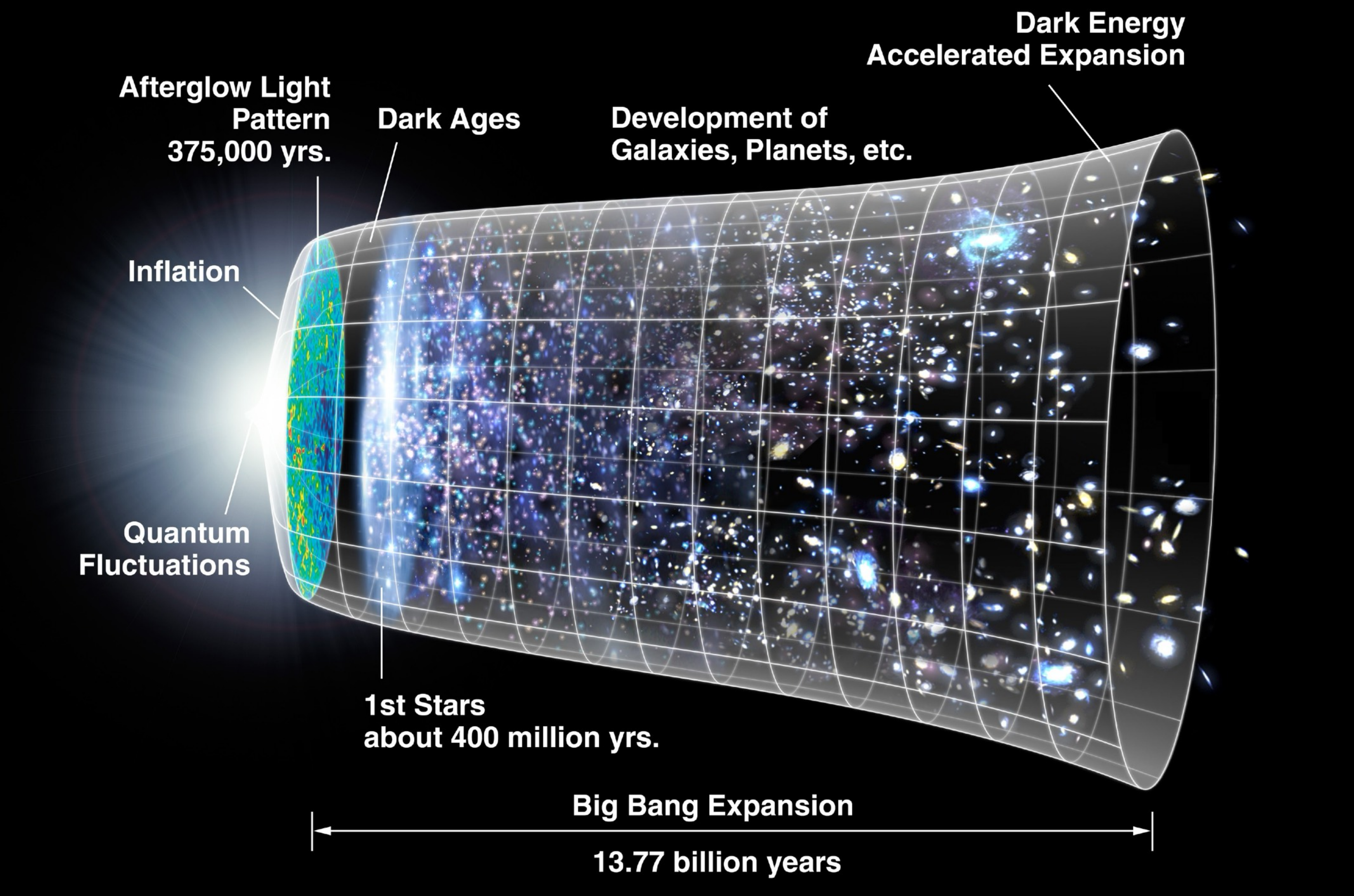
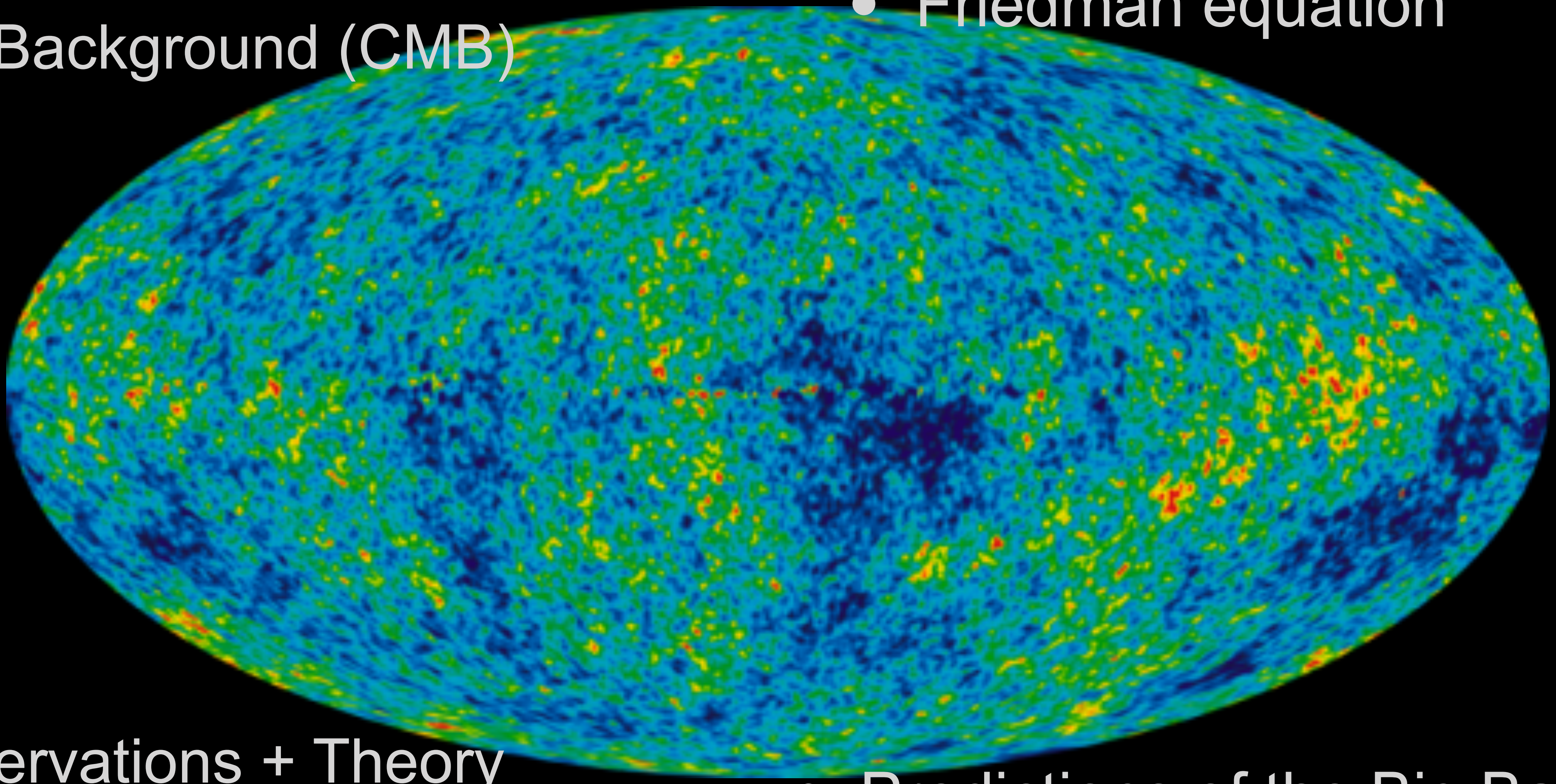
Chap 6 - The Expanding Universe



Chap 6: The Expanding Universe - Outline

- Observations (facts)
  - Hubble's Law & the Hubble "constant"
  - The Cosmic Microwave Background (CMB)
- Interpretations (theories)
  - The cosmological principle
  - Robertson-Walker metric
  - Friedman equation

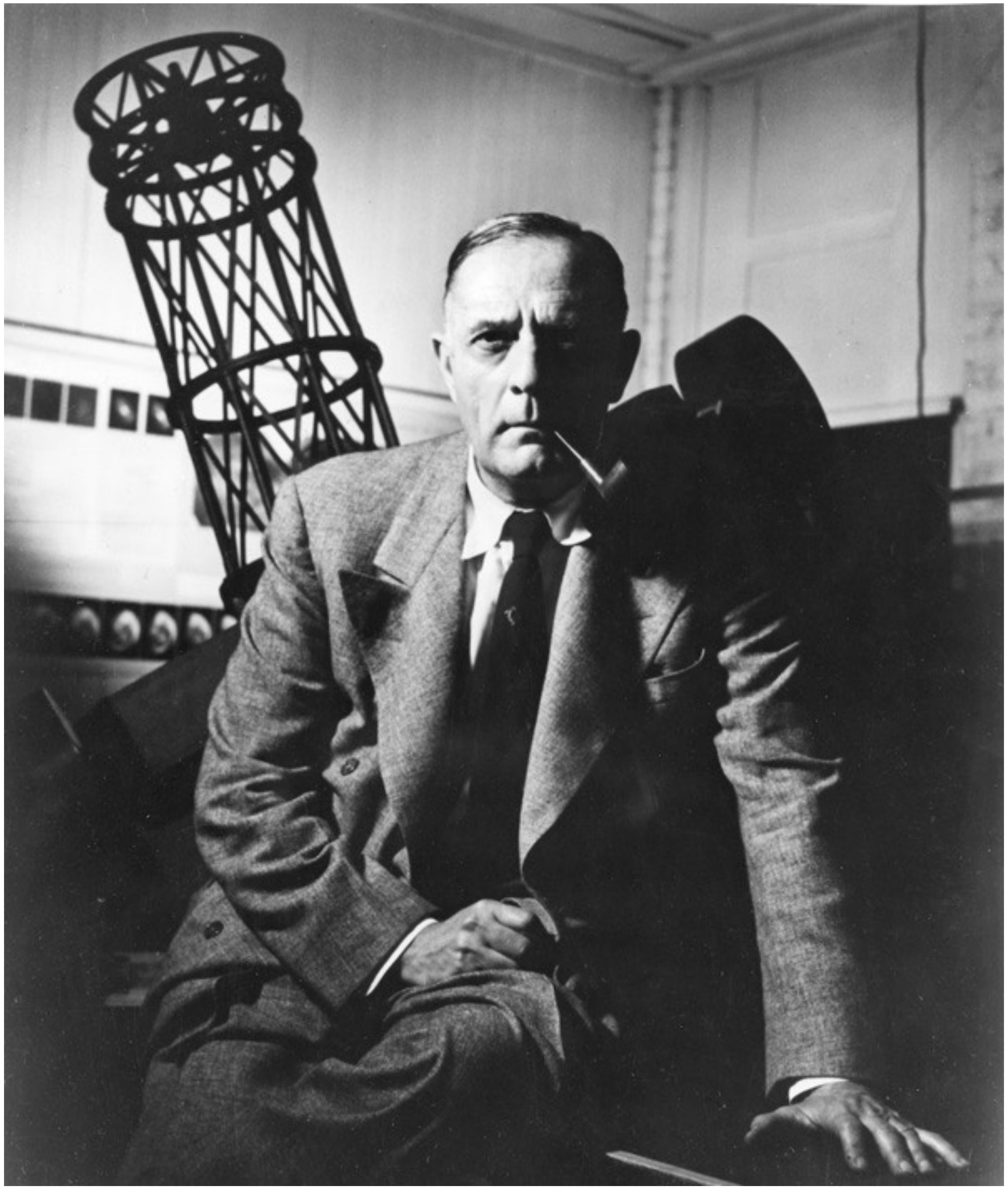


- Observations + Theory
  - Accelerating Expansion: Evidence of dark energy
  - The cosmic composition
- Predictions of the Big Bang theory: how everything began?

Evidence for an expanding Universe:

Discovery of Hubble's Law:
distance vs. redshift at $0.02 < z < 0.2$

Edwin Hubble (1889-1953)



- Born in **Marshfield, Missouri**
- B.S. & Ph.D. from University of Chicago
- Key accomplishments:
  - M31's distance: galaxies are island universes
  - Hubble's Law: the expansion of the universe
  - Hubble's sequence of galactic morphology
  - The age of the Crab nebula and its association with SN 1054.
- Photo on the left: portrait in front of the 100-in telescope on Mt Wilson, LA.

A SPIRAL NEBULA AS A STELLAR SYSTEM, MESSIER 31[1]

By EDWIN HUBBLE

ABSTRACT

Material.—The present discussion of M 31 is *based on the study of about 350 photographs* taken with the 60- and 100-inch reflectors, distributed over an interval of about eighteen years. Two-thirds of the total number were obtained by the writer during the five years 1923–1928. Since the image of the nebula is much larger than the usable fields of the telescopes, *attention was concentrated on four regions centered* on (1) the nucleus, (2) 23′ north following, (3) 17′ south, (4) 48′ south preceding the nucleus. The combined *area*, with allowance for overlapping, *represents about 40 per cent of the entire nebula.*

Resolution.—The *outer regions* of the spiral arms are *partially resolved into swarms of faint stars*, while the *nuclear* region *shows no* indications of *resolution under any conditions* with the 100-inch reflector. Intermediate regions show isolated patches where resolution is pronounced or suggested.

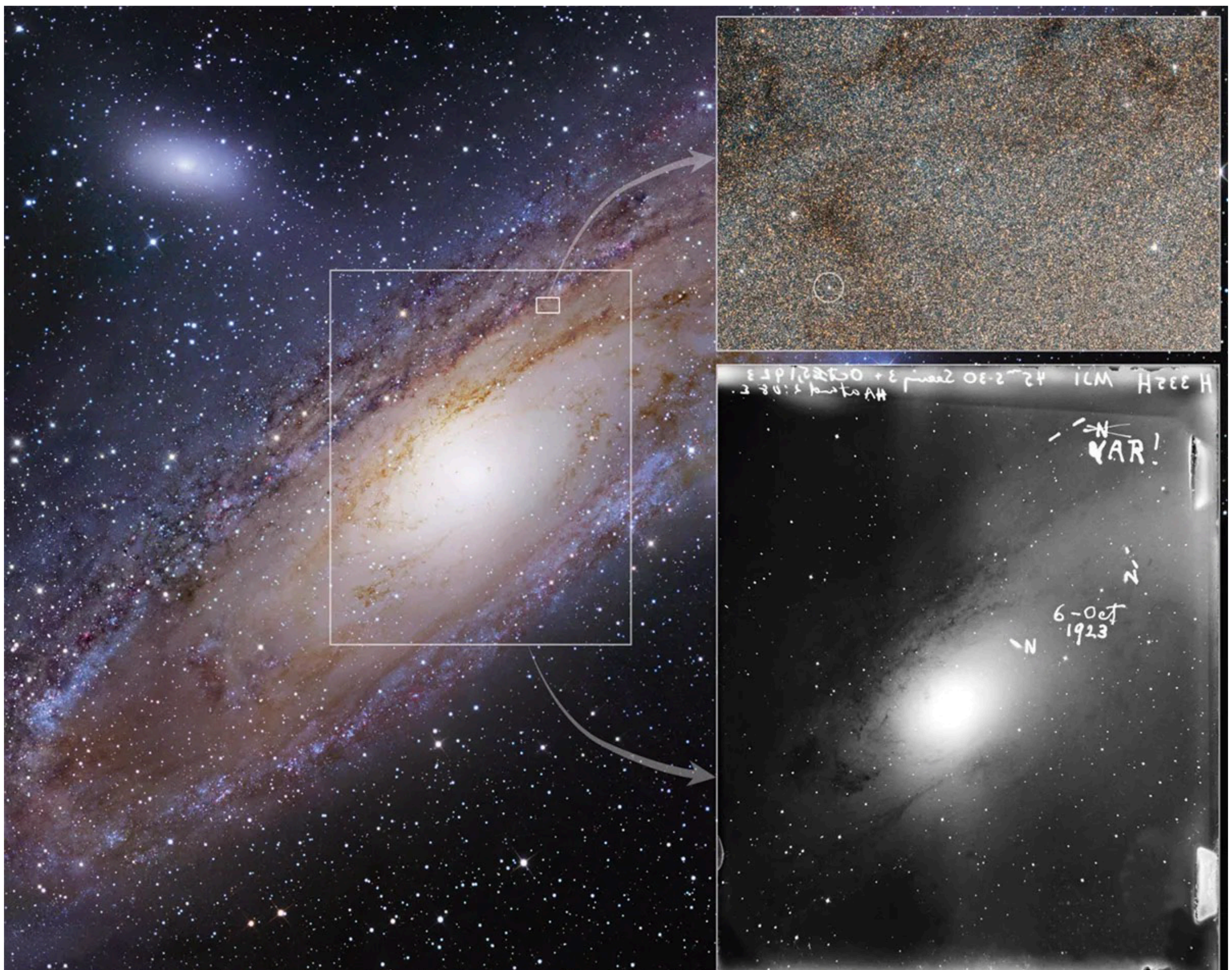
Variables.—*Fifty variables* have been found, nearly all in the outer regions where resolution is pronounced. The *survey* is believed to be *fairly exhaustive in the four selected regions down to 19.0 photographic magnitude.*

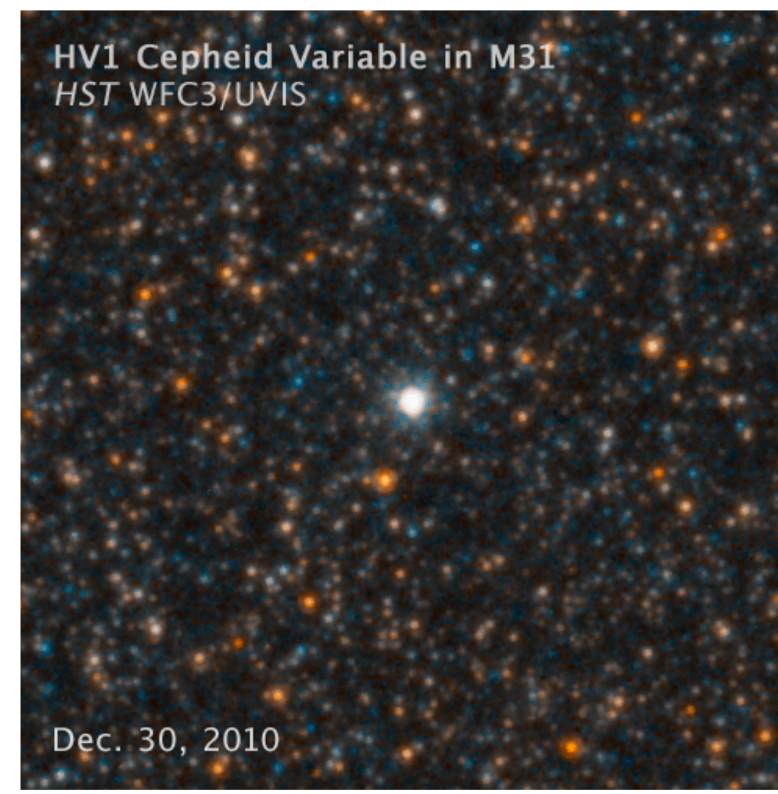
Cepheids.—*Forty* of the *variables* are *known to be Cepheids* with periods from *48 days to 10 days* and *maxima from 18.1 to 19.3 photographic magnitude;* one exceptional star varies from 17.9 to 19.2 in a period of 175 days. The *period-luminosity relation is conspicuous*, and the slope is *approximately that* found among *Cepheids in other extra-galactic systems.*

Distance of M 31 derived from Cepheid criteria.—Comparisons of period-luminosity diagrams indicate that M 31 is about 0.1 mag. or 5 per cent more distant than M 33, and about 8.5 times more distant than the Small Magellanic Cloud. Using Shapley's value for the Cloud, we find the *distance of M 31* to be *275,000 parsecs.*

Distance measurements from Cepheid variables (Standard Candles)

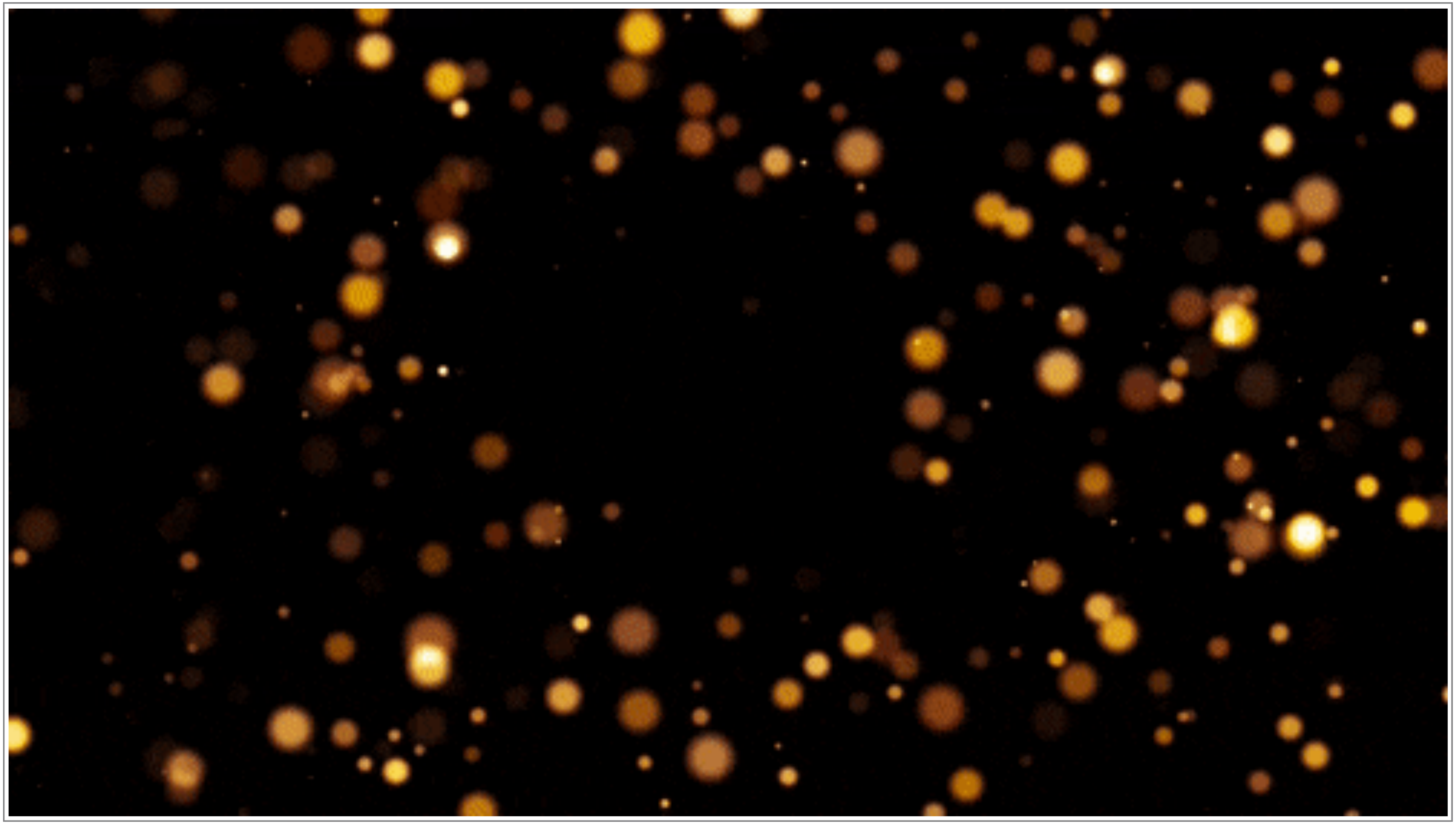
- Hubble (1929): *"A Spiral Nebula as a Stellar System, Messier 31"*
- He discovered a **Cepheid variable** inside of the Andromeda (**M31-V1**).
- He used **Leavitt** (1908)'s **Luminosity-Period relation** to calculate the distance to M31. This distance was much greater than the size of the Milky Way per Shapley.





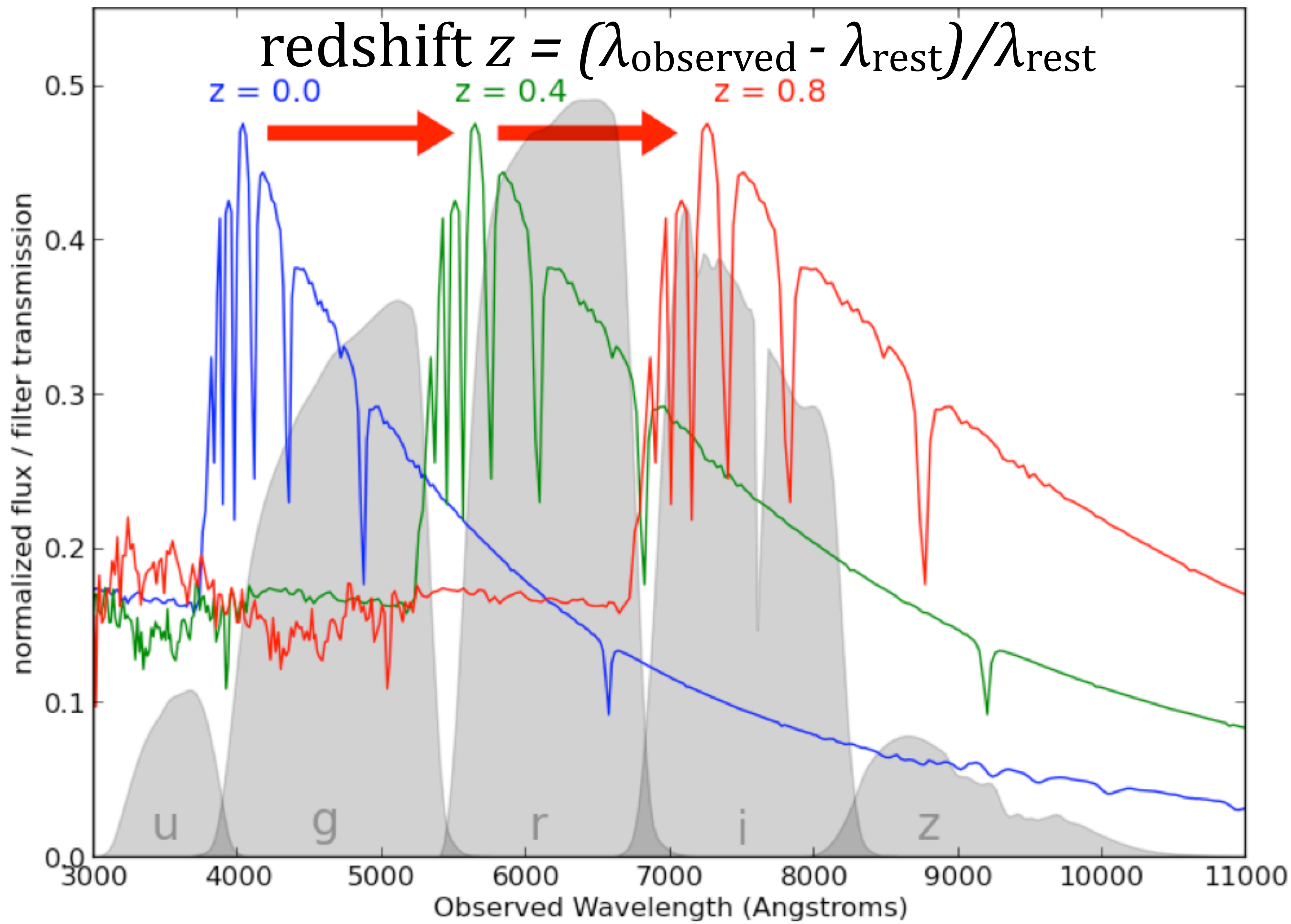
Spectroscopy: radial velocity measurements to trace the flow

- Use galaxies as “massless” particles to trace the flow caused by gravity or other things; this is similar to measuring the peculiar velocities of solar neighborhood stars in the Milky Way.



Spectroscopy: radial velocity measurements to trace the flow

- Instead of finding similar numbers of *blueshifted* and *redshifted* galaxies, Hubble found that most of the galaxies are *redshifted* — i.e., they appear to be moving away from us.



Redshift and recession velocity at $z << 1$

- In 1920s, two technological advancements enabled the discovery of Hubble's law
  - Distance ***D*** from standard candles like Cepheids (*L*-*P* relation, aka Leavitt's Law)
  - redshift ***z*** from moderate resolution spectroscopy:

$$z = (\lambda_{\mathrm{observed}} - \lambda_{\mathrm{rest}})/\lambda_{\mathrm{rest}}$$

- inspecting the definition of *z*, does it look similar to the **Doppler shift equation**?

$$v_r/c = (\lambda_{\mathrm{observed}} - \lambda_{\mathrm{rest}})/\lambda_{\mathrm{rest}}$$

- so *small* redshifts are usually converted to **recession velocities** using the following formula:

$$v_r = z \times c \text{ (when } z << 1)$$

- This relation breaks at $z > 1$, when it implies galaxies are moving away from us at speeds greater than that of light. **Cosmological redshifts are not due to Doppler effects, but due to the increase in scale.**

A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

By Edwin Hubble

Mount Wilson Observatory, Carnegie Institution of Washington

Communicated January 17, 1929

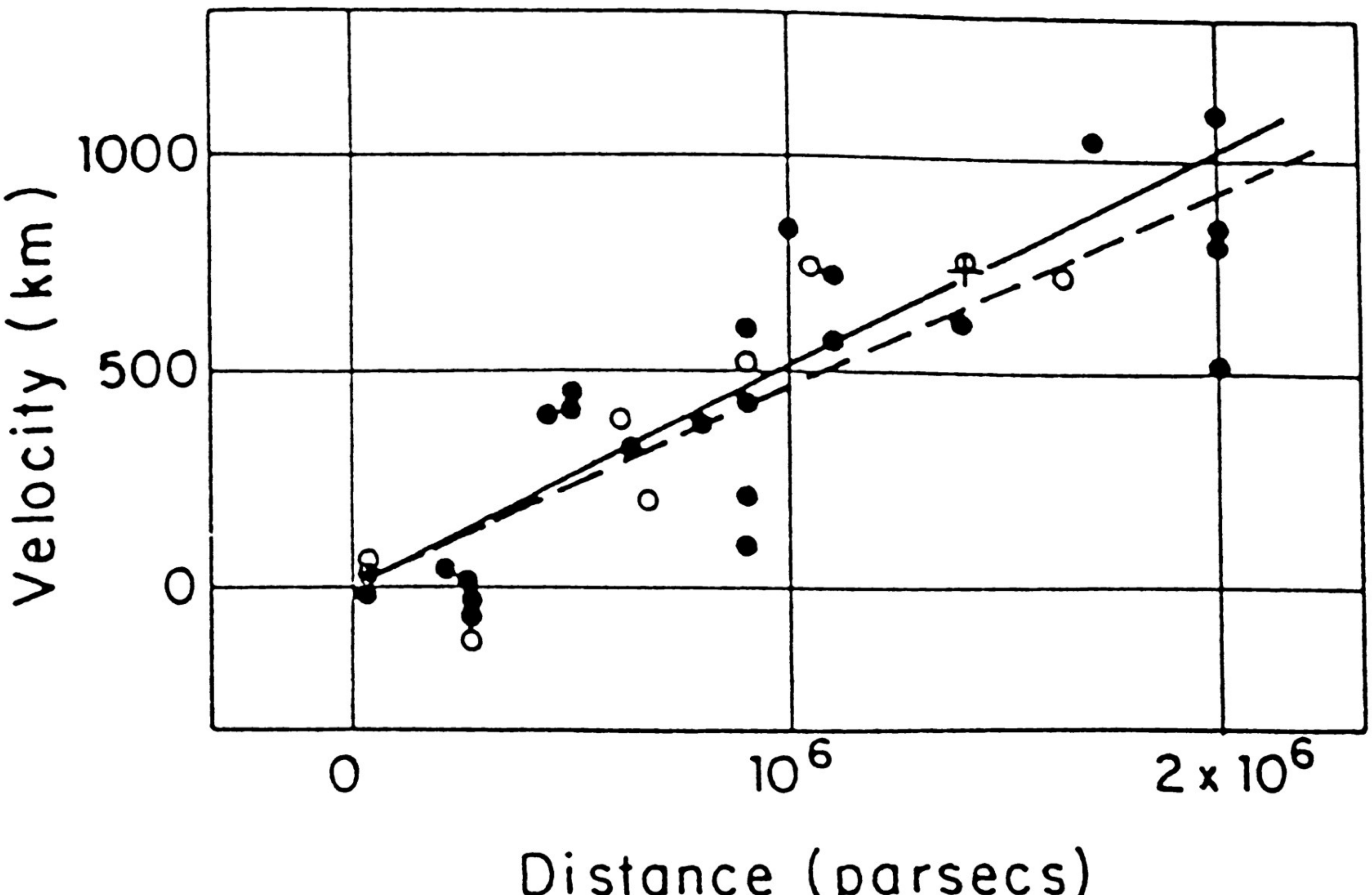
Determinations of the motion of the sun with respect to the extra-galactic nebulae have involved a K term of several hundred kilometers which appears to be variable. Explanations of this paradox have been sought in a correlation between apparent radial velocities and distances, but so far the results have not been convincing. The present paper is a re-examination of the question, based on only those nebular distances which are believed to be fairly reliable.

Distances of extra-galactic nebulae depend ultimately upon the application of absolute-luminosity criteria to involved stars whose types can be recognized. These include, among others, Cepheid variables, novae, and blue stars involved in emission nebulosity. Numerical values depend upon the zero point of the period-luminosity relation among Cepheids, the other criteria merely check the order of the distances. This method is restricted to the few nebulae which are well resolved by existing instruments. A study of these nebulae, together with those in which any stars at all can be recognized, indicates the probability of an approximately uniform upper limit to the absolute luminosity of stars, in the late-type spirals and irregular nebulae at least, of the order of M (photographic) = -6.3.[1] The apparent luminosities of the brightest stars in such nebulae are thus criteria which, although rough and to be applied with caution,

Hubble's Law, also discovered in 1929

The slope of the **velocity-distance relation** measures the expansion rate of the universe, and it's called the **Hubble constant H_0.** Initially, Hubble measured a value that is 10x too high at 500 km/s/Mpc.

Hubble (1929): "*A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae*"

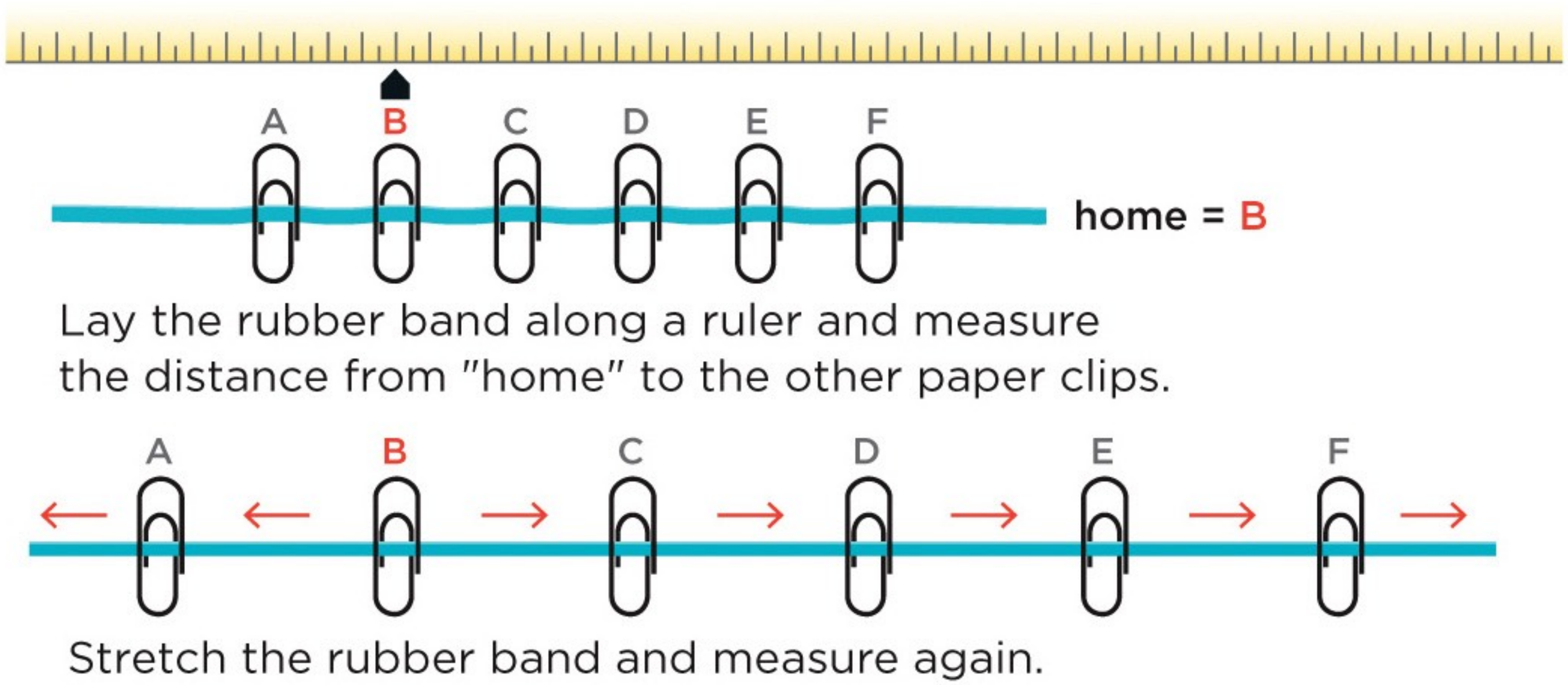


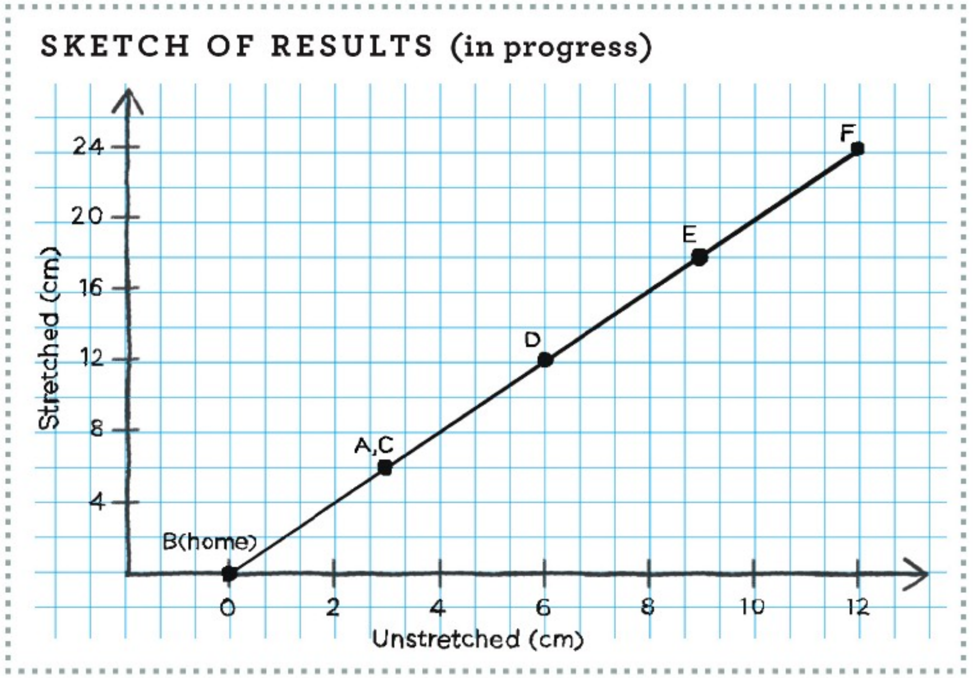
What are the implications of Hubble's Law?

- *Not only* most galaxies are **redshifted**, *but also* their **redshifts** increase **linearly** with **distance**! $cz = H_0 D$
- Hubble's Law indicates (1) the universe is expanding uniformly everywhere and (2) the universe had a beginning.

Hubble Flow: Visualizing Expansion in 1D

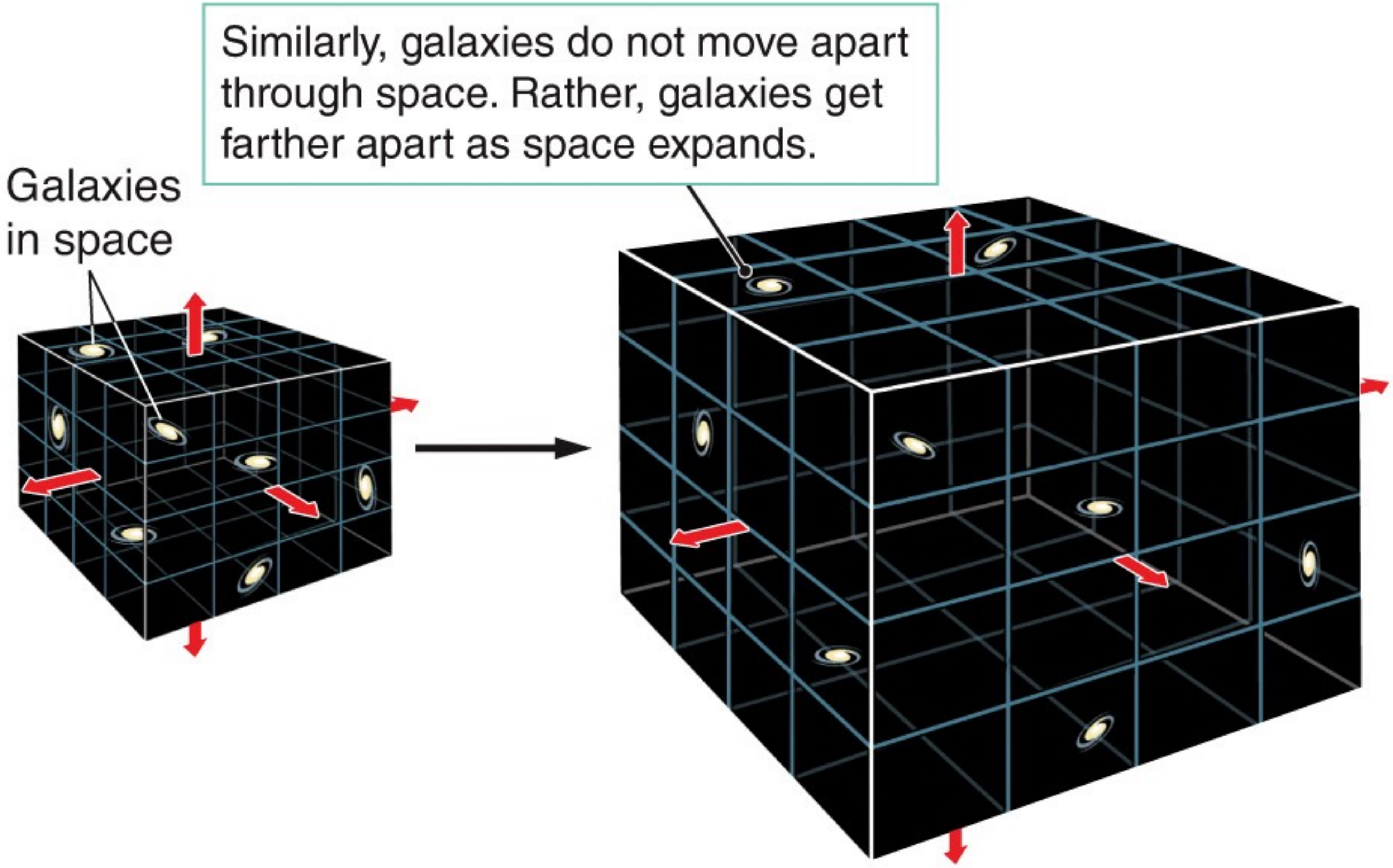
- Simple expansion model: paper clips on a rubber band
- As the rubber band stretches, an ant riding on clip B:
  - observes itself as stationary
  - observes clip F moving away twice as fast as clip D
  - observes clip A and C moving away at the same speed
- An ant on any paper clip would make similar observations.





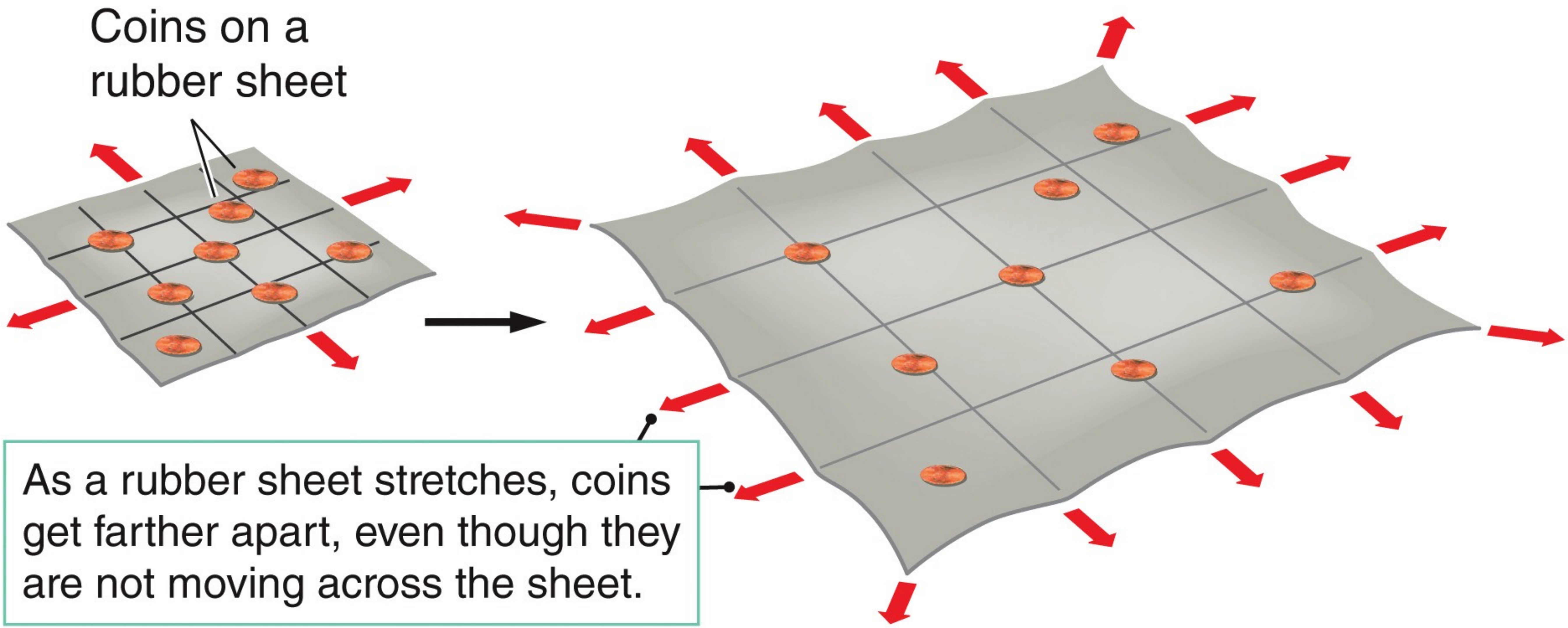
Hubble Flow: Visualizing Expansion in 3D

- Galaxies as tracers of space shows that universe is expanding.
  - Galaxies are moving away from us because space is *created*
  - New space is created *uniformly* in the Universe, leading to the *linear* proportionality between redshift and distance.



Hubble Flow: no "center" of the expansion, and non-Doppler redshifts

- It might appear that we are in the center of the universe, with all galaxies moving away. *But* there is no center: from any point in the universe, it would look the same.
- So the redshifts of galaxies are **not due to motion** (Doppler shift), but **due to space creation** (increasing in scale)



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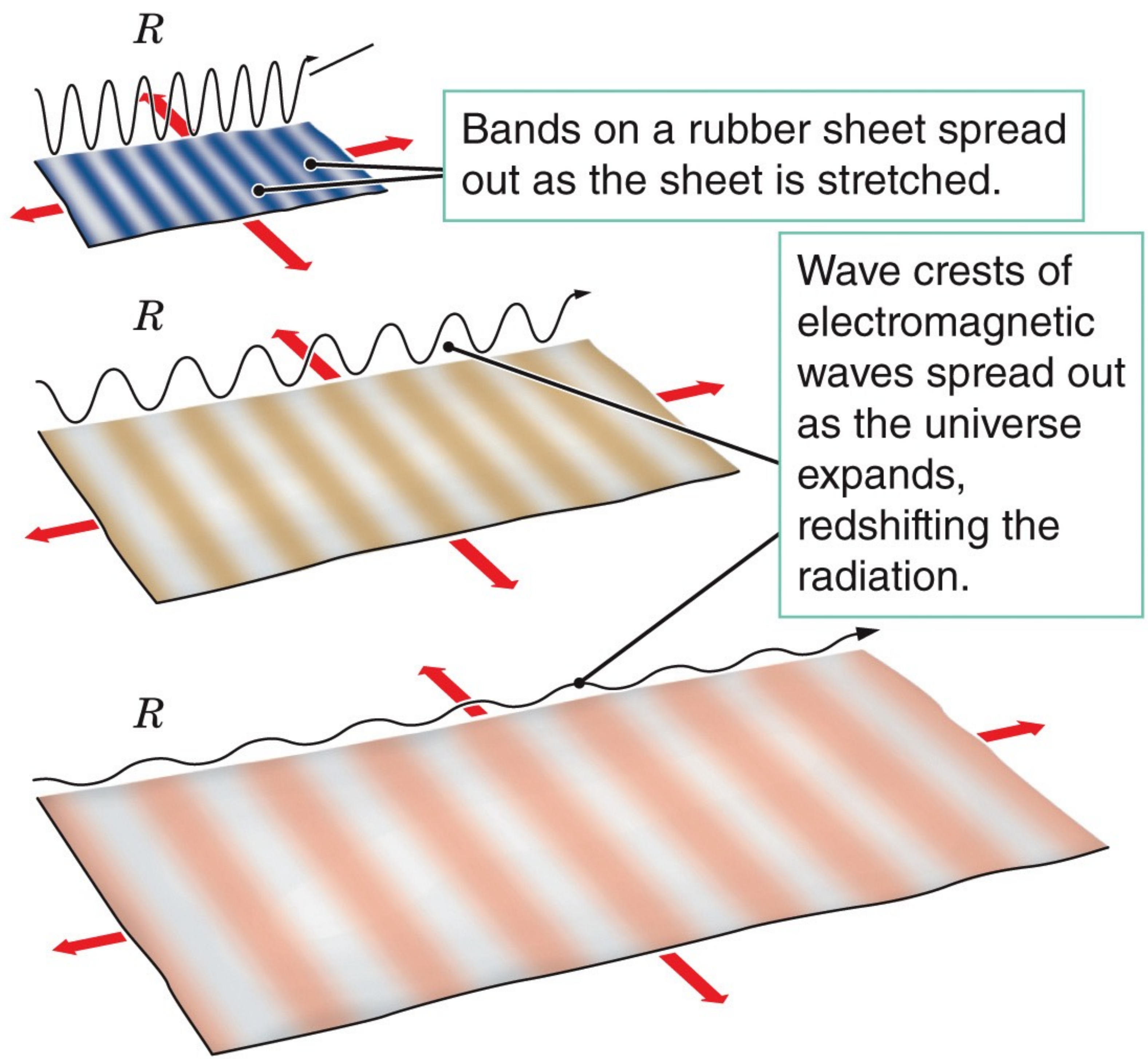
Cosmological redshift is NOT Doppler shift, it is caused by an increasing scale factor

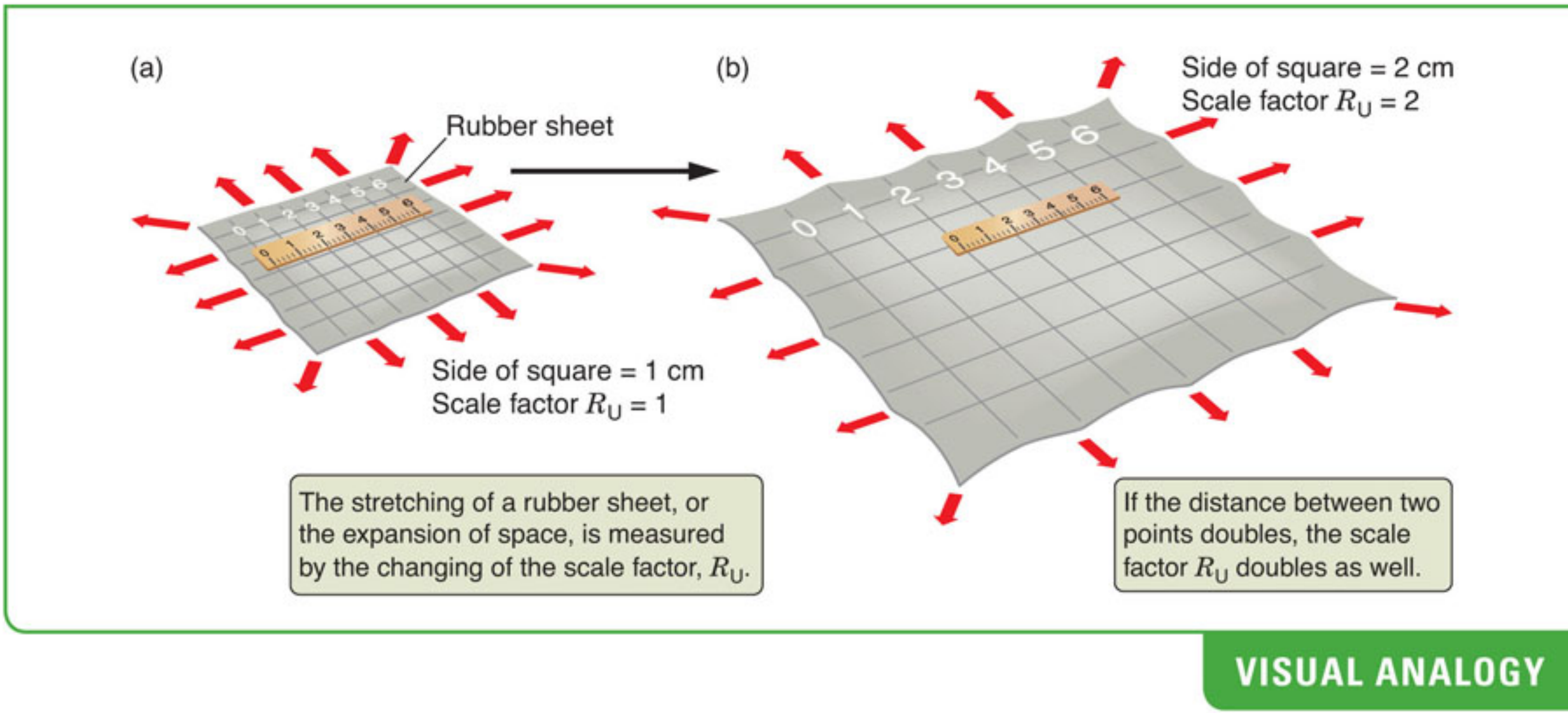
Redshift and the Expansion

- Redshifts of galaxies are **not** due to Doppler shifts (relative motions)
- Instead, the light is “stretched out” as it travels through the expanding universe: this is known as **cosmological redshift**.
- The wavelength of light is getting longer over time because the scale factor is increasing.
- A higher redshift indicates a smaller scale factor. Light emitted at high redshift will be very stretched out.



An uniform expansion of the Universe must be scalable

- Imagine space like a rubber sheet, stretching the sheet increases the scale factor everywhere on the sheet, causing distances between grid points to increase

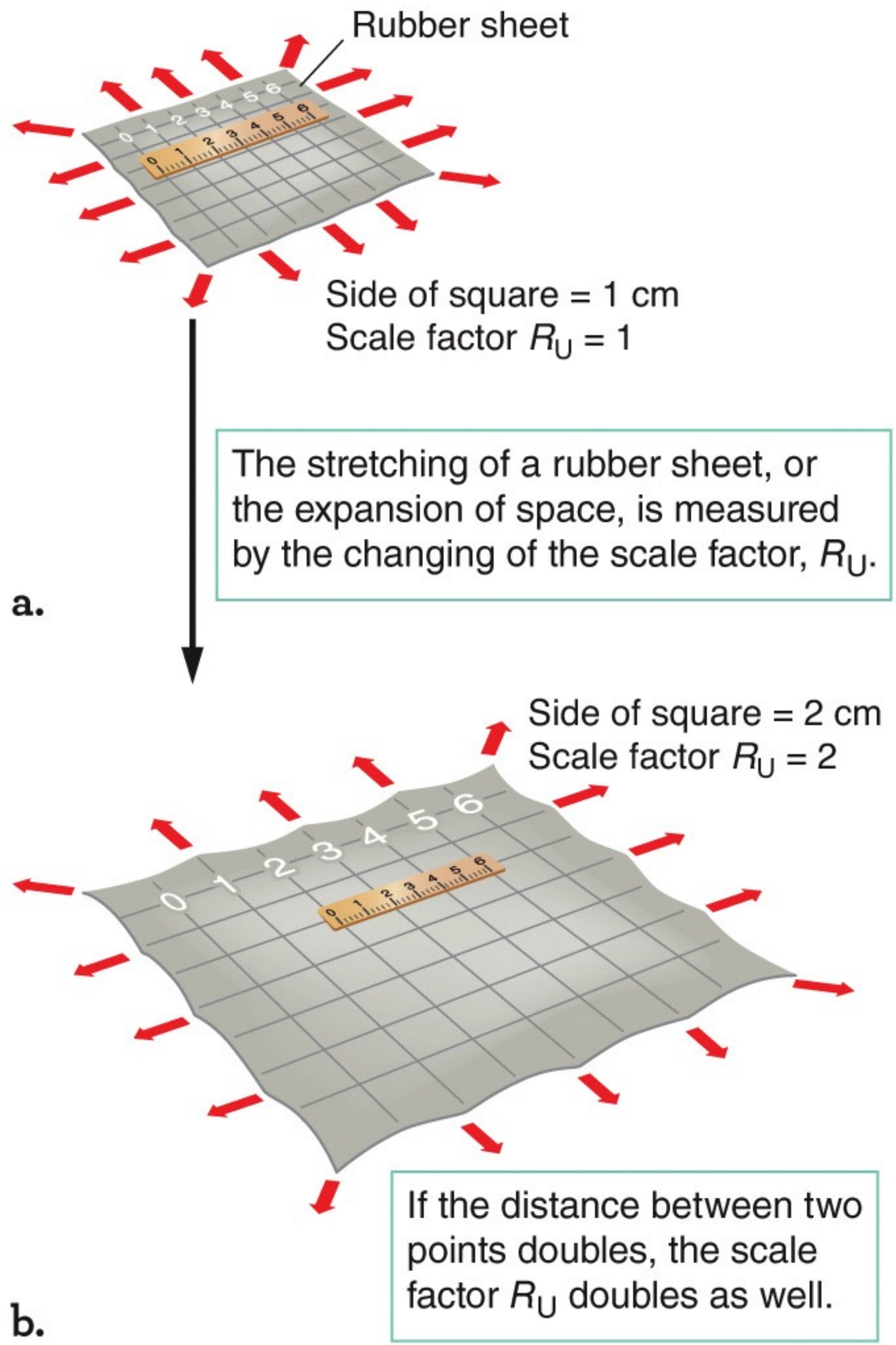


Cosmological redshift of photons emitted from a distant galaxy is caused by the increasing **scale factor** of the universe (R_u):

$$\frac{R_U(z)}{R_U(0)} = R_U(z) = \frac{1}{1+z} = \frac{\lambda_{\mathrm{rest}}}{\lambda_{\mathrm{observed}}} = \frac{\lambda(z)}{\lambda(0)}$$

The Scale Factor

- The **scale factor** (R_U) is a measure of how much the universe has expanded.
- The scale factor gets smaller as we look back in time.
  - For example, when R_U = 0.5, the universe was half of its current size.
- The expansion pulls galaxies apart but does not destroy galaxies (yet!):
  - At scales smaller than the Local Group, gravitational forces can overcome the space expansion.



Redshift gives the scale factor of the Universe at the emitted time

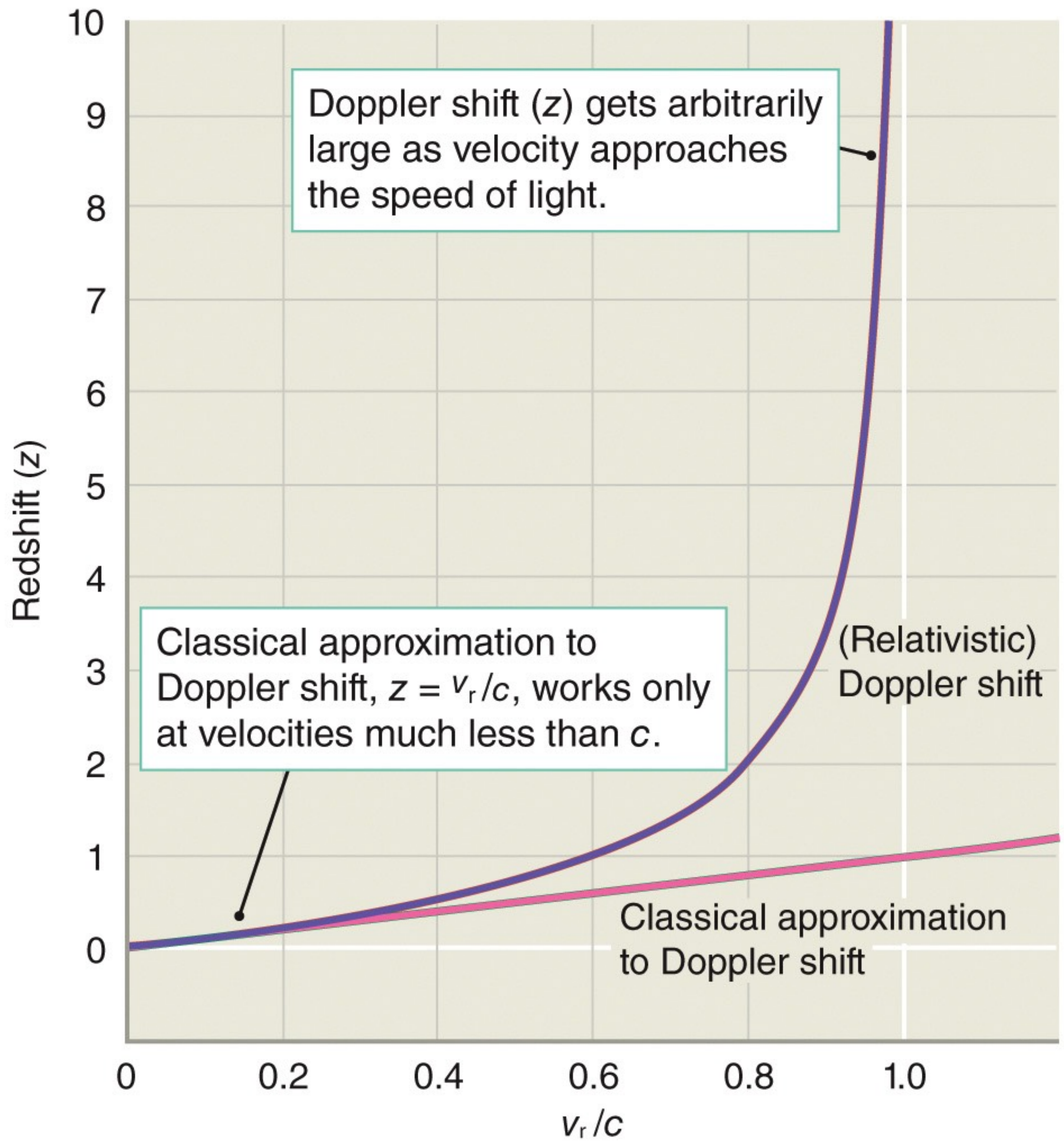
- The redshift tells us how much the universe has expanded since a galaxy's light was emitted.
- Write R_U as the *scale factor*, then

$$R_U = \frac{1}{1+z} = \frac{\lambda_{\text{rest}}}{\lambda_{\text{observed}}}$$

- Example: $z = 0$ means today, when $R_U = 1.0$. This is the maximum scale factor.
- Example: $z = 1$ means $R_U = 0.5$. The universe was half its current size when light was emitted from this galaxy.
- Example: $z = 9$ means $R_U = 0.1$

What is redshift? Classical vs. Relativistic Doppler Shift, and Scale Factors

- The **classical Doppler shift formula,**
  $$1 + z = \frac{\lambda_{obs}}{\lambda_0} = 1 + \frac{v_r}{c}$$
  gives recession velocity:
  $$v_r = cz$$
- The **relativistic Doppler shift formula,**
  $$1 + z = \frac{\lambda_{obs}}{\lambda_0} = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}}$$
  gives recession velocity:
  $$v_r = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$
- But, **cosmological redshift** should be understood as a ratio of **scale factors**:
  $$1 + z = \frac{\lambda_{obs}}{\lambda_0} = \frac{1}{R_U(z)}$$

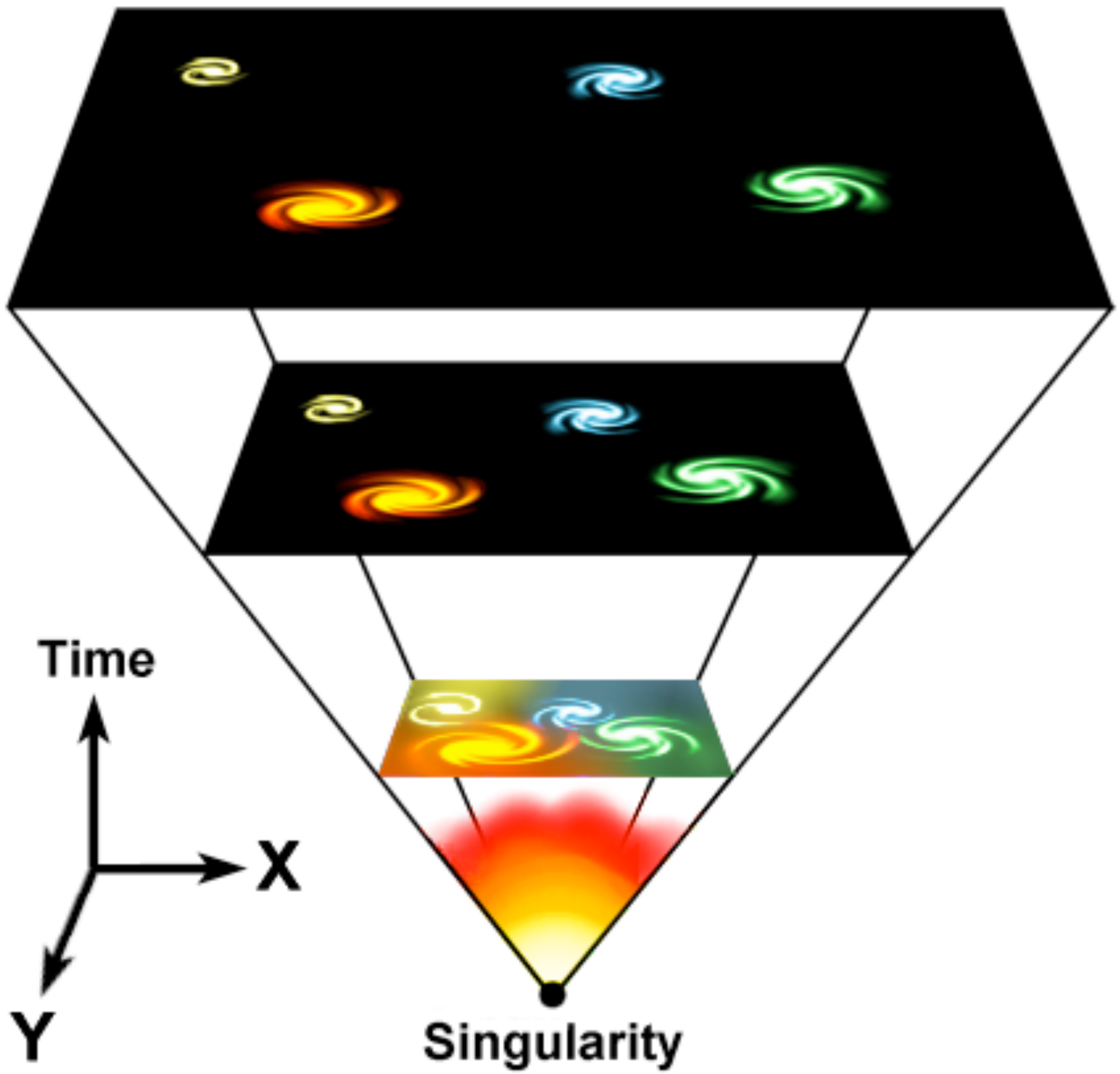


The Hubble Time ($t_H = 1/H_0$)

An estimate of the age of the Universe

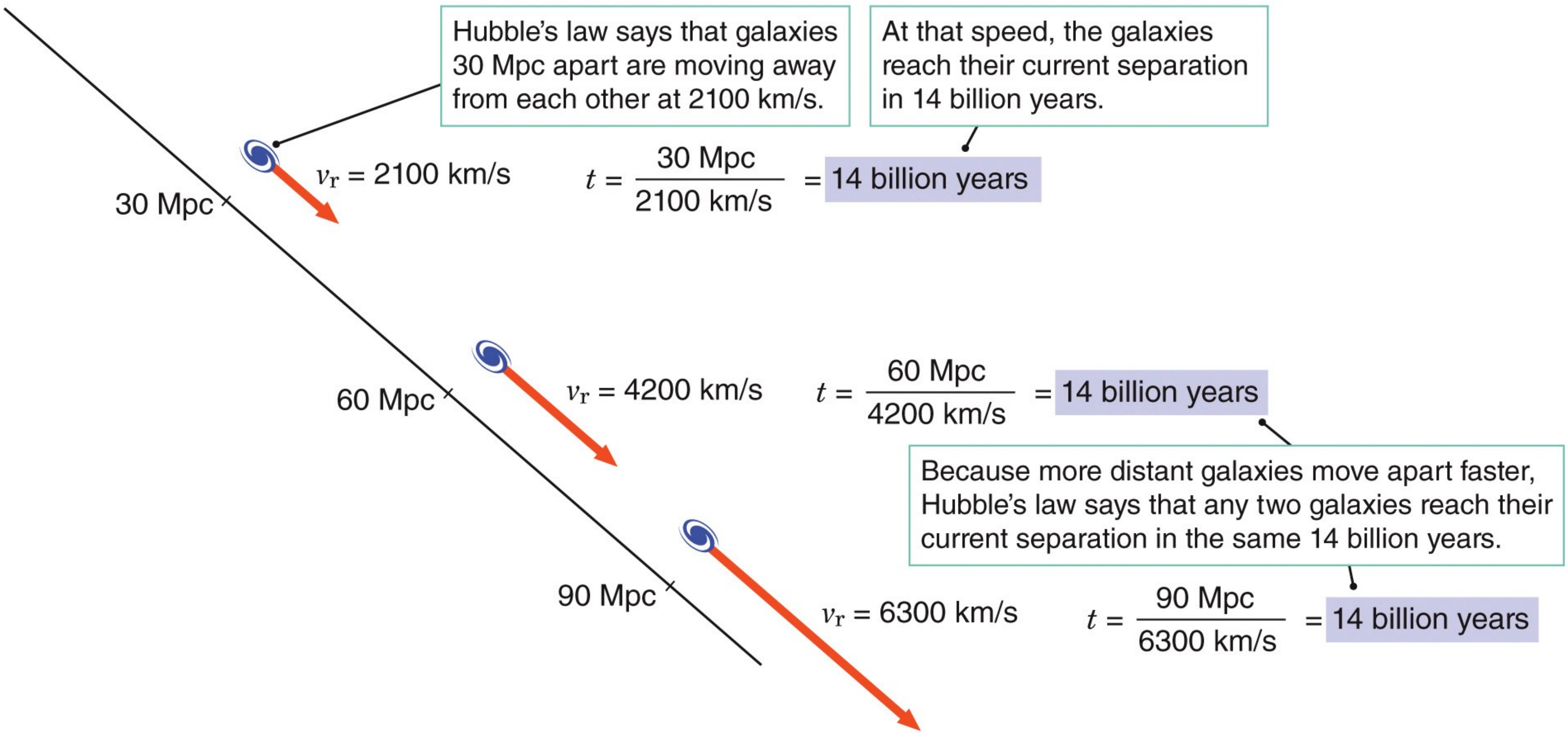
Expansion of space means that Universe was very small in distant past

- If galaxies are getting farther apart now, they must be closer together in the past. Hubble's law implies that the entire Universe started from a single point. **Can we estimate when the Universe was a single point?**



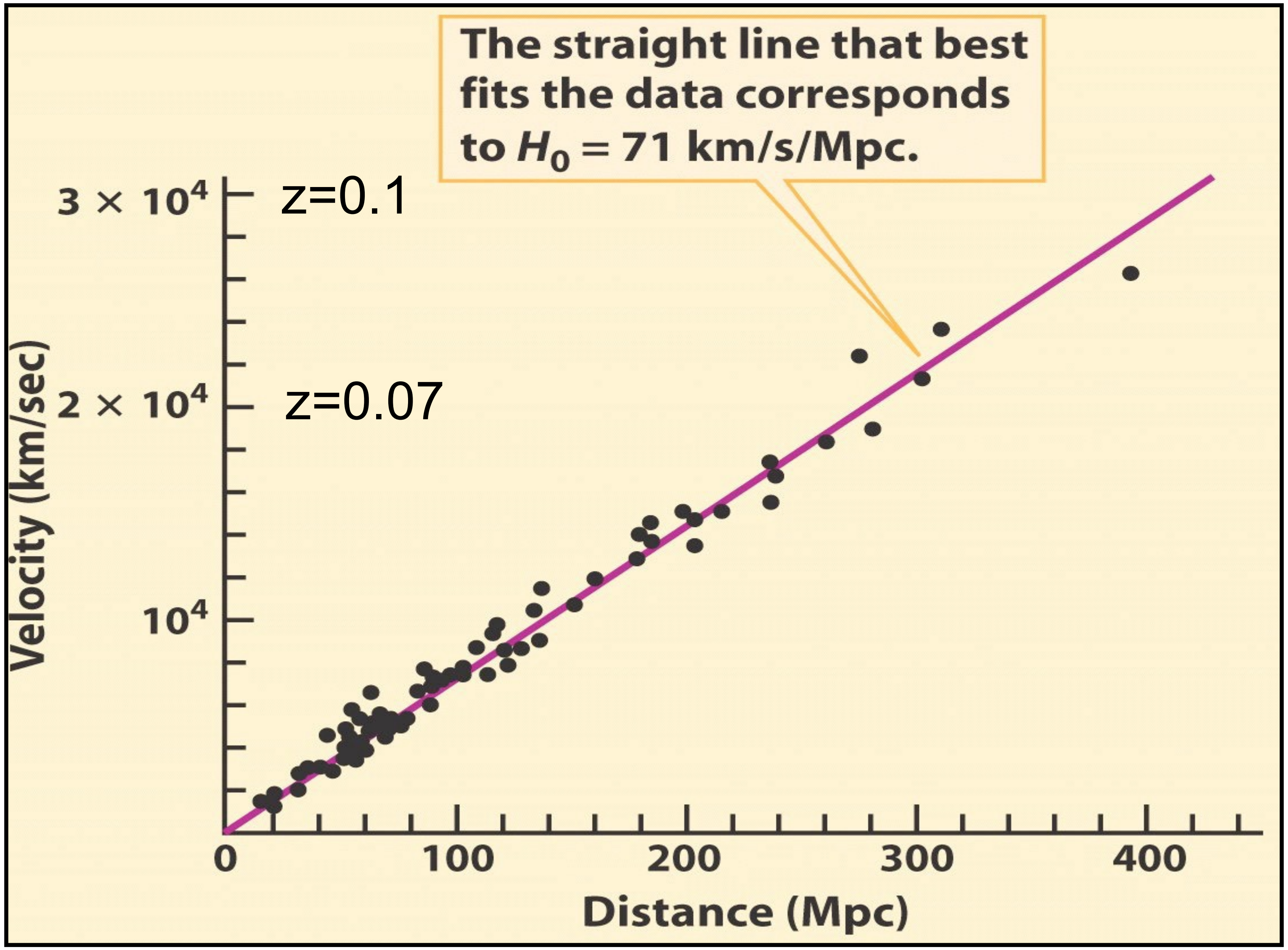
Visual Summary

- Because Hubble's law is a linear relation, it does not matter which two galaxies you use to make this measurement; the end result will be the same.



The Hubble Time: How long ago did the Universe start expanding?

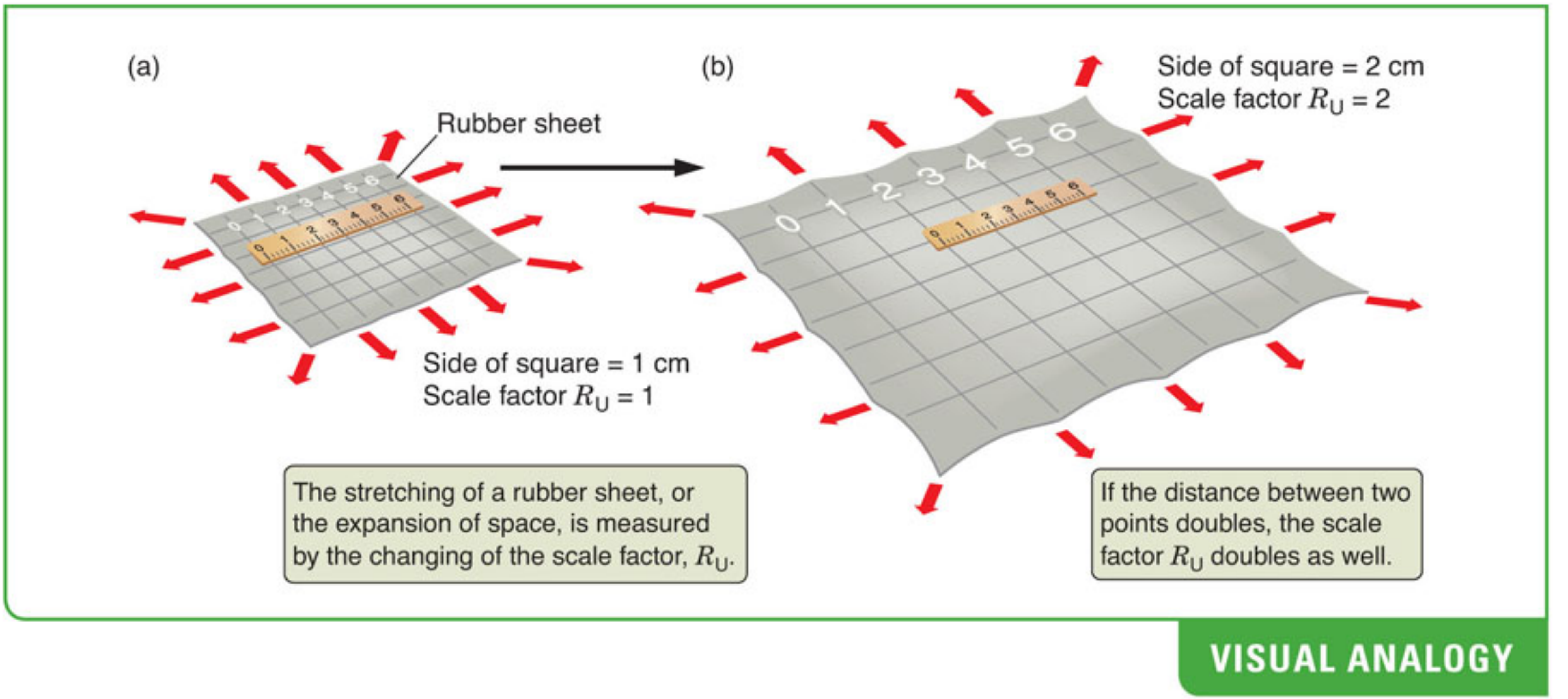
- ***Assume* space creation rate** is constant: $\frac{dD}{dt} = \text{constant} = H(t)D(t) = H_0 D(t_0)$
- Then, move ***dt*** to the right side and integrate both side from ***t=0*** to ***$t=t_0$***, we have:
  $$D(t_0) - D(t = 0) = H_0 D(t_0) \cdot (t_0 - 0)$$
- Given that ***D(t=0) = 0***, we can solve **the age of the Universe *t_0*** today:
  $$t_0 = 1/H_0 = 1/(70\ \text{km/s/Mpc}) \approx 14\ \text{Gyr}$$



Cosmological Redshifts measure the Scale Factor of the Universe

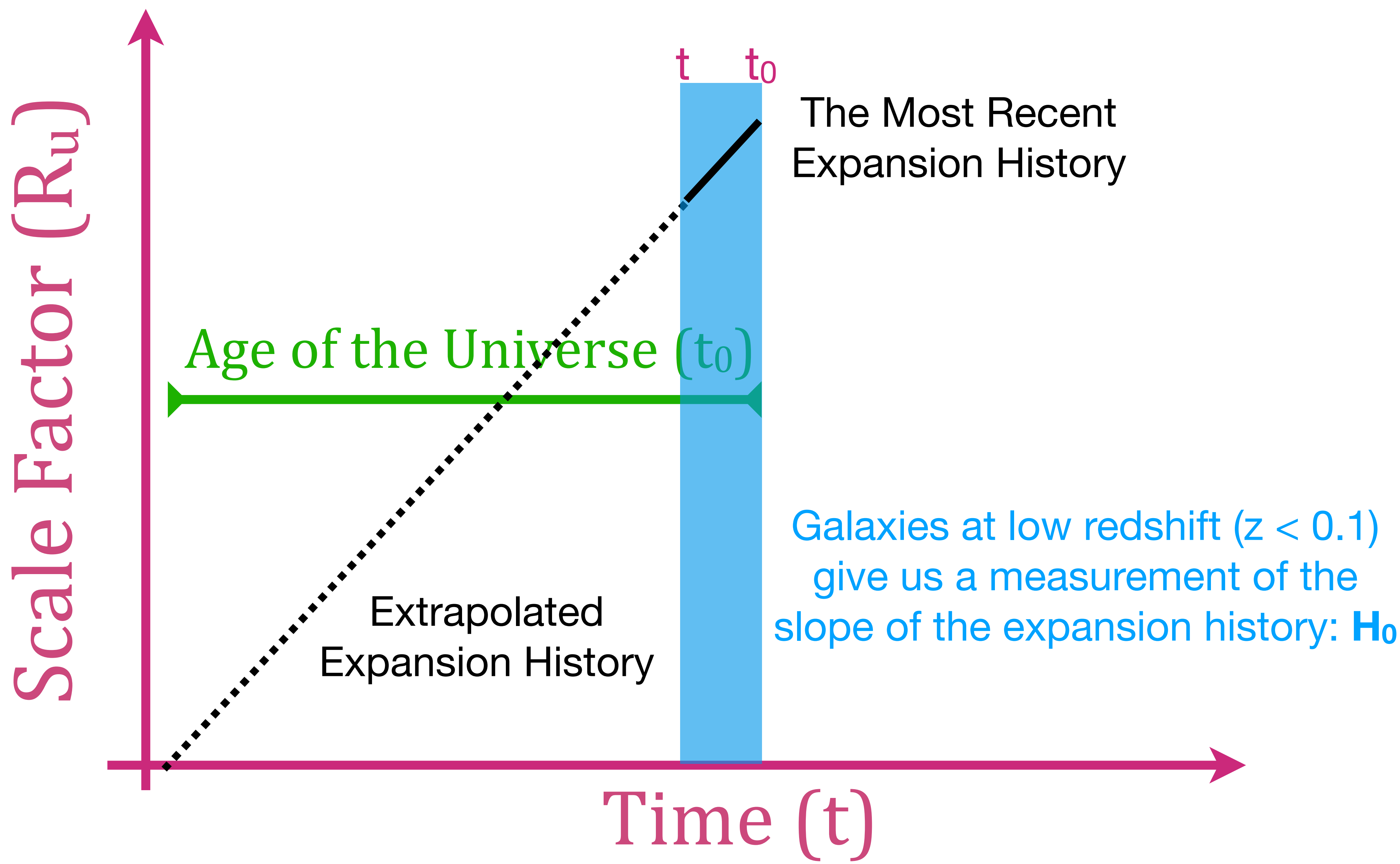
- **cosmological redshift** should be understood as a ratio of **scale factors**:

$$1 + z = \frac{\lambda_{obs}}{\lambda_0} = \frac{1}{R_U(z)}$$



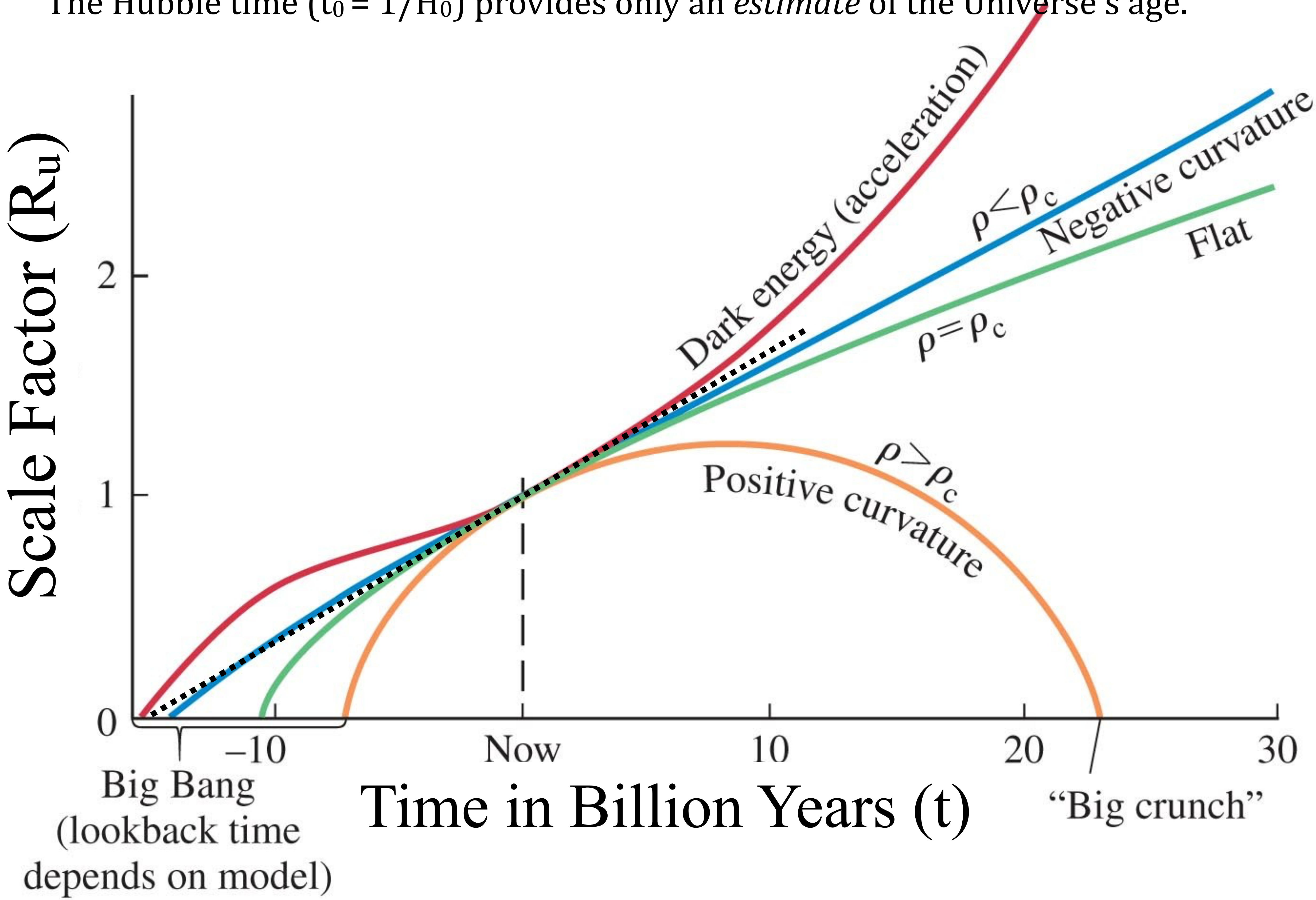
Alternative derivation of the Hubble time using Scale Factor

- **Hubble's law**: $cz = H_0 D = H_0 [c\,(t_0 - t)]$, where we used the ***lookback time*** times c to replace distance D. Canceling c on both sizes, we have $\boldsymbol{z = H_0\,(t_0 - t)}$
- **Scale factor**: $R_u = 1/(1+z) \approx 1 - z$ for $z \ll 1$
- which give the *most recent expansion history* of the Universe: $\boldsymbol{R_u = 1 - H_0\,(t_0 - t)}$,
- Extrapolating the relation to $\boldsymbol{R_u(t=0) = 0}$, we solve for *Hubble time:* $\boldsymbol{t_0 = 1/H_0}$



The various possible expansion histories of the Universe

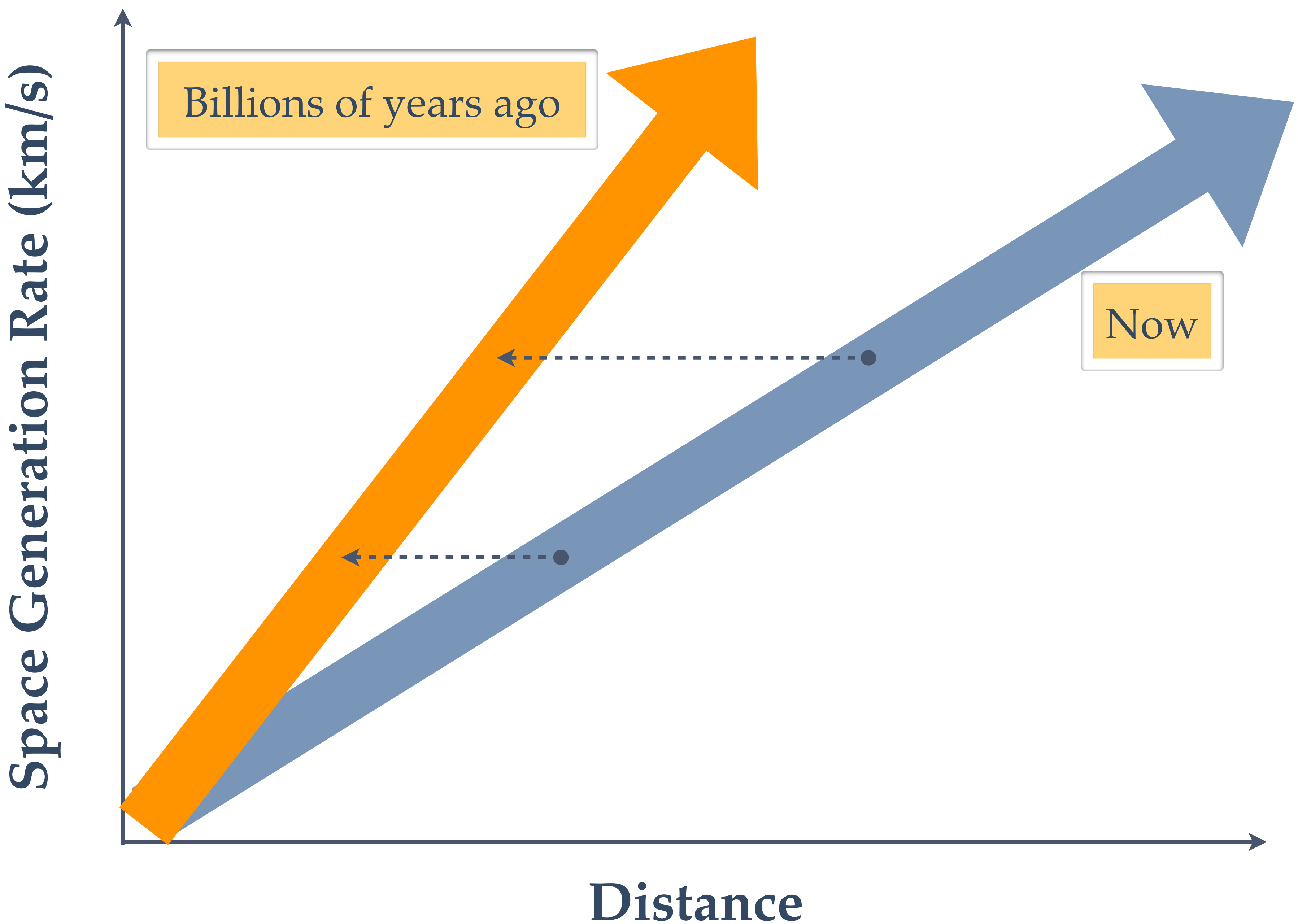
The Hubble time ($t_0 = 1/H_0$) provides only an *estimate* of the Universe's age.



A Changing Hubble Parameter H(z)

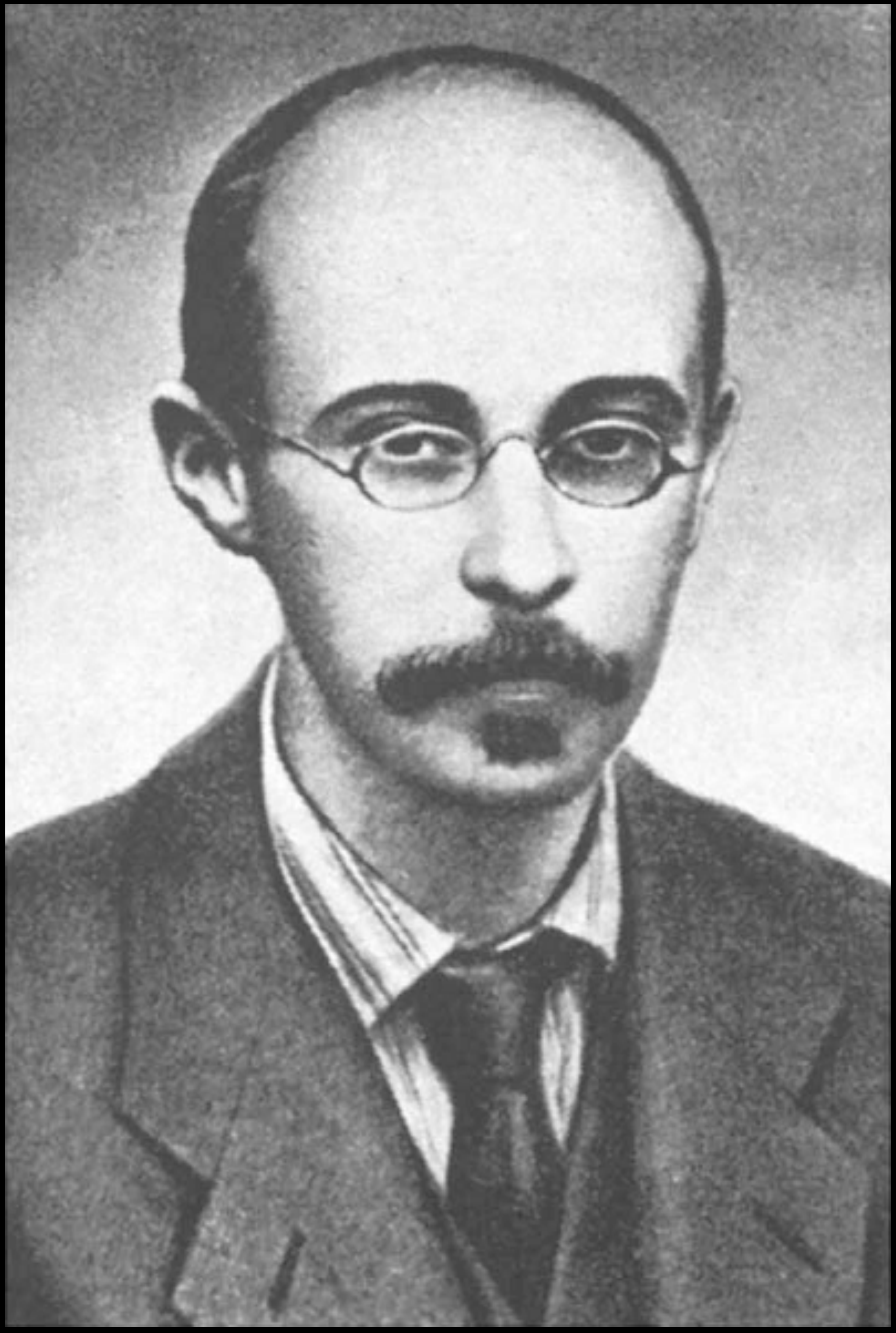
- The **Hubble constant,** $H_0 = cz/D$**, is the expansion rate of the Universe today.** In this definition, the expansion rate of the Universe is expected to change over time because the universe obviously was younger in the past.
- For the expansion rate at any time, we thus define $H(t) = \dot{R}_U(t)/R_U(t)$ as the **Hubble parameter**.
- At $\boldsymbol{t = t_0}$ (today), this definition gives **Hubble's Law**:

$$\dot{R}_U(t_0) = H(t_0)R_U(t_0) \Rightarrow \dot{D} = H_0 D \text{ where } D = R_U D_{\text{comoving}}$$



Mathematical Description of the dynamics of the Universe:

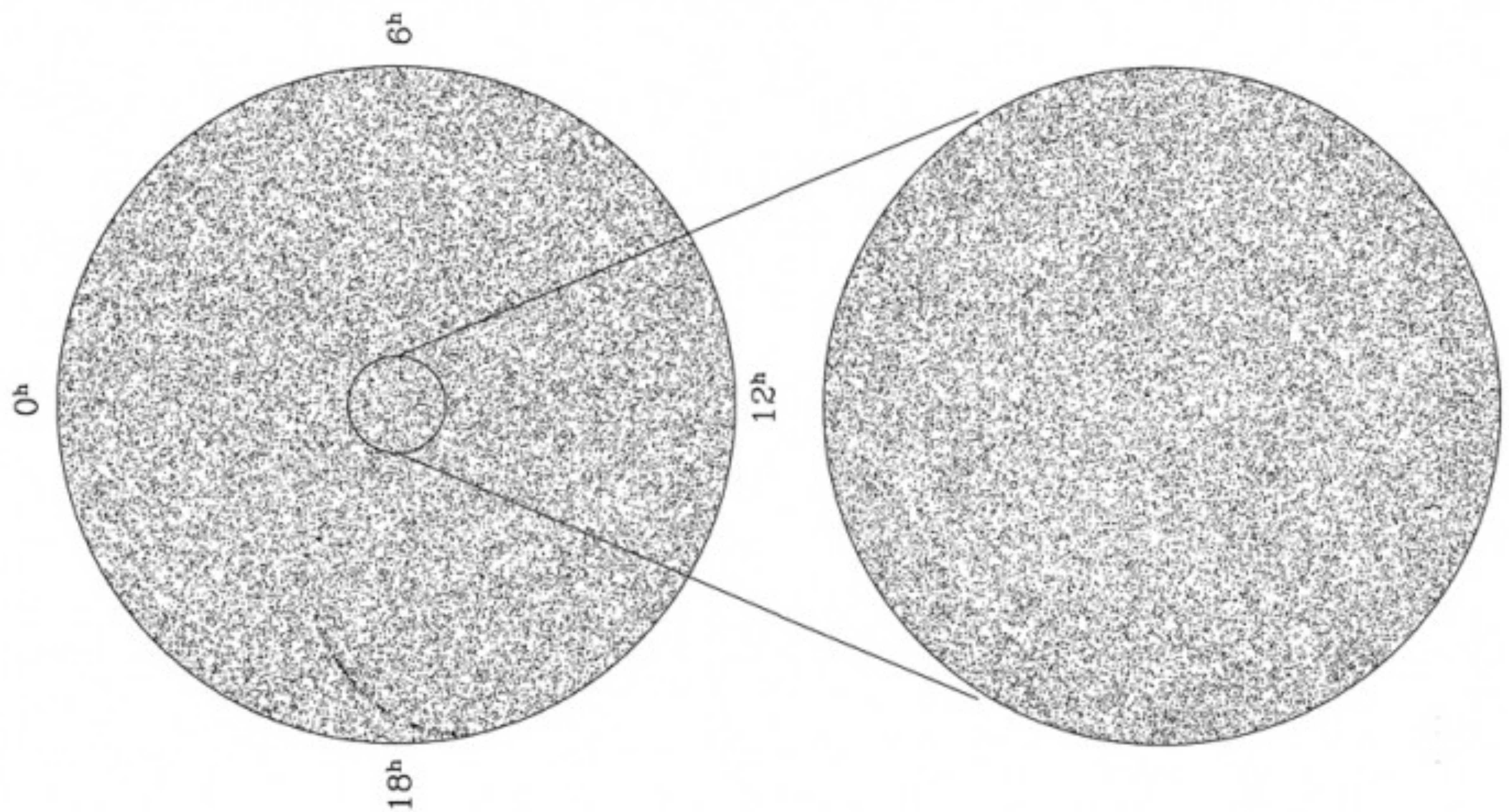
The Friedmann Equation



Alexander Friedmann (1888 – 1925) was a Russian physicist and mathematician. Fought in WWI as an aviator. Died at age 37 from typhoid fever.

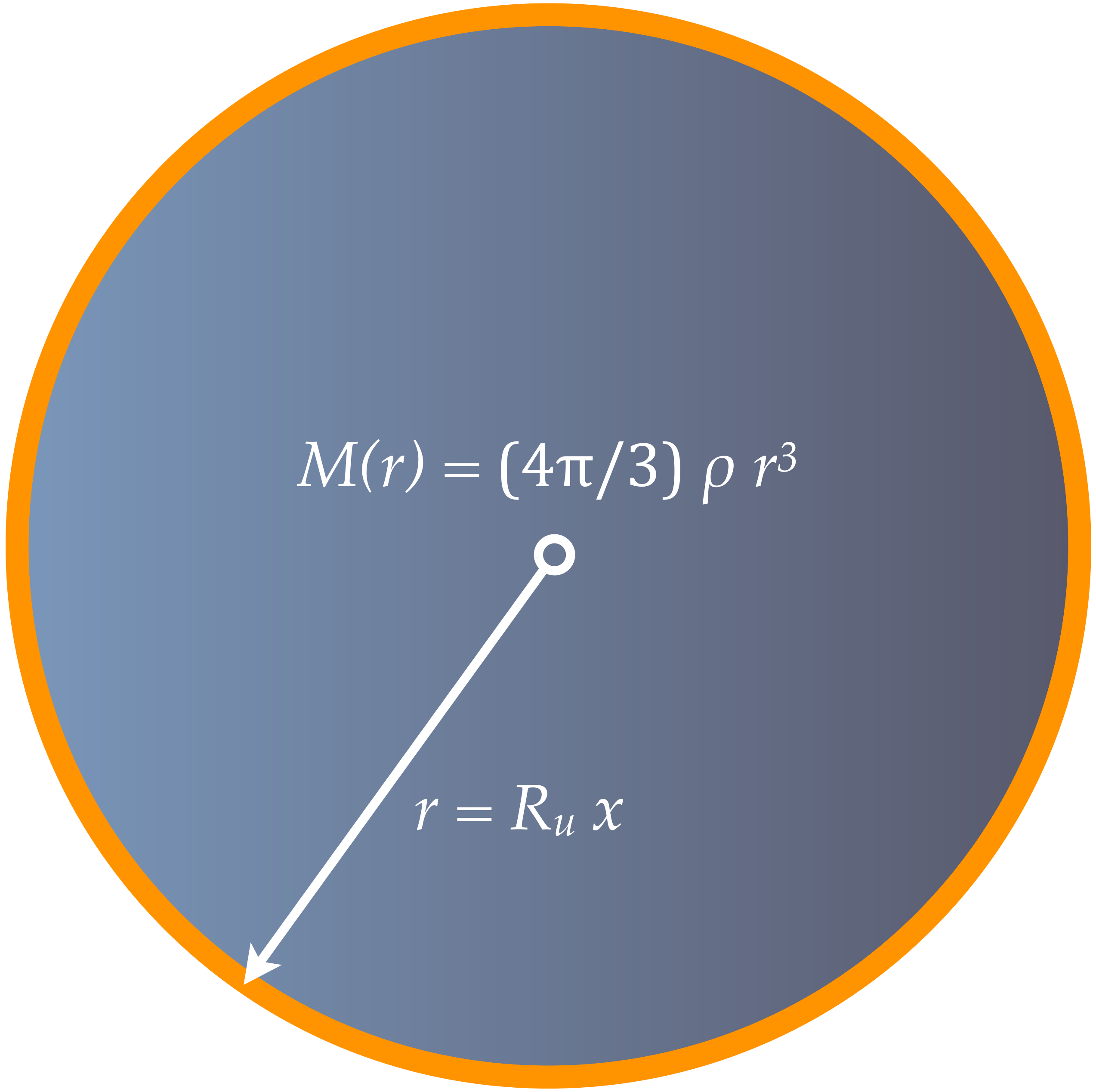
Fundamental Assumption: The Cosmological Principle

- Although galaxies tend to clump, on the largest cosmic scales, the Universe is both **homogeneous** and **isotropic**
  - **Homogeneous:** there is no preferred **location** in the Universe
  - **Isotropic**: there is no preferred **direction** in the Universe



Friedmann Equation: Classic Derivation based on Energy Conservation

- Imagine a **spherical shell** with **unit mass** in a **matter-only universe** with a **comoving radius** of **x**, as the universe expands:
  - its **physical radius** at time t is $r(t) = R_U(t)\, x$,
  - its **expanding velocity** is $v(t) = \dot{R}_U(t) \cdot x$, and
  - the **mass enclosed** in the shell is $M(r) = \frac{4\pi}{3}[R_U(t)x]^3 \cdot \rho(t)$



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  - its **physical radius** at time t is $r(t) = R_U(t)\, x$,
  - its **expanding velocity** is $v(t) = \dot{R}_U(t) \cdot x$, and
  - the **mass enclosed** in the shell is $M(r) = \frac{4\pi}{3}[R_U(t)x]^3 \cdot \rho(t)$
- We can write down the **kinetic + gravitational potential energy** for the unit-mass spherical shell:

$$E = \frac{1}{2}v^2 - \frac{GM(r)}{r} = \frac{1}{2}\dot{R}_U(t)^2 x^2 - \frac{4\pi}{3}G\rho(t)R_U(t)^2 x^2$$

- This **energy per unit mass** must be the same for every shell with the same comoving radius **x**, so we can define E with a k parameter:

$$E \equiv -\frac{1}{2}kc^2x^2$$

- Combining the two Eqs. and cancel out x^2 on both sides, we obtain:

$$\left(\frac{\dot{R}_U^{\,2}}{R_U^2} - \frac{8}{3}\pi G\rho\right) R_U^2 = -kc^2$$

Friedmann Equation: Classic Derivation based on Energy Conservation

- Recall the definition of the **Hubble parameter**:

$$H(t) \equiv \dot{R}_U / R_U$$

- define a new parameter called **critical density**:

$$\rho_c = \frac{3H^2}{8\pi G}$$

note that because H varies, the critical density is not a constant.

- we can now rewrite the energy conservation

$$\left(\frac{\dot{R}_U^{\,2}}{R_U^2} - \frac{8}{3}\pi G \rho \right) R_U^2 = -\, kc^2$$

as:

$$H^2 \left(1 - \frac{\rho}{\rho_c} \right) R_U^2 = -\, kc^2$$

- next, define the density ratio as a dimensionless **density parameter** called Omega:

$$\Omega_m \equiv \rho_m / \rho_c$$

- Finally, we have the Friedmann equation in a **matter-only universe**:

$$H^2 \; (1 - \Omega_m) \; R_U^2 = -\, kc^2$$

Calculating the Critical Density in Today's Universe

- The critical density, ρ_c, varies as the universe evolves, just like the Hubble parameter. Its value Today, $\rho_c(t_0)$, can be calculated from the Hubble constant:

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

- If we rewrite $H_0 = 70$ km/s/Mpc as $H_0 = 2.3 \times 10^{-18}$/s by converting Mpc to km, then the critical density of Today's universe is:

$$\rho_c = \frac{3 \times \left(2.3 \times 10^{-18}/\mathrm{s}\right)^2}{8 \times \pi \times \left[6.67 \times 10^{-20}\mathrm{km}^3/\left(\mathrm{kg\ s}^2\right)\right]}$$

$$\rho_c = 9.5 \times 10^{-27}\mathrm{kg/m}^3$$

- Given that $\rho = \mu m_H n$, this is equal to a hydrogen number density of **5.7 hydrogen atoms per cubic meter** (m_H = 1.67e-27 kg).
- It seems small, but the observed mass density of **ordinary matter**, *averaged over large volumes*, is less than **one hydrogen atom per cubic meter (n < 1 m^{-3} = 1e-6 cm^{-3})**.

Expansion histories predicted by the Friedmann Equation

Part I: matter-only universe

Solving the Friedmann Equation in a Matter-Only Universe

- The **Friedmann Equation (FE)** in a **matter-only universe** is:

$$H^2\ (1-\Omega_m)\ R_U^2 = -\,kc^2$$

  where the original terms were replaced by three key parameters:
  - **Hubble parameter**: $H(t) \equiv \dot{R}_U/R_U$,
  - **critical density**: $\rho_c = \dfrac{3H^2}{8\pi G}$, and
  - **density parameter**: $\Omega_m \equiv \rho_m/\rho_c$,
- To solve the Friedmann Equation, we need
  - the **boundary condition** at $t = t_0$**:**

$$H = H_0,\ R_U = 1,\ \text{thus}\ H_0^2(1-\Omega_{m,0}) = -\,kc^2$$

  - the **density relation** for matter:

$$\frac{\Omega_m}{\Omega_{m,0}} = \frac{\rho_m \rho_{c,0}}{\rho_{m,0}\rho_c} = \frac{\rho_m}{\rho_{m,0}}\frac{H_0^2}{H^2} = \frac{1}{R_U^3}\frac{H_0^2}{H^2}$$

- Replacing $-kc^2$ and Ω_m with the above two relations, we arrive at a solution of the **Hubble parameter** as a function of **redshift** or scale factor:

$$H^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_{m,0}) + \Omega_{m,0}/R_U]$$

Solutions of the Friedmann Equation: H vs. z and t vs. z

- Plug in the boundary condition and the density parameter relation:

$$H^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_{m,0}) + \Omega_{m,0}/R_U]$$

- The above is the solution for the Hubble parameter. For example, for

$$\Omega_{m,0} = 1 \Rightarrow H = H_0/R_U^{3/2} = H_0(1+z)^{3/2}$$

- To solve for the *time* evolution, we need to express $H(t) \equiv \dot{R}_U/R_U$:

$$\left(\frac{1}{R_U}\frac{dR_U}{dt}\right)^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_{m,0}) + \Omega_{m,0}/R_U]$$

- For simplicity, we assume $\Omega_{m,0} = 1$ *(a universe where density equals critical density).* Separate time *t* and scale factor R_U to two sides:

$$dt = \frac{\sqrt{R_U}dR_U}{H_0}$$

- Then integrate the differential equation from $R_U = 0$ (i.e., t = 0) to $R_U = 1/(1+z)$ [i.e., t(z)], we can solve for time as a function of scale factor or redshift:

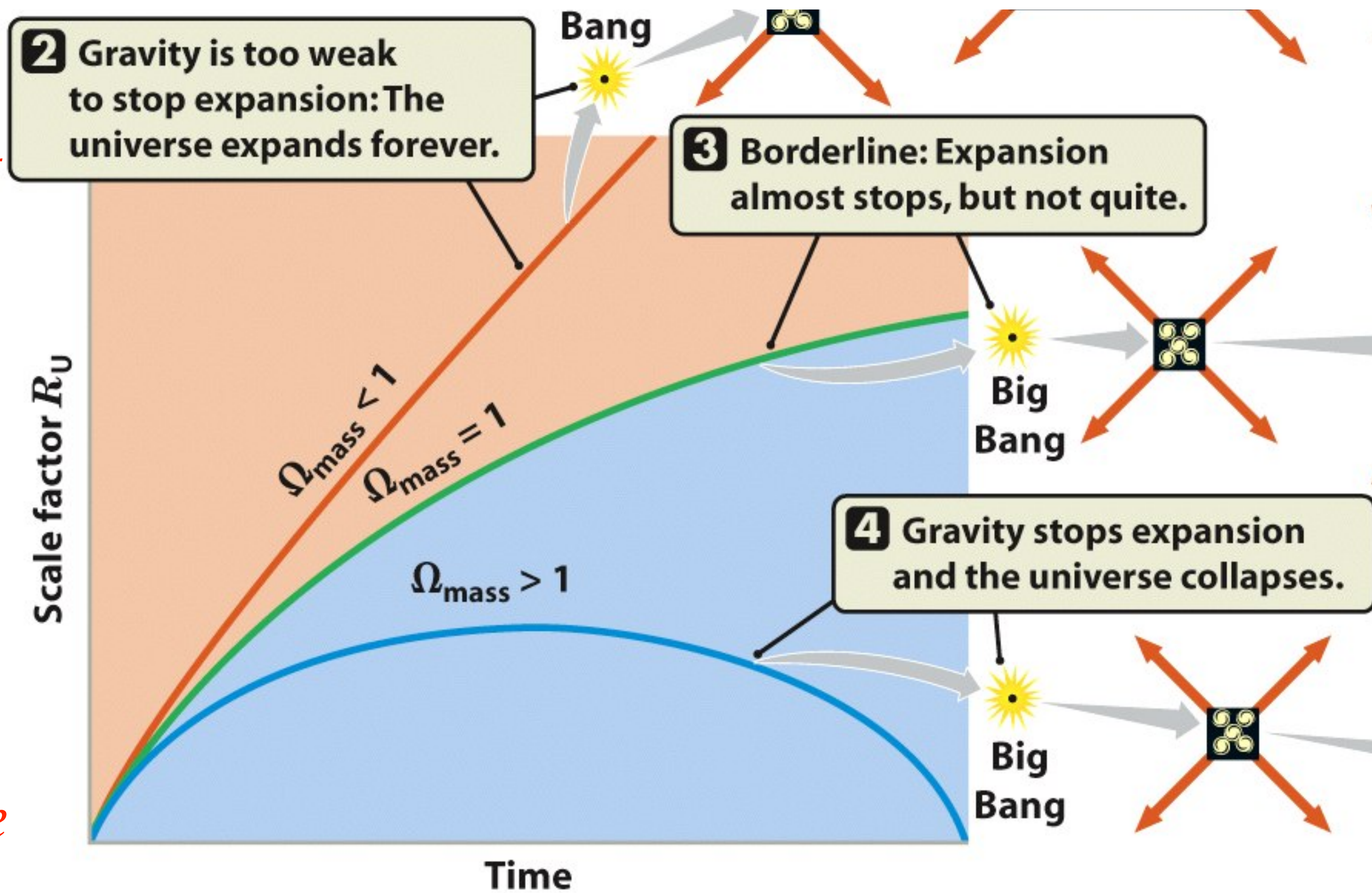
$$t = \frac{2}{3H_0}R_U^{3/2} \text{ or } t(z) = \frac{2}{3}t_H(1+z)^{-3/2} \text{ or } R_U = (3H_0t/2)^{2/3}$$

Predicted Expansion History if Only Matter Is Involved

$$H^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_{m,0}) + \Omega_{m,0}/R_U] \qquad t = t_H \int_0^{R_U} \frac{dR_U}{\sqrt{(1-\Omega_{m,0}) + \Omega_{m,0}/R_U}}$$

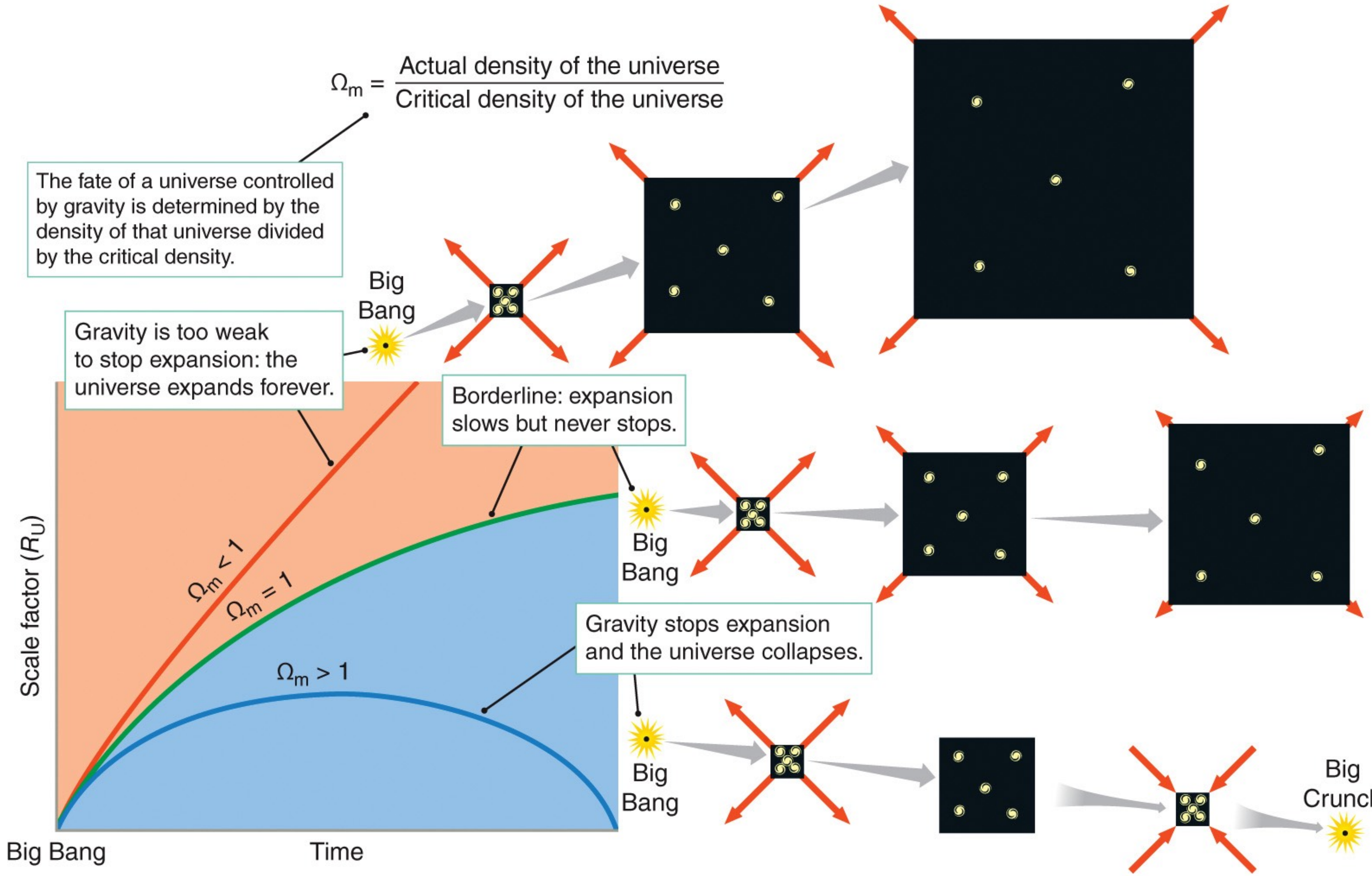
The expansion history depends on $\Omega_{m,0}$, while H_0 sets the overall scale.

- *$\Omega_{m,0} < 1$: sub-critical, expanding forever.*
- *$\Omega_{m,0} = 1$: critical, expanding forever, but expansion rate approaches zero as time goes. This critical universe is called **Einstein-de Sitter** universe*
- *$\Omega_{m,0} > 1$: super-critical expansion stops and the universe collapse. **(Big Crunch)***



The Total Normal Matter Density is Sub-Critical

- Ordinary matter in galaxies, IGM, and ICM: $\Omega_{m,0} = 4.5\%$ (today).
- Dark matter increases $\Omega_{m,0}$ to 32% (today).



Space-Time Geometry of the Universe

the *Robertson-Walker Metric*

Redshift—Scale-factor Relation from the Robertson-Walker Metric

- In **General Relativity**, a **metric** is a function which measures ***differential space-time distance*** between two events and is **Lorentzian invariant**. The **Robertson-Walker metric** is the metric that describes the geometry of a **homogeneous, isotropic, expanding** universe. The metric in spherical coordinate system is:

$$(ds)^2 = (c \cdot dt)^2 - R_U^2(t)\left[\left(\frac{dx}{\sqrt{1-kx^2}}\right)^2 + (xd\theta)^2 + (x\sin\theta d\phi)^2\right]$$

  where ***R**$_U$* is the scale factor, ***x*** is the **comoving** radial distance $x \equiv r(t)/R_U(t)$, ***k*** is the **comoving** curvature $k \equiv K(t)R_U(t)^2$. The same terms are in Friedmann Equation.

- Photons travel along **null geodesics** ($ds = 0$, i.e., proper time is frozen). Along the **radial direction** ($d\theta = d\phi = 0$), we have:

$$\frac{dx}{\sqrt{1-kx^2}} = \frac{c\ dt}{R_U(t)}$$

- Follow the path of two adjacently emitted photons (separated by one wavelength: $\delta t_e = \lambda_e/c,\ \delta t_o = \lambda_o/c,\ 1+z = \lambda_o/\lambda_e$) by integrating from ***emitter*** to ***observer***:

$$\int_{x_e}^{x_o} \frac{dx}{\sqrt{1-kx^2}} = \int_{t_e}^{t_o} \frac{cdt}{R_u(t)} = \int_{t_e+\delta t_e}^{t_o+\delta t_o} \frac{cdt}{R_u(t)} = \int_{t_e}^{t_o} \frac{cdt}{R_u(t)} + \int_{t_o}^{t_o+\delta t_o} \frac{cdt}{R_u(t)} - \int_{t_e}^{t_e+\delta t_e} \frac{cdt}{R_u(t)}$$

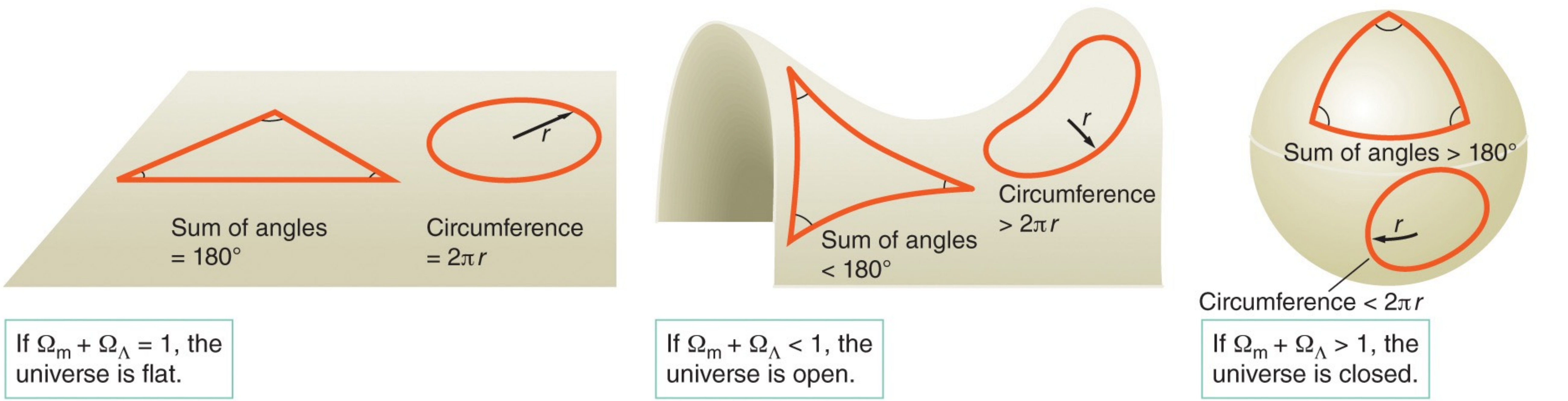
  which gives us the **redshift—scale-factor relation**

$$\Rightarrow \frac{c\delta t_o}{R_U(t_o)} = \frac{c\delta t_e}{R_U(t_e)} \Rightarrow R_U(t_e) = \frac{\delta t_e}{\delta t_o} = \frac{\lambda_e}{\lambda_o} = \frac{1}{1+z}$$

The curvature parameter *k* in the Friedmann Equation

- The universe has three possible geometry types determined by Ω_0 & H_0, given the boundary condition today $k = -H_0^2(1-\Omega_0)/c^2$ (Unit of k: $1/\mathrm{Mpc}^2$)
  - $k = 0$ ($\Omega_0 = 1$): Flat universe, infinite.
  - $k < 0$ ($\Omega_0 < 1$): Open universe, infinite, like the surface of a saddle.
  - $k > 0$ ($\Omega_0 > 1$): Closed universe, finite, like the surface of a sphere.

2D Analogies



a. Flat geometry **b.** Open (saddle) geometry **c.** Closed (spherical) geometry

Expansion histories predicted by the Friedmann Equation

Part II: matter + radiation + dark energy universe

The Complete Friedmann Equation with matter and dark energy

- By defining a new parameter called **critical density**: $\rho_c = \dfrac{3H^2}{8\pi G}$,
  we have derived the Friedmann Equation for matter-only universe:

$$H^2 \left(1 - \frac{\rho}{\rho_c}\right) R_U^2 = -kc^2$$

- The full General Relativity version of the Friedmann Equation is:

$$H^2 \left[1 - \left(\frac{\rho_m}{\rho_c} + \frac{\rho_\gamma}{\rho_c} + \frac{\Lambda c^2}{8\pi G \rho_c}\right)\right] R_U^2 = -kc^2$$

- There are now three density ratios, i.e., define three Omega's:
  - $\Omega_m \equiv \rho_m/\rho_c$, **ordinary matter** (baryons and dark matter)
  - $\Omega_\gamma \equiv \rho_\gamma/\rho_c$, **relativistic matter** (light and neutrinos)
  - $\Omega_\Lambda \equiv \Lambda c^2/(8\pi G \rho_c)$, **dark energy** ($\Lambda$ is the cosmological constant and has the same physical unit as the curvature constant k)

- Replacing those, we have the final Friedmann Equation:

$$H^2 \left[1 - (\Omega_m + \Omega_\gamma + \Omega_\Lambda)\right] R_U^2 = -kc^2$$

How to Solve the Complete Friedmann Equation? H(z) solution

- **Boundary Condition at t = t_0:**

  $H = H_0$, $R_U = 1$, thus $H_0^2(1-\Omega_0) = -kc^2$

- **Relations between density parameters and scale factor**:

$$\frac{\Omega_m}{\Omega_{m,0}} = \frac{\rho_m \rho_{c,0}}{\rho_{m,0}\rho_c} = \frac{\rho_m}{\rho_{m,0}}\frac{H_0^2}{H^2} = \frac{1}{R_U^3}\frac{H_0^2}{H^2}$$

$$\frac{\Omega_\gamma}{\Omega_{\gamma,0}} = \frac{1}{R_U^4}\frac{H_0^2}{H^2} \quad \text{and} \quad \frac{\Omega_\Lambda}{\Omega_{\Lambda,0}} = \frac{H_0^2}{H^2}$$

- Write down the Friedmann Equation with the boundary condition:

$$H^2\ [1-(\Omega_m+\Omega_\gamma+\Omega_\Lambda)]\ R_U^2 = -kc^2 = H_0^2(1-\Omega_0)$$

  then plug in the density parameter relations and rearrange:

$$H^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_0) + \Omega_{m,0}/R_U + \Omega_{\gamma,0}/R_U^2 + \Omega_{\Lambda,0}R_U^2]$$

- Examples:
  - For **an empty universe**:

$$\Omega_0 = \Omega_{m,0} = \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0 \Rightarrow H = H_0/R_U = H_0(1+z)$$

  - For **a matter-only flat universe (Einstein-de Sitter universe)**:

$$\Omega_0 = \Omega_{m,0} = 1, \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0 \Rightarrow H = H_0/R_U^{3/2} = H_0(1+z)^{3/2}$$

How to Solve the Complete Friedmann Equation? t(z) or $R_u(t)$ solution

- Write down the Friedmann Equation with the boundary condition and replace Hubble parameter with scale factor, $H(t) \equiv \dot{R}_U/R_U$, we have

$$\left(\frac{1}{R_U}\frac{dR_U}{dt}\right)^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_0) + \Omega_{m,0}/R_U + \Omega_{\gamma,0}/R_U^2 + \Omega_{\Lambda,0}R_U^2]$$

- For simplicity, assume a **flat universe:** $\Omega_0 = \Omega_{m,0} + \Omega_{\gamma,0} + \Omega_{\Lambda,0} = 1.$
- Separate time and scale factor into two sides of the equation:

$$dt = \frac{1}{H_0}\frac{R_U dR_U}{\sqrt{\Omega_{m,0}R_U + \Omega_{\gamma,0} + \Omega_{\Lambda,0}R_U^4}}$$

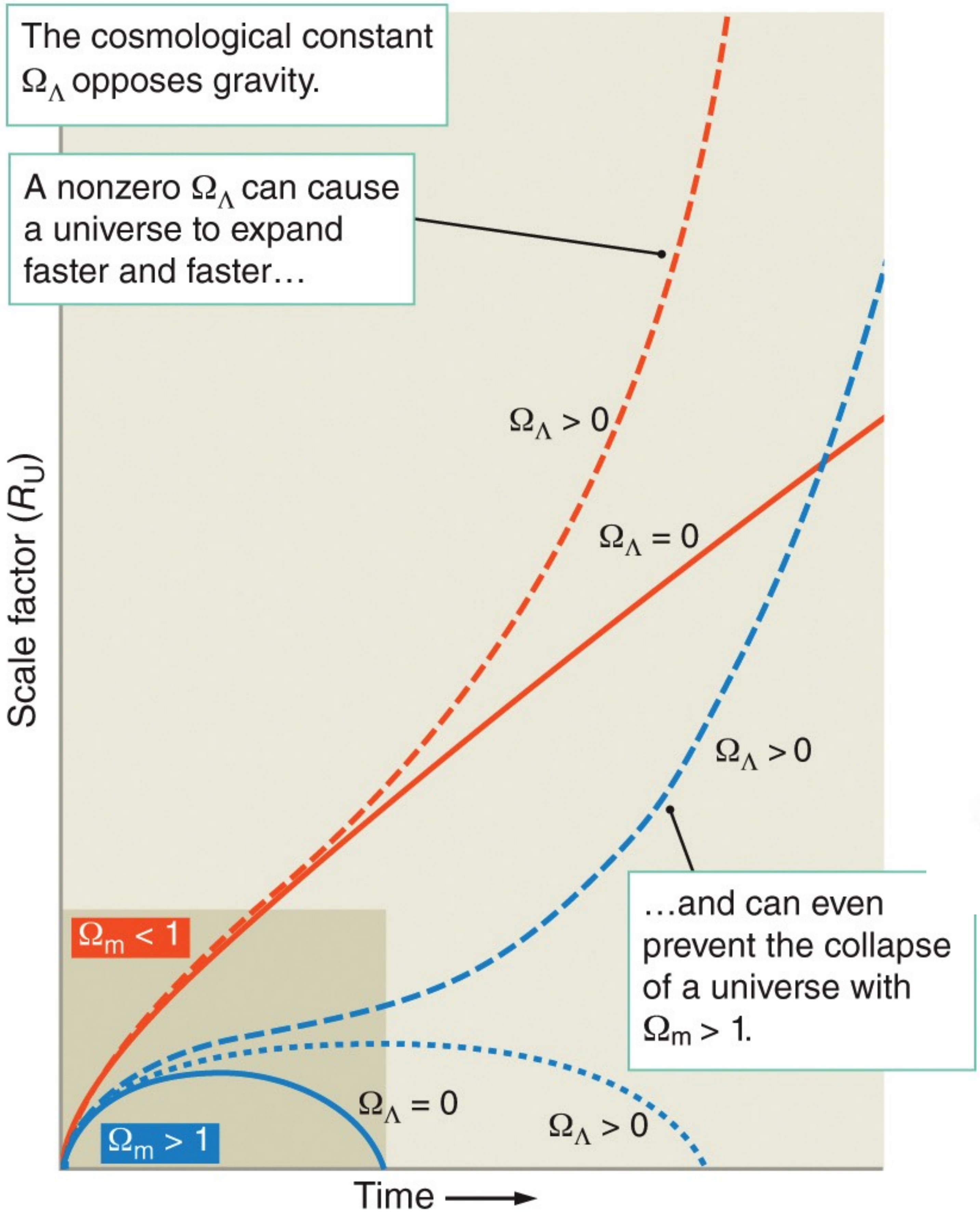
- Integrating it from R_U=0 (i.e., t=0) to R_U = 1/(1+z) [i.e., t(z)], we can solve for the t(z) relation for *any given values of the density parameters*.
- For example, for **a matter-only critical/flat universe (a.k.a. the Einstein-de Sitter universe),** we have solved for both H(z) and t(z):

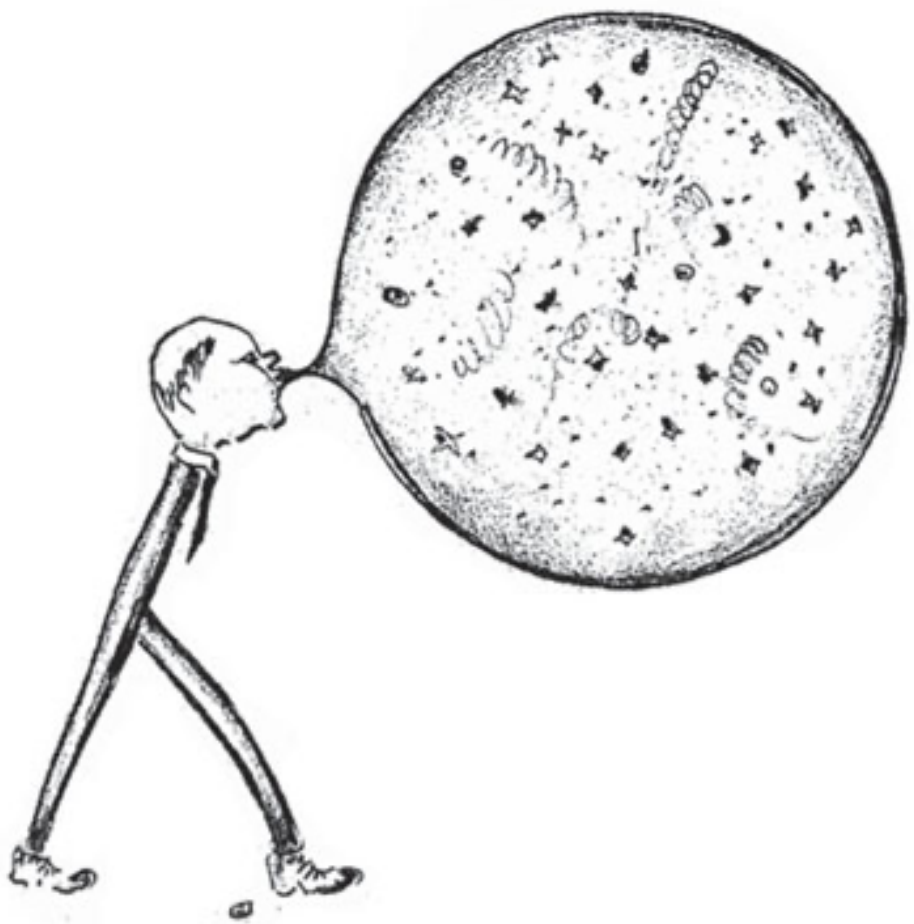
$$\Omega_0 = \Omega_{m,0} = 1,\ \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0$$

$$\Rightarrow H = H_0/R_U^{3/2} = H_0(1+z)^{3/2}$$

$$\Rightarrow t(z) = \frac{2}{3}t_H(1+z)^{-3/2}$$

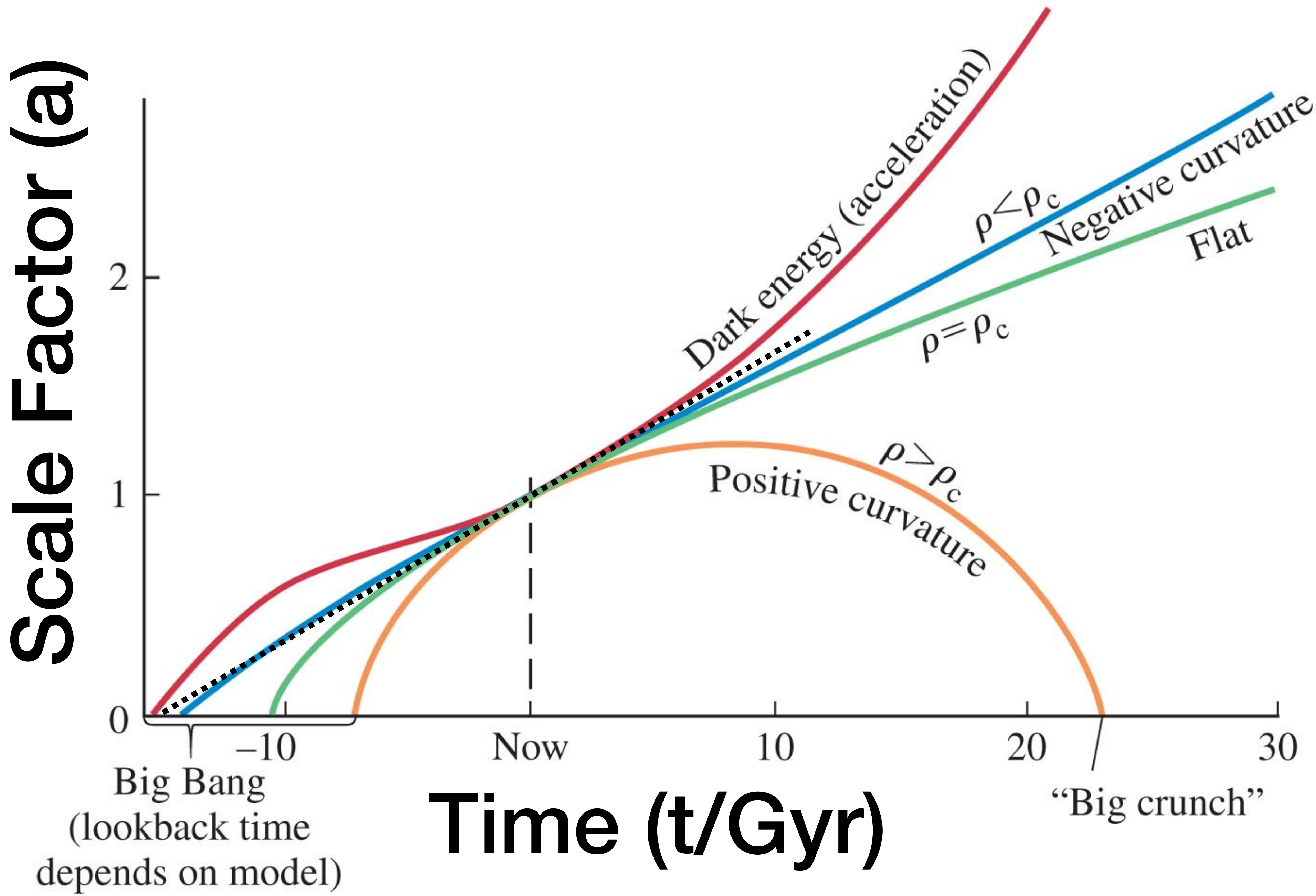
$R_u(t)$ solutions: Expansion history with the cosmological constant





Λ as the source of cosmic expansion: De Sitter in 1930

$R_u(t)$ solution from FE: The Expansion History



Constraints on Cosmological Parameters:

distance-redshift relation up to $z \sim 1$

Distance Modulus Measurements from Standard Candles

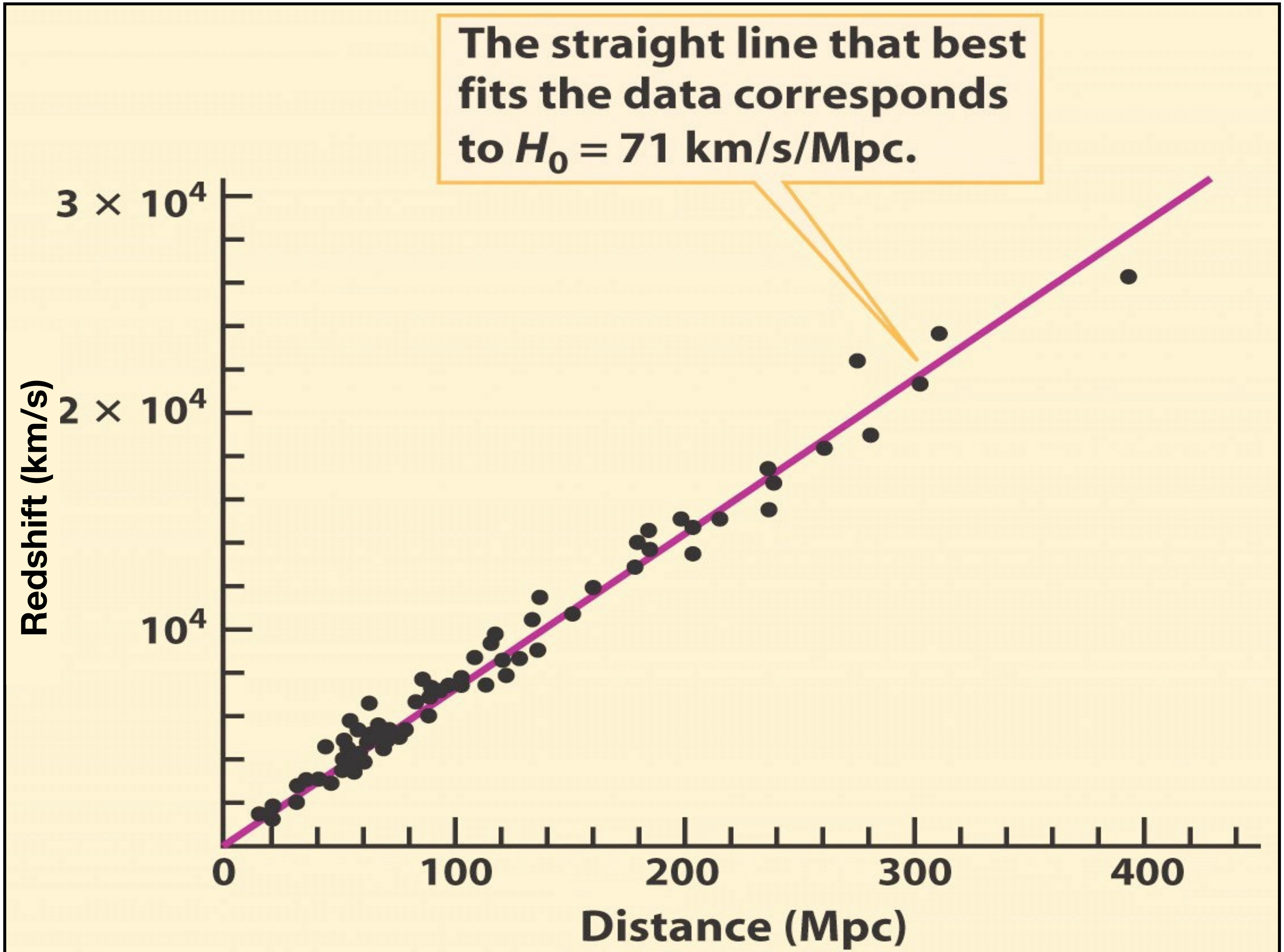
- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach 1.44 solar mass (the Chandrasekhar mass) to trigger the thermonuclear explosion
- At its peak, the absolute magnitude in V-band (550 nm) is ***M_V = -19***, and you measured a peak apparent magnitude of ***m_V = 10***, what's the distance in parsec?

$$m - M = 5\ [\log d(\text{parsec}) - 1]$$

$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

10 parsec * 10^(0.2*(10-(-19))) = 6.3 Mpc

Simple Distance Predictions from Hubble's Law: $cz = H_0 D \rightarrow D = cz/H_0$, this approximation is valid at 0.02 < z < 0.2 (why there is a lower limit?)

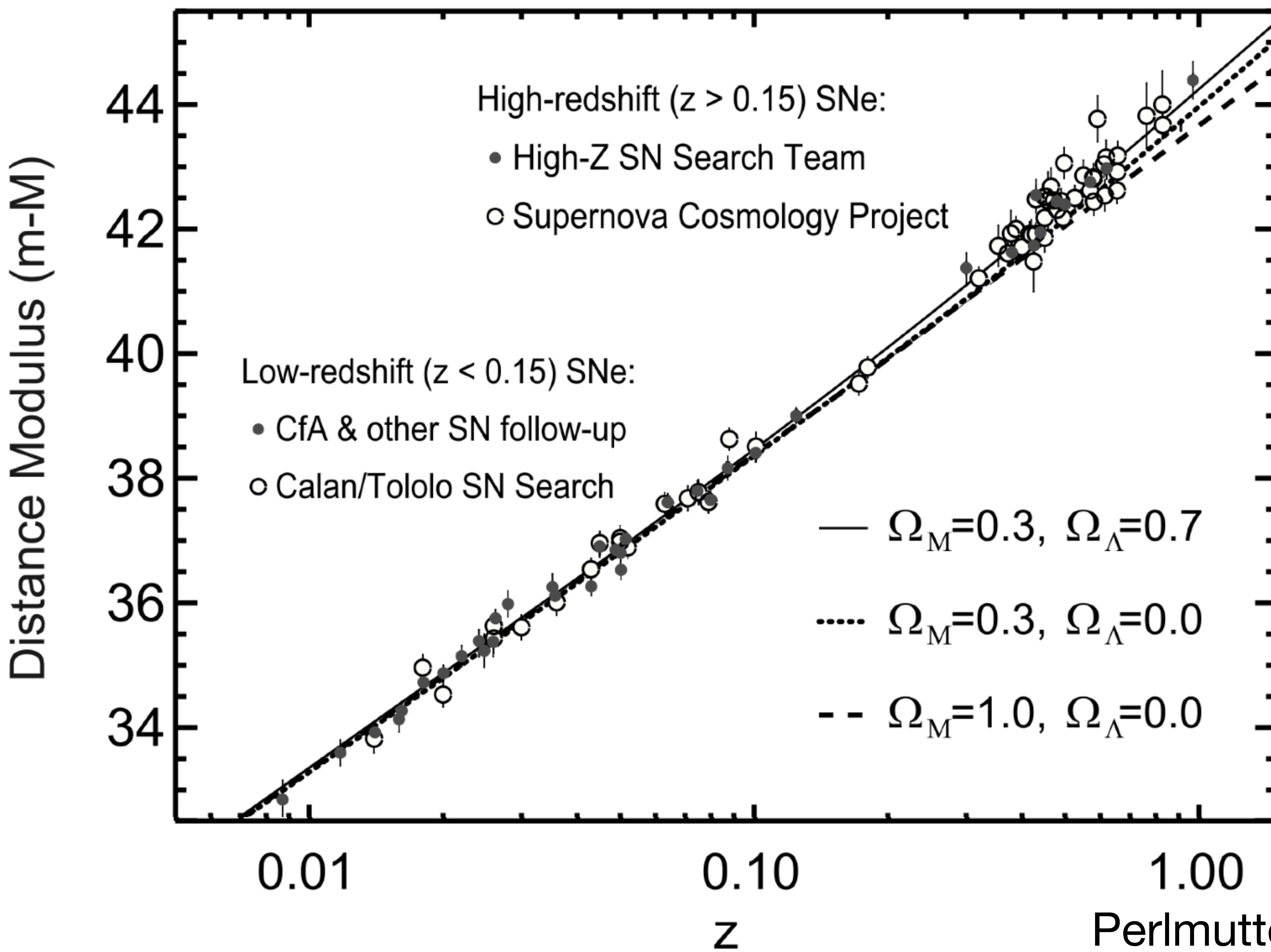


Hubble Diagram: Distance Modulus vs. Cosmological Redshifts

Cosmological parameters can be constrained by **comparing DM measurements (data points) with model predictions (curves)** for a range of redshifts. For standard model, one uses these Eqs:

$$D_L = (1+z)\int_0^z \frac{c\,dz'}{H(z')} \qquad H^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_0) + \Omega_{m,0}/R_U + \Omega_{\gamma,0}/R_U^2 + \Omega_{\Lambda,0}R_U^2]$$

$$\mu = m - M = 5\log D_L^{Mpc}(z; H_0, \Omega_M, \Omega_\Lambda) + 25$$



Perlmutter & Schmidt 2003

2011 Nobel Prize in Physics

The Supernova Cosmology Project

Photo: Lawrence Berkeley National Lab

Saul Perlmutter

The High-z Supernova Search Project

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Scanpix/AFP

Adam G. Riess

The Nobel Prize in Physics 2011 was awarded *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"* with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt and Adam G. Riess.

The Density Parameters of the Universe Today

How do they evolve over time?
What’s the predicted future?

Density Parameters Today

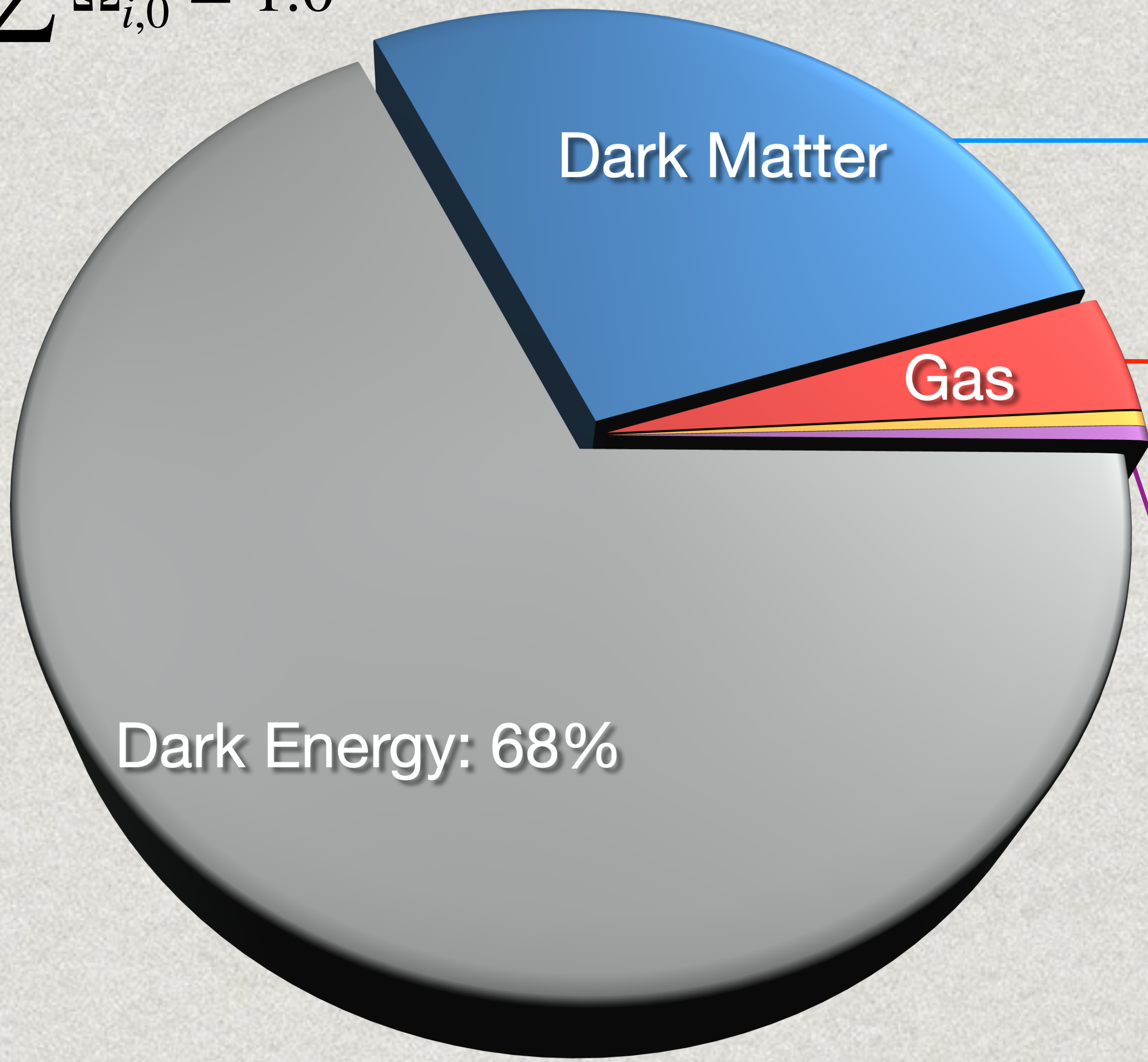
$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

Critical density as a function of time. Value below is present value, based on present value of the Hubble parameter H

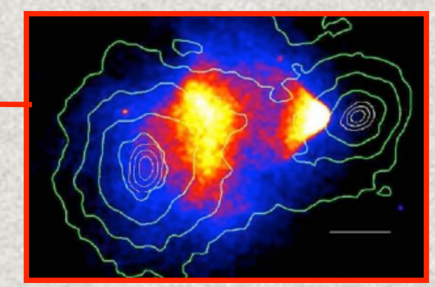
$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 9.47x10^{-27}\,kg\,/\,m^3$$

Planck Collaboration (2013)

$$\sum \Omega_{i,0} = 1.0$$

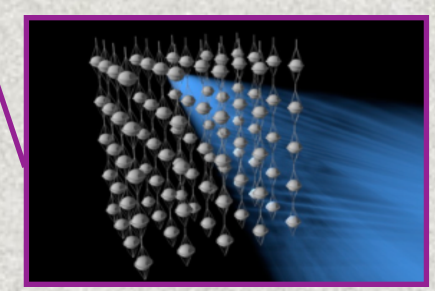


Dark matter: 27%



Gas: 4%

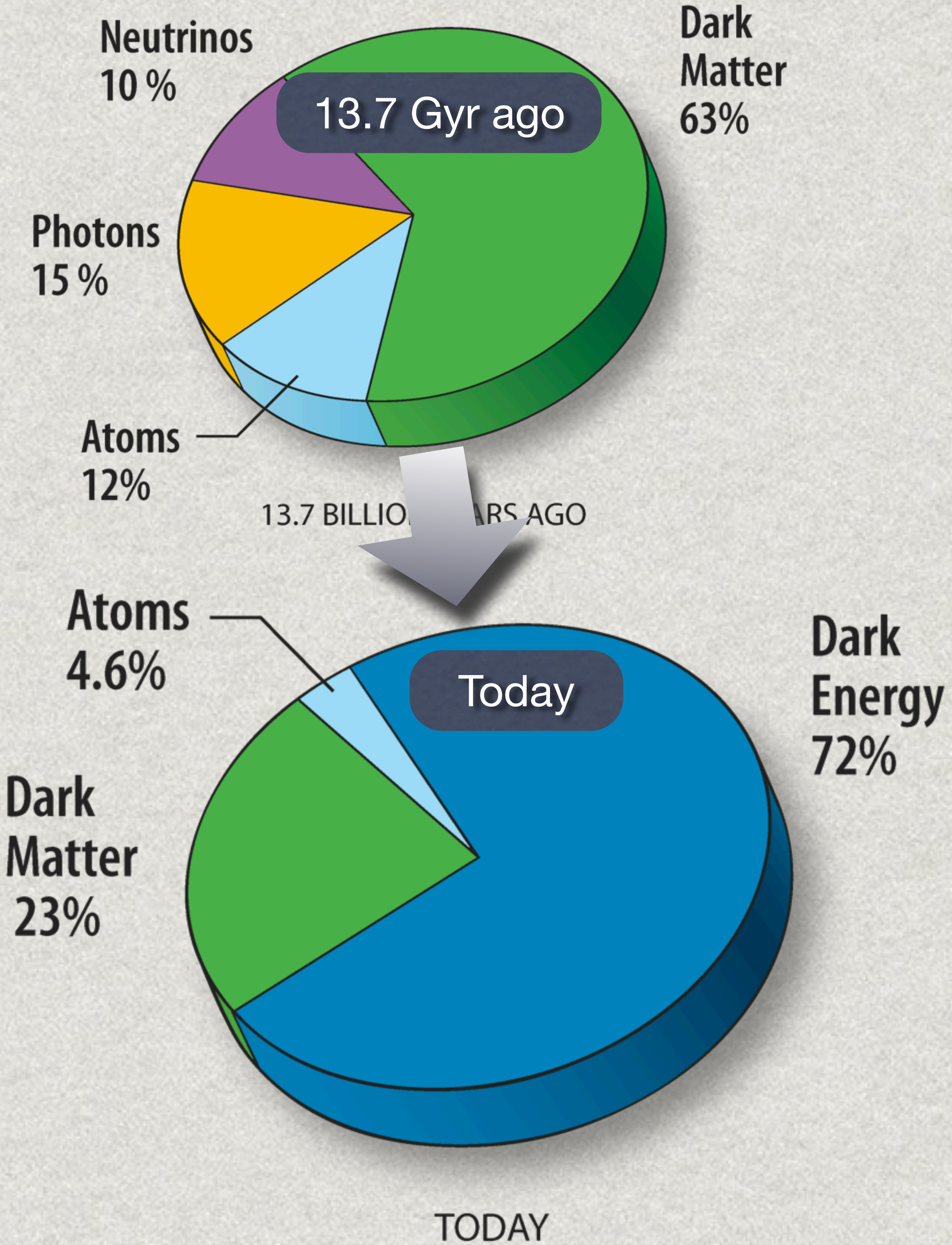
Galaxies: 0.5%

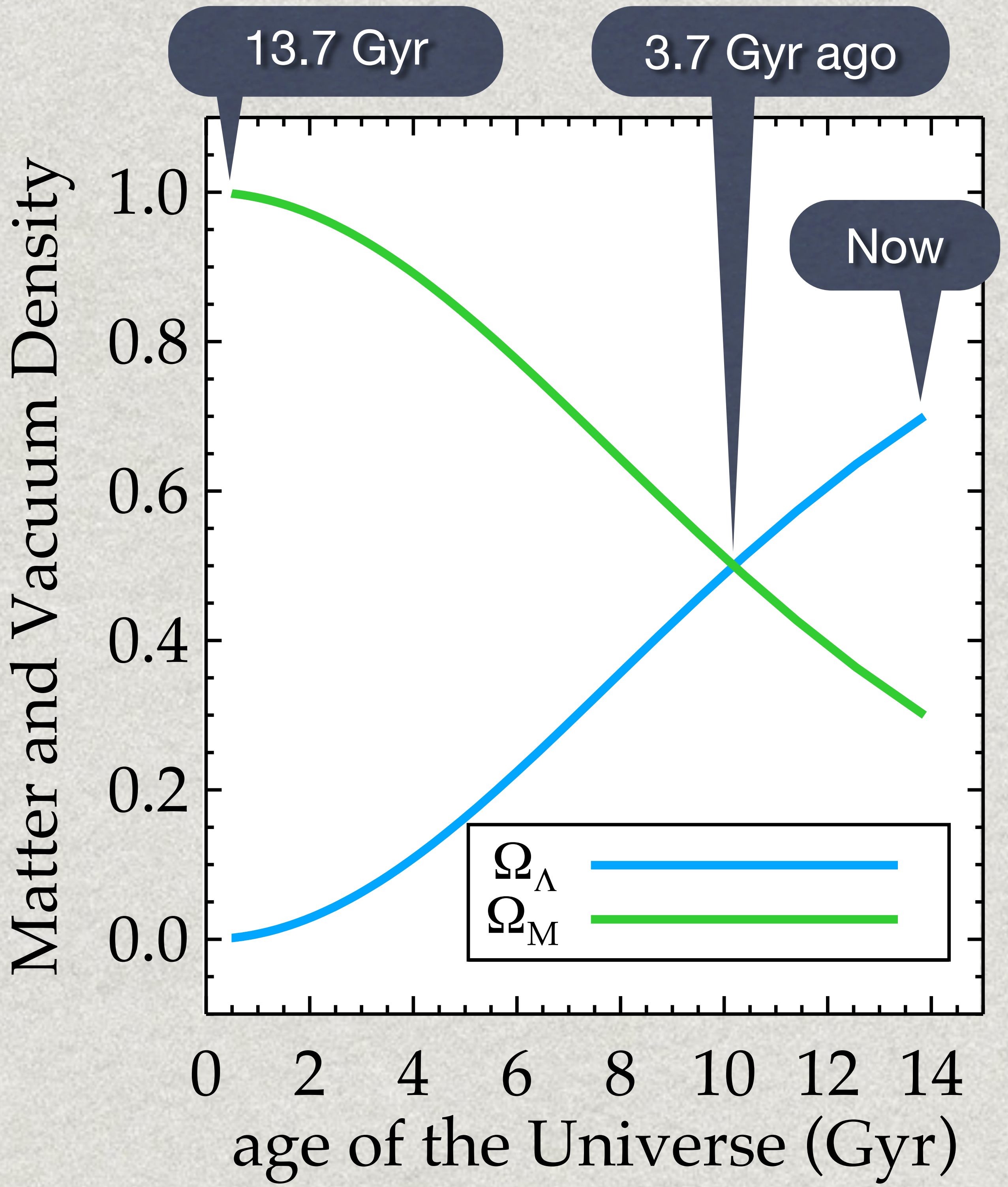


Neutrinos: <0.5%

Density Parameters vs. Time

- Dark Matter-dominated in the first 10 Gyrs, then Dark Energy dominated





Predicted Evolution of Density Parameters

- Dark Matter-dominated in the first 10 Gyrs, then Dark Energy dominated

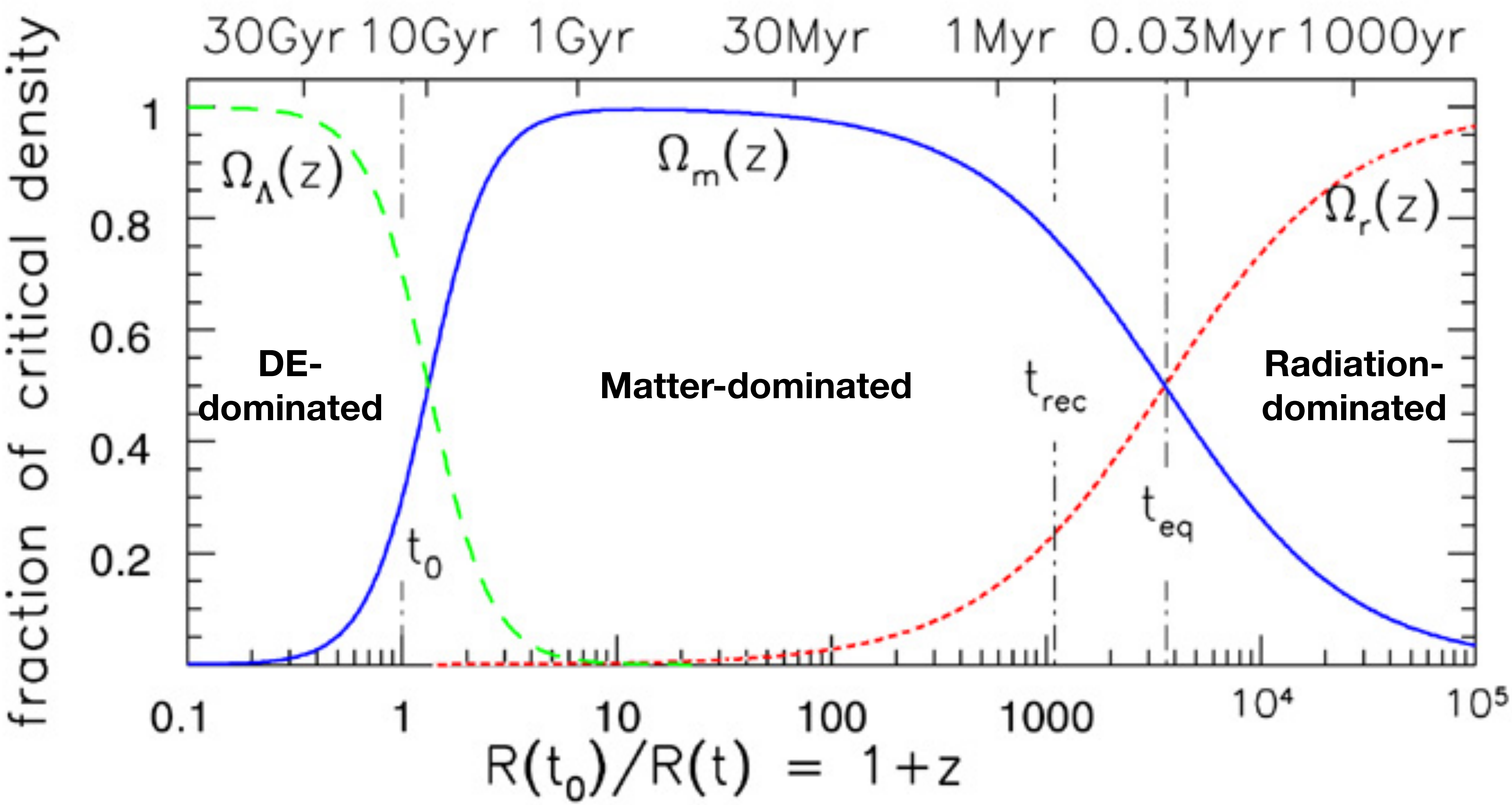


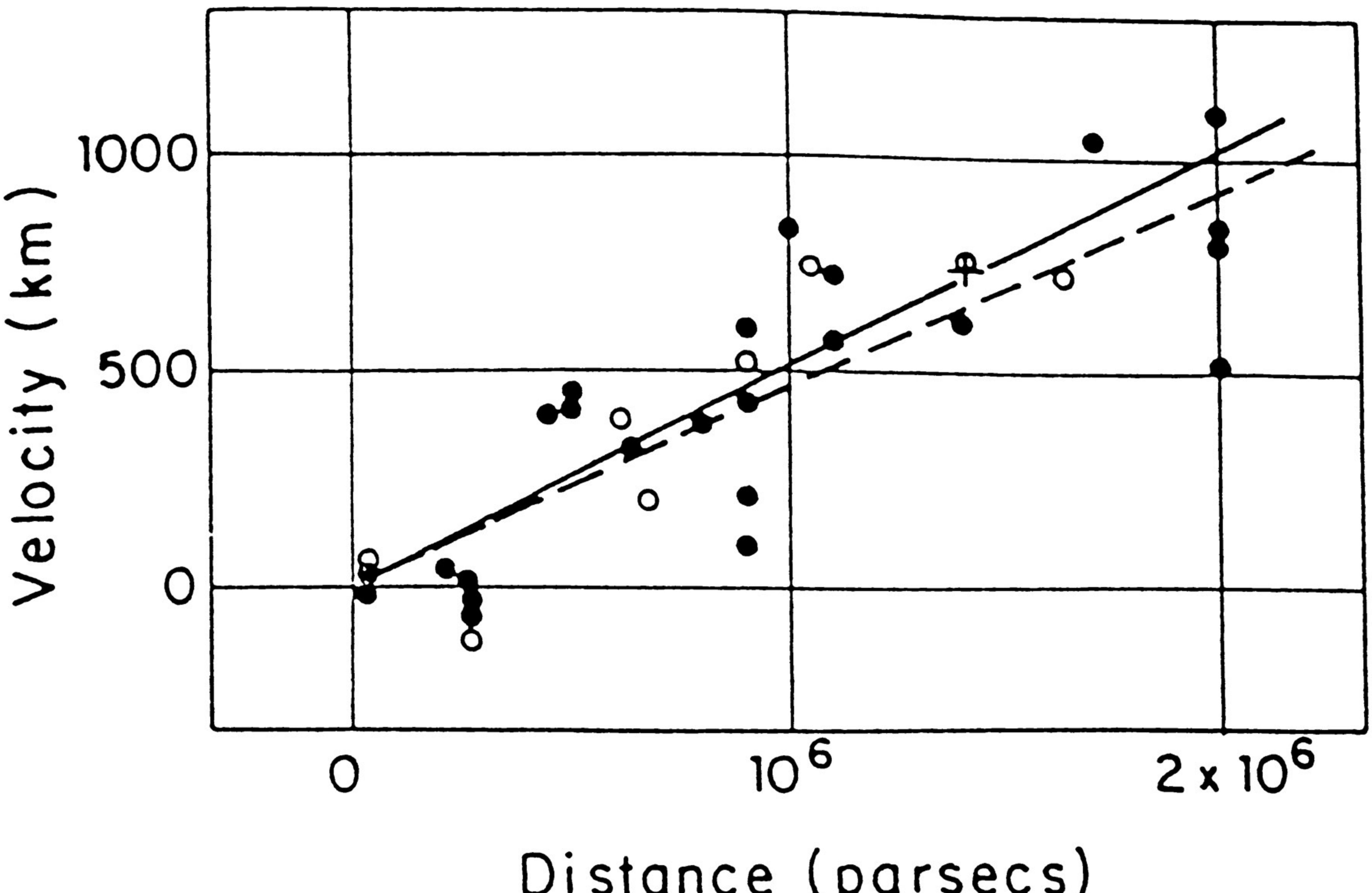
Fig 8.7 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

The Struggle to Determine the Hubble Constant (H_0)

Why H_0 is so important? Because it determines not only the current expansion rate, but also the geometry of the Universe.

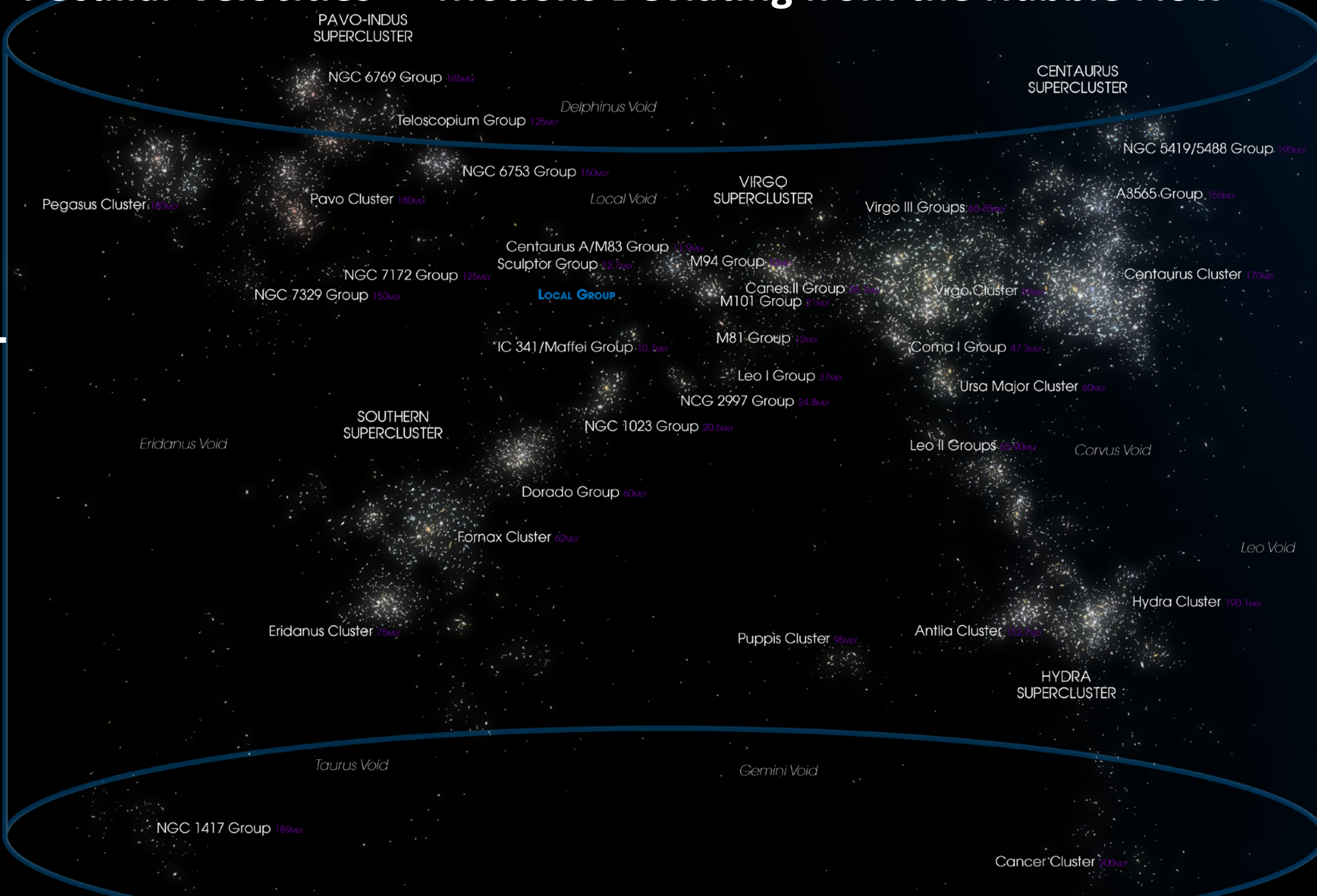
Hubble (1929)

Hubble initially got **H_0 = 500 km/s/Mpc,**
which led to a young universe: **$1/H_0$ = 2 Gyrs.**



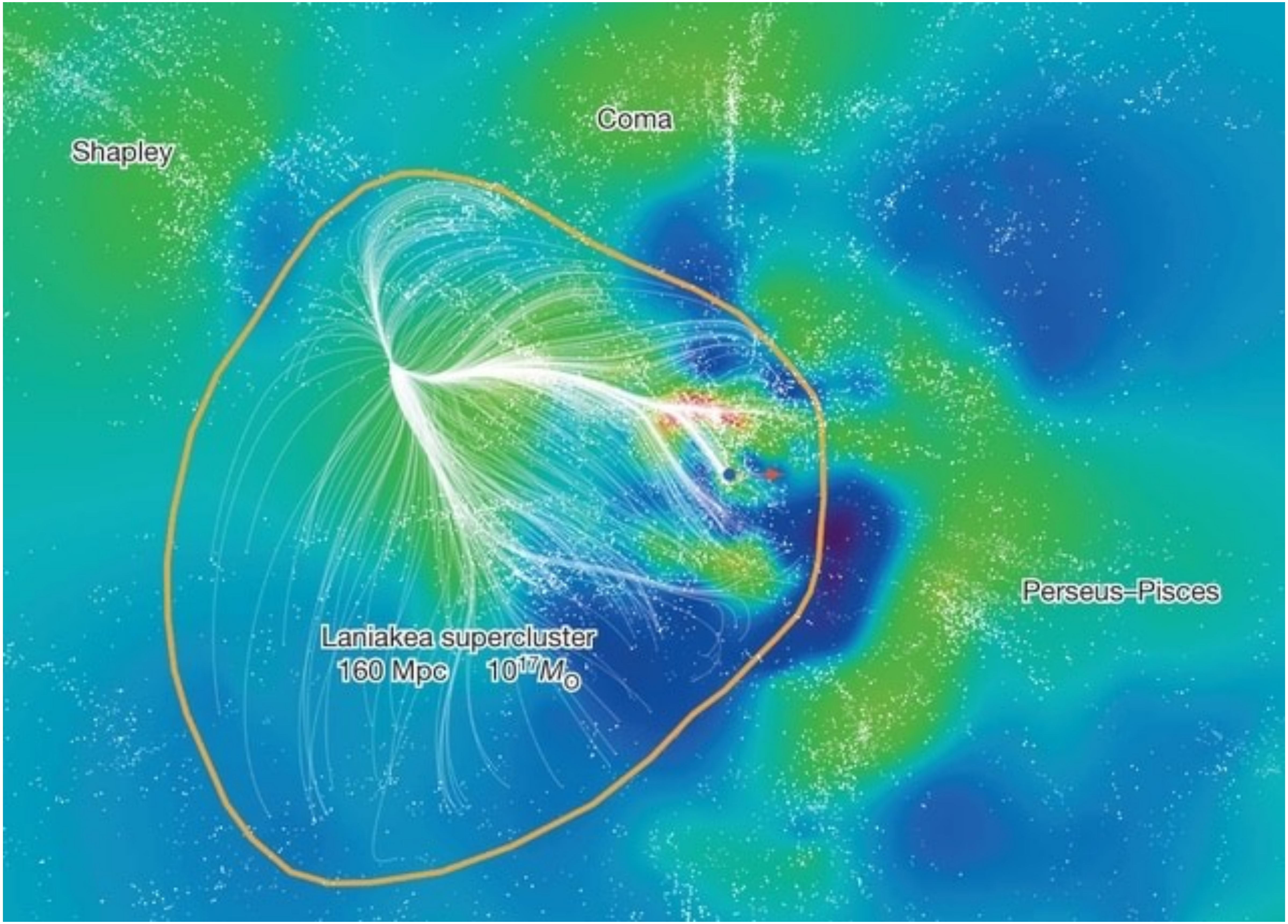
Peculiar Velocities — Motions Deviating from the Hubble Flow

LANIAKEA Supercluster

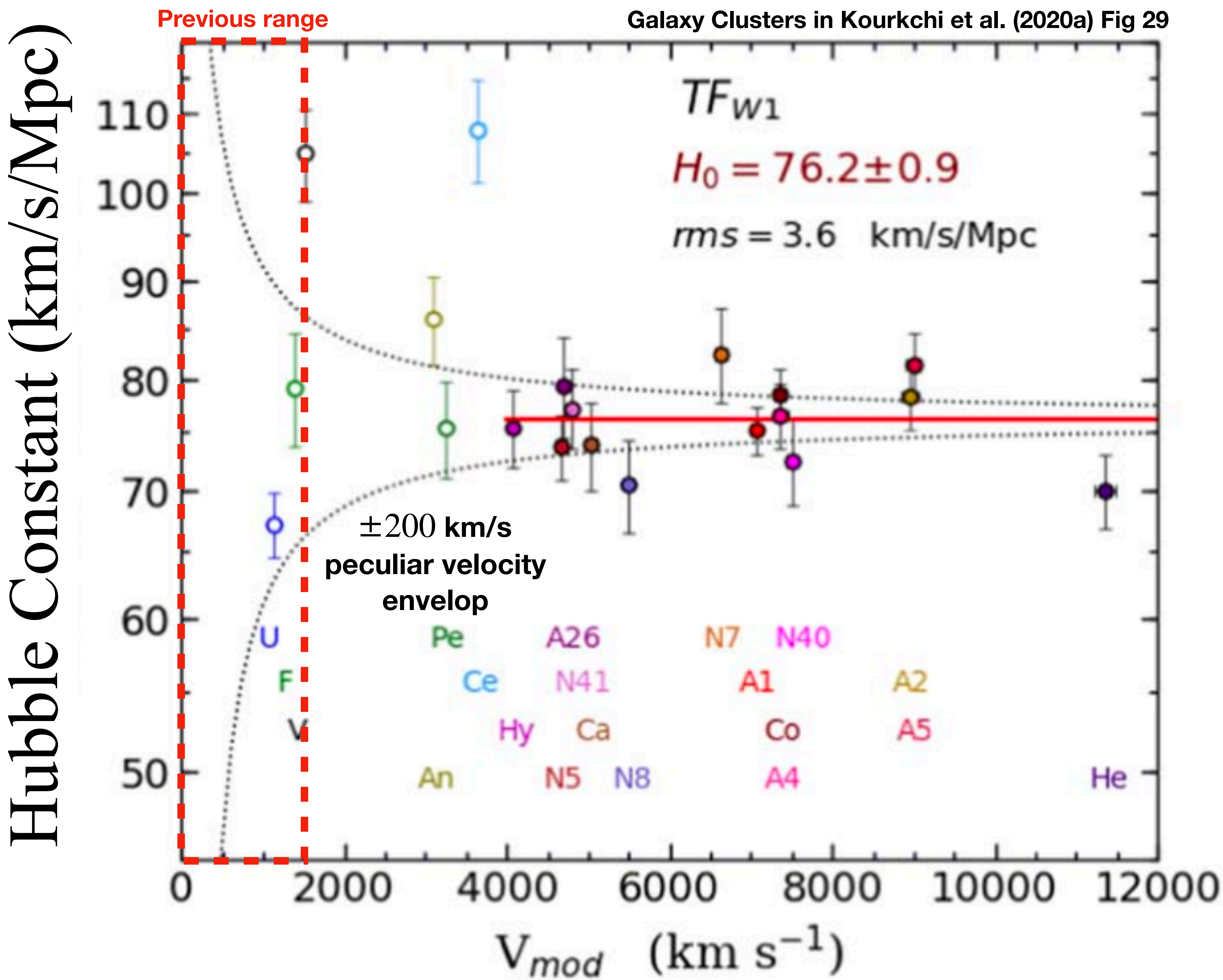


10^{17} Solar Masses, 10^{5} Member Galaxies, 520 million light years across

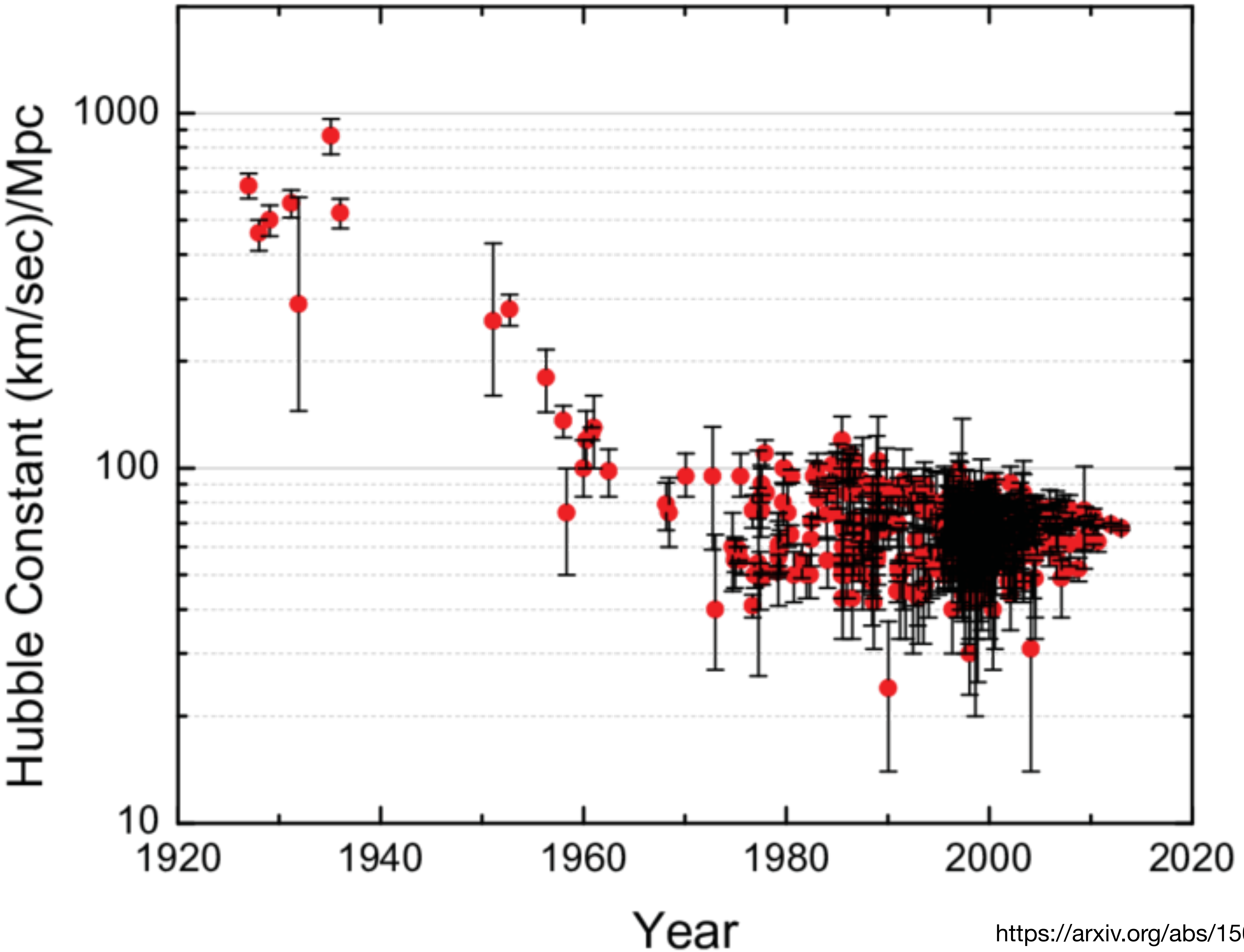
Peculiar Velocities — Motions Deviating from the Hubble Flow



Galaxies at greater distances provide more accurate H_0



Historical evolution of Hubble constant measurements



https://arxiv.org/abs/1506.02978

Allan Sandage (1958) - The First Reasonable Determination of H_0

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CURRENT PROBLEMS IN THE EXTRAGALACTIC DISTANCE SCALE*

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Mount Wilson and Palomar Observatories
Carnegie Institution of Washington, California Institute of Technology
Received February 3, 1958

ABSTRACT

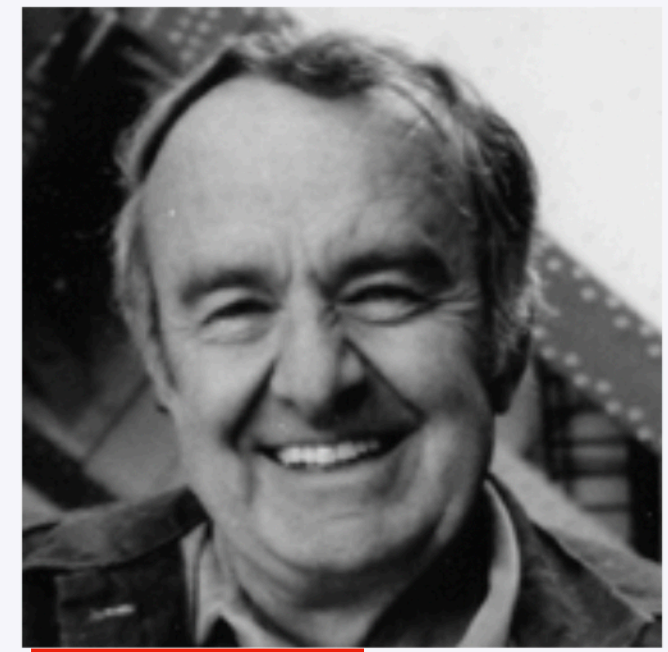
In principle, a decision between the simplest cosmological models (exploding cases with $\Lambda = 0$, $k = +1, 0, -1$, or the steady-state case) is possible from the observed velocity-distance relation. Two numbers are needed. These are H and the deceleration parameter, $\ddot{R}_0/R_0H^2$. This paper discusses the determination of H.

Problems connected with the use of cepheid variables as distance indicators are discussed. Because of a finite width of the instability region for cepheids in the color-magnitude (C-M) diagram, intrinsic scatter in the period-color and period-luminosity (P-L) relation is expected. The observed period-color relation at median light for field cepheids in our Galaxy shows that Eggen's type C variables are oscillating in a higher mode than cepheids of Eggen type A, B. The data show $Q_{A,B}/Q_C \approx 1.9$. If the region of instability for cepheids in the C-M diagram has a width of $\Delta(B-V) = 0.2$ mag, then the P-L relation is expected to have a scatter of 1.2 mag. The intrinsically bluest cepheids should be the brightest. Arp's two-color data for cepheids in the SMC confirm these predictions.

The brightest stars are discussed as distance indicators for galaxies beyond the local group. It is probable that knots identified by Hubble as brightest stars in more distant resolved galaxies are really H II regions. From data in M100, the stars appear to be 1.8 mag. fainter than the knots. This correction, together with a correction of 2.3 mag. to Hubble's moduli for galaxies in the local group, suggests a total correction of about 4.1 mag. to the 1936 scale of distances. This gives $H \approx 75$ km/sec 10^6 pc or $H^{-1} \approx 13 \times 10^9$ years, with a possible uncertainty of a factor of 2. The connection of this value of H with the time scale of exploding cosmologies is briefly discussed.

Allan is a student of Edwin Hubble

Allan Sandage



Born	June 18, 1926 Iowa City, Iowa
Died	November 13, 2010 (aged 84) San Gabriel, California
Nationality	American
Alma mater	California Institute of Technology University of Illinois
Known for	Physical cosmology

Evidence for a Hot Beginning:

Discovery of the Microwave Background

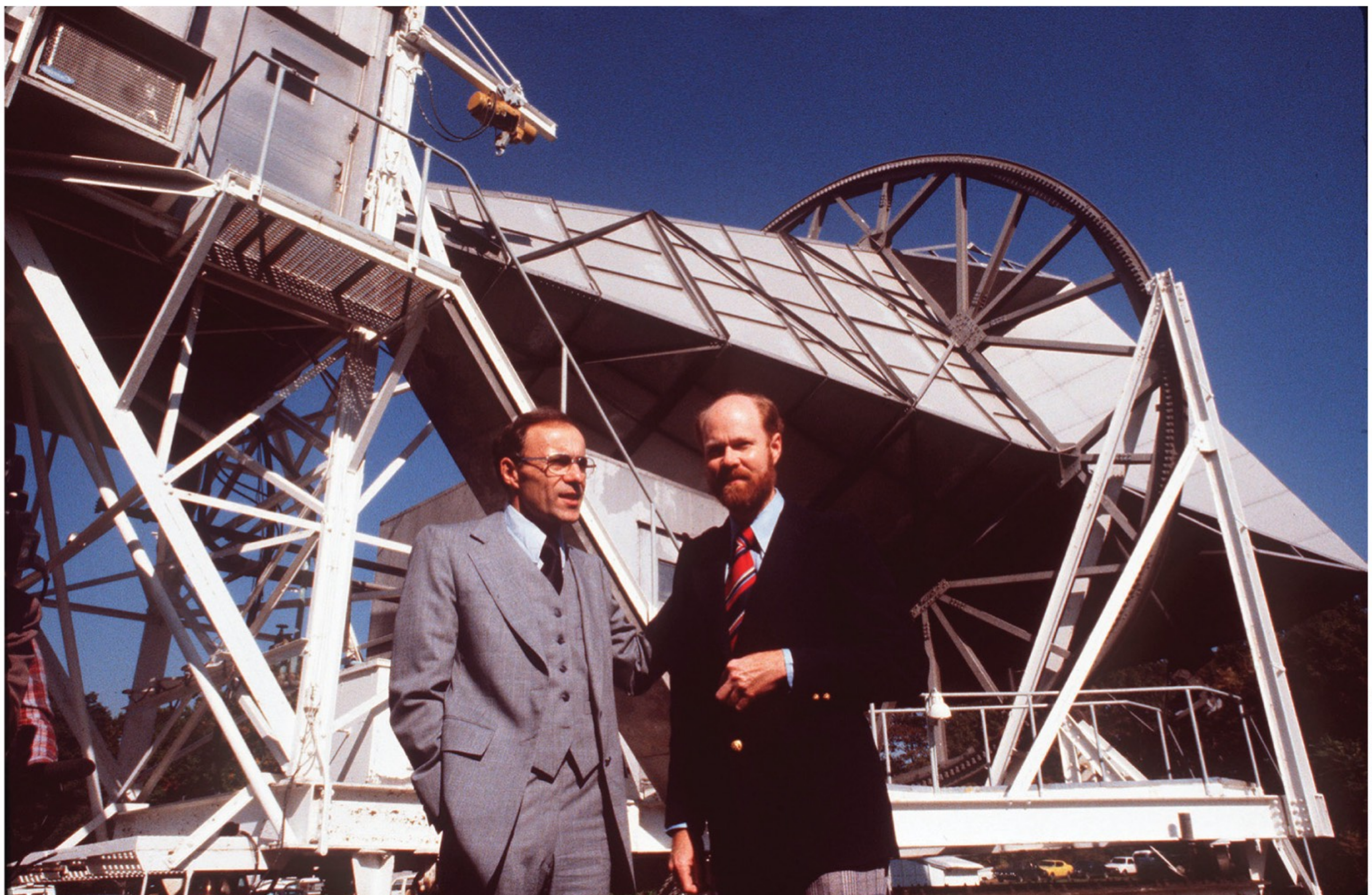
If the universe started from a Big Bang, the hot early universe should have generated a blackbody radiation field that is detectable everywhere

- If all matter started in a small volume, conditions would be very hot. This hot, dense gas would have emitted blackbody radiation, which should be detectable even today (after the long expansion stretched these photons).
- In **1948**, **Alpher, Gamow**, and **Herman** made a prediction: The Planck spectrum of the thermal emission should be everywhere and it would be uniformly redshifted by the expansion of the universe to a temperature of about **5–50 K**, peaking at **microwave wavelengths.** *Their prediction was ignored or mostly forgotten.*

- Wien's displacement law: $\lambda_{\rm peak} = (2.9\ {\rm mm\ K})/T$
- Cosmological redshift: $\lambda_{\rm obs} = \lambda_{\rm emit}(1+z)$
- Combining the above two, we have the relation the blackbody temperature when emitted and the observed temperature:
$$T_{\rm obs} = T_{\rm emit}/(1+z)$$
in other words, the observed temperature today is much lower than the original temperature due to cosmic expansion.

Measuring the Cosmic Microwave Background

- This predicted spectrum was *accidentally* discovered in **1963** by **Arno Penzias** and **Robert Wilson**. They measured microwave emission in all directions and the observed temperature of the emission was about **3 K**.
- We call this predicted spectrum the **cosmic microwave background (CMB)**.
- This was the first clear evidence of **the Big Bang**.



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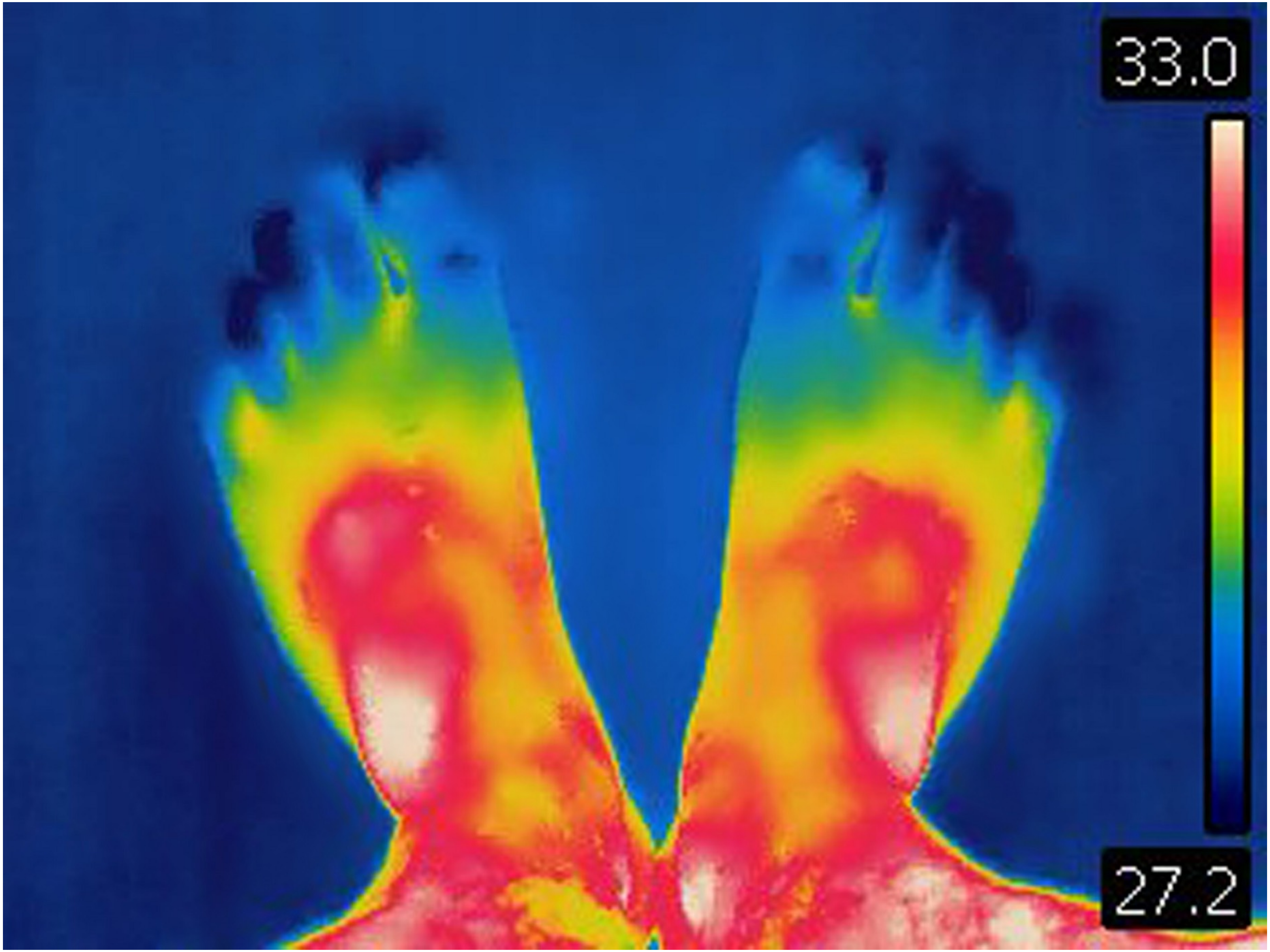
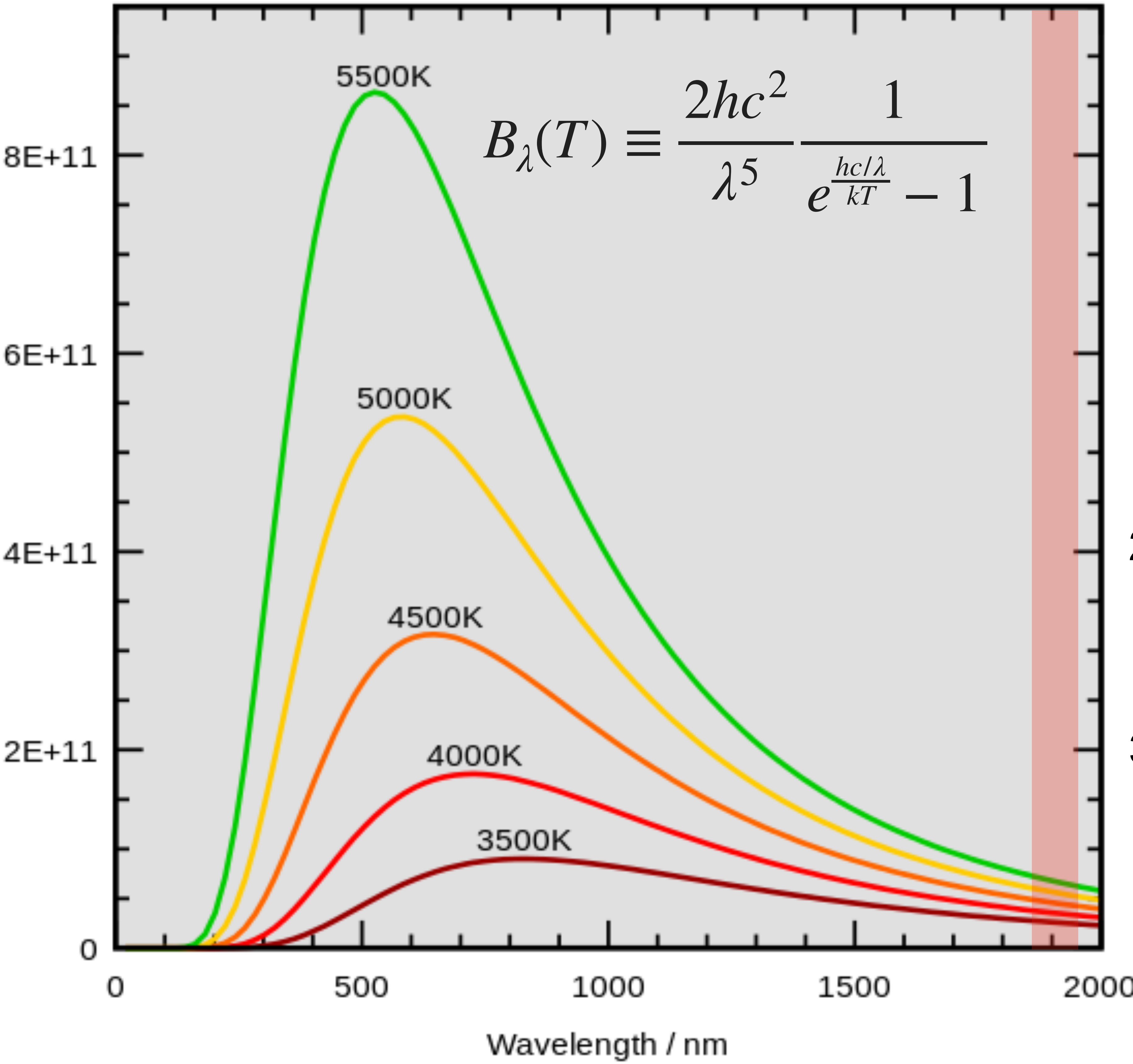
Temperature Map from Single-wavelength Intensity Measurement



The Planck Curve Predict *Intensity* as a function of *T* and λ



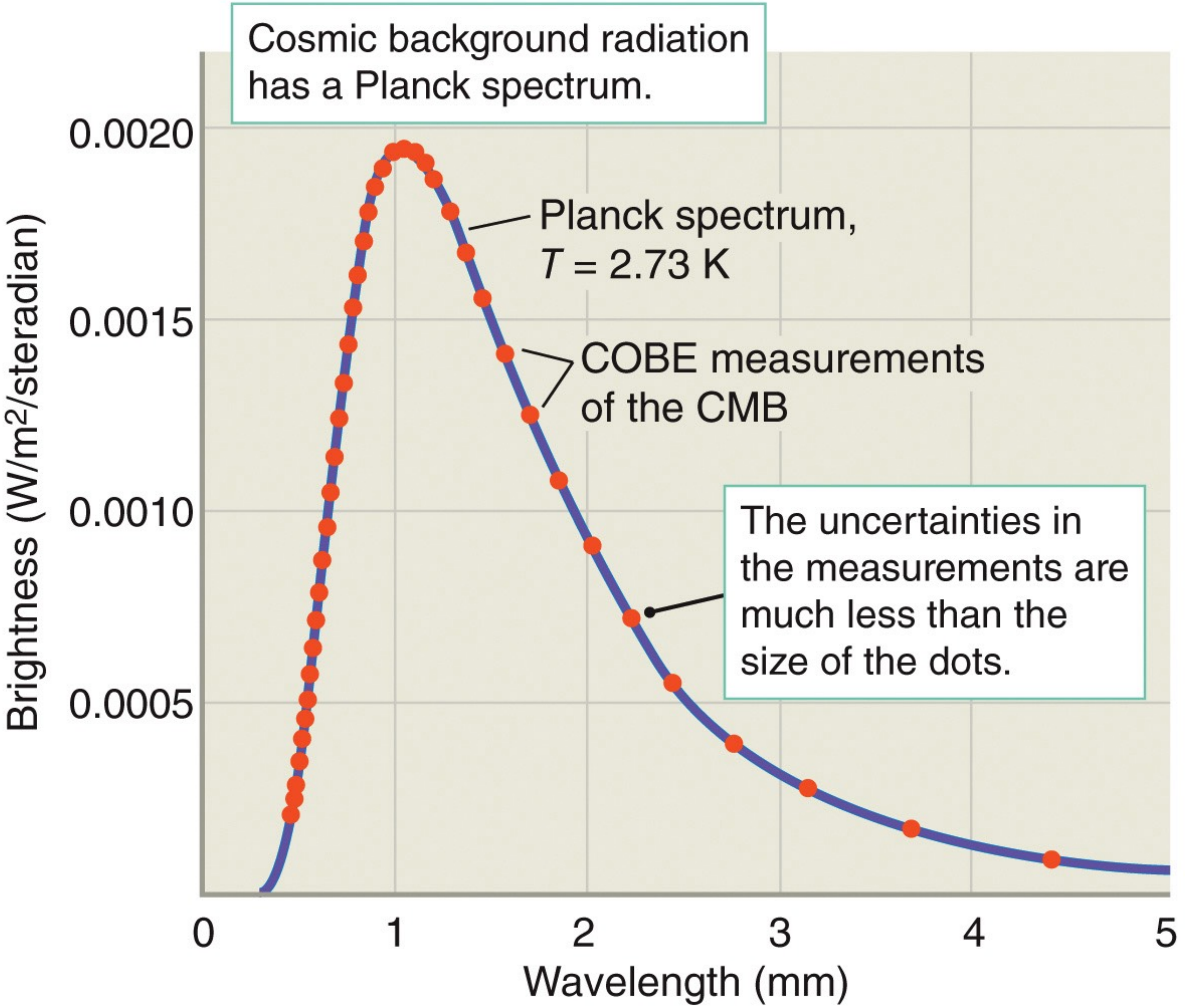
- It is determined by ***Temperature***, so blackbody emission is also called **thermal emission**

- **When *T* increases:**

1. **Peak** shifts to shorter wavelength - **Wien's Displacement Law**
2. **Surface Flux,** the total area under each Planck curve, increases rapidly - **Stefan-Boltzmann Law**
3. **Surface brightness (intensity)** increases at all wavelengths - e.g., **infrared thermometers**

The CMB Temperature from the Full Blackbody Spectrum

- **The COBE satellite (launched in 1989)** was the first instrument to provide very accurate measurements of **the CMB spectrum**. It determined the temperature of the CMB today is **2.73 K** by fitting a beautiful Planck curve to the data.



Problem is that the observed CMB temperature can't tell the original temperature of the blackbody

- Wien's displacement law: $\lambda_{\rm peak} = (2.9\ {\rm mm\ K})/T$
- Cosmological redshift: $\lambda_{\rm obs} = \lambda_{\rm emit}(1+z)$
- Combining the above two, we have the relation the blackbody temperature when emitted and the observed temperature:

$$T_{\rm obs} = T_{\rm emit}/(1+z)$$

  in other words, the observed temperature today is much lower than the original temperature due to cosmic expansion.

- The above equation cannot constrain the temperature when the cosmic radiation background first emerged. In fact, infinite number of (T_{emit}, z) combinations could give us the same 3 K observed temperature, for examples:
  - T_{emit} = 10 K, z = 2.3
  - T_{emit} = 100 K, z = 32
  - T_{emit} = 1000 K, z = 330
- We call that **the two parameters (T_{emit}, z)** are **degenerate.**

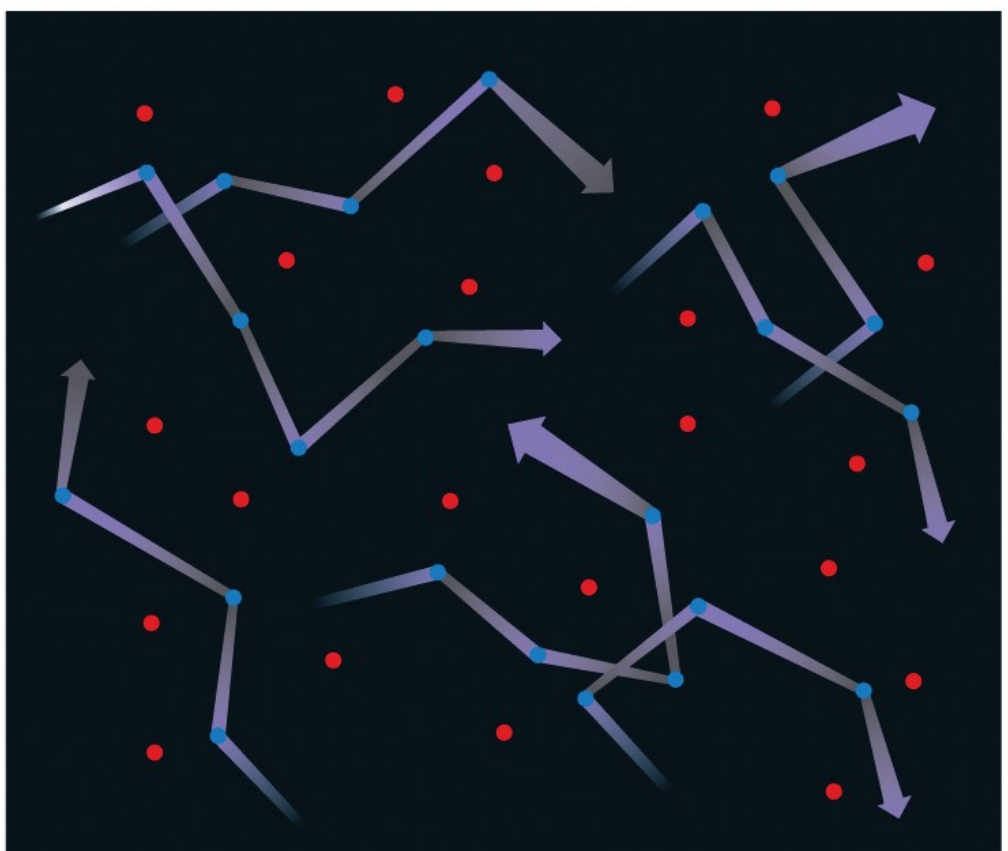
How do we know when the CMB first started to propagate in the universe?

The epoch of hydrogen recombination

How did the CMB emerge?
First, EM waves are trapped in an ionized universe

- When the universe was hot and the gas was ionized, photons were trapped with matter.
  - Free electrons interact strongly with photons (***Thomson scattering***).
  - We cannot observe anything during this era. It's as if the universe is filled with a dense fog.

In the ionized early universe, light was trapped by free electrons. Radiation had a blackbody spectrum.



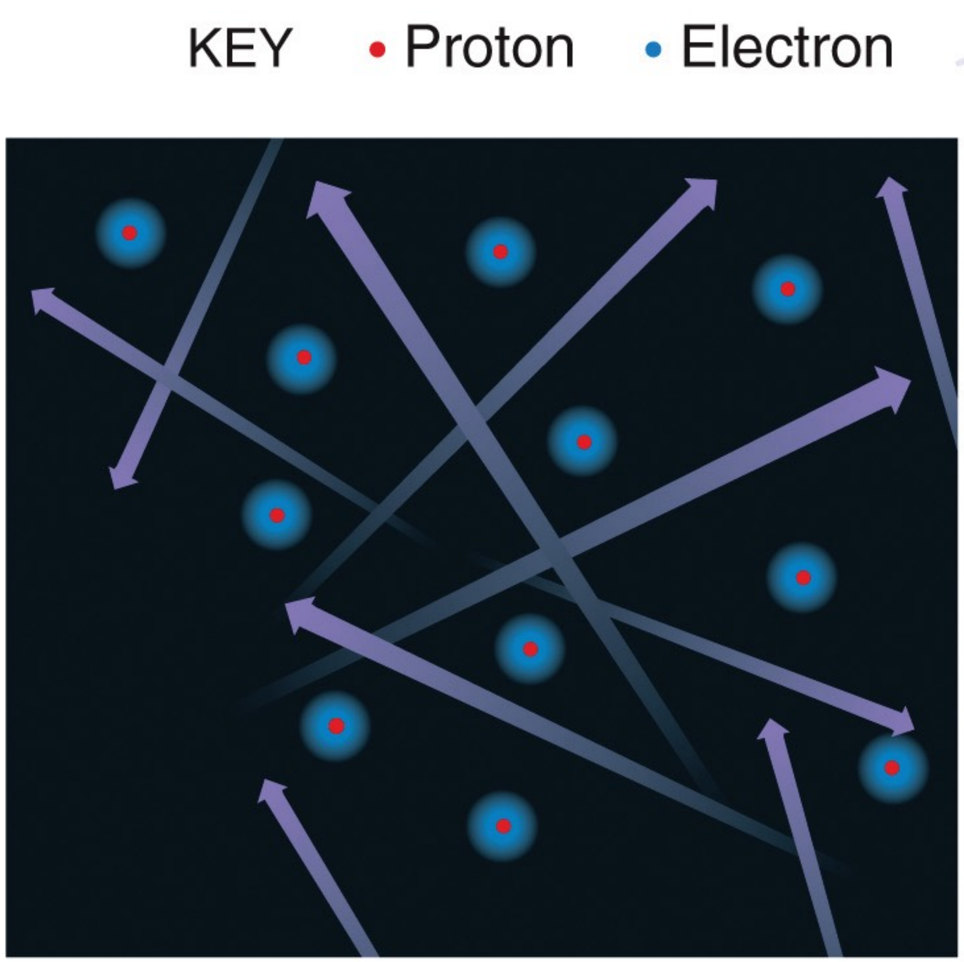
At that time, it was as though the universe was filled with a thick fog.

Emil Oprisa/Alamy Stock Photo

KEY • Proton • Electron Path of photon

How did the CMB emerge?
Then, protons and electrons recombined to form hydrogen

- Eventually, the expansion causes the temperature to cool enough that **protons** and **electrons** could form **neutral H atoms**: this phase-transition of the Universe is called **the epoch of recombination (EoR)**.
- At that time, light was no longer blocked from its travel by free electrons.
- **EoR** marks the earliest point in the universe that we can observe.

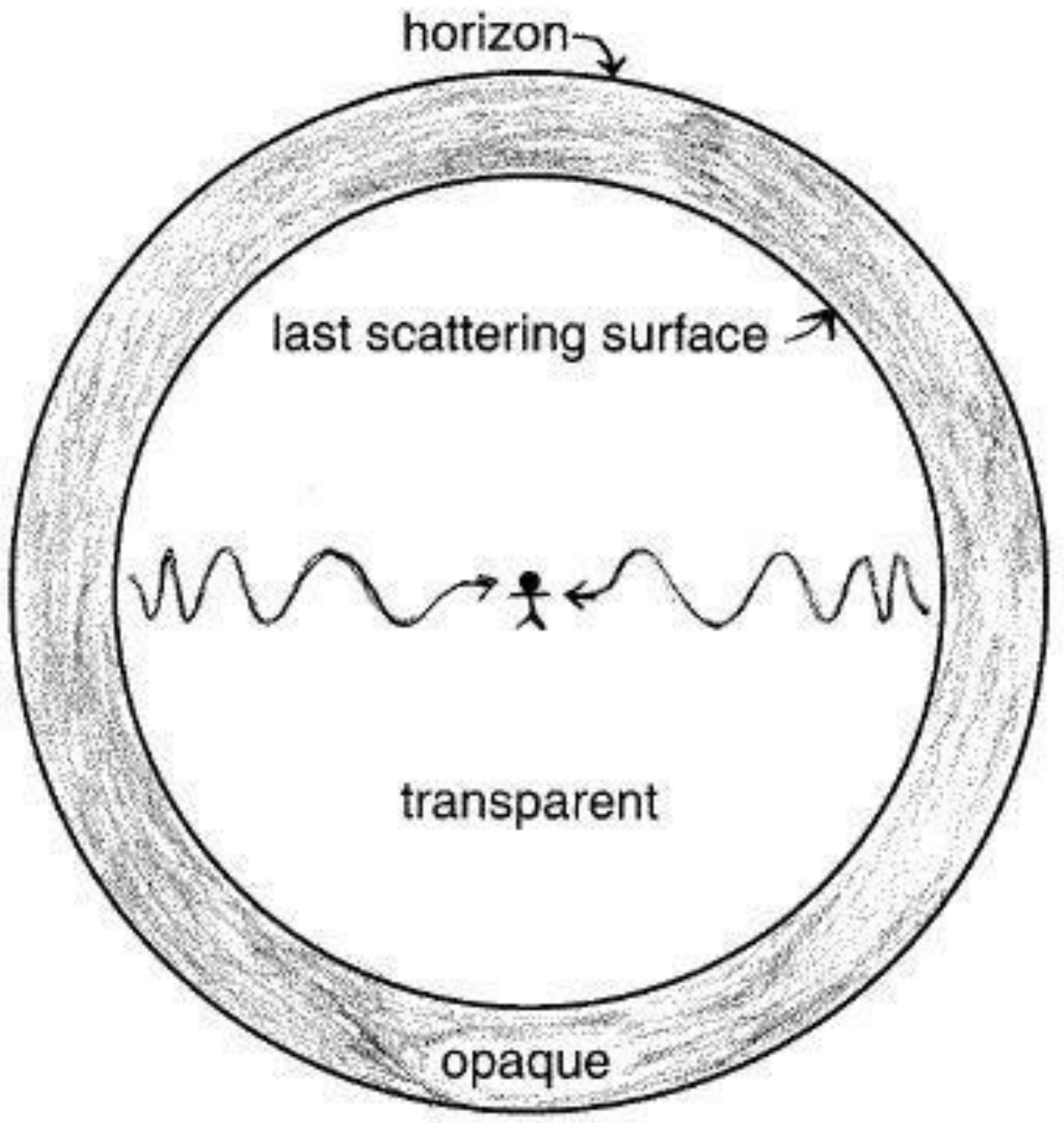


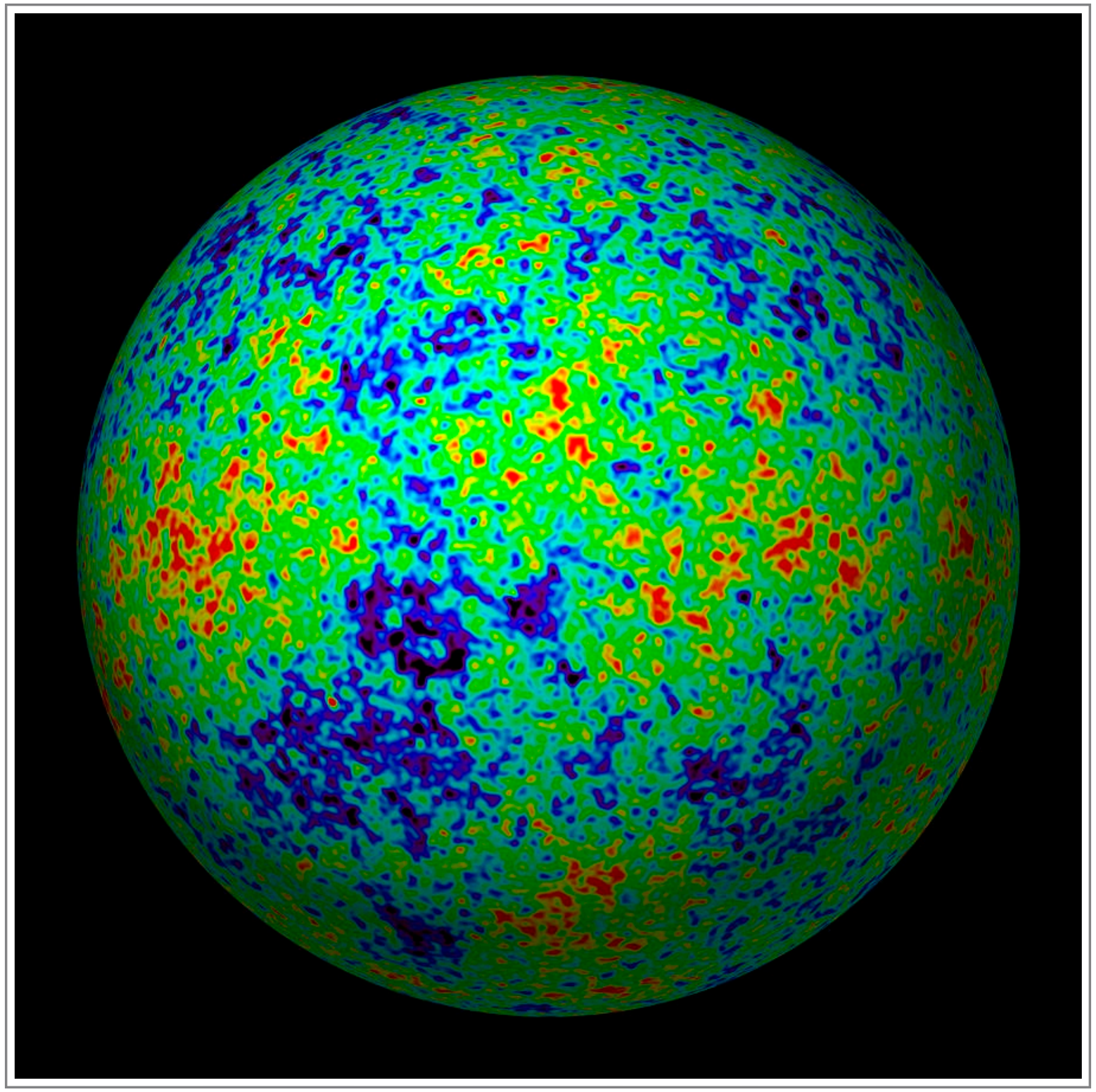
At recombination, the universe became transparent, and the blackbody radiation traveled freely through the universe.

Recombination was like the fog suddenly clearing.

CMB Photons travel straight to us from the last scattering surface

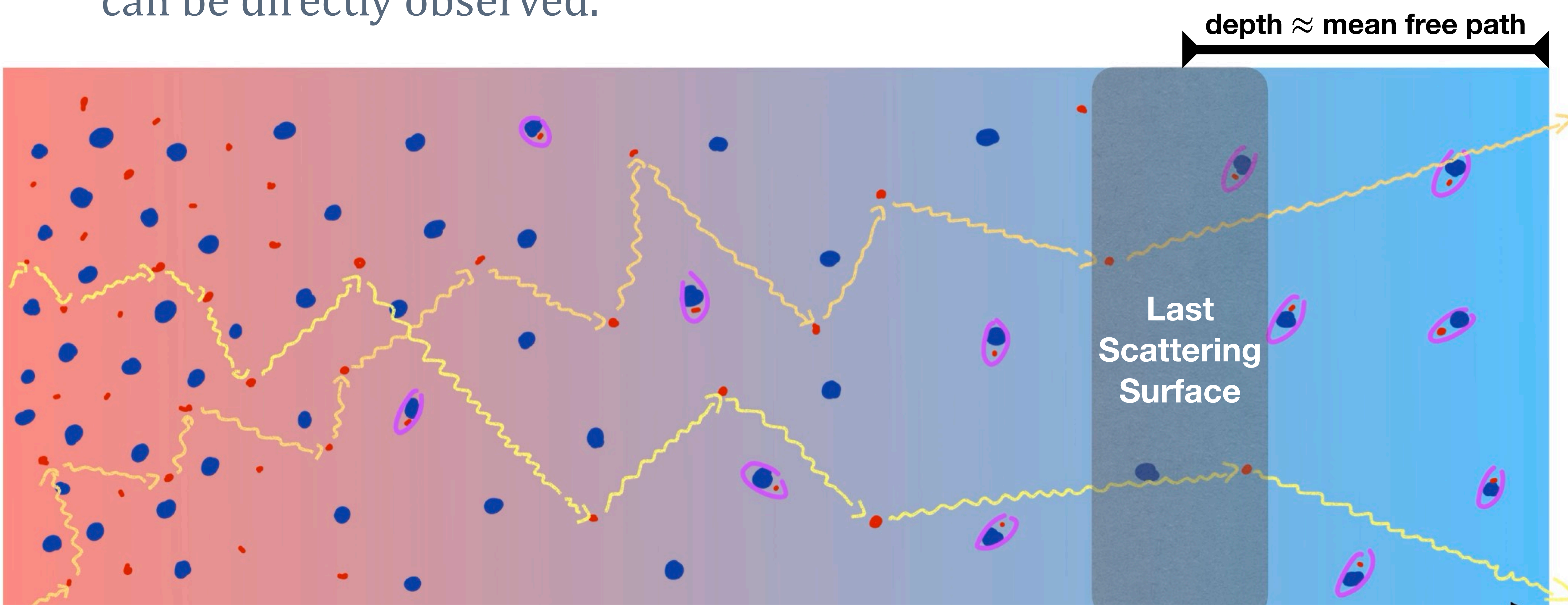
- Analogous to the *last scattering surface* that marks the surface of the Solar photosphere





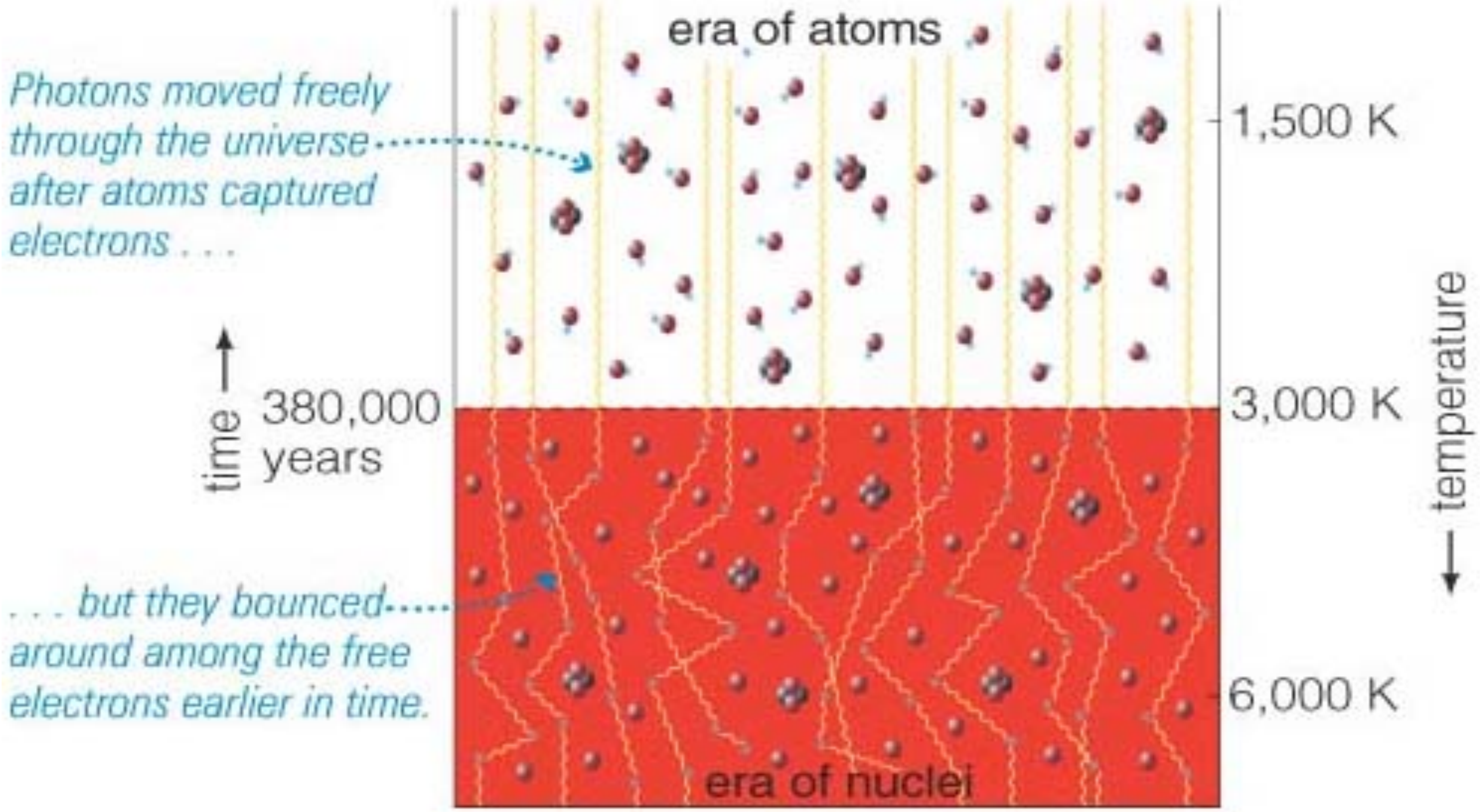
Recall this slide in Chap 2 - The Sun? Last Scattering Surface

- The Sun has no solid surface, but the apparent surface of the Sun is the surface at which light can directly escape into space.
- Let's call this surface the **last scattering surface** (a concept also used in **cosmology**). Note that its depth depends on **(1) the angle we look into the Sun and (2) the wavelength of the photons**
- The layers above this point are known as the **atmosphere**, which can be directly observed.

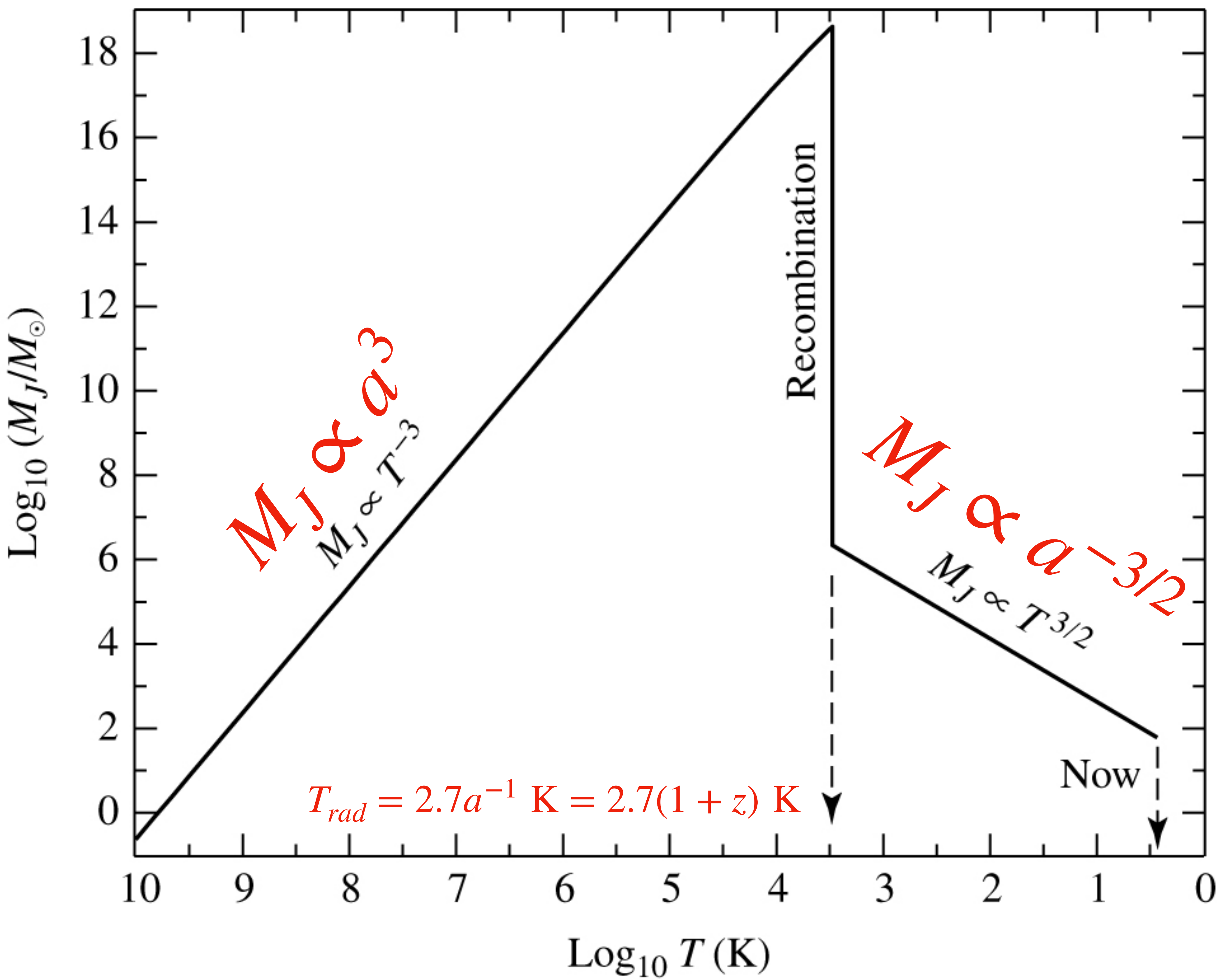


The EM radiation background emerges when recombination completed

- Before **recombination**, photons cannot travel far before it is scattered by e-; after **recombination**, photons can freely travel and eventually reach us.
- Given the baryon density of the Universe, it can be shown that **Hydrogen recombination** completed when the universe was ~**3000 K.** This is **the original temperature of the cosmic EM radiation field**.
- Since we have proven $T_{\text{observed}} = T_{\text{emitted}}/(1+z)$, and we know the CMB has a temperature is 2.7 K today, the redshift at which the CMB emerged must be around 1100: $1+z = T_{\text{emitted}}/T_{\text{observed}} = 3000\text{ K}/\ 2.7\text{ K} \sim 1100$

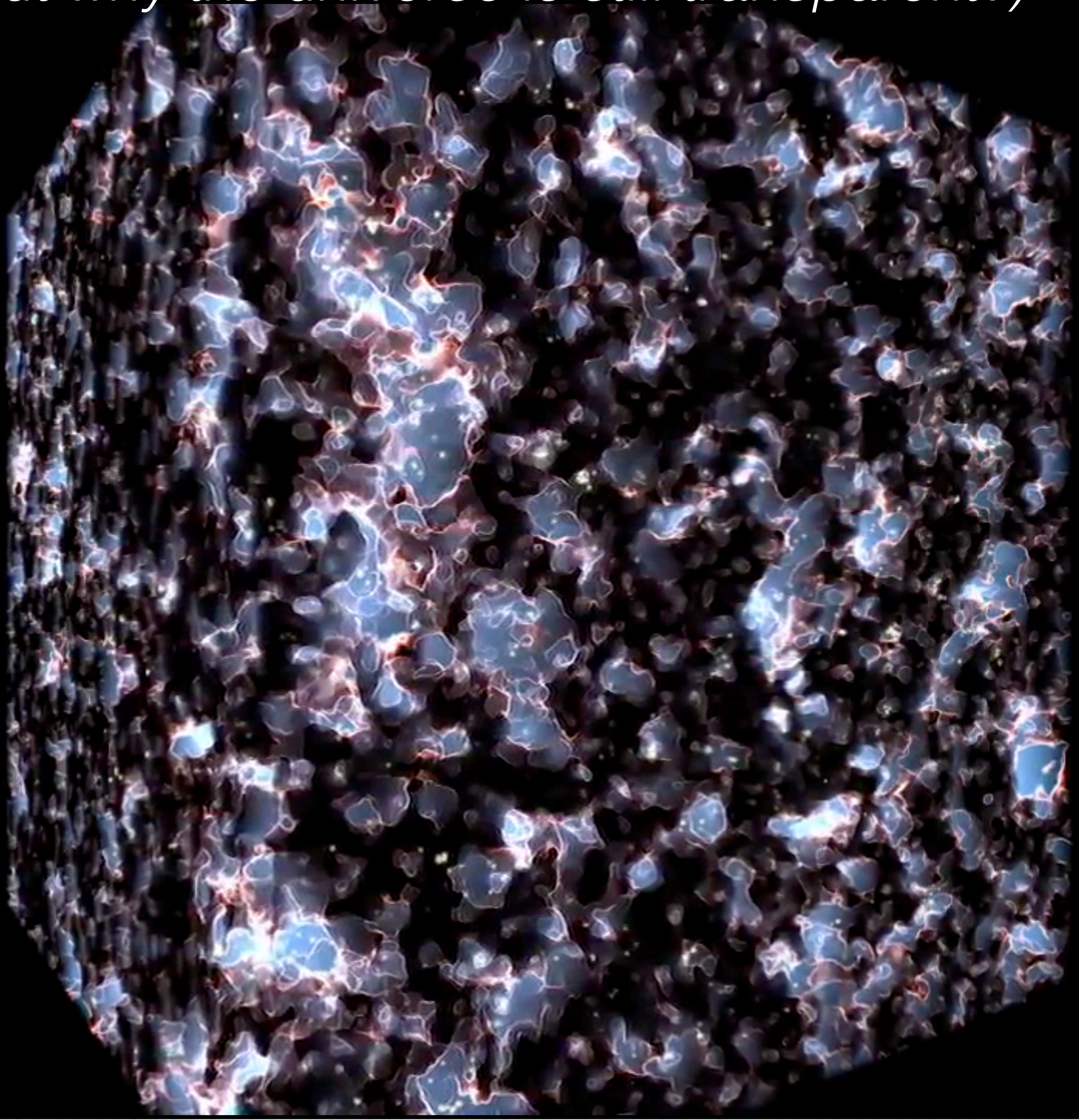


Recombination drastically reduced the Jeans mass for gravitational instability, allowing galaxies to form. ***What would happen next?***

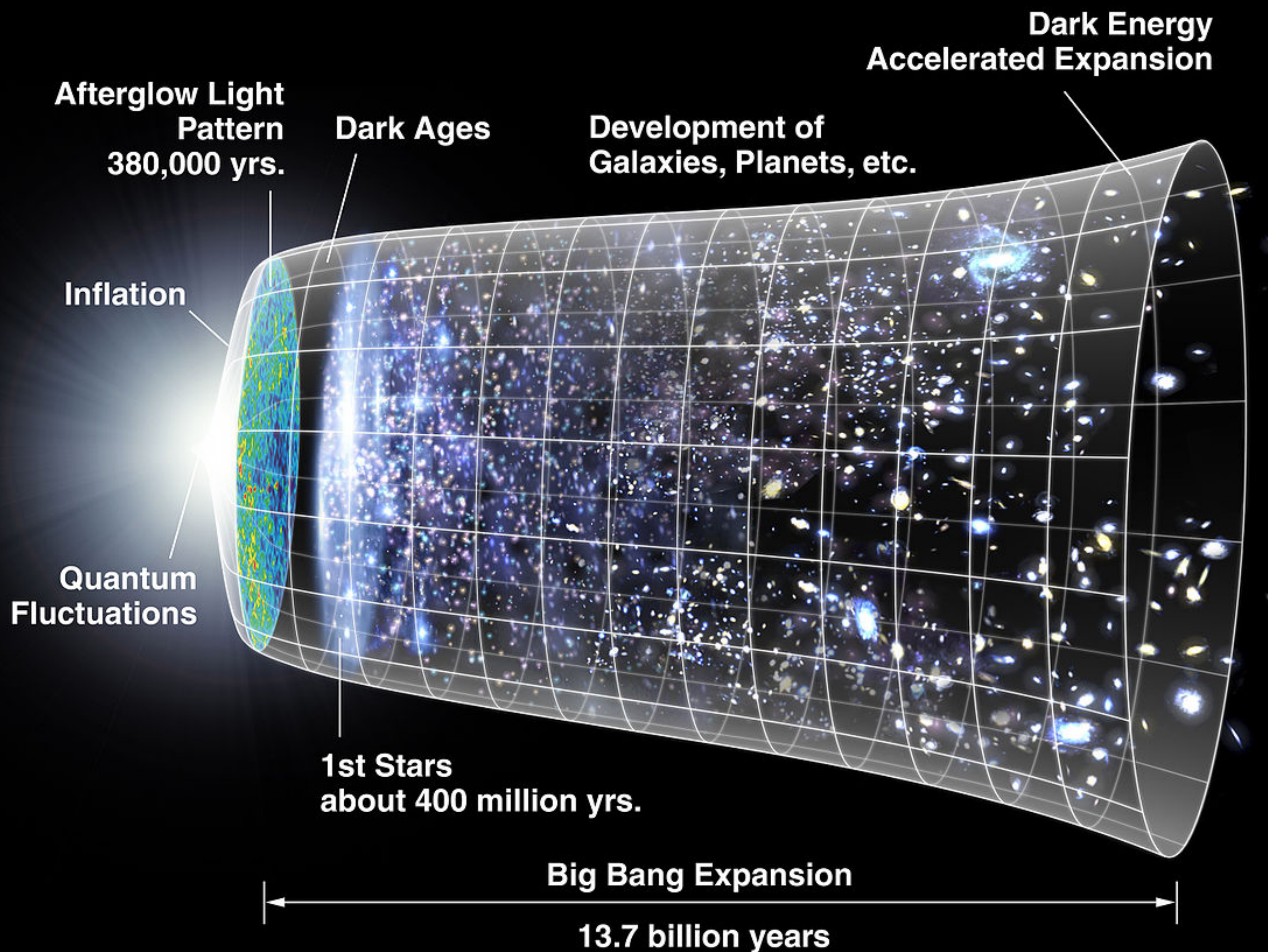


Reionization of the Universe by Galaxies

(but why the universe is still transparent?)

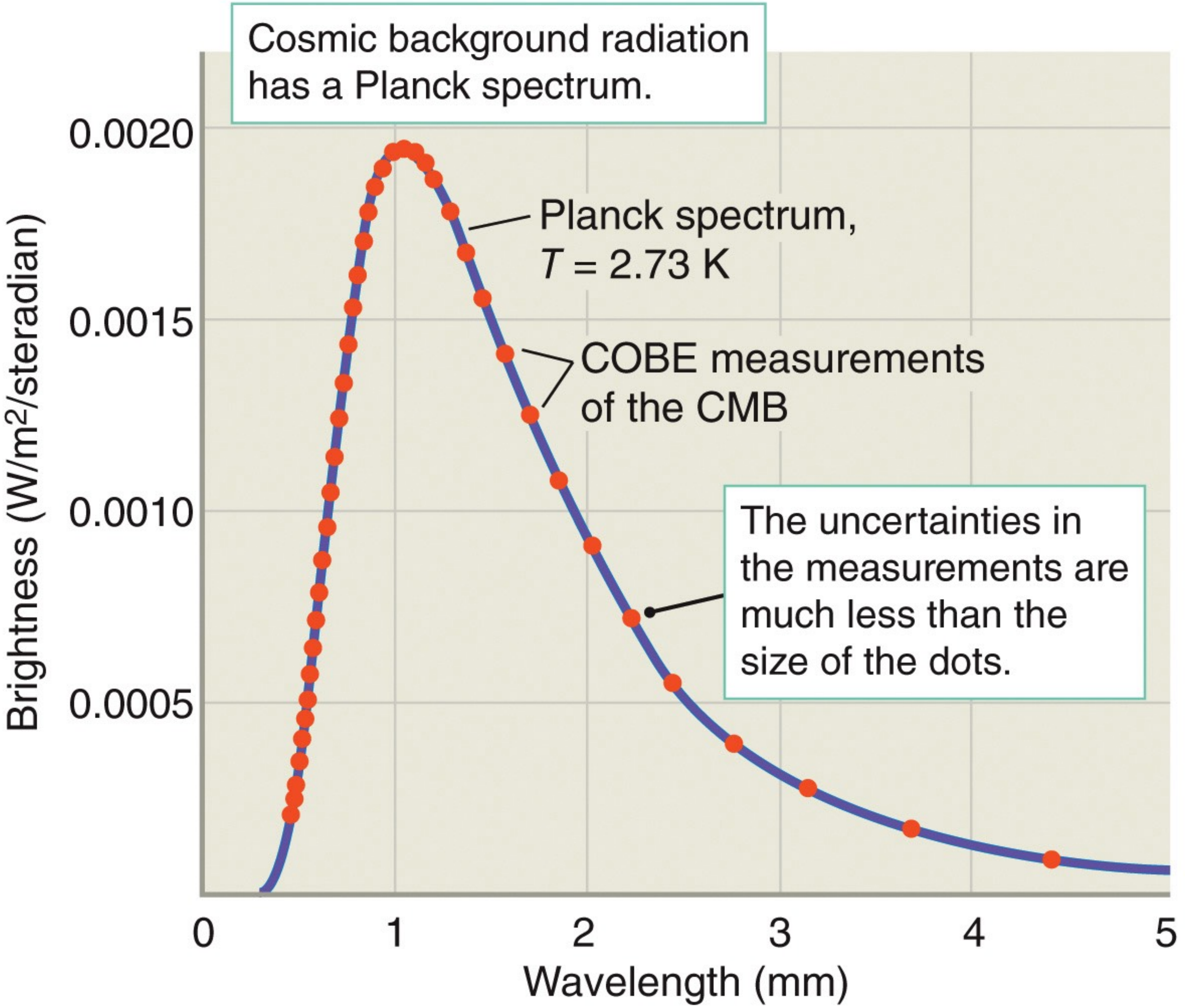


Big Bang - Particles created - Ionized universe (opaque) - Recombination (z ~ 1100) - Dark Ages - Reionization (z ~ 20-7)



The CMB Temperature from the Full Blackbody Spectrum

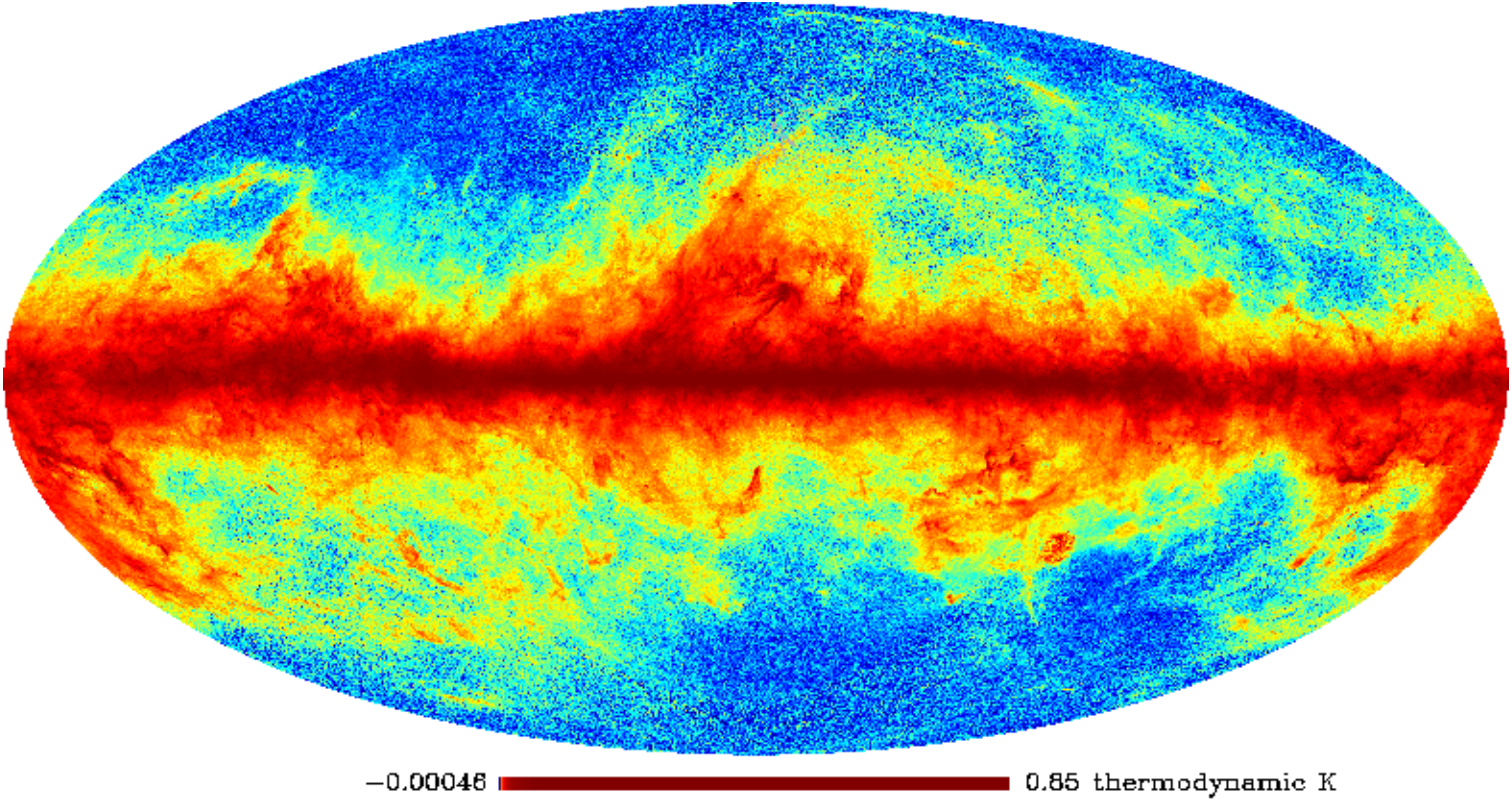
- **The COBE satellite (launched in 1989)** was the first instrument to provide very accurate measurements of **the CMB spectrum**. It determined the temperature of the CMB today is **2.73 K** by fitting a beautiful Planck curve to the data.



If the entire sky glows in microwave radiation, why not get an all-sky map of the CMB?

The discovery of CMB anisotropies

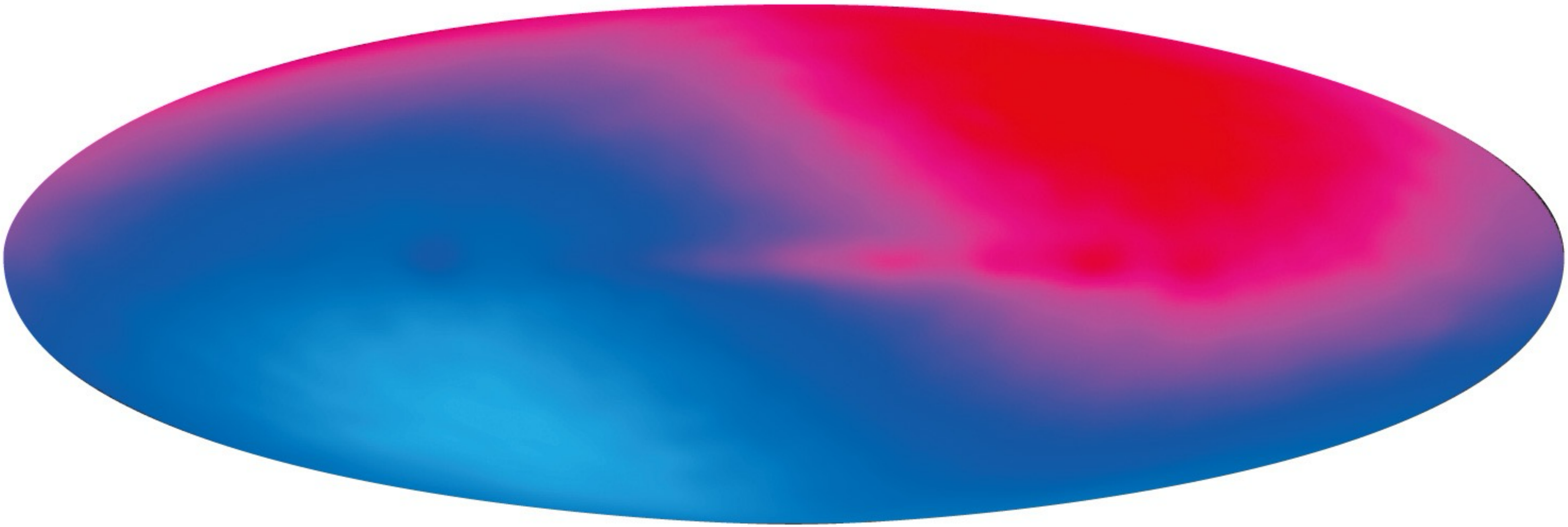
The All-sky Temperature Map of the CMB in Mollweide (equal-area) projection



After subtracting the Milky Way, there is a strong dipole signal in the CMB (mean $T = 2.7\text{K}$)

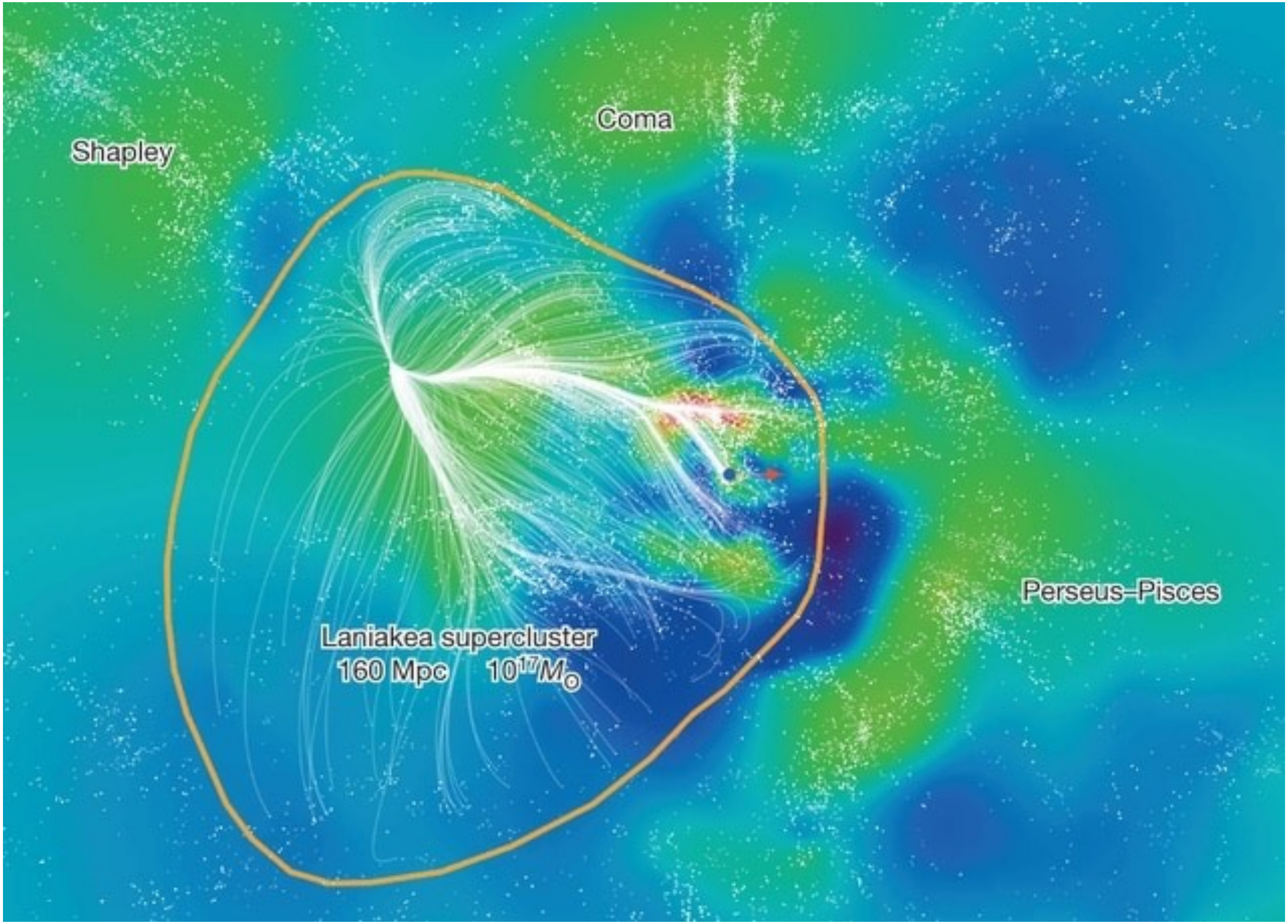
Dipole Maximum Direction: RA = $167^\circ\!.942 \pm 0^\circ\!.007$, Dec = $-6^\circ\!.944 \pm 0^\circ\!.007$ (J2000).

Dipole Maximum Amplitude: 3.362 mK

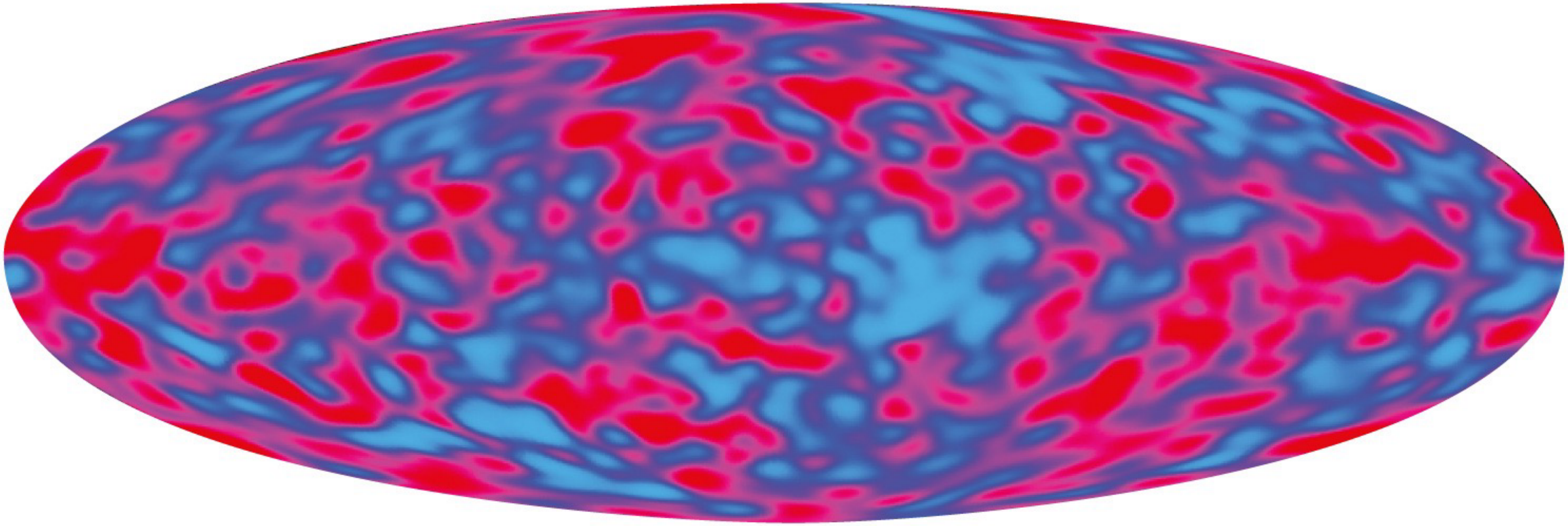


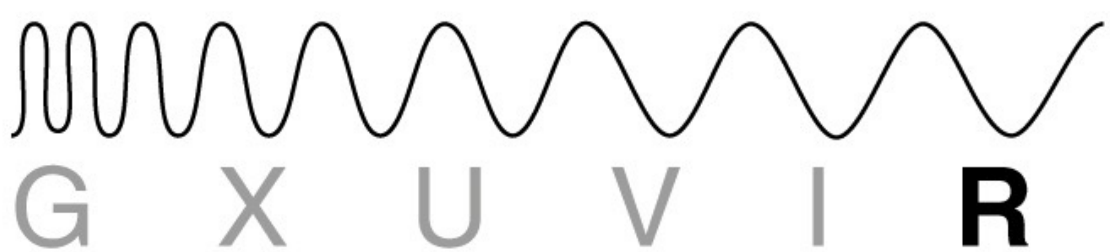
What is the relative velocity between the Solar System and the CMB rest frame?

The CMB Solar Dipole shows the Combined Motion of (1) the Solar System in the Milky Way and (2) the Milky Way Galaxy in the Local Supercluster

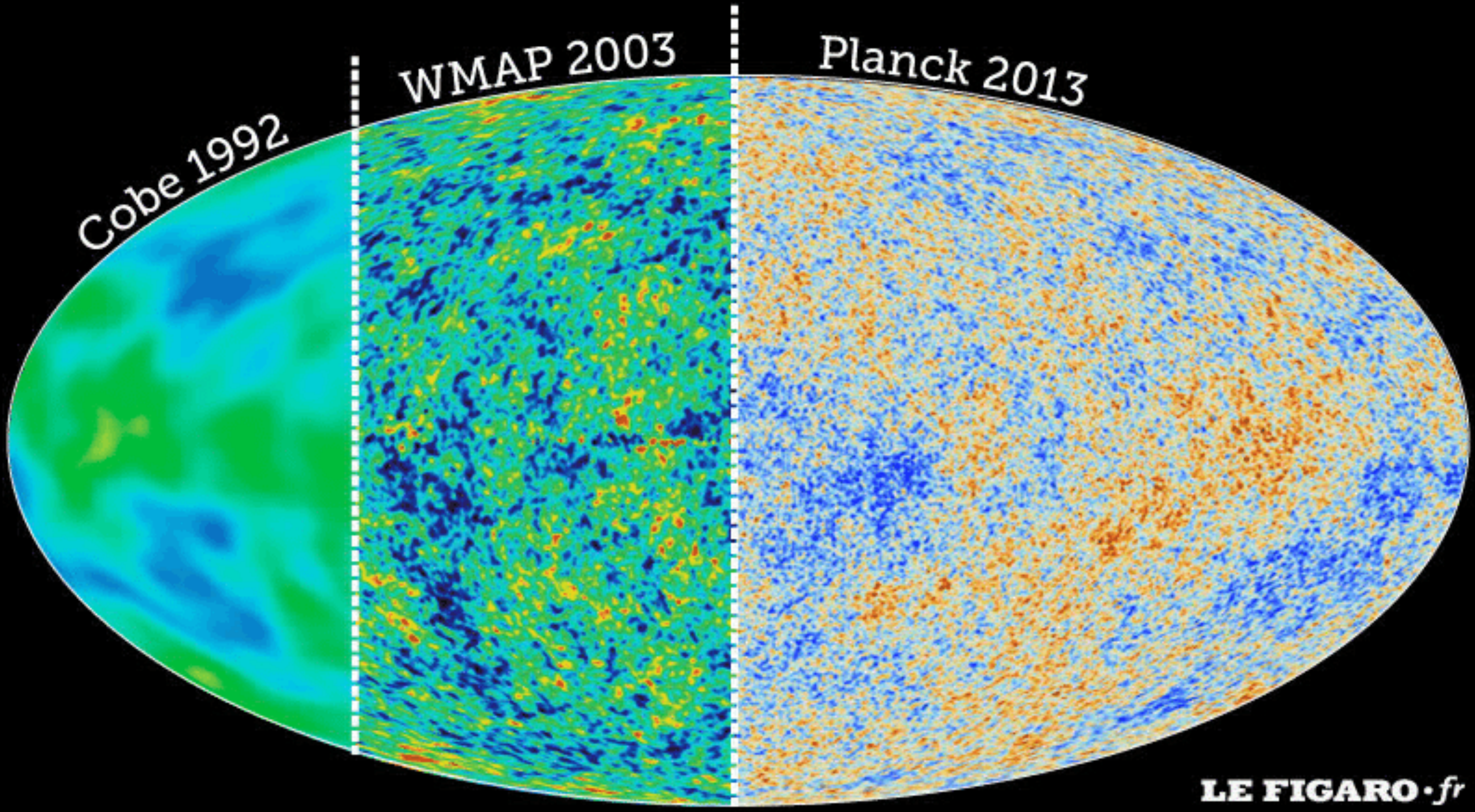


Subtraction of the dipole reveals smaller scale fluctuations in the CMB (the anisotropies)

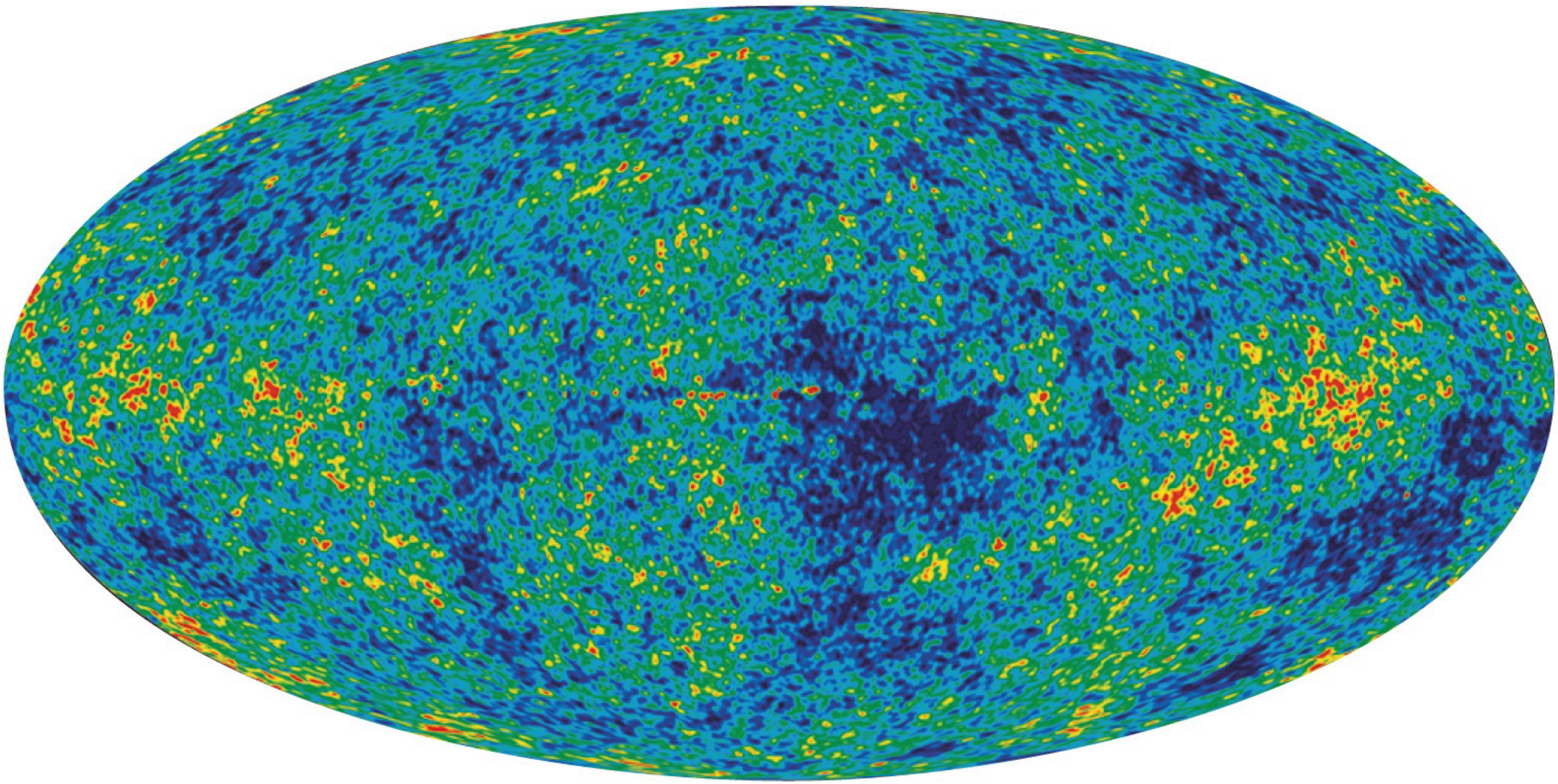




Improved angular resolutions over three generations of satellites



A much sharper map showing temperature fluctuations on the level of $\delta T/T \approx 0.00001$



CMB anisotropy shows density fluctuations of $3\mathrm{x}10^{-5}$ at $z_{rec} \sim 1000$

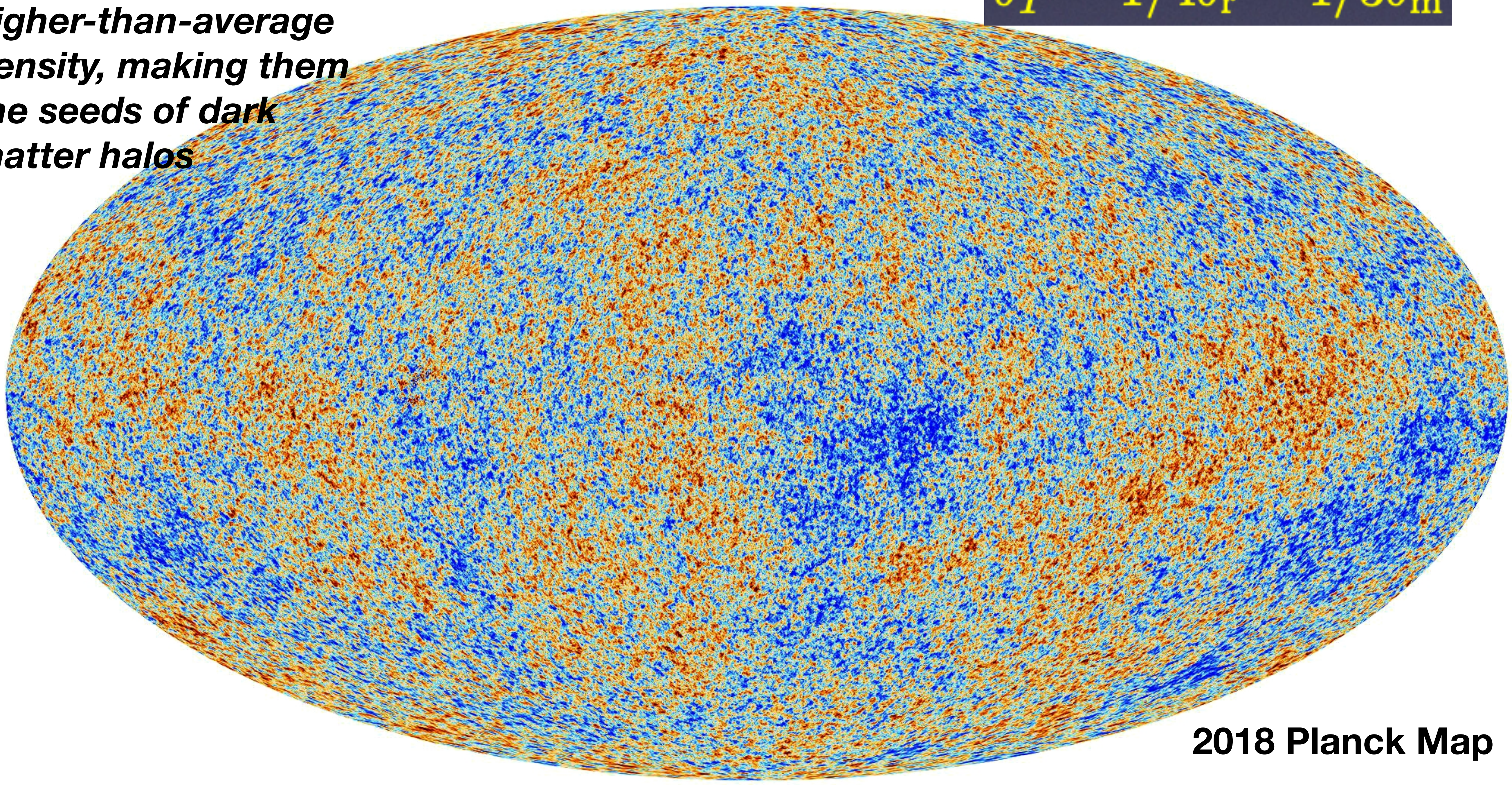
isentropic/adiabatic perturbations: entropy per unit mass is preserved:

$$S = \frac{s}{\rho_{\mathrm{m}}} \propto \frac{\rho_{\mathrm{r}}^{3/4}}{\rho_{\mathrm{m}}}$$

$$\boxed{\delta_S = \frac{\partial S}{S} = \frac{1}{S}\left[\frac{\partial S}{\partial \rho_{\mathrm{r}}}\,\partial\rho_{\mathrm{r}} + \frac{\partial S}{\partial \rho_{\mathrm{m}}}\,\partial\rho_{\mathrm{m}}\right] = \frac{3}{4}\delta_{\mathrm{r}} - \delta_{\mathrm{m}}}$$

$$\delta_T = 1/4\delta_{\mathrm{r}} = 1/3\delta_{\mathrm{m}}$$

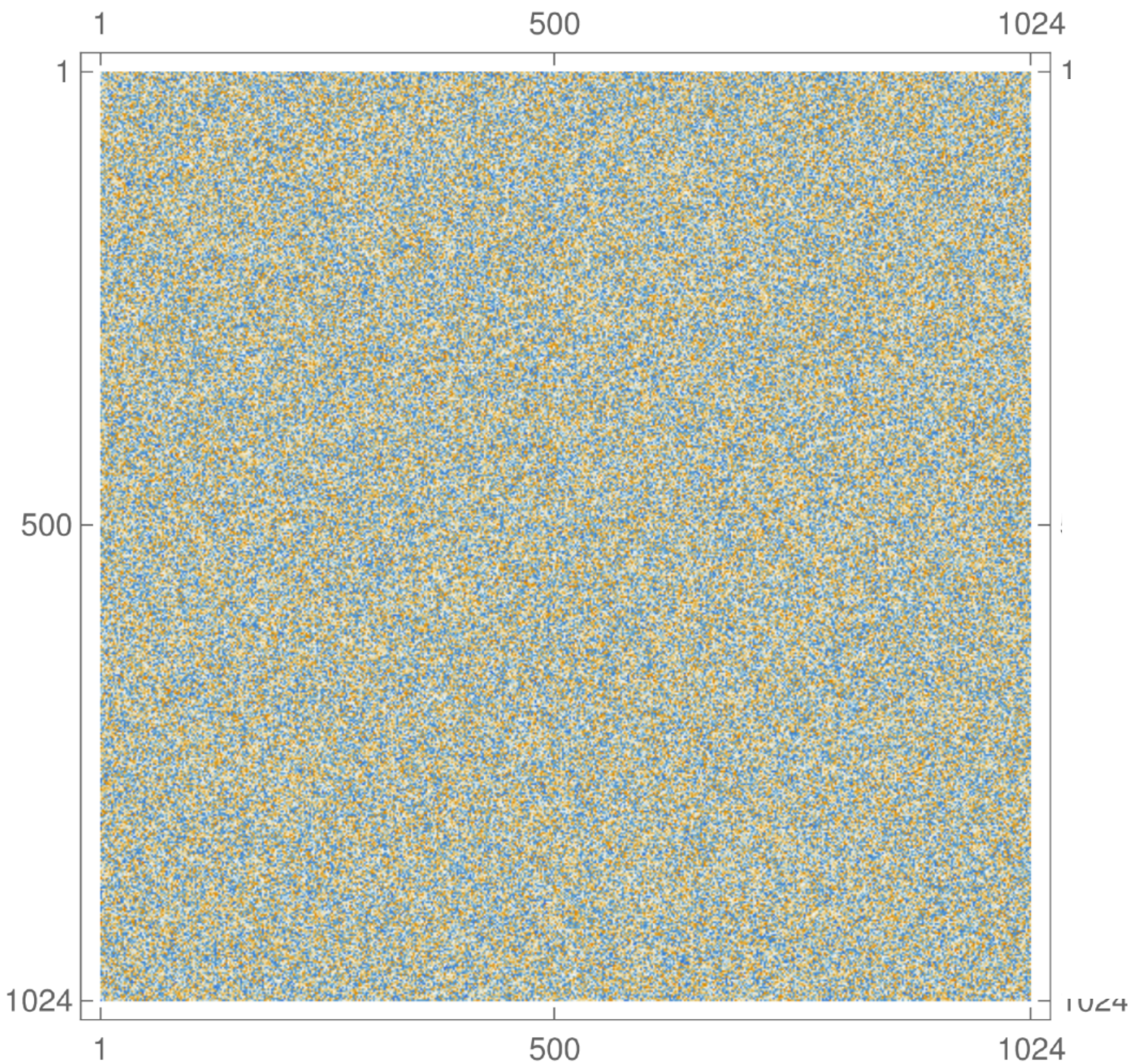
Hotter T regions have higher-than-average density, making them the seeds of dark matter halos



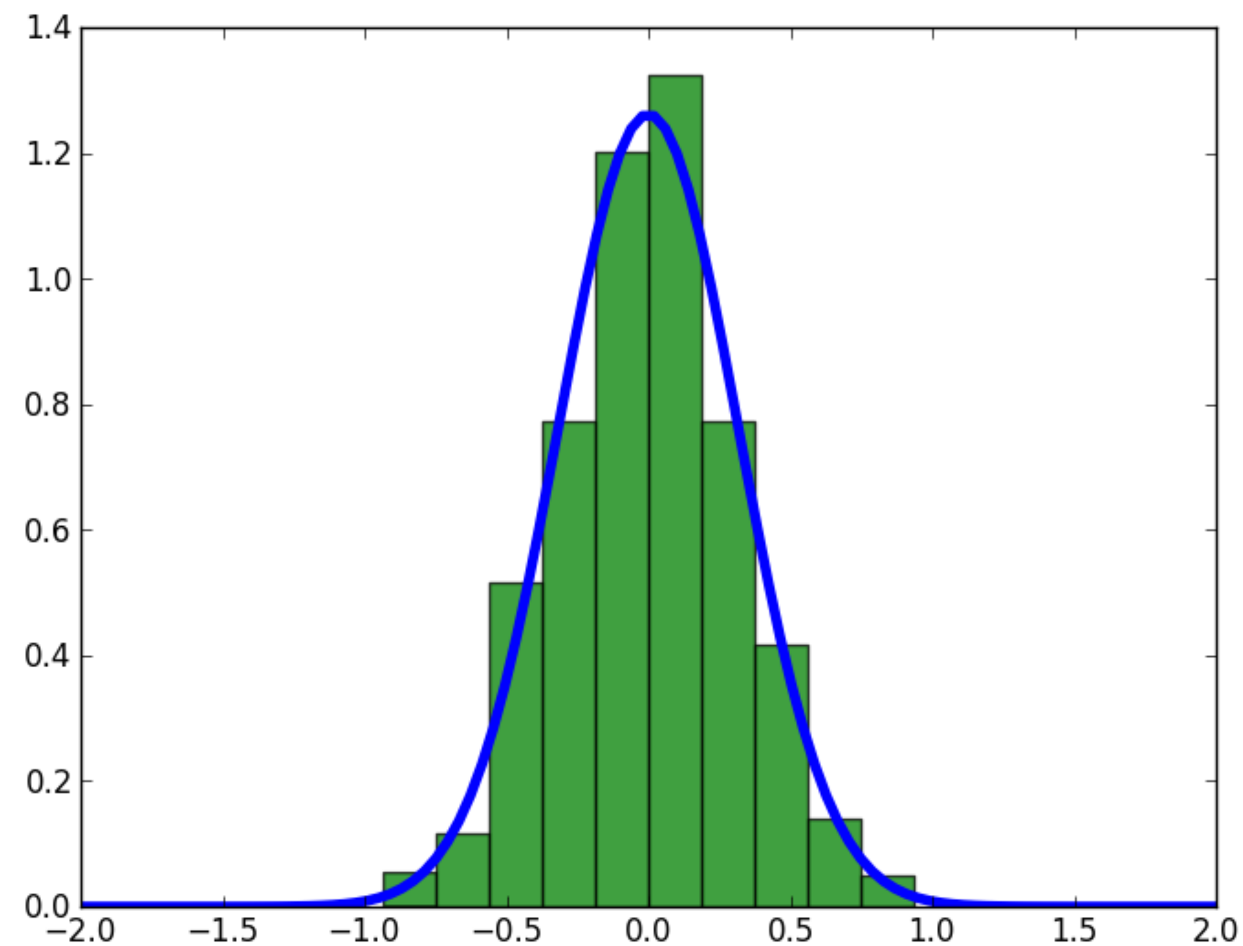
2018 Planck Map

Gaussian random field: white noise

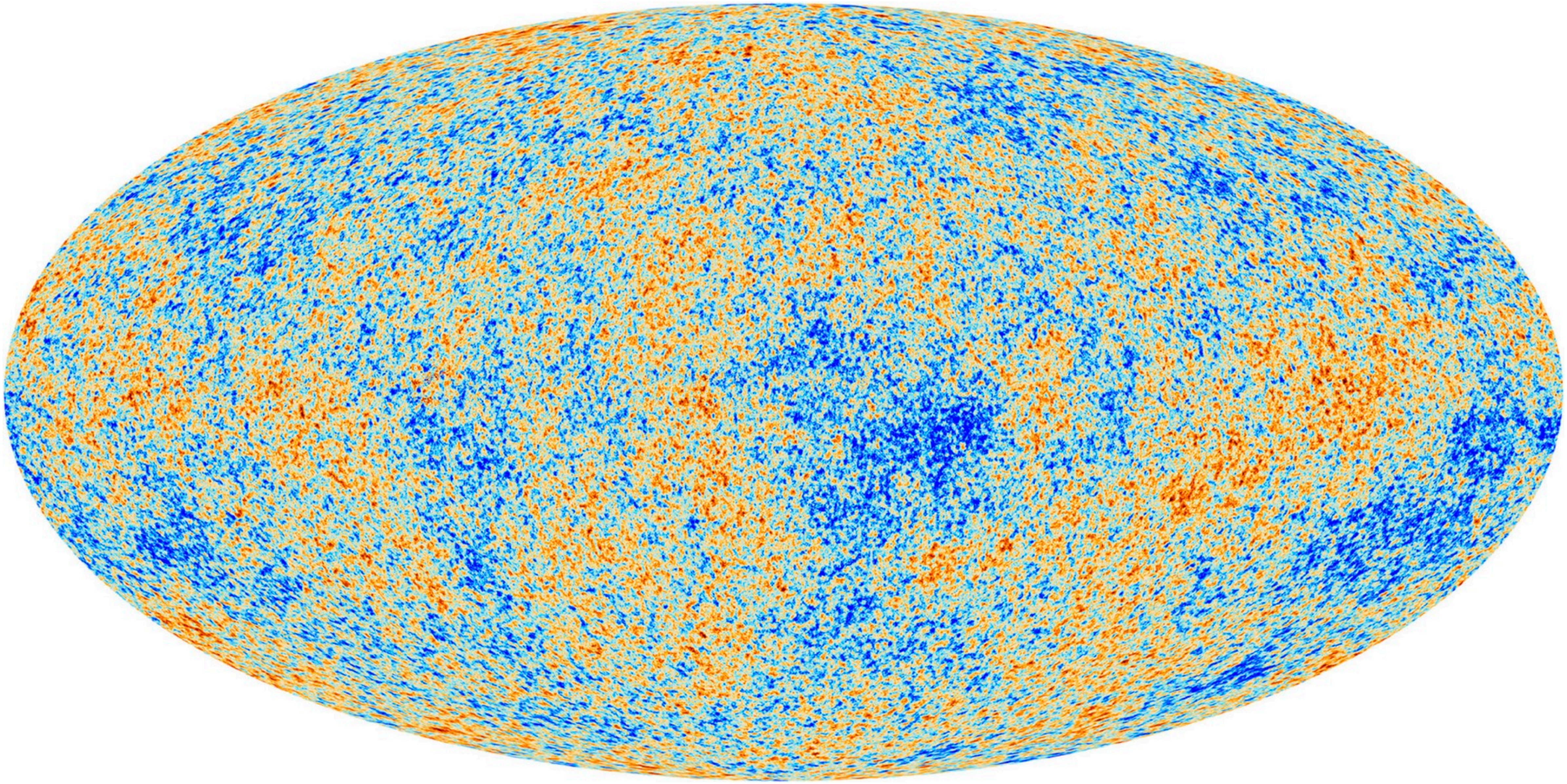
Img = randomn(seed, nx=1024, ny=1024, /normal, **sigma**=0.2)



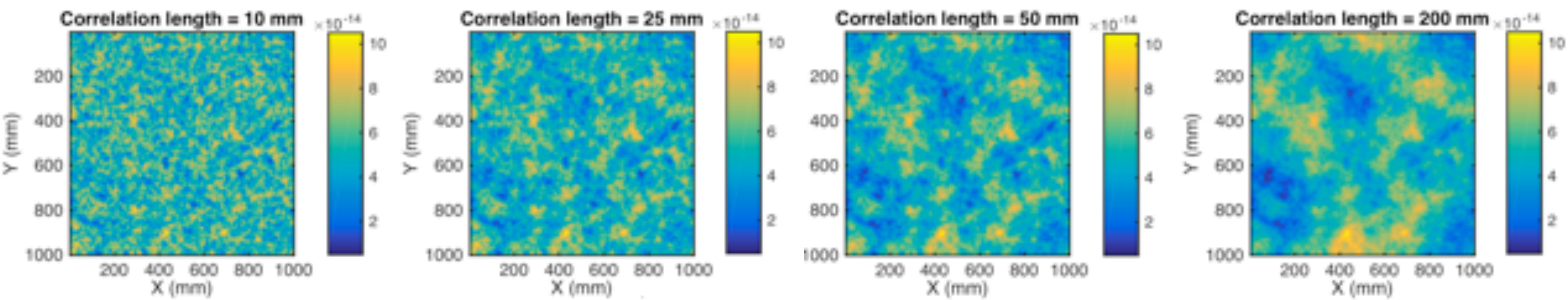
histogram(img, bin=0.2)



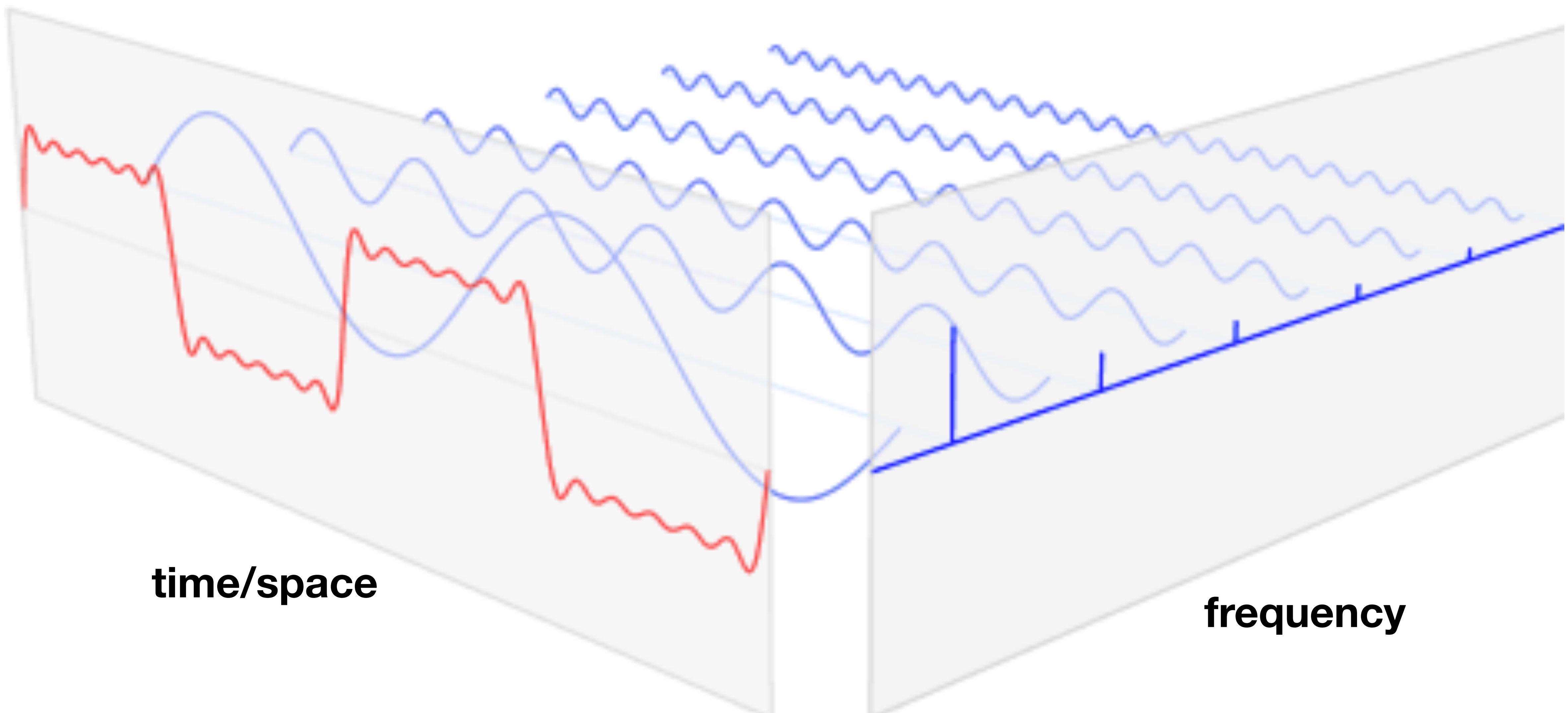
Planck CMB $\delta T/T$ map is clearly not white noise



Gaussian random fields w/ increasing correlation lengths



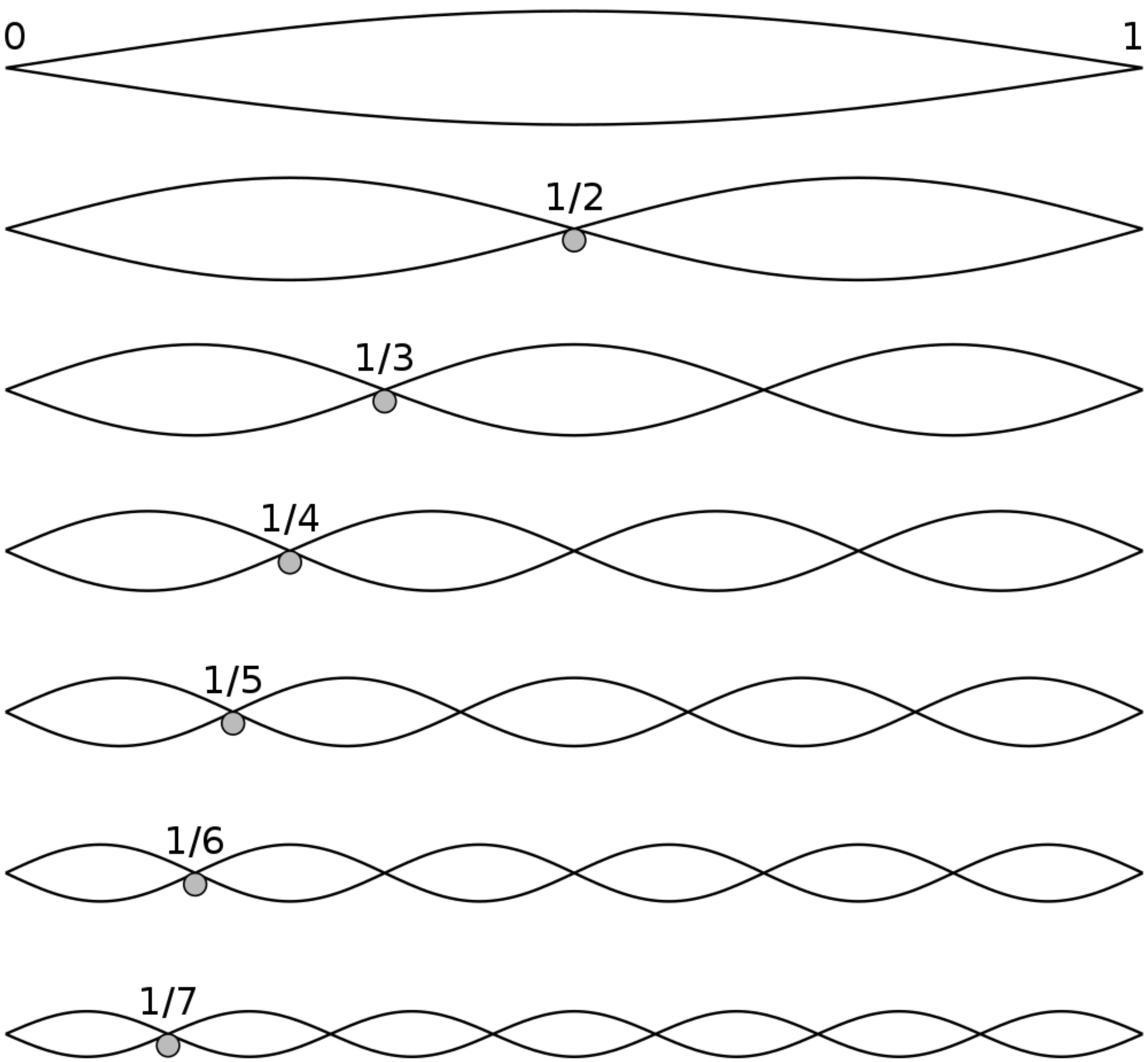
Fourier Transform: from time domain to frequency domain



Decomposing a *periodic* time/space signal into series of sine & cosine functions

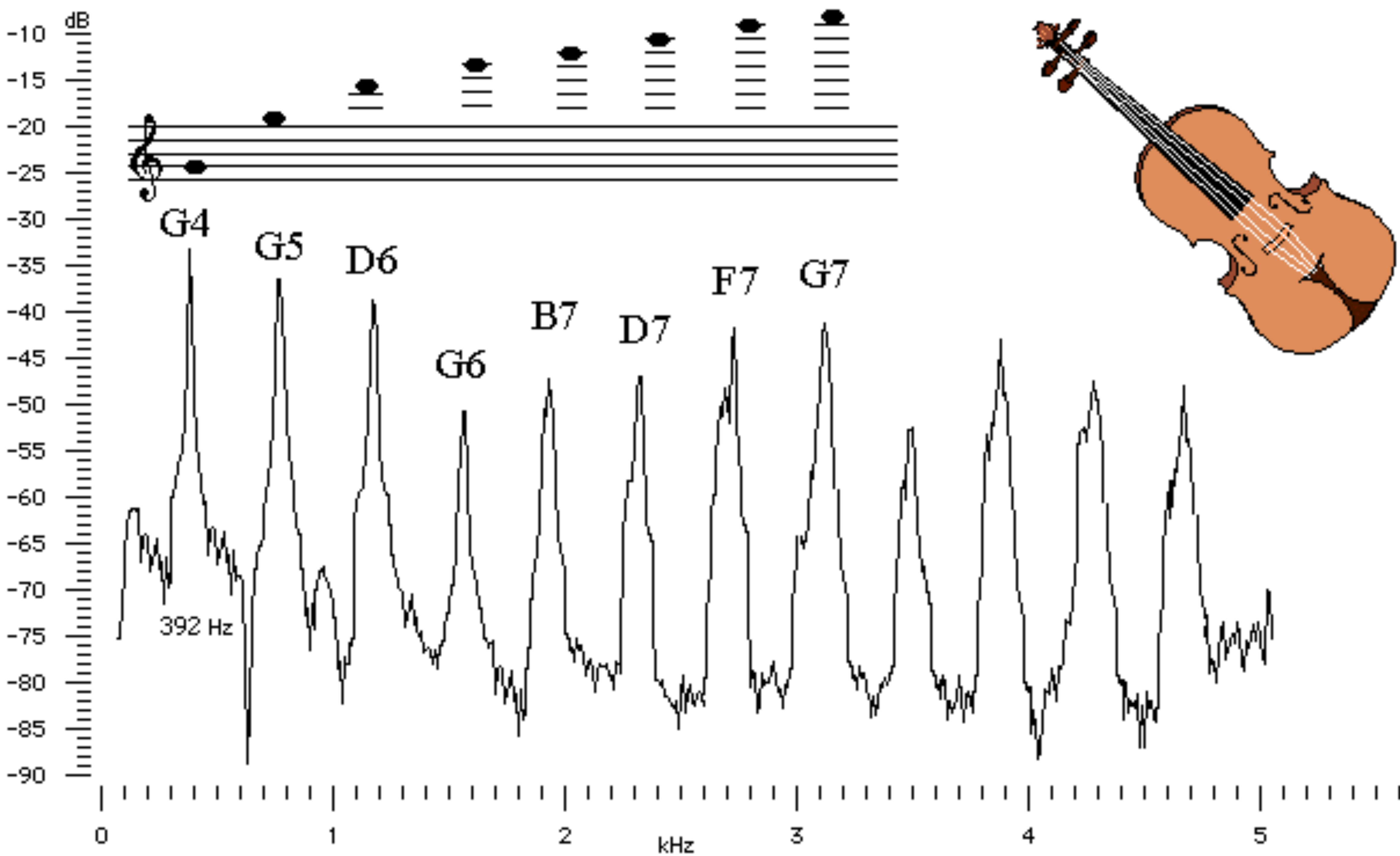
$$\widehat{f}(\xi) = \underbrace{Ae^{i\theta}}_{\text{polar coordinate form}} = \underbrace{A\cos(\theta) + iA\sin(\theta)}_{\text{rectangular coordinate form}}.$$

Harmonics of a string showing the periods of the pure-tone harmonics



acoustic resonators based on strings, which some call “music instruments”

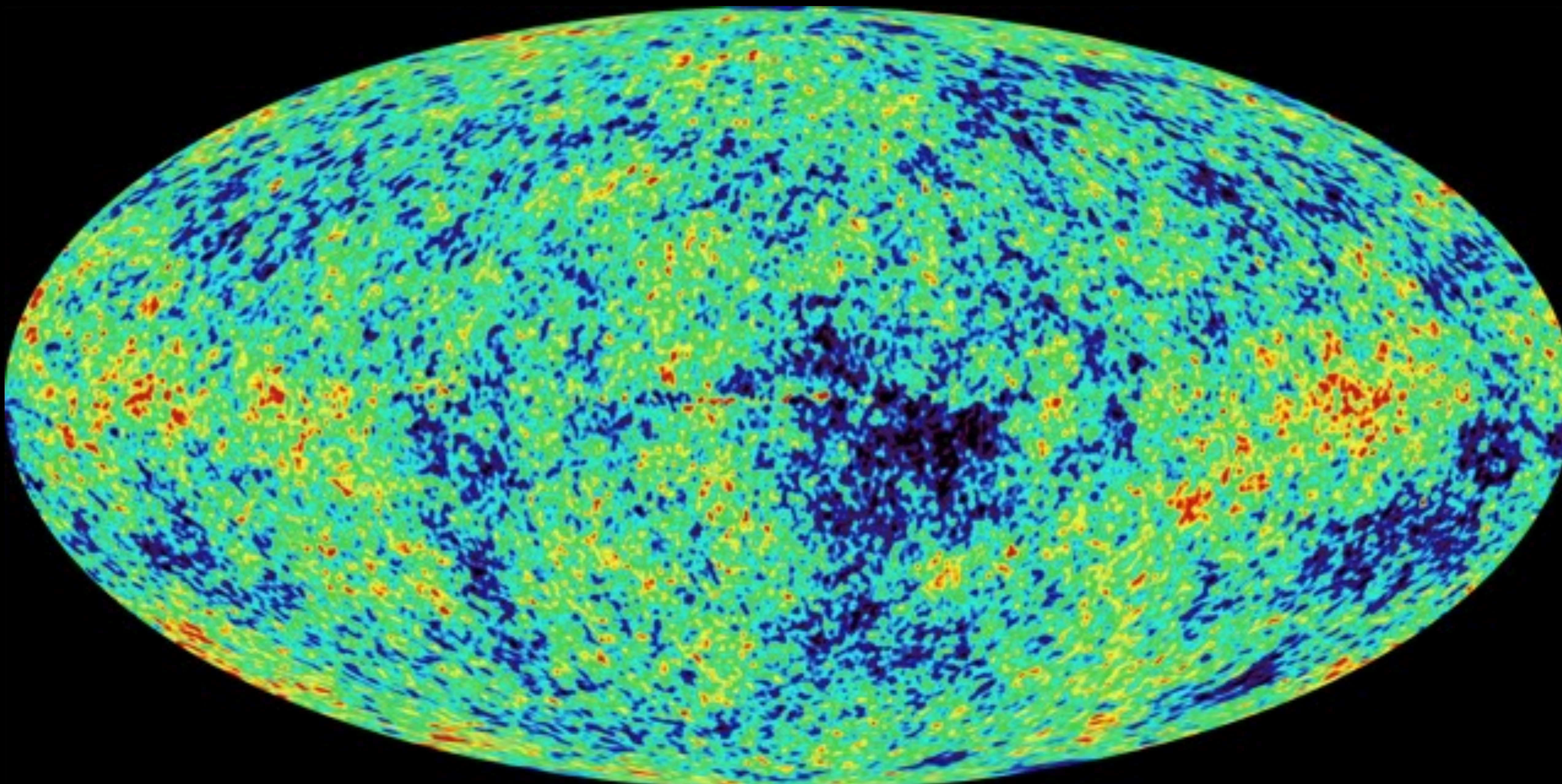
The harmonic spectrum of a Violin



https://www.intmath.com/fourier-series/6-line-spectrum.php

Quantifying CMB anisotropies w/
angular power spectrum

CMB Anisotropy in Mollweide (equal-area) projection



$\delta T/T \approx 0.00001$

Quantum Mechanics: spherical harmonics $Y_l^m(\theta, \phi)$

eigenfunctions that describe
the angular distribution of electrons:
l: orbital angular momentum
m: z-axis projection of l

$$l \approx \pi/\theta$$

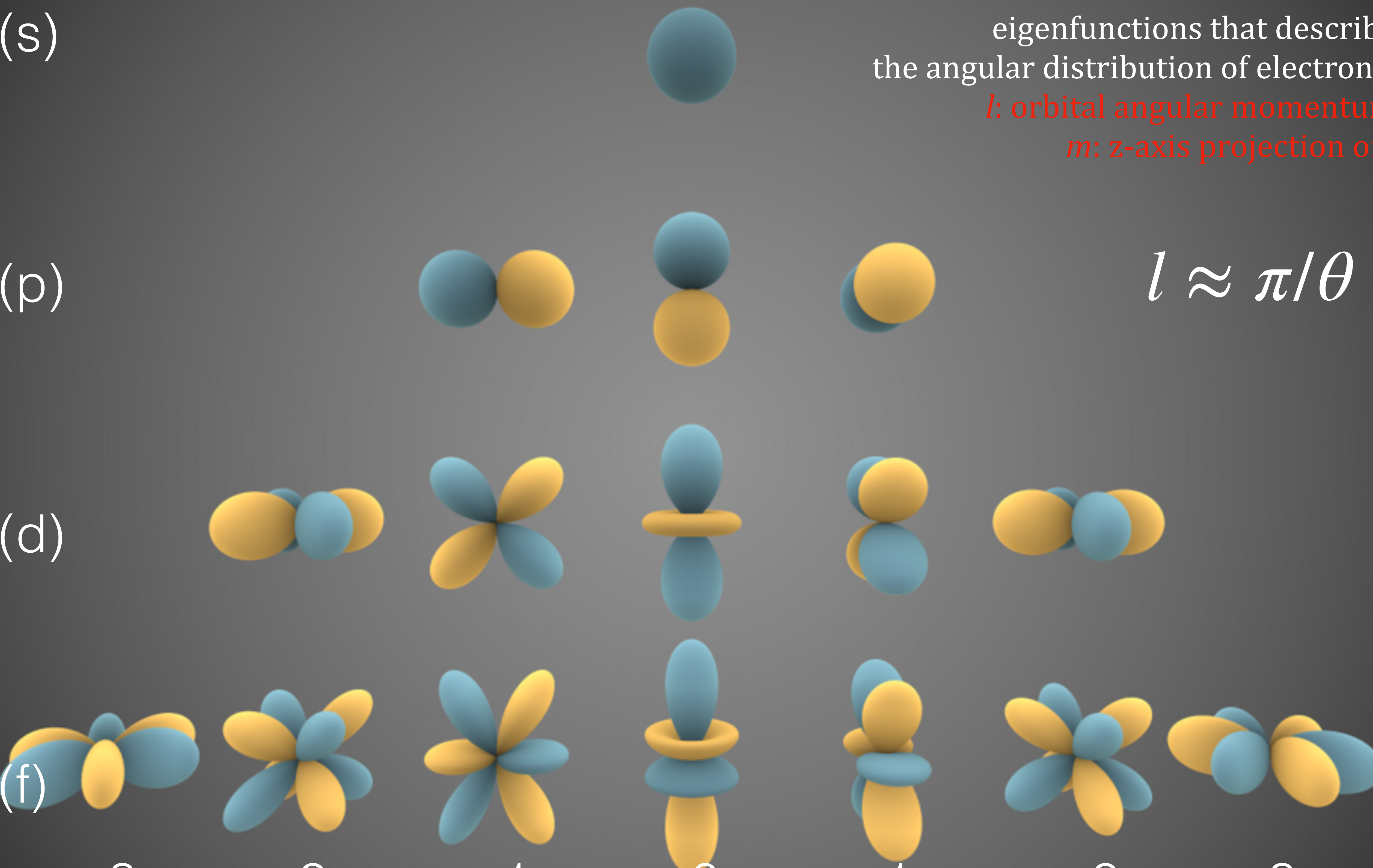
$l =$

0 (s)

1 (p)

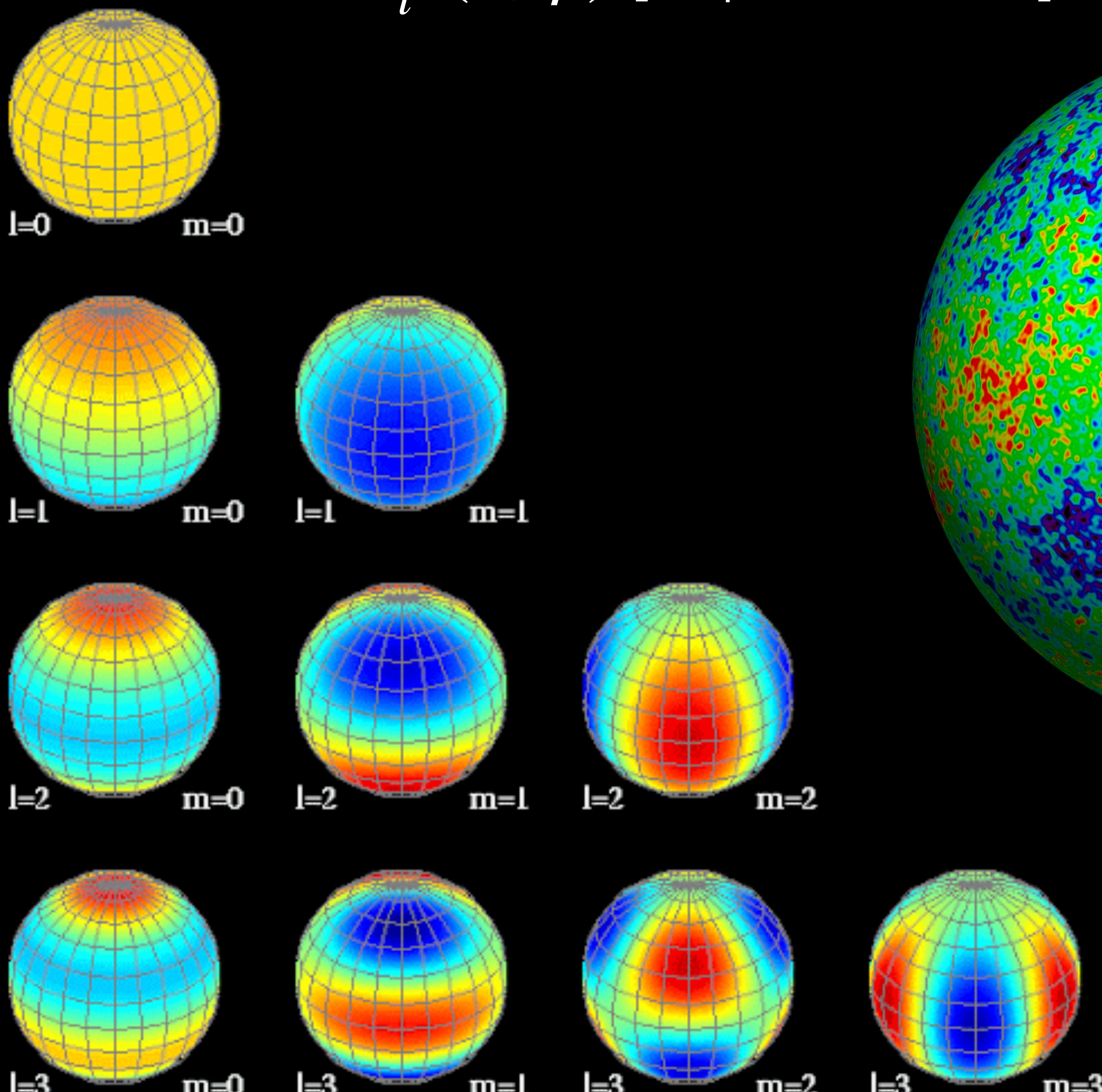
2 (d)

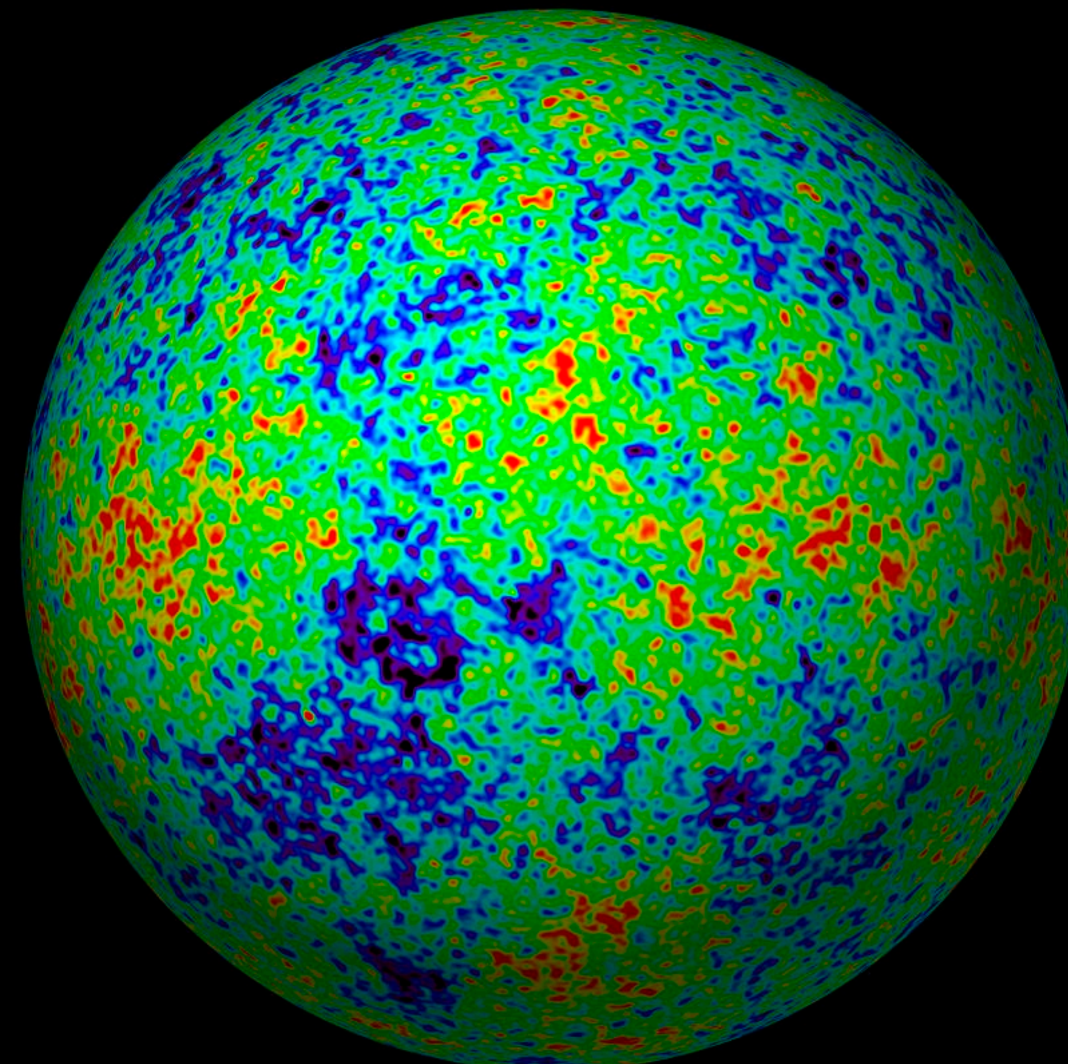
3 (f)

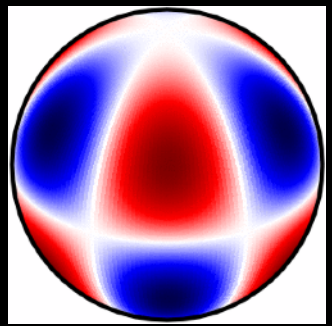


$m =$ 3 2 1 0 -1 -2 -3

Representing CMB anisotropies as a sum of spherical harmonics $Y_l^m(\theta, \phi)$ [Laplace 1782]

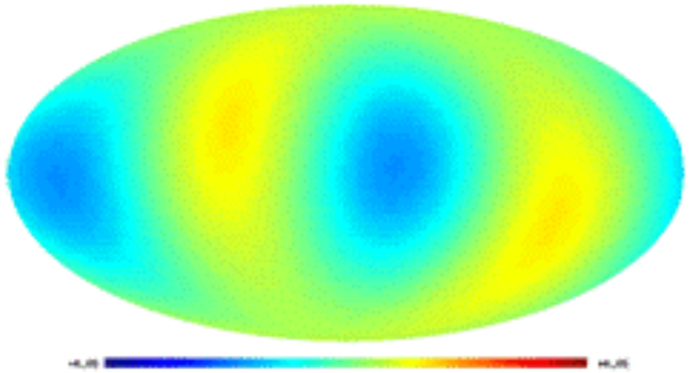




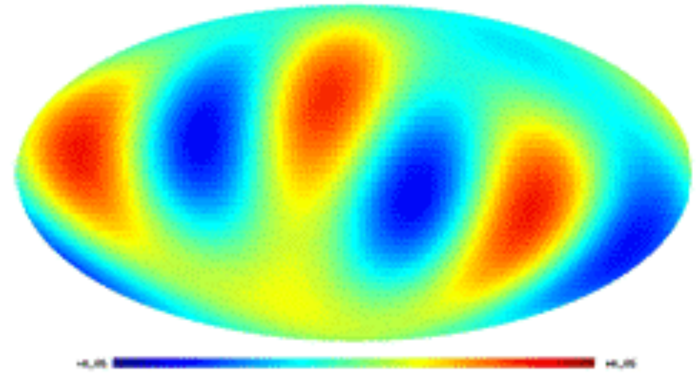


Spherical harmonics in Mollweide projection

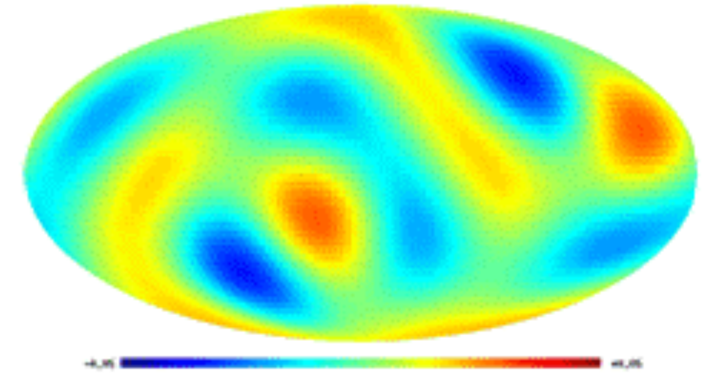
$m = l$



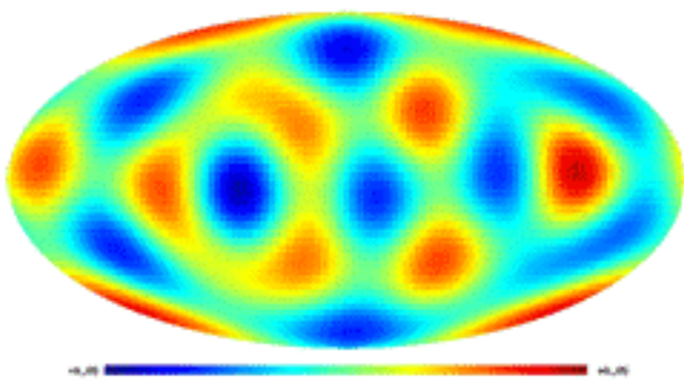
$\ell = 2$



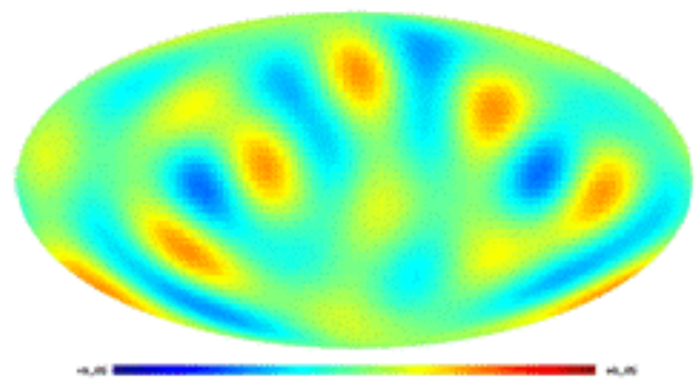
$\ell = 3$



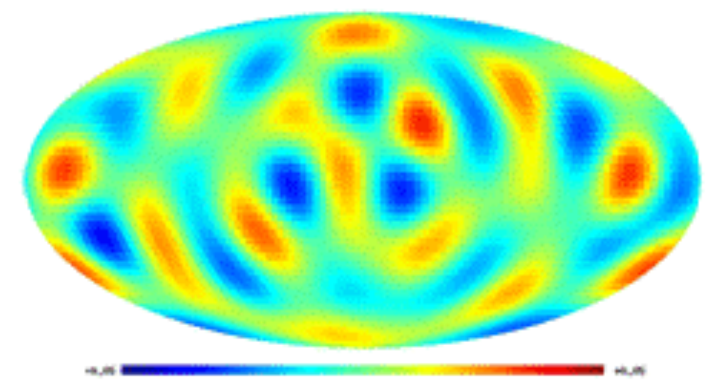
$\ell = 4$



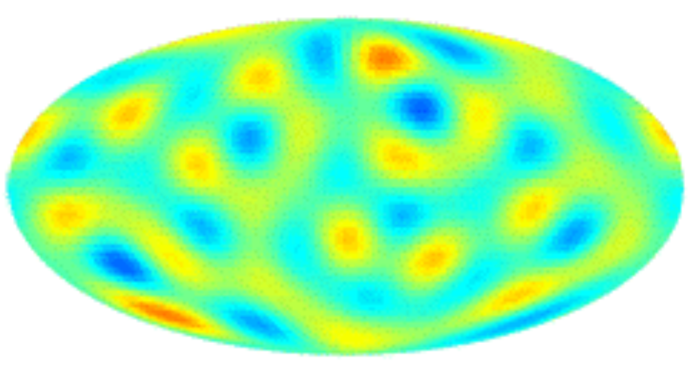
$\ell = 5$



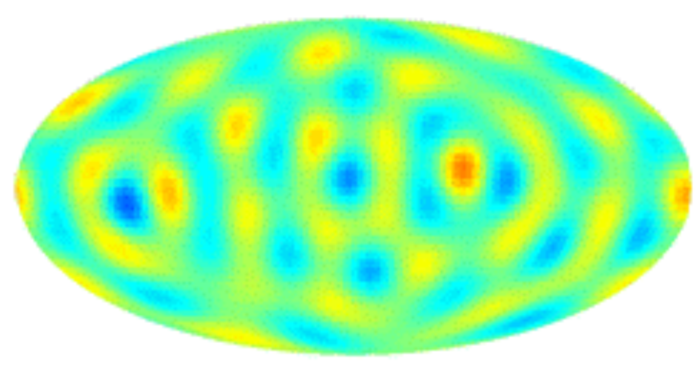
$\ell = 6$



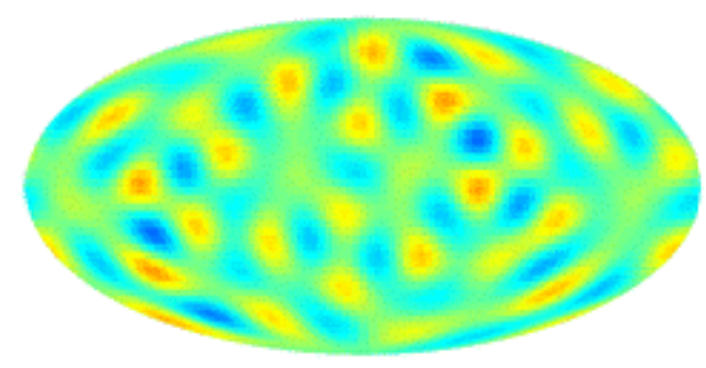
$\ell = 7$



$\ell = 8$

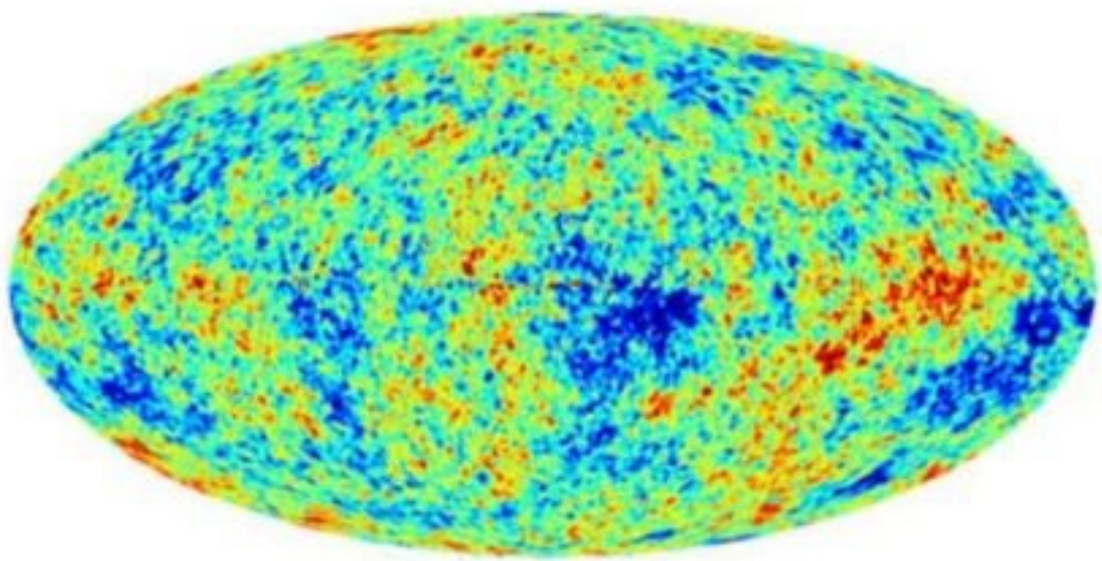


$\ell = 9$



$\ell = 10$

Expressing anisotropies as sum of spherical harmonics



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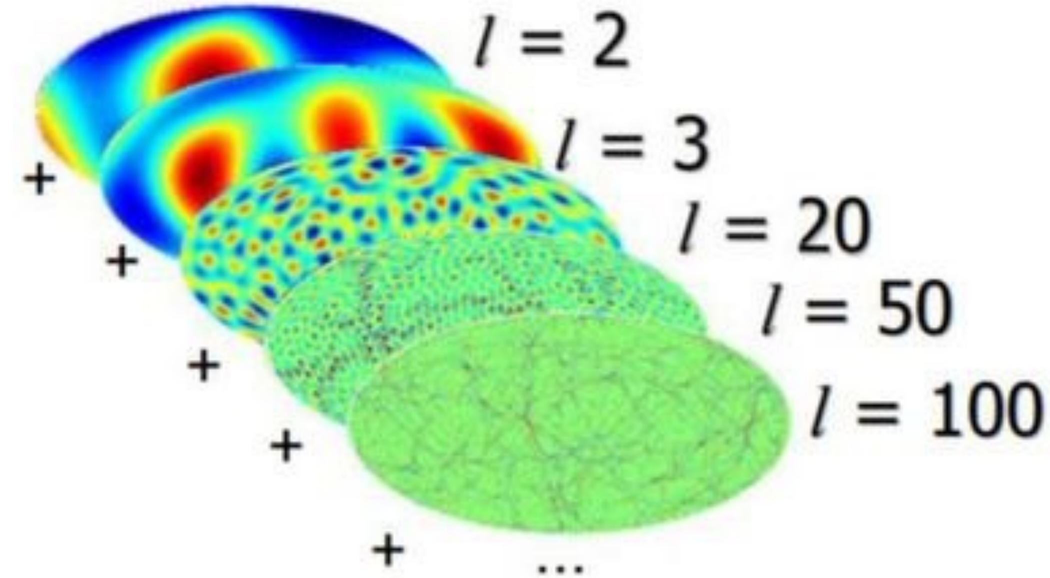


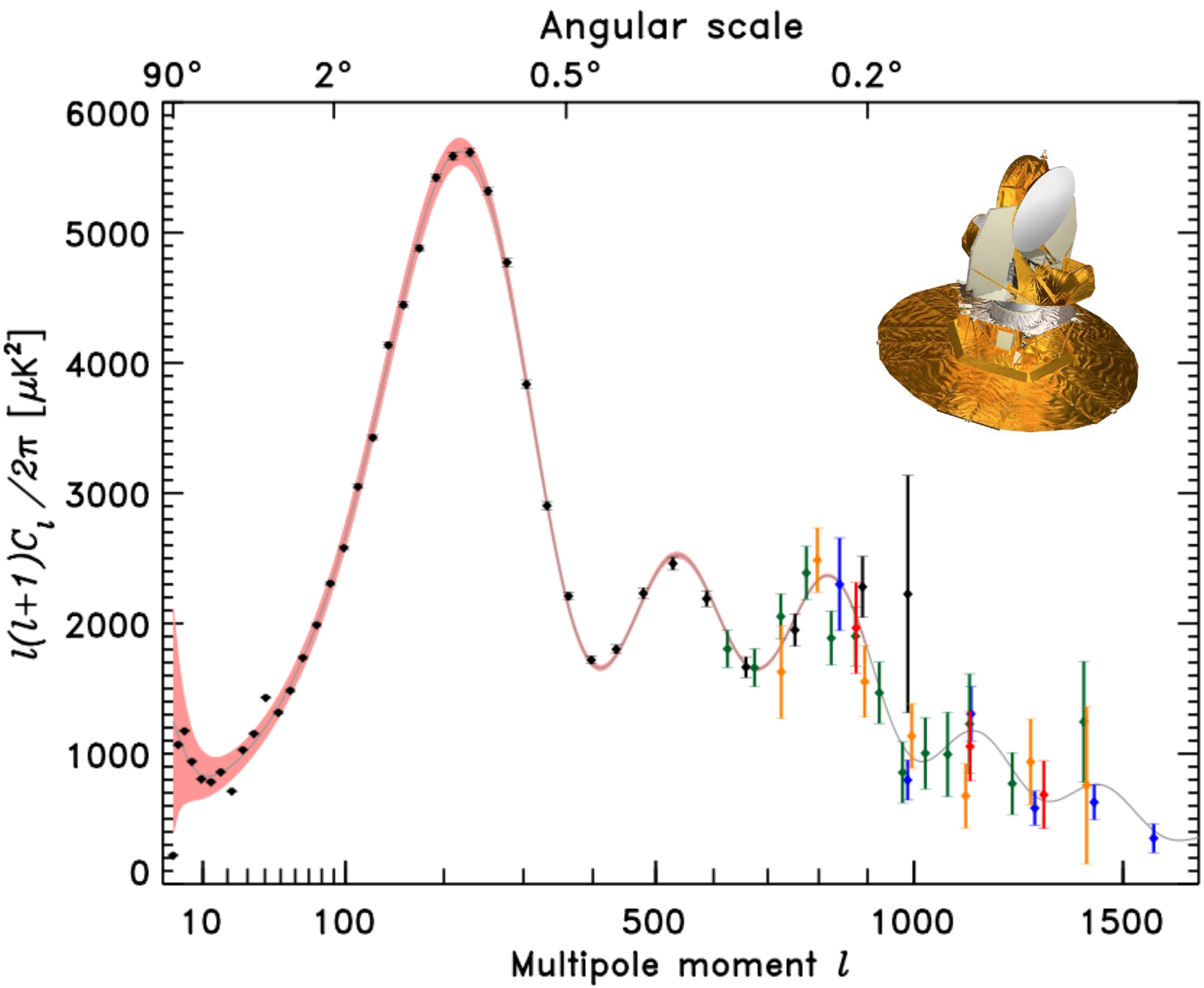
Harmonic Decomposition:

$$\delta_T(\theta,\phi) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{l,m} Y_l^m(\theta,\phi)$$

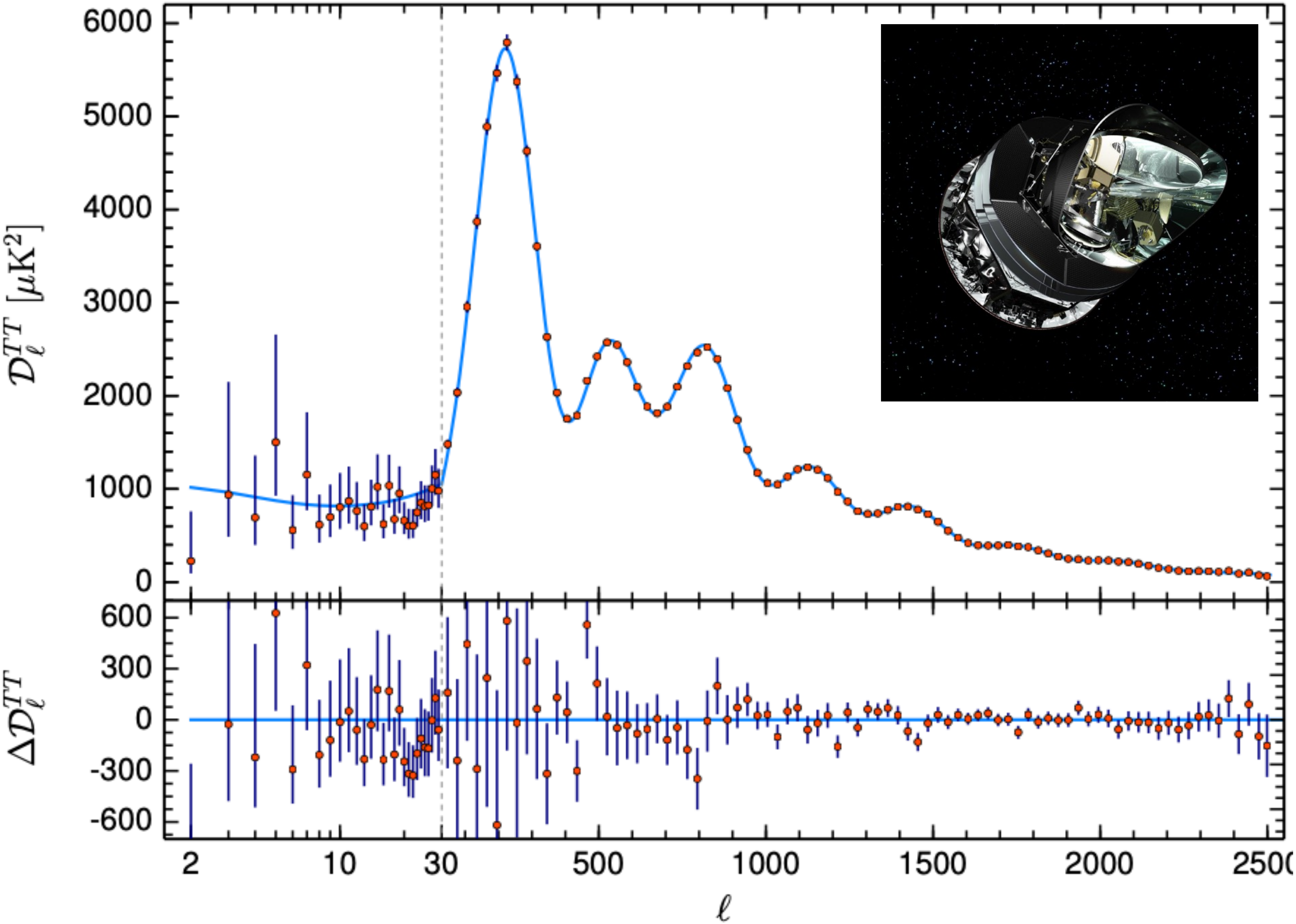
Temp. Power Spectrum:

$$P(l) = \frac{l(l+1)}{2\pi} C_l = \frac{l(l+1)}{2\pi}\frac{1}{2l+1}\sum_{m=-l}^{l} |a_{l,m}|^2$$

Power spectrum of CMB anisotropy (WMAP: launched in 2001)



Power spectrum of CMB anisotropy (Planck: launched in 2009)

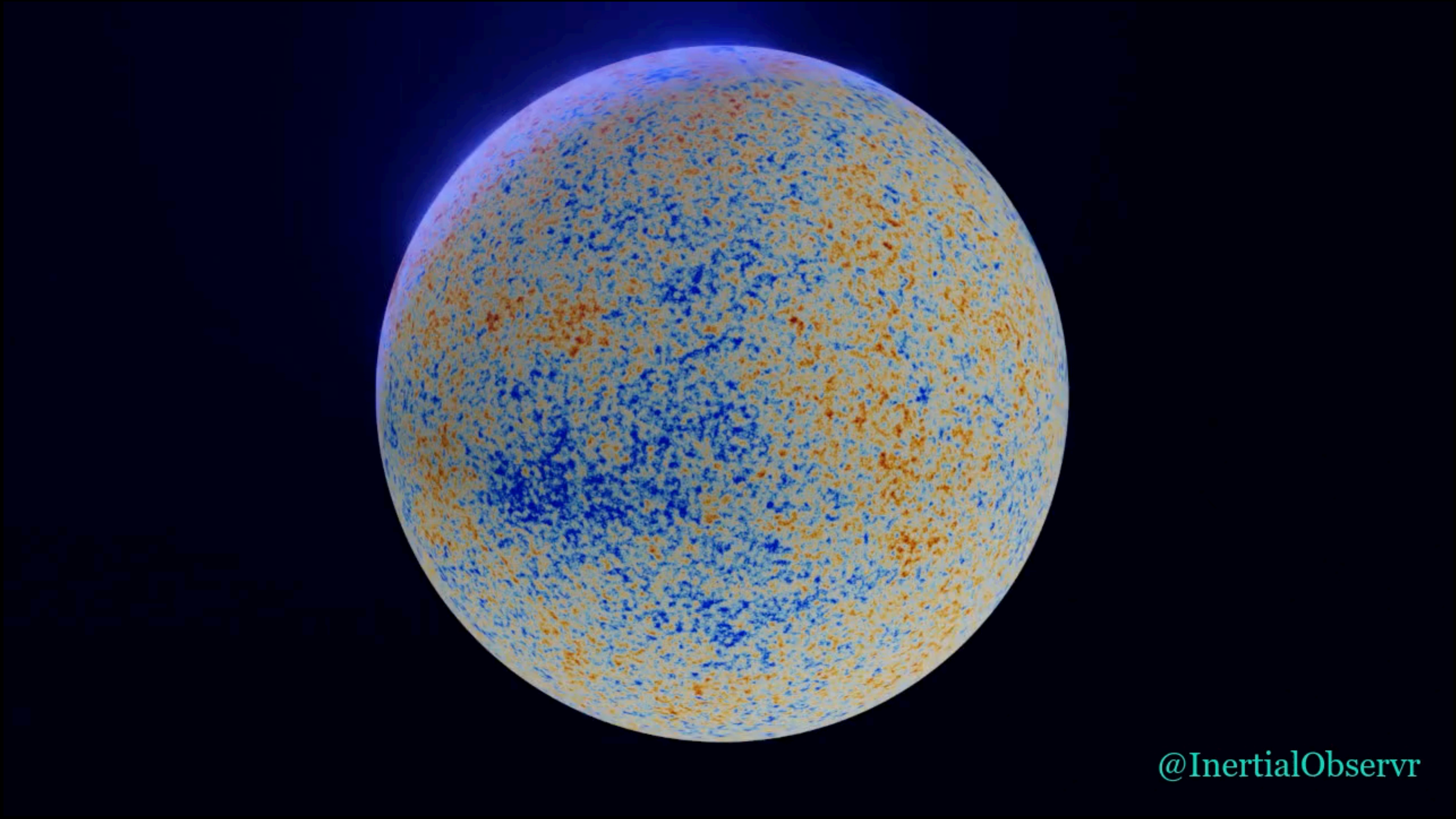


Constraints on Cosmological Parameters:

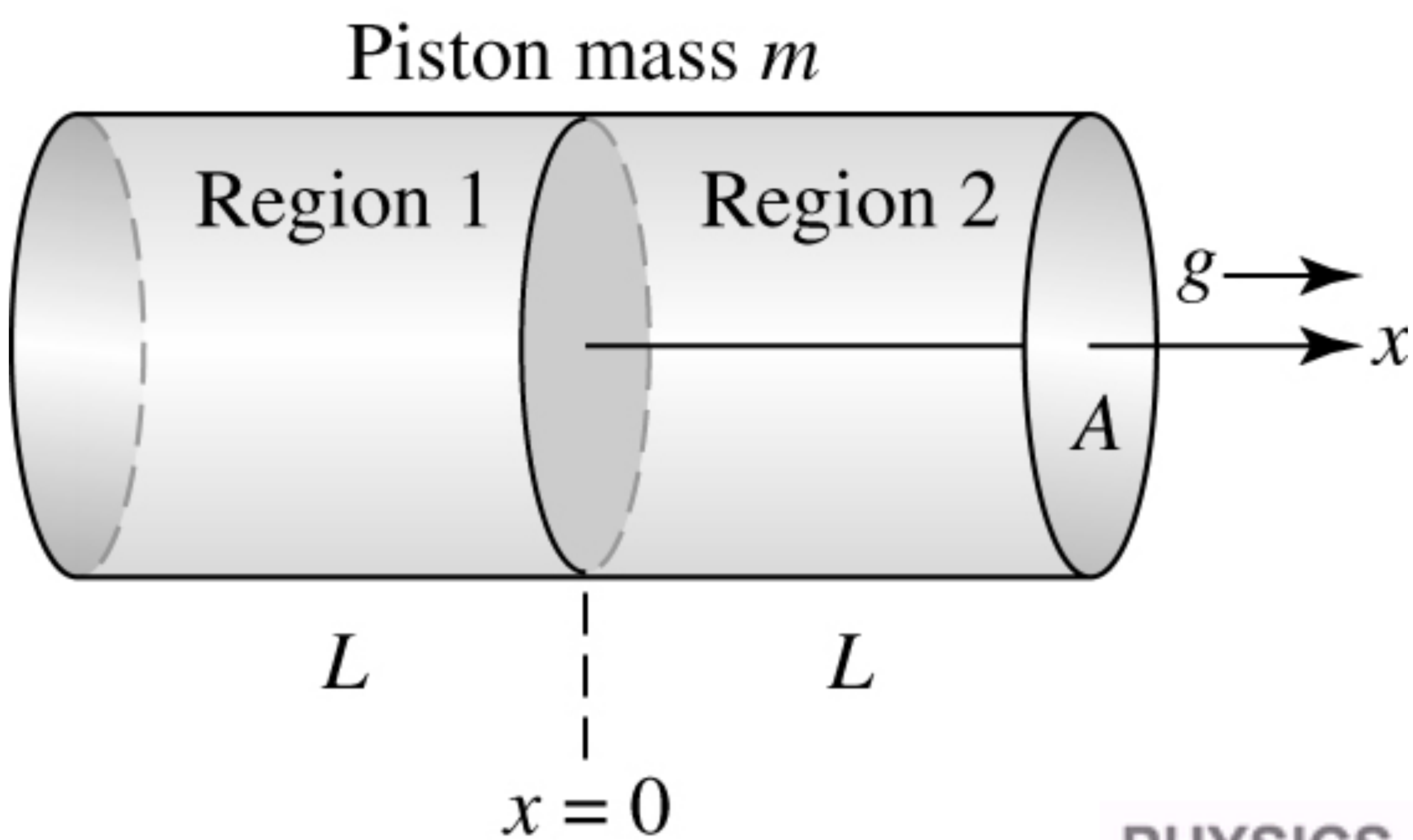
CMB Anisotropies

The cosmic harmonics frozen in time

"What makes the music of heaven?" - Chuang Tzu (300 BC)



Because overdensities of the baryon+photon fluid cannot collapse (Jeans length > Horizon size), they undergo acoustic oscillations



Ideal Gas Solution

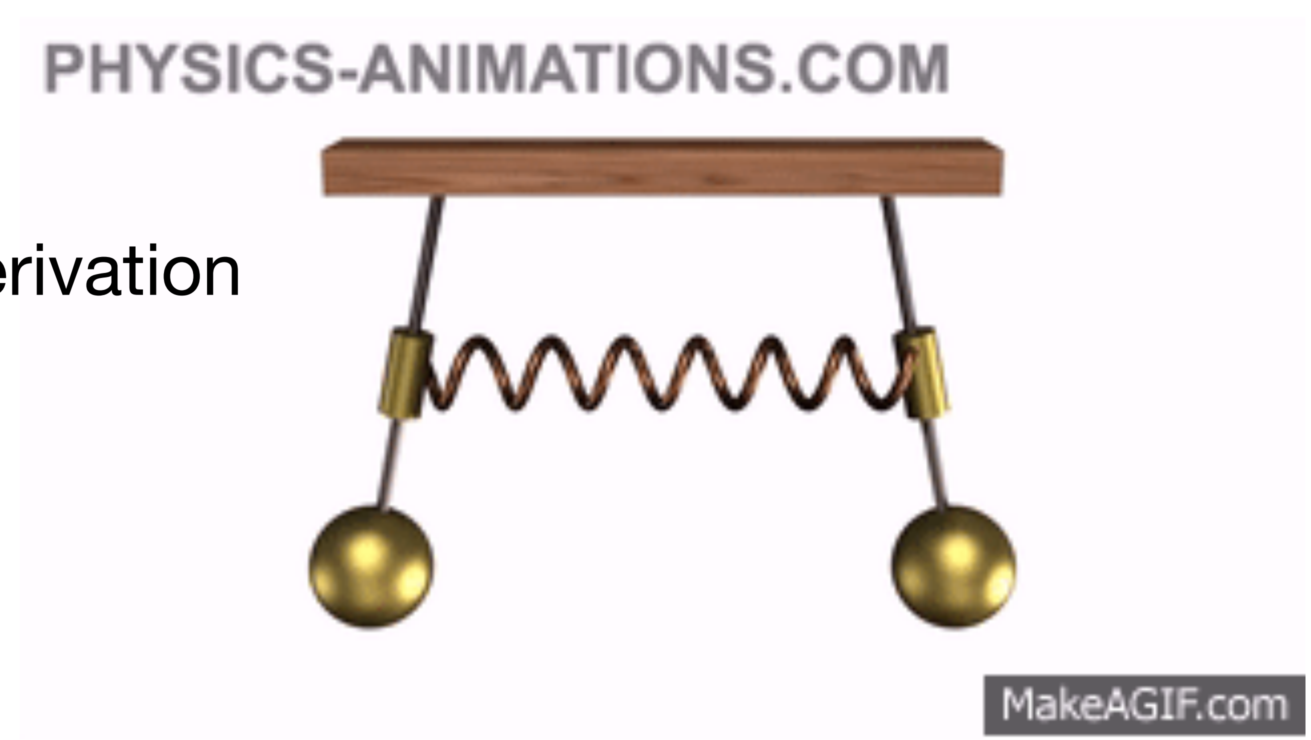
$$x(t) = x_0 \sin(c_s t/L)$$

$$\tau = 2\pi L/c_s$$

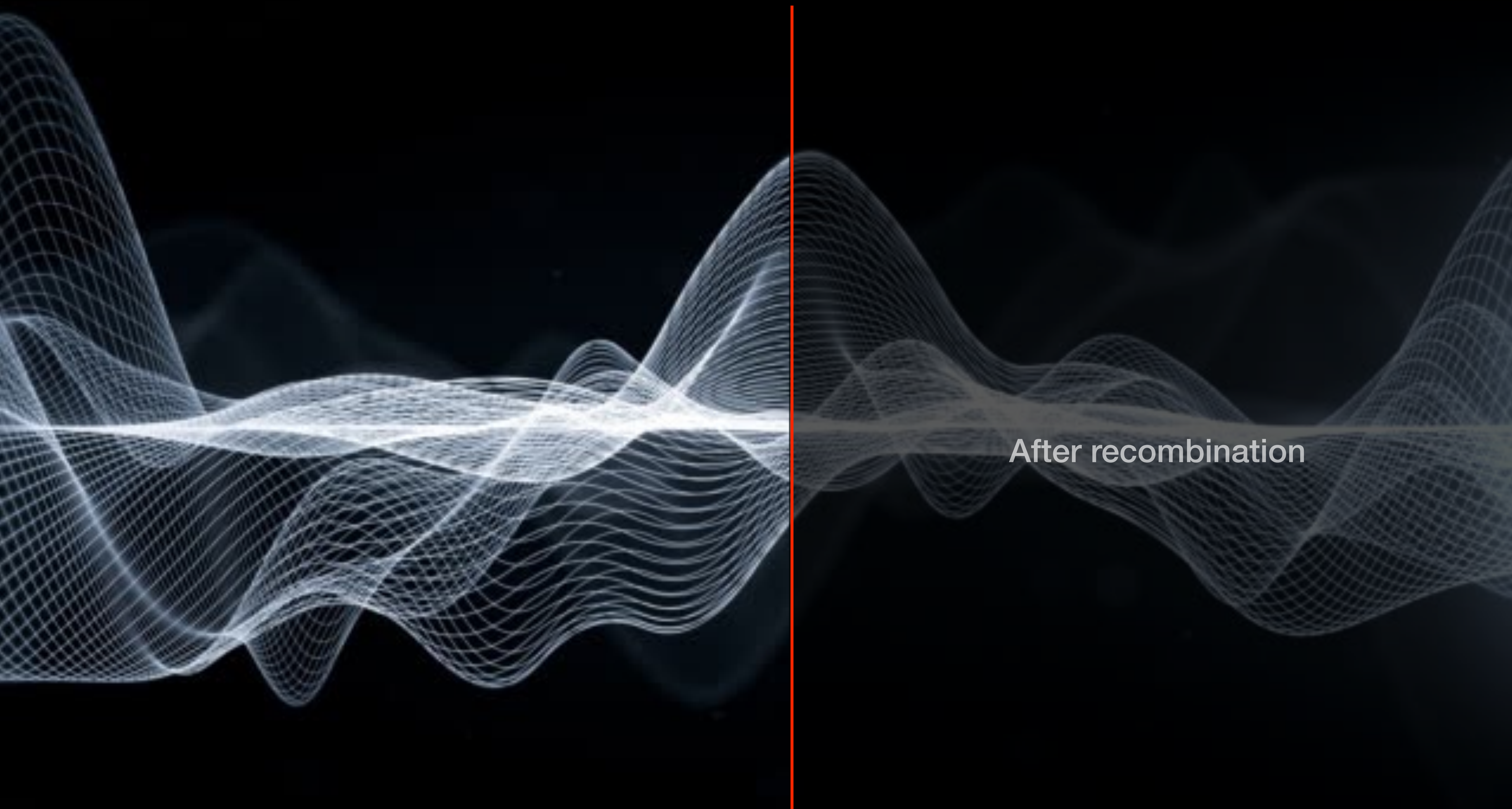
$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{\gamma k T}{\mu m_H}$$

$$\gamma = C_P/C_V$$

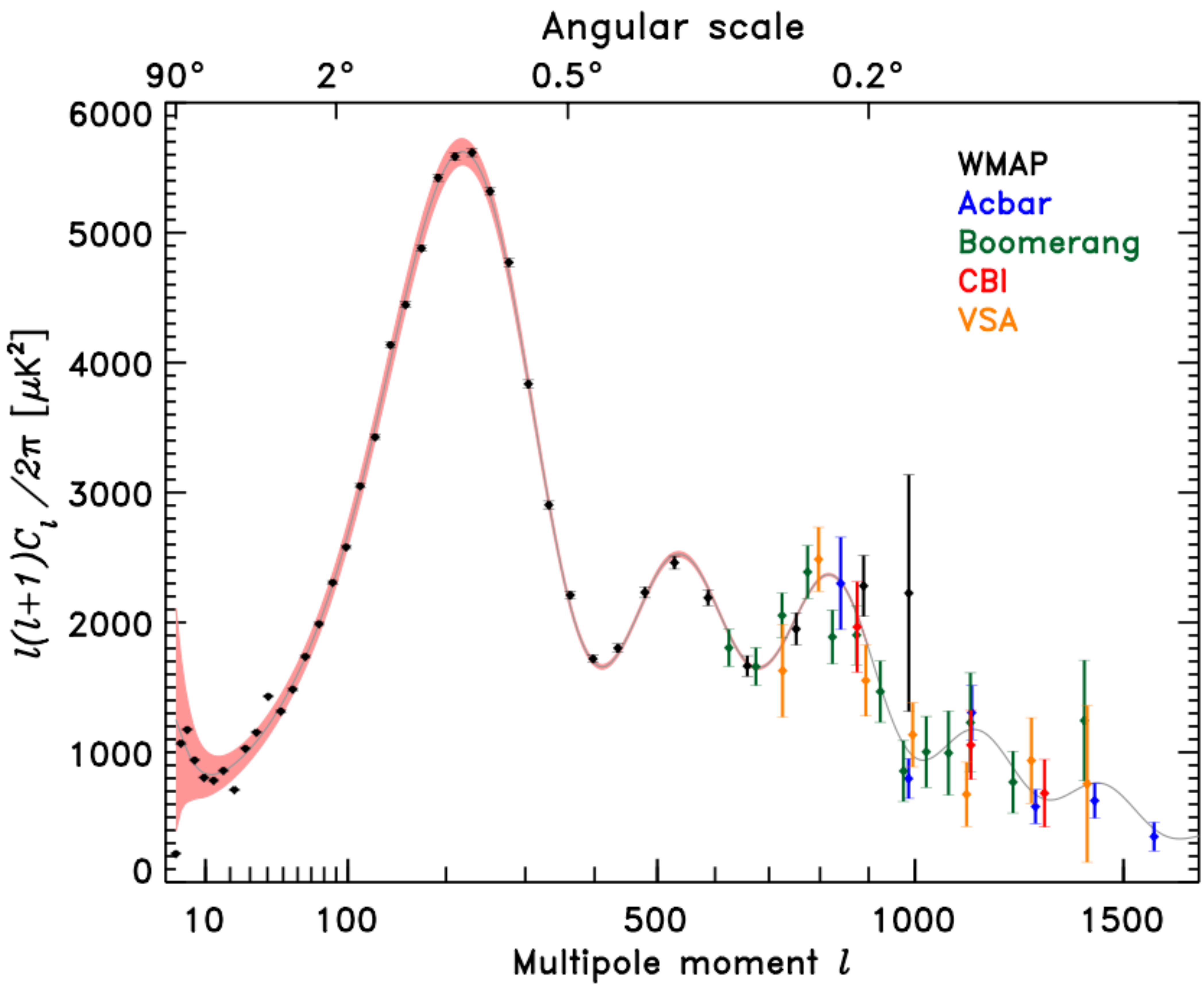
Simple gas cylinder + piston model derivation



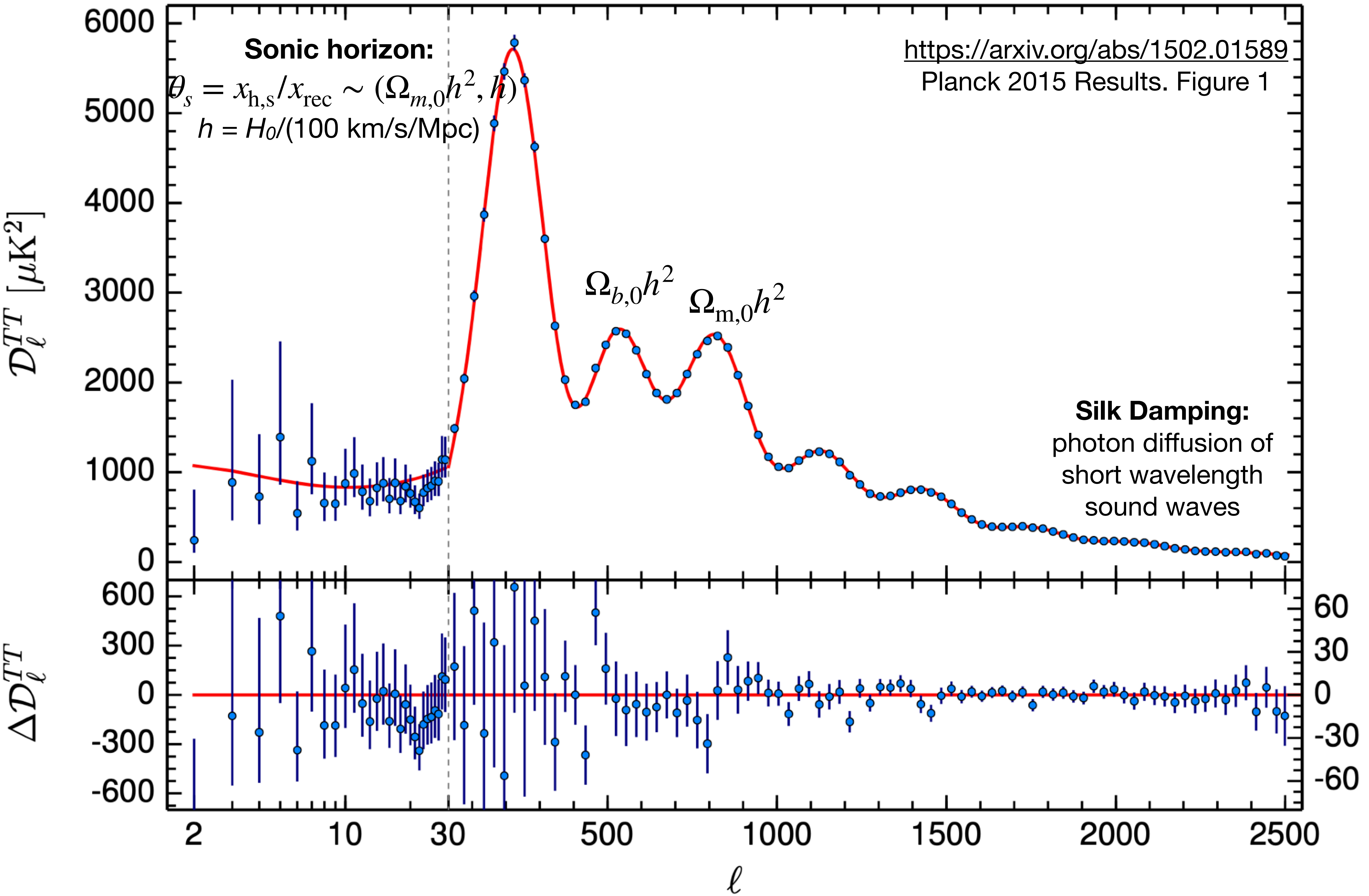
random acoustic oscillations frozen at recombination



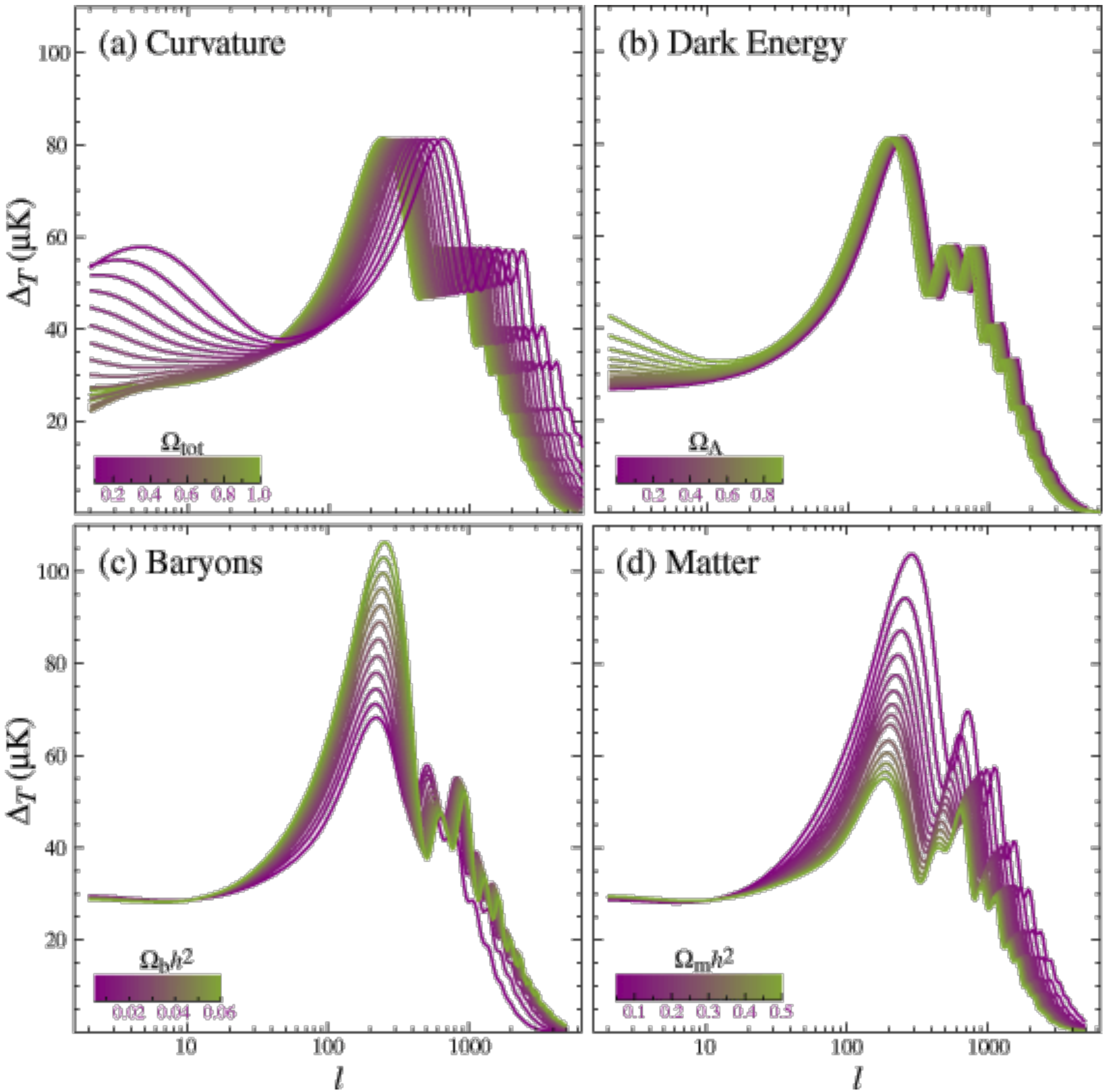
The Power Spectrum of the CMB from WMAP (2003)



The Power Spectrum of the CMB from Planck (2015)



The Power Spectrum: Sensitivities to Cosmological Parameters



Hu & Dodelson (2002)

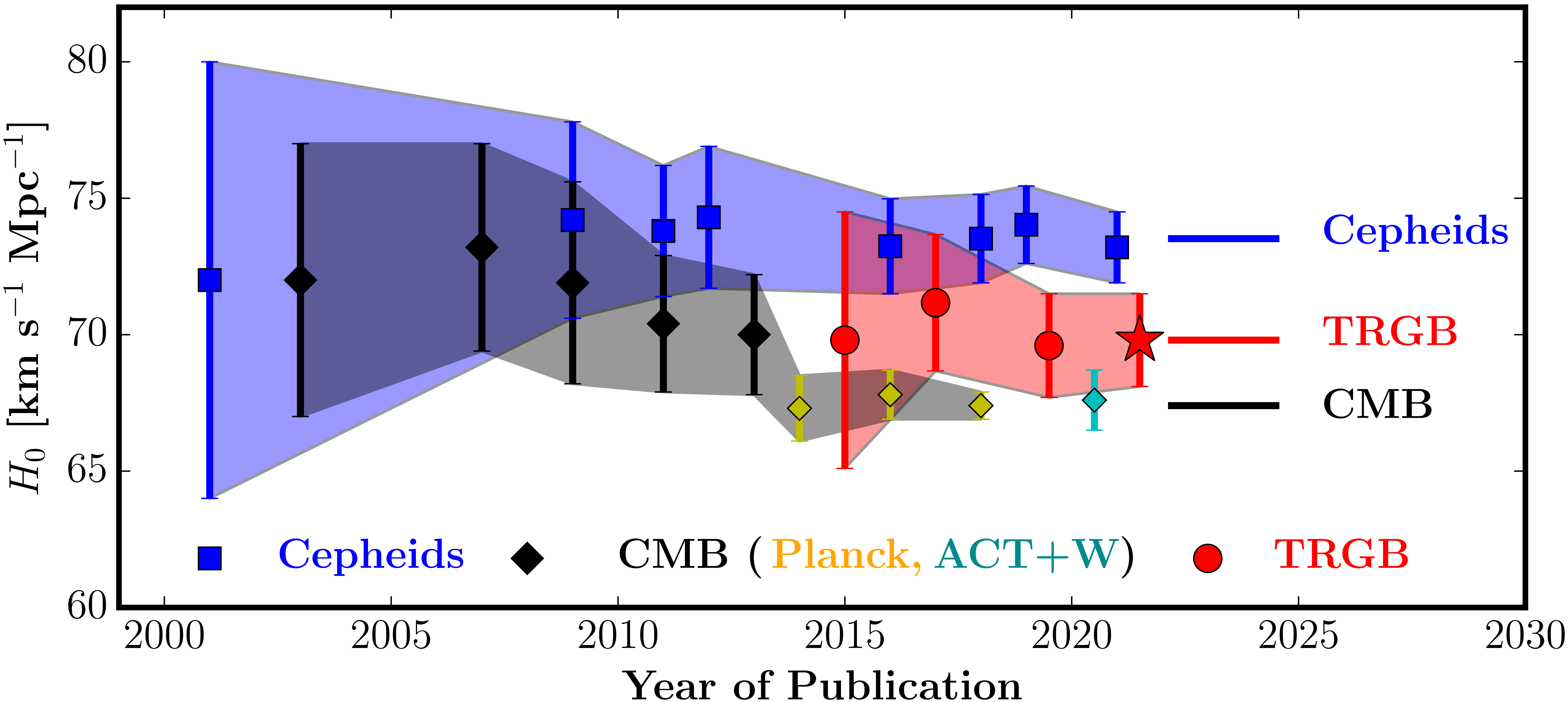
The Era of Precision Cosmology (1-2% errors)

Parameter	TT+lowP+lensing 68% limits	TT,TE,EE+lowP+lensing+ext 68% limits
n_s	0.9677 ± 0.0060	0.9667 ± 0.0040
H_0	67.81 ± 0.92	67.74 ± 0.46
Ω_Λ	0.692 ± 0.012	0.6911 ± 0.0062
Ω_m	0.308 ± 0.012	0.3089 ± 0.0062
$\Omega_b h^2$	0.02226 ± 0.00023	0.02230 ± 0.00014
$\Omega_c h^2$	0.1186 ± 0.0020	0.1188 ± 0.0010
σ_8	0.8149 ± 0.0093	0.8159 ± 0.0086
z_{re}	$8.8^{+1.7}_{-1.4}$	$8.8^{+1.2}_{-1.1}$
Age/Gyr	13.799 ± 0.038	13.799 ± 0.021

https://arxiv.org/abs/1502.01589
Planck 2015 Results. Table 4

The Tension between local and CMB measurements of H_0

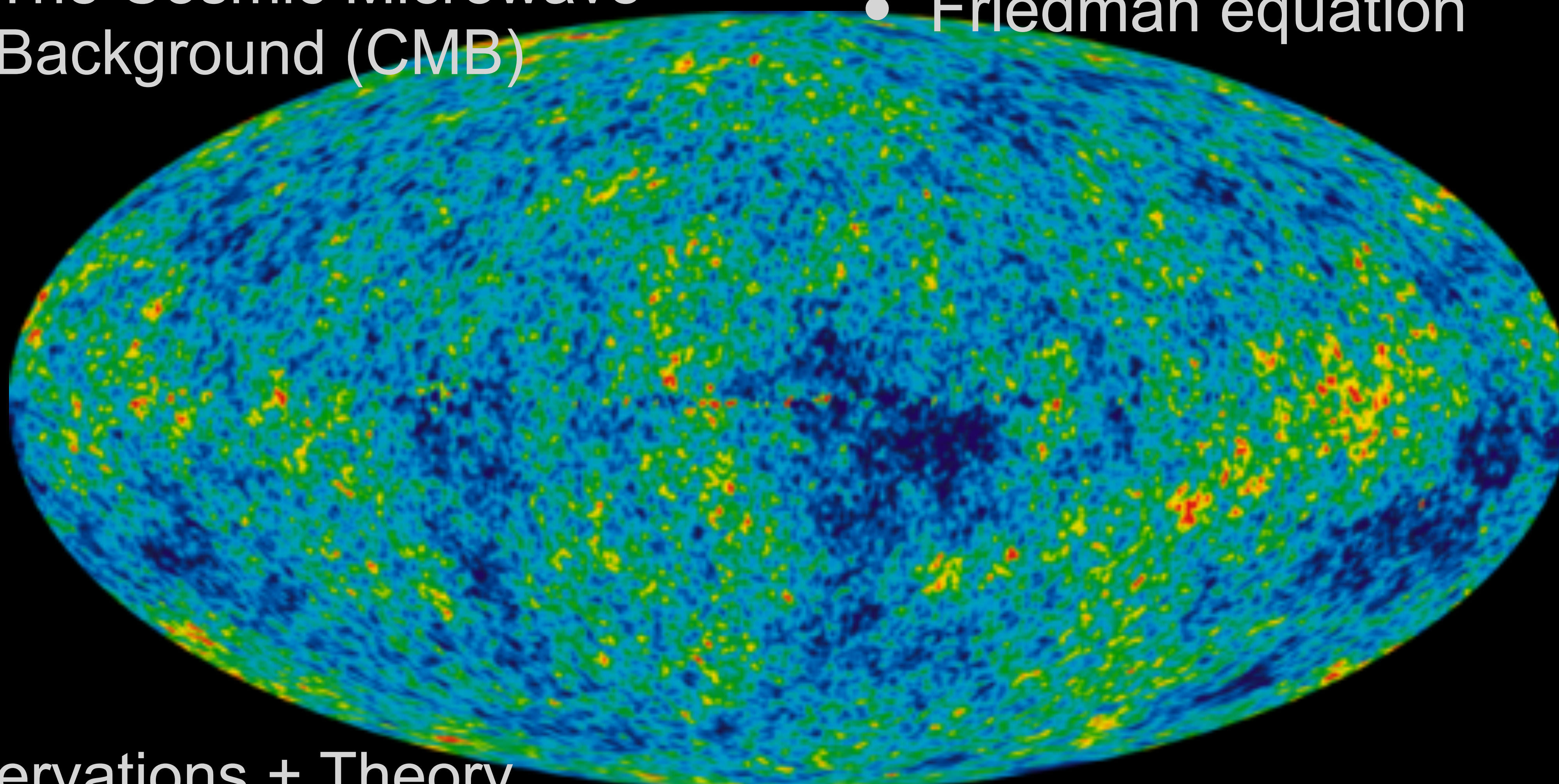
The tension between **68** and **73 km/s/Mpc** in $\boldsymbol{H_0}$
could be reconciled by small systematic errors of
0.154 in magnitude or **0.0001** in redshift



Freedman et al. (2019)

The Expanding Universe

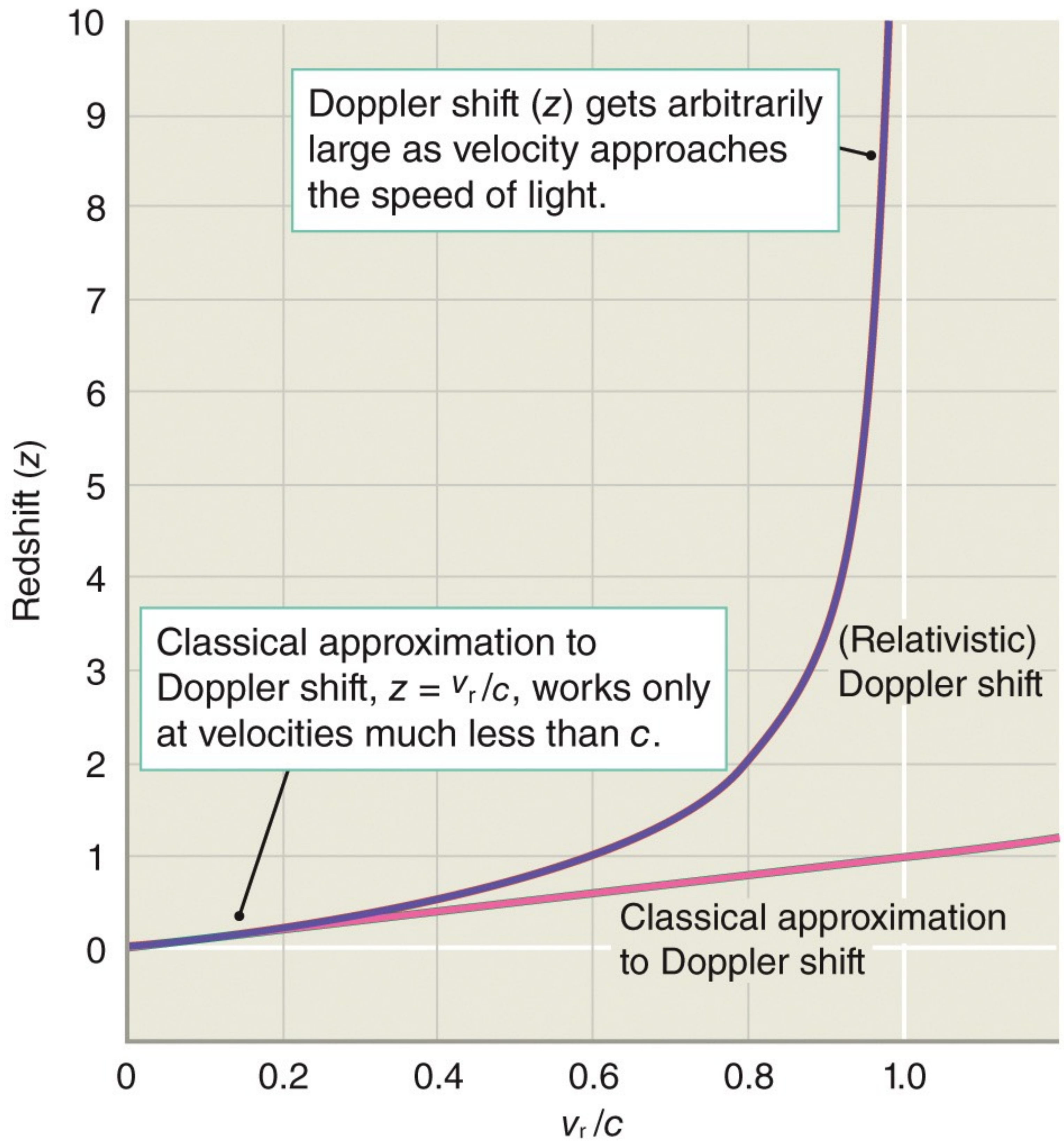
- Observations (facts)
  - Hubble-Lemaître Law & the Hubble "constant"
  - The Cosmic Microwave Background (CMB)
- Interpretations (theories)
  - The cosmological principle
  - Robertson-Walker metric
  - Friedman equation



- Observations + Theory
  - Accelerating Expansion: Evidence of dark energy
  - The cosmic composition
- Predictions of the Big Bang theory: how everything began?

What is redshift? Classical vs. Relativistic Doppler Shift, and Scale Factors

- The **classical Doppler shift formula,**
$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_0} = 1 + \frac{v_r}{c}$$
gives recession velocity:
$$v_r = cz$$
- The **relativistic Doppler shift formula,**
$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_0} = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}}$$
gives recession velocity:
$$v_r = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$
- But, **cosmological redshift** should be understood as a ratio of **scale factors**:
$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_0} = \frac{1}{R_U(z)}$$



How to Solve the Friedmann Equation? H(z) solution

- The full GR version of the Friedmann (1922) Equation is:

$$H^2 \left[1 - \left(\frac{\rho_m}{\rho_c} + \frac{\rho_\gamma}{\rho_c} + \frac{\Lambda c^2}{8\pi G \rho_c}\right)\right] R_U^2 = -kc^2$$

where $\rho_c = \dfrac{3H^2}{8\pi G}$ is the **critical density** at redshift z and $H \equiv \dot{R}_U / R_U$ the **Hubble parameter** at time t or redshift z.

- Define Ω's as critical density ratios:
  - $\Omega_m \equiv \rho_m / \rho_c$, **ordinary matter** (baryons and dark matter)
  - $\Omega_\gamma \equiv \rho_\gamma / \rho_c$, **relativistic matter** (light and neutrinos)
  - $\Omega_\Lambda \equiv \Lambda c^2 / (8\pi G \rho_c)$, **cosmological constant** (dark energy)
- Replacing those, we have the final Friedmann Equation:

$$H^2 \left[1 - (\Omega_m + \Omega_\gamma + \Omega_\Lambda)\right] R_U^2 = -kc^2$$

- Apply the boundary condition today, the **Hubble parameter** is:

$$H^2 = \frac{H_0^2}{R_U^2}\left[(1 - \Omega_0) + \Omega_{m,0}/R_U + \Omega_{\gamma,0}/R_U^2 + \Omega_{\Lambda,0} R_U^2\right]$$

How to Solve the Friedmann Equation? t(z) or $R_u(t)$ solution

- Write down the Friedmann Equation with the boundary condition and replace Hubble parameter with scale factor, $H(t) \equiv \dot{R}_U/R_U$, we have

$$\left(\frac{1}{R_U}\frac{dR_U}{dt}\right)^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_0) + \Omega_{m,0}/R_U + \Omega_{\gamma,0}/R_U^2 + \Omega_{\Lambda,0}R_U^2]$$

- For simplicity, assume a **flat universe:** *k = 0 and* $\Omega_0 = 1$*.*
- Separate time and scale factor into two sides of the equation:

$$dt = \frac{1}{H_0}\frac{R_U dR_U}{\sqrt{\Omega_{m,0}R_U + \Omega_{\gamma,0} + \Omega_{\Lambda,0}R_U^4}}$$

- Integrating it from R_U=0 (i.e., t=0) to R_U = 1/(1+z) [i.e., t(z)], we can solve for the t(z) relation for *any given values of the density parameters*.
- For example, for **a matter-only flat universe (Einstein-de Sitter universe),** we have solved for both H(z) and t(z):

$$\Omega_0 = \Omega_{m,0} = 1, \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0$$

$$\Rightarrow H = H_0/R_U^{3/2} = H_0(1+z)^{3/2}$$

$$\Rightarrow t(z) = \frac{2}{3}t_H(1+z)^{-3/2}$$

Temperature-Redshift Relation of the Cosmic Background Radiation

- Wien's displacement law: $\lambda_{\text{peak}} = (2.9\ \text{mm K})/T$
- Cosmological redshift: $\lambda_{\text{obs}} = \lambda_{\text{emit}}(1+z)$
- Combining the above two, we have the relation the blackbody temperature when emitted and the observed temperature:
$$T_{\text{obs}} = T_{\text{emit}}/(1+z)$$
in other words, the observed temperature today is much lower than the original temperature due to cosmic expansion.

- The above equation cannot constrain the temperature when the cosmic radiation background first emerged. In fact, infinite number of (T_{emit}, z) combinations could give us the same 3 K observed temperature, for examples:
  - T_{emit} = 10 K, z = 2.3
  - T_{emit} = 100 K, z = 32
  - T_{emit} = 1000 K, z = 330
- We call that **the two parameters (T_{emit}, z)** are **degenerate.**