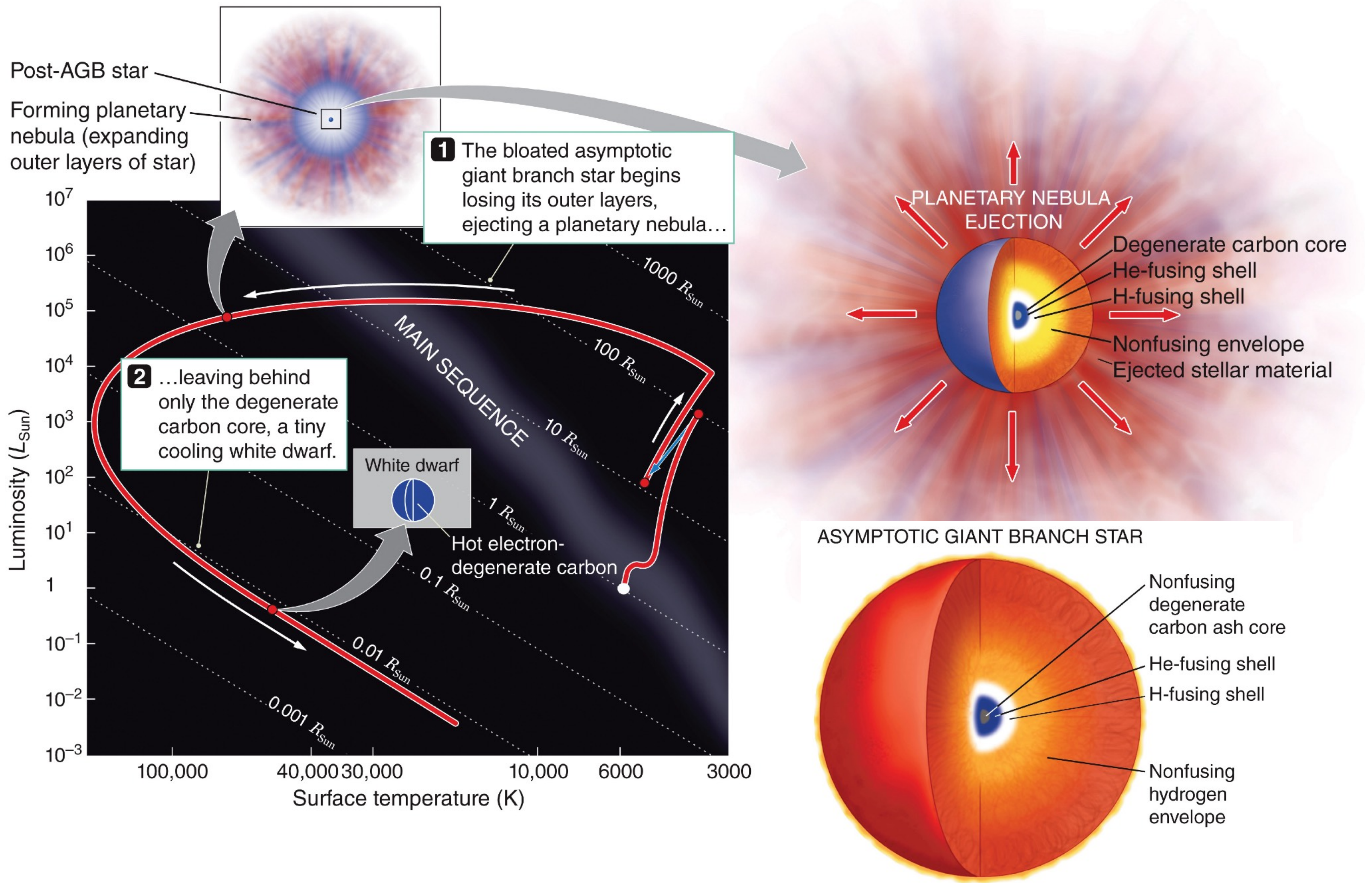




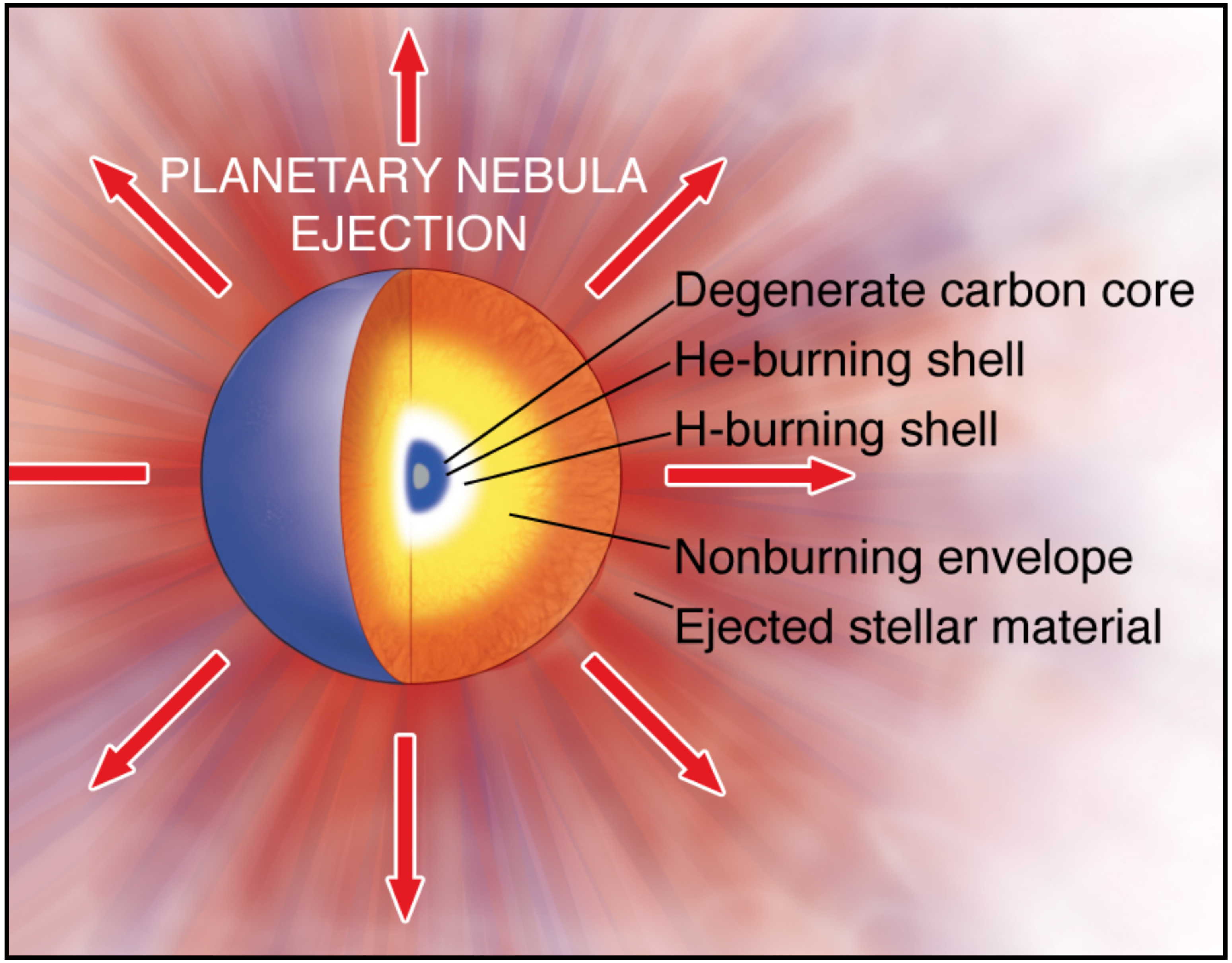
- Star gets more luminous and cool, and enters the asymptotic giant branch (AGB).
- Electron-degenerate Carbon core with no nuclear fusion (T not hot enough)
- H^- keeps the surface temperature almost constant, like in the Red Giant Branch (RGB)

Post-**AGB** Mass Loss

- Asymptotic Giant Branch – 100 Kyr
- Post-AGB / Planetary Nebulae - 10 Kyr



Post-AGB - No "Carbon Flash" but a Planetary Nebula



- Temperature never gets high enough to initiate C burning (**500 million K** at core density)

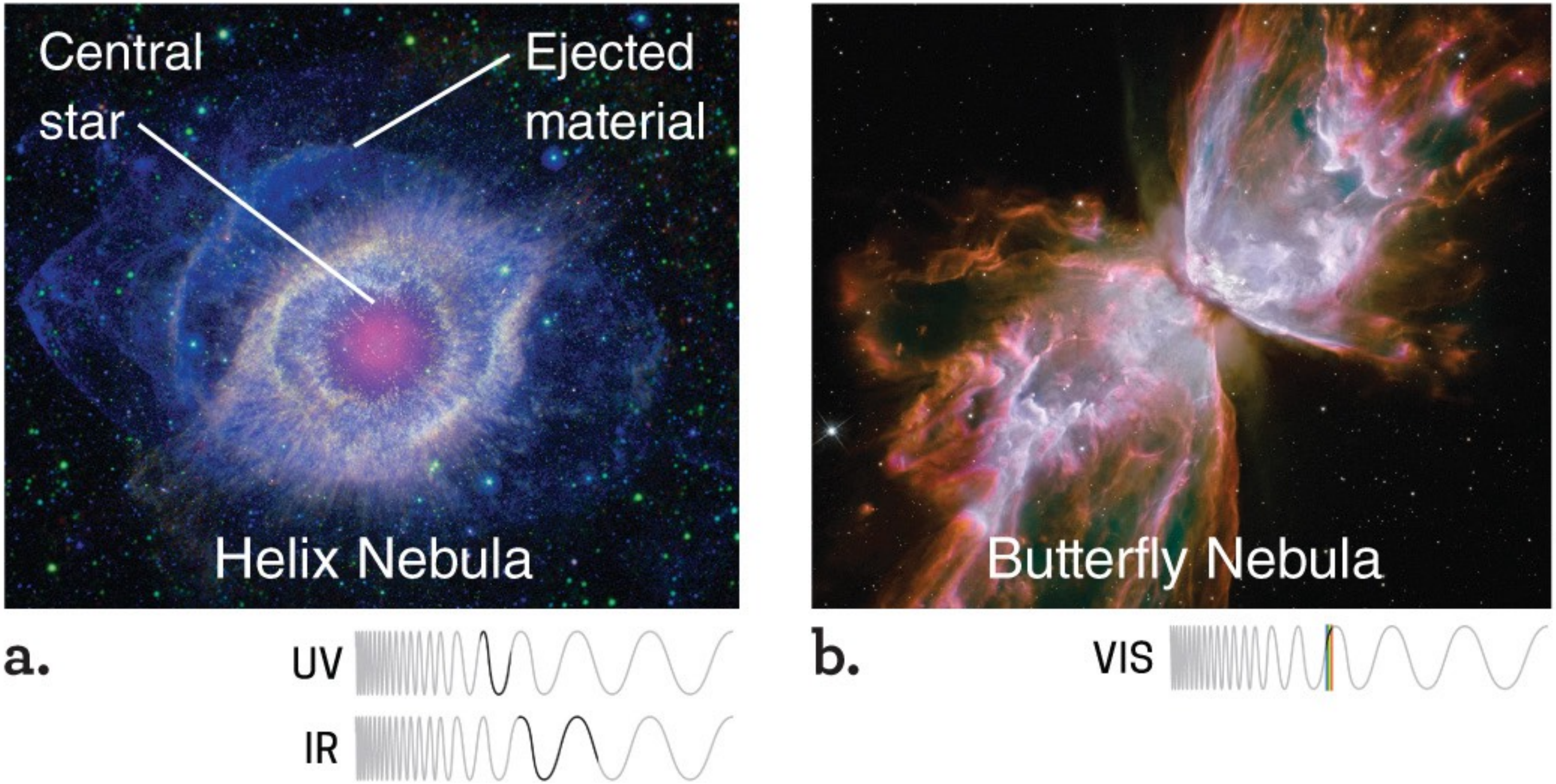
$$^{12}_{6}C + ^{12}_{6}C \rightarrow ^{24}_{12}Mg + \gamma$$

- **Mass loss becomes a runaway process** – forming **planetary nebulae**
- mass loss -> star puffs up -> less gravity -> more mass loss

M57 - the Ring Nebula in Lyra near Vega

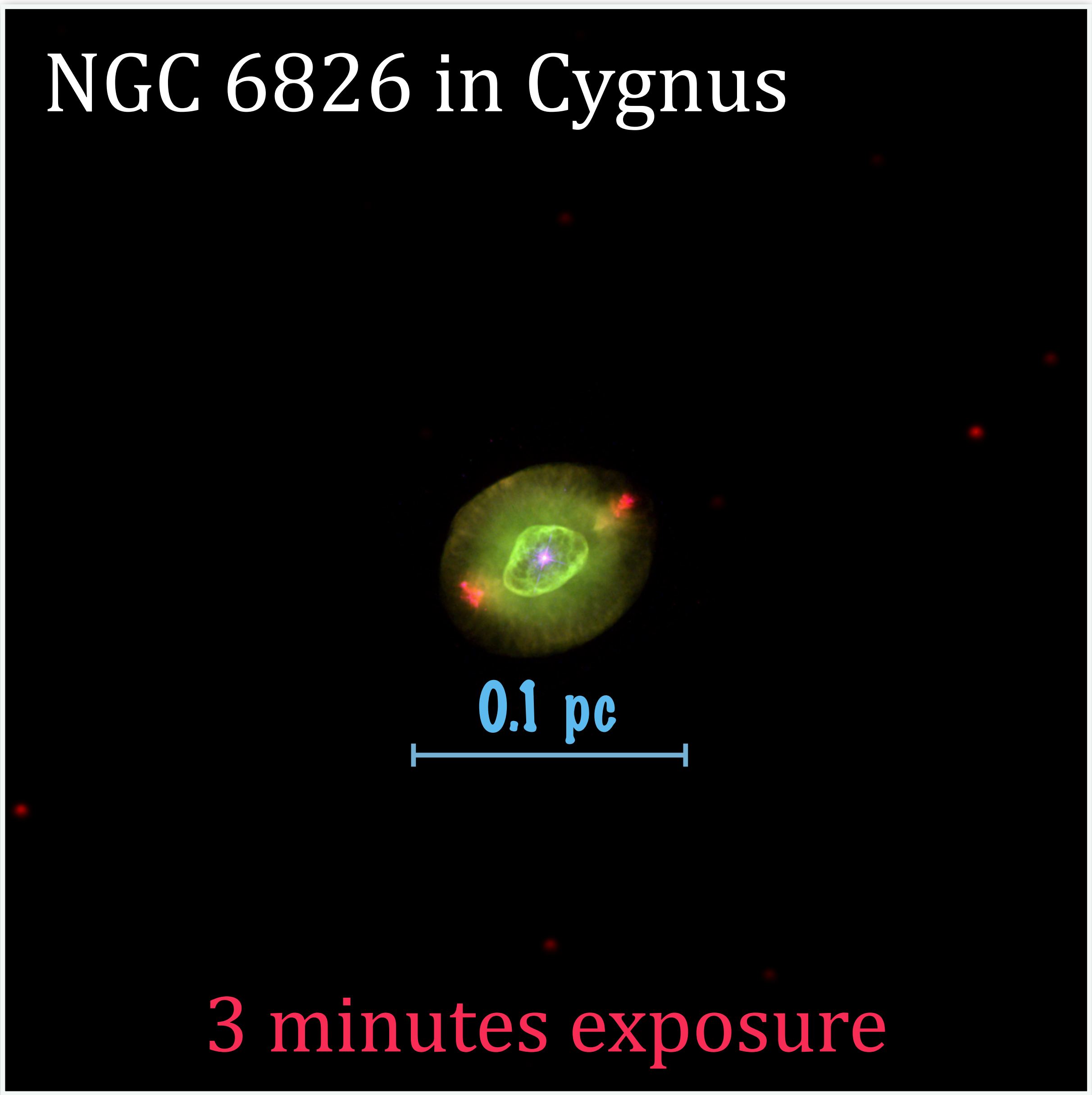
Planetary Nebulae

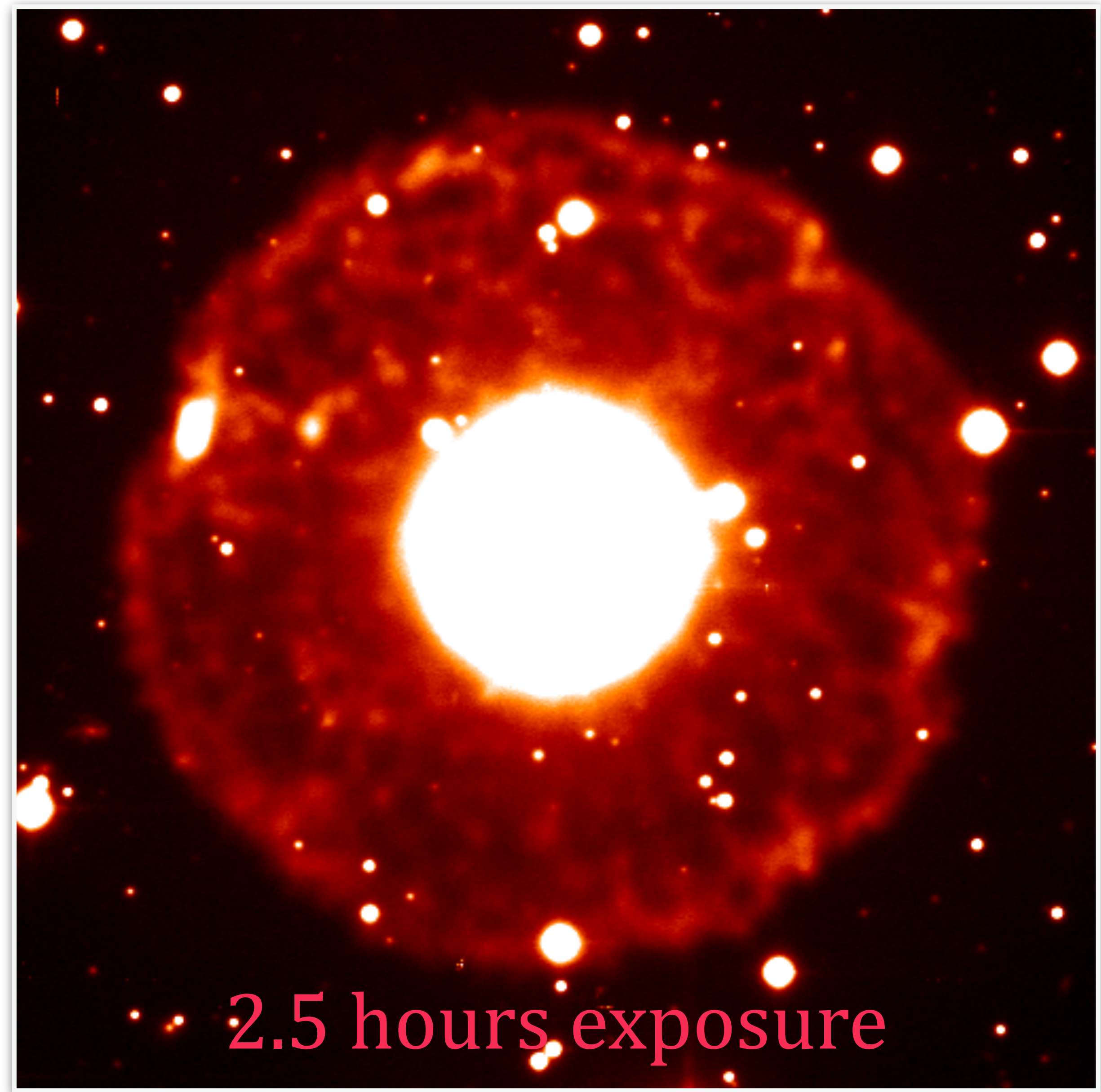
- Material farther from the star was ejected earlier.
- Radiation from the white dwarf ionizes the gas. The colors are due to specific atoms and bright spots indicate areas of denser gas.

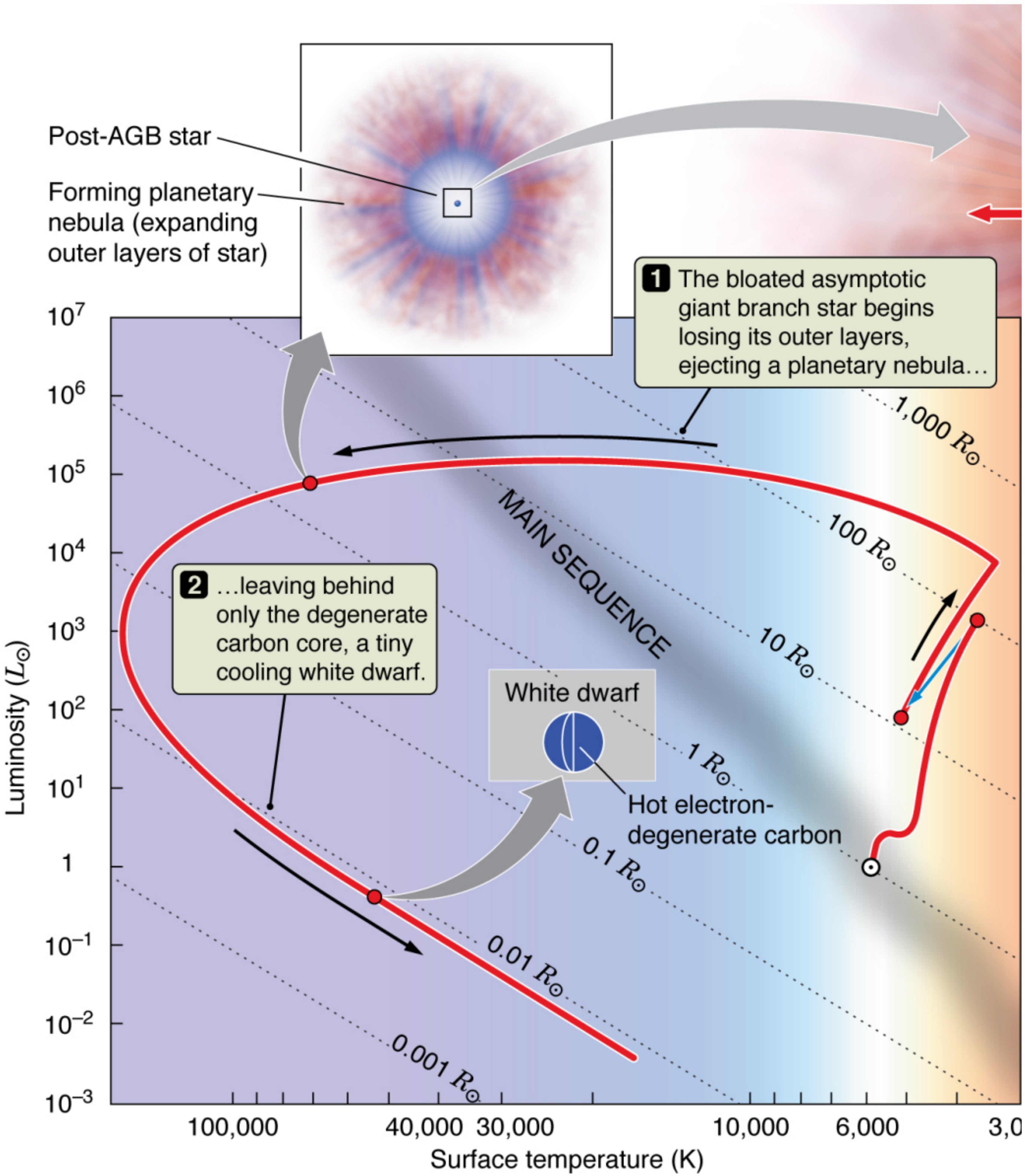


Severe Stellar Mass Loss illustrated in a Planetary Nebula

The planetary nebula extends much farther than the central bright area



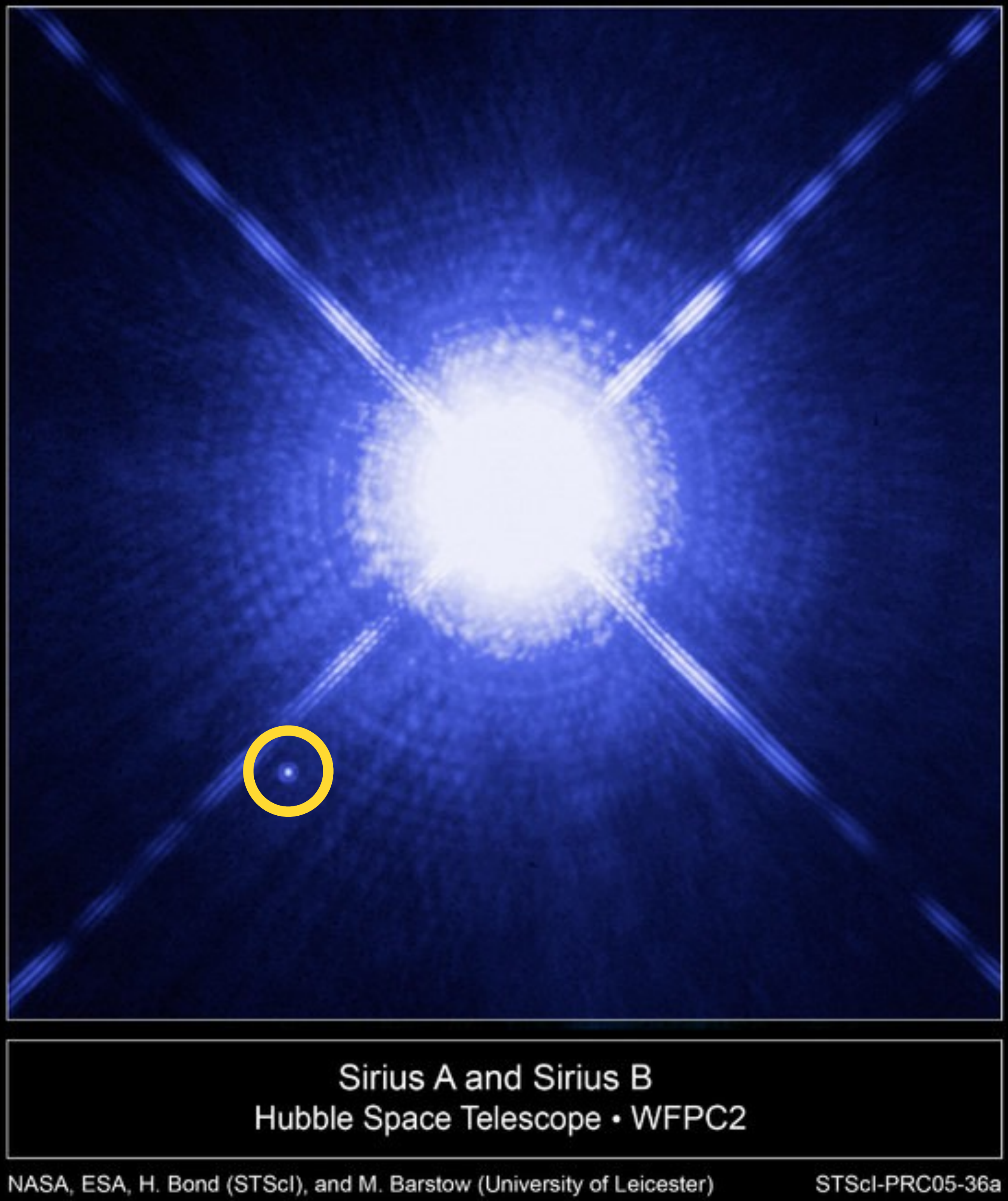




Post-AGB and White Dwarfs

- Post-AGB lasts **~10,000 yrs** before the gas expands too far and disperses into the ISM
- The hot **electron-degenerate Carbon core** gradually reveals itself as the star's outer envelope disperses.
- A **white dwarf** is born.

The Evolution End Point - White Dwarfs



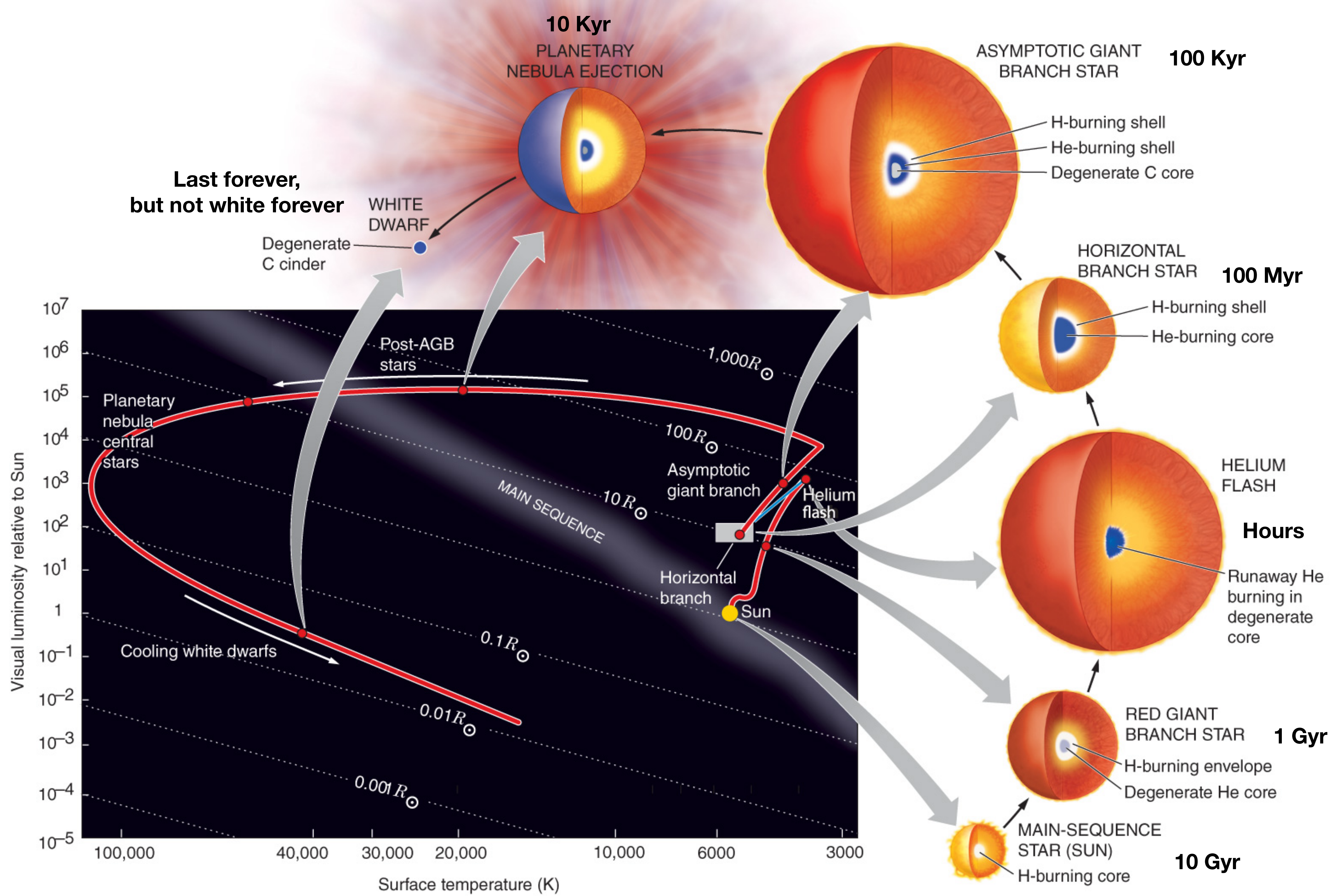
- Inferred properties of Sirius B:
  - 1 Solar Mass
  - 0.03 Solar Luminosity
  - 27,000 K surface temperature
  - 5500 km radius (Earth-size)

- The physical conditions of WDs are extreme:
  - extreme density ($\rho \approx$ 3e9 kg/m^3)
    ($n_e \sim$ 1e36 /m^3)
  - extreme surface gravity
  - extreme pressure at the center:

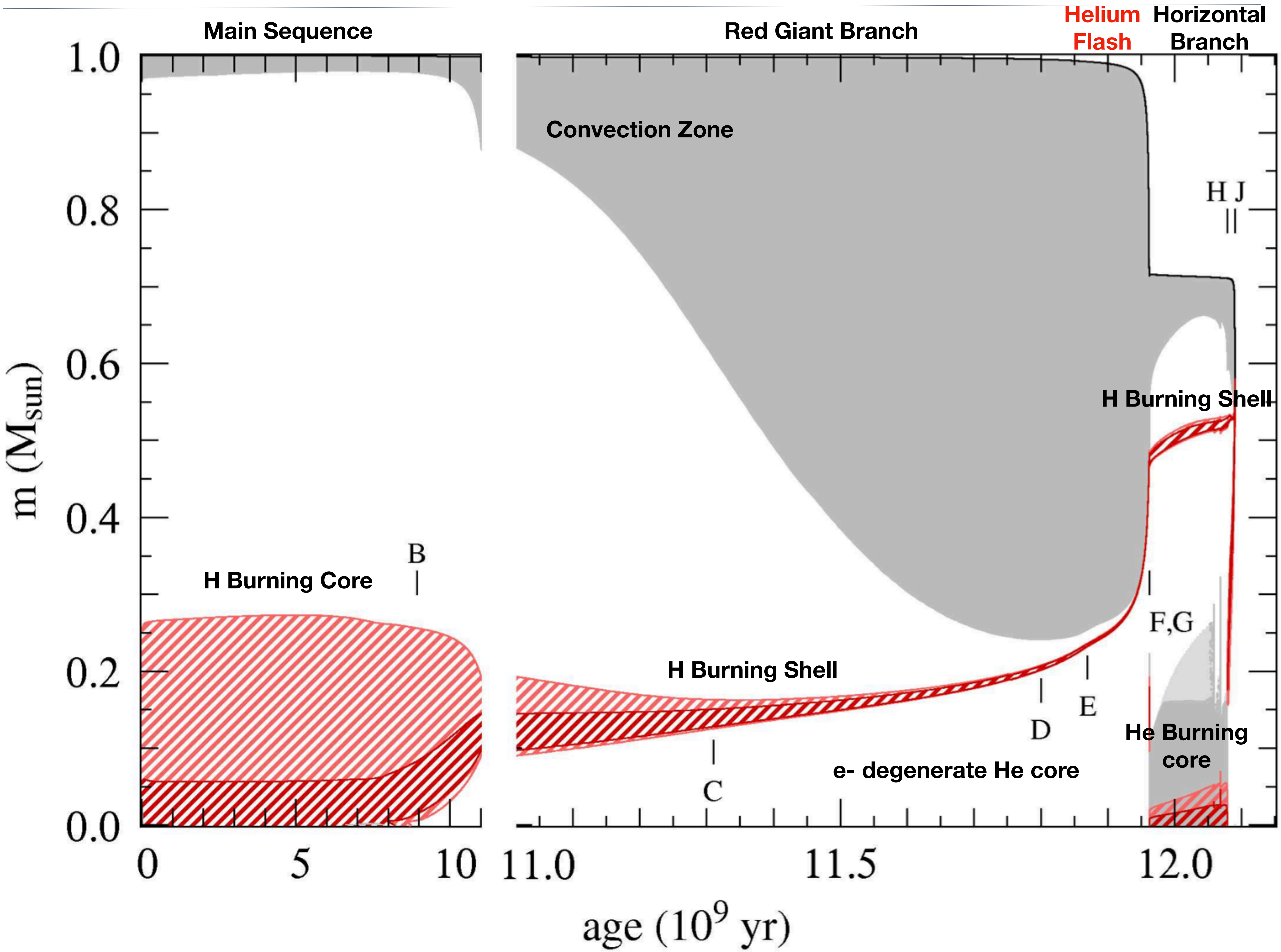
* a **white dwarf** will cool for eternity, without changing its size.
How would you calculate the time it takes for the temperature to half?

A Summary of Low-Mass Stellar Evolution

The evolutionary track of a 1 Solar mass star



Kippenhahn Diagram of a Star with an Initial Mass of 1.0 Solar Mass



Post-**MS** evolution:

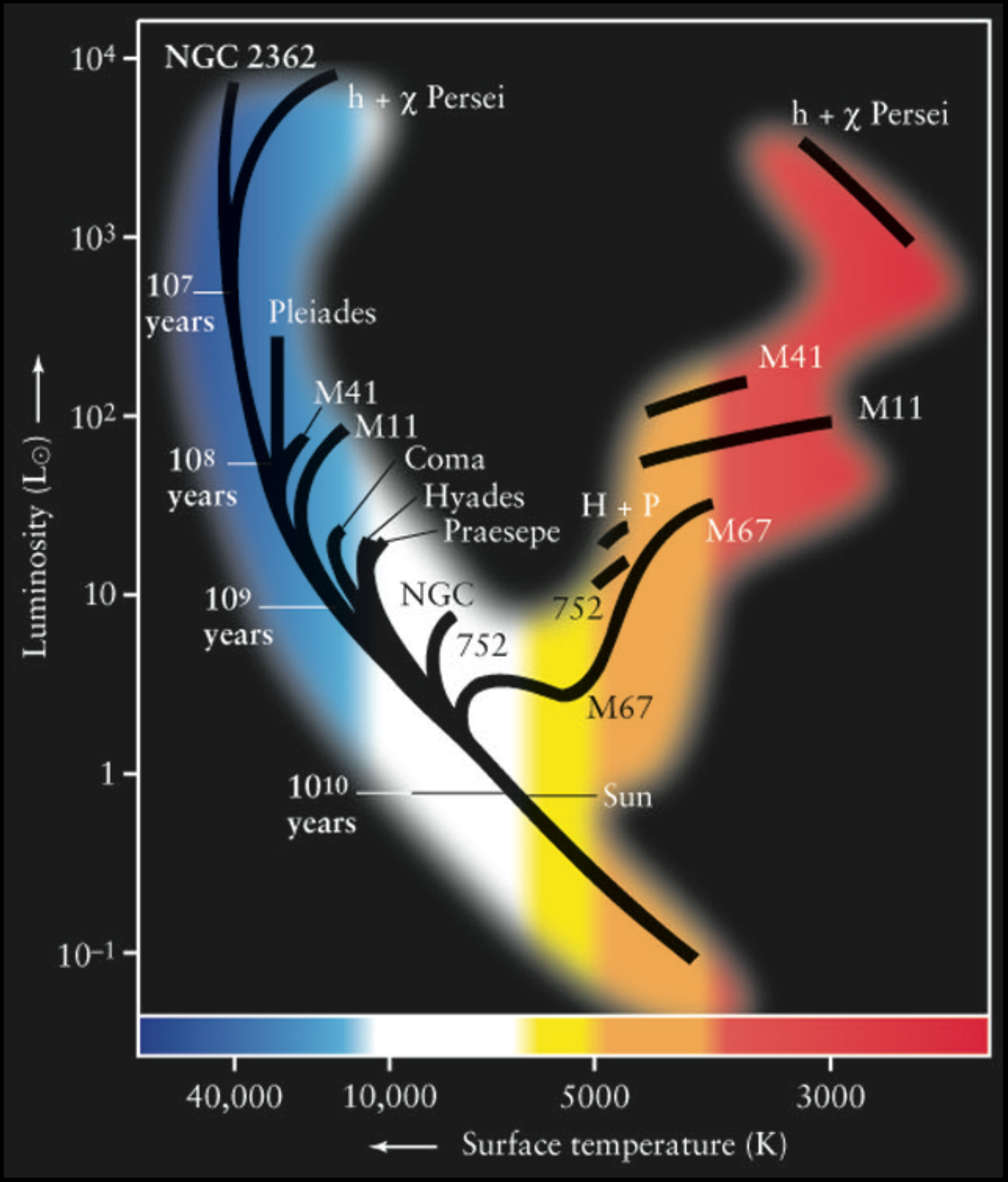
- non-fusing Helium core, H-burning shell
- He-core contracts and become **e- degenerate**
- **Red giant** phase (H^- controls surface T as L increases)
- uncontrolled Helium-burning in the e- degenerate core (thermonuclear runaway, **Helium flash**)
- core expands and become non-degenerate, allowing steady Helium burning (**Horizontal branch**)

Post-**HB** evolution:

- non-fusing C core, He-burning shell, H-burning shell
- C-core contracts and become **e- degenerate**
- **Asymptotic Giant Branch** Phase (H^- controls surface T as L increases, the radius of the star increases even more)
- core temperature never reaches 500 million K needed for Carbon-burning (**no Carbon flash**)
- But eventually the AGB star becomes so bloated that it loses its envelope (**post-AGB** phase) and reveals the **WD**

Chap 3: Key Concepts

- Observations
  - Nothing last forever, even stars
  - **H-R diagram of star clusters**
- Numerical Models
  - Equations of stellar structure and evolution
  - Stellar **evolutionary tracks**
- Fine-Tune Models
  - **Isochrones** (equal-age lines)
  - Fitting cluster H-R diagrams
  - Cluster age estimates
- Model Inferences
  - Main stages and rough lifetimes
  - Changes in the interiors of the stars: **e- degenerate core** + fusion shells



Chap 3: Key Equations

- Hydrostatic Equilibrium:

$$\frac{dP}{dr} = -\rho g(r) = -\rho \frac{GM_r}{r^2}$$

- The pressure from non-relativistic degenerate gas is:

$$P_{\text{degen}} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

- The pressure from ideal gas is:

$$P_{\text{ideal}} = \frac{2}{3} n \left(\frac{3}{2} kT \right) = nkT$$

- The condition for degeneracy is

$$\frac{h^2}{4\pi^2} \frac{n^{2/3}}{m} > kT$$

- Mass-Radius relation:
  - $R \propto M^{-1/3}$ for white dwarfs (Chandrasekhar limit: 1.4 M_{sun})
  - $R \propto M^{0.7}$ for main-sequence stars