Chap 4: The Evolution of High-Mass Stars

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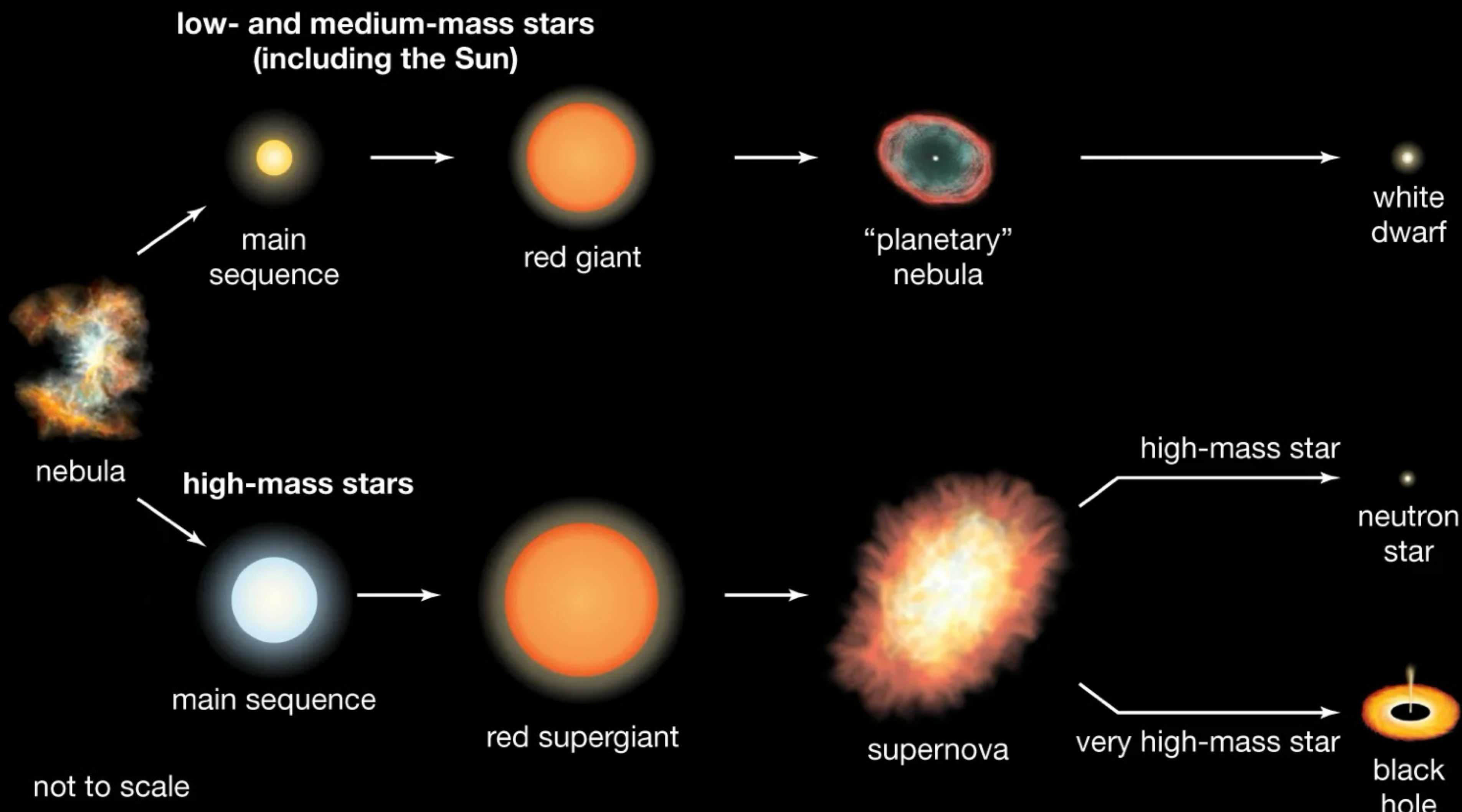
- CNO Cycles
- Convective cores
- Consecutive fusion shells
- End of fusion - Binding Energy
- Core collapse supernovae
- Neutron stars and Pulsars
- Supernova Remnants (SNR): expansion parallax method
- The Origin of Elements: six primary astrophysical sources
- Periodic variables: L-P relations (distance measure)



- Mass-Transfer Binaries
  - Roche Lobe, Lagrange Points
  - Novae, Type Ia SNe, Blue Stragglers

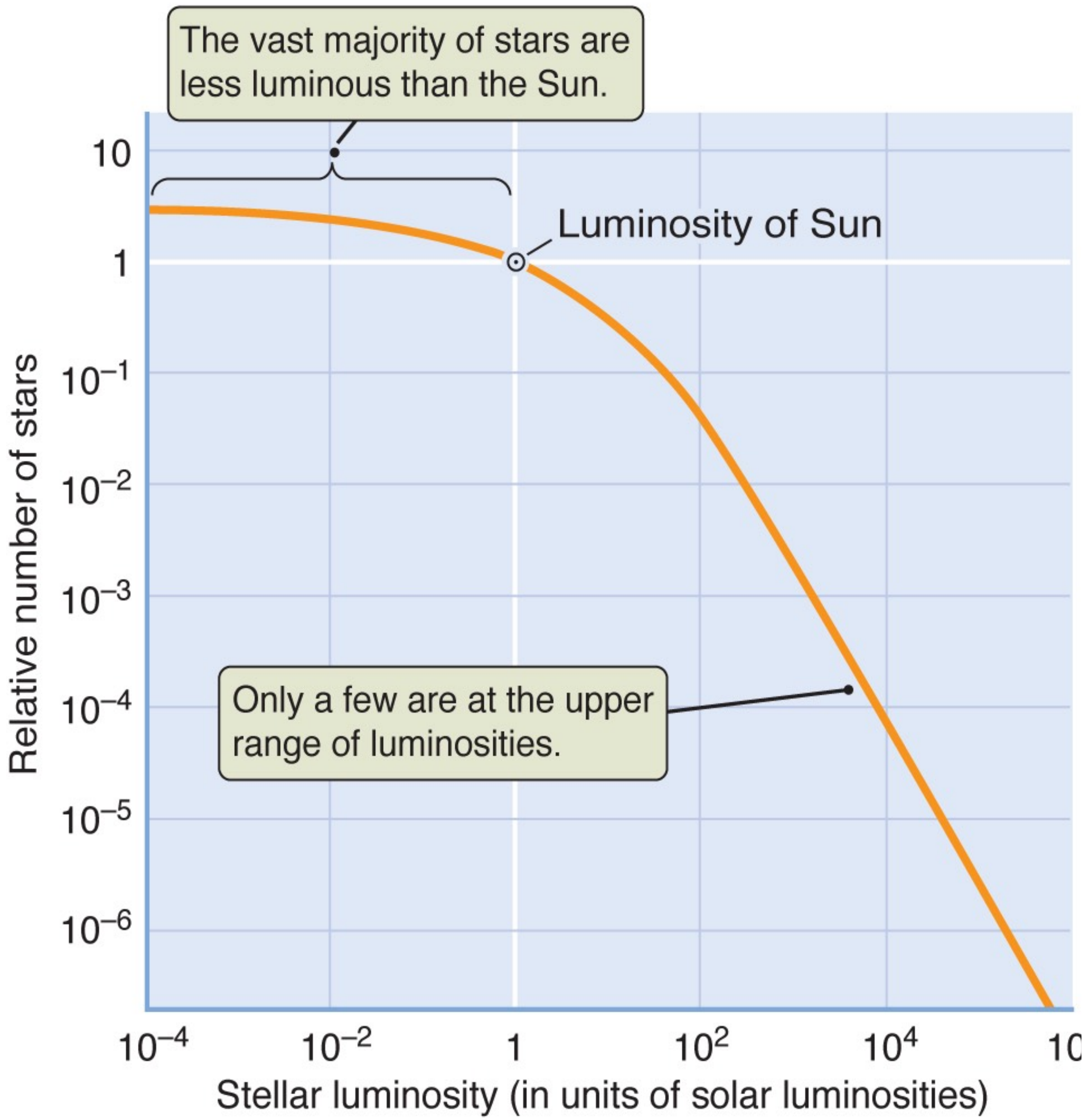
Chap 4 deals with stars of initial masses greater then 3 M_{sun}

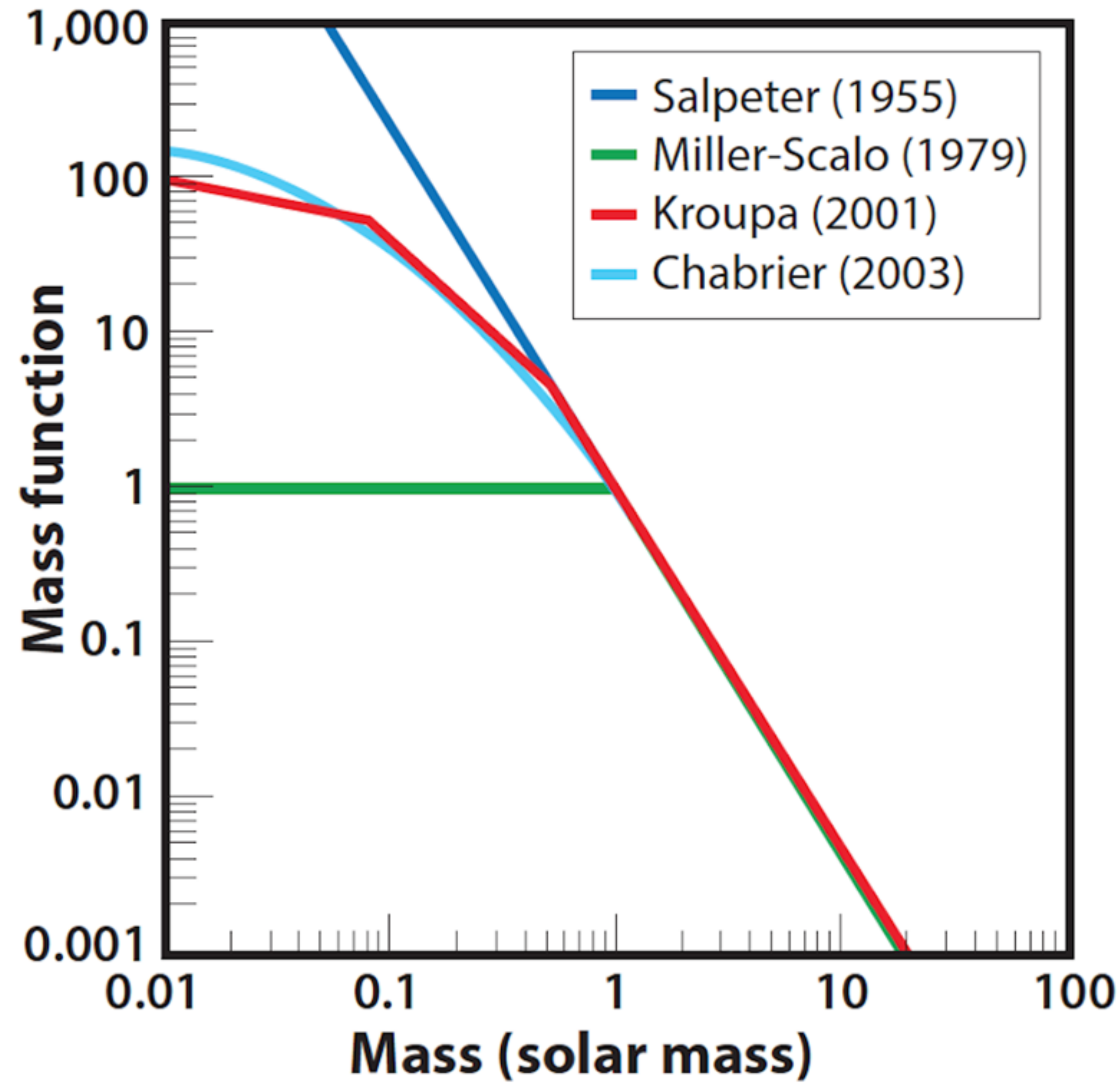
- A star's life depends on mass and composition because the rates and types of fusion depend on the star's mass.
- Stars of different masses evolve differently. There are three categories of stars:
  - **low-mass stars** (Mass < 3 M_{Sun})
  - **intermediate-mass stars** (Mass between 3 M_{Sun} and 8 M_{Sun})
  - **high-mass stars** (Mass > 8 M_{Sun})

Name	High-mass stars	Medium-mass stars	Low-mass stars	Very low-mass stars	Brown dwarfs
Spectral type	O, B	B	A, F, G, K	M	M, L, T, Y
Minimum mass	8 M_{Sun}	3 M_{Sun}	0.5 M_{Sun}	0.08 M_{Sun}	~0.01 M_{Sun} (~13 $M_{Jupiter}$)

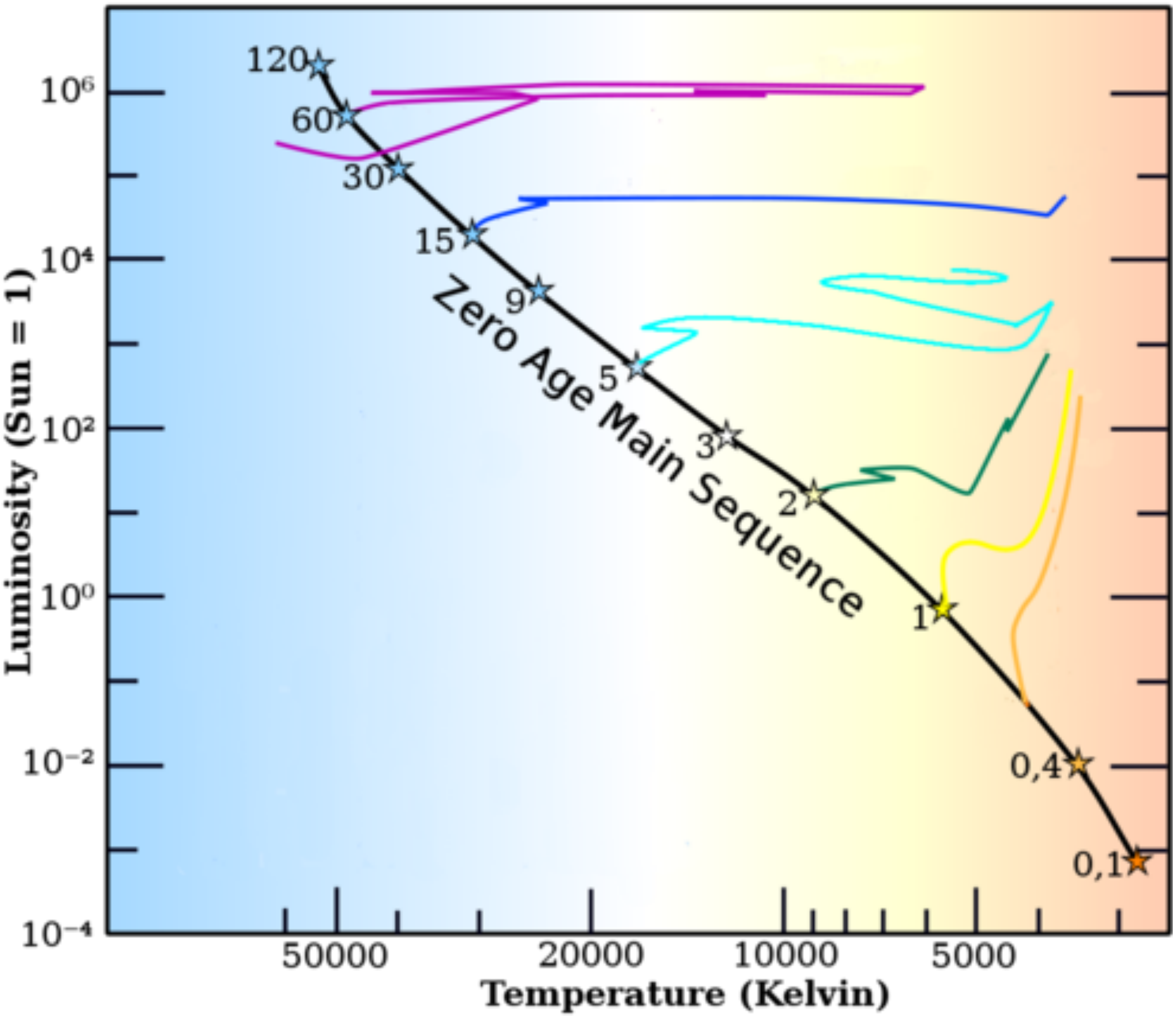
Massive stars are rare, not only because of their short lifespan

- *Initial Mass Function* shows the distribution of stellar masses at birth





The division at 3 M_{sun} marks an rough transition in the shape of the computed evolutionary tracks

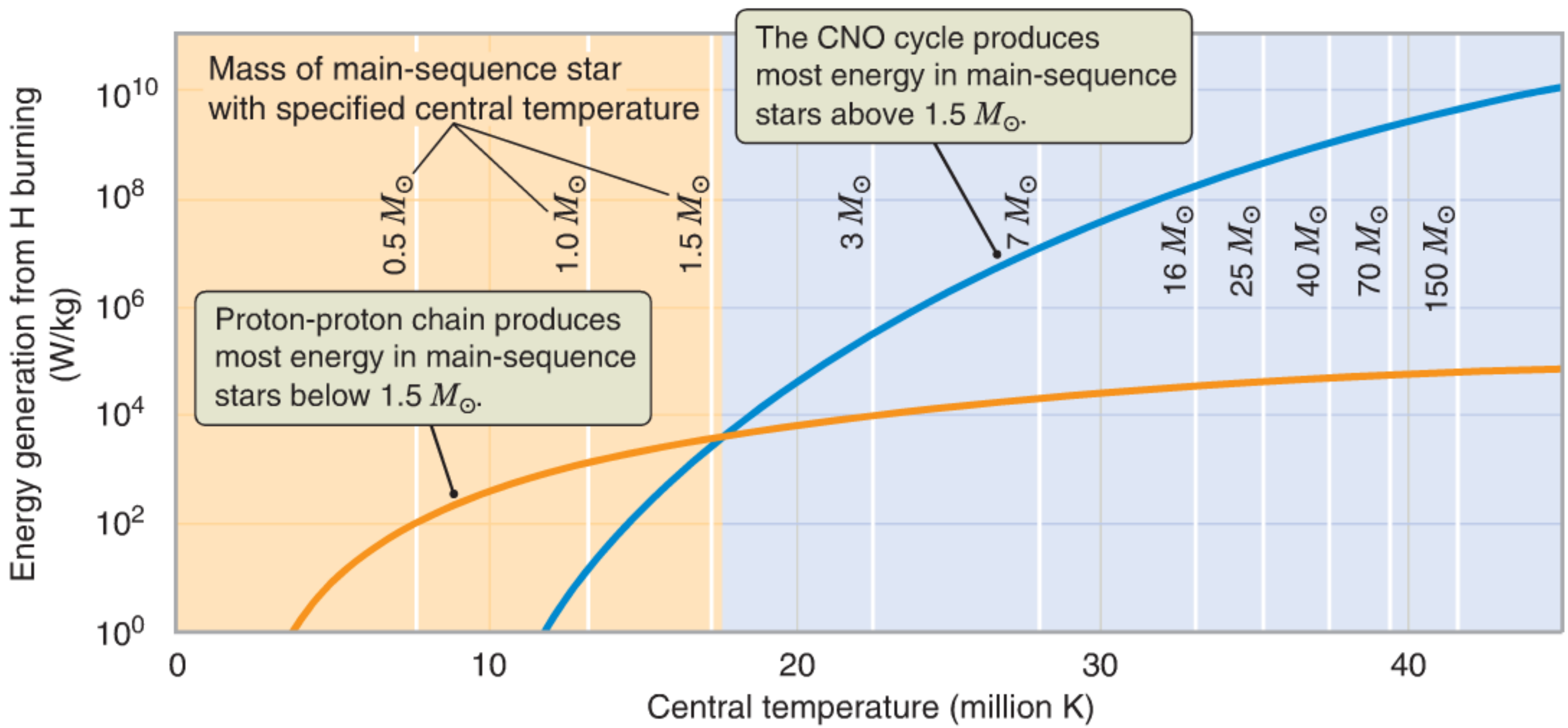


High-mass stars on the main sequence:

CNO cycle and convective core

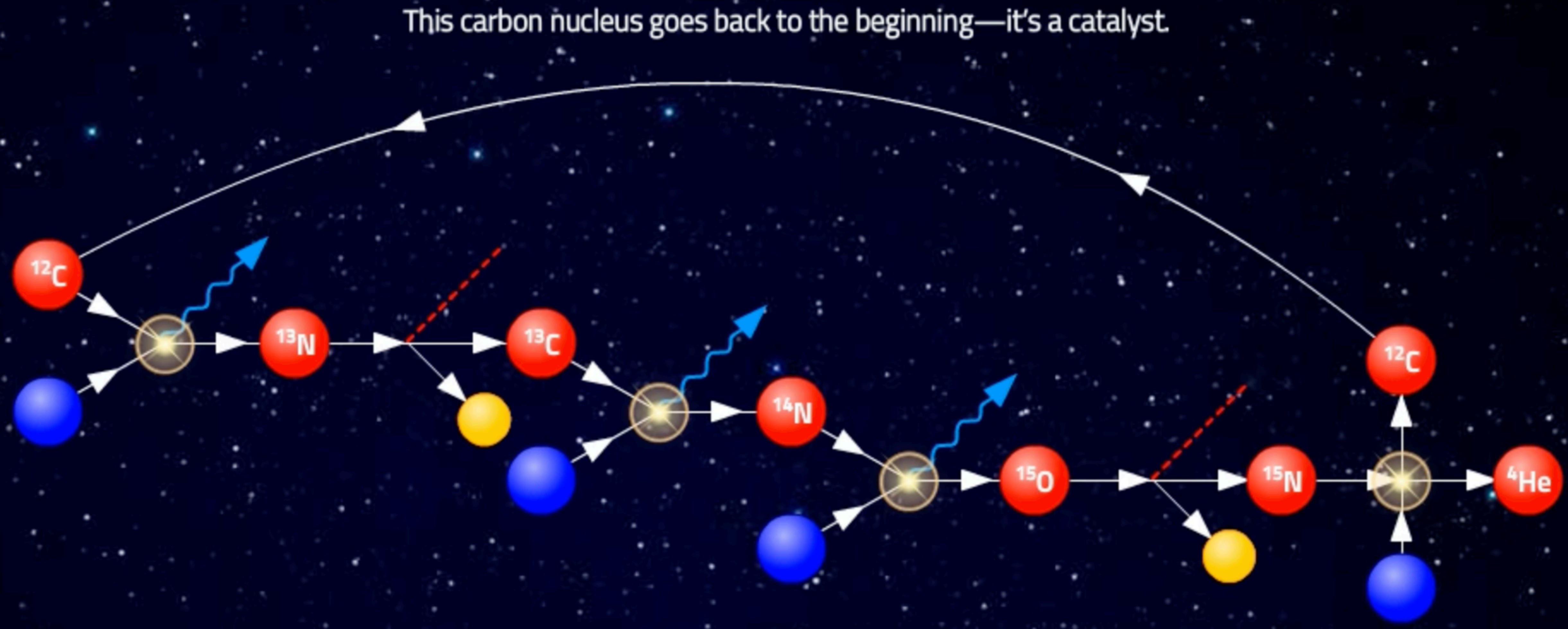
Massive MS stars have higher core temperature but lower core pressure

- Core temperature can be estimated using the **virial theorem**:
$$kT_c \approx GM\mu m_H/R$$
- Core pressure can be estimated from a **force balance**:
$$4\pi R^2 P_c \approx GM^2/R^2 \Rightarrow P_c \approx GM^2/(4\pi R^4)$$
- Main sequence stars show a **mass-radius relation** of:
$$R \propto M^{0.7}$$
- Therefore, $T_c \propto M^{0.3}$ and $P_c \propto M^{-0.8}$



The CNO Cycle: step-by-step

CNO Cycle



Legend: Proton · Nucleus · Positron · Neutrino · Gamma ray

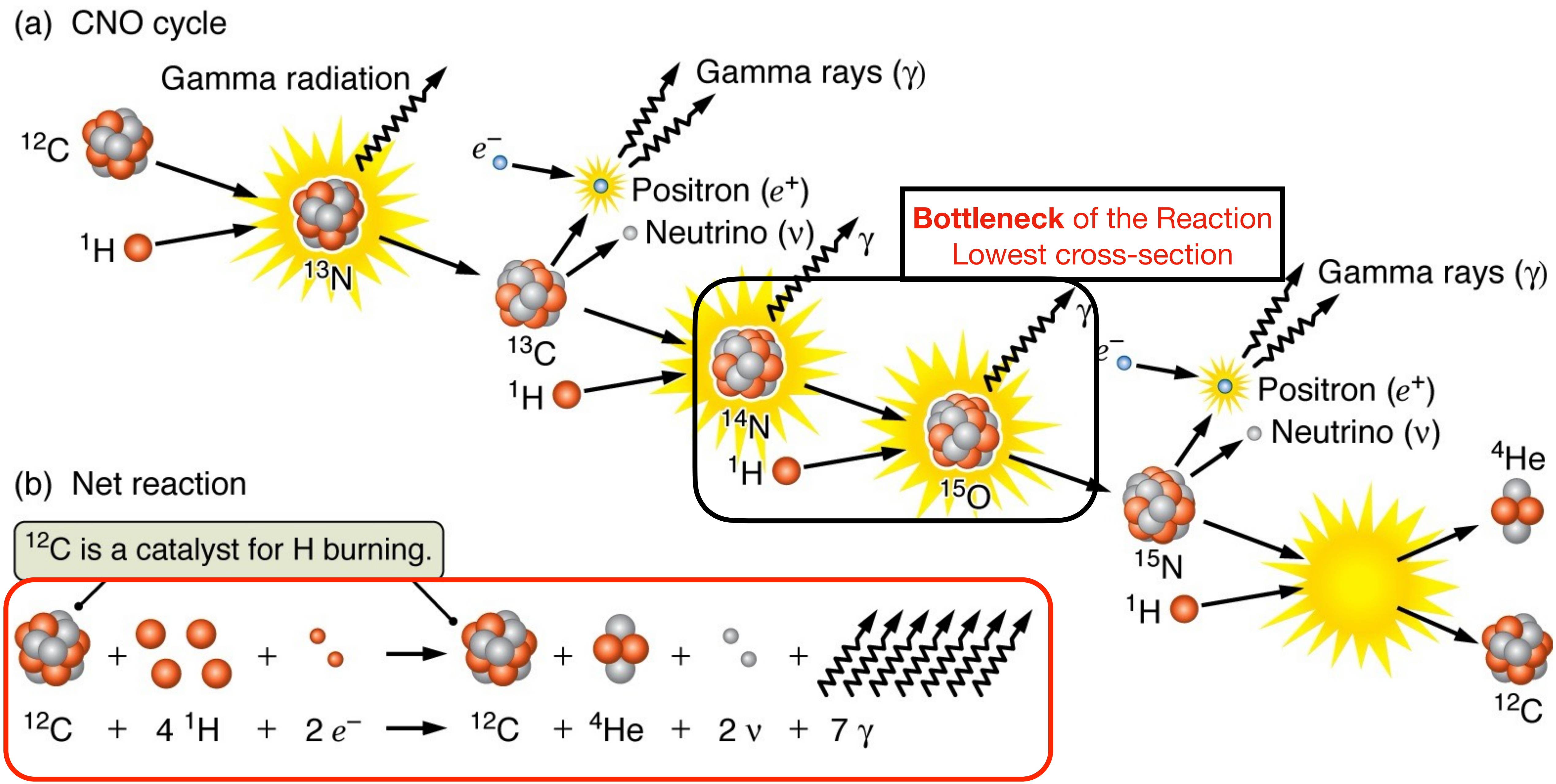
The net result is that four hydrogen nuclei were turned into a helium nucleus, two neutrinos, and seven gamma rays.

Play Reset

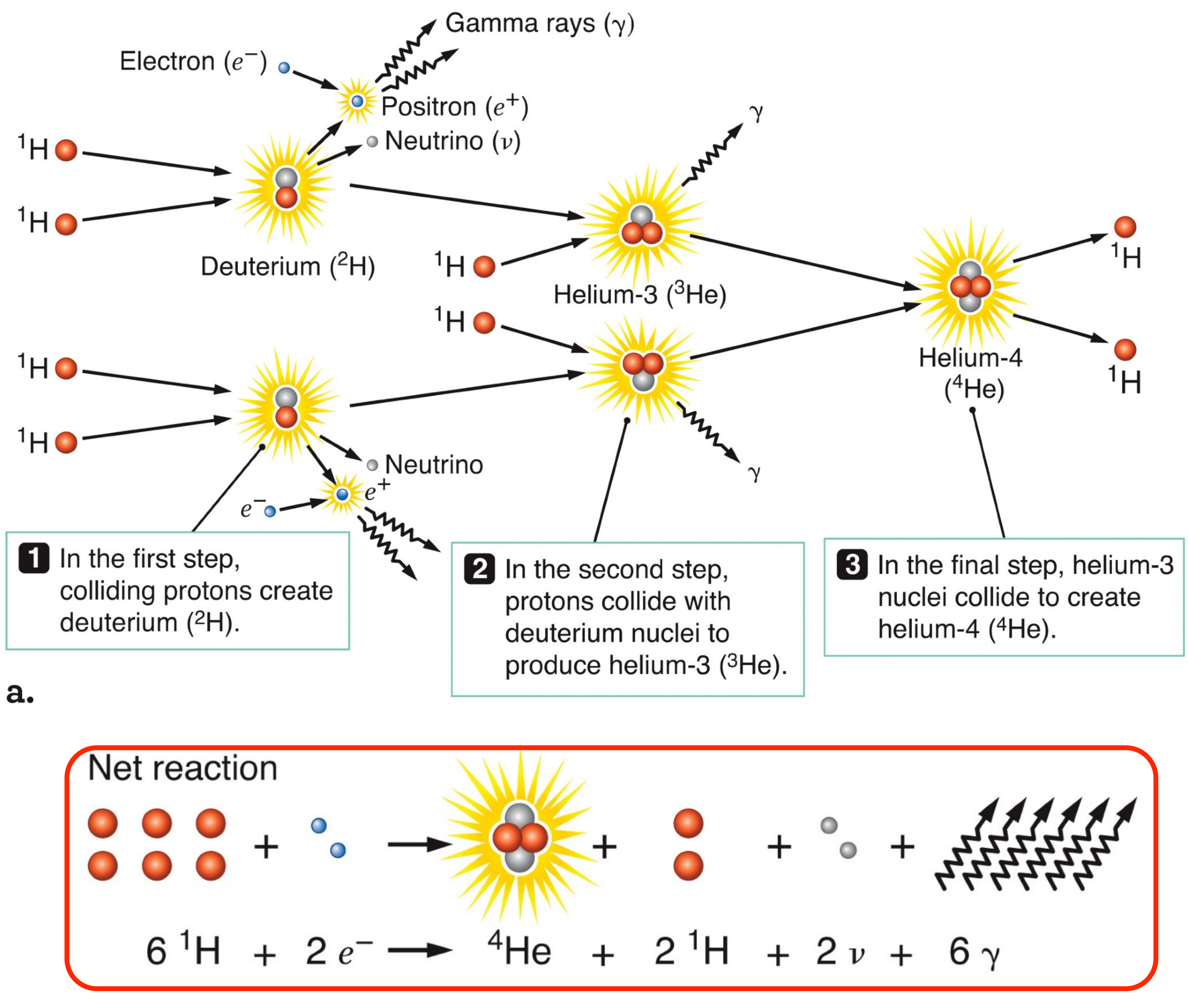
PREVIOUS NEXT

Net reaction of the CNO cycle and the Origin of Nitrogen

- In high-mass stars and the midlife Sun, hydrogen burning proceeds in the CNO cycle instead of the pp chain, due to higher core temperatures.
- Due to the **bottleneck** of the reaction chain ($^{14}N \rightarrow ^{15}O$), **Nitrogen** accumulates to high levels in the core while depleting **Carbon** (this is the origin of N)

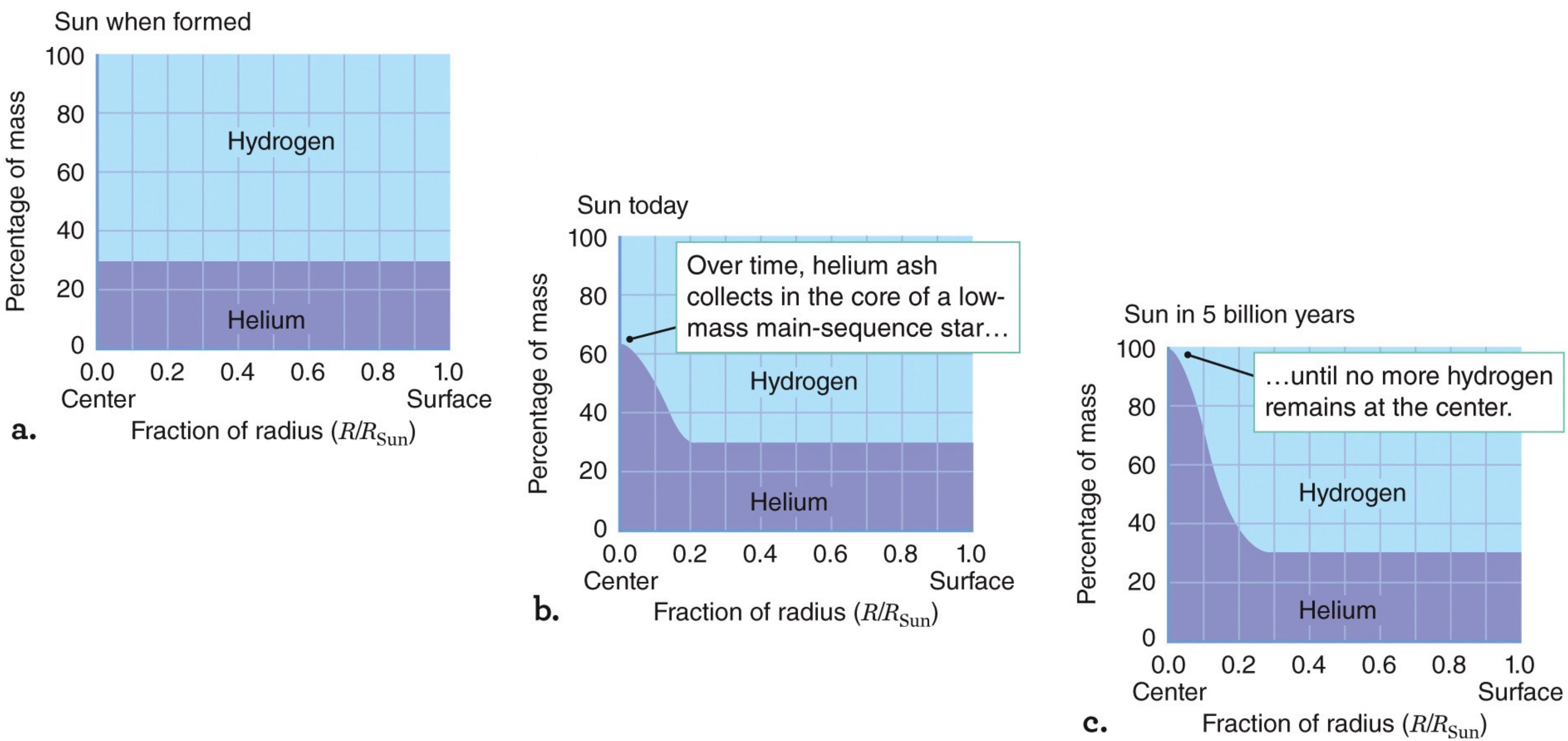


Net reaction of the Proton-Proton chain



Changes on the Main Sequence due to Fuel Exhaustion

- The chemical composition inside a star changes over time as hydrogen is fused into helium.
- The Sun started with 70 percent hydrogen by mass, but now contains only 35 percent hydrogen in the core.
- What will happen when the hydrogen is exhausted in the core?

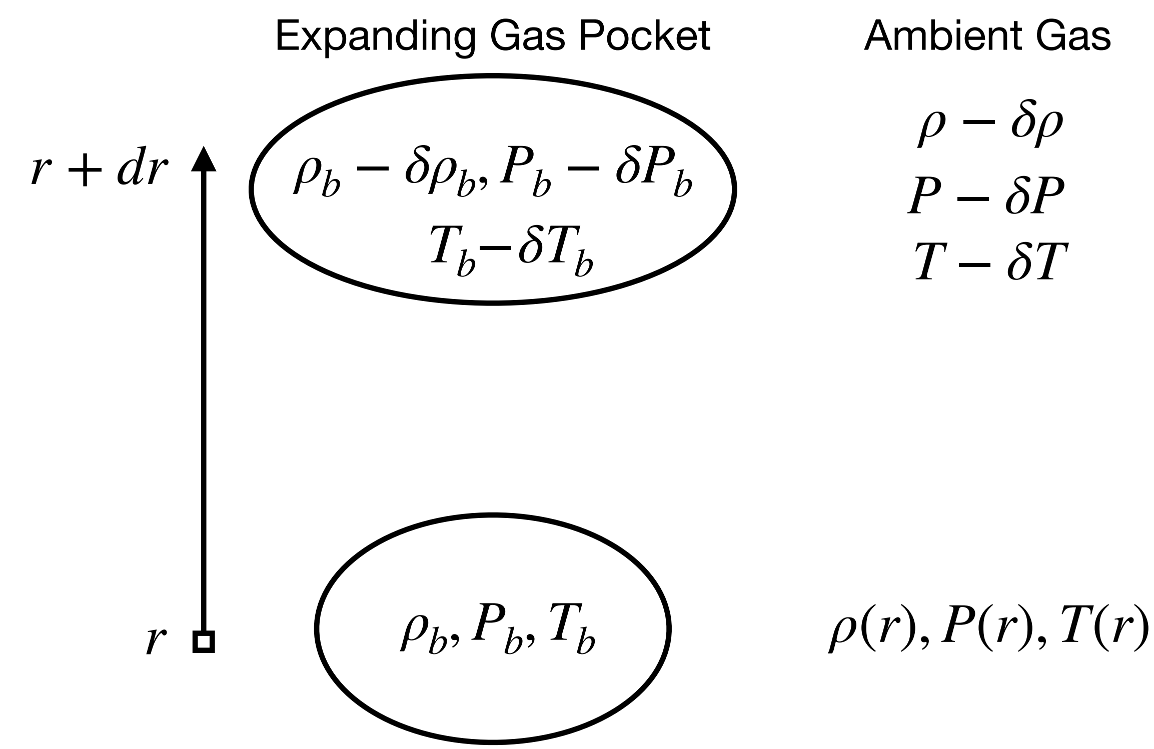


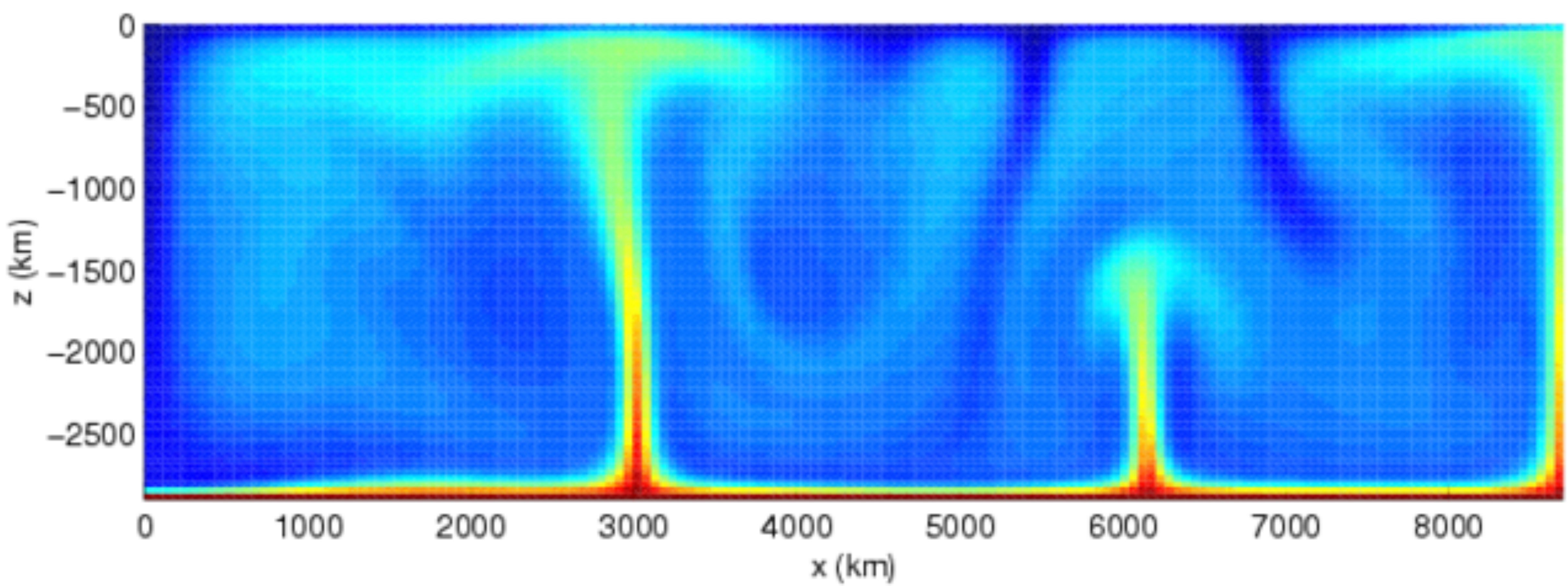
Condition for Convection: Large Temperature Gradients

- When **adiabatic expansion** of a gas pocket causes its temperature to drop ***less*** than that of the ambient gas, **convection** ensues:

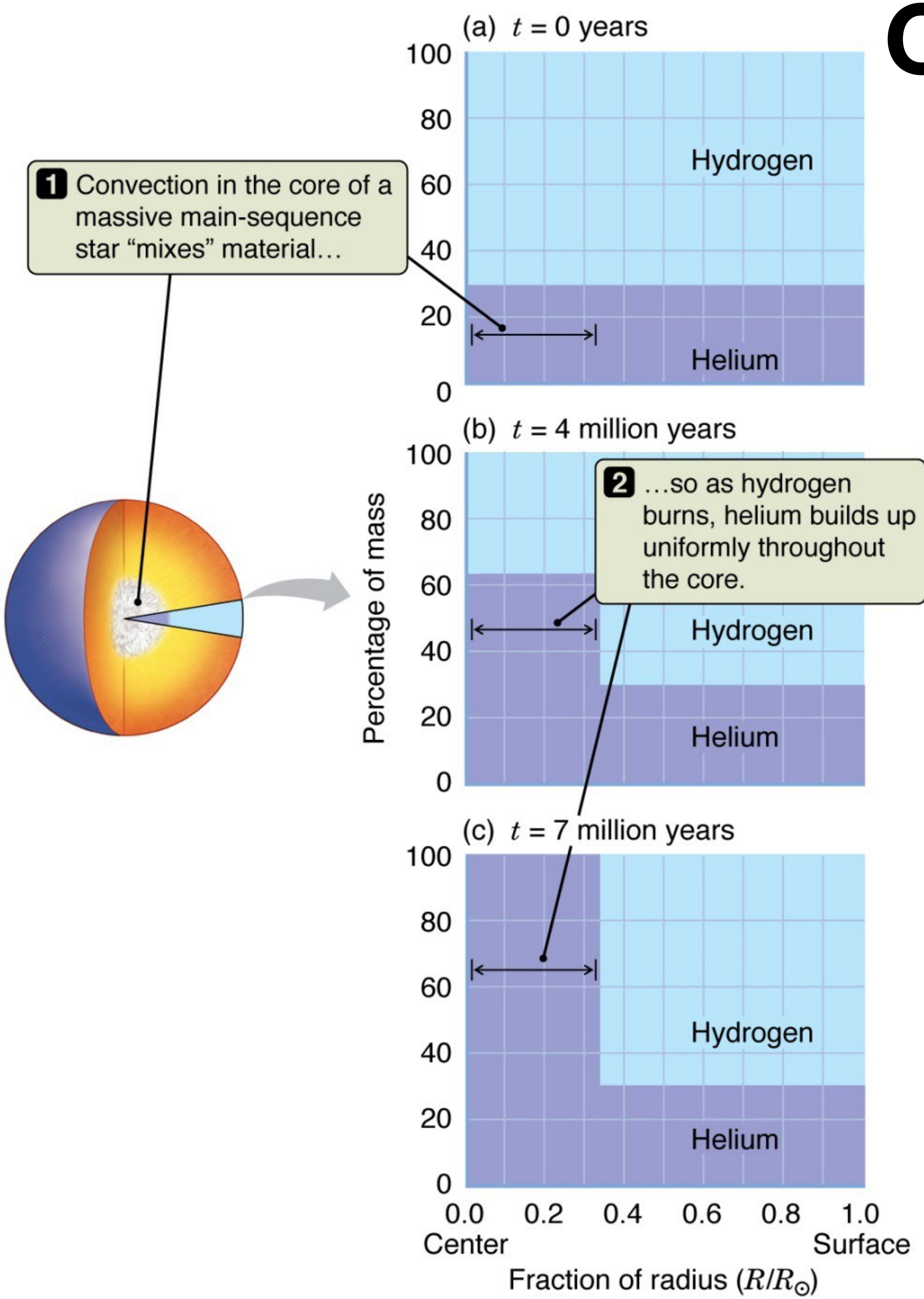
$$-(1-\frac{1}{\gamma})\frac{T}{P}\frac{dP}{dr} < -\frac{dT}{dr}$$

- **Why**? $P = nkT$, warmer gas at the same pressure as colder gas will have lower density. So the pocket will continue to rise due to buoyancy





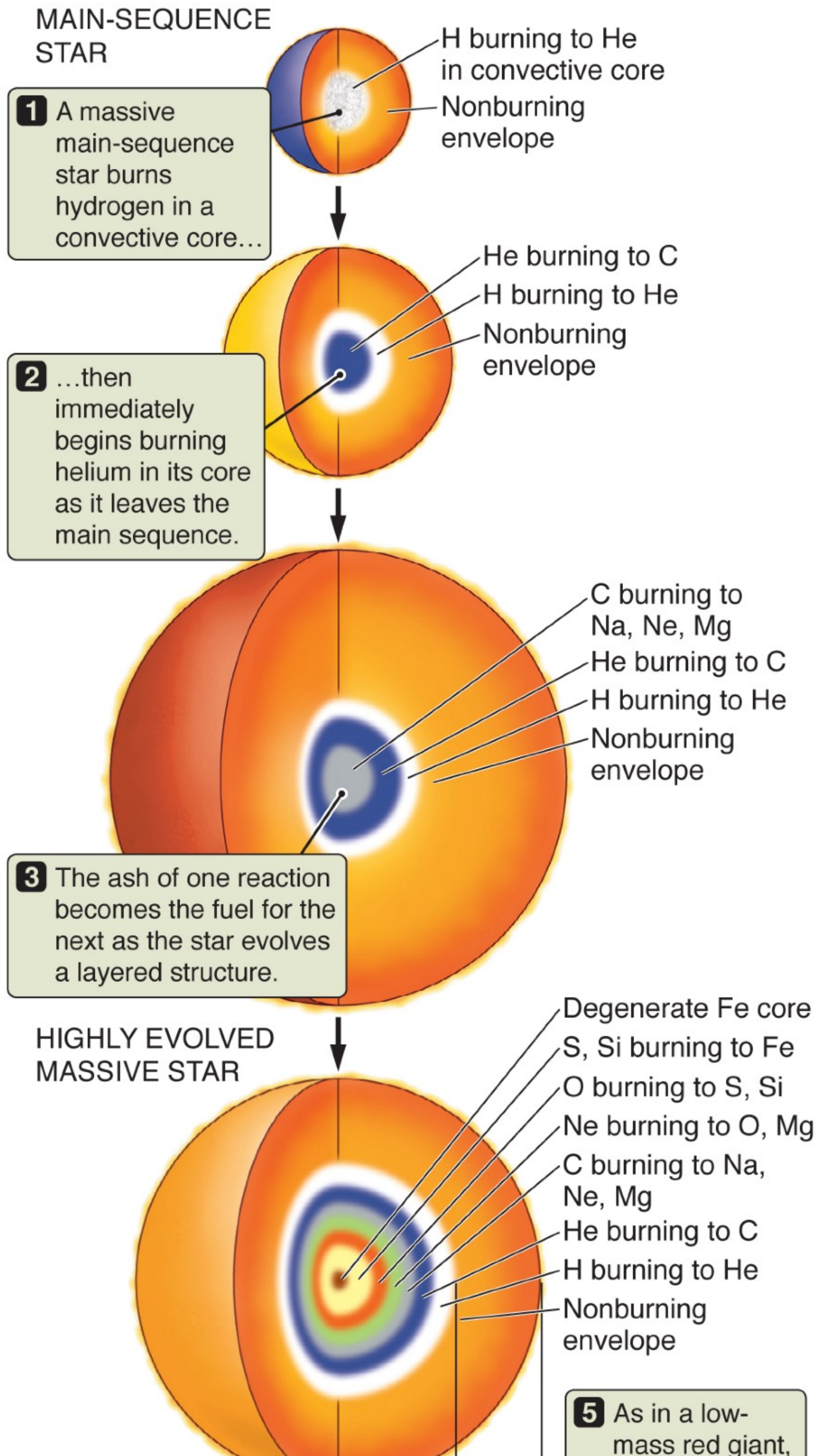
Convective Cores



- High-mass stars have large T gradient, so convection mixes the core effectively.
- Increases the amount available for fusion, but still cannot sustain the high luminosity very long
- High-mass stars live shorter lives (3 Myr for 25 M_{sun}).
- Once H is exhausted from the core, the star leaves the MS and expands and cools

High-mass stars' post-MS evolution:

Onion layers of burning shells,
nearly horizontal evolution on the HRD,
and the end of fusion (Iron)



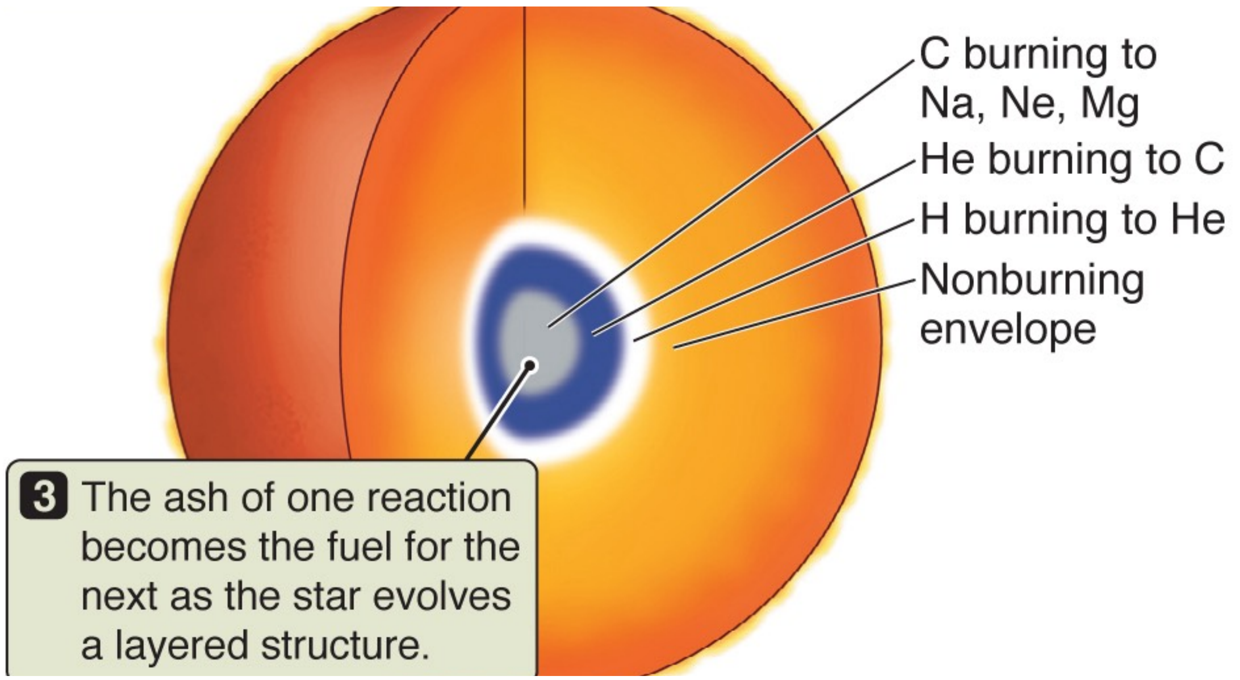
Burning Core and Burning Shells

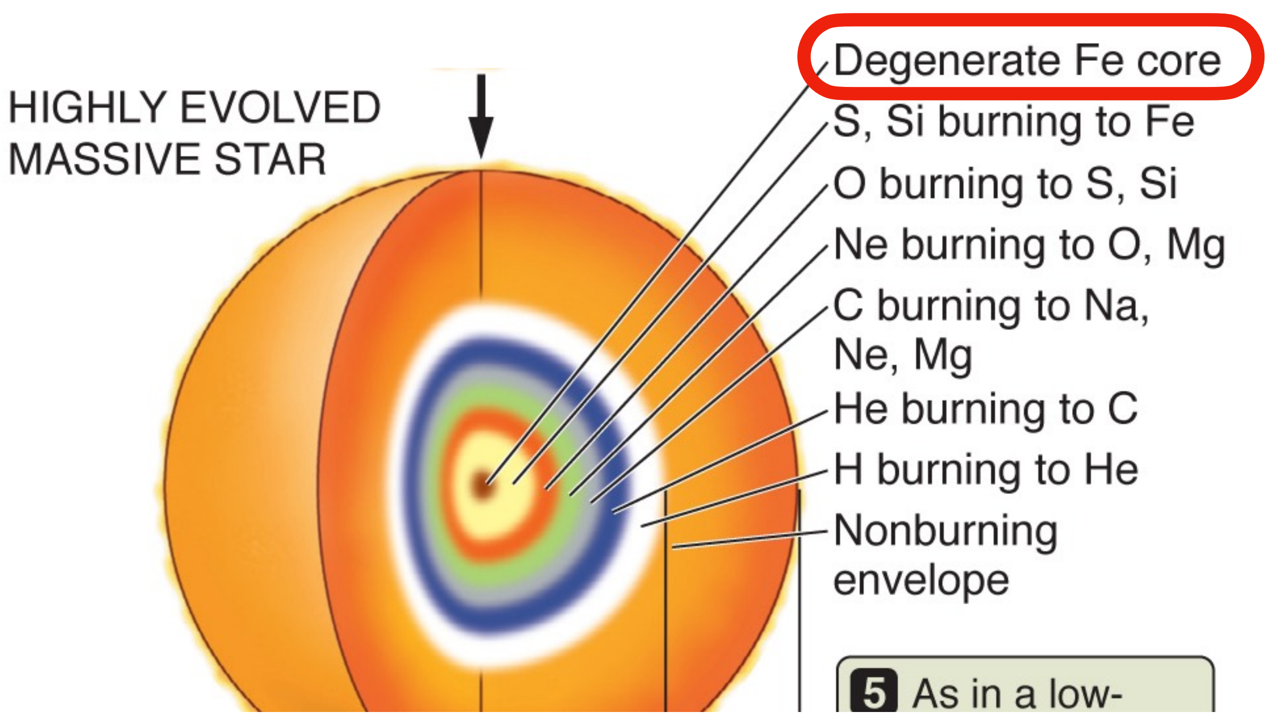
- The compression of the **core** ignite **He-burning before it becomes degenerate**, unlike low-mass stars.
- The fusion shells build up like the **layers of an onion**.
- The more massive the star, the heavier the elements that can fuse.
- Cores of high-mass stars will **fuse elements up until iron (Fe)**.

Burning Stages in High-Mass Stars

Harder** to fuse **heavier** elements, with **diminishing returns

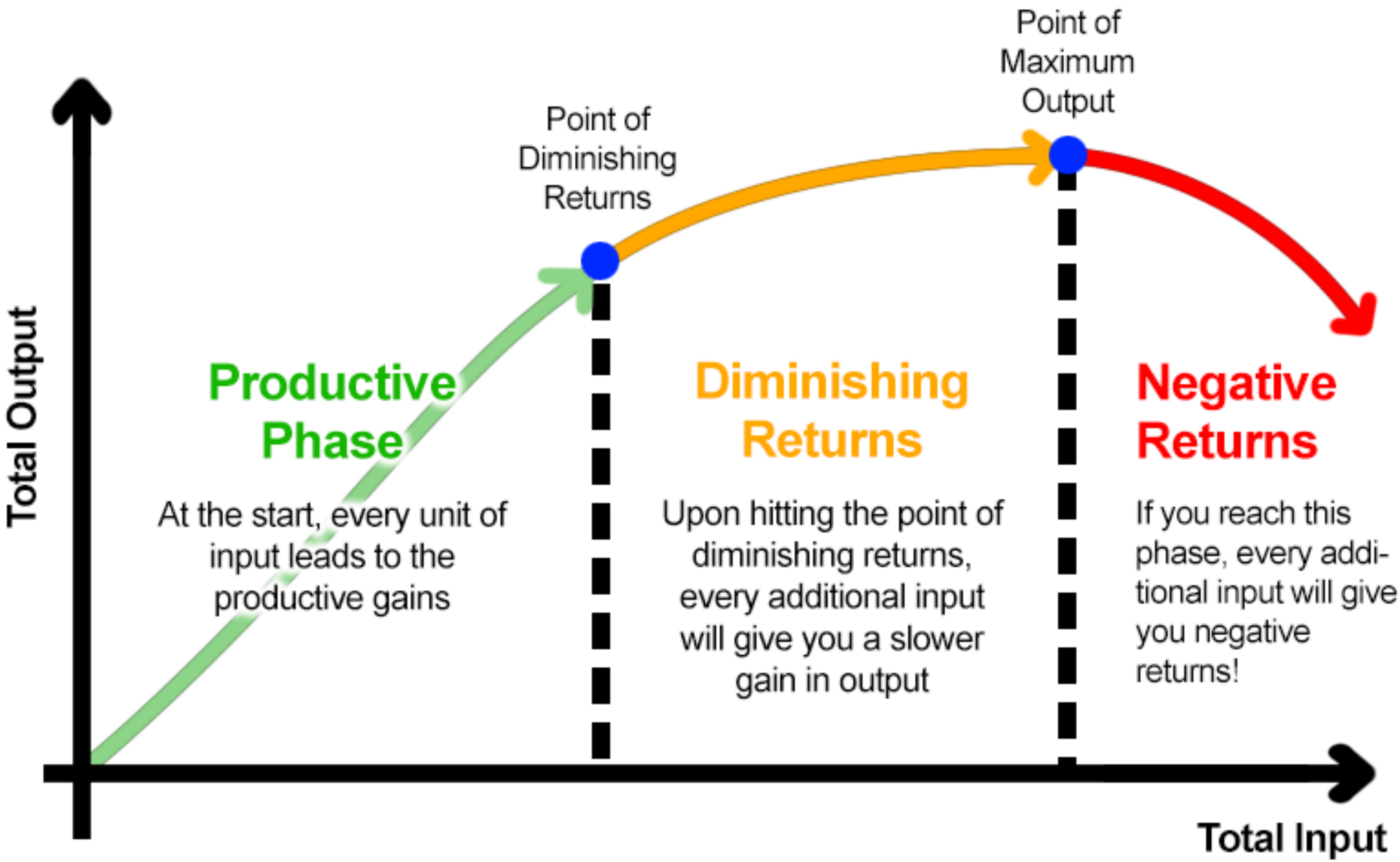
Core Burning Stage	9-$M_\odot$ Star	25-$M_\odot$ Star	Typical Core Temperatures
Hydrogen (H) burning	20 million years	7 million years	$(3–10) \times 10^7$ K
Helium (He) burning	2 million years	700,000 years	$(1–7.5) \times 10^8$ K
Carbon (C) burning	380 years	160 years	$(0.8–1.4) \times 10^9$ K
Neon (Ne) burning	1.1 years	1 year	$(1.4–1.7) \times 10^9$ K
Oxygen (O) burning	8 months	6 months	$(1.8–2.8) \times 10^9$ K
Silicon (Si) burning	4 days	1 day	$(2.8–4) \times 10^9$ K





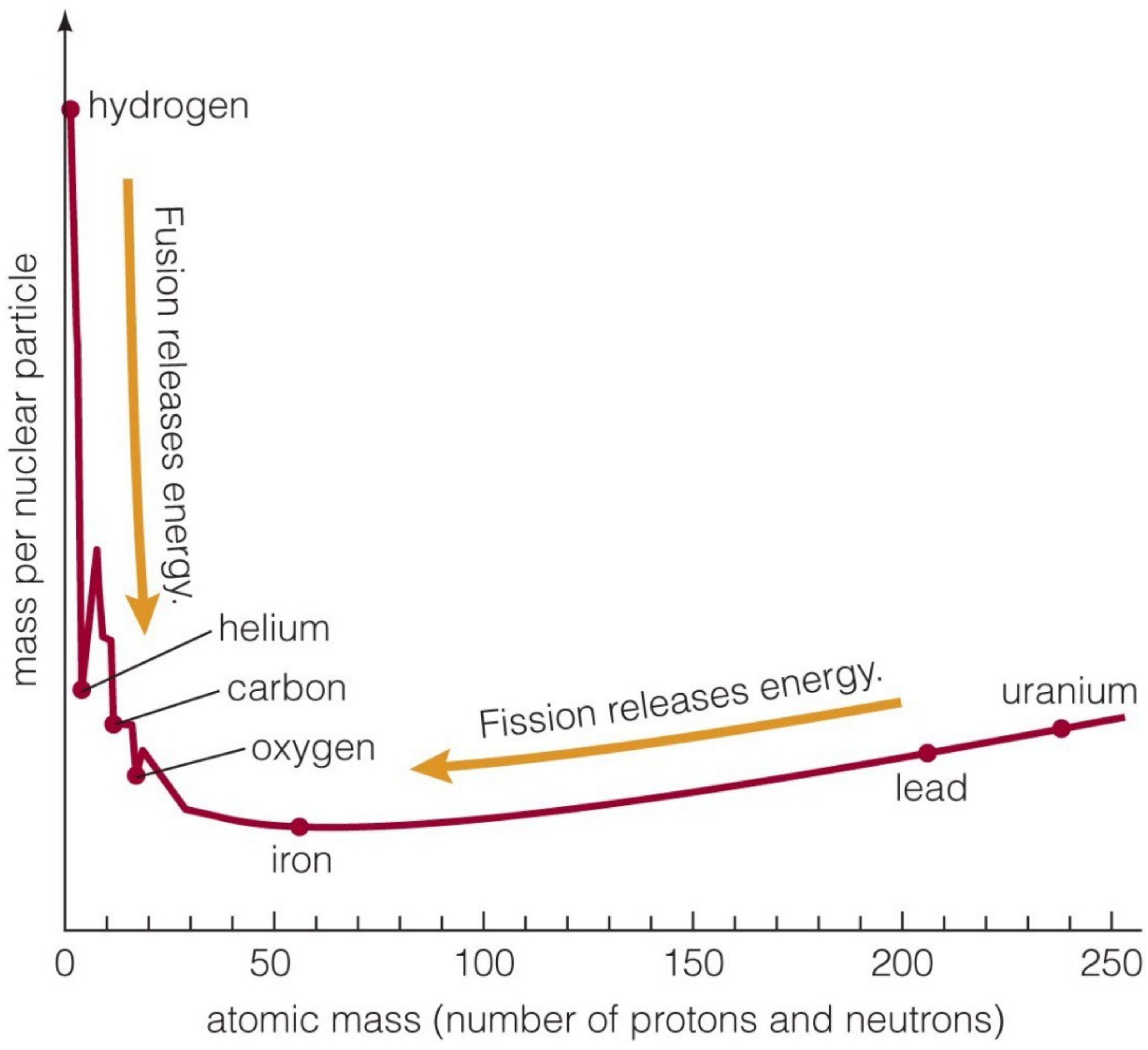
The Law of Diminishing Returns

In Economics, the law of diminishing returns states that in productive processes, increasing a factor of production by one, while holding all others constant, **will at some point return lower output per incremental input unit**



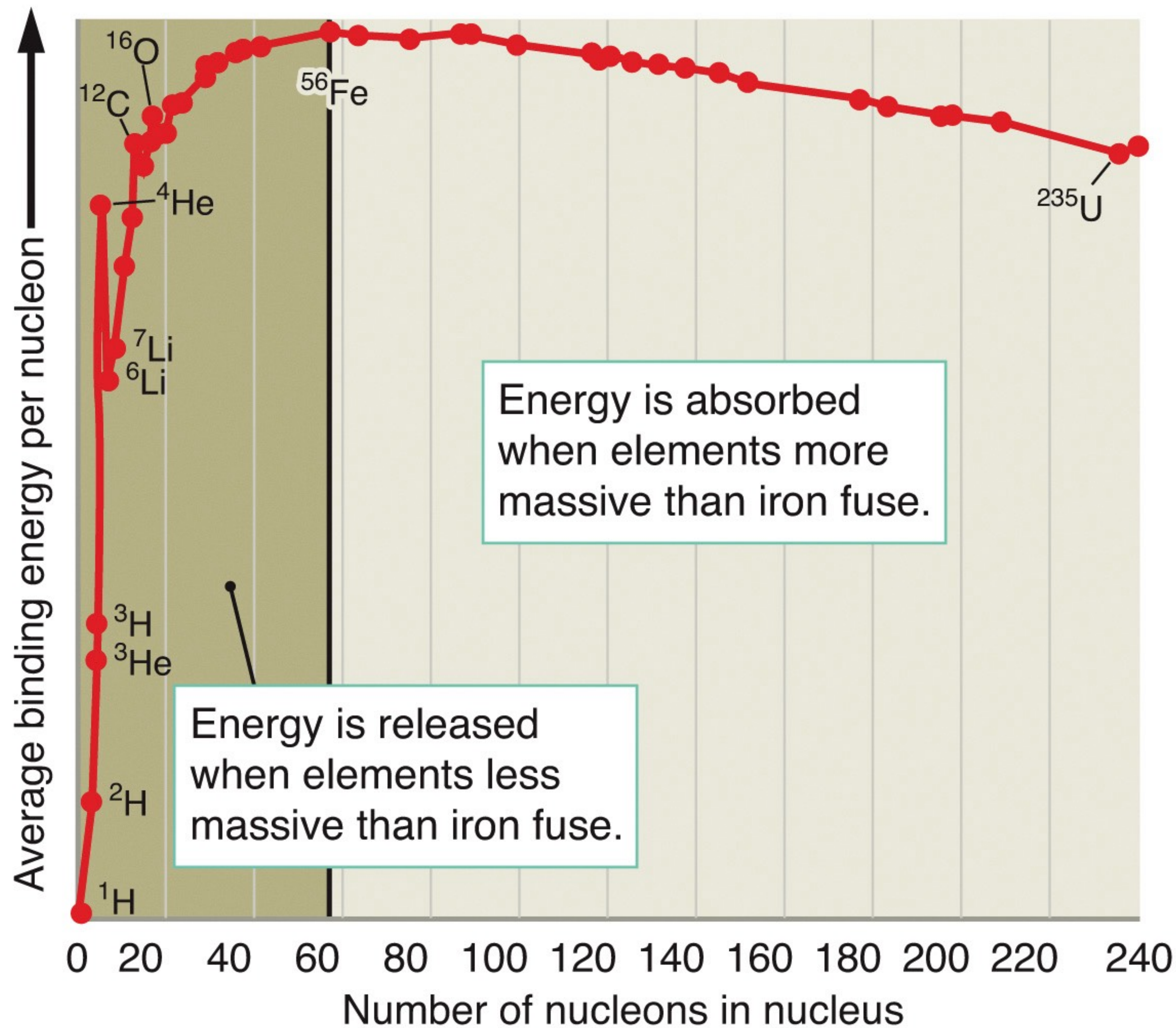
Fusion Energy: the mass per nucleon decreases from H to Fe

- Nuclear fusion produces energy from the mass loss occurred when fusing lighter elements into heavier elements ($E = \Delta m \cdot c^2$). This works from Hydrogen (H), to Helium (He-4), up to **Iron (Fe-56)**, which has the *lowest* mass per nucleon.



An alternative way to understand fusion energy: nuclear binding energy

- **Binding Energy** is the minimum energy required to **disassemble** the nucleus of an atom into its constituent nucleons.
- **Fusion energy** is produced from **the difference in binding energy** between products and reactants.
- **Fe-56** has the highest binding energy per nucleon, so it marks **the end of both fusion and fission.**



Example: Binding Energy of Atomic Nuclei

- The net energy released by a nuclear reaction is the difference between the **binding energy** of the products and the binding energy of the reactants.
- For the triple-alpha process:

$$\begin{pmatrix}\text{Net energy from}\\ \text{fusing 1 kg of He}\end{pmatrix} = \begin{pmatrix}\text{Binding energy}\\ \text{of C formed}\end{pmatrix} - \begin{pmatrix}\text{Binding energy}\\ \text{of He fused}\end{pmatrix}$$

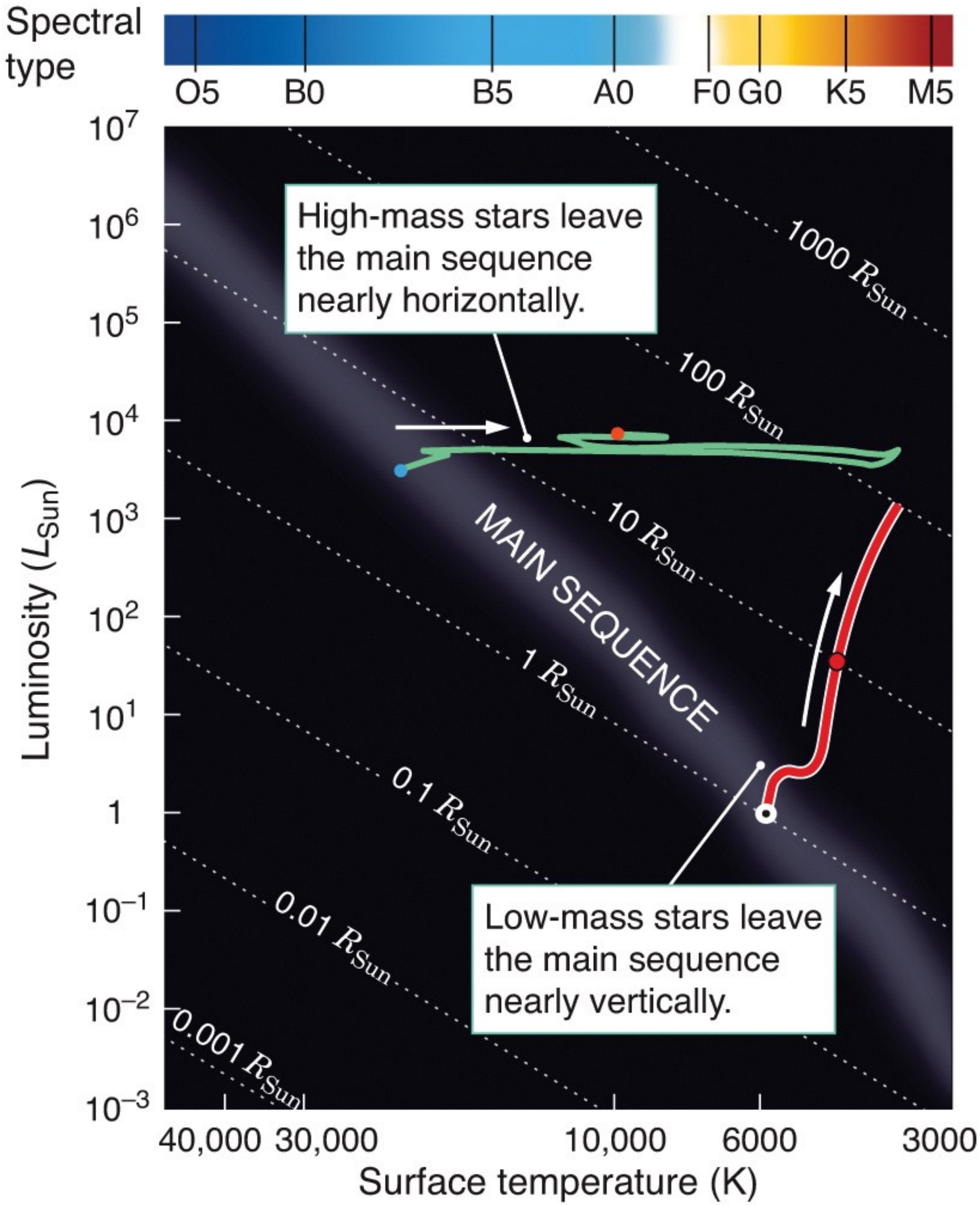
$$= (7.402 \times 10^{14}\ \text{J}) - (6.824 \times 10^{14}\ \text{J})$$

$$= 5.780 \times 10^{13}\ \text{J}$$

- For the fusion of **iron**, the binding energy of the products is less than that of the reactants, so the net energy is **negative**.

Evolution Tracks on H-R Diagram

Once leaving the MS, the star starts to expand to a supergiant. While keeping almost constant luminosity, its temperature varies by ~10x



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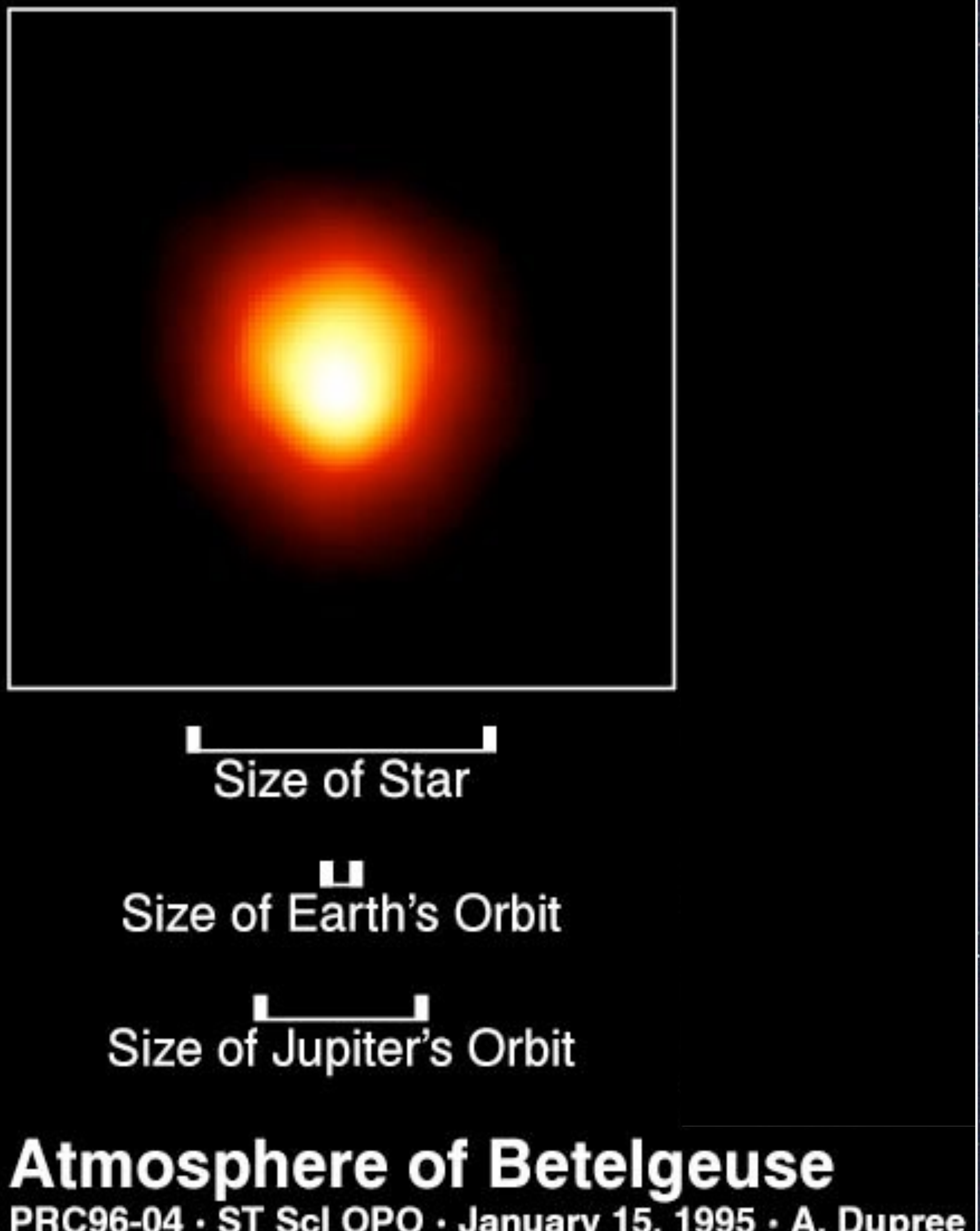
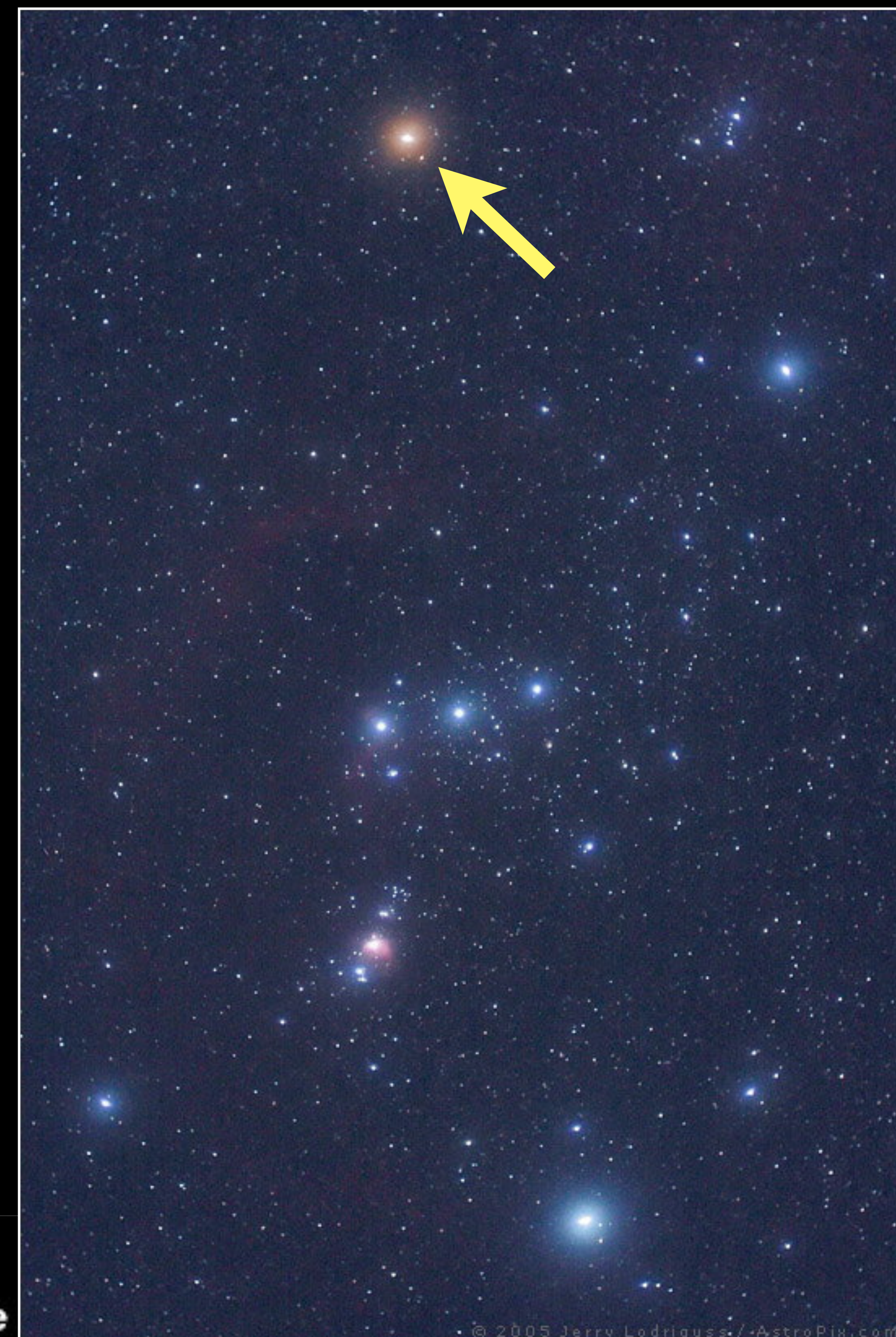
Betelgeuse

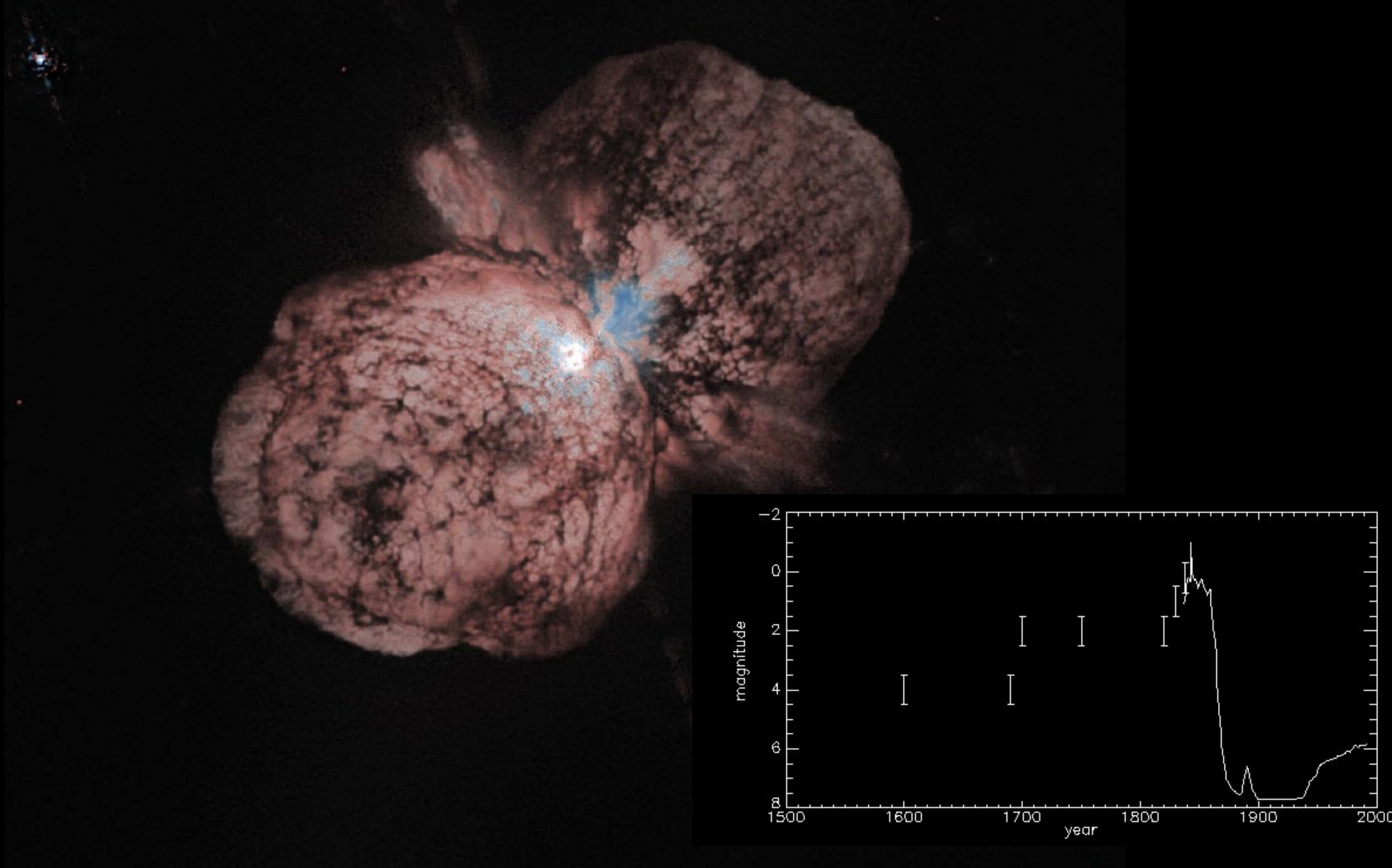
The familiar red star in Orion constellation. Betelgeuse is a red supergiant.



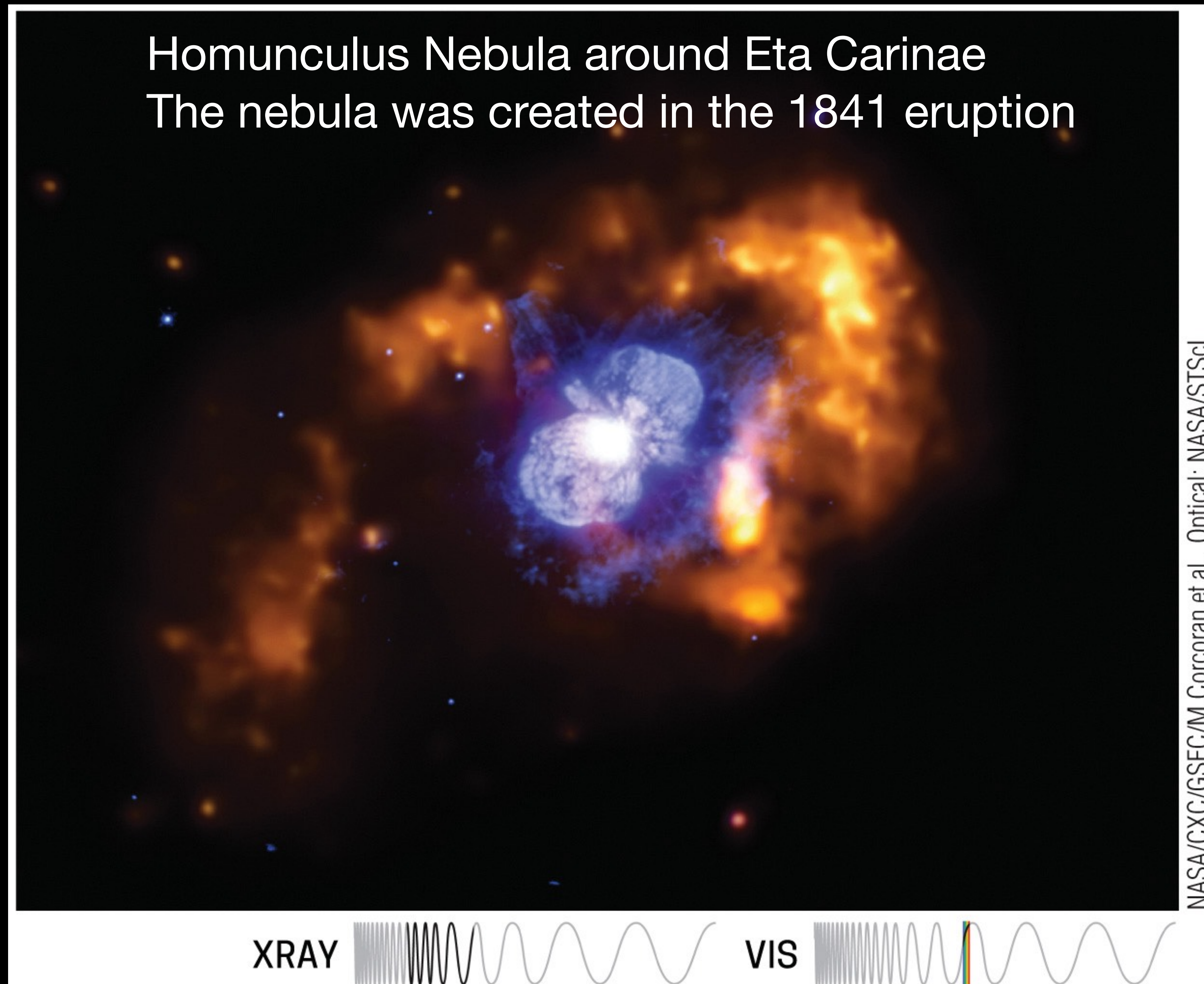


distance = 640 light years, $M = 7.7\ M_{sun}$, $R = 1{,}200\ R_{sun}$

- Eta Carinae (dis ~ 2.4 kpc): 150 M_{sun} primary + 80 M_{sun} secondary
- The primary may have already lost 30 M_{sun} by now.
- Homunculus Nebula: ~22,000 AU in radius (bipolar lobes), likely formed in the 1841 outburst.



Severe Mass Loss of High Mass Stars



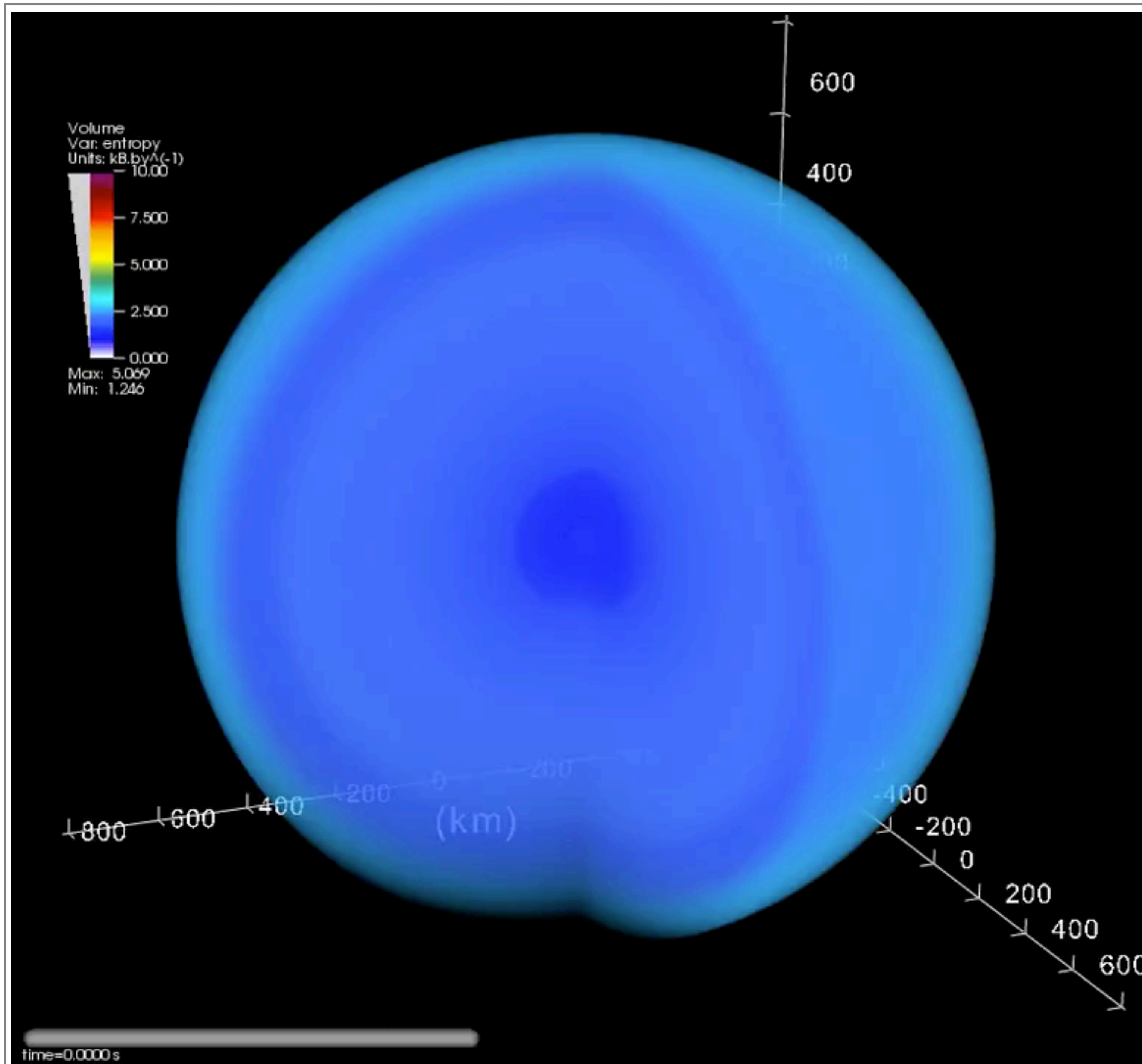
- Stars with 20 M_{sun} loses its mass quickly: 10^{-7} to 10^{-5} M_{sun}/yr because of low gravity and radiation pressure, and occasional eruptions (the Sun loses 10^{-14} to 10^{-13} M_{sun}/yr to wind)

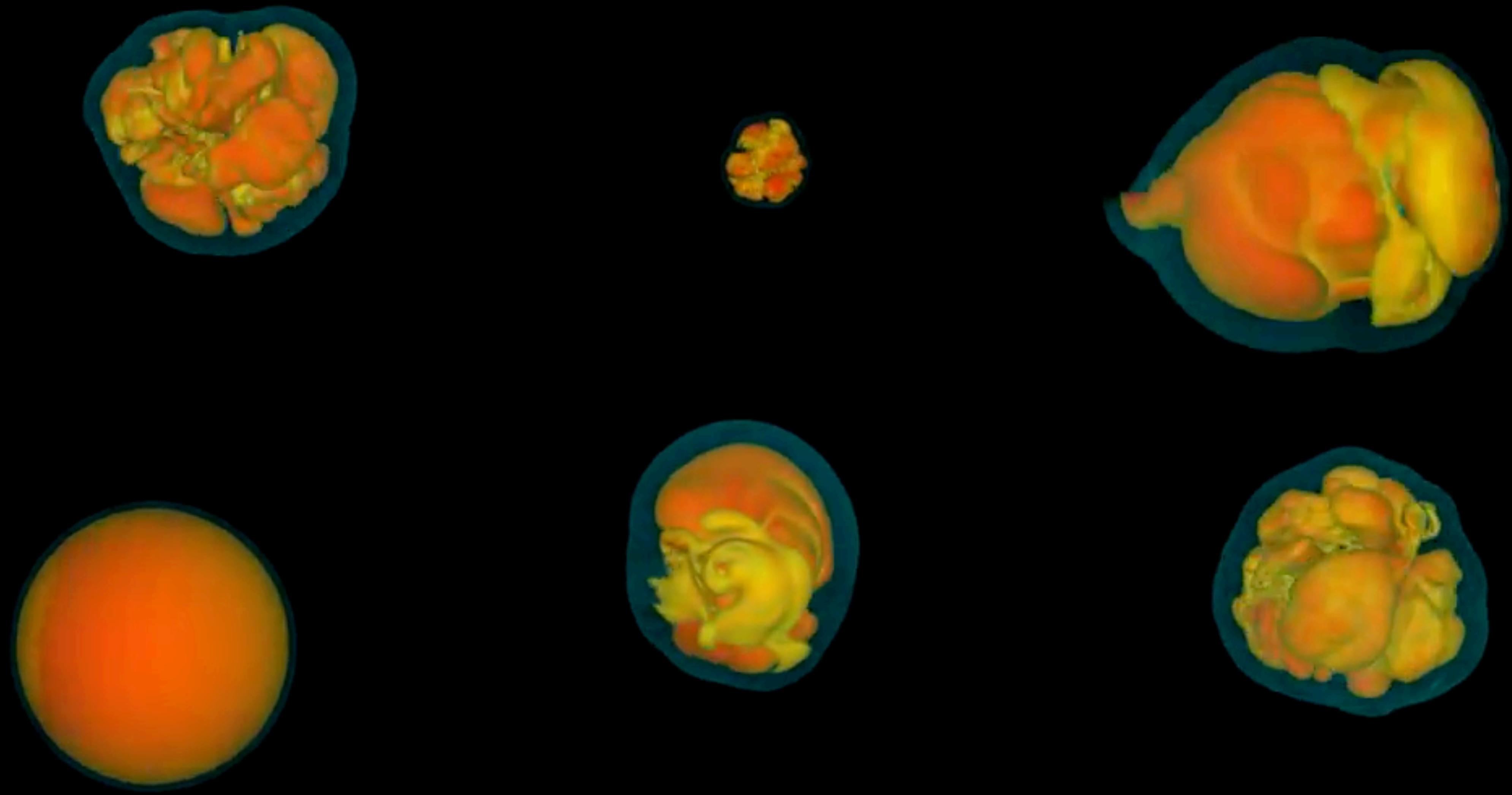
Core-Collapse Supernovae

Type II SNe

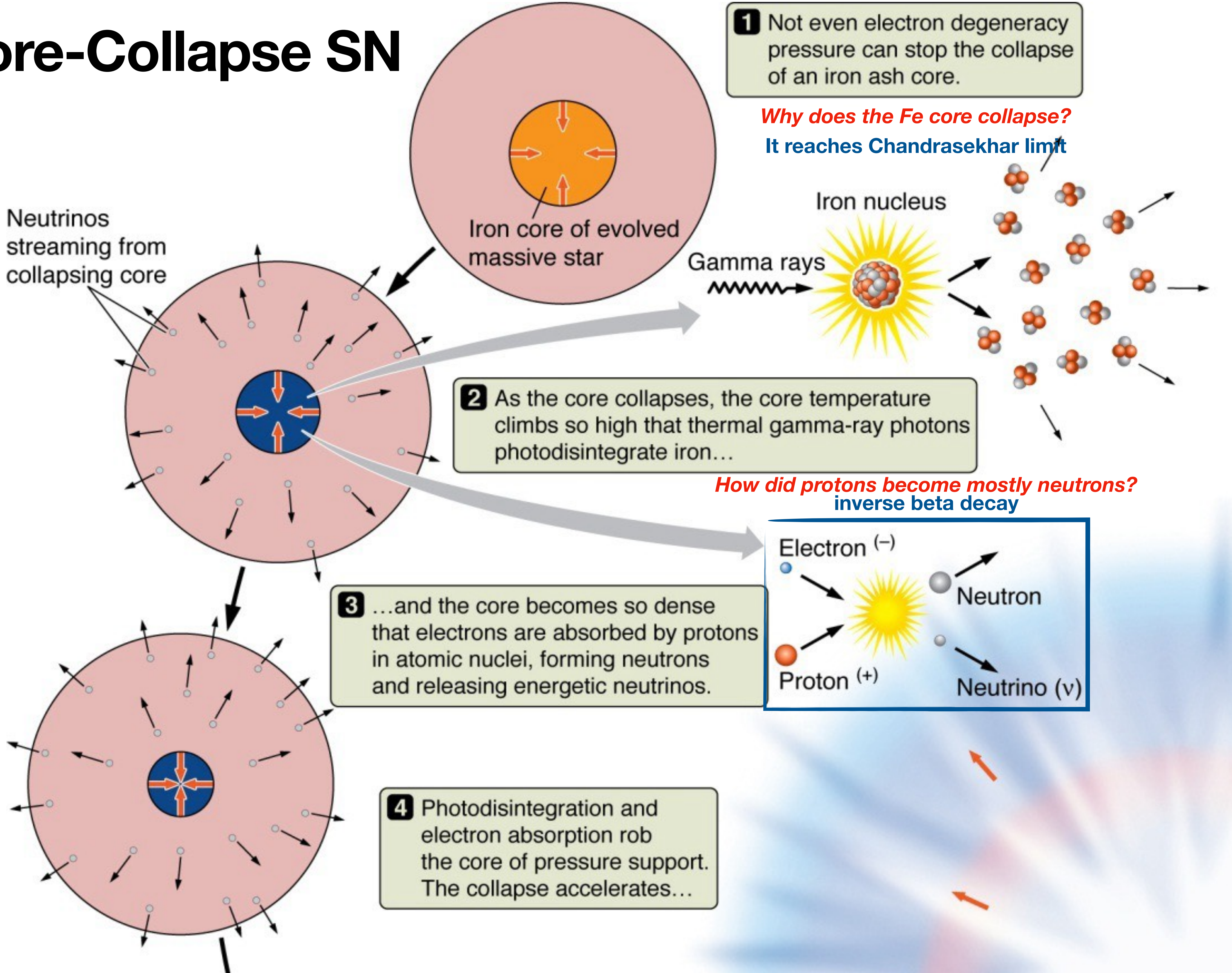
Simulation of a Core-Collapse SN explosion

- e- degenerate iron core collapses as required *P* surpasses e^- degenerate
- Fe -> He, *photodisintegration*
- $p^+ + e^- = n + \nu$
- core collapse accelerates
- strong nuclear force becomes repulsive
- core **bounces** and send shock waves outwards
- trapped **neutrinos** further accelerate shock waves
- 0.1*c* shocks reach surface and heat it to 500,000 K



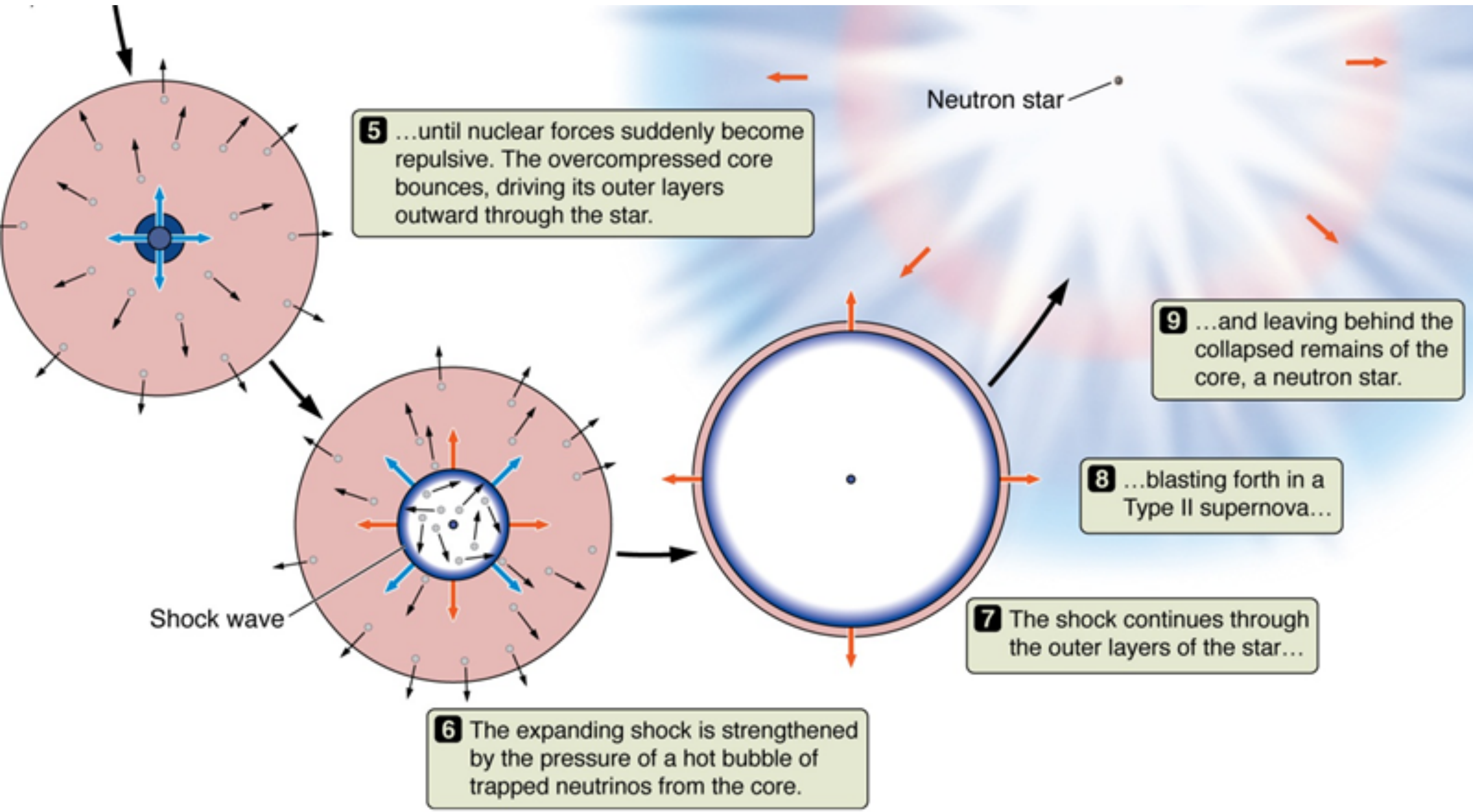


Core-Collapse SN

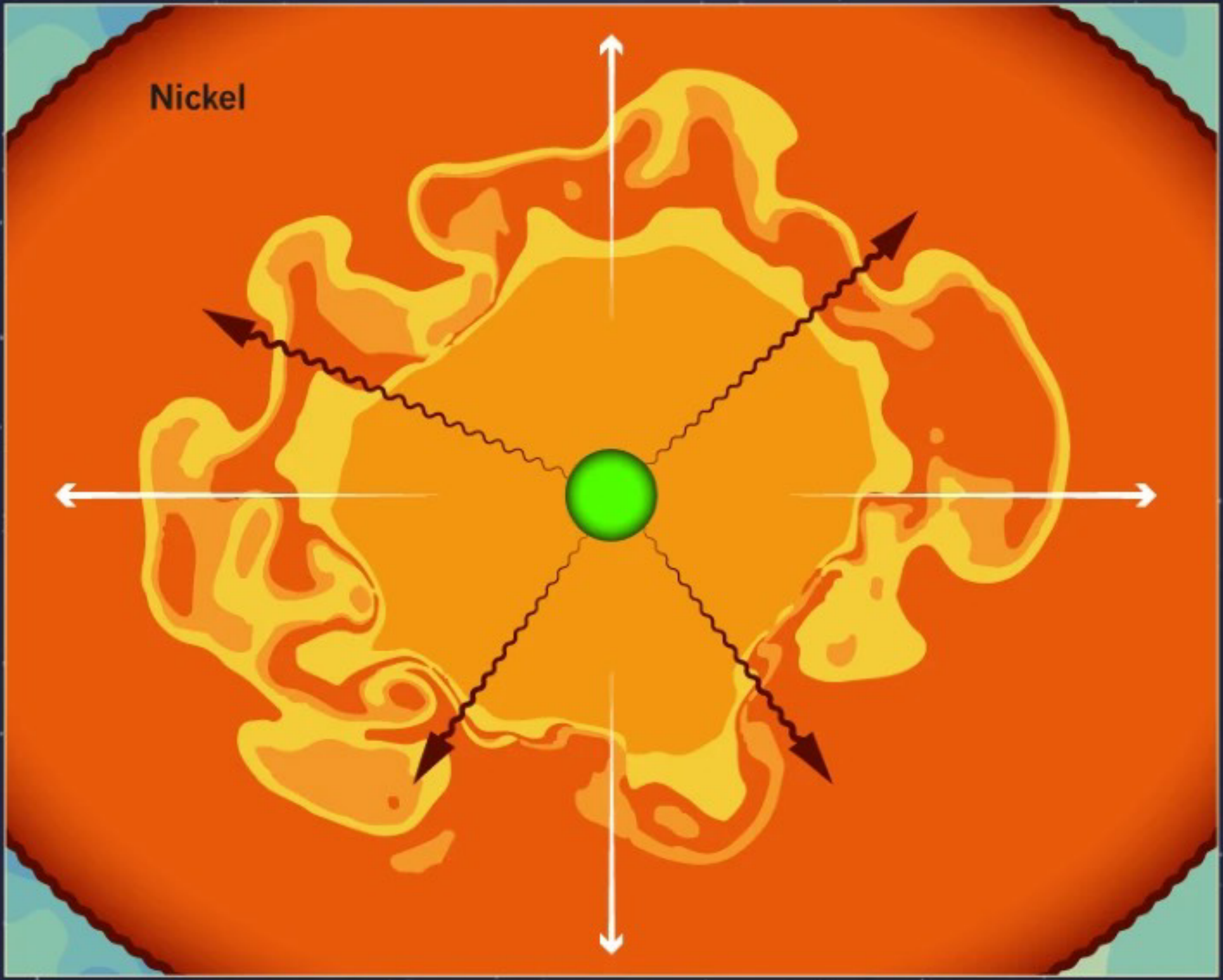


Core Bounce and Explosion

leaving behind a blast nebula and a neutron star



5. EXPLOSION AND NUCLEOSYNTHESIS



Just a few hundred milliseconds after the shock wave first forms, it accelerates out of the core — although it can take as long as a day to reach the star's surface. The energy of the shock wave creates new elements, such as radioactive nickel. In the neutrino-heated, inner part of the explosion, nuclei also capture free neutrons or protons to form elements heavier than iron.

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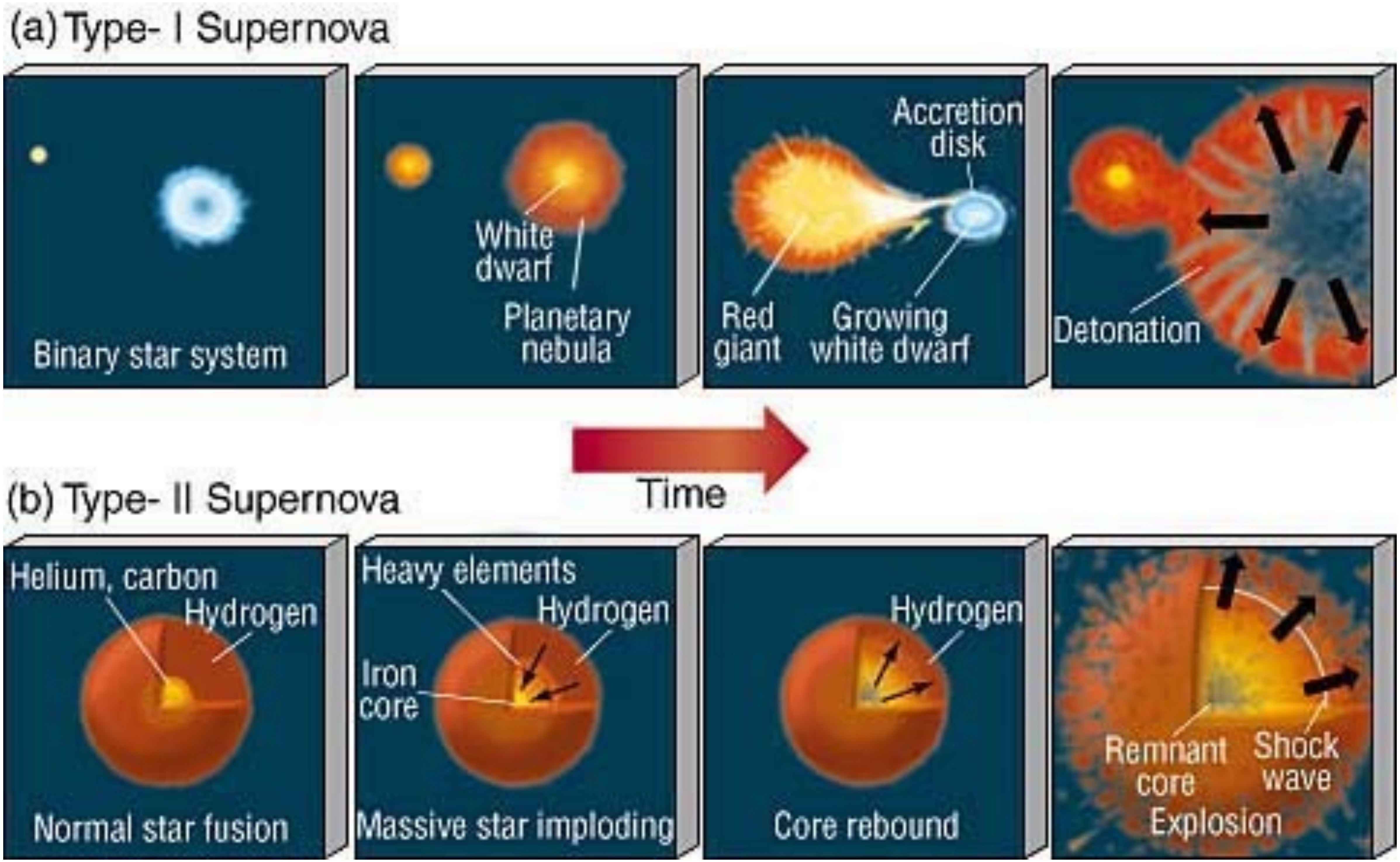
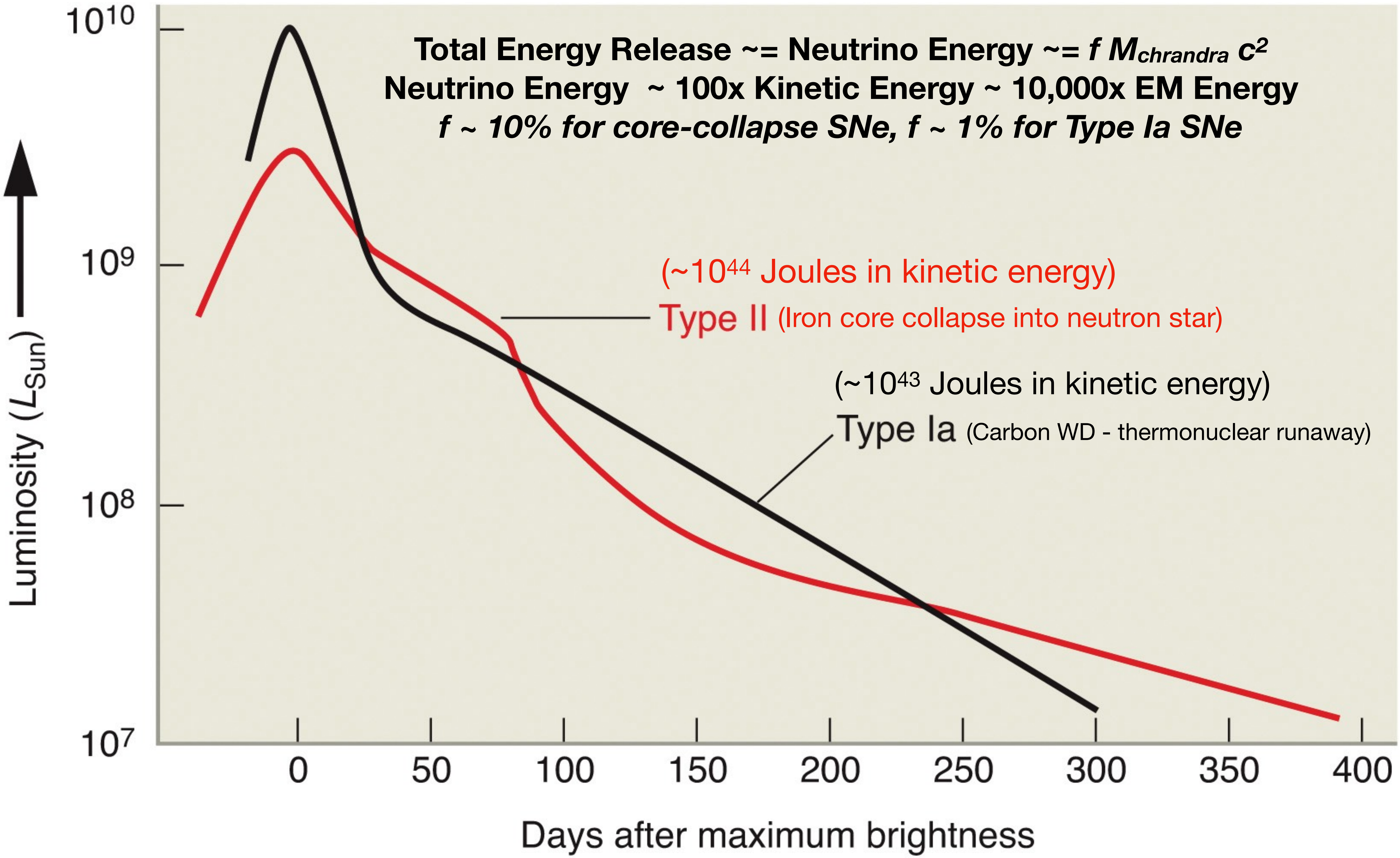
How to distinguish between the two main types of SNe?



Light Curves & Energy Output: Type Ia vs. Type II Supernovae



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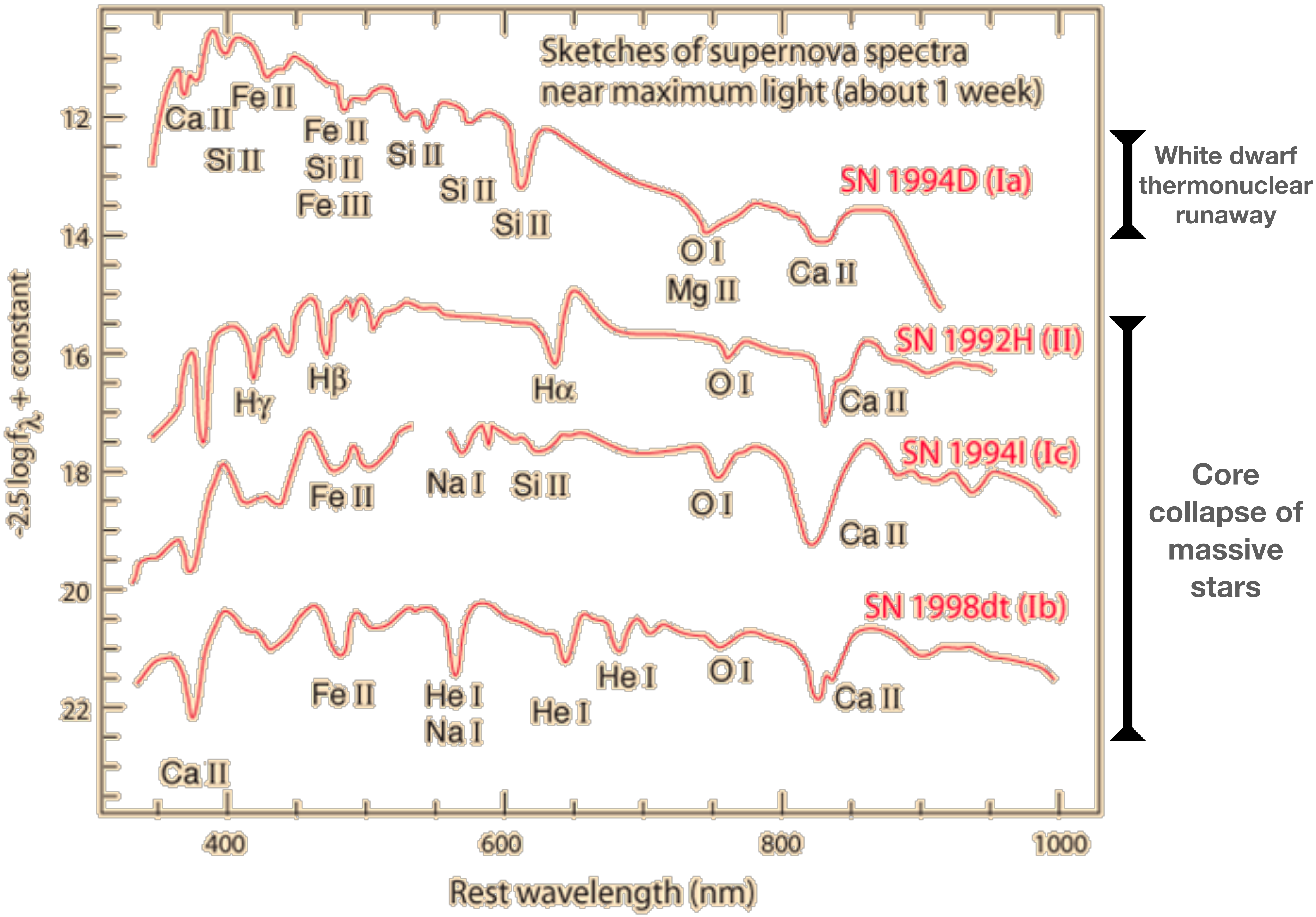
Optical Spectra of Supernovae near Peak: Type I (No Hydrogen lines) vs. Type II (Hydrogen lines)



Sketches of spectra from Carroll & Ostlie, data attributed to Thomas Matheson of National Optical Astronomy Observatory.

Supernova Remnants are beautiful objects, but what can they tell us about the supernovae?

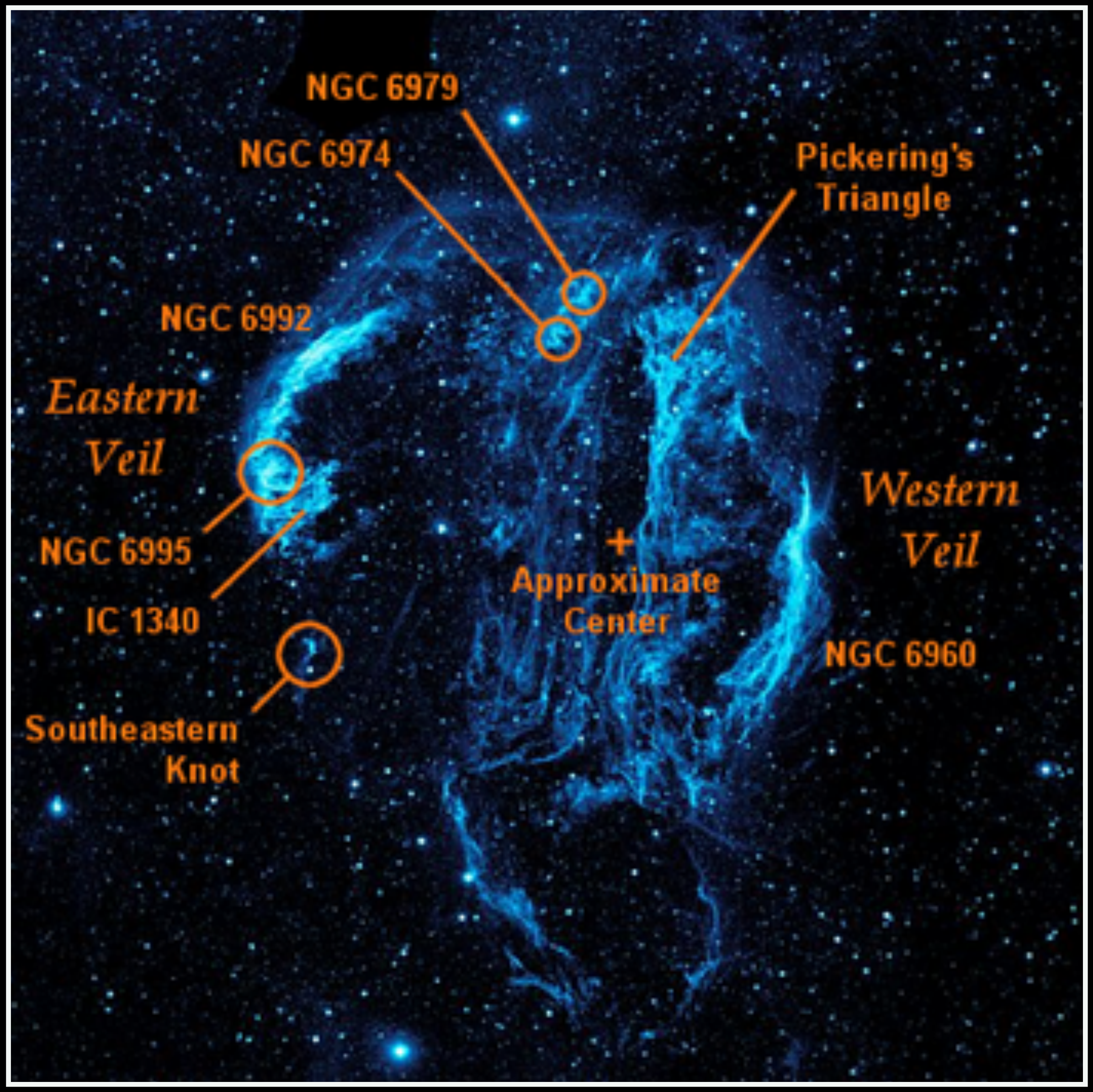
Zoom into Veil Nebula (a SN remnant formed ~10,000 yrs ago)

How to Measure the Age and Distance of a Supernova Remnant?

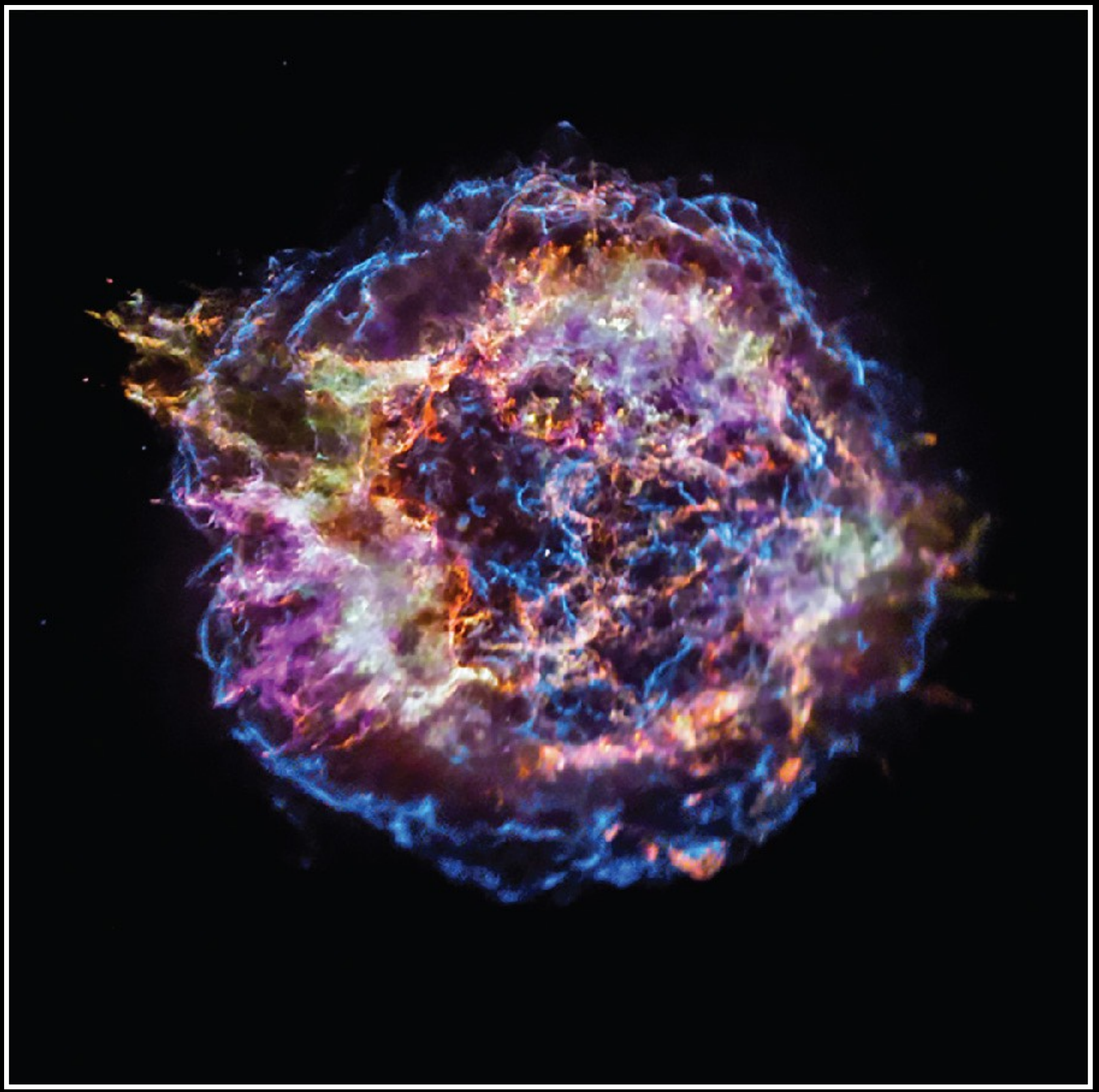
the "Expansion Parallax" method

When we see a SN remnant, how could we measure the age of the remnant and thus estimate when the SN exploded?

Veil Nebula



Cassiopeia A



Detecting the Expansion of SN remnants

How to associate a remnant with a supernova in the past?

Expansion of the Crab Nebula

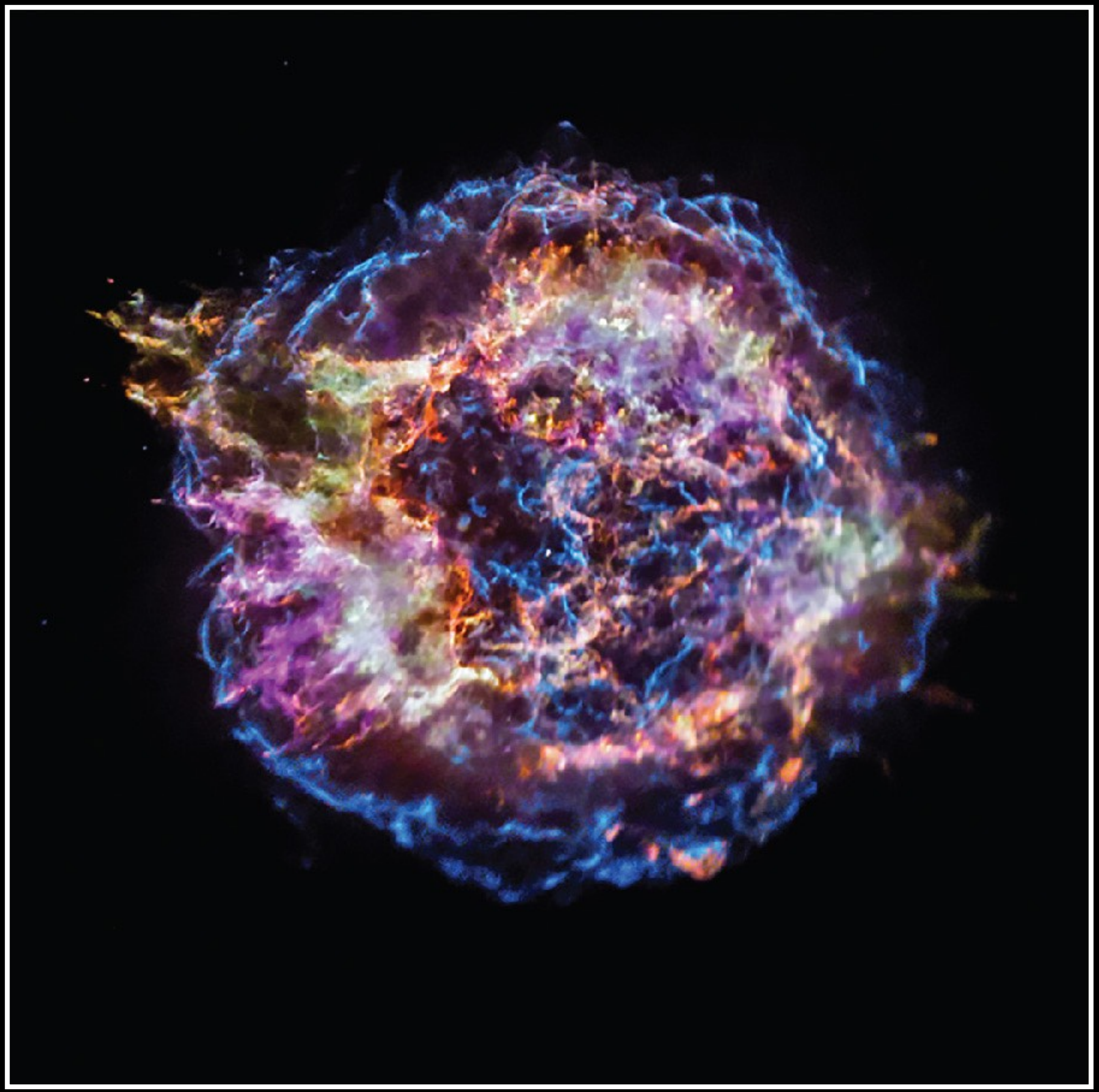
Years 1999 and 2012

Estimating Age from Angular Expansion Rate (assuming constant angular expansion rate)

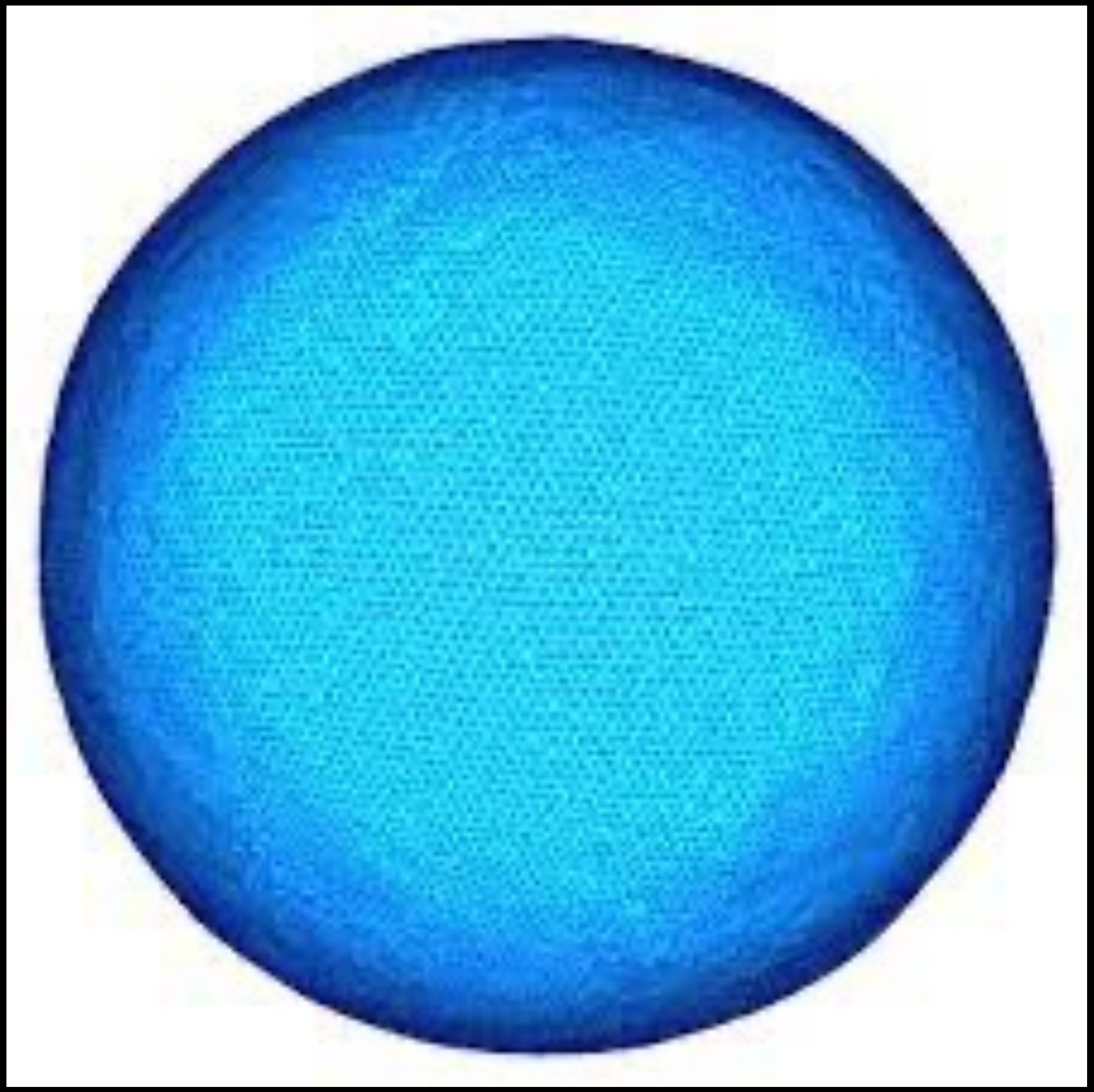
Age = Angular Size / Angular Expansion Rate

$$t = \theta/\dot{\theta}$$

Data: Cassiopeia A

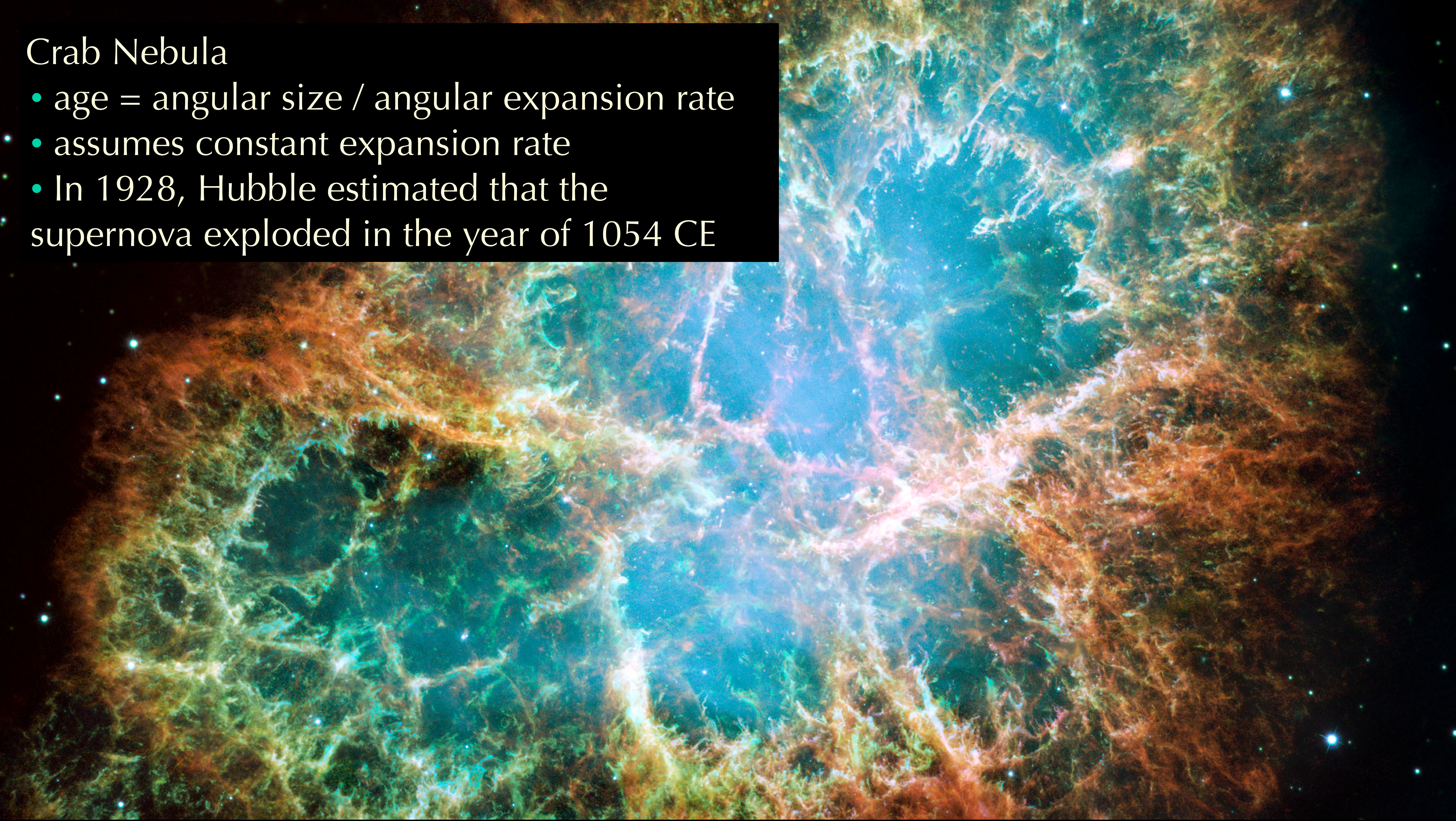


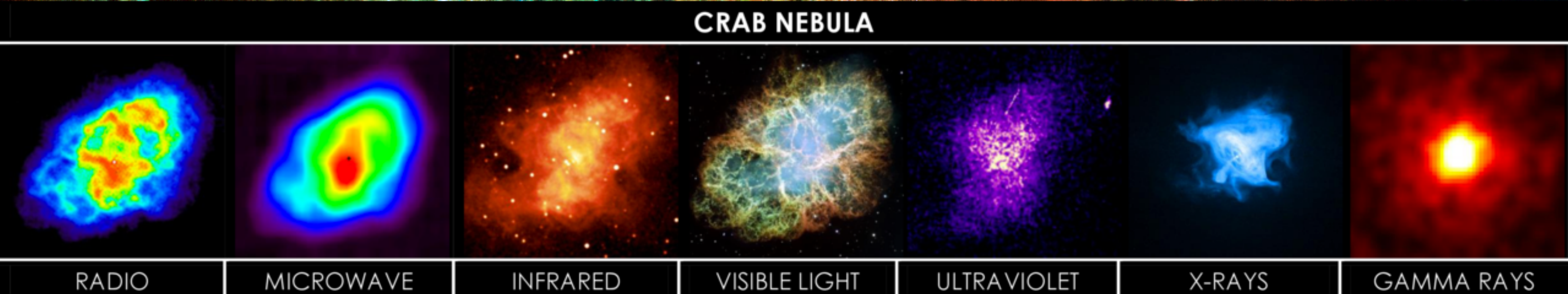
Model: spherical expansion of a shell



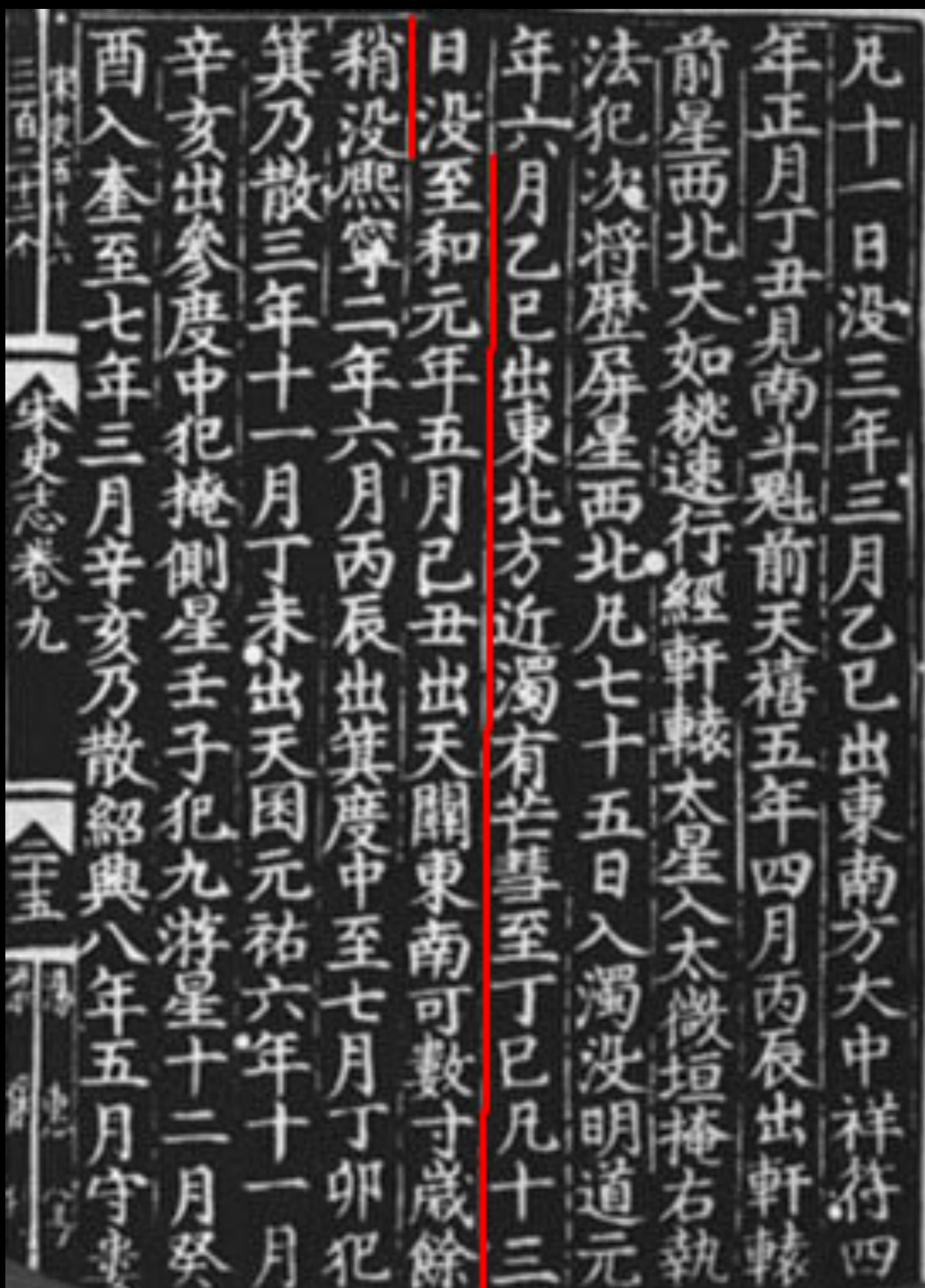
Crab Nebula

- age = angular size / angular expansion rate
- assumes constant expansion rate
- In 1928, Hubble estimated that the supernova exploded in the year of 1054 CE



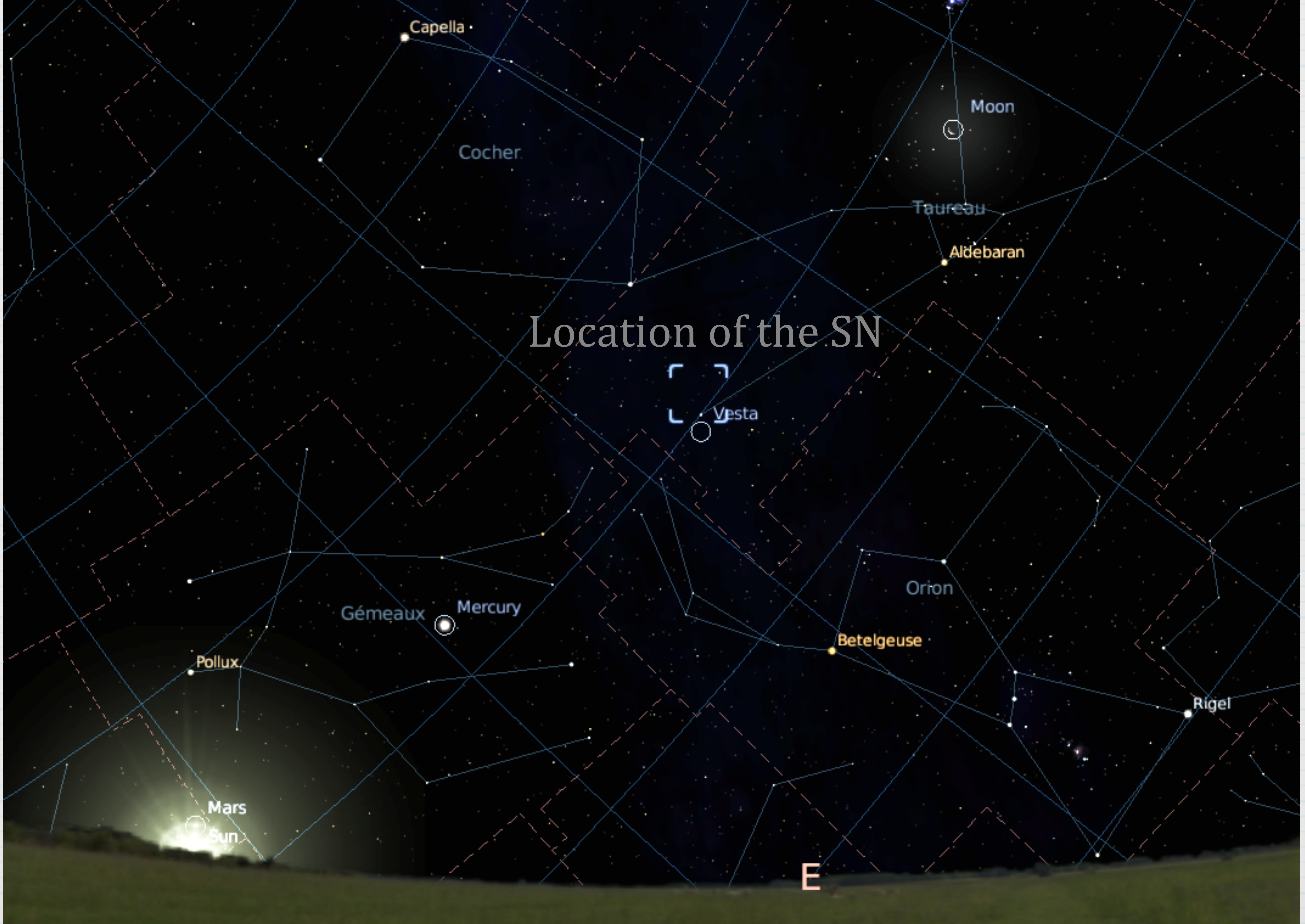


- The supernova was seen by the entire world in July 1054 CE.
- Peak magnitude between -7 and -4.5 (brighter than Venus)
- The event was documented by astronomers in Song Dynasty
- There is also some drawing evidence in Native American ruins in New Mexico, Chaco Canyon


凡十一日沒三年三月乙巳出東南方大中祥符四
年正月丁丑見南斗魁前天禧五年四月丙辰出軒轅
前星西北大如桃速行經軒轅大星入太微垣掩右執
法犯次將歷屏星西北凡七十五日入濁沒明道元
年六月乙巳出東北方近濁有芒彗至丁巳凡十三
日沒至和元年五月己丑出天關東南可數寸歲餘
稍沒熙寧二年六月丙辰出箕度中至七月丁卯犯
箕乃散三年十一月丁未出天囷元祐六年十一月
辛亥出參度中犯掩側星壬子犯九游星十二月癸
酉入奎至七年三月辛亥乃散紹興八年五月守婁

宋史志卷九

The Sky in the morning of July 4, 1054



Determining Distance from “Expansion Parallax”

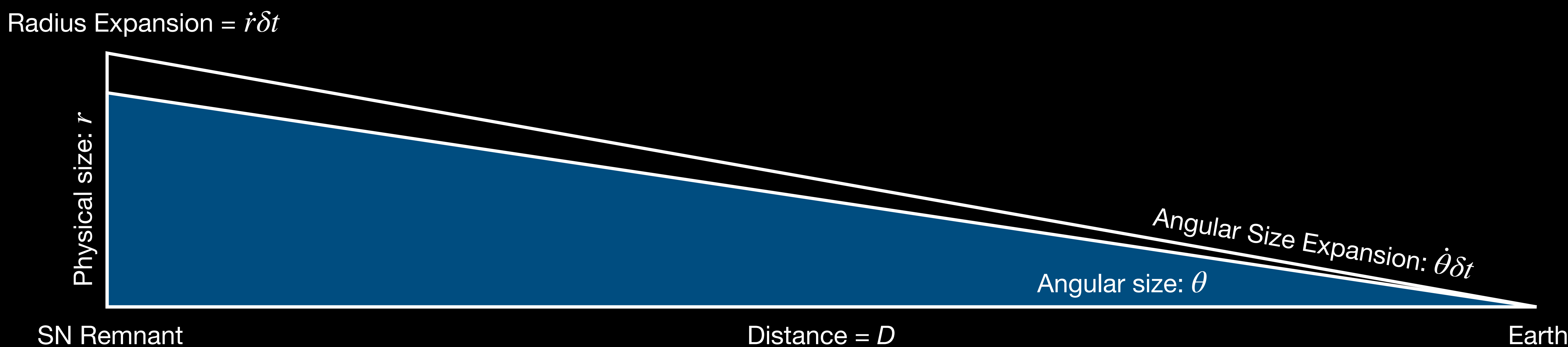
Physical Size vs. Angular Size: $r = D\theta$

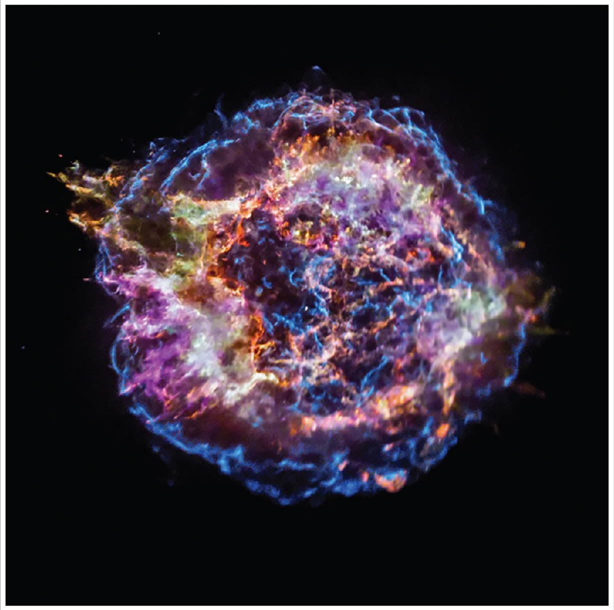
Physical Expansion Rate vs. Angular Expansion Rate:

$$\dot{r}\delta t = D\dot{\theta}\delta t \rightarrow \dot{r} = D\dot{\theta}$$

Distance = Physical Expansion Rate / Angular Expansion Rate

$$D = \dot{r}/\dot{\theta}$$

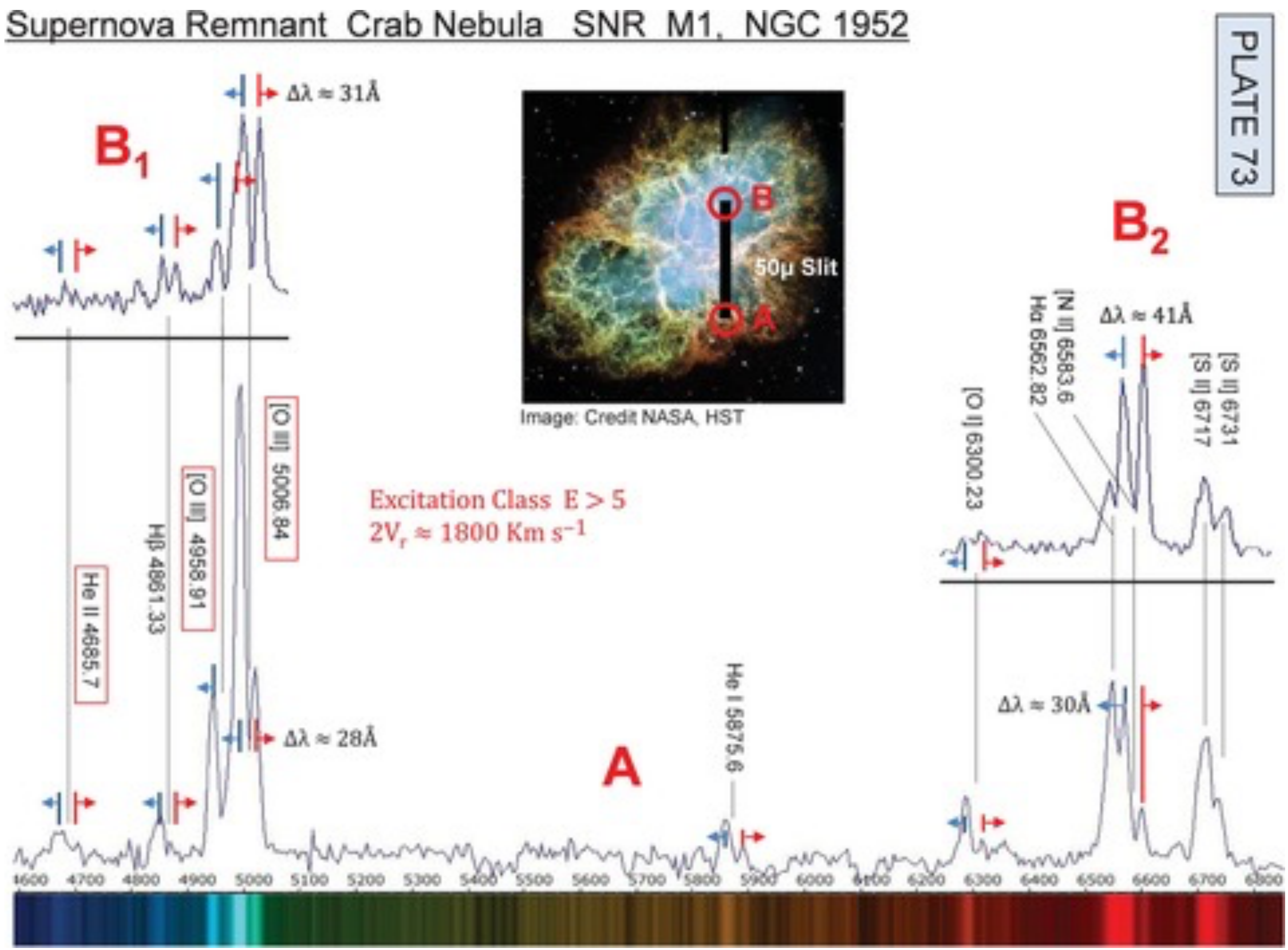




e.g., $\dot{\theta} = 0.01''/yr$, and $\dot{r} = 250$ km/s, what is the distance?

But how do we measure the physical expansion rate $\dot{r}$ in km/s?

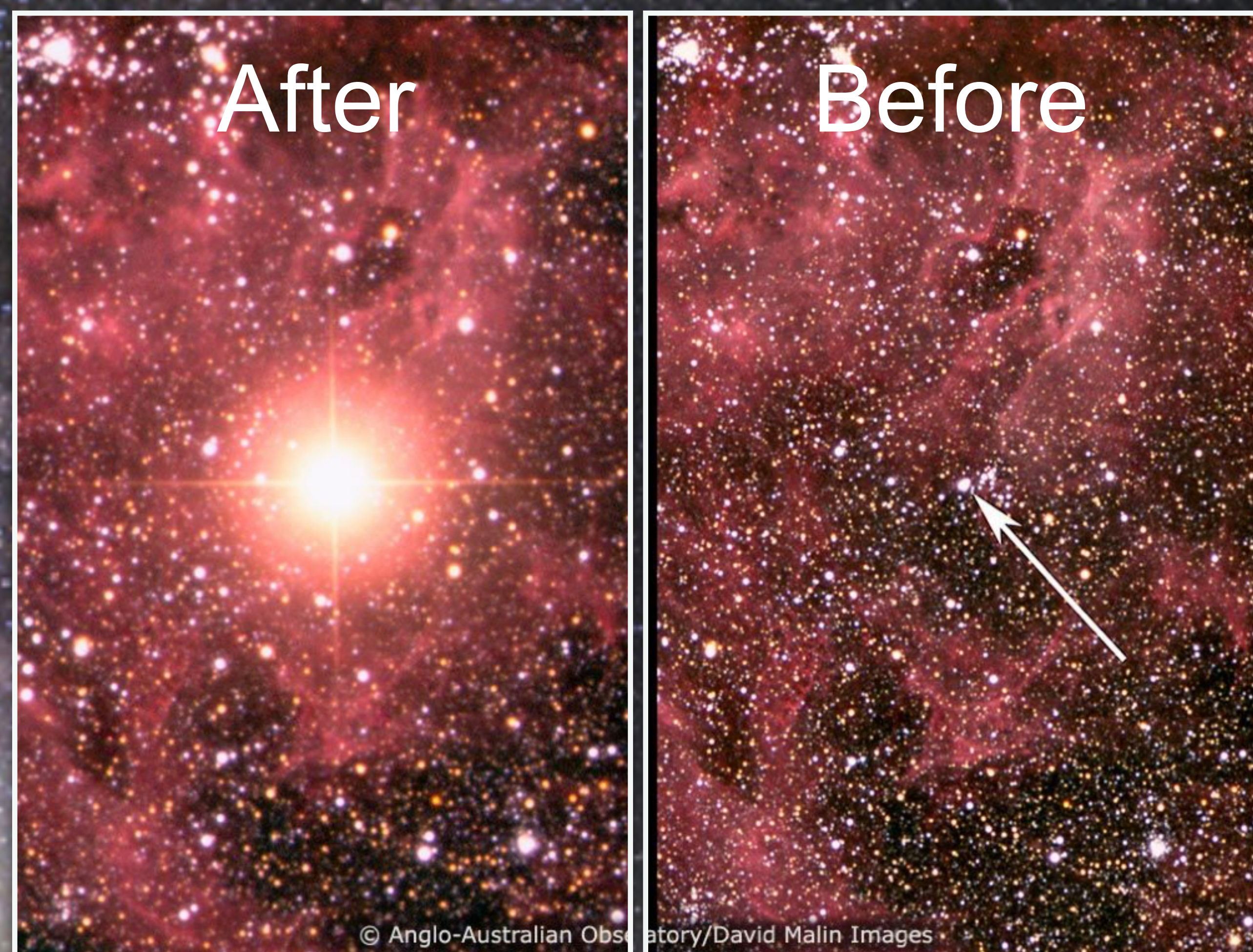
Physical Expansion Rate from Doppler Shifts: Each Line Splits into Multiples



Witnessing the formation of
a Supernova Remnant (SNR)

SN 1987A

SN 1987A



LMC

d = 50 kpc

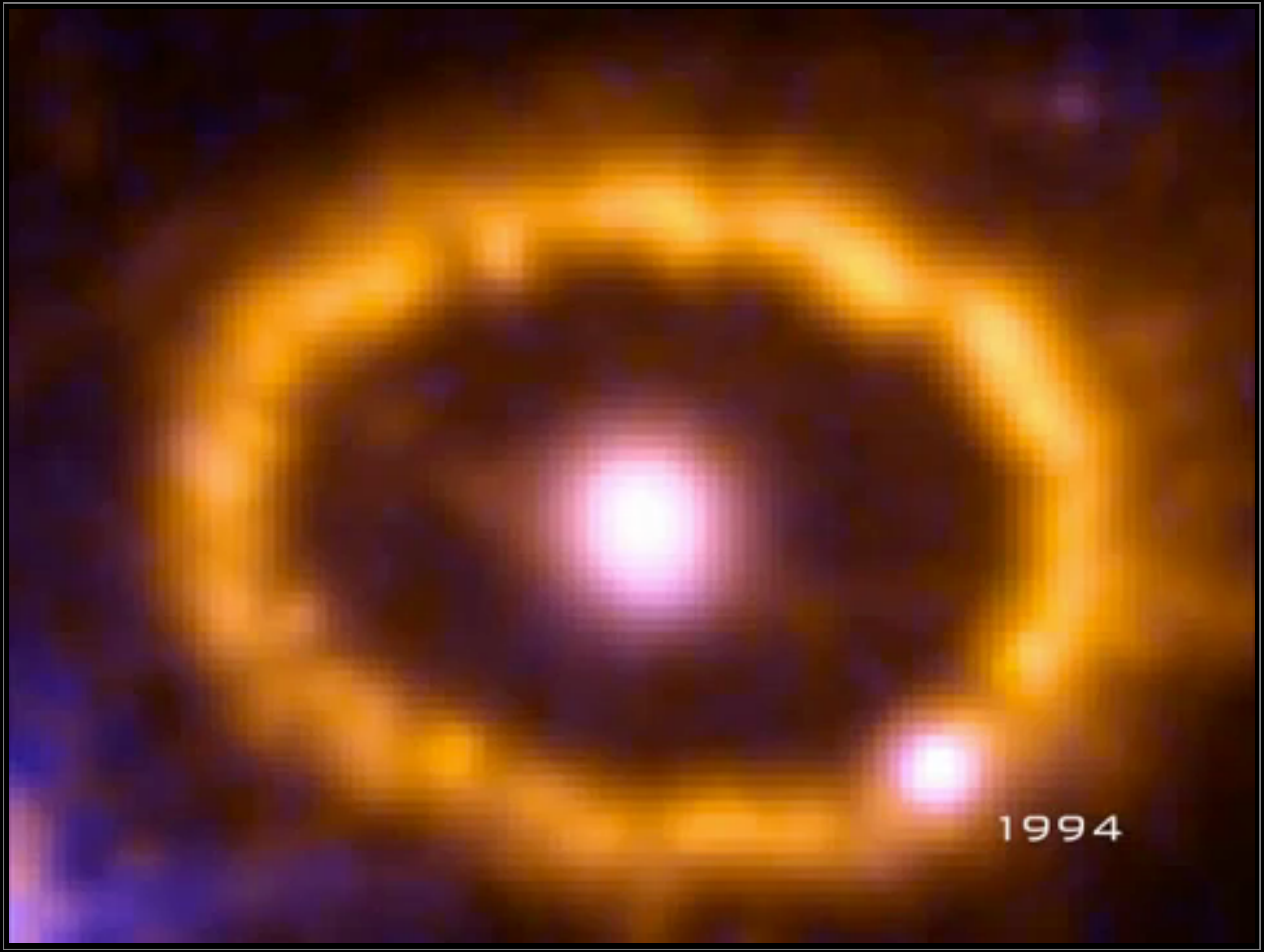
Dec = -70d

M* = 1e10 M_{sun}

Zoom in onto SN 1987A

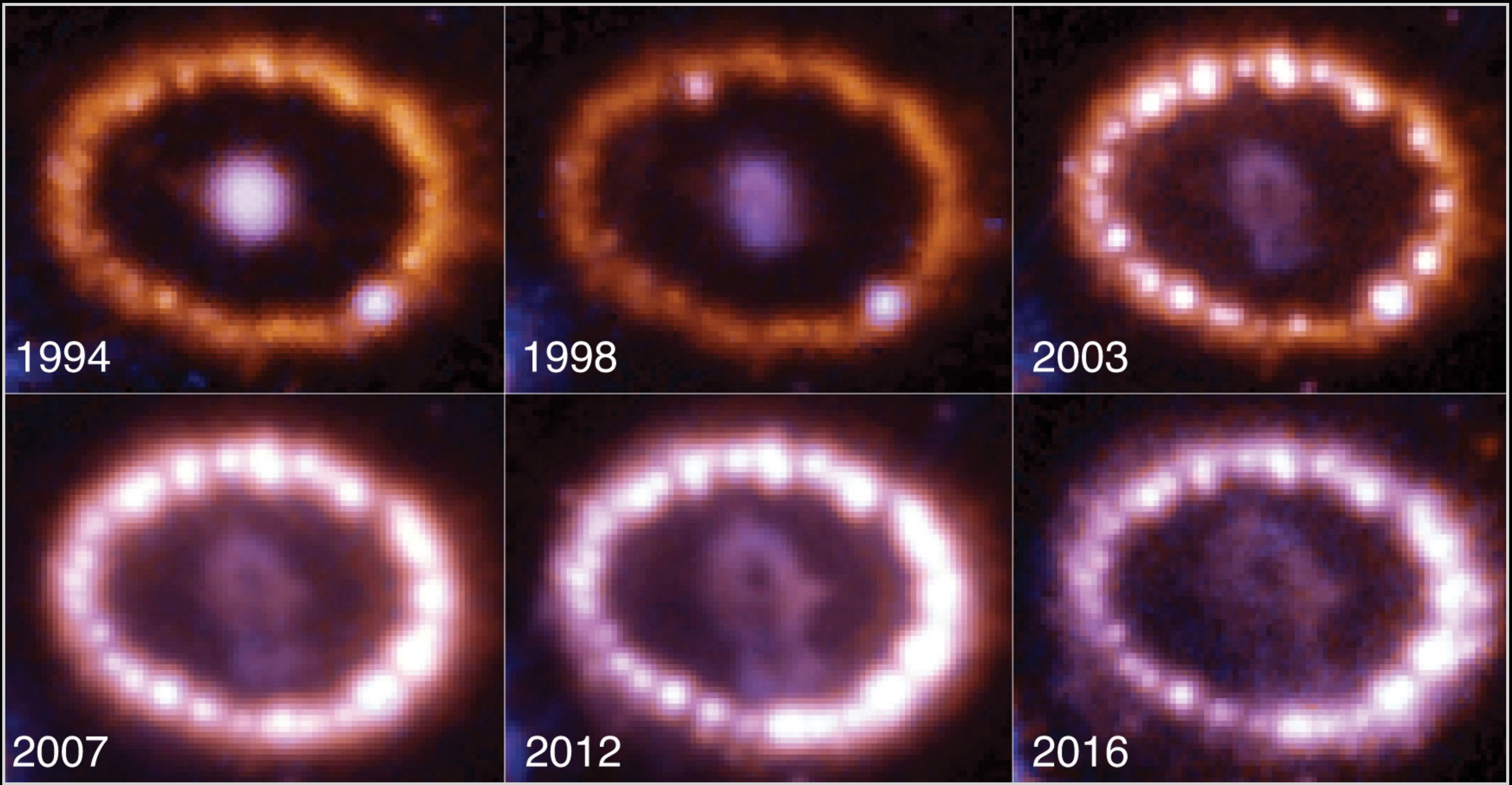
The firework after the explosion

SN ejecta catching up with the mass loss from stellar winds



The firework after the explosion

SN ejecta catching up with the mass loss from stellar winds



Neutrinos from SN 1987A

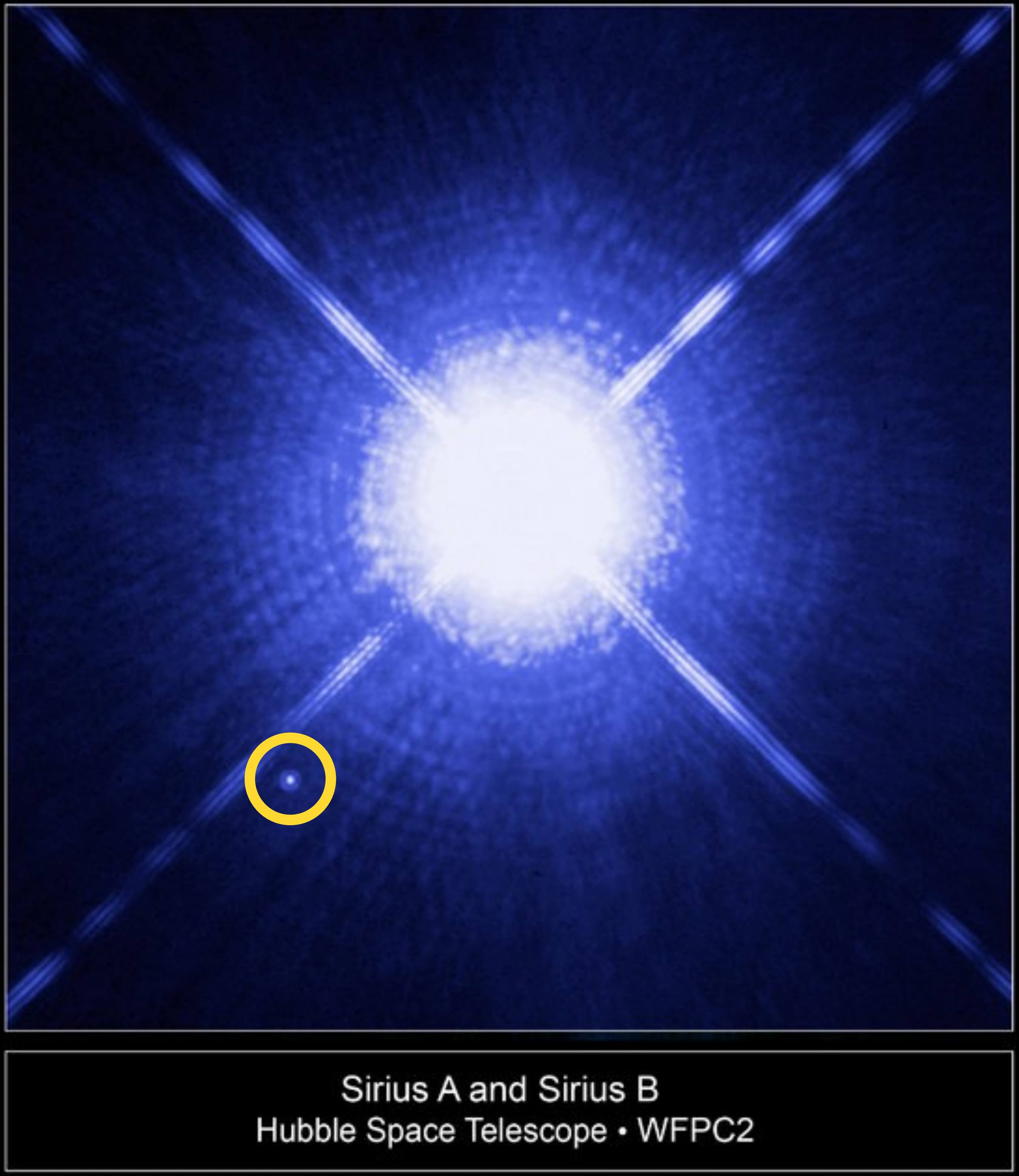
19 neutrinos were detected (in Japan & Ohio) the day ***before*** the optical explosion was seen

In a core-collapse SN (i.e., type II), neutrinos are produced when protons and electrons combine to form neutrons

At the center of the SN explosion, a Neutron Star is born

Theoretical Models

Sirius B - the white dwarf companion of the Dog Star

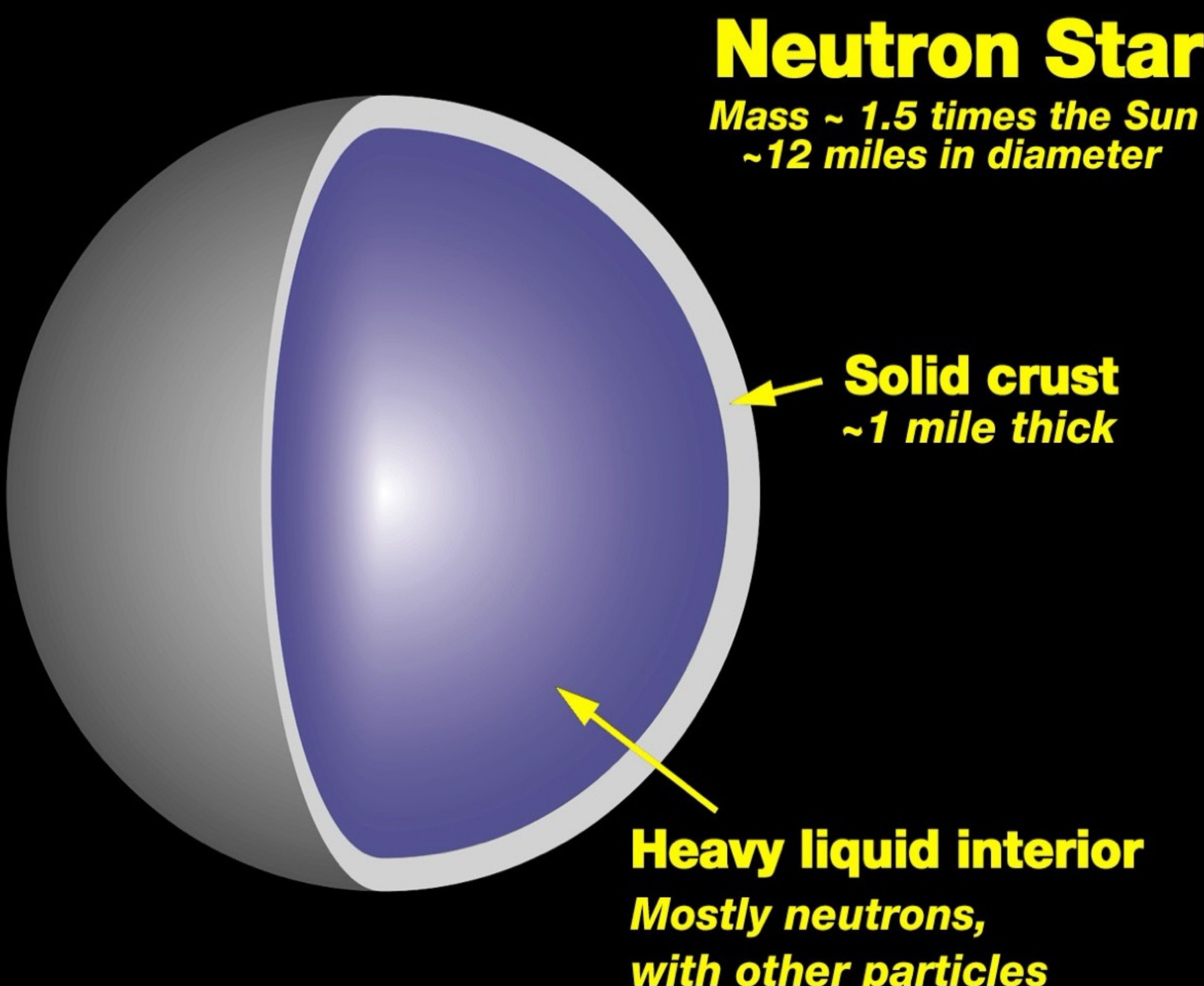


NASA, ESA, H. Bond (STScI), and M. Barstow (University of Leicester) STScI-PRC05-36a

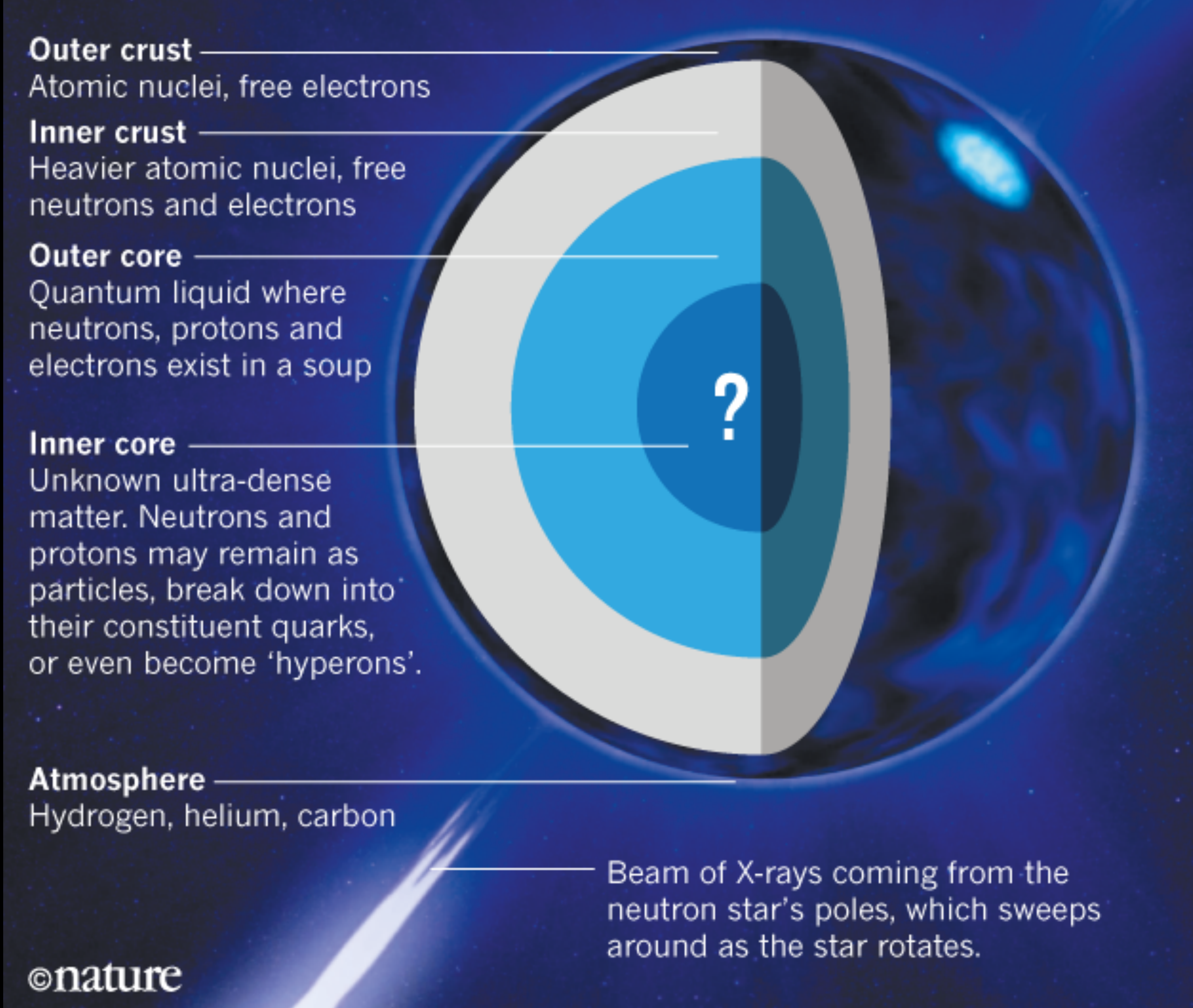
- Inferred properties of Sirius B:
  - 1 Solar Mass
  - 0.03 Solar Luminosity
  - 27,000 K surface temperature
  - 5500 km radius (Earth-size)
- Sirius B represent a class of objects called **White Dwarfs (WDs)**

- The physical conditions of WDs are extreme:
  - extreme density ($\rho \approx$ 3e9 kg/m^3) ($n_e \sim$ 1e36 /m^3)
  - extreme surface gravity (HW)
  - extreme pressure at the center

Neutron stars are extremely compact and dense, even compared to white dwarfs



- packing a solar mass into a ball of ~10 km in radius results in a density of $\sim 10^9$ tons per teaspoon (compared to ~10 tons on a white dwarf)
- surface gravitational field – 300,000 times that of Earth ($g = GM/R^2$)
- To escape from a neutron star, an object would have to reach 50% the speed of light!


Outer crust
Atomic nuclei, free electrons
Inner crust
Heavier atomic nuclei, free neutrons and electrons
Outer core
Quantum liquid where neutrons, protons and electrons exist in a soup
Inner core
Unknown ultra-dense matter. Neutrons and protons may remain as particles, break down into their constituent quarks, or even become 'hyperons'.
?
Atmosphere
Hydrogen, helium, carbon
Beam of X-rays coming from the neutron star's poles, which sweeps around as the star rotates.
©nature

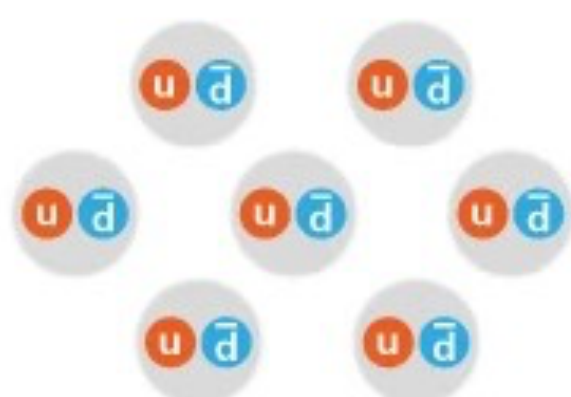
Core scenarios

A number of possibilities have been suggested for the inner core, including these three options.

u Up quark
s Strange quark
d Down quark
d̄ Anti-down quark

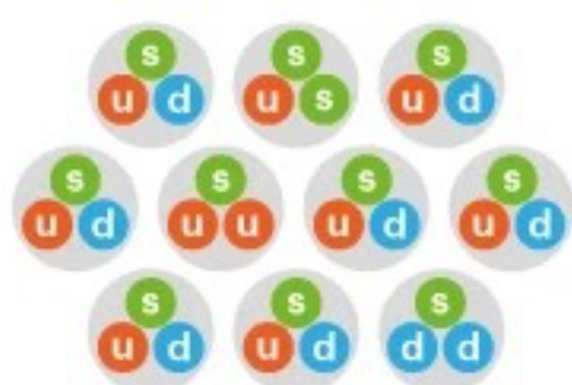
Quarks

The constituents of protons and neutrons — up and down quarks — roam freely.



Bose–Einstein condensate

Particles such as pions containing an up quark and an anti-down quark combine to form a single quantum-mechanical entity.



Hyperons

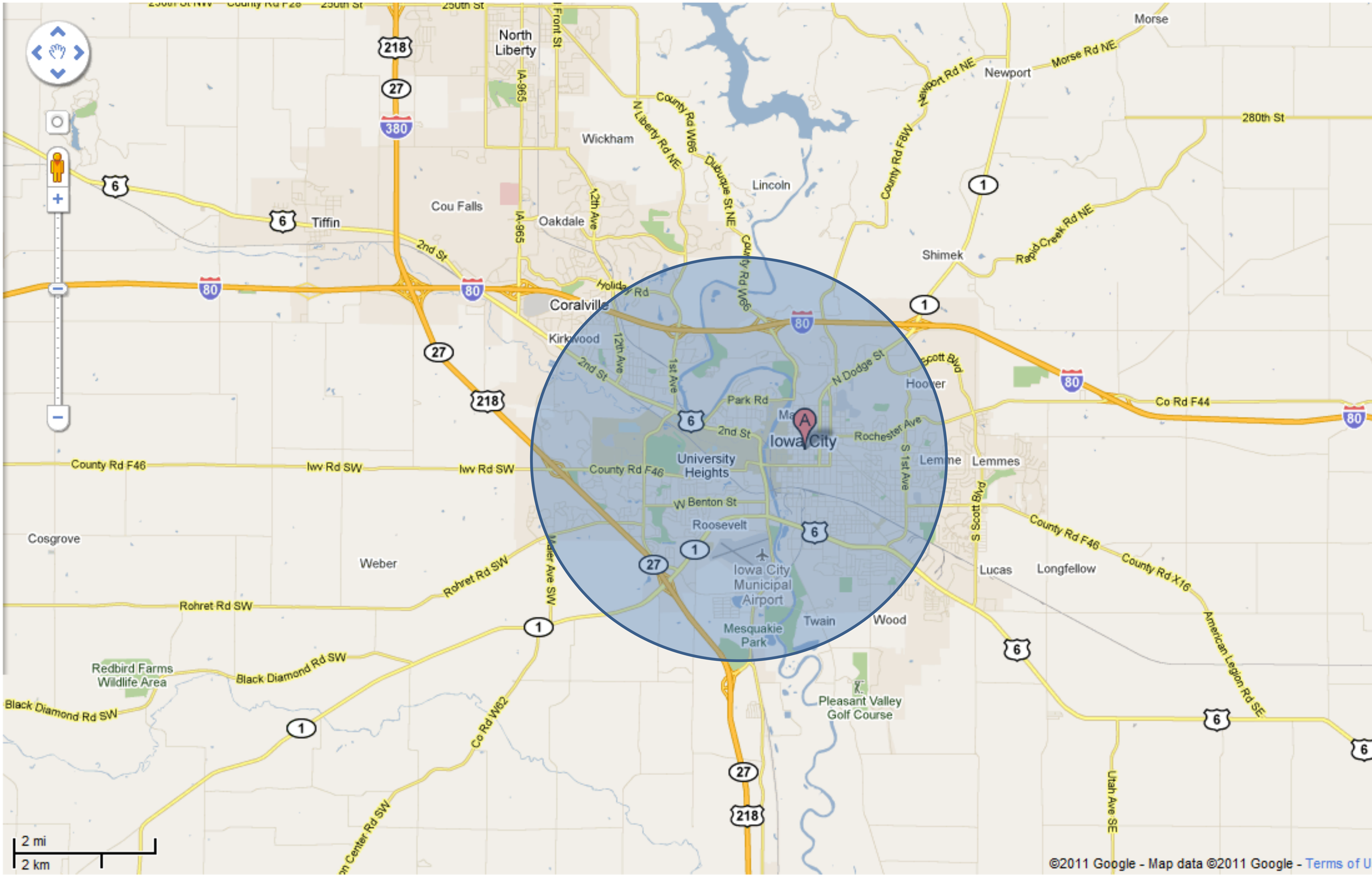
Particles called hyperons form. Like protons and neutrons, they contain three quarks but include 'strange' quarks.

©nature

Observable Neutron Stars I:

Pulsars

Neutron star has a size of a small city - How can we find them?



What are the unique observational properties of neutron stars?

- How can we tell if any of the objects in the image is a neutron star?

The Discovery of Pulsars in 1967

1967: Jocelyn Bell Burnell
PhD student of **Anthony Hewish**
at Cambridge University, England

Helped build a radio telescope to study the solar wind by looking at "twinkling" of background radio sources as their emission passes through the solar wind

In Nov 1967, she discovered a repeating radio signal with period~1s

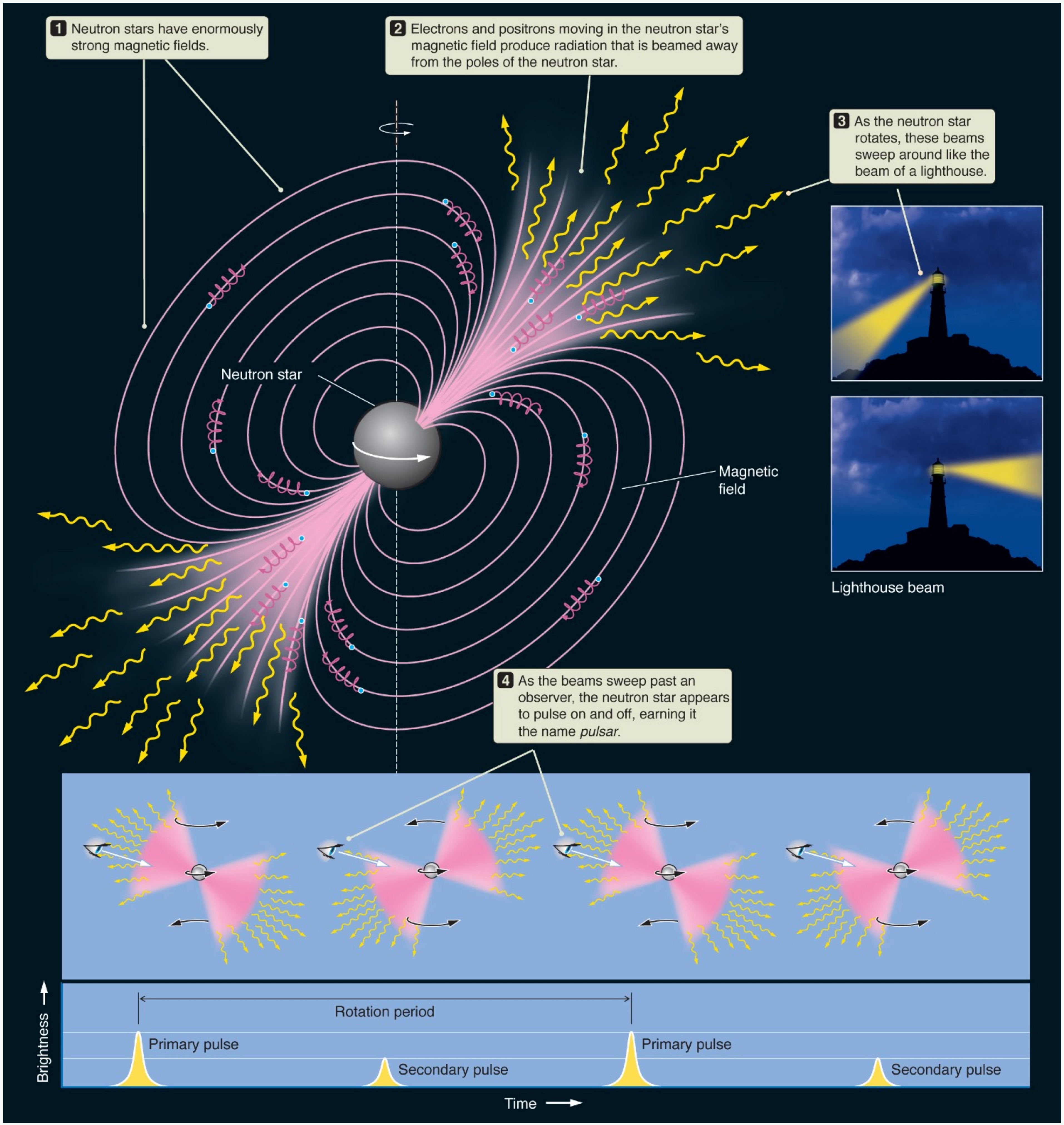
Thought it could be detection of ET – "LGM1"
extraterrestrial (ET)
little green men (LGM)

Now known as PSR 1919+21

Pulsars

- Charged particles around rapidly rotating, highly magnetized *object* produce beam of **synchrotron radiation**

- The beams sweep by Earth like a lighthouse beam.

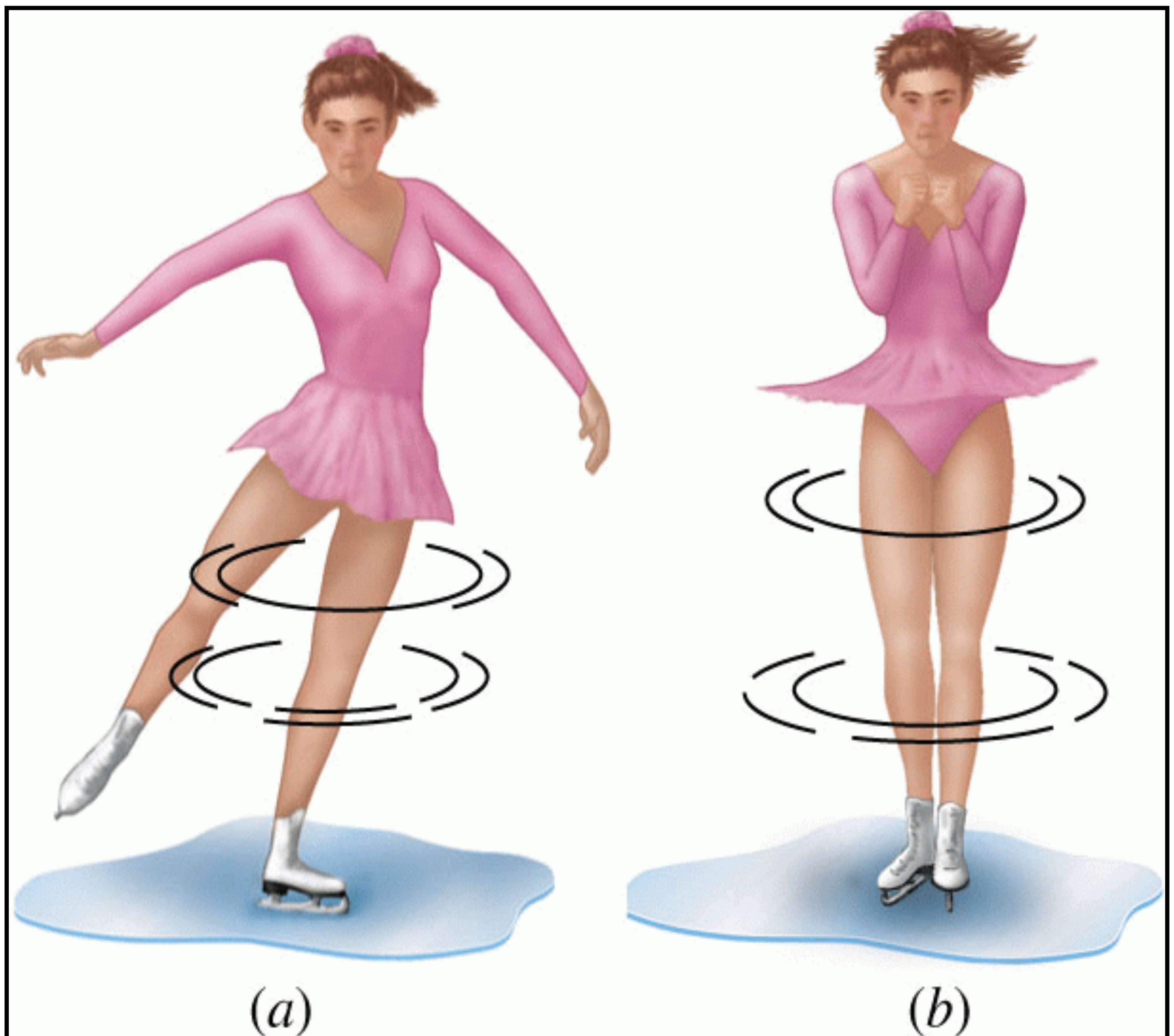


Neutron stars are expected to spin very fast!

Angular momentum of a uniform sphere, and *L* is conserved during collapse

$$L = I\omega = \frac{2}{5}Mr^2\omega \Rightarrow \omega \propto r^{-2} \Rightarrow P \propto r^2$$

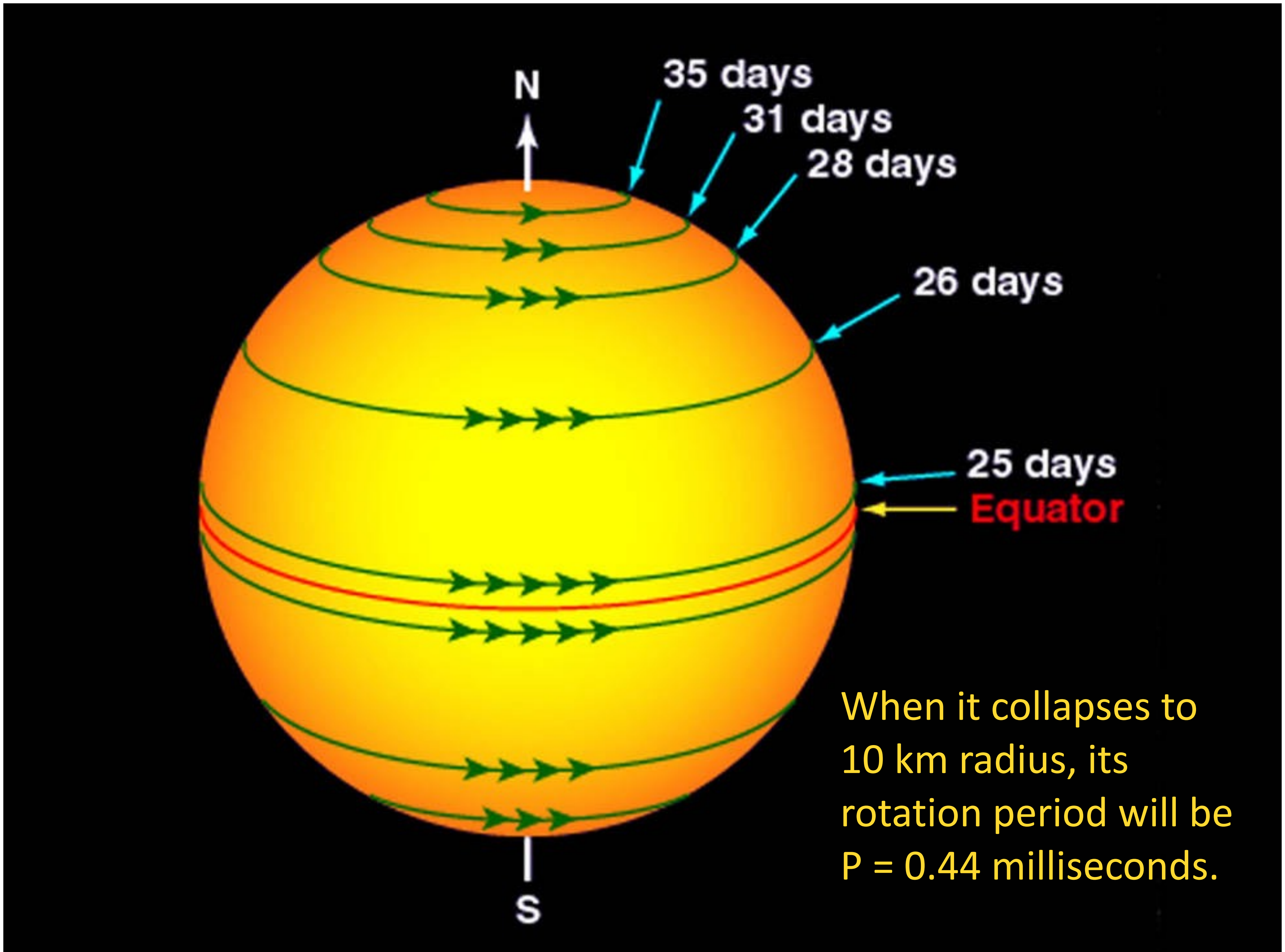
where *I* is the moment of inertia, and ω the angular velocity.



When the Sun (R = 7e5 km) collapses into a neutron star (R = 10 km):

What's the current angular velocity?
What will be the rotating period?

$$\omega_{\odot} = \frac{2\pi}{P} = 14 \text{ deg/day for } P = 25 \text{ days}$$



When it collapses to 10 km radius, its rotation period will be P = 0.44 milliseconds.

Strengths of Magnets

The **tesla** (symbol: **T**) is the SI unit of magnetic flux density (also called magnetic B-field strength). **1 T = 1e4 Gauss**
A particle, carrying a charge of one coulomb (C), and moving perpendicularly through a magnetic field of one tesla, at a speed of one meter per second, experiences a force with magnitude one newton (N), according to the Lorentz force law:
$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,.$

Smallest value in a magnetically shielded room	10^-14 Tesla	10^-10 Gauss
Interstellar space	10^-10 Tesla	10^-6 Gauss
Earth's magnetic field	0.00005 Tesla	0.5 Gauss
Small bar magnet	**0.01 Tesla**	100 Gauss
Within a sunspot	0.15 Tesla	1500 Gauss
Small NIB magnet	0.2 Tesla	2000 Gauss
Big electromagnet	1.5 Tesla	15,000 Gauss
Strong lab magnet	10 Tesla	100,000 Gauss
Surface of neutron star	**100,000,000 Tesla**	10^12 Gauss
Magstar	**100,000,000,000 Tesla**	10^15 Gauss

Conservation of magnetic flux:
Neutron stars are expected to have strong magnetic fields

The collapse of a star also concentrates the magnetic field on the surface

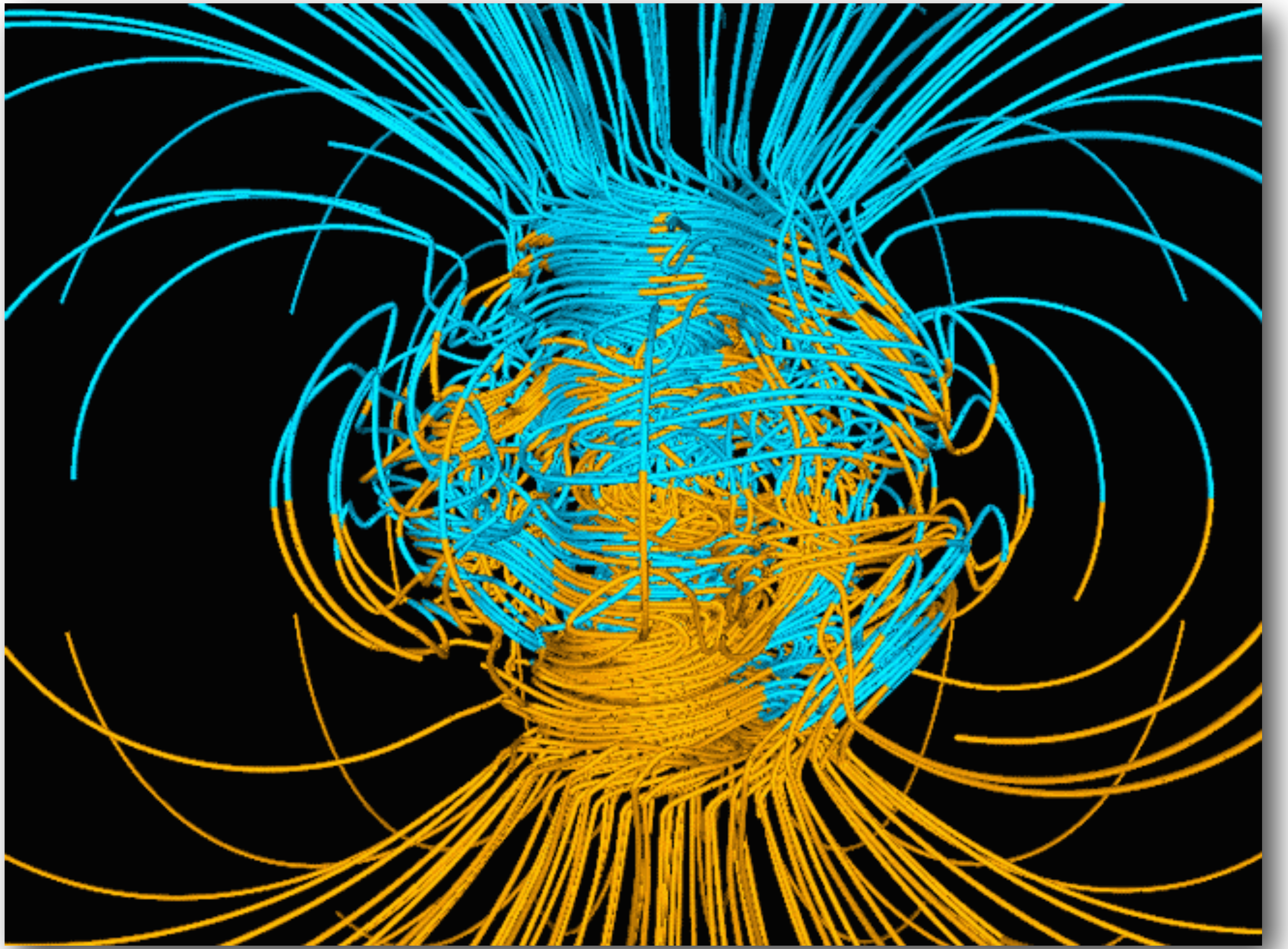
Similar to angular velocity, $B \propto R^{-2}$

shrinking to a radius of 10 km, the Sun's magnetic field would be 10^{10} times as strong!

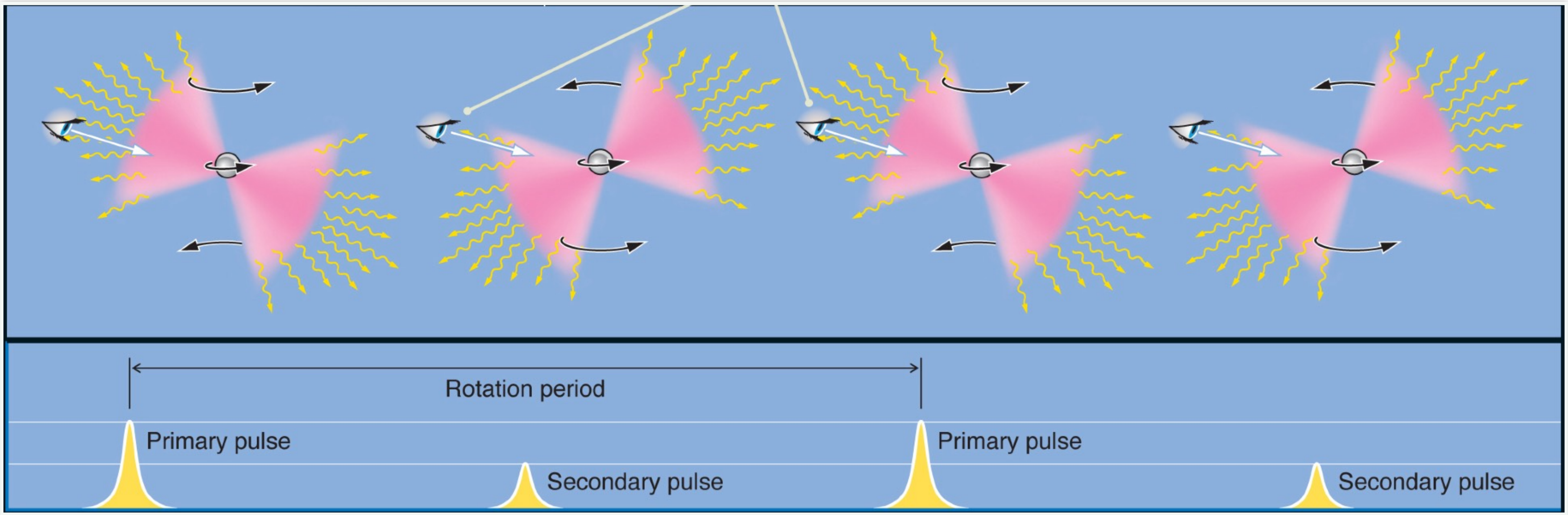
Magnetic field for normal star (e.g. Sun): a few Gauss (G)

Magnetic field for neutron star: up to 10^{15} G

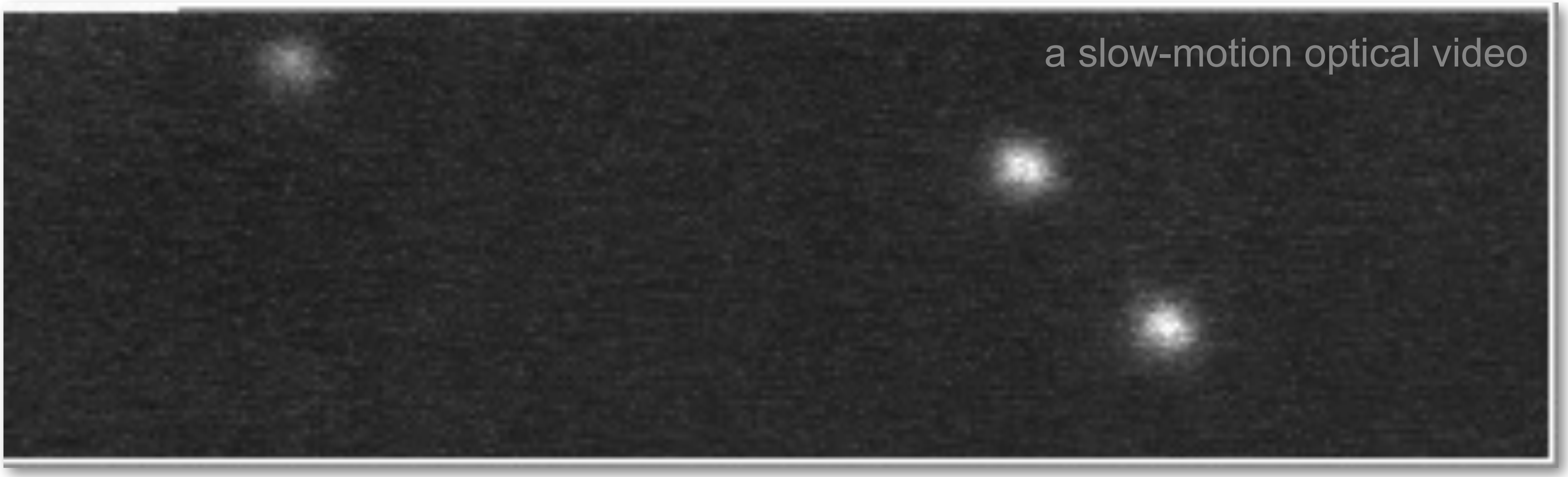
Strongest magnetic field produced in a laboratory (for a few seconds): 10^{6} G



Optical Pulsars and Primary vs. Secondary Pulses



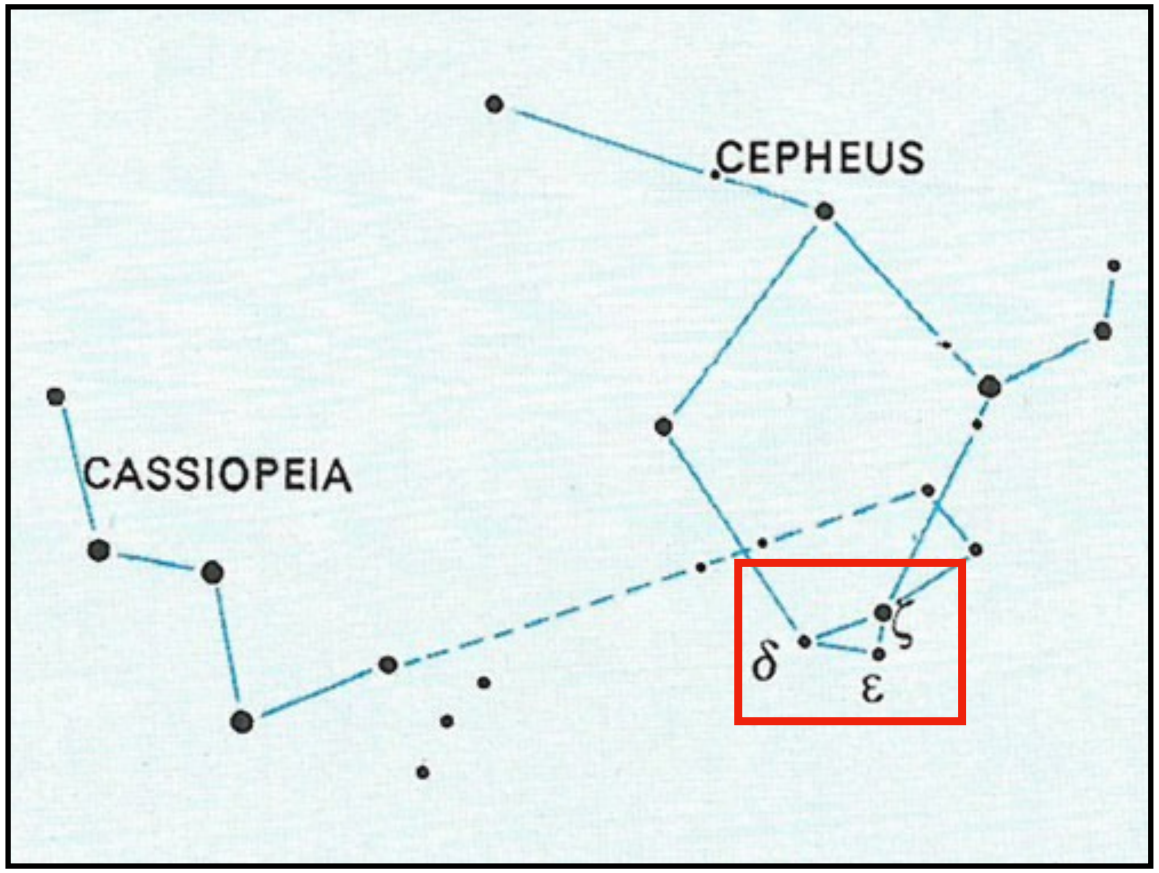
The pulsar near the center of the Crab Nebula

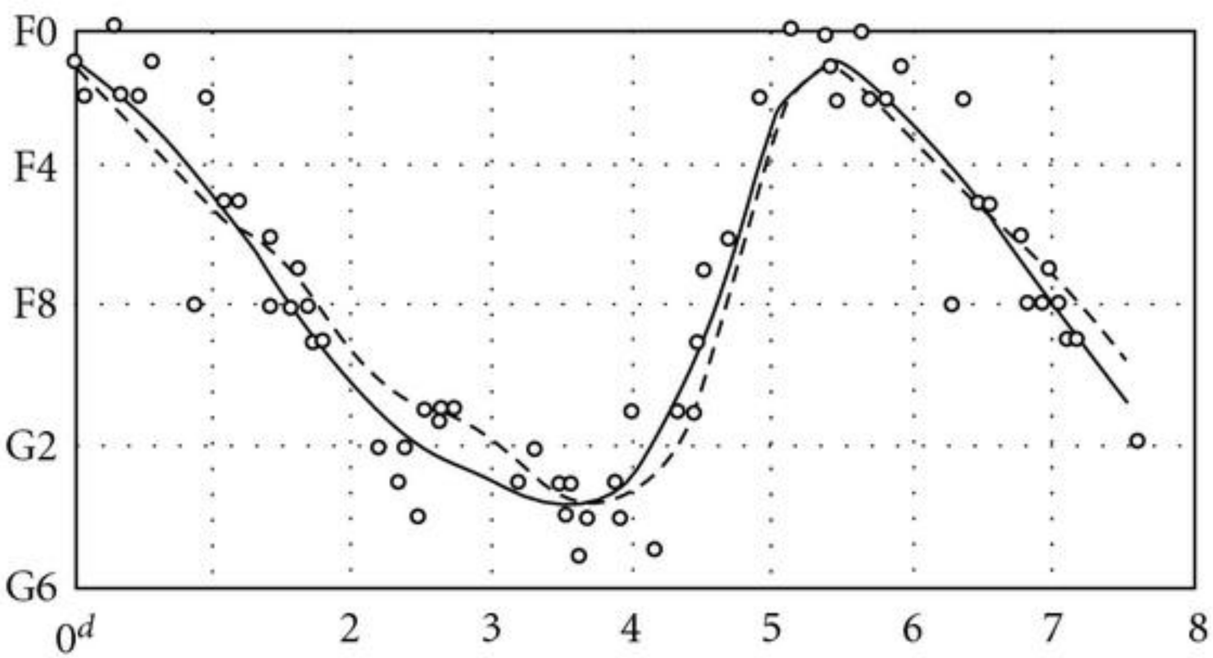


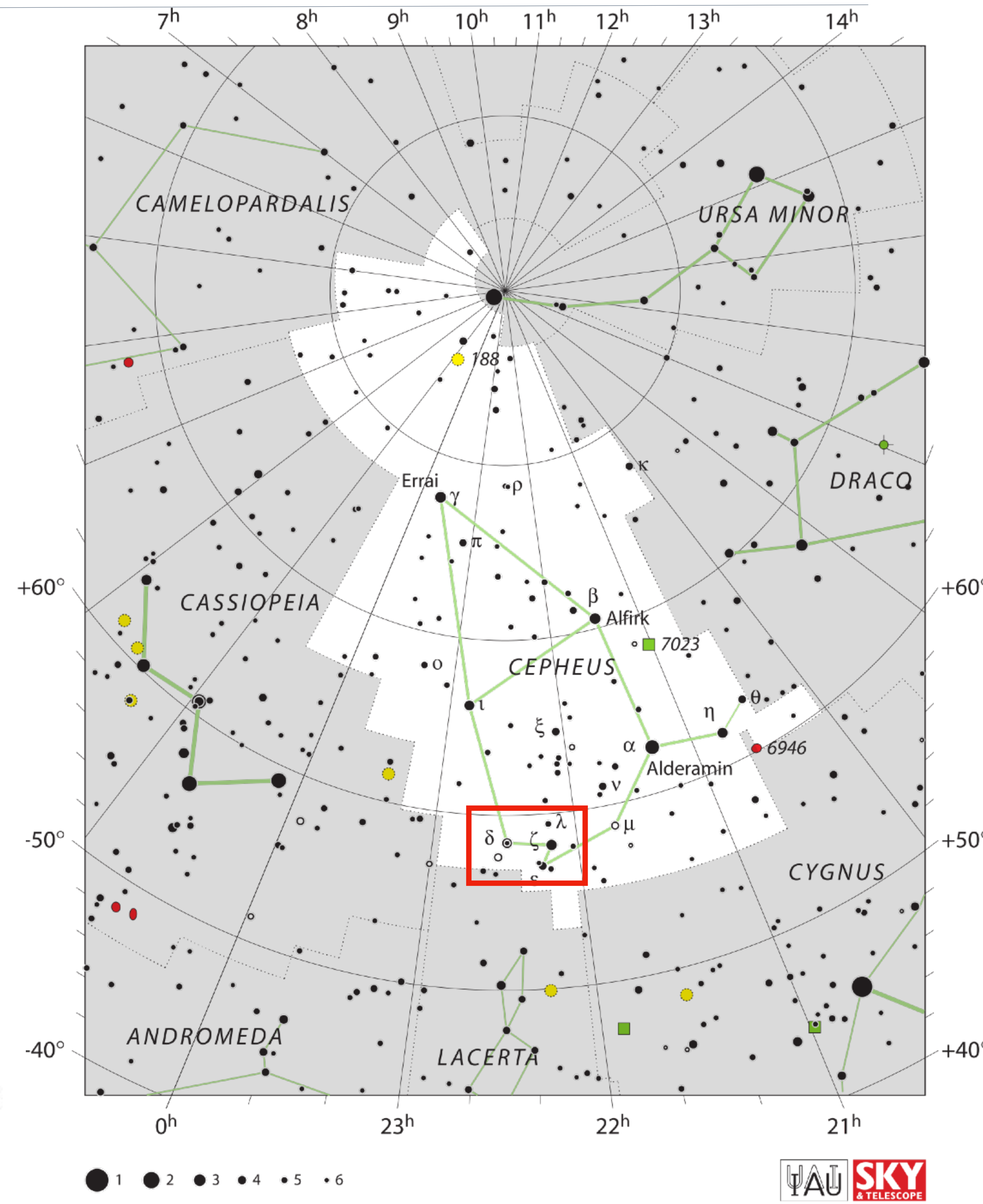
Pulsars animation

The Discovery of Cepheid Variables and the P-L Relation

Delta Cephei - the Prototype Cepheid Variable Star

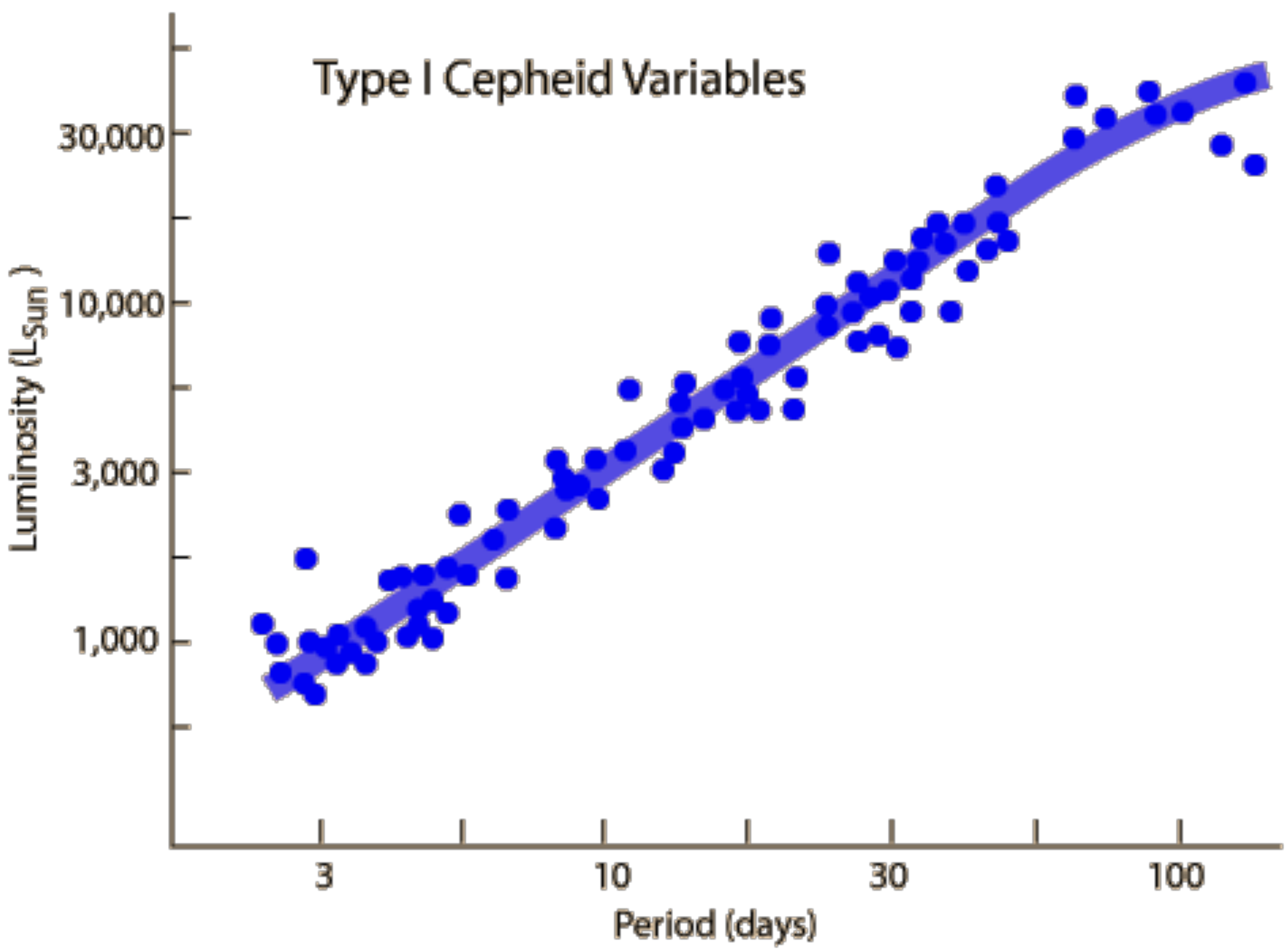






Period-Luminosity Relations (Leavitt's Law discovered in 1912)

- $M_V = -2.43\log(P_{\mathrm{day}}) - 1.62$ (Type I Cepheids, Fritz et al. 2007)
- This is critical for determining distances to other galaxies: $d_{\mathrm{pc}} = 10^{\frac{(m-M)+5}{5}}$



Henrietta Swan Leavitt

Born	July 4, 1868 Lancaster, Massachusetts, U.S.
Died	December 12, 1921 (aged 53) Cambridge, Massachusetts, U.S.
Education	Oberlin College Harvard University (BS)
Known for	Leavitt's law: the period-luminosity relationship for Cepheid variables

Annie Cannon and Henrietta Leavitt in 1913

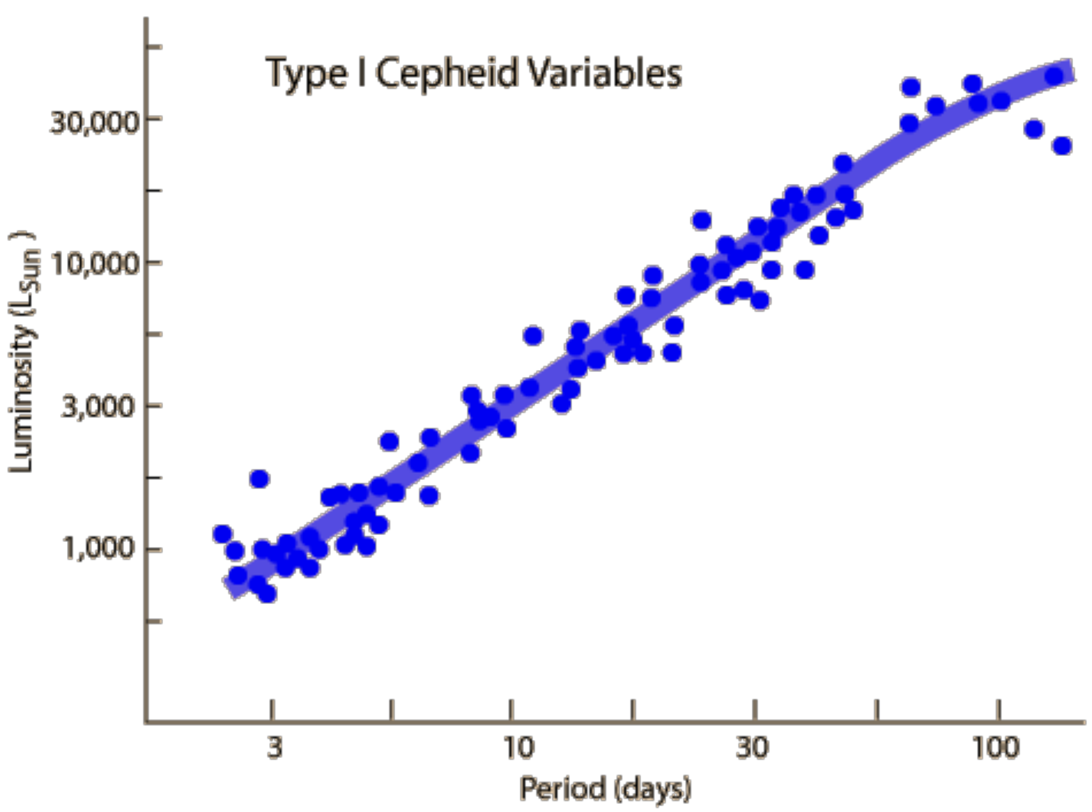
Annie Jump Cannon

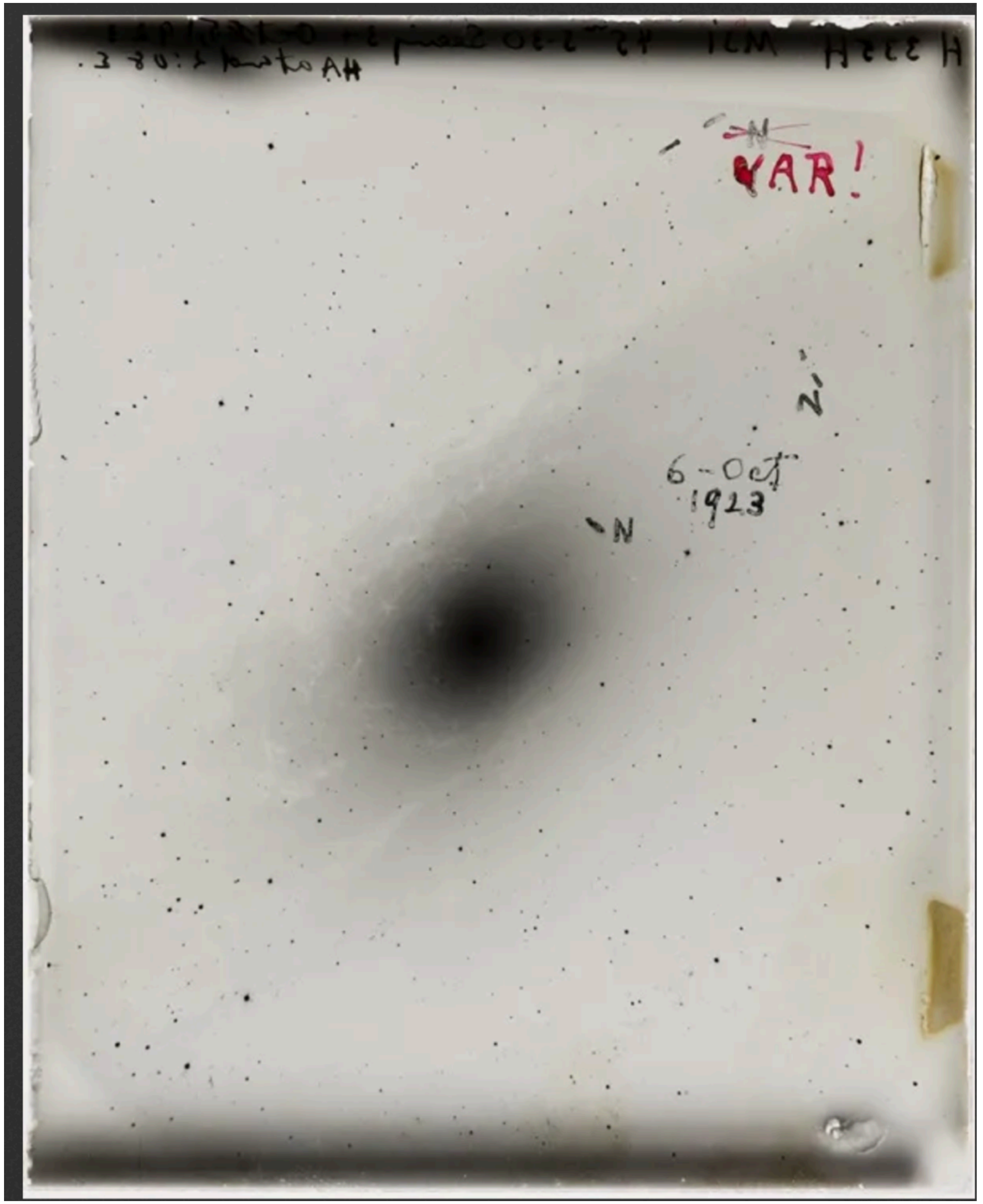
Cannon in 1922

Born	December 11, 1863 Dover, Delaware, U.S.[1]
Died	April 13, 1941 (aged 77) Cambridge, Massachusetts, U.S.
Alma mater	Wellesley College, Wilmington Conference Academy, Radcliffe College
Known for	Stellar classification

In 1923, Edwin Hubble used a Cepheid to determine distance to M31

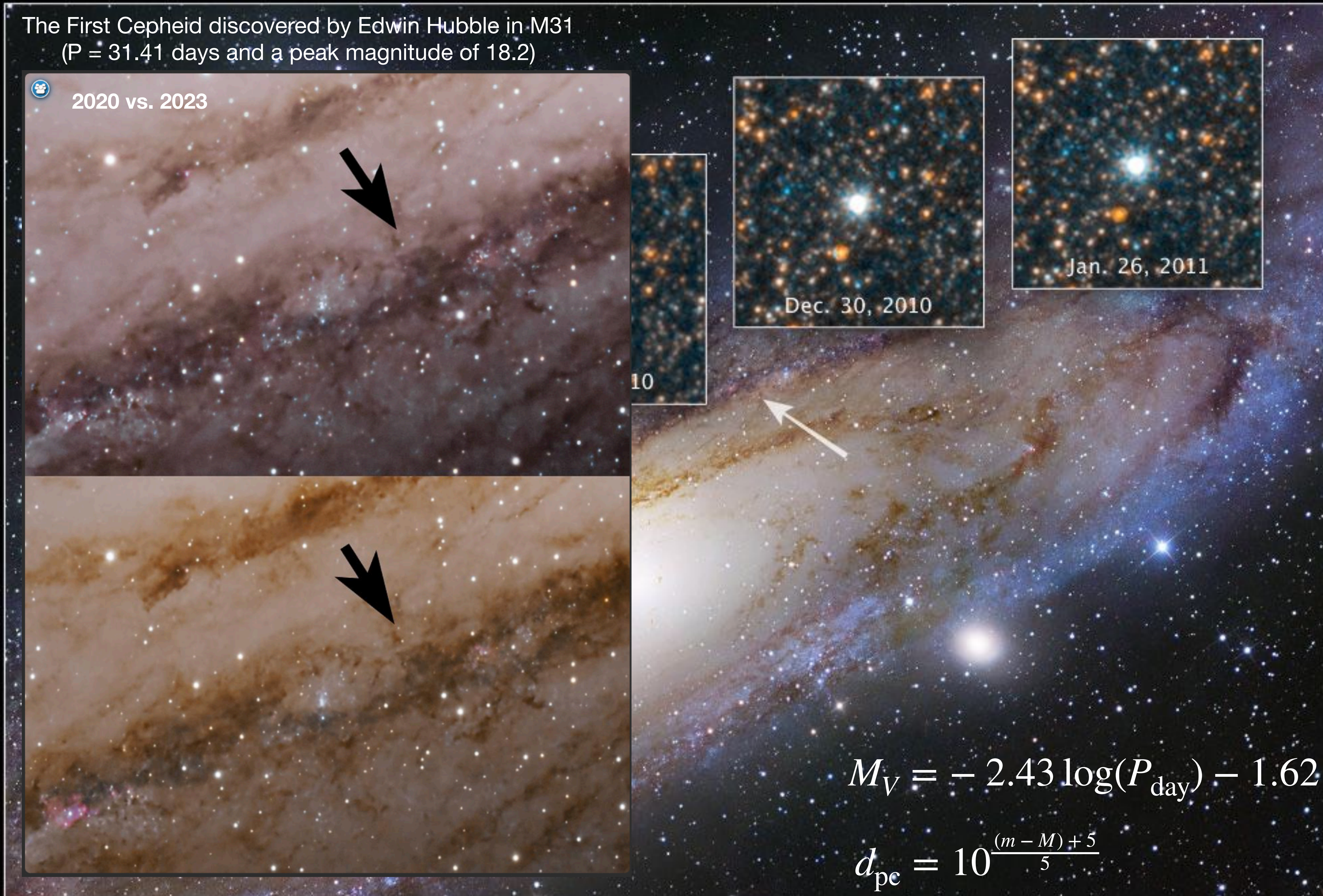
- $M_V = -2.43\log(P_{\mathrm{day}}) - 1.62$
  (Type I Cepheids, Fritz et al. 2007)
- This is critical for determining distances to other galaxies:
  $d_{\mathrm{pc}} = 10^{\frac{(m-M)+5}{5}}$





Cepheid Variable Star V1 in M31

Hubble Space Telescope ▪ WFC3/UVIS

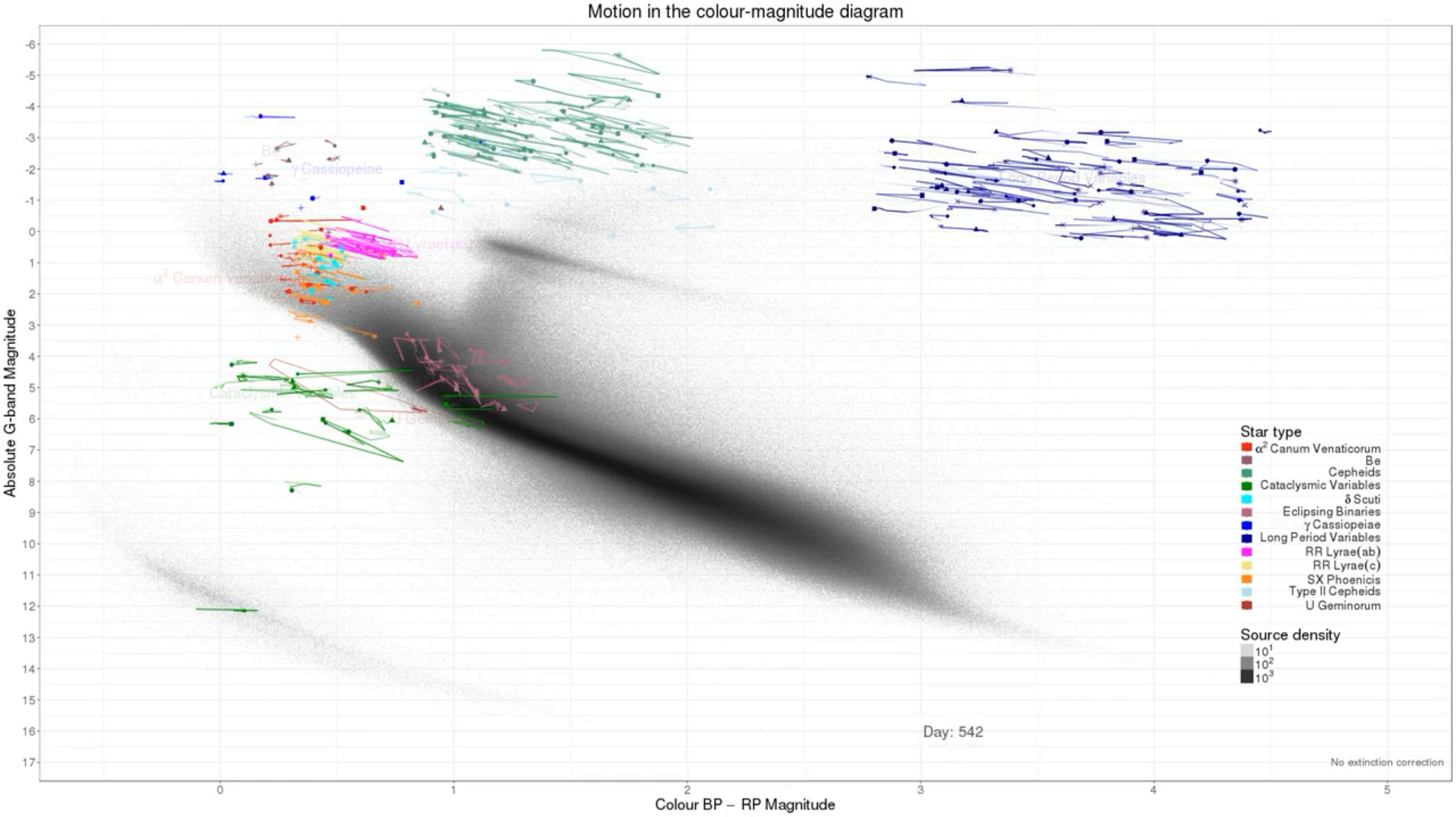


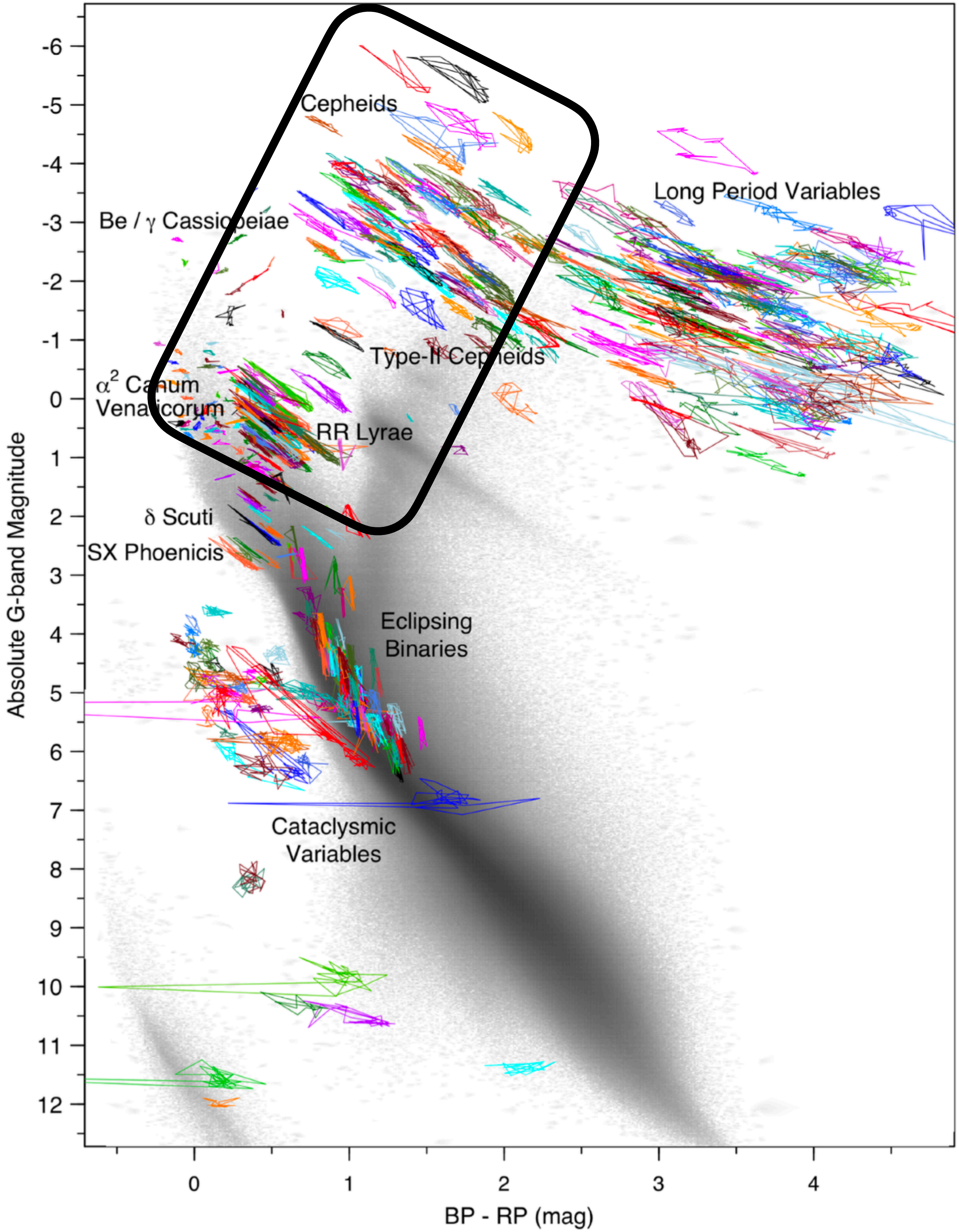
NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

STScI-PRC11-15a

The Instability Strip on the H-R Diagram

GAIA data release 2: Variable Stars on the HR Diagram

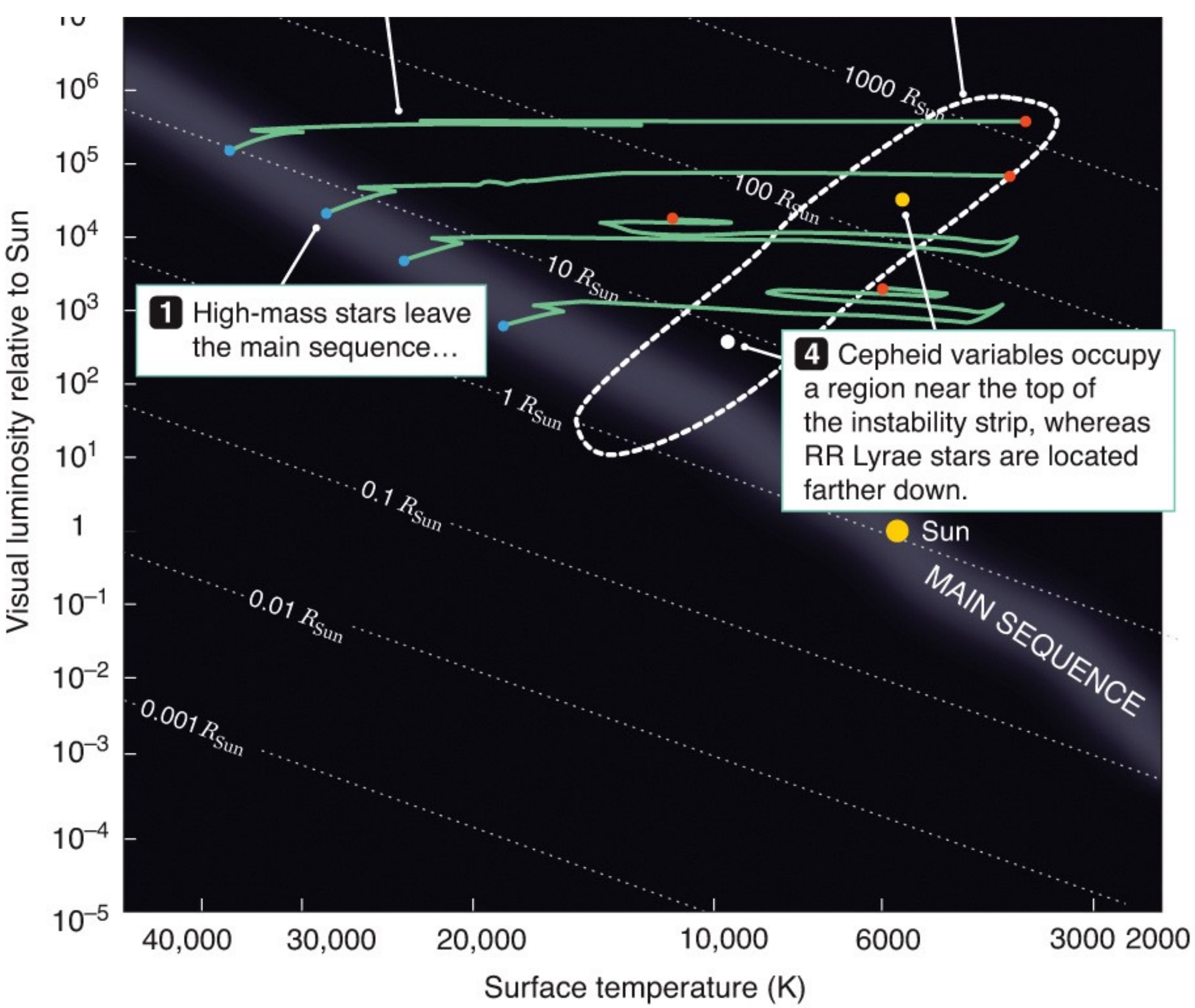


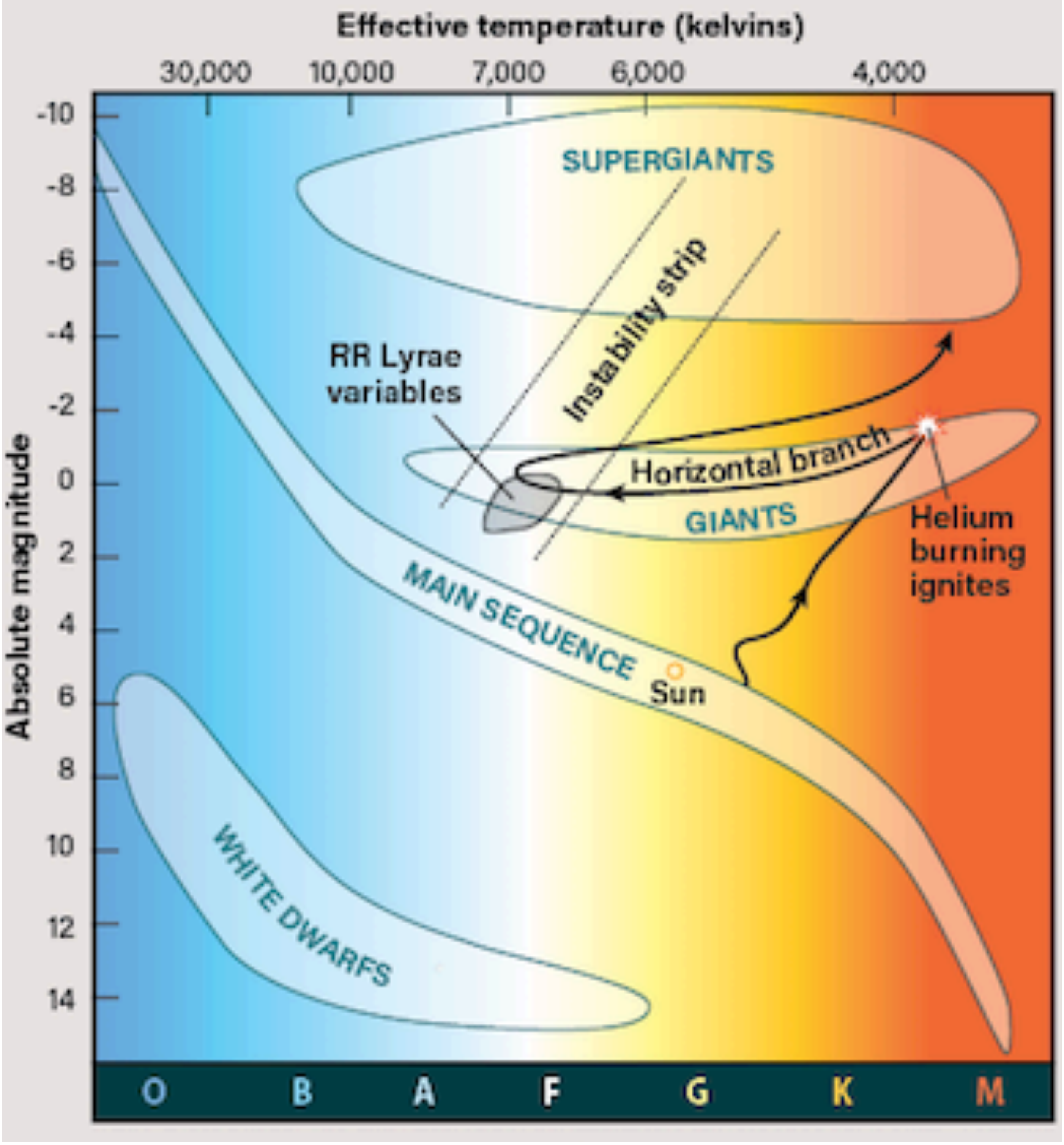


Cepheids
Long Period Variables
Be / γ Cassiopeiae
Type-II Cepheids
α2 Canum Venaticorum
RR Lyrae
δ Scuti
SX Phoenicis
Eclipsing Binaries
Cataclysmic Variables
Absolute G-band Magnitude
BP - RP (mag)

The Instability Strip on the HR Diagram

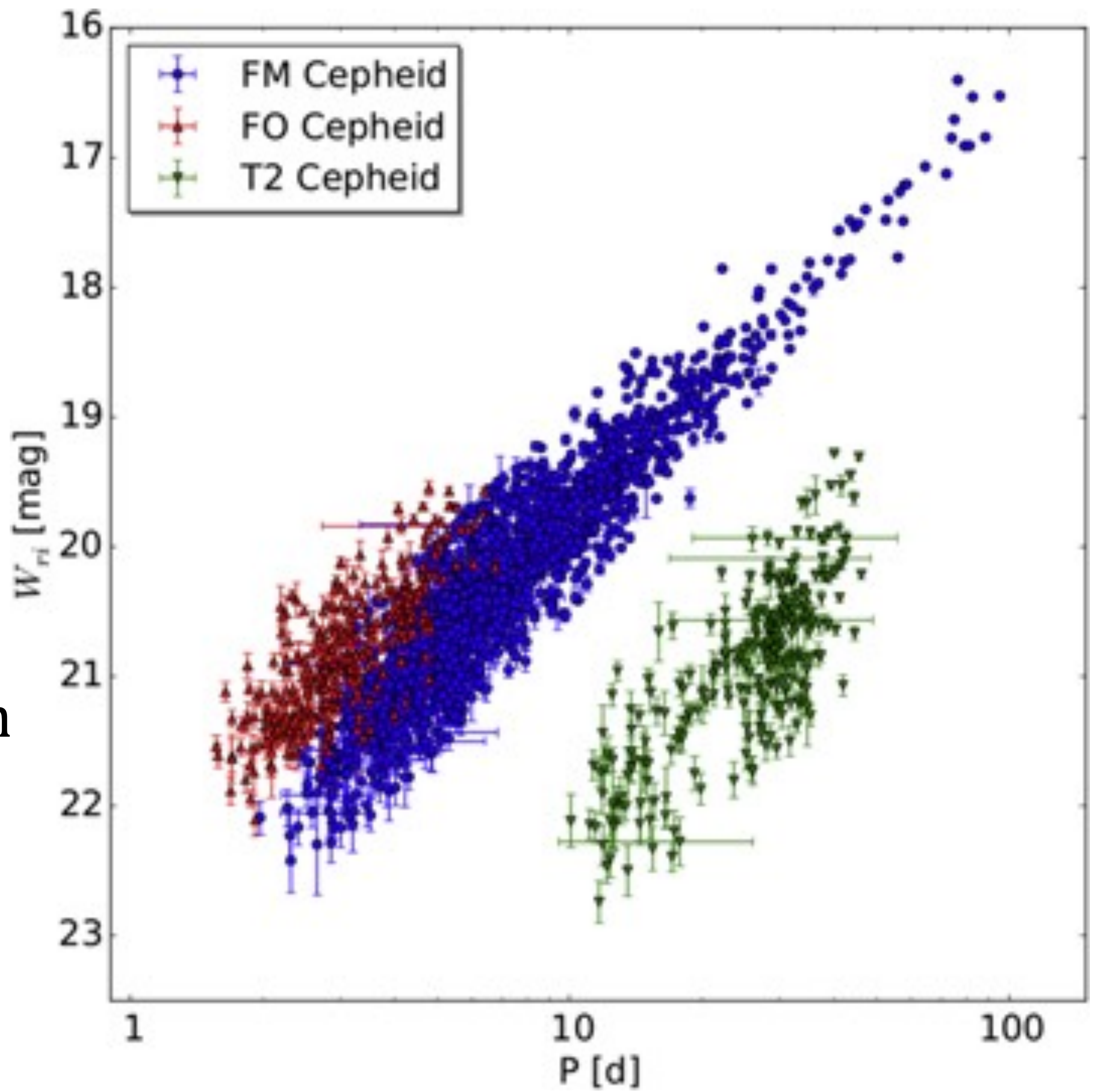
- **Pulsating variables** populate the **instability strip** on the HR diagram.
- During the pulsation, the stars change in both **radius** and **temperature**





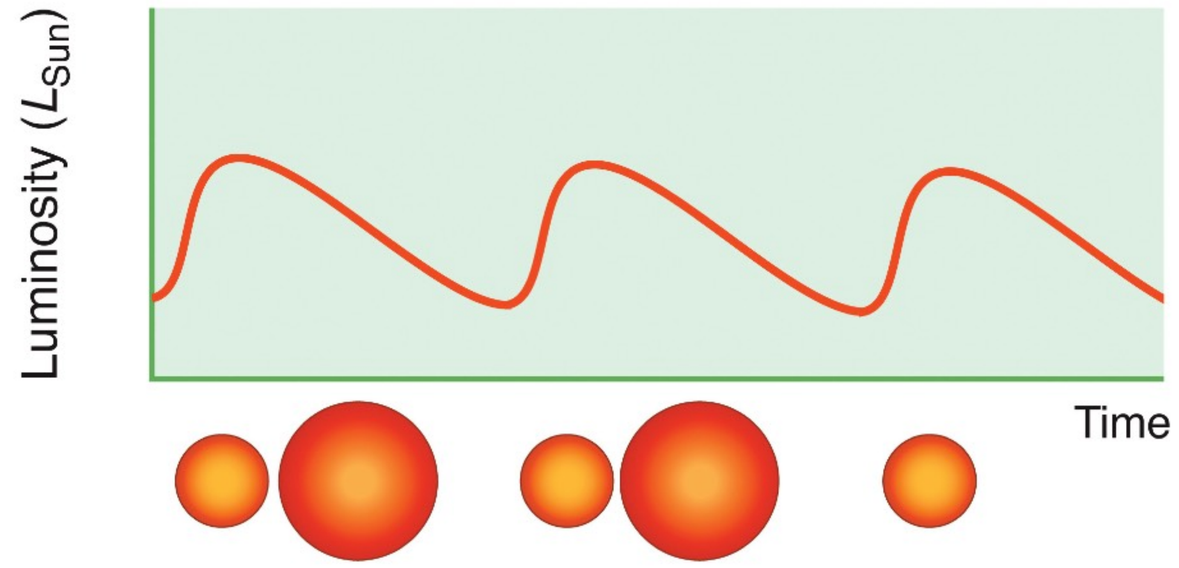
Period-Luminosity Relation of Different Types of Cepheids

- Based on the shape of the light curve, astronomers have classified three main types of Cepheids:
  - FM - Fundamental Mode
  - FO - First Overtone
  - T2 - Type II
- The P-L relations of the three types differ from each other, as illustrated on the diagram using ~2000 Cepheids in M31 (Kodric+2018; Fig 10).

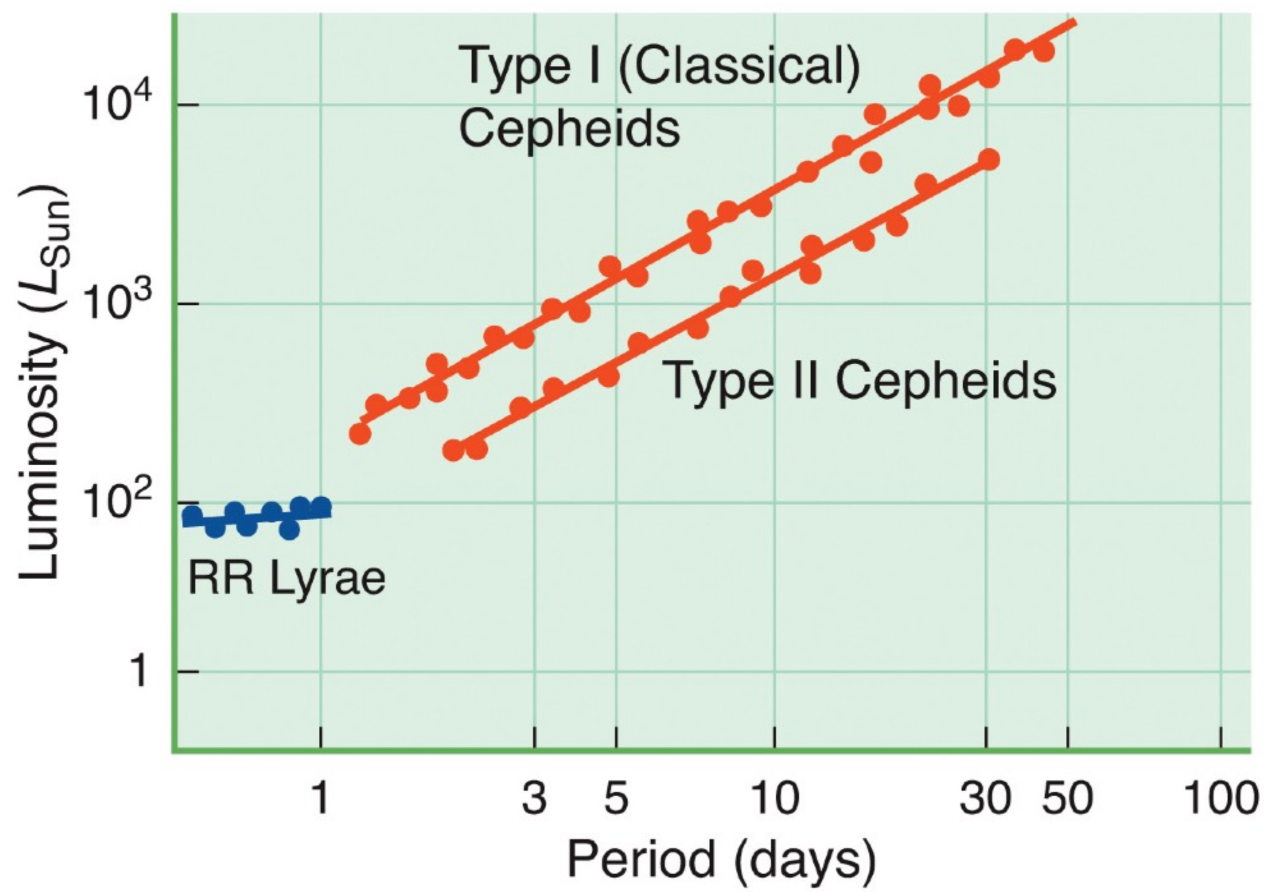


How to Tell Apart the Different Types of Pulsating Variable Stars?

- **Type I Cepheid variables**
  - Classic Cepheid variables are high-mass stars becoming supergiants.
  - They have periods from 1 to 100 days.
- **RR Lyrae variables & Type II Cepheid variables**
  - These are low-mass stars on the horizontal branch.
  - They are less luminous than Cepheid variables.
  - They follow different L-P relations

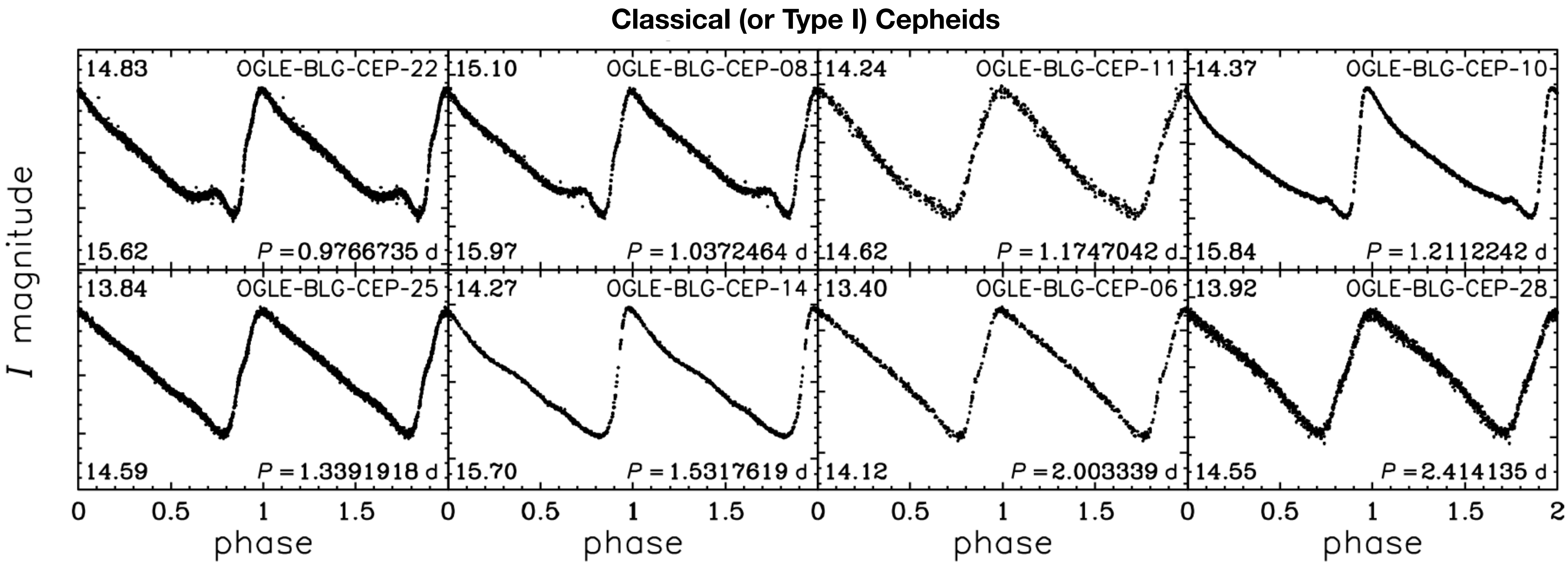


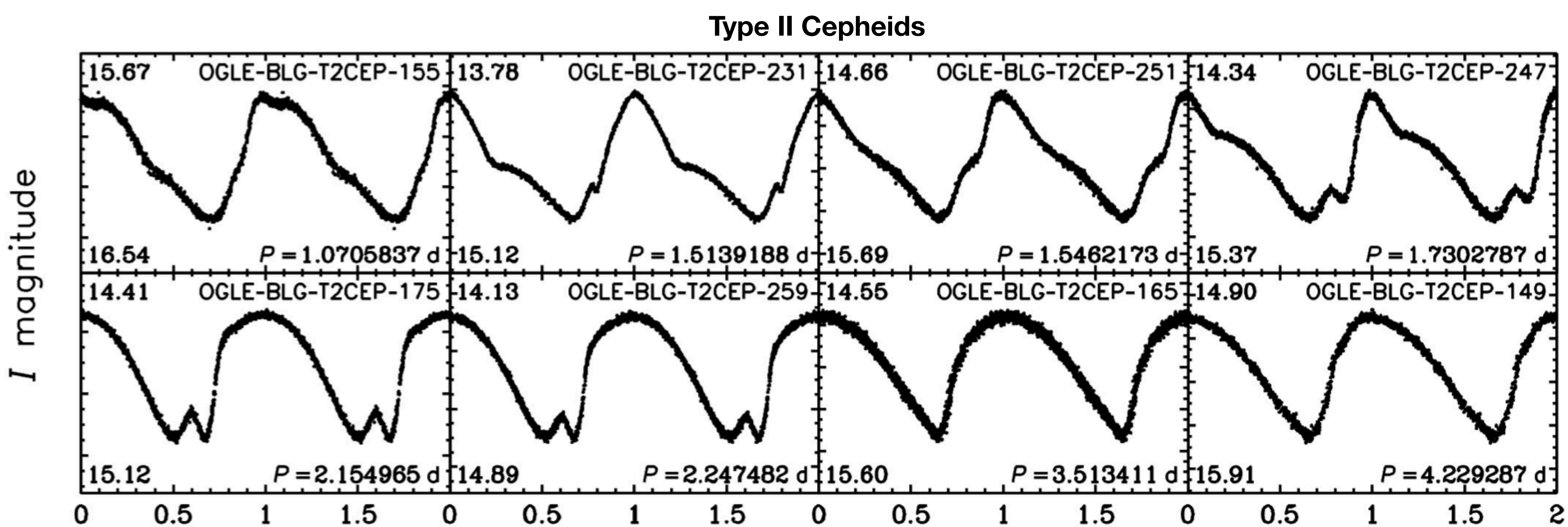
Luminosity vs. time



Period-luminosity relationship

Cepheids Light Curves - Type I vs. Type II Cepheids

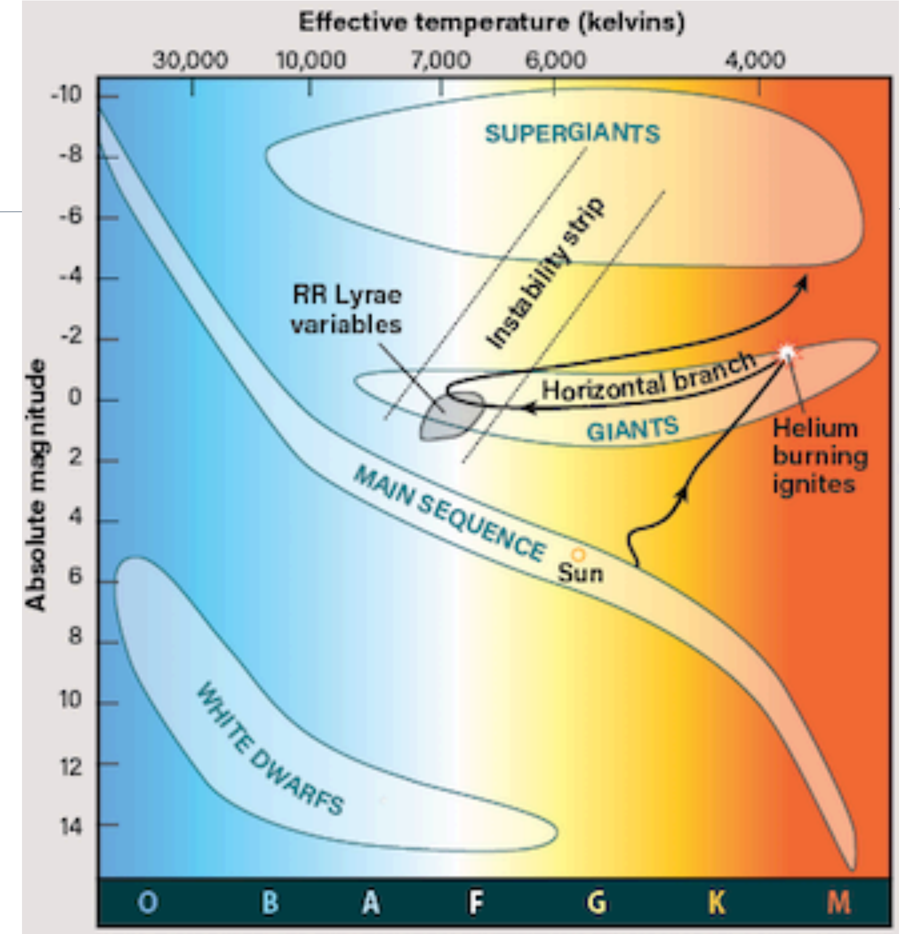




https://arxiv.org/abs/1112.1406

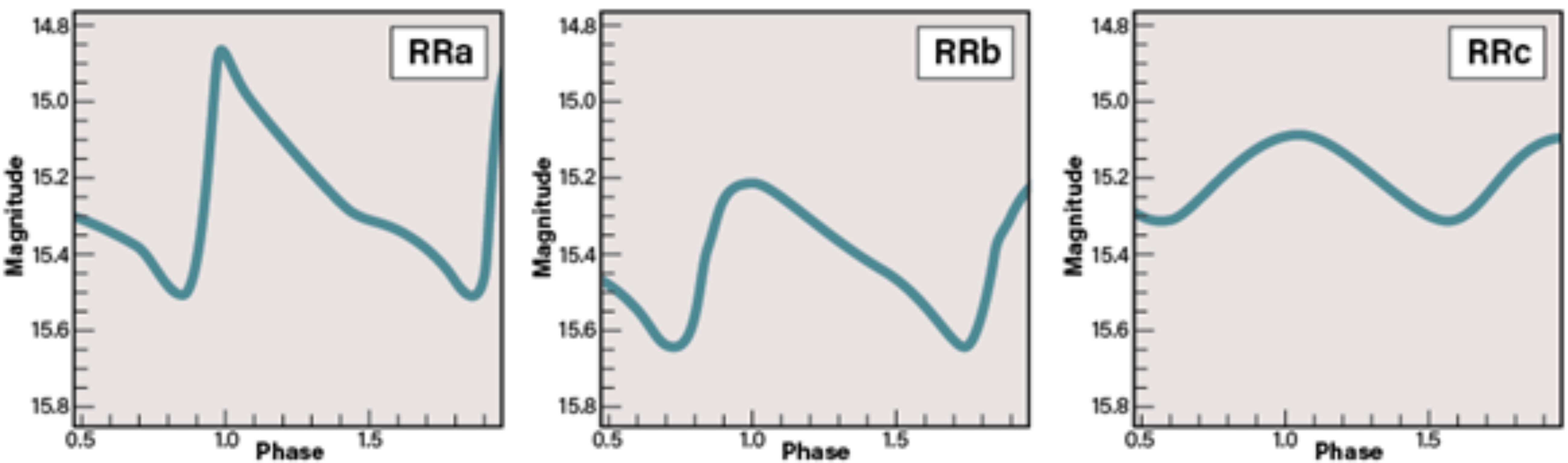
RR Lyrae - Shorter Periods

- RR Lyrae variables have periods shorter than one day.
- Like Cepheids, their light curves show a variety shapes.
- They are low-mass stars in the horizontal branch phase.



RR LYRAE LIGHT CURVES

There are two major classes of RR Lyrae stars, based on the shape of their light curve, which measures a star's brightness over time: RRab- (left, middle) and RRc-type stars. ASTRONOMY: ROEN KELLY

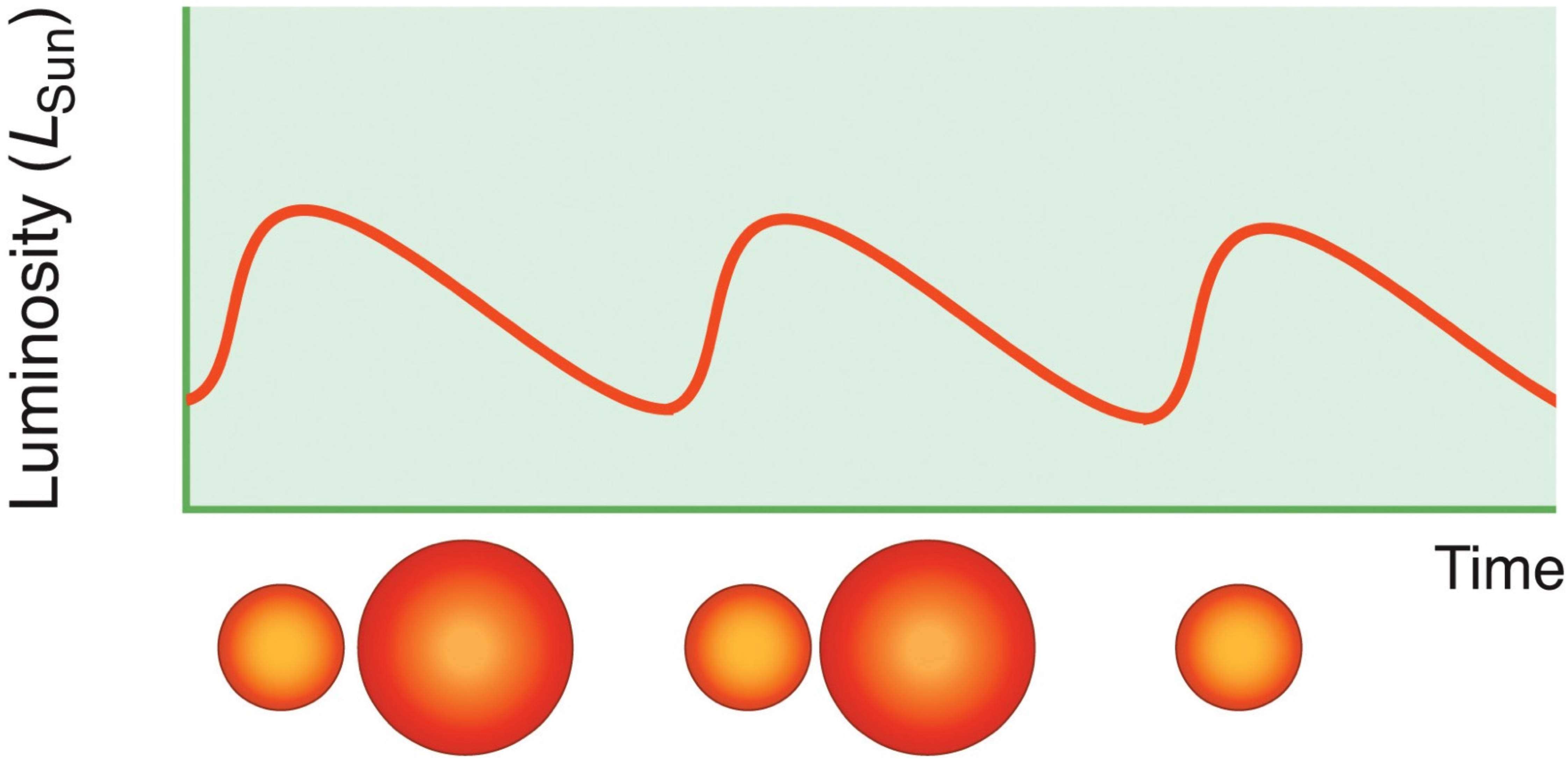


https://astronomy.com/magazine/news/2020/07/how-pulsating-stars-unlock-our-universe

What's the physical mechanism that drives the pulsation?

Pulsating Variable Stars Change in Size and Temperature

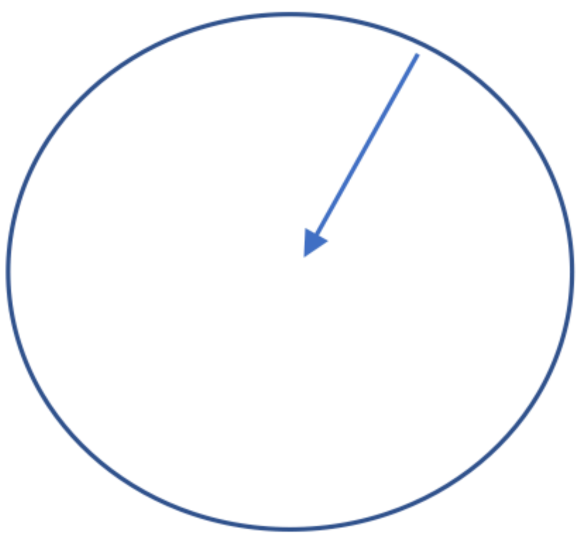
- The star's **luminosity** changes as their **radius** and **temperature** changes at a regular period.
- A star can evolve into a **pulsating variable star when its interior fails to** achieve a steady balance between pressure and gravity (i.e., it overshoots and undershoots).



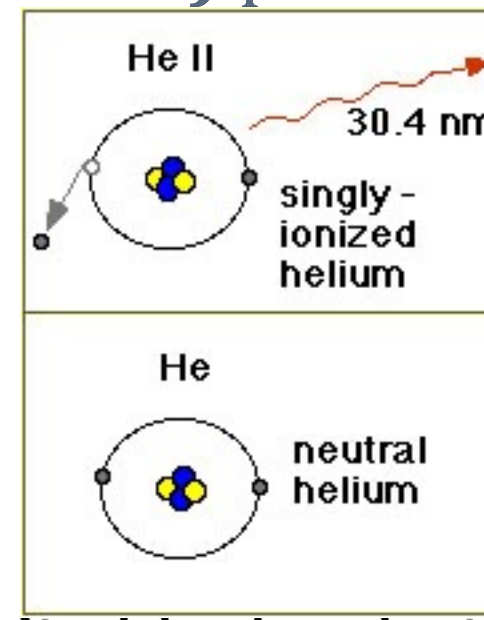
Pulsation Caused by Helium (He) Ionization and Recombination

- **Pulsations** are caused by the atmosphere oscillating between **more ionized (more opaque)** and **more neutral (more transparent)** phases.

- Photons go through gas
- Gravity condenses star

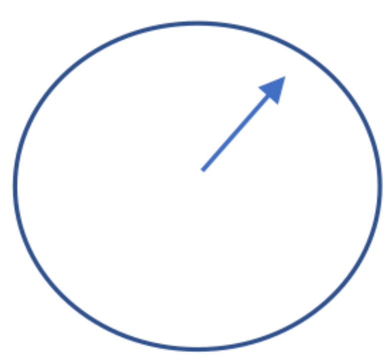


- Temperature increases
- He is ionized
- Gas becomes opaque



- Opacity blocks photon flow
- Pressure increases

- Less ionization means less blocked photons
- Decreases pressure



- Pressure expands star
- He cools and becomes less ionized

Peter Wysocki

Pulsation Caused by Gas Ionization and Recombination

The **kappa(opacity)-mechanism** works as follows:

- As the star contracts, its temperature rises, causing the outer layers to become more ionized.
- The **increased** ionization leads to a corresponding **increase** in **opacity** in these layers.
- This increase in opacity inhibits the outward flow of radiation, leading to a buildup of pressure.
- The increased pressure causes the outer layers to expand, which in turn causes the temperature to decrease.
- As the temperature decreases, the outer layers become **less ionized**, reducing the **opacity**.
- With reduced opacity, radiation can escape more easily, leading to a decrease in pressure.
- The decreased pressure allows the outer layers to contract again, starting the cycle anew.

This cyclical process results in periodic expansions and contractions of the star's outer layers, which manifest as changes in brightness observed from Earth. The period of these pulsations is related to the time it takes for a pressure wave to travel through the star's interior and back to the surface.

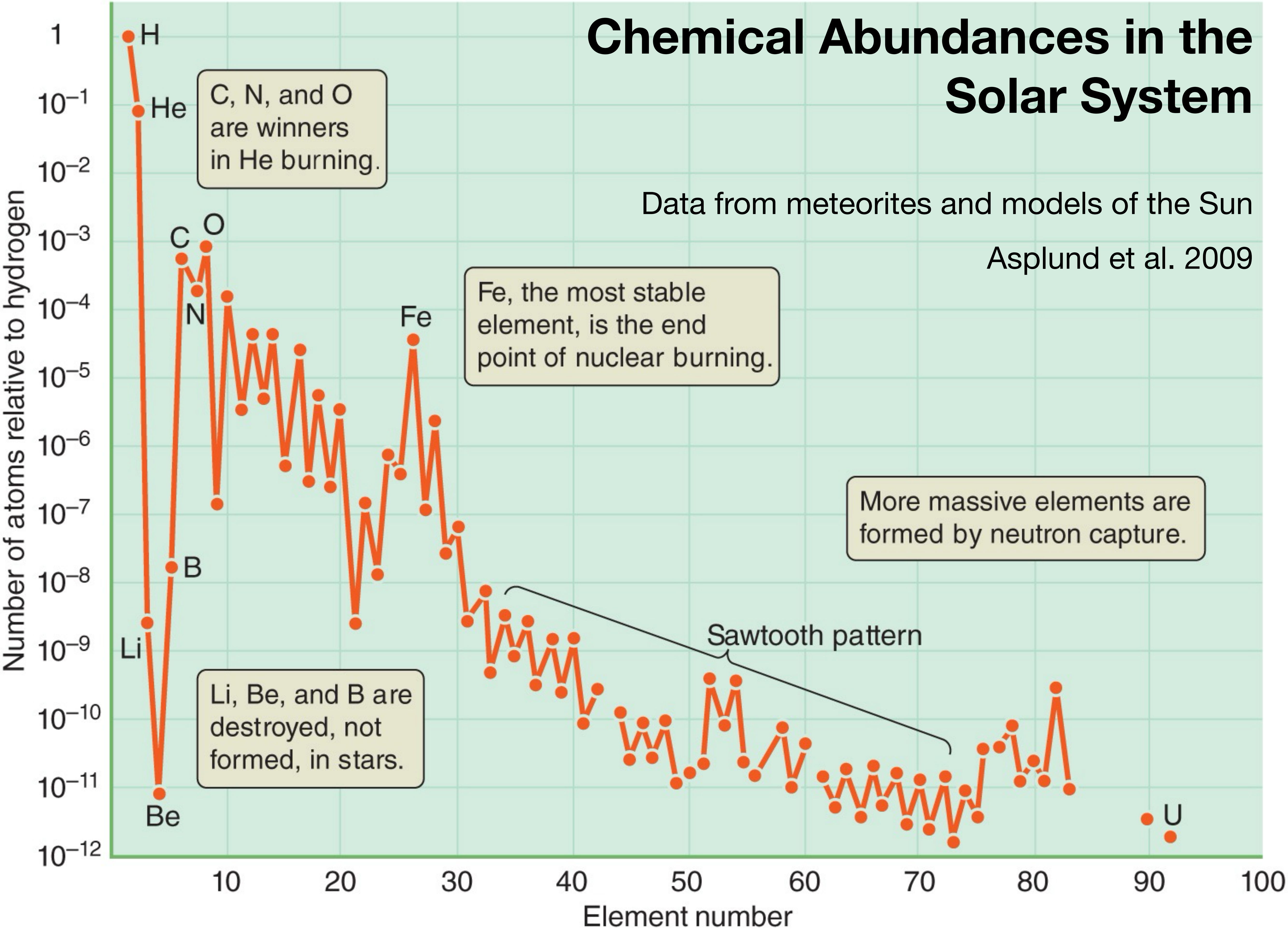
The Origin of Elements: from H to Fe

Nucleosynthesis in the Big Bang, stellar cores, and supernovae explosions

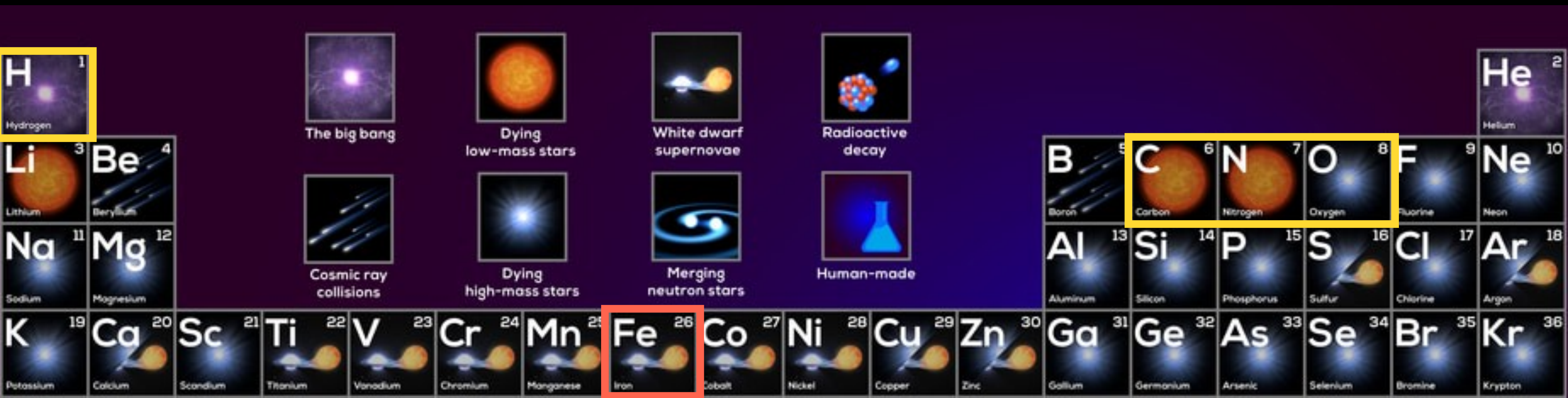
Chemical Abundances in the Solar System

Data from meteorites and models of the Sun

Asplund et al. 2009



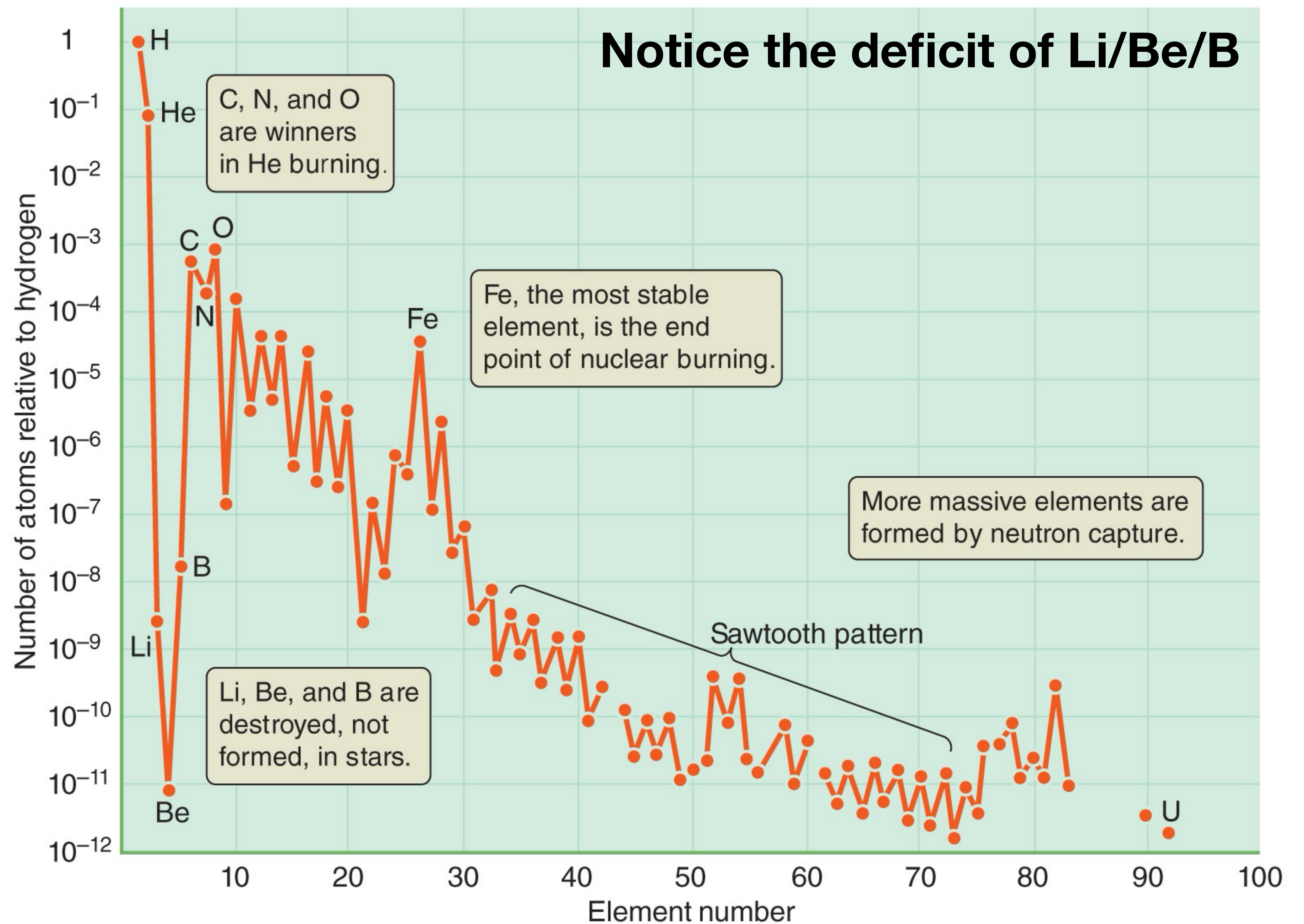
The Primary Origin of Each Element
(up to two main contributors are shown for each element)



- The **hydrogen** atoms in water were created 13.7 billion years ago in the Big Bang (t = 3 min, T = *1e9 K, density = 1e5 kg/m*^{3})
- The **oxygen** atoms in the air you breathe and the water you drink were created by nucleosynthesis in the cores of high mass stars and released into the ISM via type II SNe.
- The **carbon** atoms were formed in the cores of low-to-intermediate mass stars and released into the ISM in the Post-AGB phase.
- The **iron** atoms that are a key element of hemoglobin, which makes up the red blood cells that carry oxygen from your lungs to the rest of your body, formed in the explosion of white dwarfs (type Ia SNe).

The Origin of Elements: Li, Be, and B

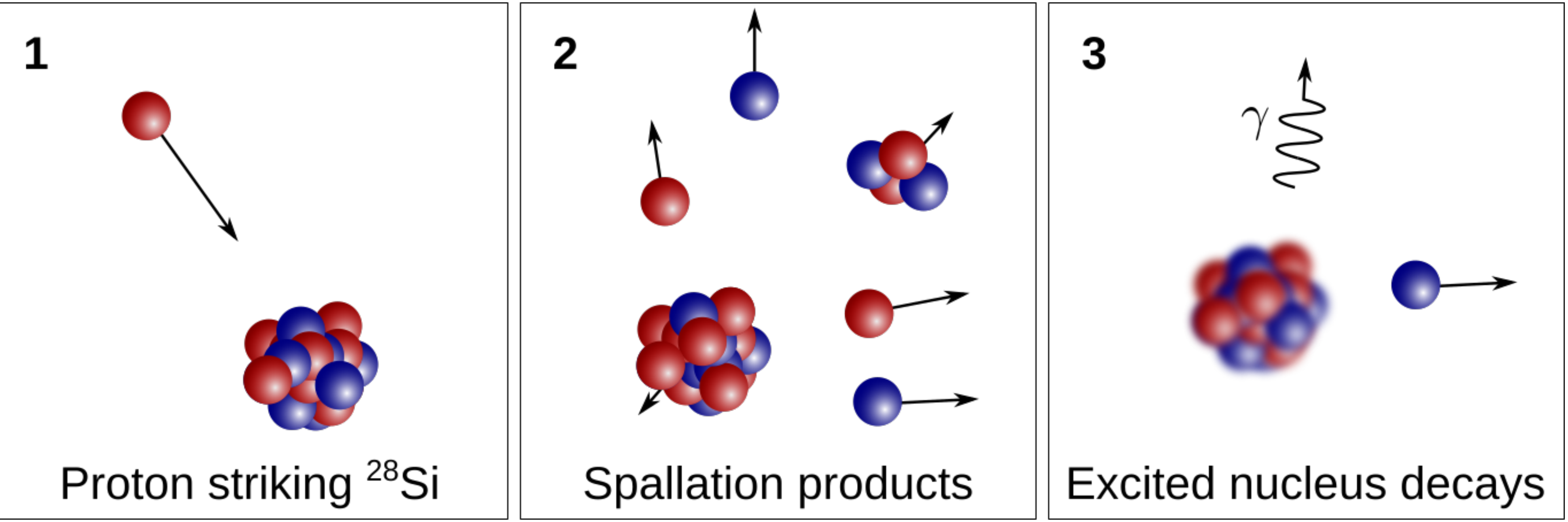
Classical Nova, AGB stars, and Cosmic Ray Spallation



Notice the deficit of Li/Be/B
C, N, and O are winners in He burning.
Fe, the most stable element, is the end point of nuclear burning.
More massive elements are formed by neutron capture.
Li, Be, and B are destroyed, not formed, in stars.
Sawtooth pattern
H
He
Li
Be
B
C
N
O
Fe
U
Number of atoms relative to hydrogen
Element number
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10
20
30
40
50
60
70
80
90
100

Cosmic Ray Spallation (x-process): Be and B

- **Cosmic rays** are high energy particles emitted by astrophysical sources like our Sun, supernovae, and active galactic nuclei (1936 Nobel Prize)
- Cosmic rays can hit other nuclei and cause them to split. This process is called **Cosmic Ray Spallation** or **x-process**
- **Lithium, Beryllium, and Boron** are *destroyed* in stars, the x-process is responsible for their abundances:
  e.g., $n + {}^{14}N \rightarrow p + {}^{4}He + {}^{10}Be$, then ${}^{10}Be \rightarrow {}^{10}B + e^-$ (beta decay)



Lithium Production in AGB and Classical Nova (Cameron & Fowler 1971)

- *As part of the triple-alpha process in Helium burning, Beryllium forms:*
  - ^{3}He + ^{4}He -> ^{7}Be + photon
- *if Be is transported to cooler regions (10^{6} K) by convection, it can form Lithium:*
  - ^{7}Be + e^{-} -> ^{7}Li + neutrino
- *Otherwise, Beryllium fuses with hydrogen to form Boron:*
  - ^{7}Be + p -> ^{8}B + photon

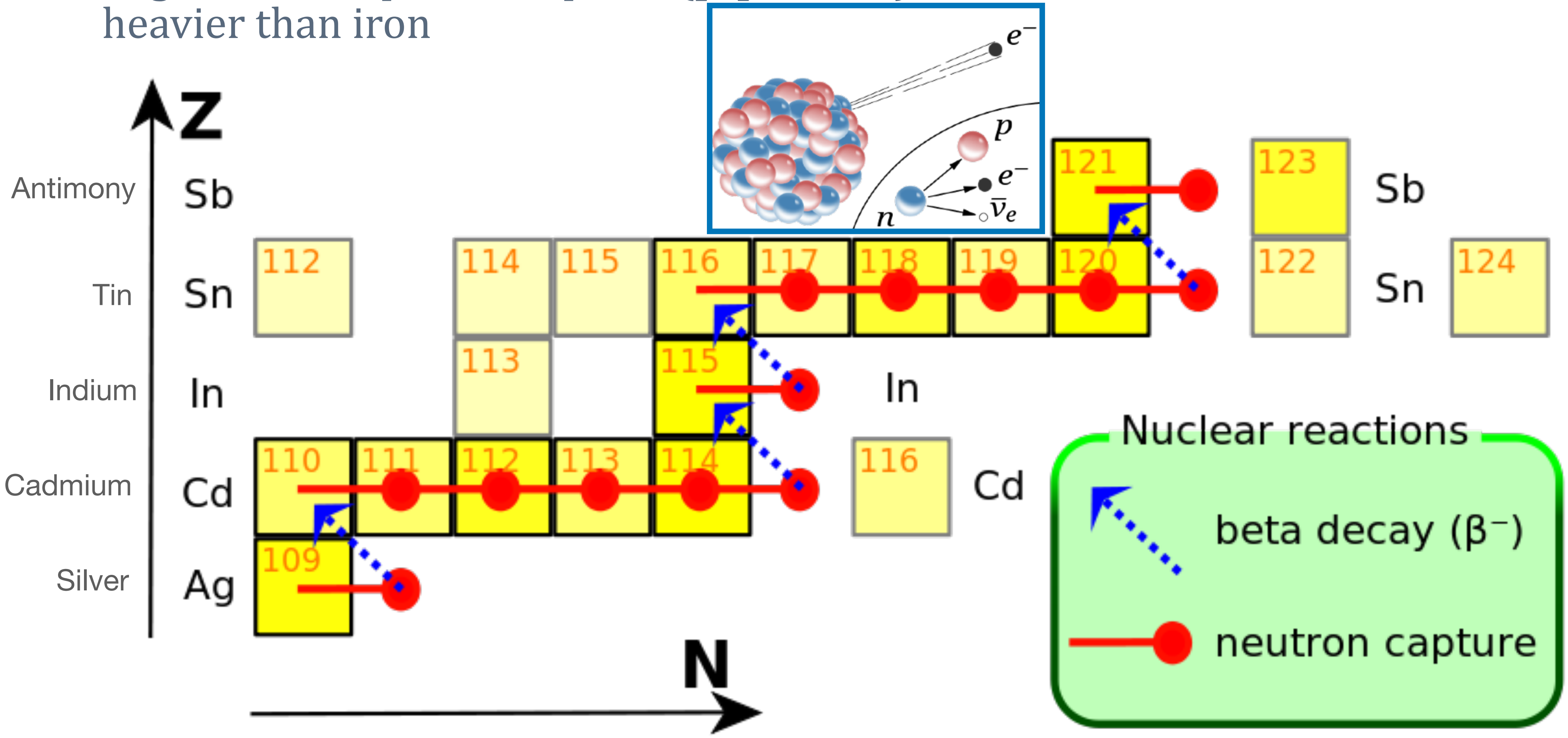
Illustration by David A. Hardy

The Origin of Elements: beyond Fe

Neutron star mergers and AGB Stars

Rapid and Slow Neutron Capture Processes (r- / s-process)

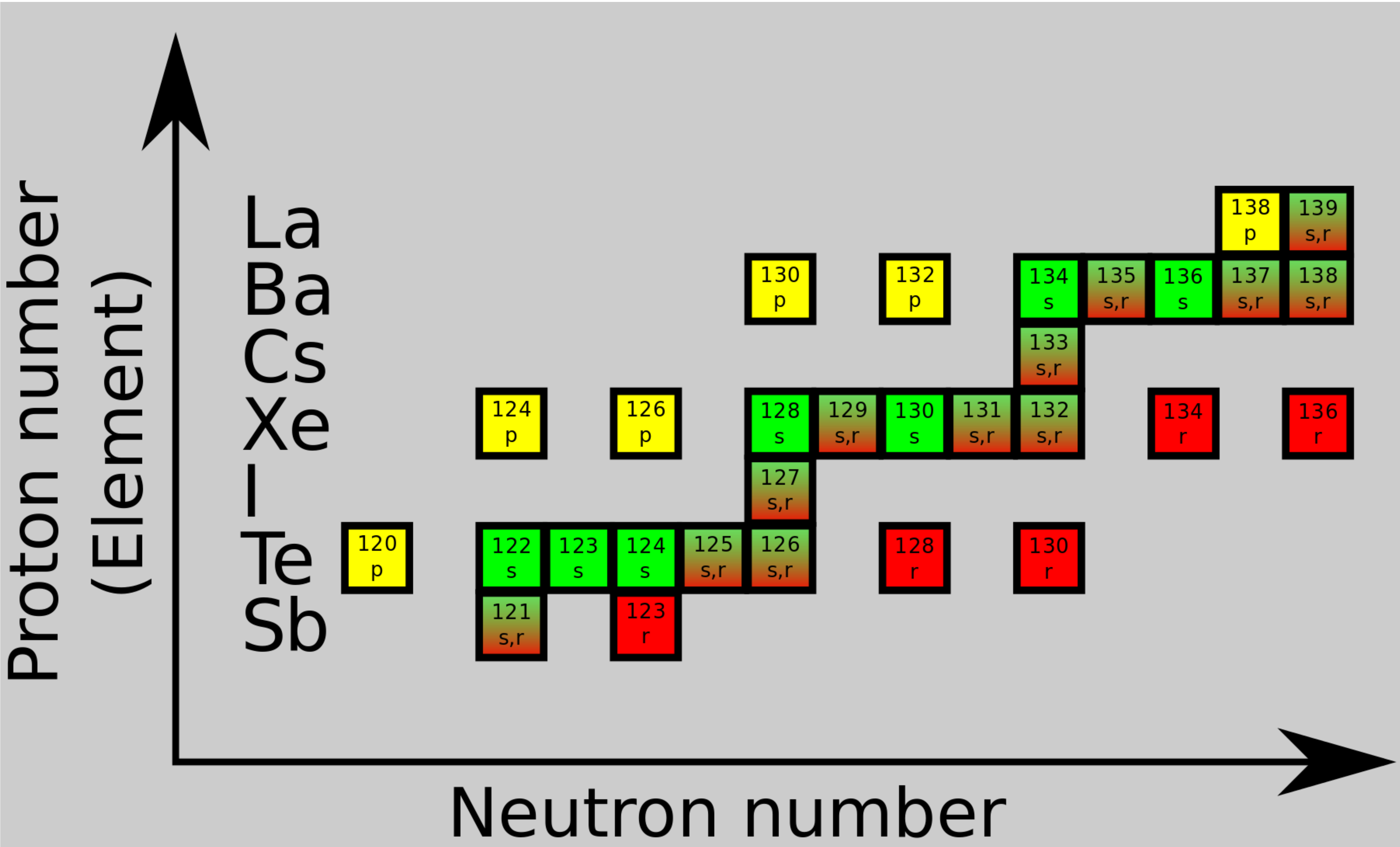
- ***Rapid*** neutron capture: the nuclei can capture multiple neutrons before the **beta decay** (emission of an electron)
  - **neutron star mergers** (short Gamma-ray bursts), given the high neutron fluxes
  - Makes half of the nuclei heavier than iron (mostly **neutron-rich isotopes**)
- ***Slow*** neutron capture: the nuclei undergo **beta decay** before another neutron can be captured.
  - Important in **AGB stars**, neutron flux comes from $^{13}C(\alpha, n)^{16}O$ & $^{22}Ne(\alpha, n)^{25}Mg$
  - Together with proton capture (**p-process**), makes the other half of the nuclei heavier than iron



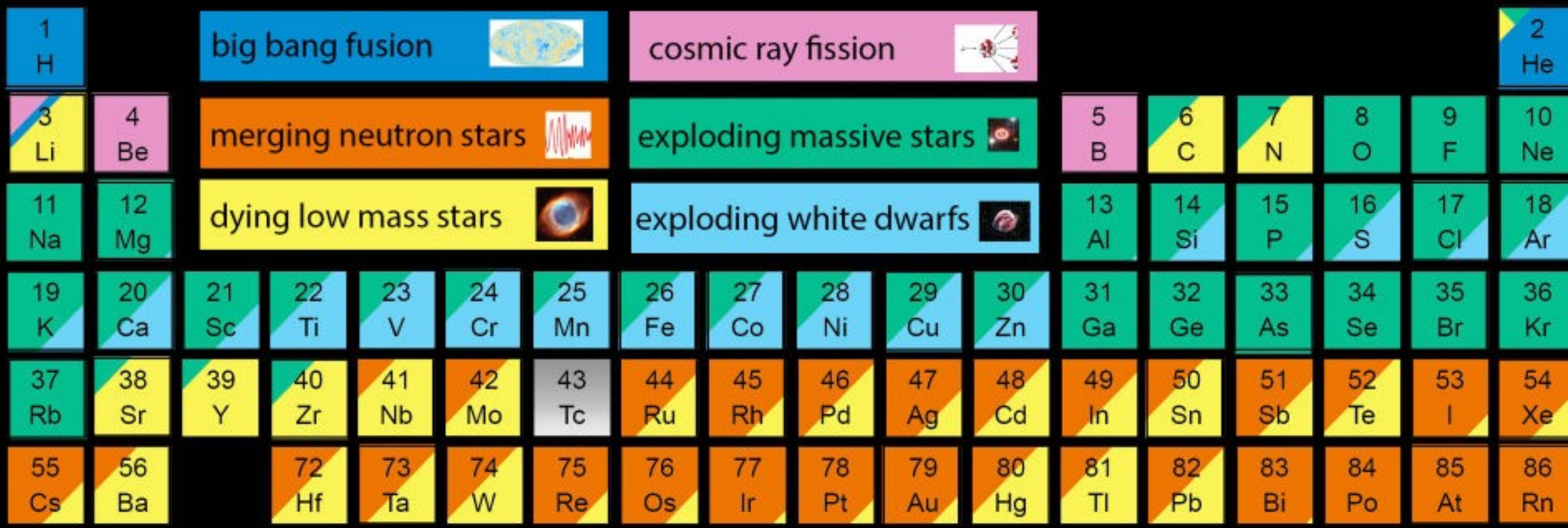
Neutron Star Merger: Gravitational Wave & Gamma-Ray Burst

The Proton Capture Process (p-process)

- makes **neutron-deficit** isotopes from selenium (Se-34) to mercury (Hg-80)

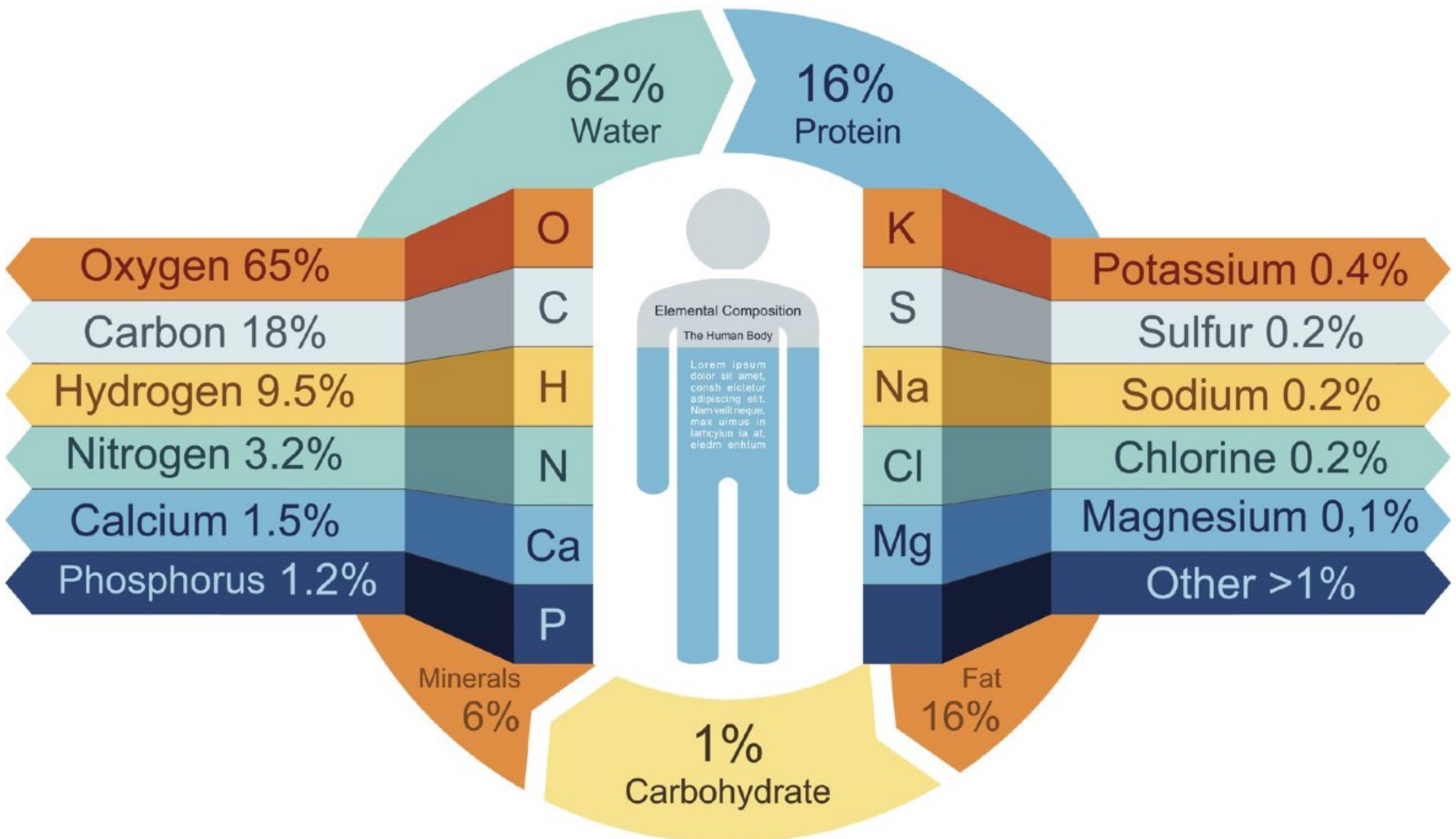


Summary: The Six Astrophysical Sources of Elements

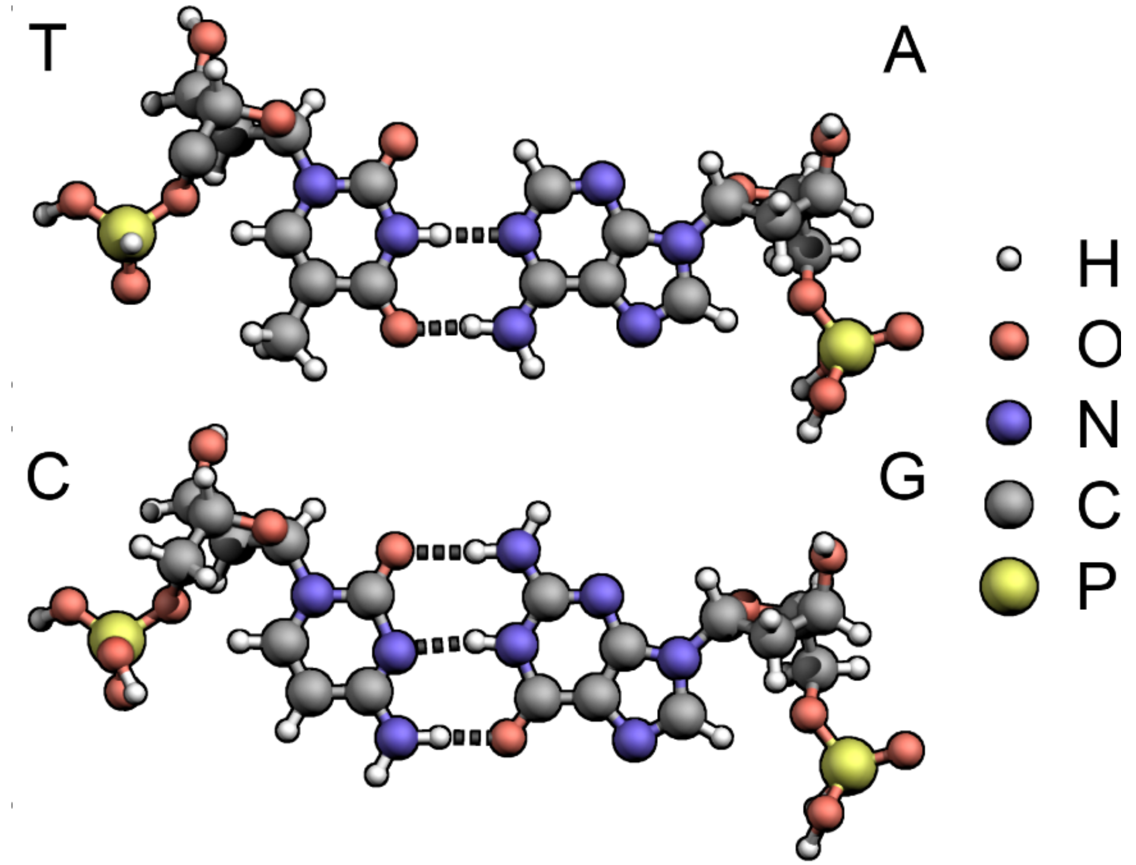


(top two/three main contributors are shown foreach element)

THE HUMAN BODY



DNA

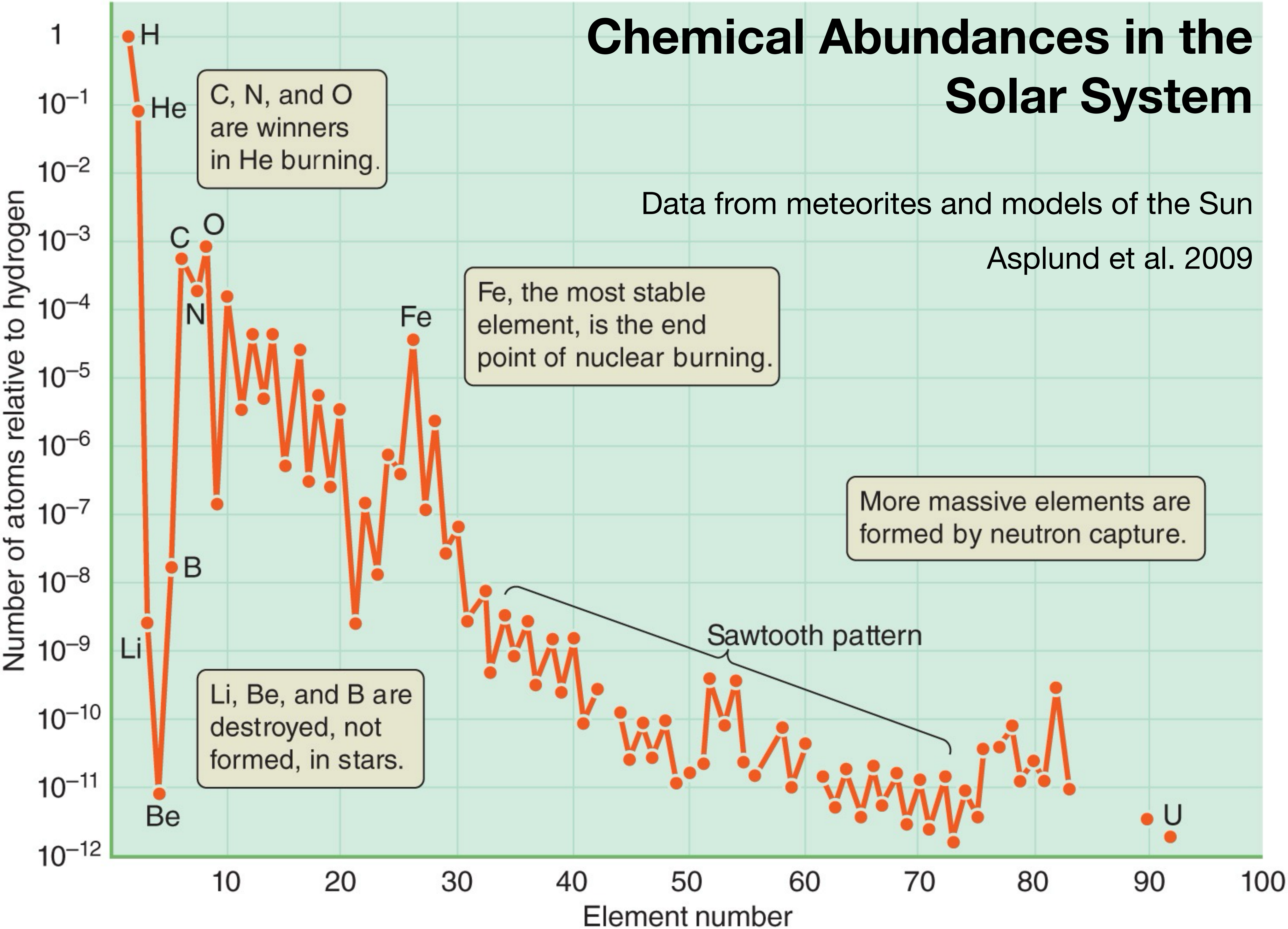


"We are all made of stardust" — Carl Sagan

Chemical Abundances in the Solar System

Data from meteorites and models of the Sun

Asplund et al. 2009

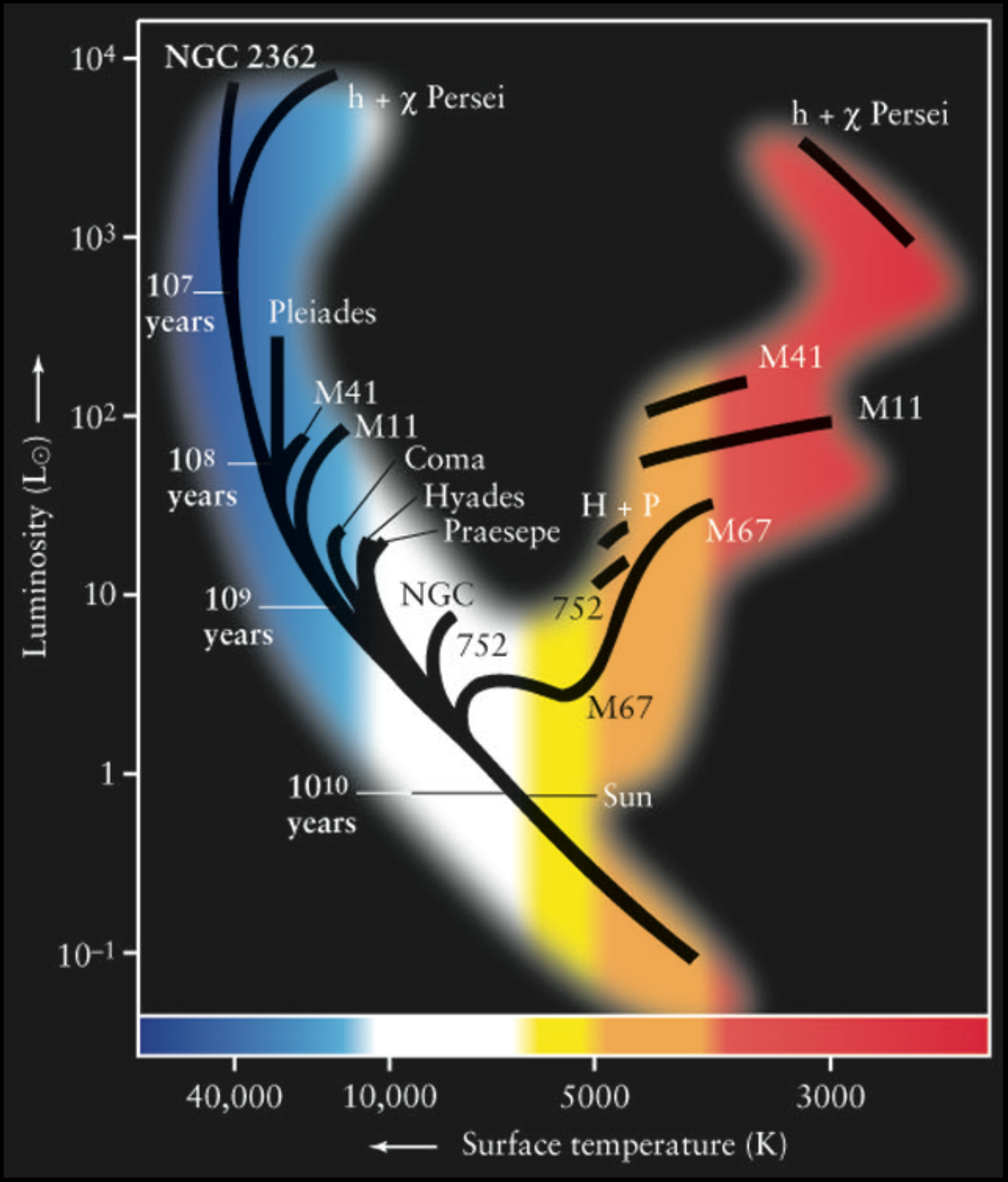


Midterm 2

Key concepts to be tested

Chap 3: The Evolution of Low-Mass Stars

- Observations
  - Nothing last forever, even stars
  - H-R diagram of star clusters
- Numerical Models
  - Equations of stellar structure and evolution
  - Stellar evolutionary tracks
- Fine-Tune Models
  - Isochrones (equal-age lines)
  - Fitting cluster H-R diagrams
  - Cluster age estimates
- Model Inferences
  - Main stages and lifetimes
  - Changes in the interiors of the stars: e- degenerate core + fusion shells



Chap 3: Key Equations

- Hydrostatic Equilibrium:

$$\frac{dP}{dr} = -\rho g(r) = -\rho \frac{GM_r}{r^2}$$

- The pressure from non-relativistic degenerate gas is:

$$P_{\text{degen}} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

- The pressure from ideal gas is:

$$P_{\text{ideal}} = \frac{2}{3} n \left(\frac{3}{2} kT \right) = nkT$$

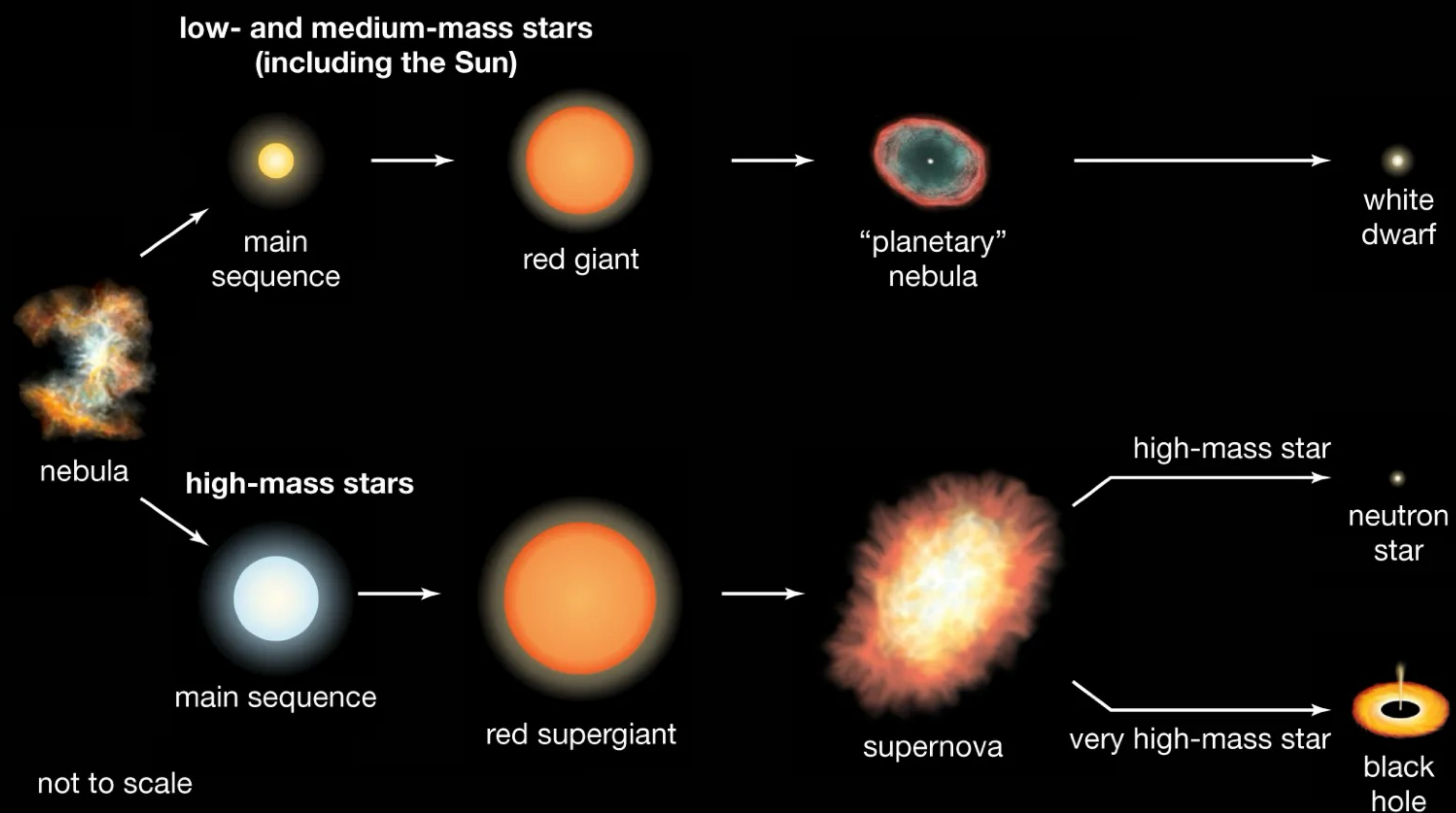
- The condition for degeneracy is

$$\frac{h^2}{4\pi^2} \frac{n^{2/3}}{m} > kT$$

- Mass-Radius relation:
  - $R \propto M^{-1/3}$ for white dwarfs (Chandrasekhar limit: 1.4 M_{sun})
  - $R \propto M^{0.7}$ for main-sequence stars

Chap 4: The Evolution of High-Mass Stars

- CNO Cycles
- Convective cores
- Consecutive fusion shells
- End of fusion - Binding Energy
- Type I vs. II supernovae
- Neutron stars and Pulsars
- Supernova Remnants (SNR): expansion parallax method
- The origins of elements: six primary astrophysical sources
- Periodic variables: Leavitt's law (standard candles)



- Mass-Transfer Binaries
  - Roche Lobe, Lagrange Points
  - Novae, Type Ia SNe, Blue Stragglers

Determining Distance from "Expansion Parallax"

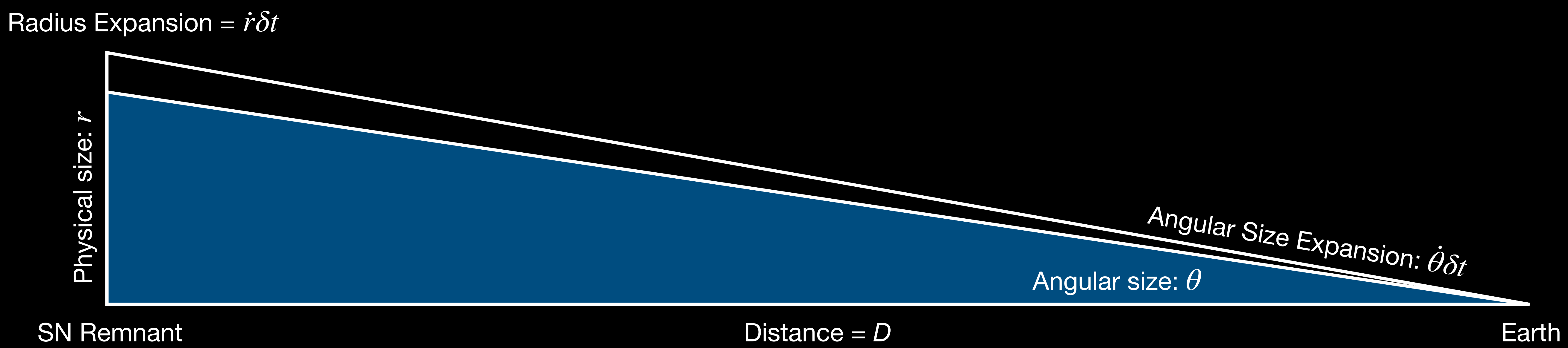
Physical Size vs. Angular Size: $r = D\theta$

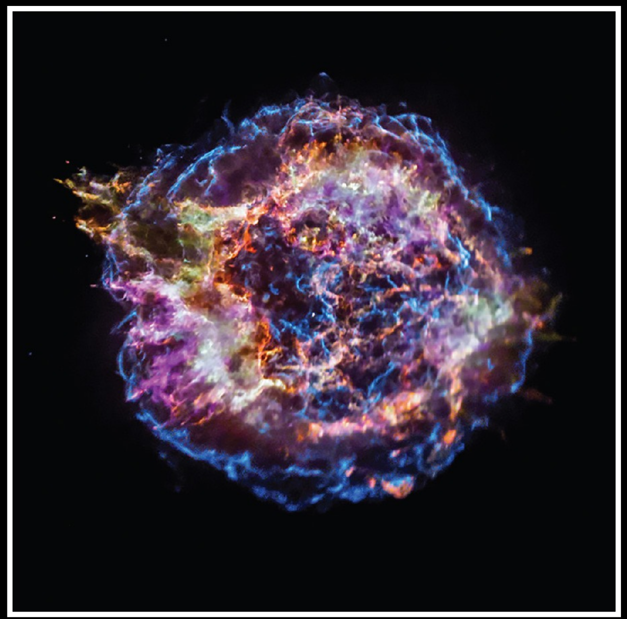
Physical Expansion Rate vs. Angular Expansion Rate:

$$\dot{r}\delta t = D\dot{\theta}\delta t \rightarrow \dot{r} = D\dot{\theta}$$

Distance = Physical Expansion Rate / Angular Expansion Rate

$$D = \dot{r}/\dot{\theta}$$



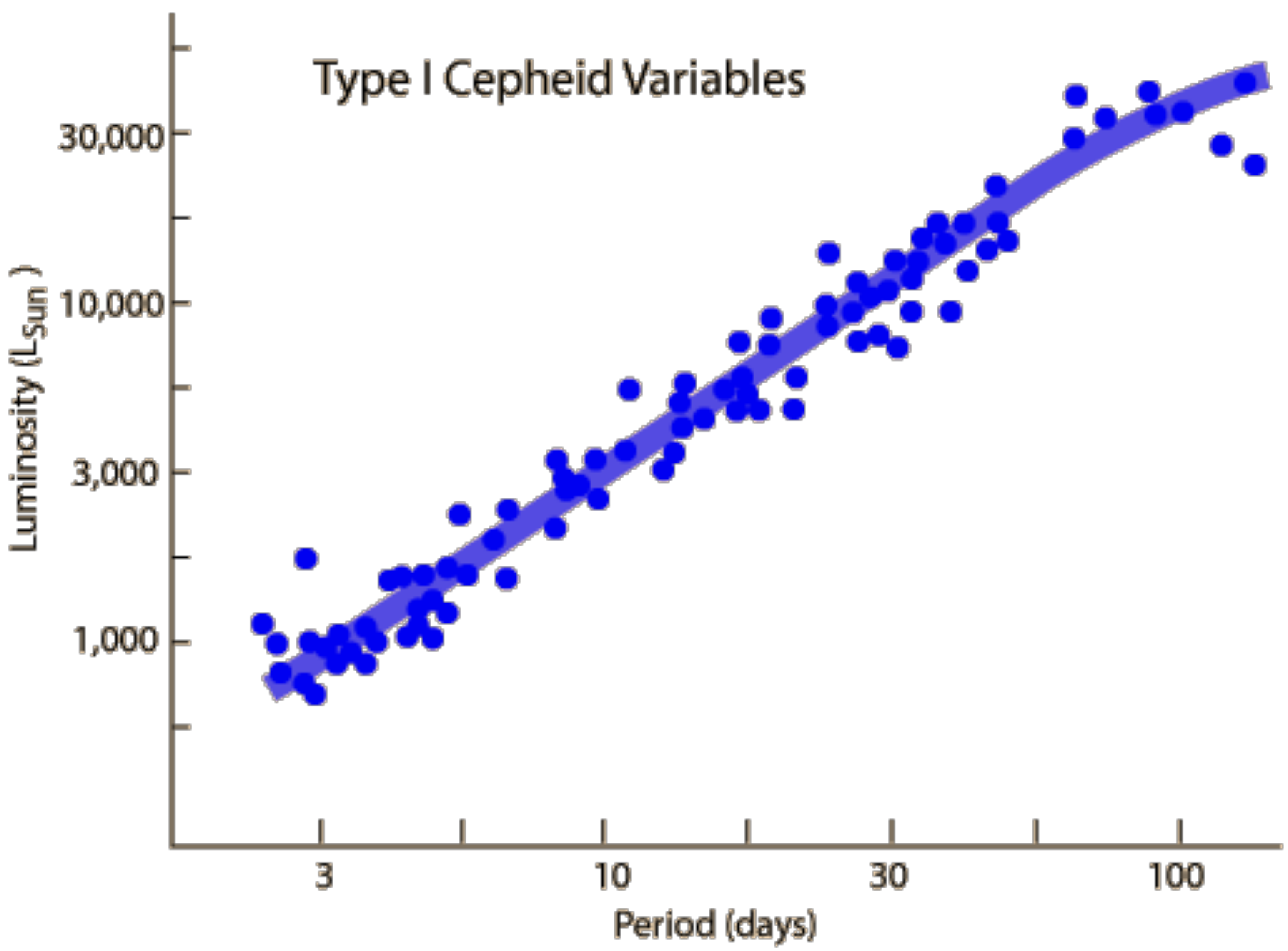


e.g., $\dot{\theta} = 0.01''/yr$, and $\dot{r} = 250$ km/s, what is the distance?

But how do we measure the physical expansion rate $\dot{r}$ in km/s?

Period-Luminosity Relations (Leavitt's Law discovered in 1912)

- $M_V = -2.43\log(P_{\mathrm{day}}) - 1.62$
  (Type I Cepheids, Fritz et al. 2007)
- This is critical for determining distances to other galaxies: $d_{\mathrm{pc}} = 10^{\frac{(m-M)+5}{5}}$



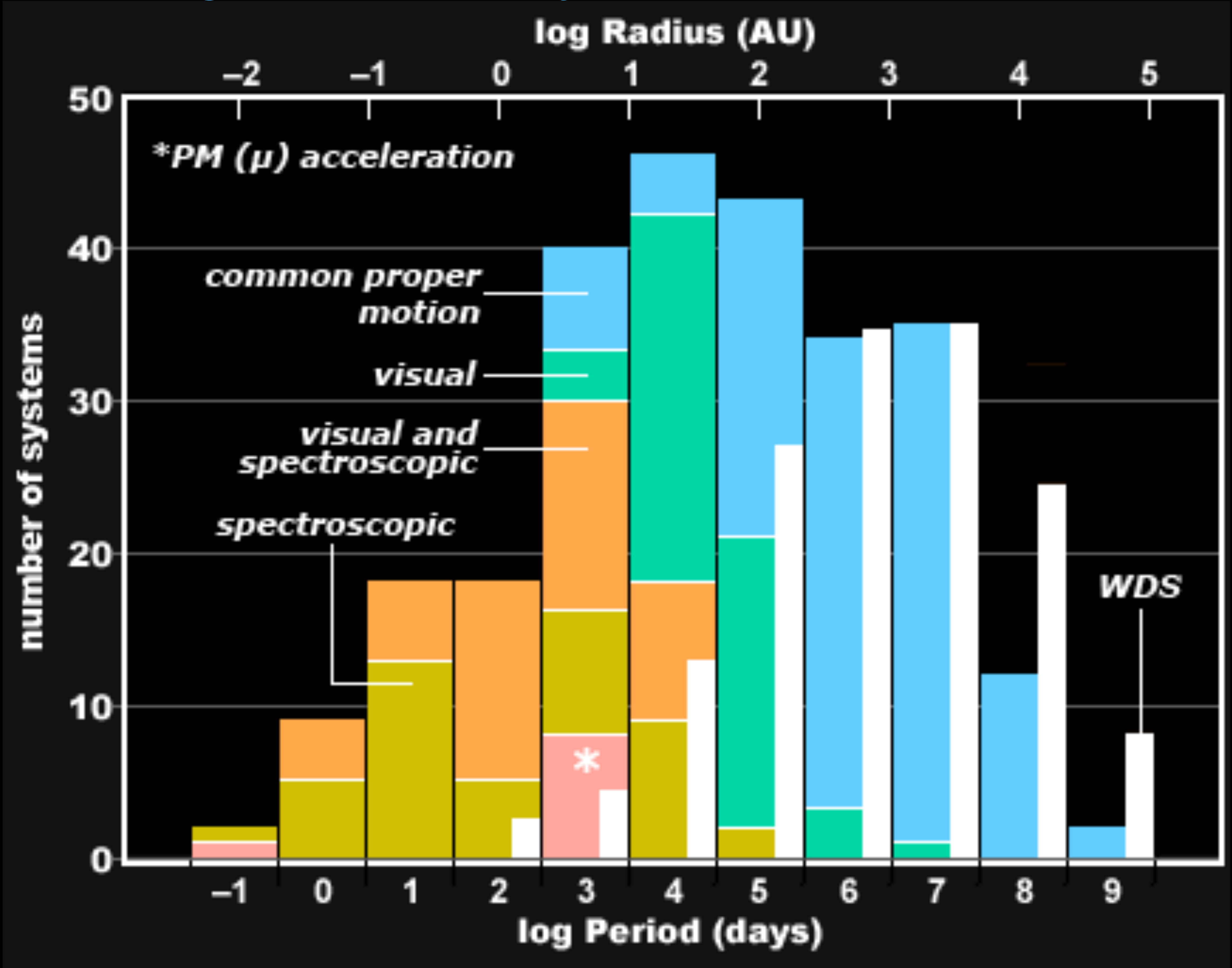
Henrietta Swan Leavitt

Born	July 4, 1868 Lancaster, Massachusetts, U.S.
Died	December 12, 1921 (aged 53) Cambridge, Massachusetts, U.S.
Education	Oberlin College Harvard University (BS)
Known for	Leavitt's law: the period-luminosity relationship for Cepheid variables

Mass-Transfer Binary Stars

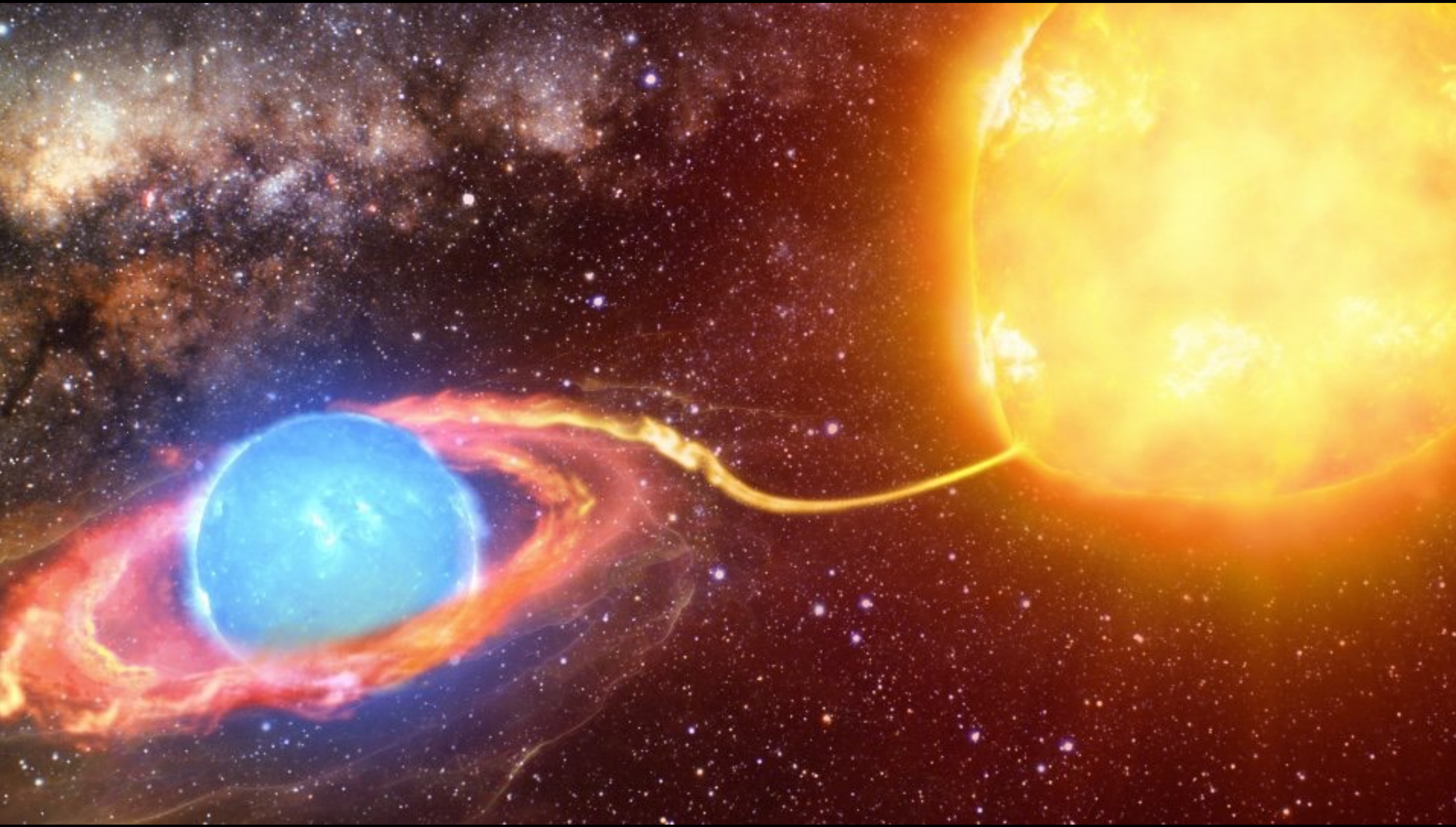
Blue stragglers and Type-Ia SNe

The logarithmic of Binary Periods follow a “Bell” curve



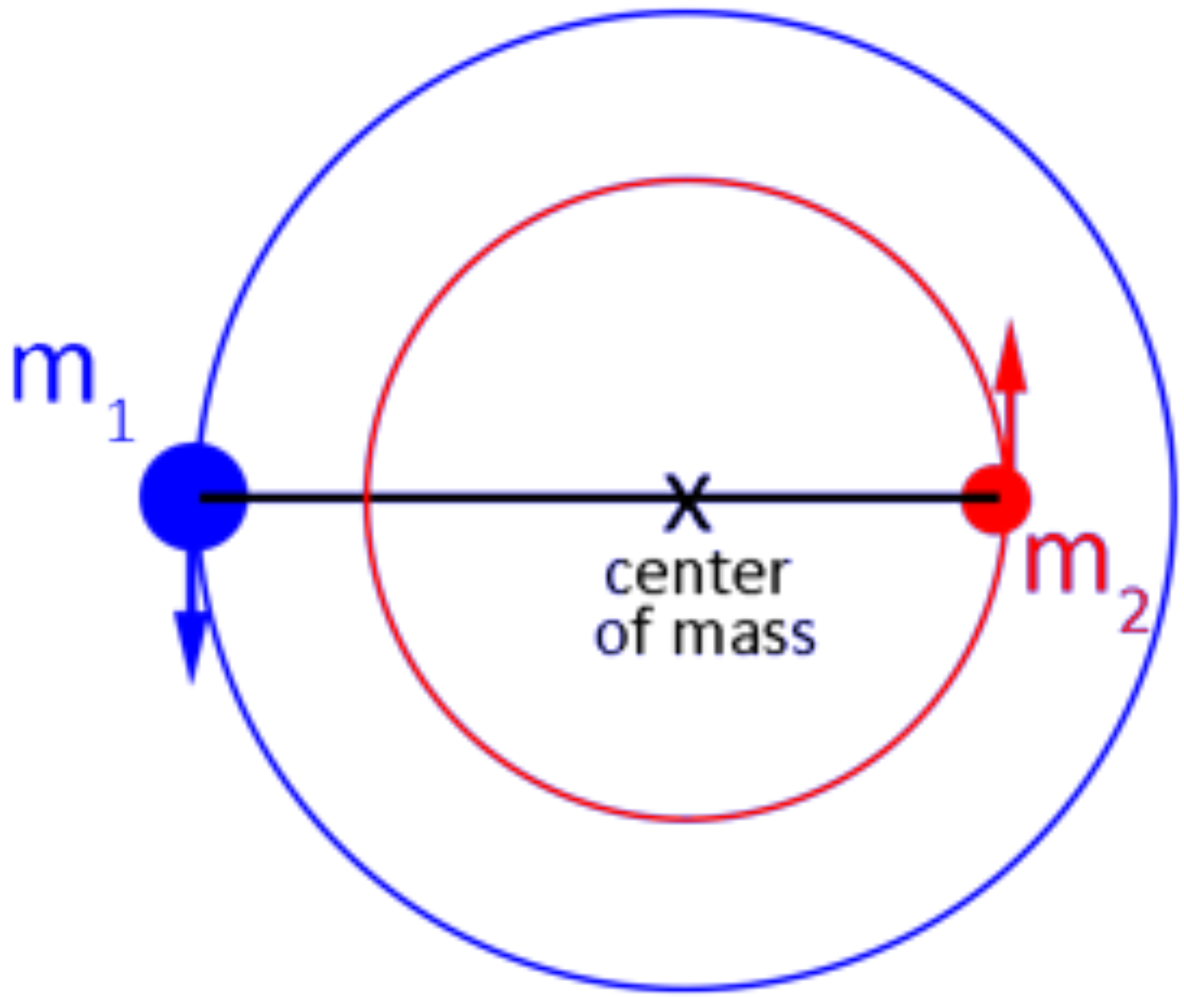
What about stars in close binary systems?

"Breaking the isolation and starting to share": co-evolution

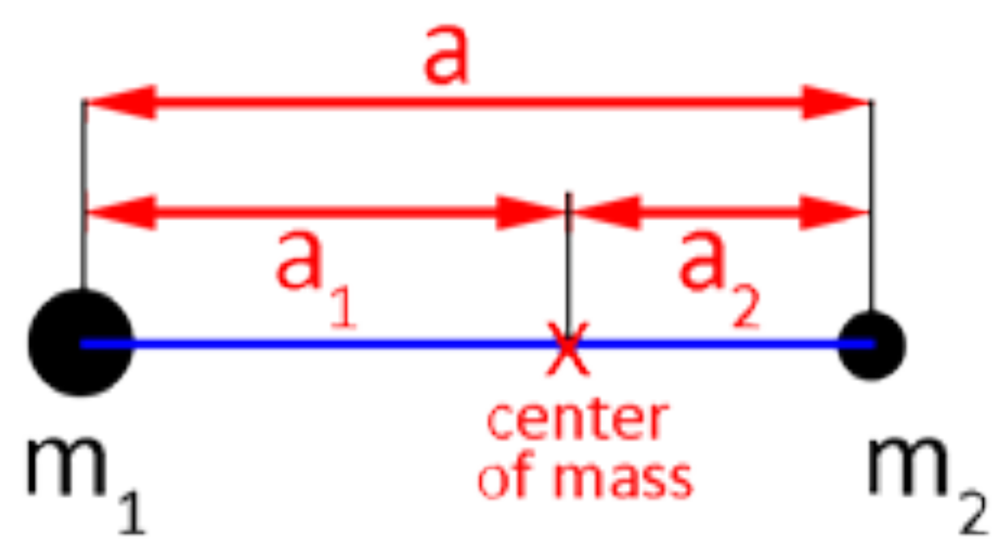


Introducing the non-inertial co-rotating reference frame

- Inertial reference frame for binaries on circular orbits



- Non-inertial co-rotating reference frame



A **non-inertial reference frame** (also known as an **accelerated reference frame**[1]) is a frame of reference that undergoes acceleration with respect to an inertial frame. In classical mechanics it is often possible to explain the motion of bodies in non-inertial reference frames by introducing additional fictitious forces (also called inertial forces, pseudo-forces[5] and d'Alembert forces) to Newton's second law. Common examples of this include the Coriolis force and the centrifugal force.

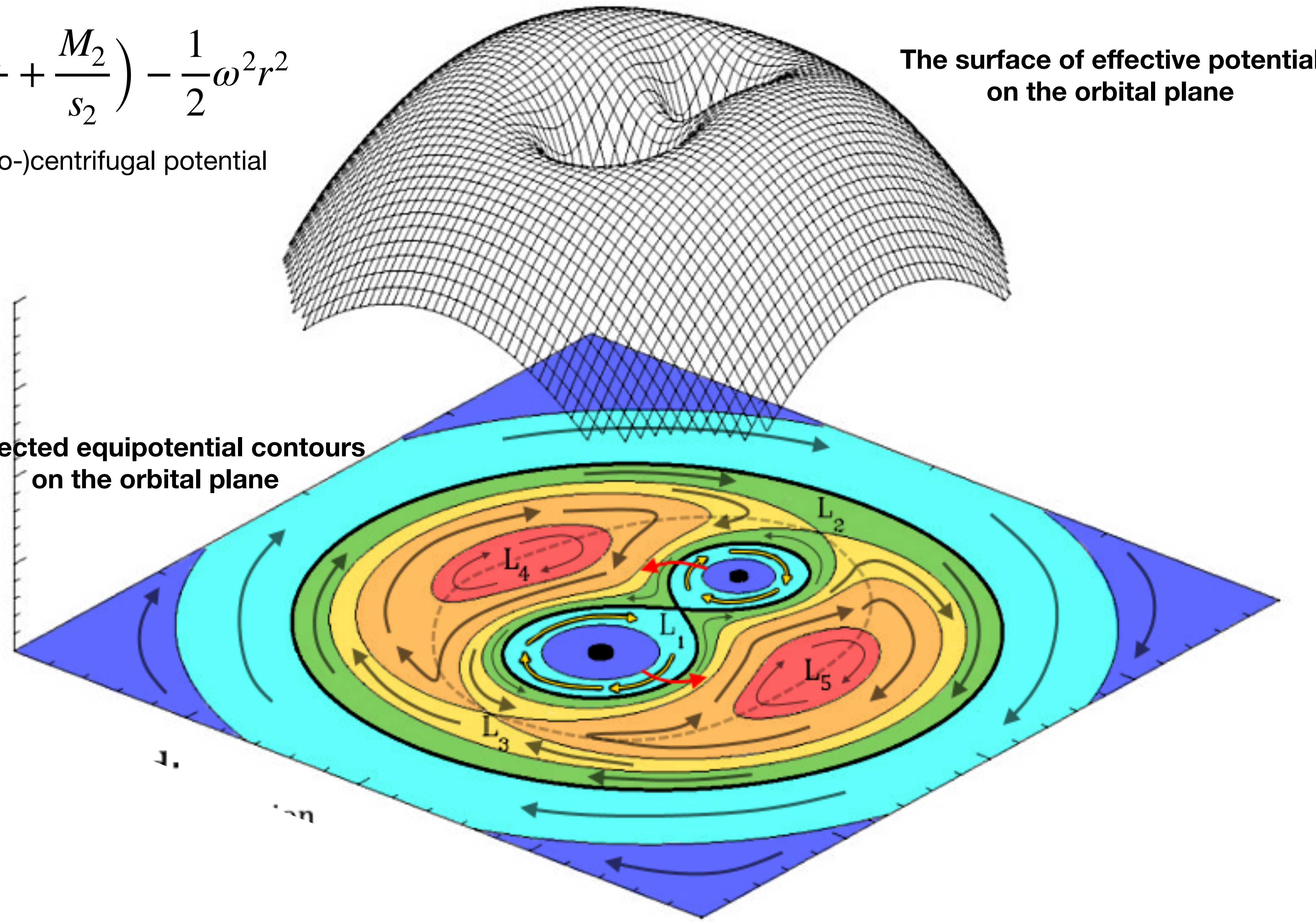
Effective potential in a co-rotating reference frame

The contours of **equipotential** show the ***effective* gravitational potential** on the orbital plane, the **Lagrange points** are the **local maxima** where the gradient of effective potential is zero (**no acceleration** in the **co-rotating non-inertial reference frame**).

$$\Phi_{\text{eff}} = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}\omega^2 r^2$$

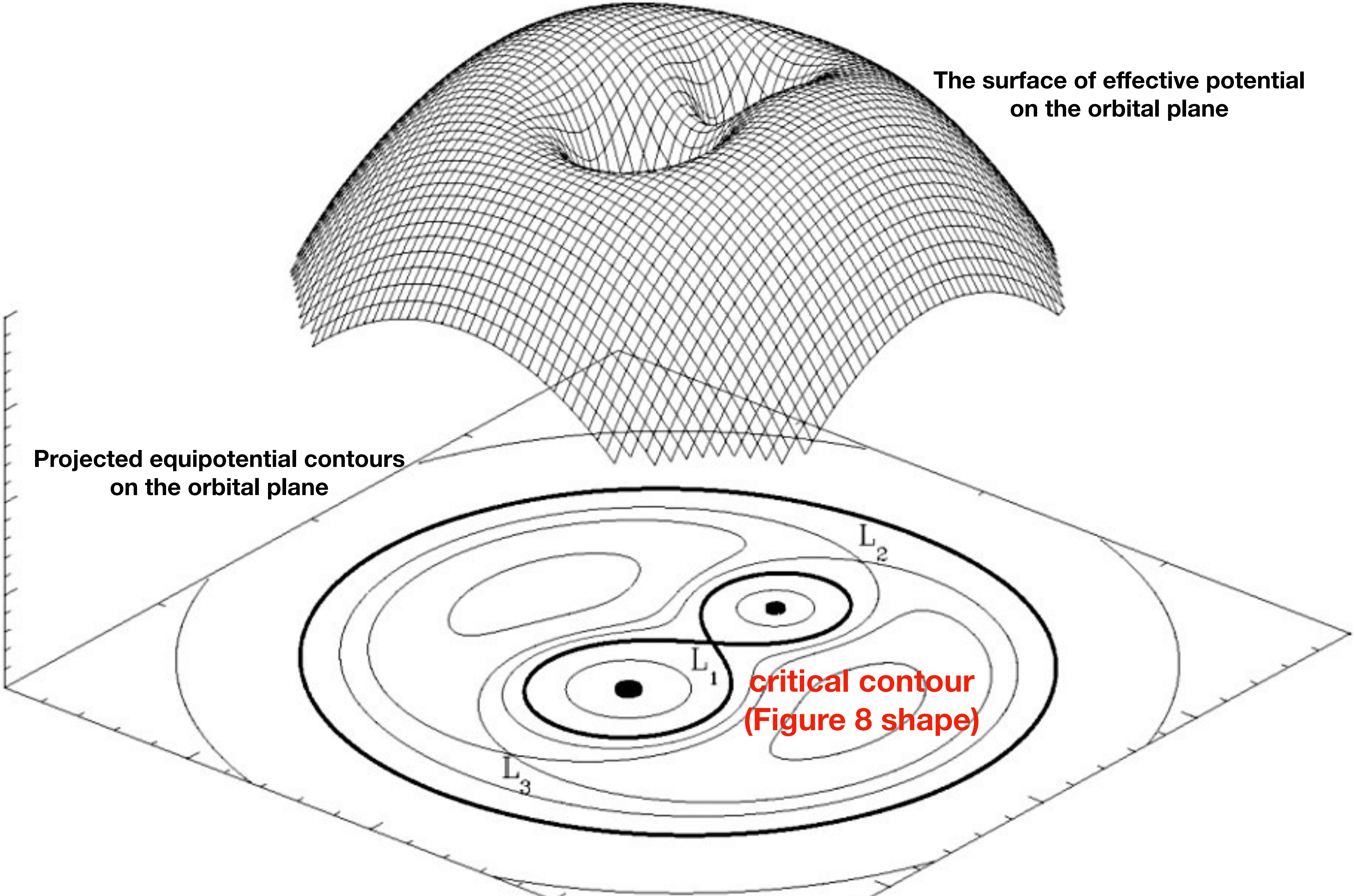
2nd term is (pseudo-)centrifugal potential

$$\vec{a}_{\text{eff}} = -\nabla\Phi_{\text{eff}}$$



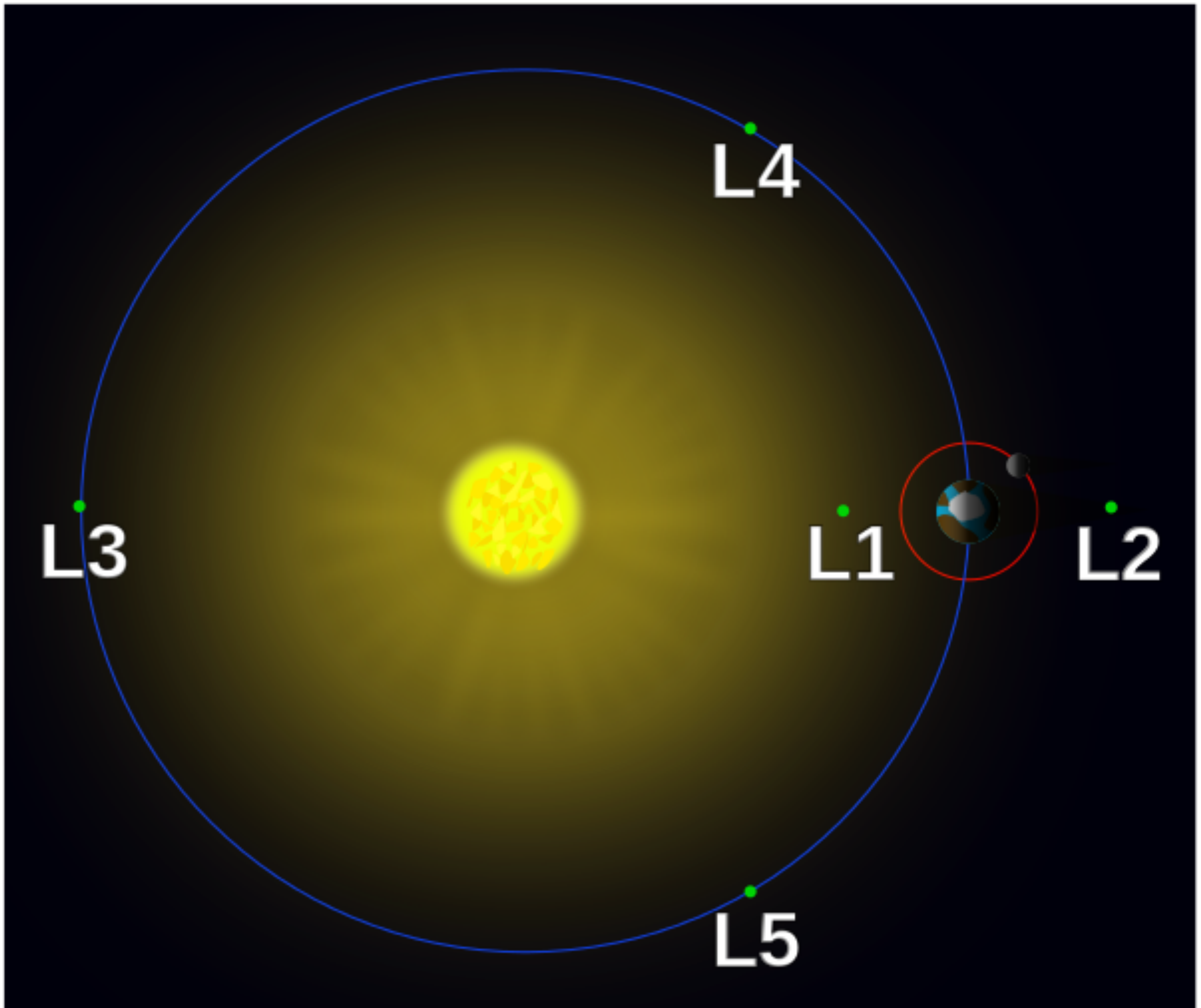
Roche Lobe (or Roche Surface)

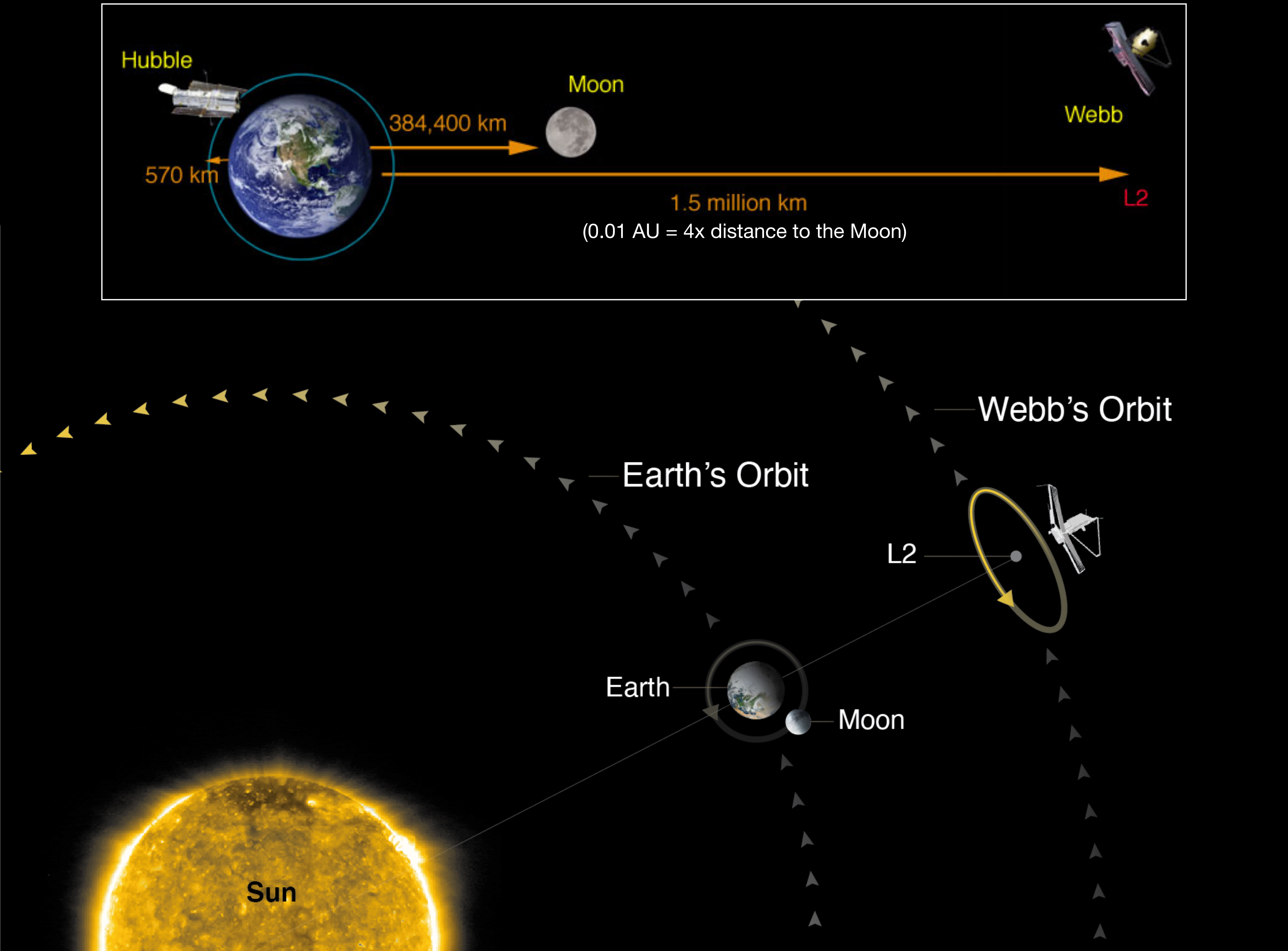
- On the orbital plane, there is a **critical equipotential contour** that intersects itself at the **L1 point**, forming a figure-of-eight.



Lagrange Points of a Binary System

- **Equilibrium points** for small mass objects under the influence of two massive orbiting bodies in a **co-rotating frame of reference (non-inertial)**

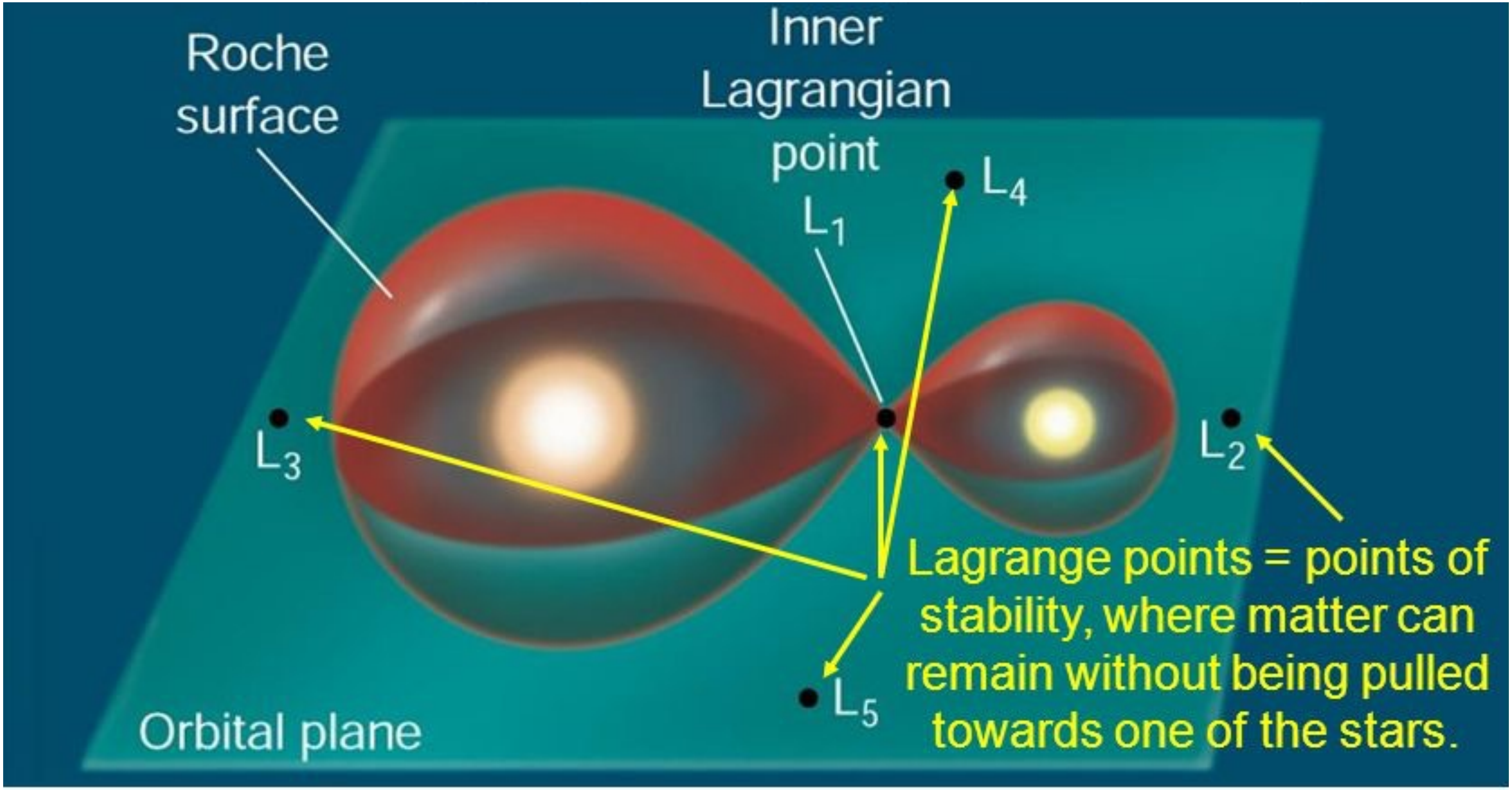




Hubble
Moon
Webb
384,400 km
570 km
1.5 million km
L2
(0.01 AU = 4x distance to the Moon)
Webb's Orbit
Earth's Orbit
L2
Earth
Moon
Sun

Roche Lobe (or Roche Surface)

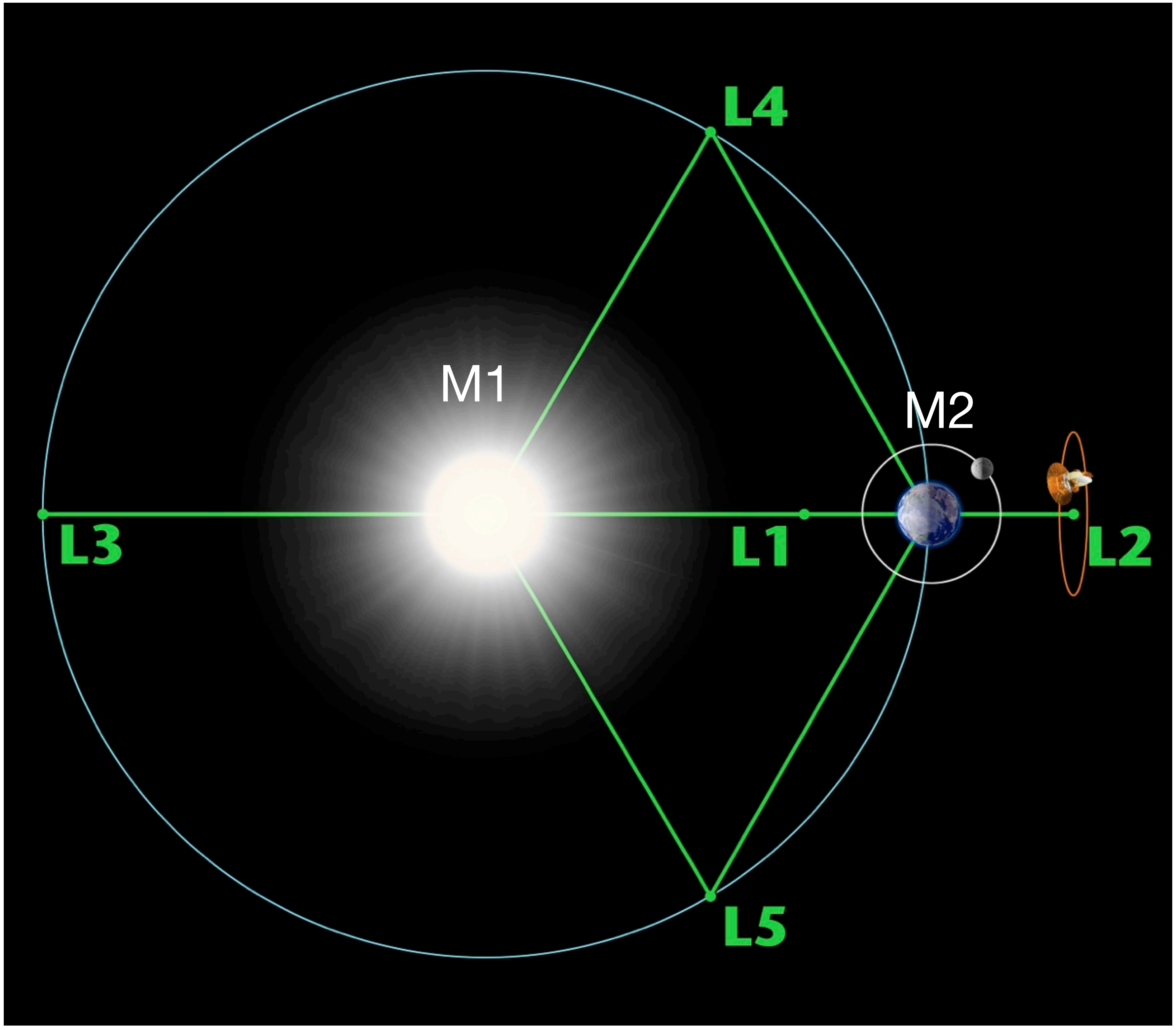
- In 3D, the critical equipotential surface delineates two lobes in a binary system. In each lobe, small-mass objects are gravitationally bounded to the massive object at the center.



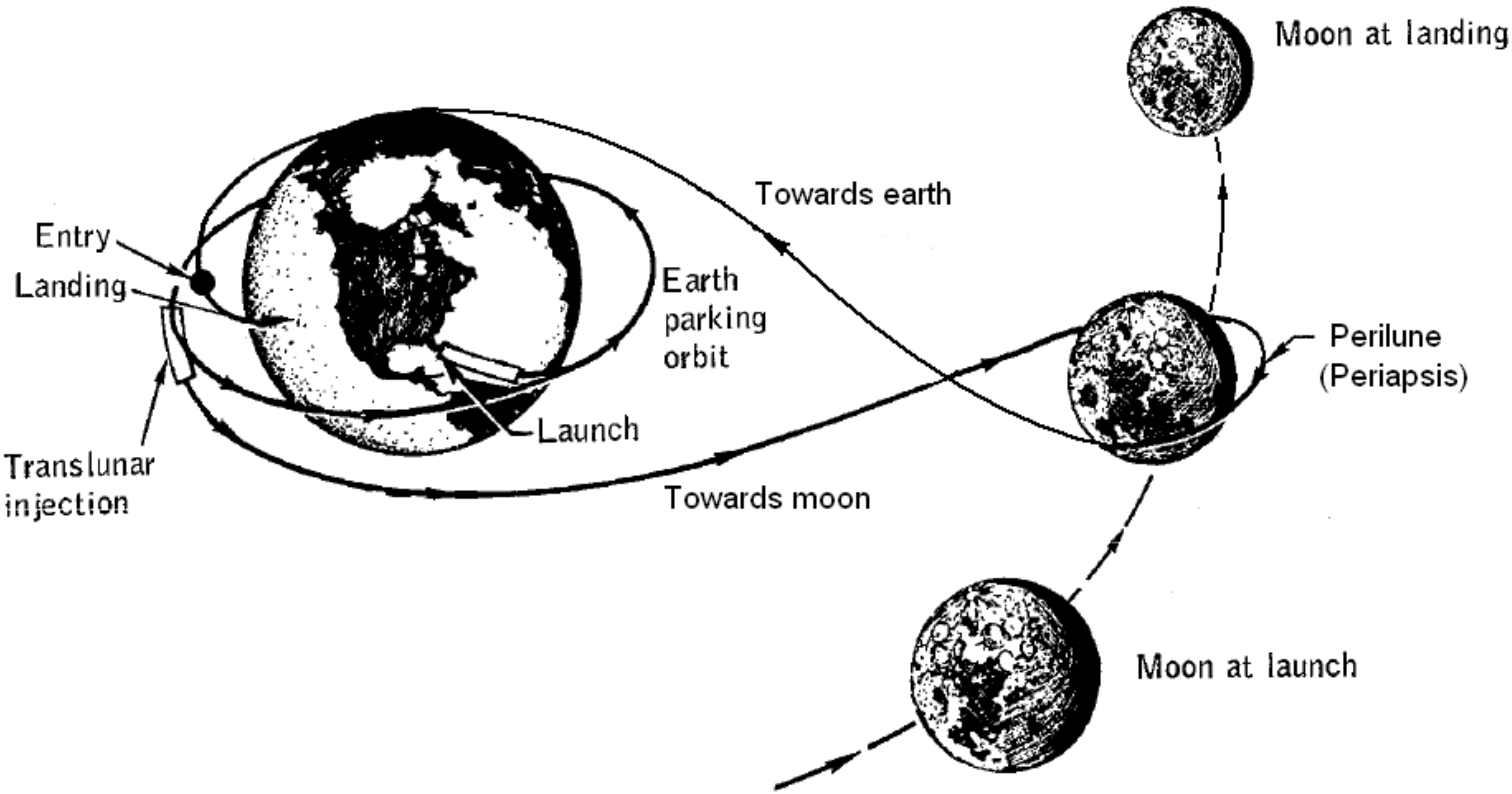
Location of the L1 Point: approximate formulae for large mass ratios ($M2/M1 > 0.01$), this provide a rough estimate of the sizes of Roche Lobes

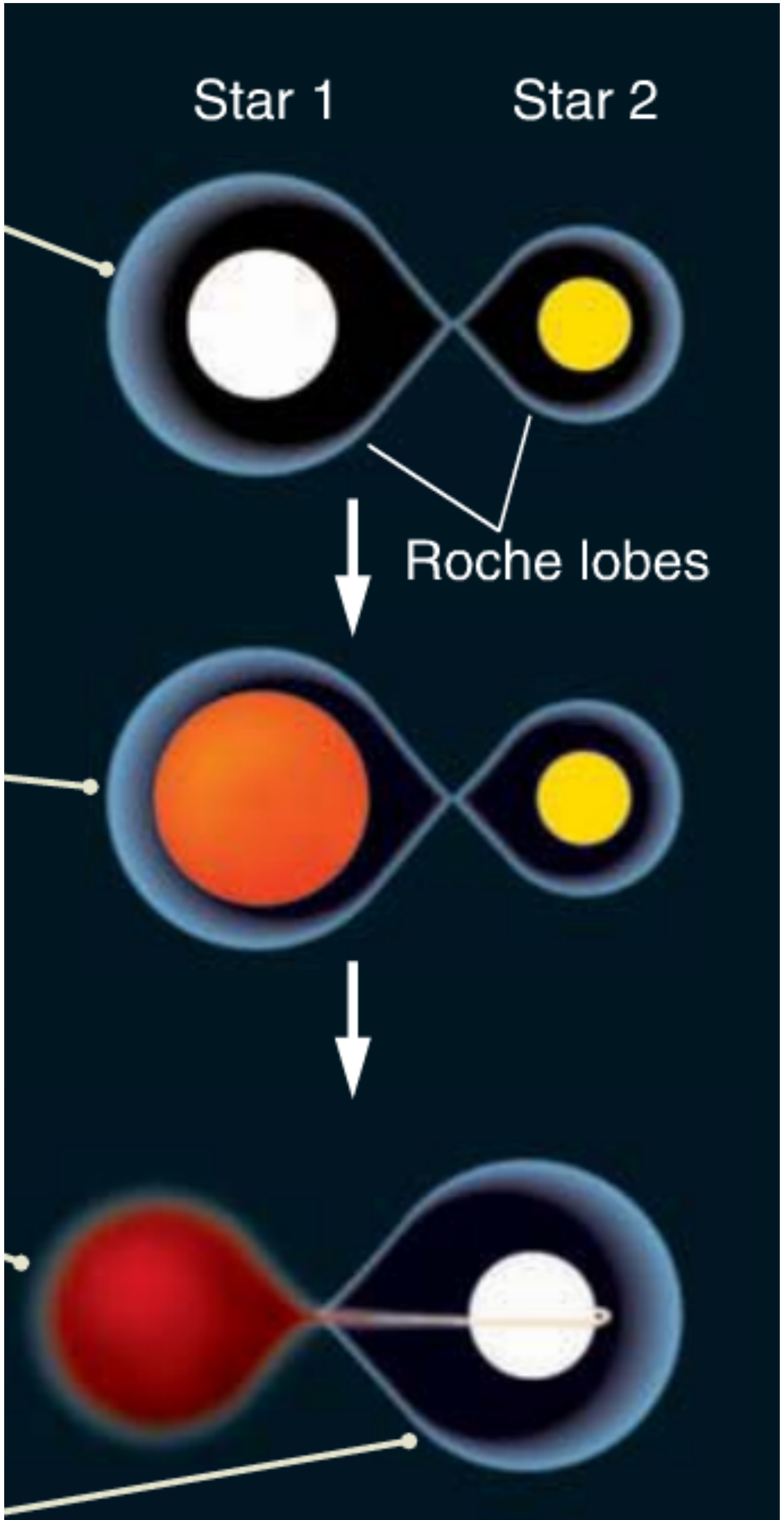
$$d(L1 \rightarrow M_1) = a[0.5 - 0.227 \log(M_2/M_1)]$$
$$d(L1 \rightarrow M_2) = a[0.5 + 0.227 \log(M_2/M_1)]$$

Note that as the mass ratio changes, the **L1 point** moves in the ***opposite direction*** as the **center of mass.**



Earth-Moon free-return trajectory: 1959 Soviet Luna 3

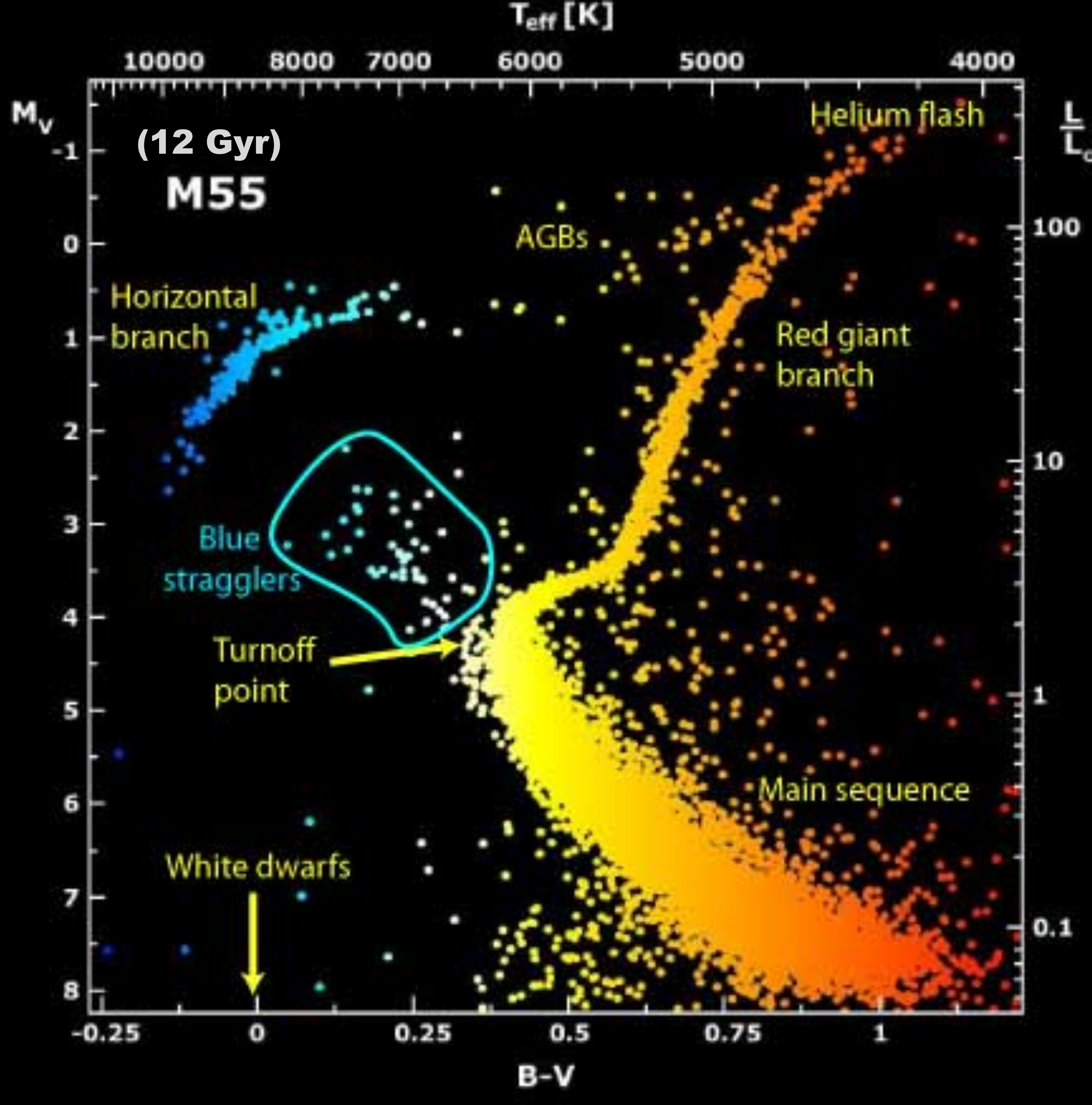


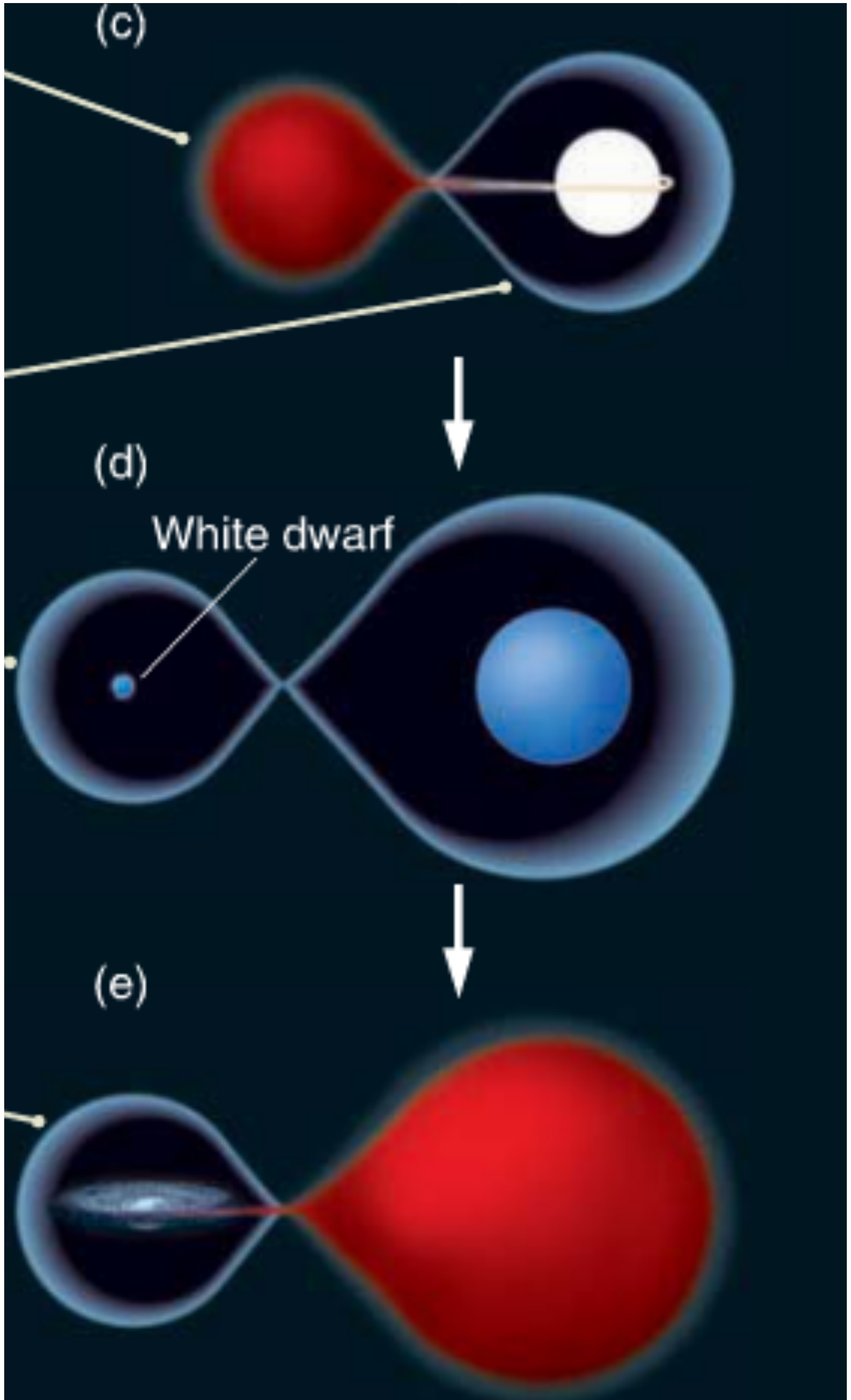


Mass-Transfer Binary Stars

- ~60% of stars are in binaries, a small fraction of which are very close binaries.
- The two stars in a binary have **different MS lifetimes** because of their different *initial* masses
- The more massive **primary** evolves into a **RGB** while the less massive **secondary** remains on the **MS** (middle figure)
- If the **Roche lobe** is smaller than the possible size of the RGB, the **red giant primary** can only expand so much before material is lost to the **MS secondary's** gravity (bottom fig)

Blue Stragglers:
MS stars in a cluster beyond the turnoff point

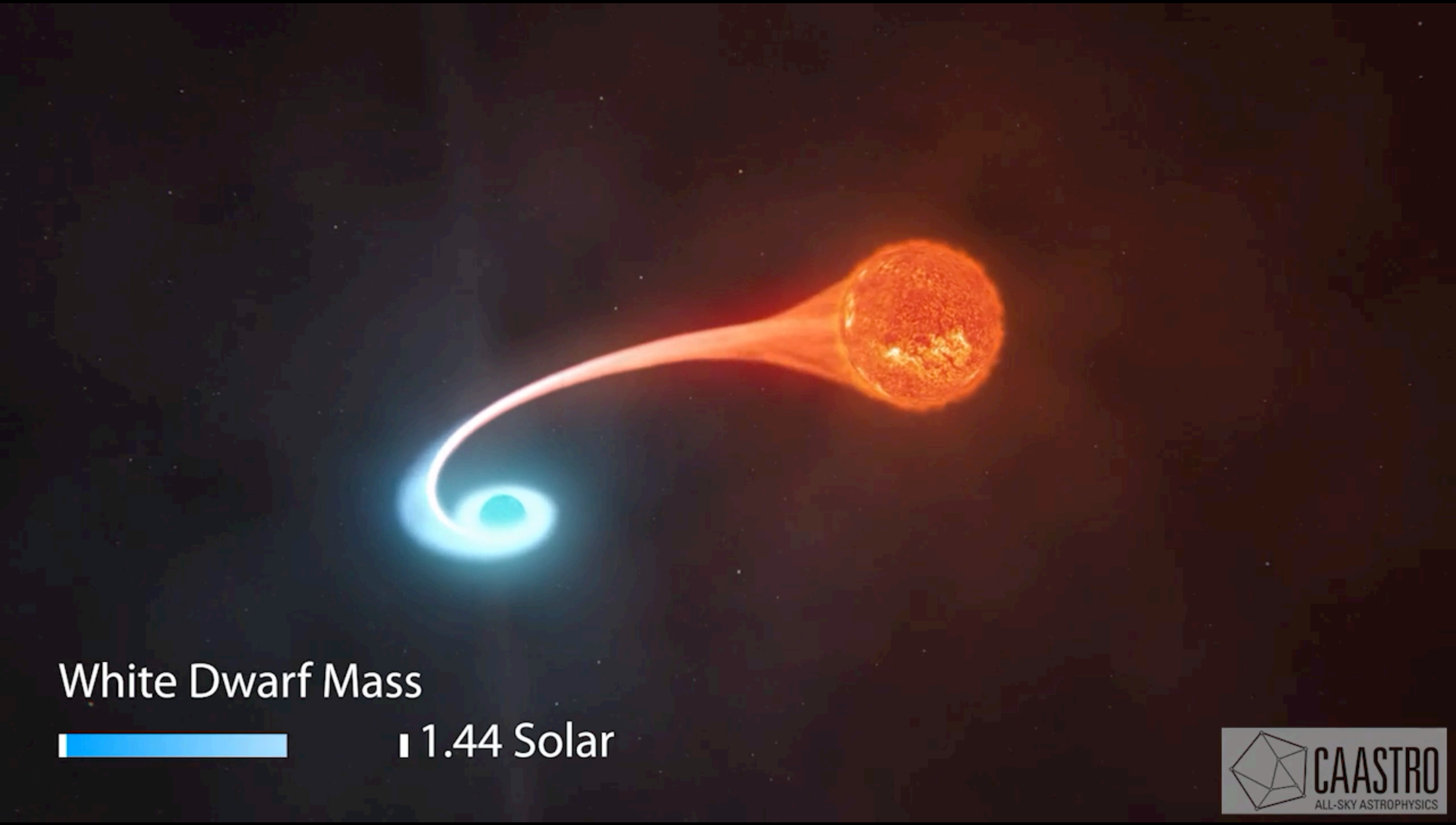


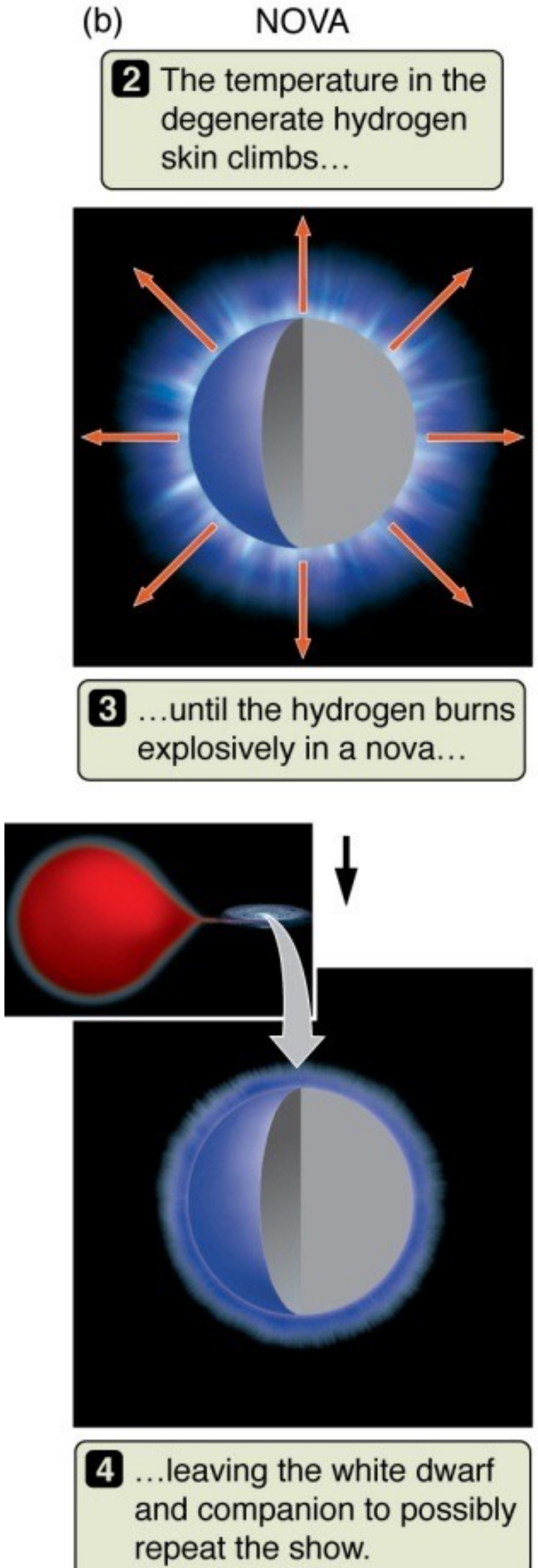


Mass-Transfer Binary Stars

- It's likely that by the time the less massive star evolves to a red giant, the originally more massive star already evolved into a WD (Fig d)
- So mass transfer reverses: the secondary star begins to lose its envelope to the WD's gravity, forming an accretion disk (Fig e)
- As the WD grows in mass because of accretion, there are two possible consequences.

Mass Transfer Binary Stars and Type Ia Supernovae



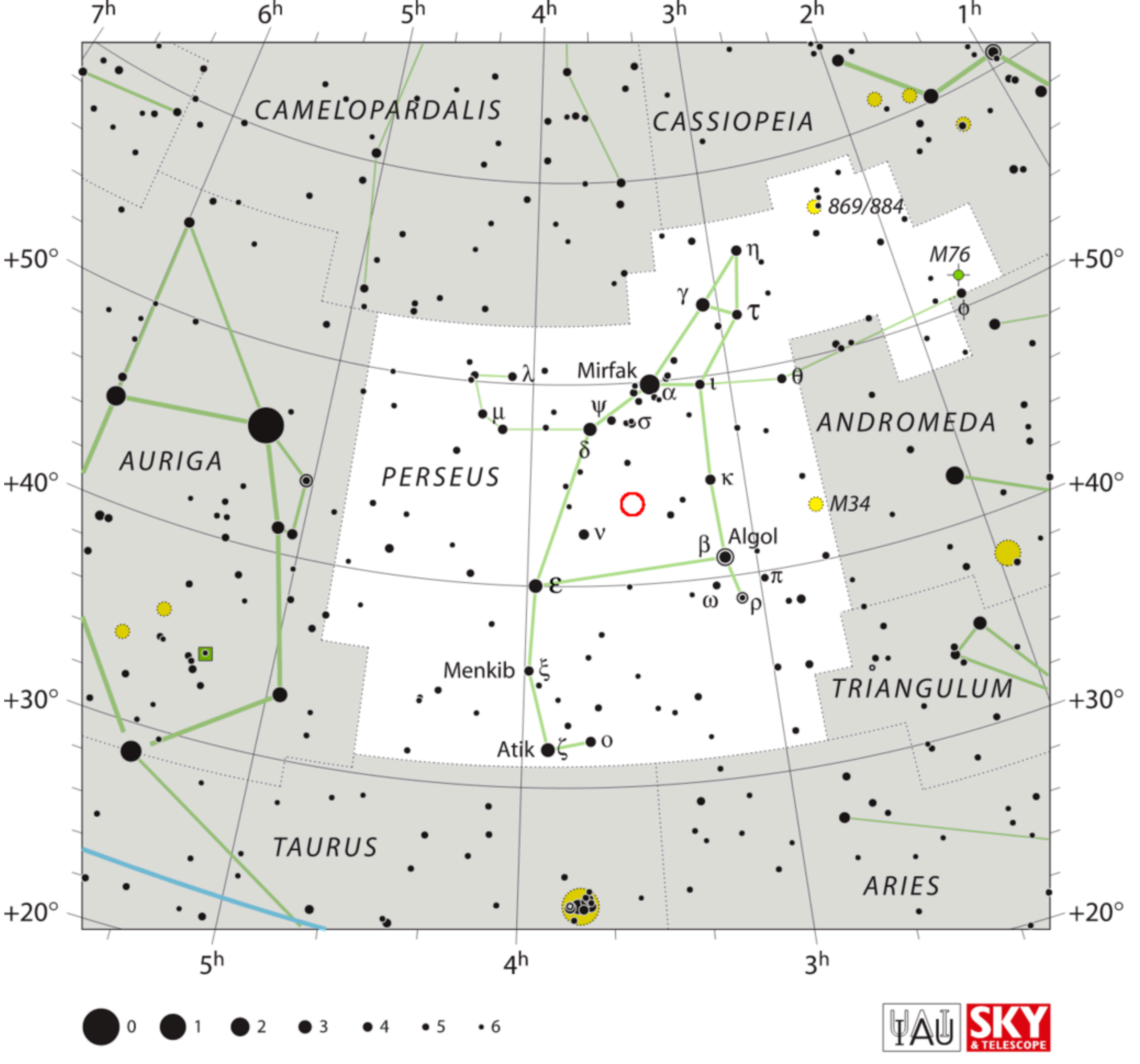


I: Classical Novae

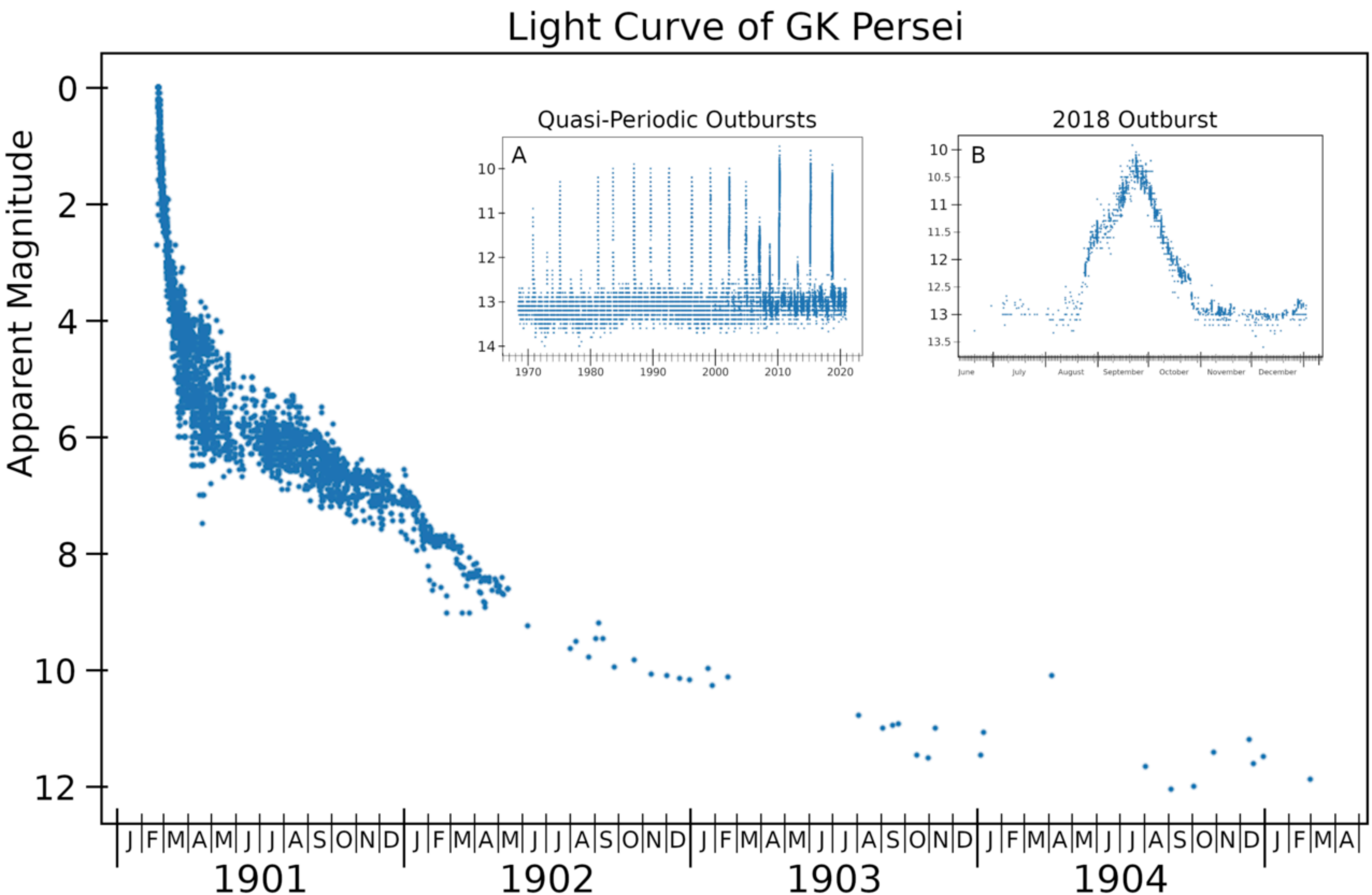
- H deposition on the surface of the white dwarf from the red giant star
- Condenses onto degenerate core and explosively burns episodically: **Nova**
- For a few hours, a Nova can be 10^5 times more luminous than the Sun.

GK Persei:
Nova Persei 1901
red circle on the chart

m ~ 14
d ~ 442 pc

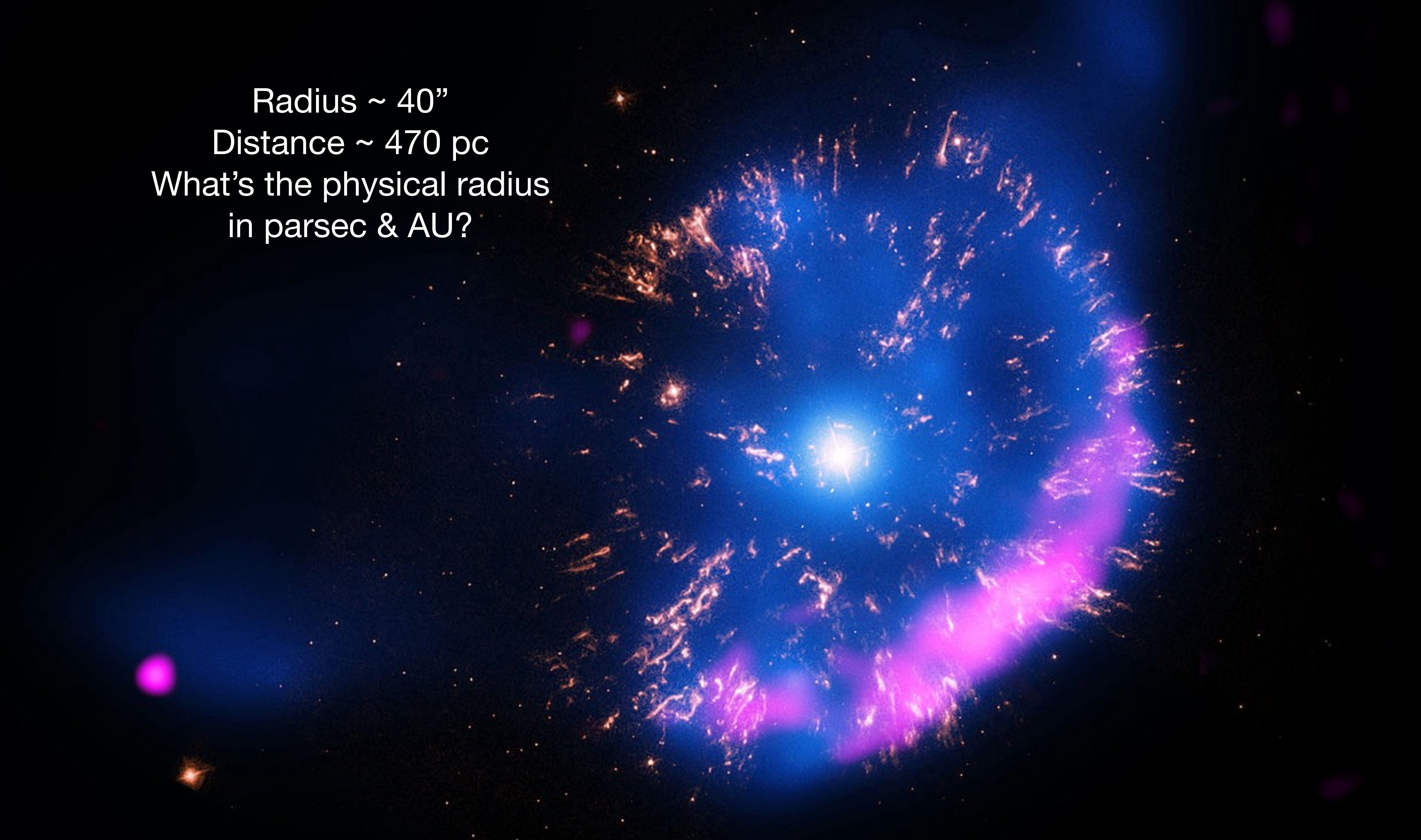


In 1901, GK Persei was one of the brightest star on the sky (for a few days)

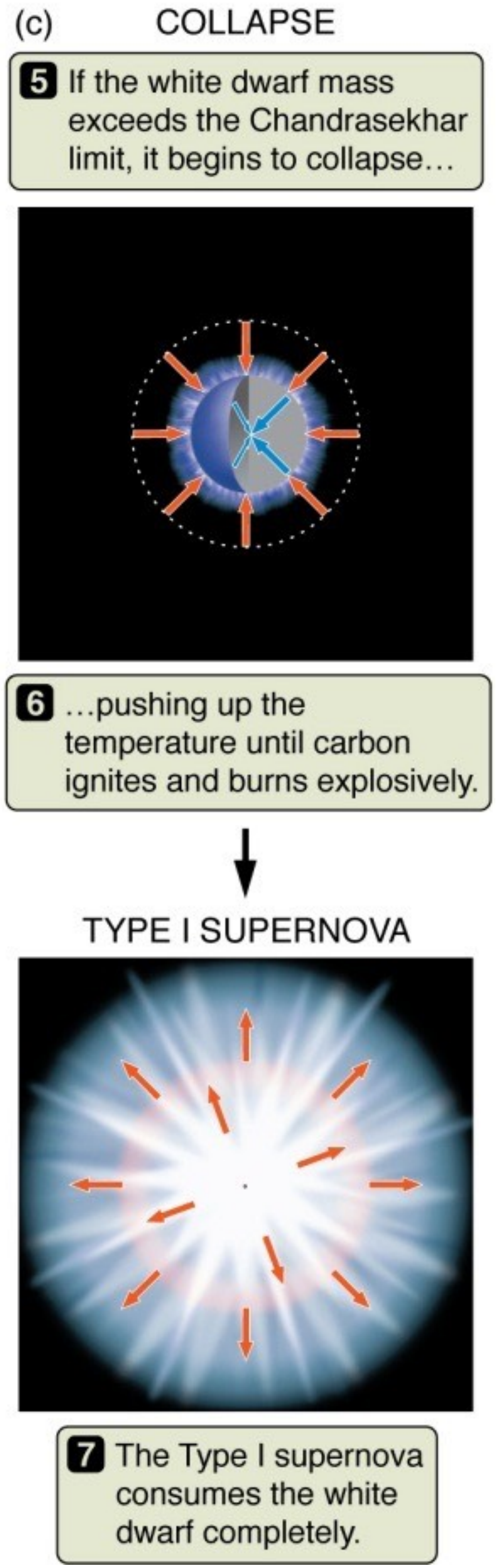


GK Persei: Nova of 1901
X-ray (blue), optical (yellow), radio (pink)

Radius ~ 40”
Distance ~ 470 pc
What’s the physical radius
in parsec & AU?



Takei et al. 2015 ApJ (2013 data)

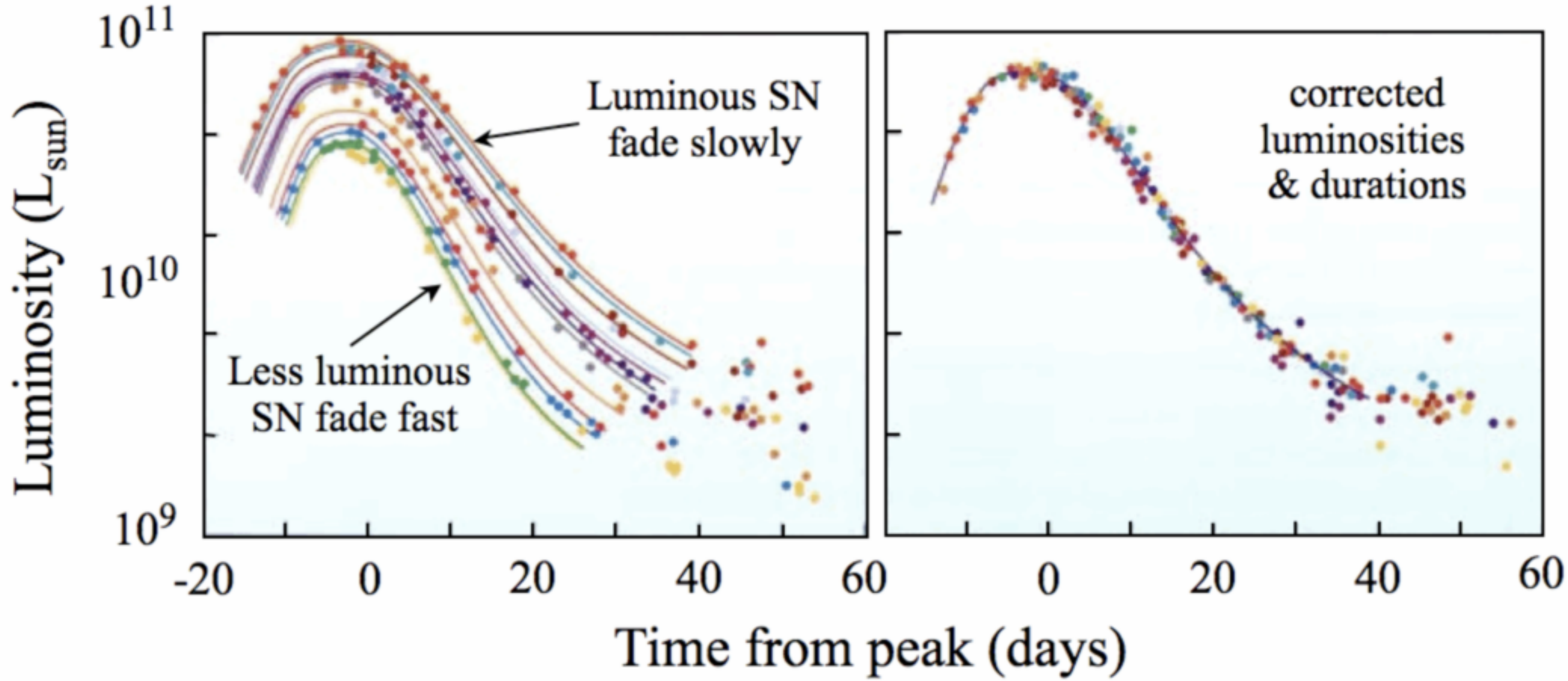


II: Type Ia Supernovae

- WD *mass increases over time* because of accretion from the RGB
- When its mass reaches 1.4 M_{sun}, the **Chandrasekhar limit**, gravity overcomes the ***relativistic* electron degeneracy pressure**
- The WD collapses, heats up and triggers a **thermonuclear runaway**:
  - **"C Flash"**: C core burns out in <1 sec! Forms Mg, Ne, Na, Ni, Fe.
  - This is a Type Ia supernova.
- 10^{10} times brighter than the Sun – comparable to the luminosity of a galaxy!
- Note: Type Ia may also be WD mergers

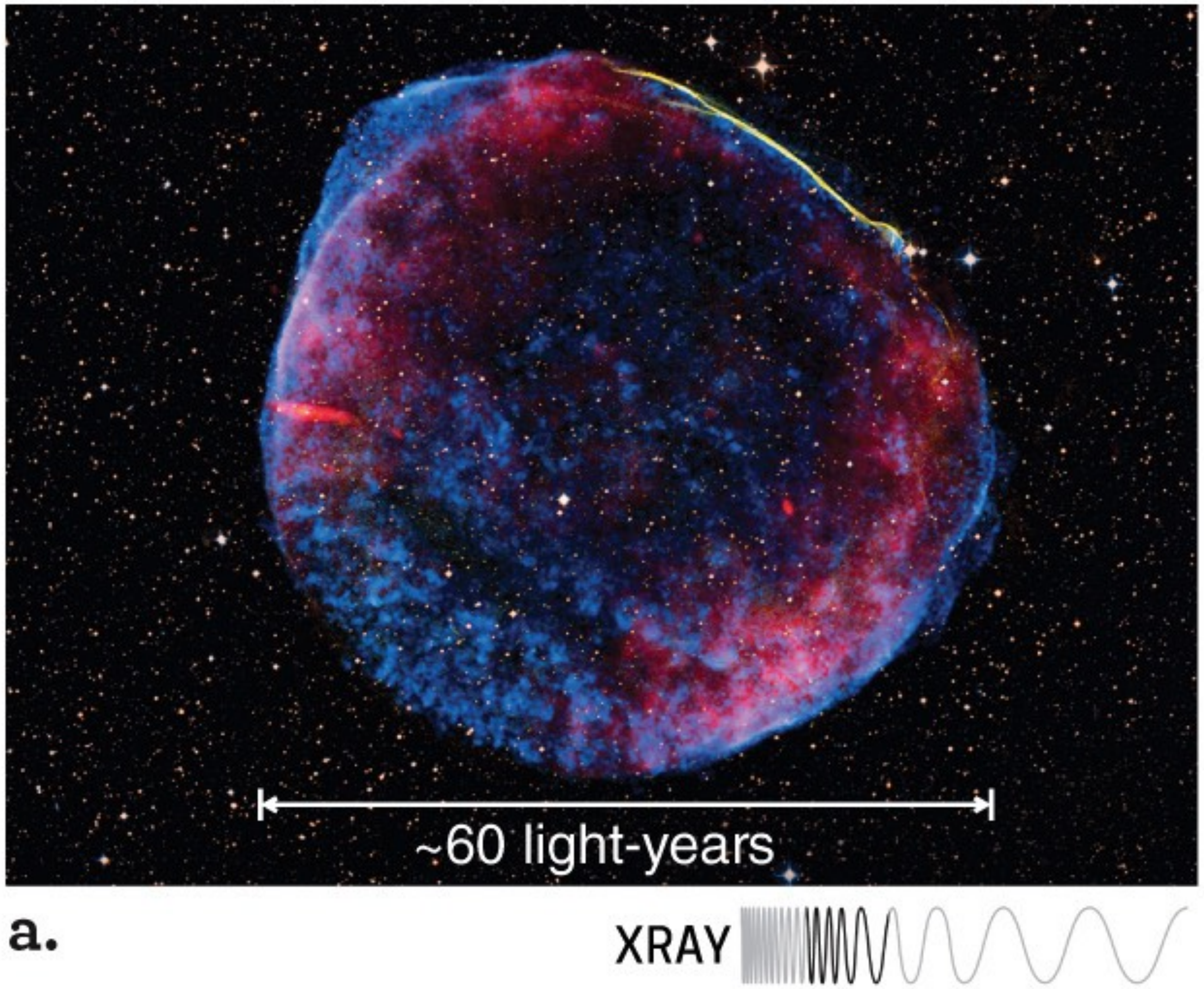
Type Ia Supernovae

- Over just a few days, the explosion releases about the same amount of energy as the Sun does over its entire main-sequence lifetime (10^{44} Joules).
- Type Ia supernovae are excellent **distance indicators** because they are **standard candles** (luminosity can be inferred from the shape of its light curve)

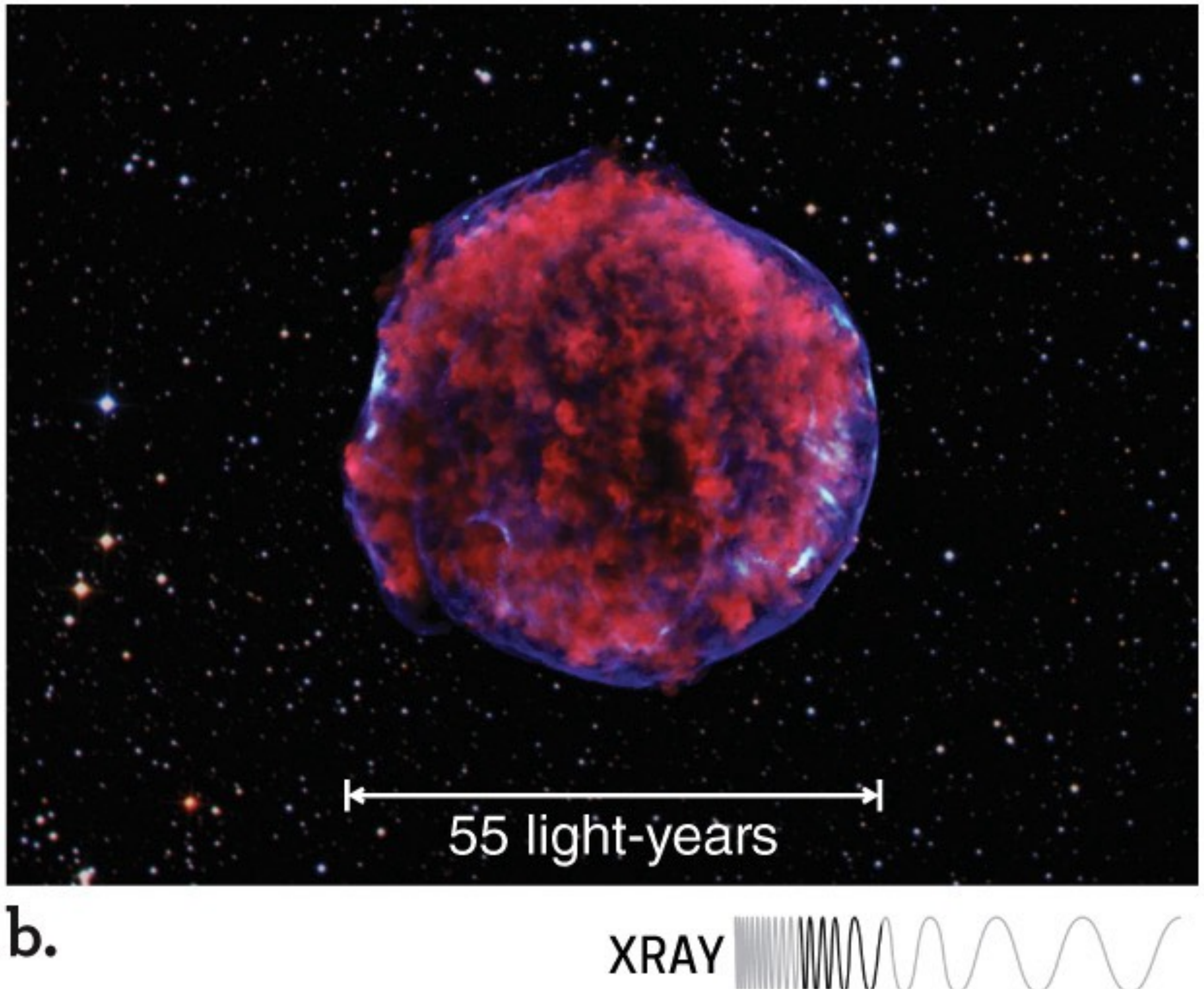


Type Ia Supernova Remnants

- The entire star explodes in the thermonuclear runaway — there is no central star left (unlike planetary nebulae and type II SNe)
- **Supernova remnants** are leftover shells of dust and gas from the explosion

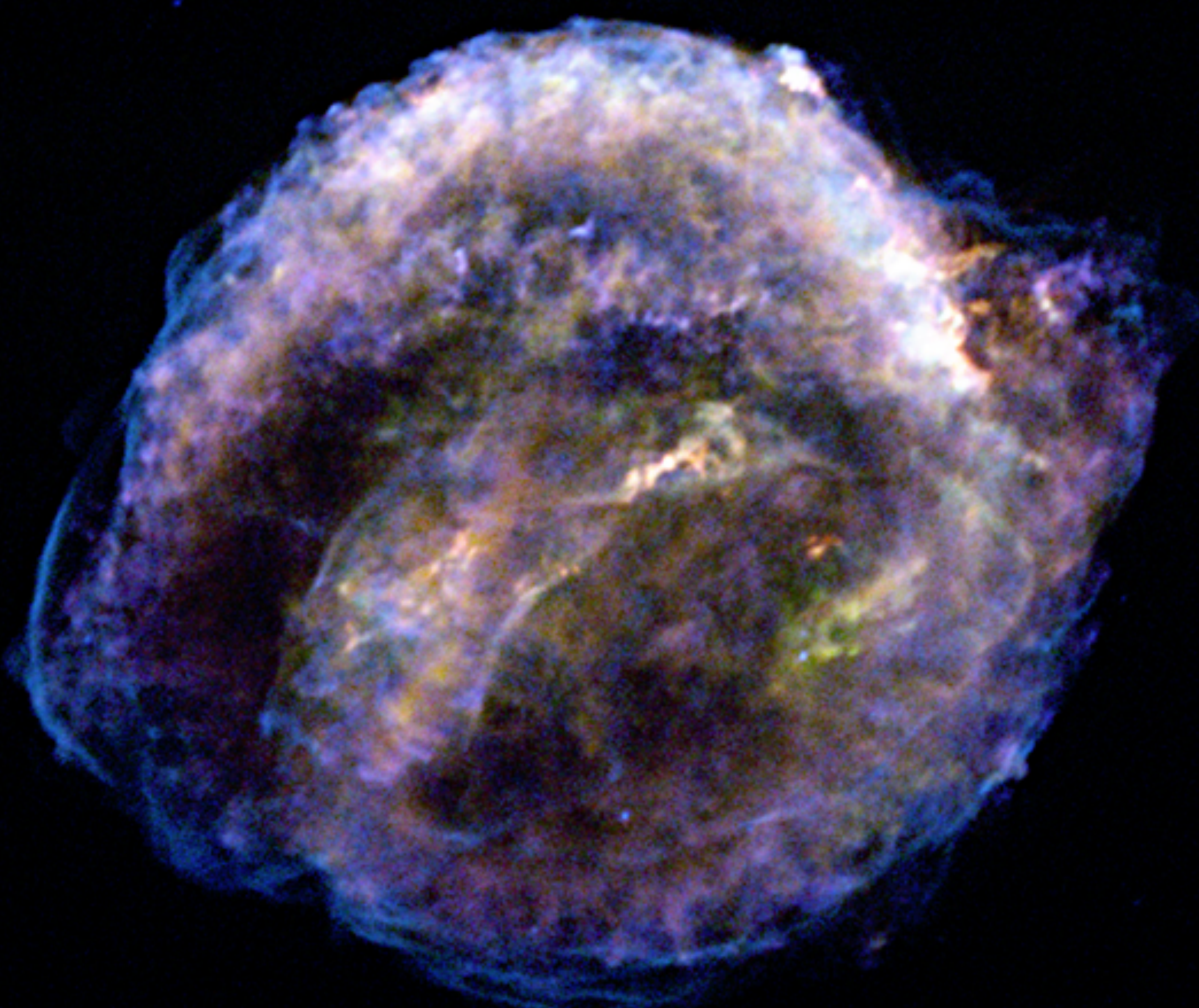


a.

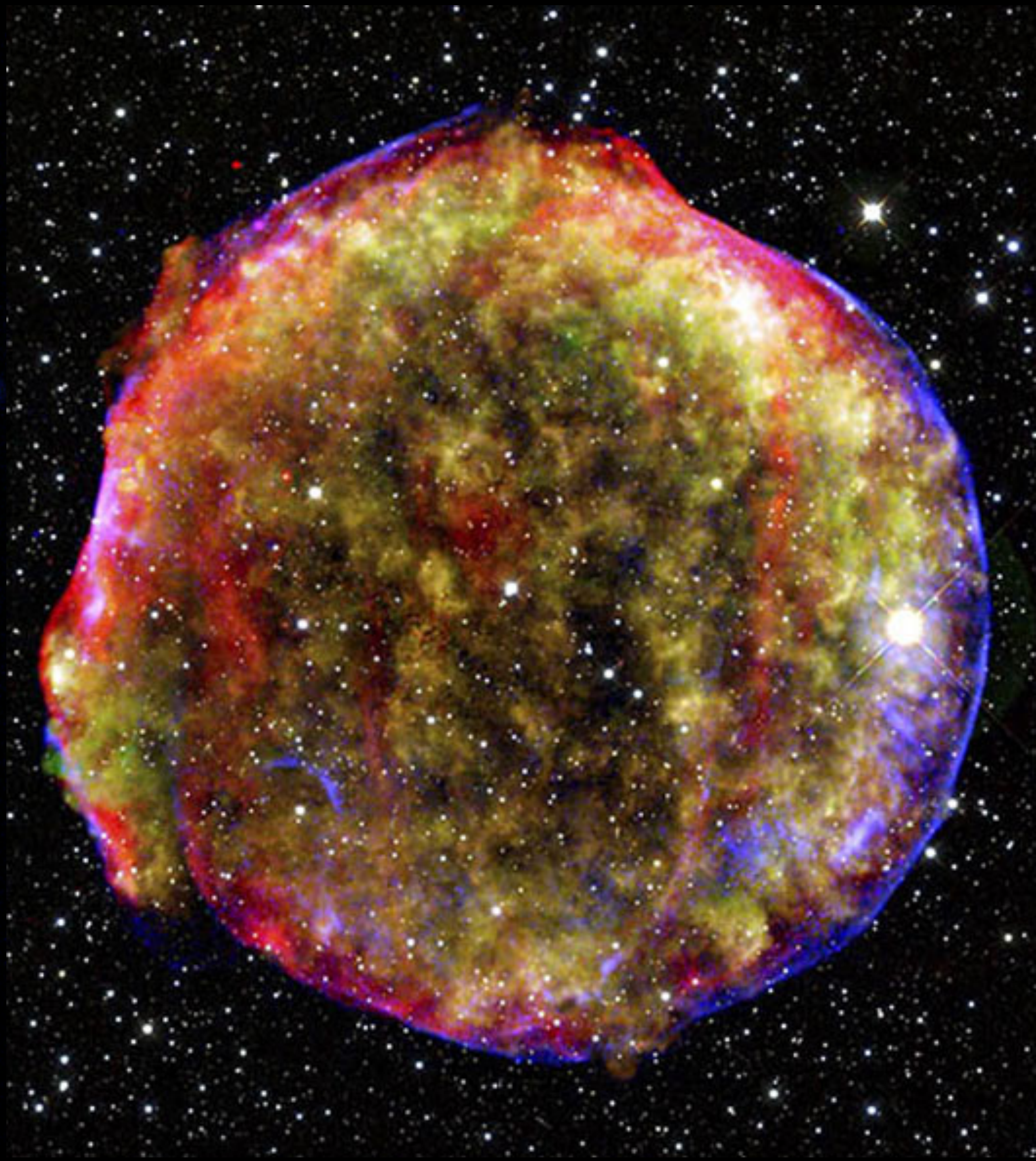


b.

More X-ray Images of Type Ia Supernova Remnants



Kepler supernova (1604)



Tycho supernova (1572)

Chap 4: The Evolution of High-Mass Stars

- CNO Cycles
- Convective cores
- Consecutive fusion shells
- End of fusion - Binding Energy
- Type I vs. II supernovae
- Neutron stars and Pulsars
- Supernova Remnants (SNR): expansion parallax method
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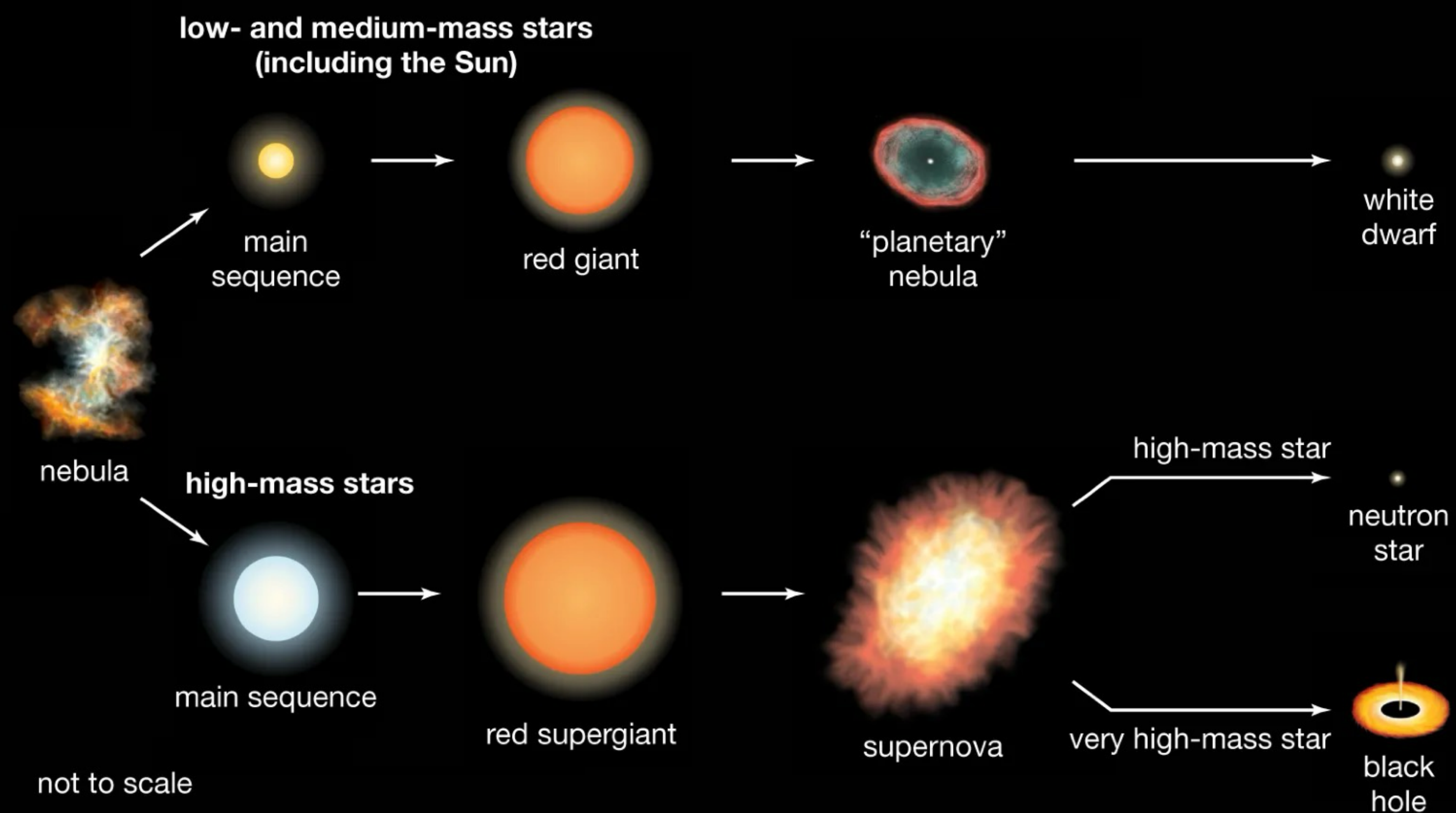

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