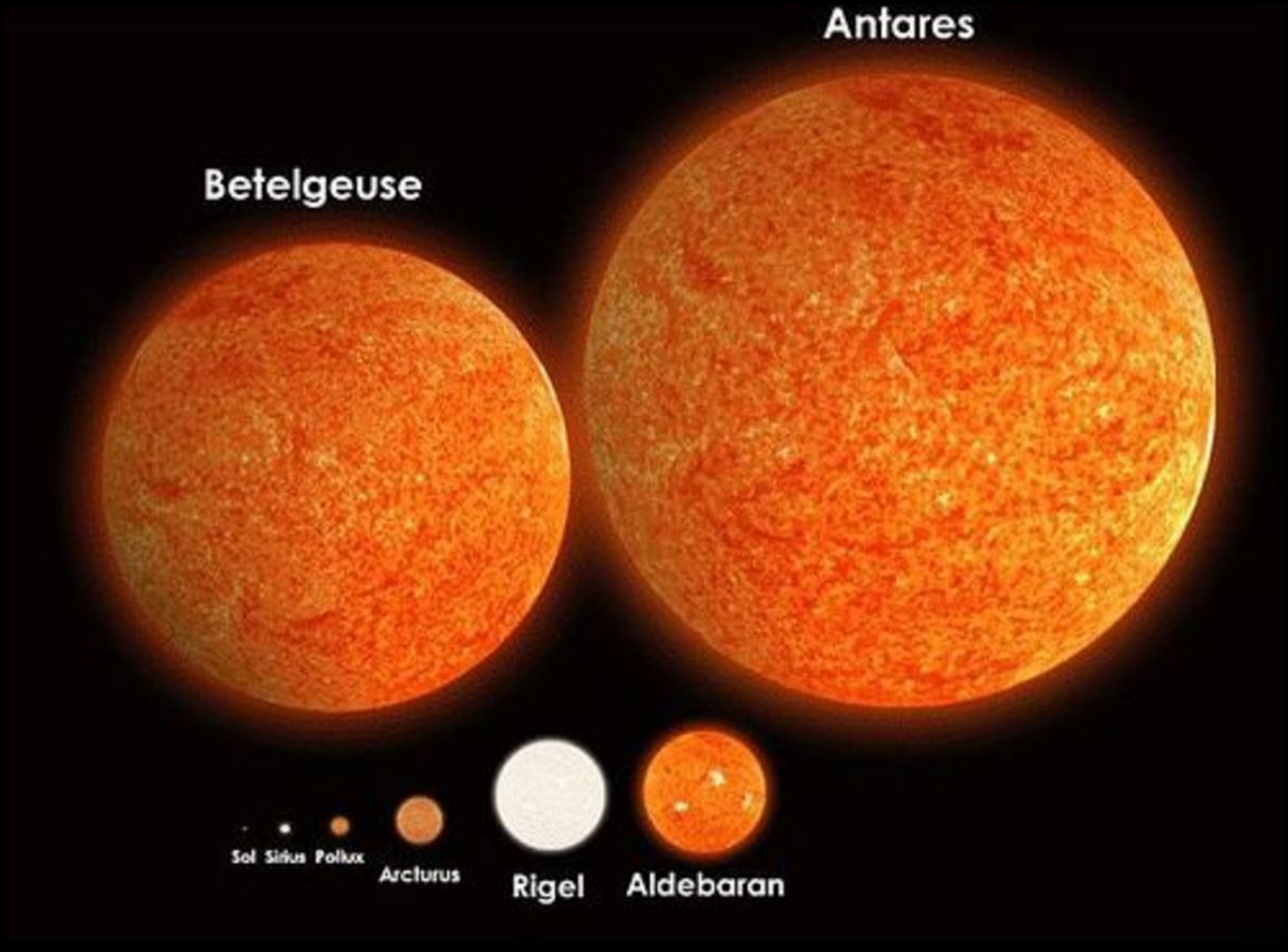
Chap 13: Taking the Measure of Stars



Chap 13: Taking the Measure of Stars

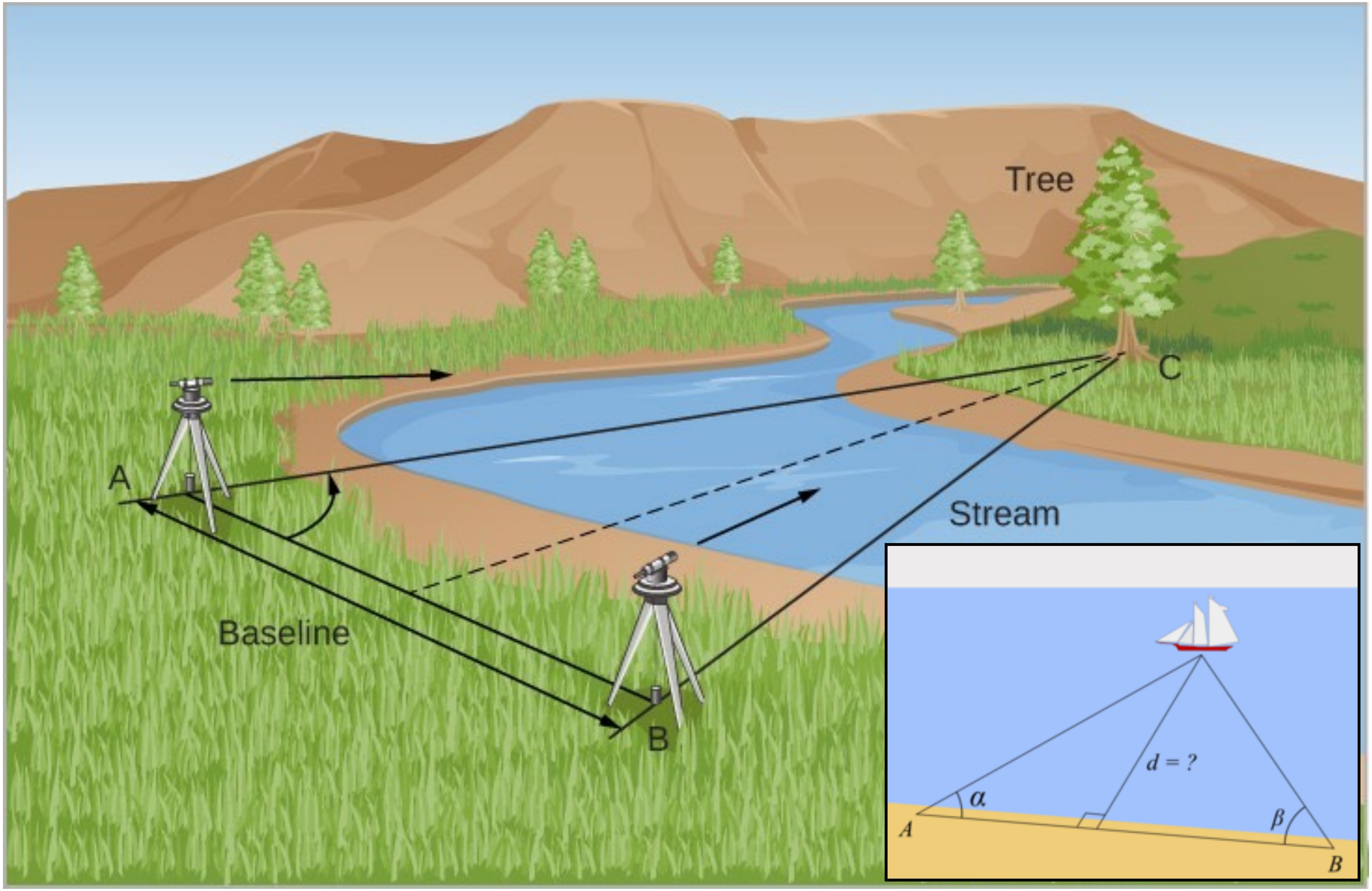
- How do we use parallax to determine distance? Astrometry.
- How do we measure brightness? Photometry.
- How do we combine distance (d) with brightness (apparent magnitude, m) to determine luminosity (absolute magnitude, M)?
- How do we measure temperature (T)? color index
- The Hertzsprung-Russell (H-R) diagram: M vs. color index
- Key concepts:
  - **parallax, magnitude system, distance modulus**
  - **H-R diagram and the distribution of stars on the diagram**
- Other measurements: size & mass of stars

Distance Measurements: Parallax

Geological Survey Method

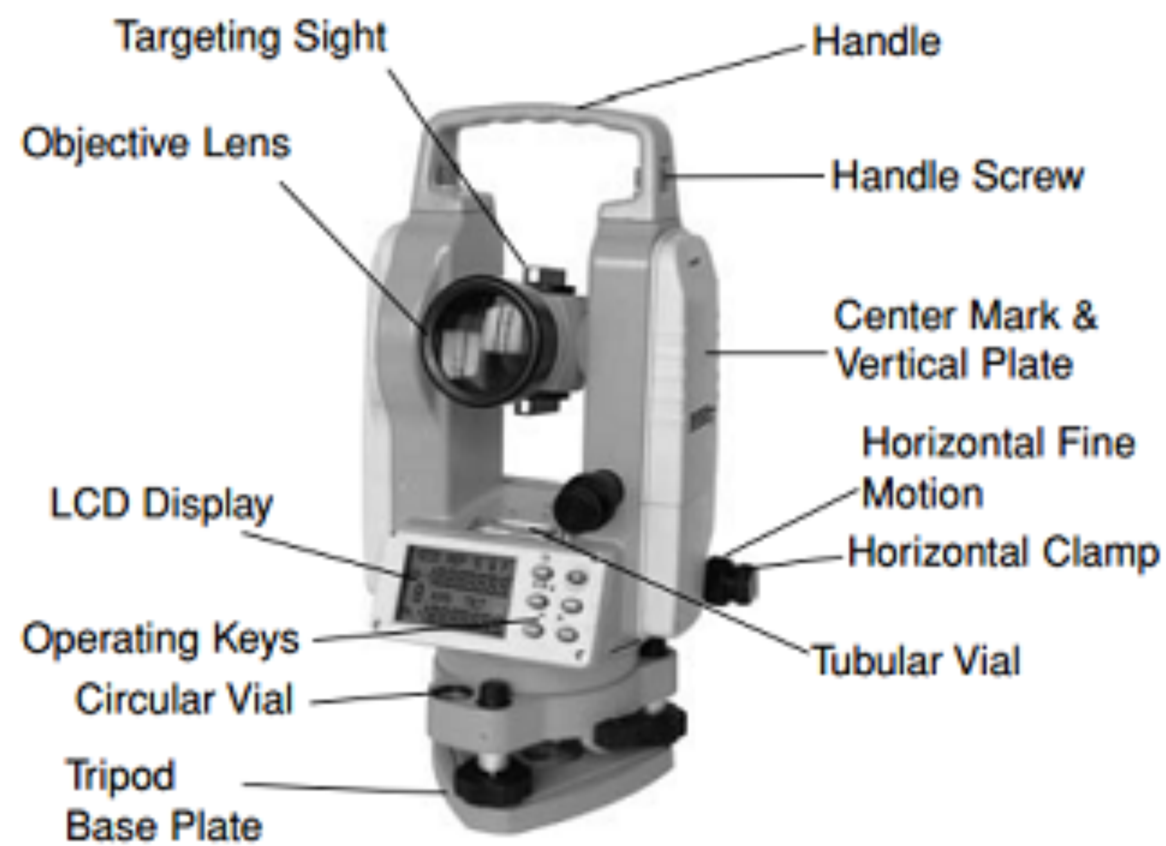
- Measurements of distance and elevation

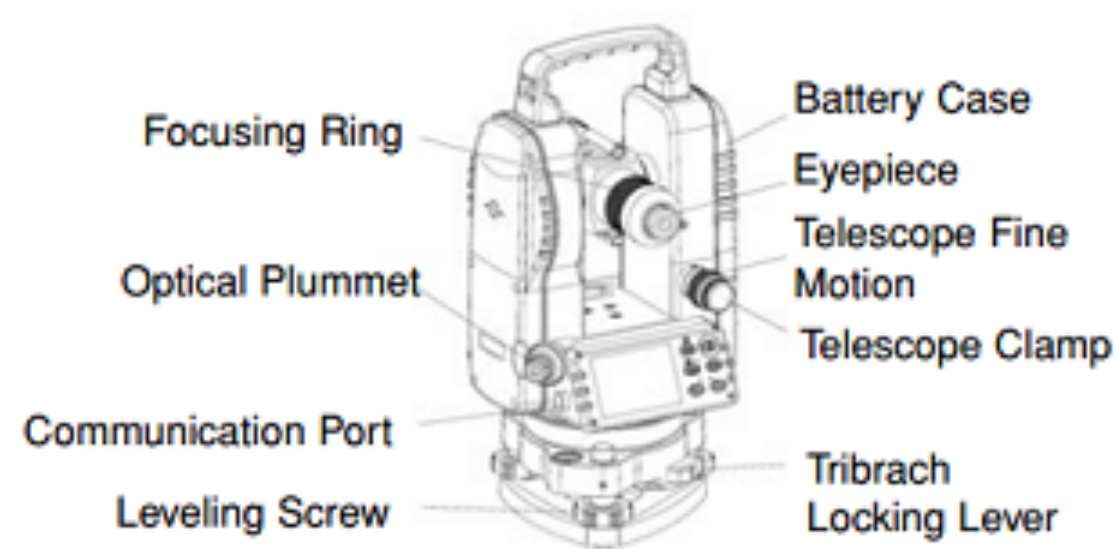
Geological Survey Method



Geological Survey Instrument: Theodolite

a surveying instrument with a rotating telescope for measuring horizontal and vertical angles.





Geological Survey Method: Theodolite measurements

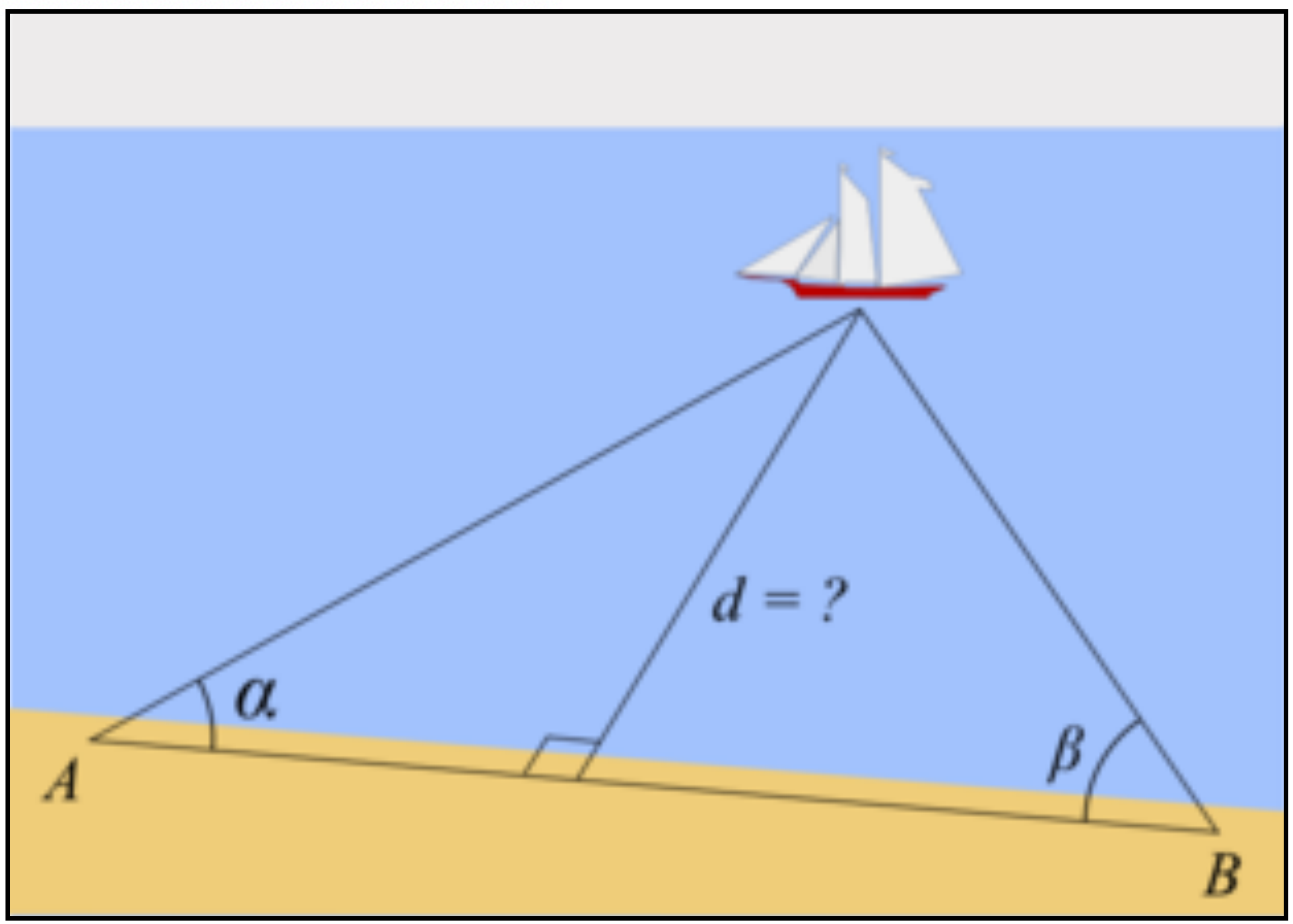
need to know the baseline length ($l = AB$) and the two angles (α, β)

$$\ell = d\left(\frac{\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta}\right)$$

$$\ell = d\,\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$$

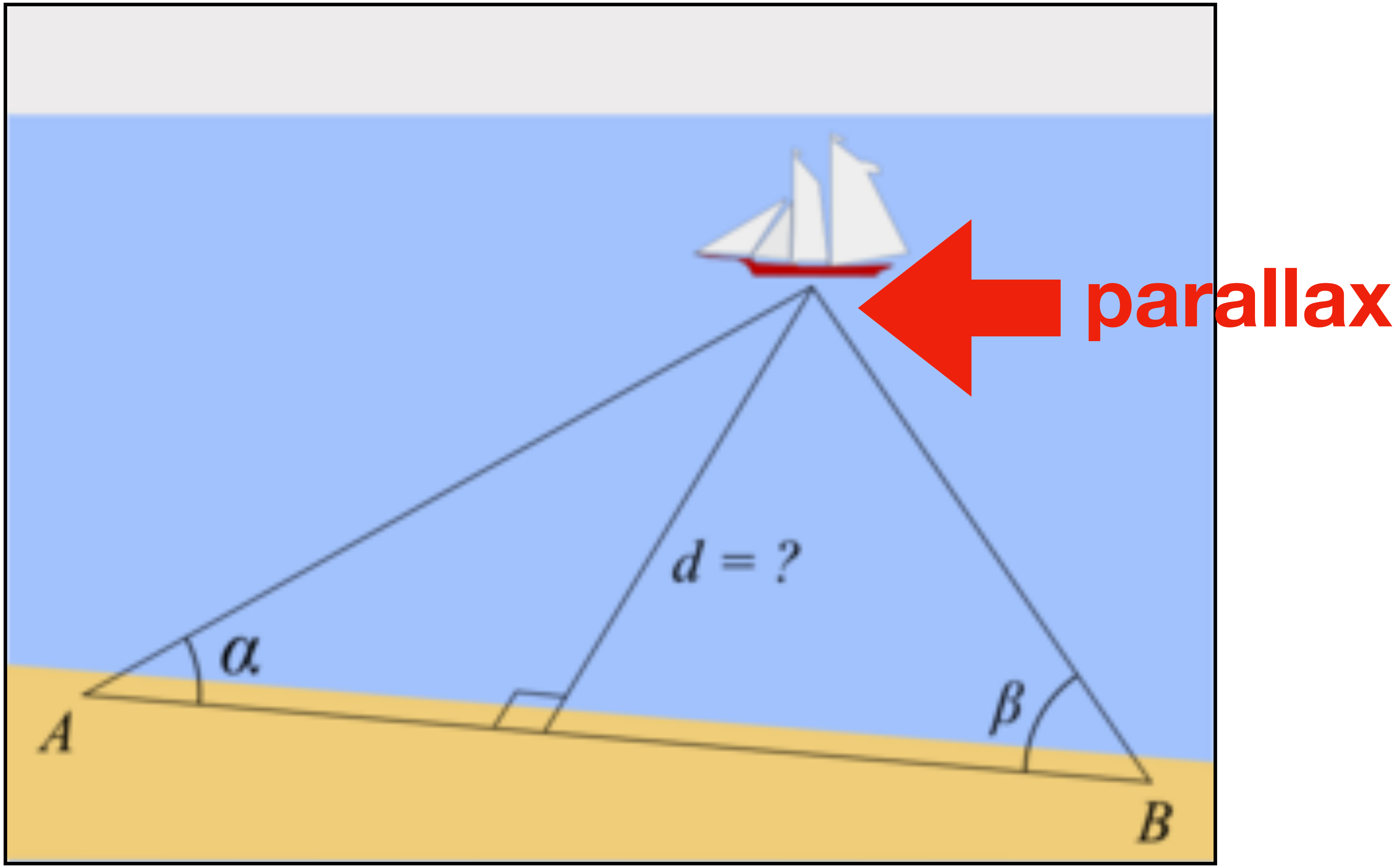
therefore:

$$d = \ell\,\frac{\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$$



Geological Survey Method: Theodolite measurements

What would the angles become when d is much much greater than AB?

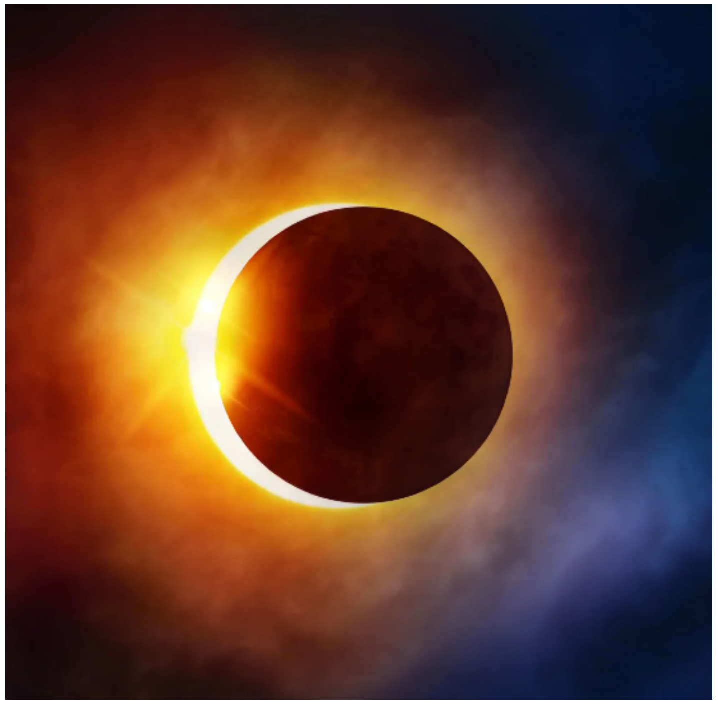


To measure greater distances, we need:
(1) longer baselines and (2) the ability to measure tiny angles

The Earliest Parallax Measurement by Hipparchus (~150 BC): Baseline limited by the diameter of the Earth

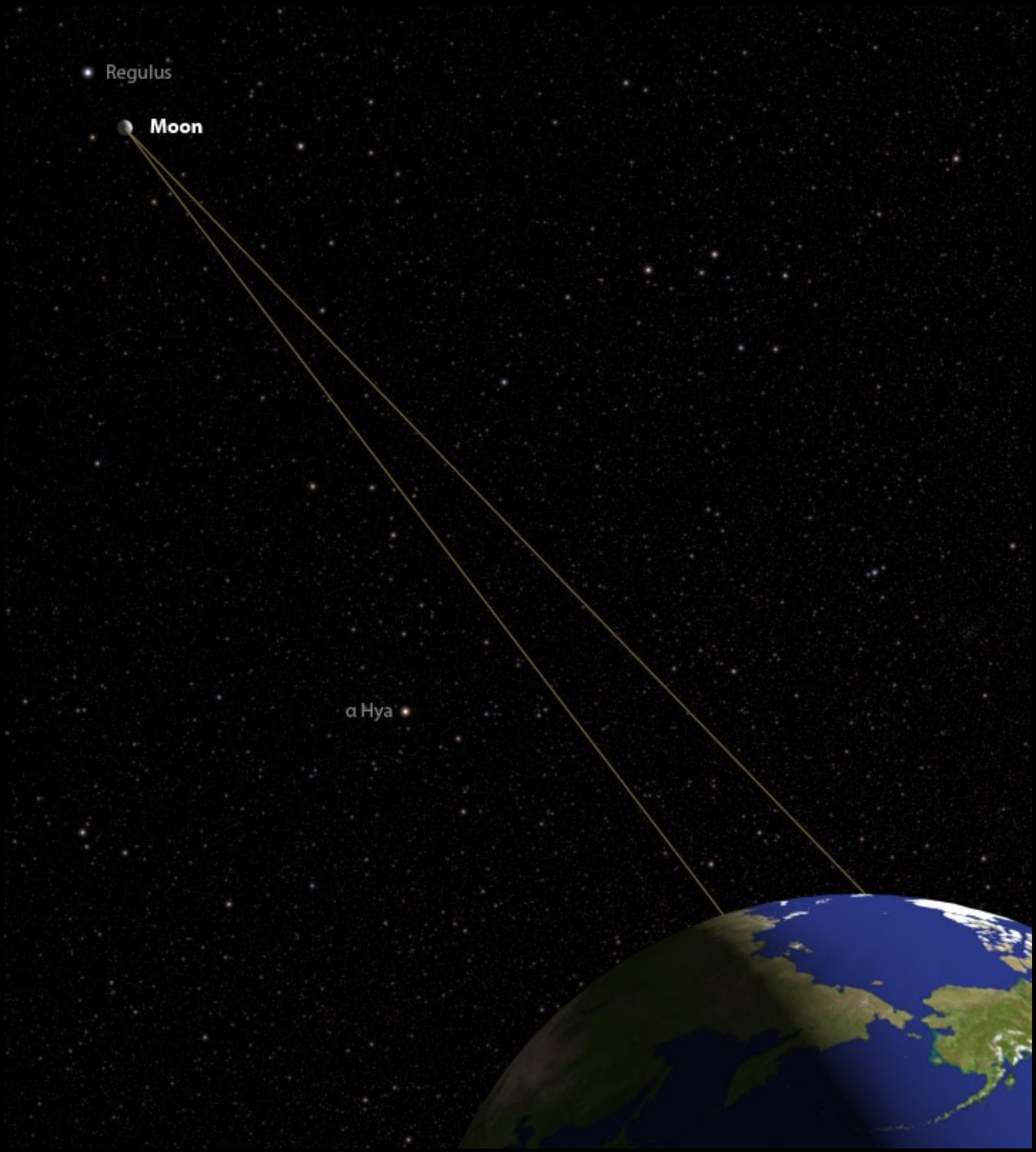
seen in Hellespont (100% obscured)

seen in Alexandria (80% obscured)



The Solar Eclipse on Mar 14, 190 BC

Parallax of the Moon using background stars

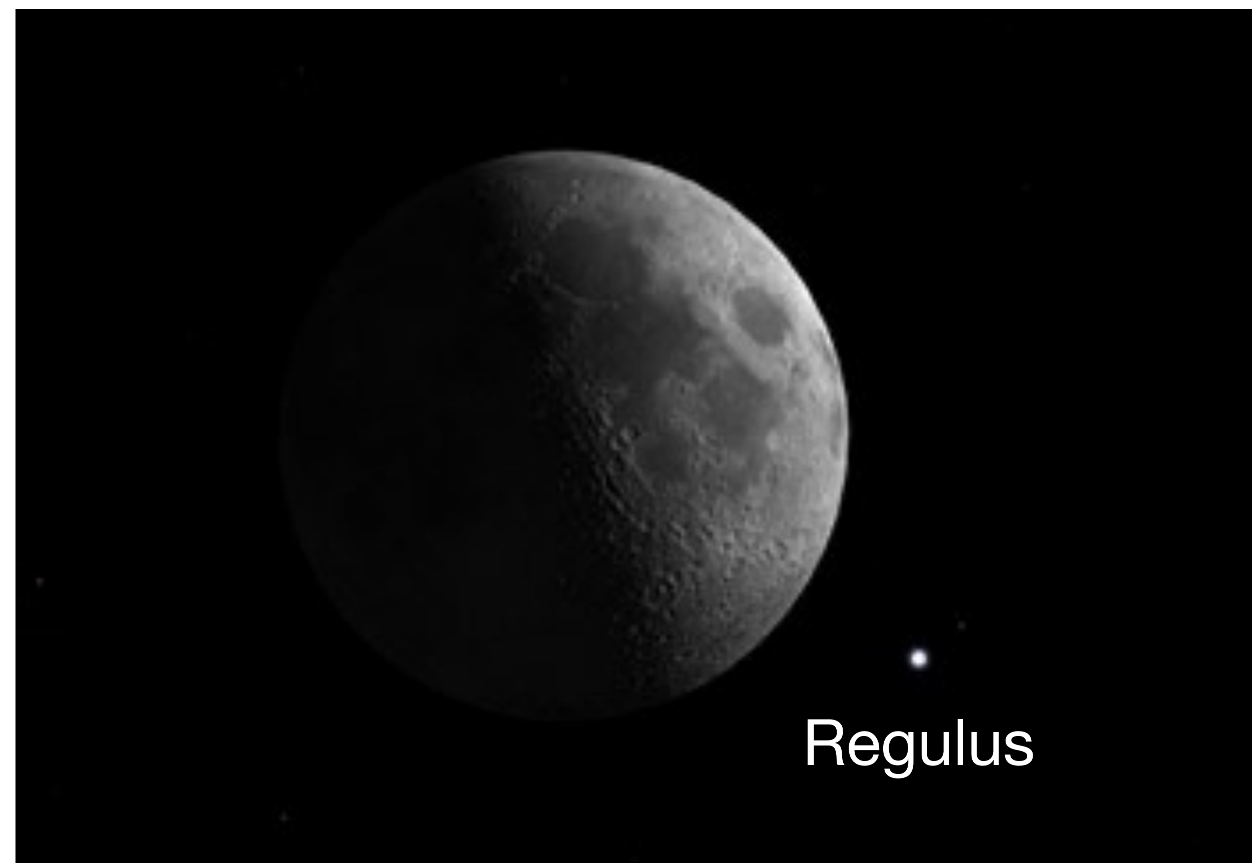


Night-time Parallax Measurement of the Moon

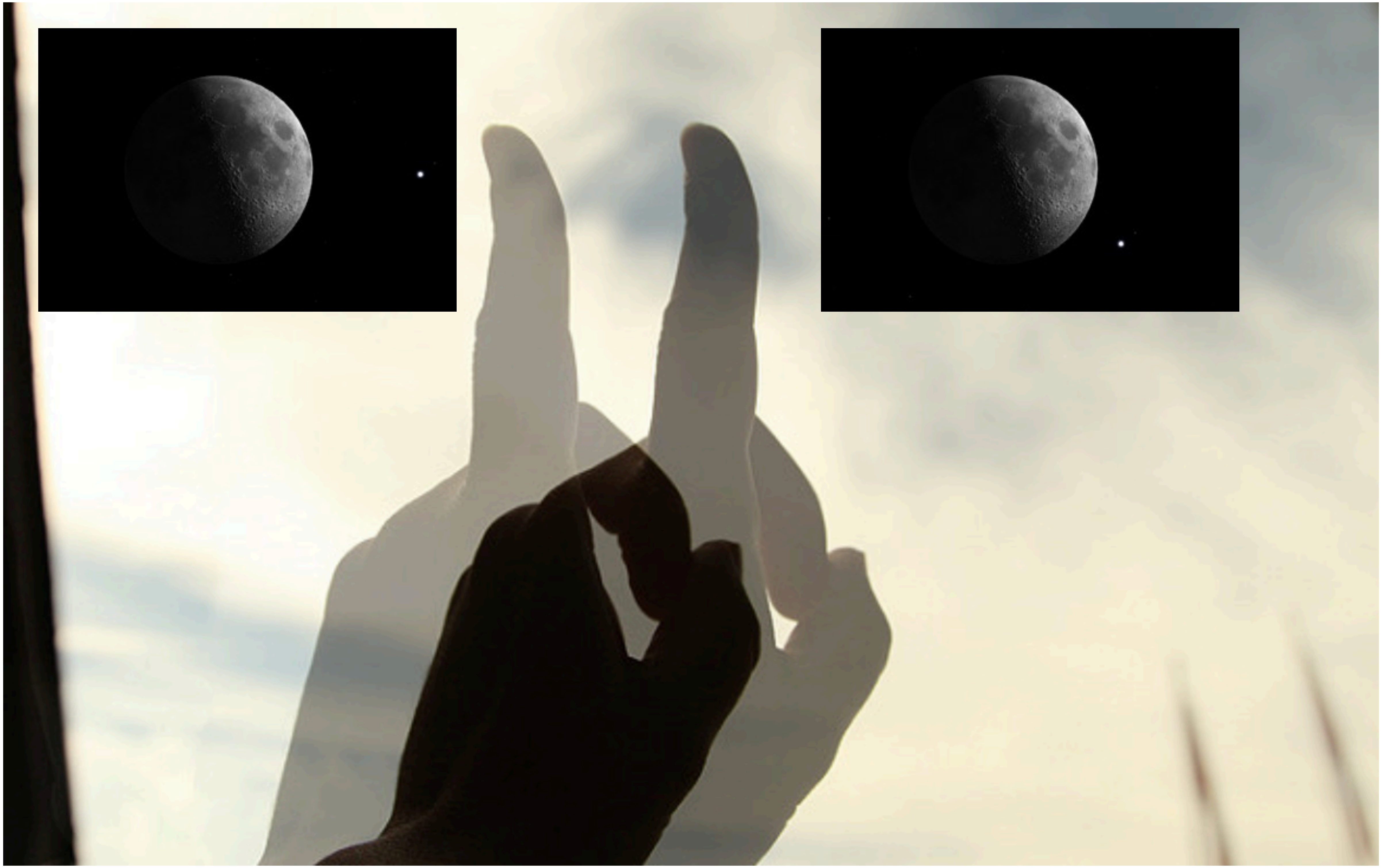
On May 23, 2007, at Athens, the moon appears closer to the bright star (**Regulus**) by 18 arcmin compared to the image taken in Selsey. The separation of the two locations is 2360 km. This difference seen in the direction of the moon against distance stars is the parallax.

Selsey, UK

Athens, Greece

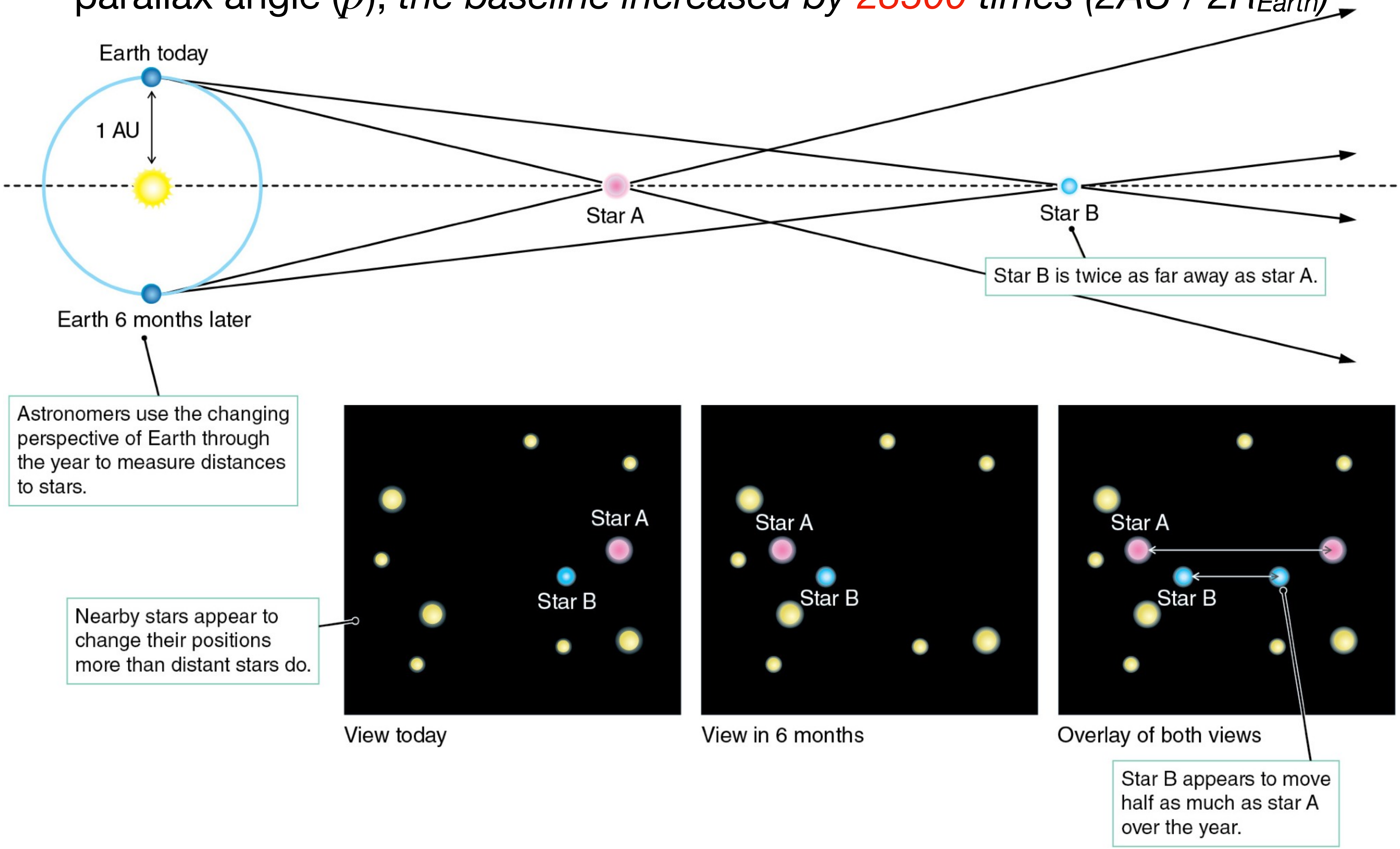


Same Concept as our Stereoscopic Vision



Extend the Baseline from Earth Size to Earth's Orbit Size: Stellar Parallax

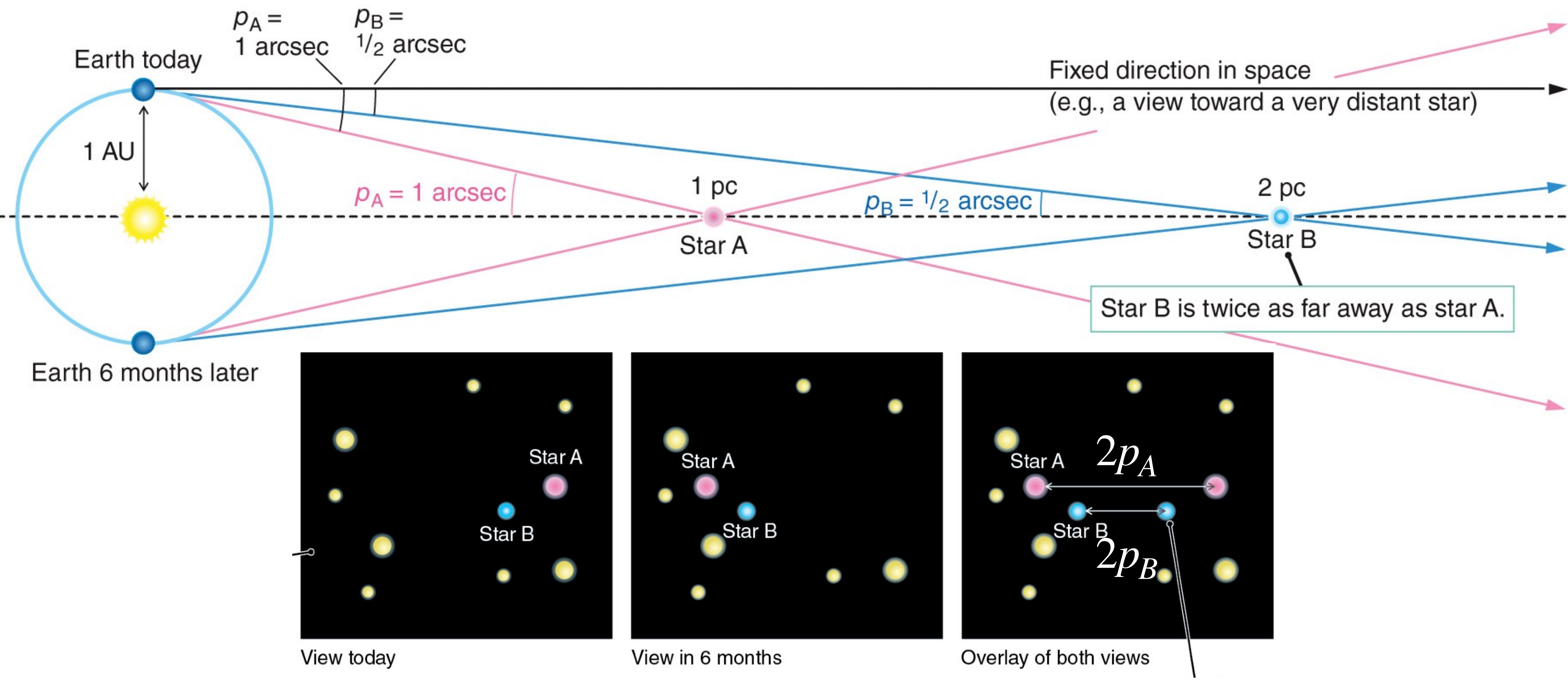
distance can be measured given the baseline length ($l = 2\ \mathrm{AU}$) and the parallax angle (p); *the baseline increased by 23500 times ($2AU / 2R_{Earth}$)*



The Definition of Parallax in Astronomy

Any directional shift due to a positional shift is a parallax effect, but in astronomy, **parallax** is defined as **half of the maximum directional shift** due to Earth's orbital motion.

From this diagram, it's clear that **parallax** is inversely proportional to **distance**: $p \sim 1/d$



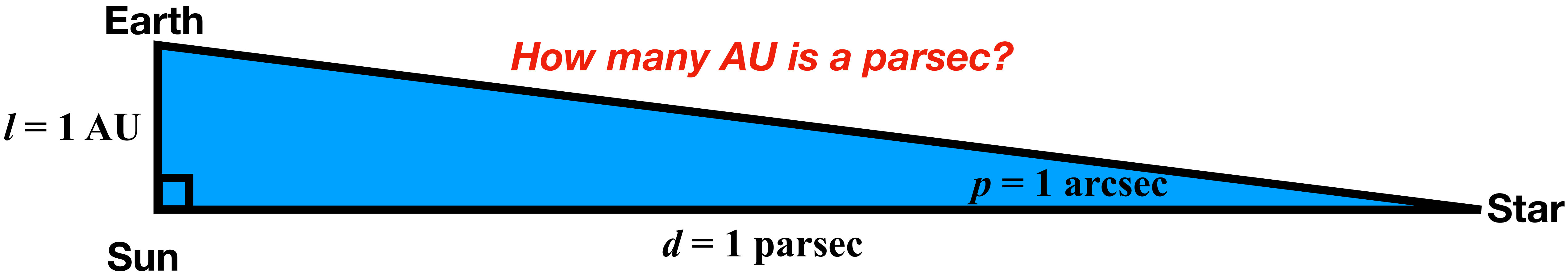
Definition of the unit parsec: the distance at which *p* = 1 arcsec

Let p be the parallax in arcseconds.

Let d be the distance in parsecs; the unit **parsec** is defined as the distance at which ***p* = 1 arcsec**

Given this definition we have:

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p} \right)$$

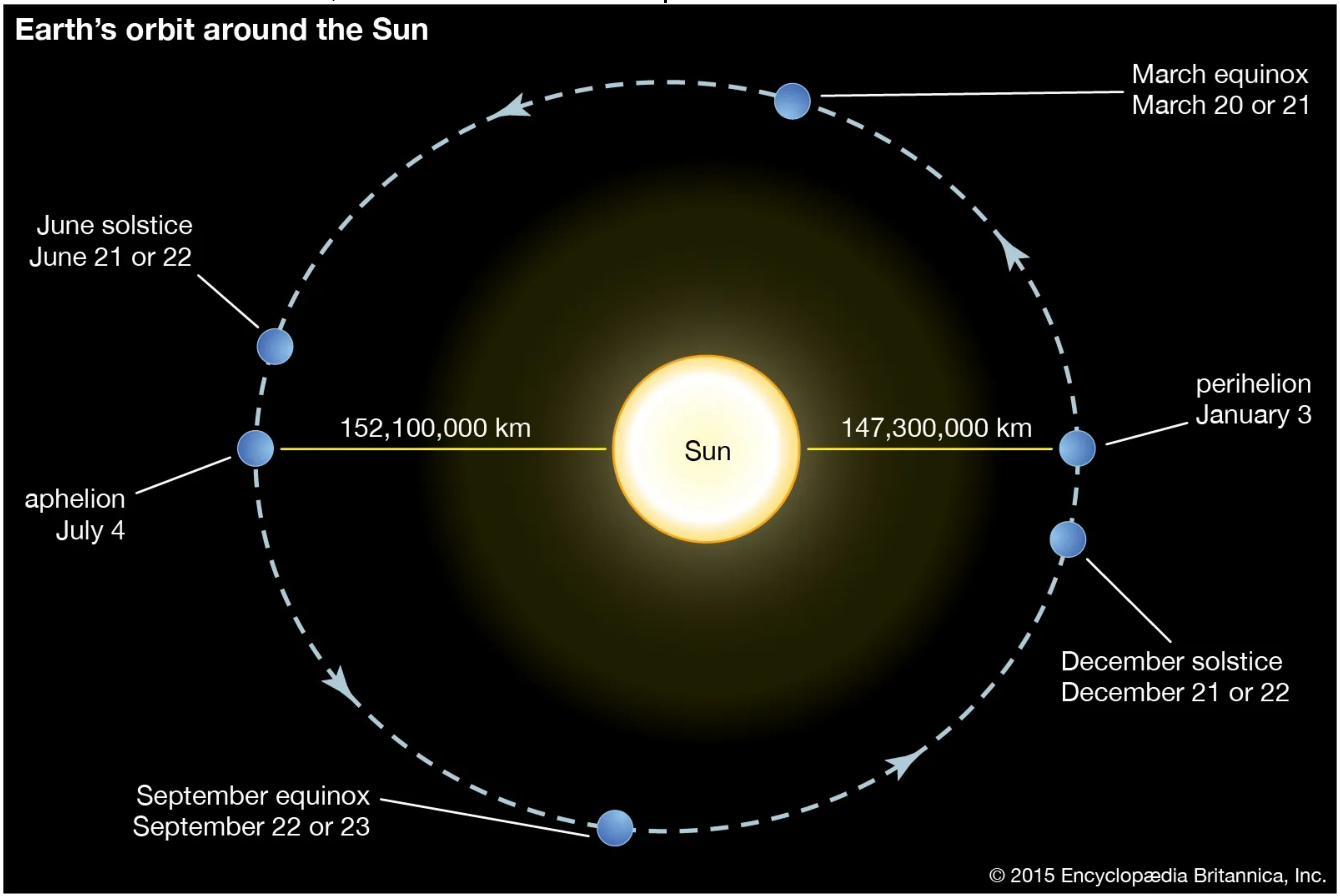


From the diagram above, we derived:

1 parsec = 206,205 AU since $l/d = \tan p \sim p$ (in radian)

How would you determine the length of the Astronomical Unit?

To learn more, see the scanned Chap 18 of Abell's textbook on ICON



Practice: convert parallax to distance

The greater the parallax, the smaller the distance.

A star with a parallax of 1 **arcsecond (arcsec)** is at a distance of 1 **parsec (pc)**.

- 1 arcsec = 1/3,600 degree
- 1 pc = 3.26 light-years

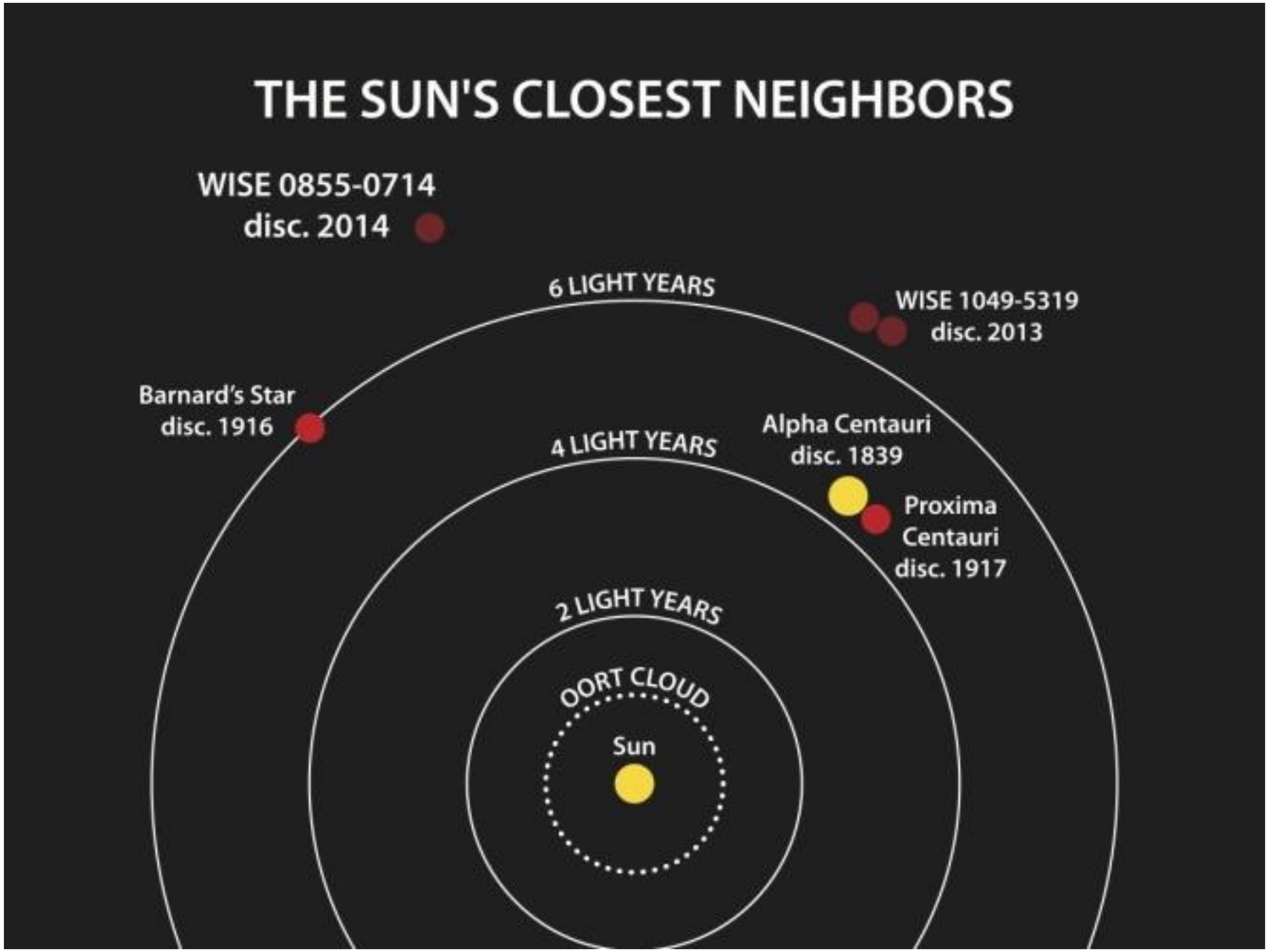
Parallax angles have been measured for >1 billion stars.

The first star with measured parallax was 61 Cygni by Friedrich Bessel in **1838**.
It had a parallax of **0.314 arcsec**, what is its distance in parsec and light-year?

Bessel functions in Mathematics are named after him.

Practice: Convert distance to parallax (WIO 13.1)

Let’s try a reversed problem. After the Sun, the closest star to Earth is Proxima Centauri, which is 4.24 light-years away. What is the star’s parallax in arcsec? (1 pc = 3.26 ly)



Practice: Convert distance to parallax (WIO 13.1)

Let’s try a reversed problem. After the Sun, the closest star to Earth is Proxima Centauri, which is 4.24 light-years away. What is the star’s parallax in arcsec?

First, we convert light-years to parsecs:

$$d = 4.24 \text{ light−years} \times \frac{1 \text{ parsecs}}{3.26 \text{ light−years}} = 1.30 \text{ parsecs}$$

Then, we plug in to find the distance:

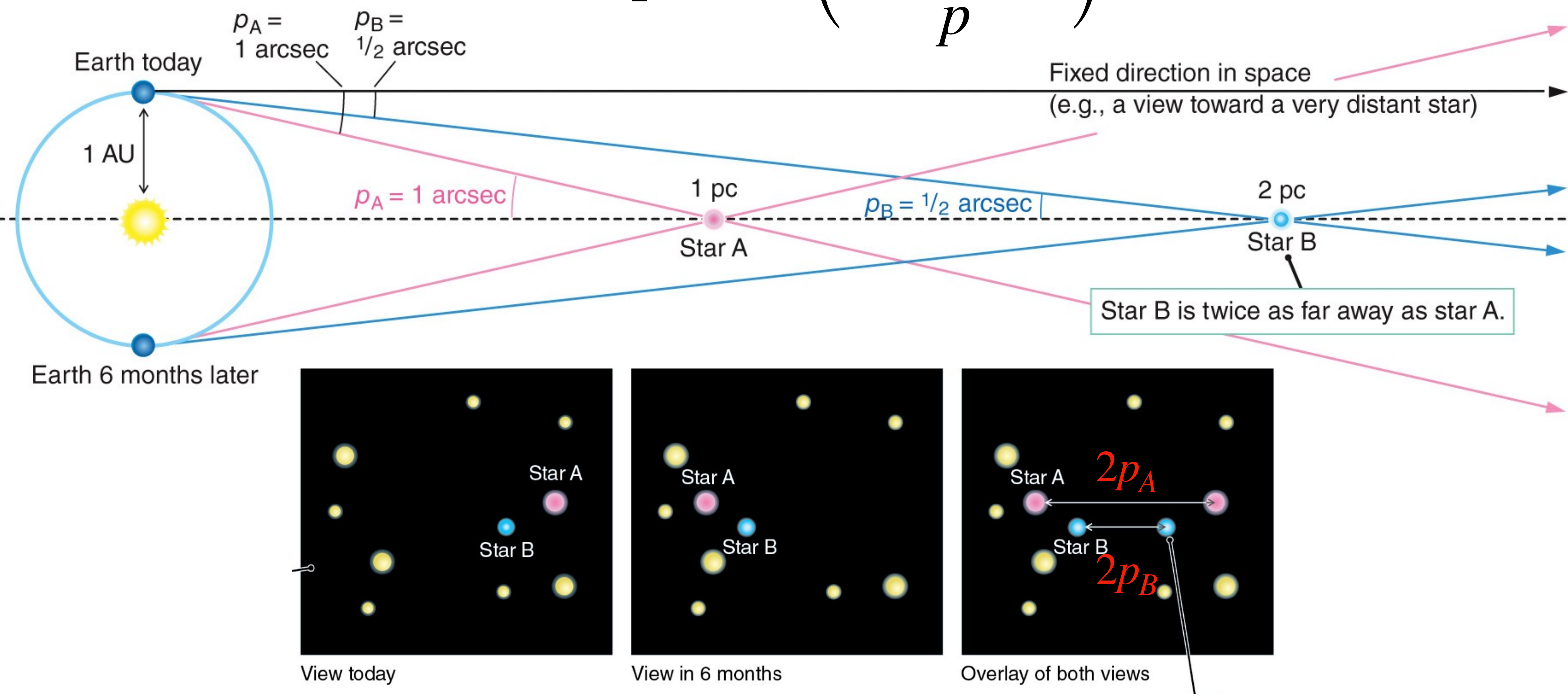
$$p \text{ (arcsec)} = \frac{1}{1.30 \text{ pc}} = 0.77 \text{ arcsec}$$

The closest star to the Sun has a parallax smaller than 1 arcsec!

Stellar Parallax: One Slide Summary

Any directional shift due to a positional shift is a parallax effect, but in astronomy, **parallax (p)** is defined as **half of the maximum directional shift** due to Earth's orbital motion. With this definition, we have the following **parallax-distance relation**:

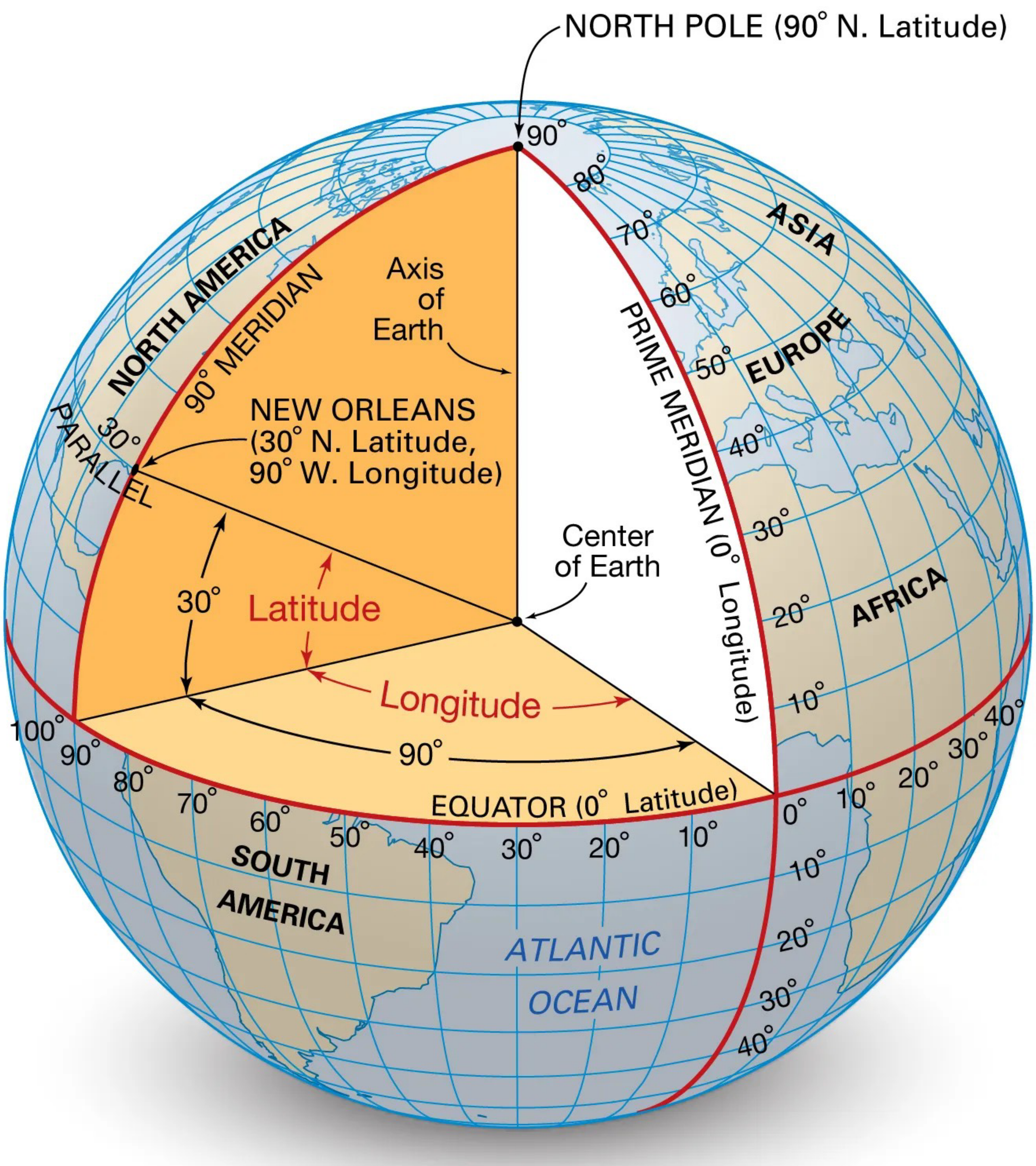
$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p} \right)$$



How to Calculate Parallax from Coordinates?

A star's position is recorded in celestial coordinates (RA, Dec), how to calculate the angular offset between two coordinates?

Celestial Coordinates are similar to the Longitude and Latitude system on Earth's surface

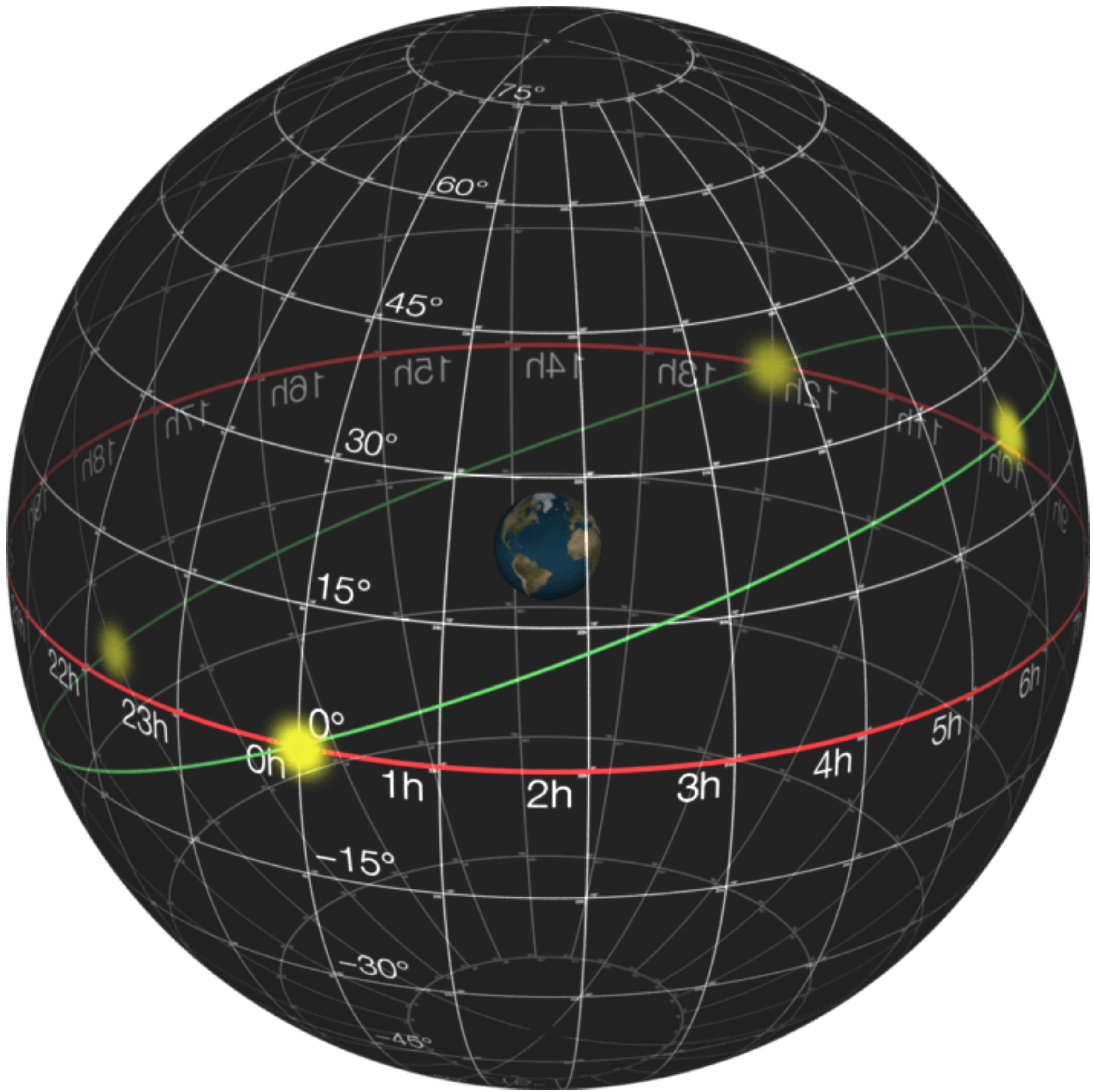


Equatorial coordinates

right ascension (RA)
declination (Dec)

RA's units
(hour, minute, second)

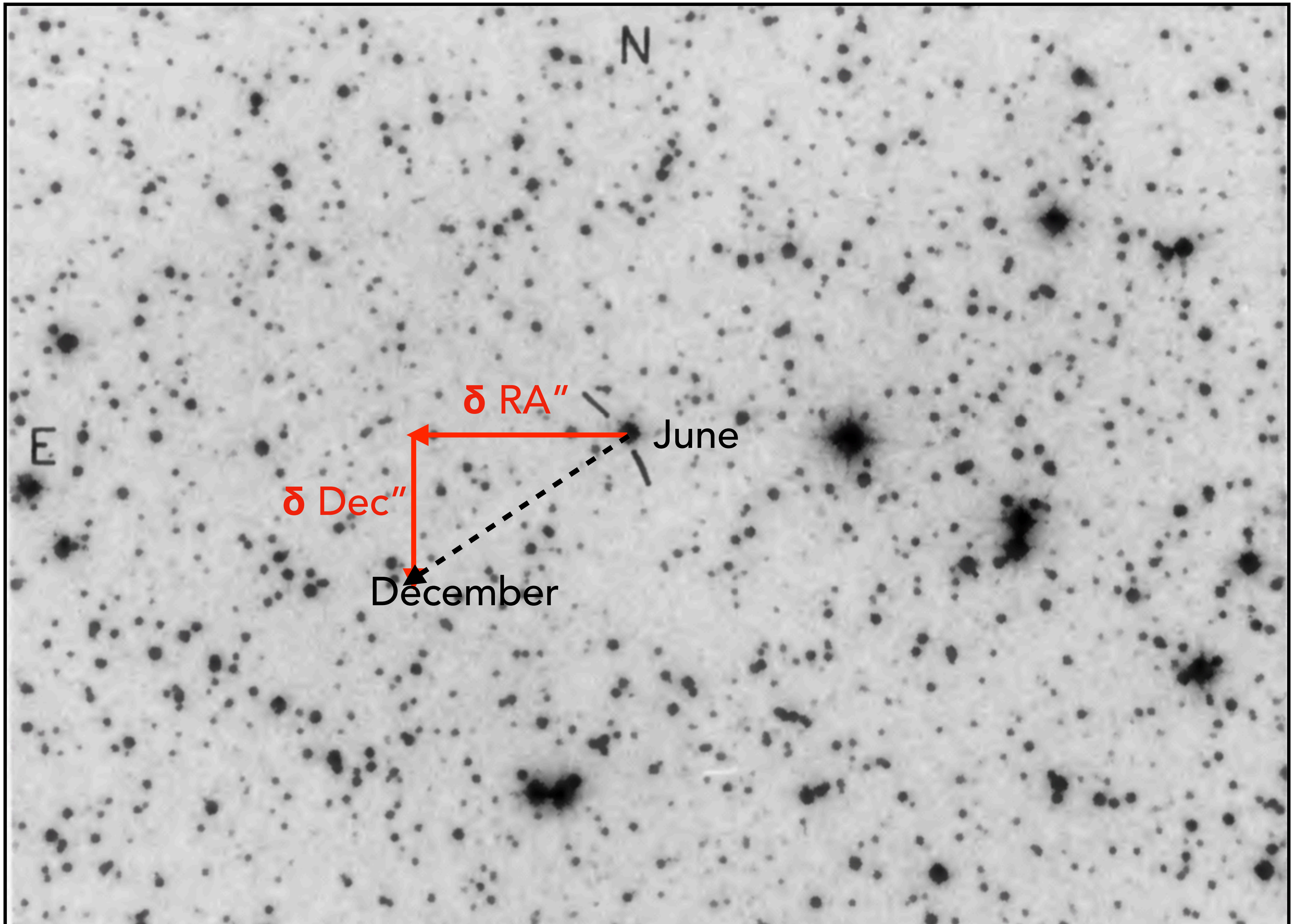
Dec's units:
(deg, arcmin, arcsec)



Given two (RA, Dec) coordinates, calculate their angular offset

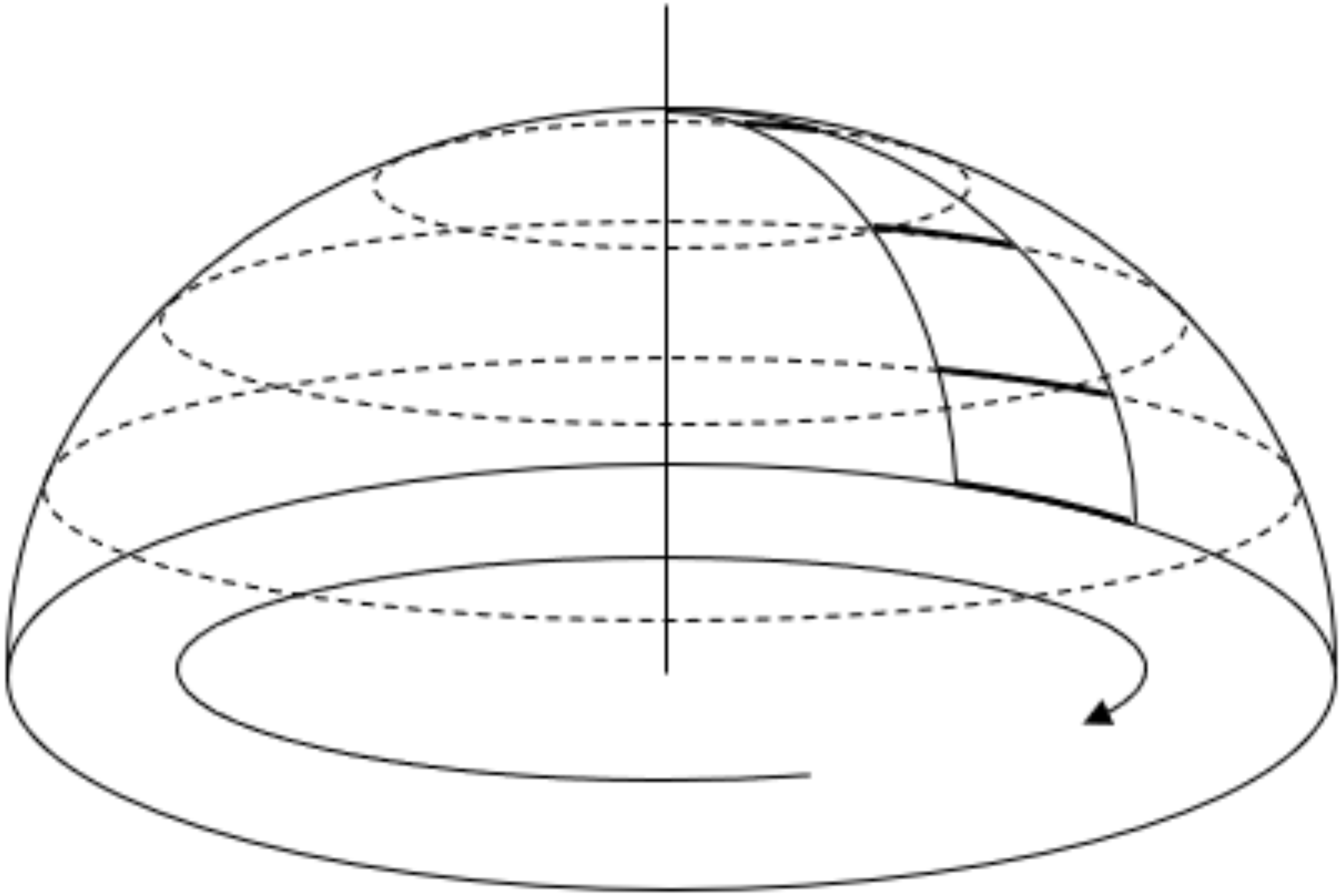
When the two coordinates are close together, we can use plane trigonometry to approximate spherical trigonometry:

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$



Given two (RA, Dec) coordinates, calculate their angular offset

- Obj 1: RA = 2hr, Dec = 0deg — Obj 2: RA = 3hr, Dec = 0deg; what's their angular distance in degrees?
- Obj 1: RA = 2hr, Dec = 60deg — Obj 2: RA = 3hr, Dec = 60deg; what's their angular distance in degrees?



$$\delta \mathrm{RA}^\circ = (\mathrm{RA}_1^h - \mathrm{RA}_2^h) \cdot \cos(\mathrm{Dec}_1^\circ) \cdot 15^\circ/\mathrm{hour}$$

$$\delta \mathrm{Dec}^\circ = \mathrm{Dec}_1^\circ - \mathrm{Dec}_2^\circ$$

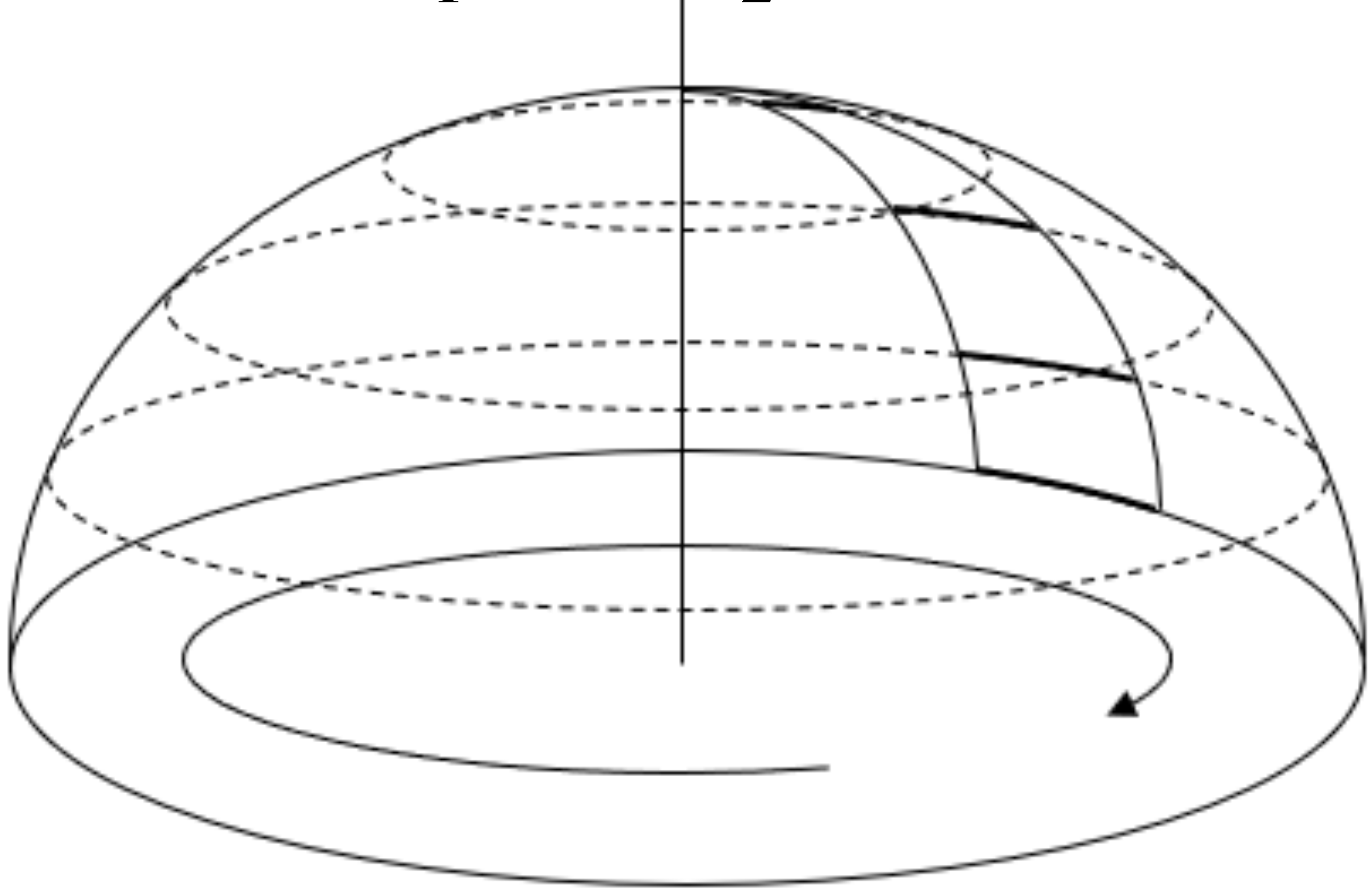
Given two (RA, Dec) coordinates, calculate their angular offset

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$

Note that (1) RA's units are (hour, minute, second), and Dec's units are (deg, arcmin, arcsec), and (2) the angular distance between two meridians ***decreases*** from the equator to the poles. As a result, we have the following formulae to calculate both the RA offset and the Dec offset in arcsec:

$$\delta \mathrm{RA}'' = (\mathrm{RA}_1 - \mathrm{RA}_2) \cdot \cos(\mathrm{Dec}) \cdot 15\ ''/\mathrm{s}$$

$$\delta \mathrm{Dec}'' = \mathrm{Dec}_1 - \mathrm{Dec}_2$$



Practice: Given two (RA, Dec) coordinates, calculate their angular offset

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$

$$\delta \mathrm{RA}'' = (\mathrm{RA}_1 - \mathrm{RA}_2) \cdot \cos(\mathrm{Dec}) \cdot 15''/\mathrm{s}$$

$$\delta \mathrm{Dec}'' = \mathrm{Dec}_1 - \mathrm{Dec}_2$$

dRA = 0.03 * cos(23.5 deg) * 15 = 0.413"
dDec = 0.005"
=> p = 0.413"/2 => d = 2.4*2 parsec

A star's coordinates have been recorded based on images taken on the following dates:

Mar 21 2022: 06h00m15.205s 23d29'15.155"
Sep 21 2022: 06h00m15.235s 23d29'15.160"

- How far has the star moved in RA & in Dec (both in arcsec)?
- How large is the parallax? What's the distance in parsec?

How to Plan Parallax Observations?

Given a star's position in equatorial coordinates (RA, Dec), how to decide when to make the two observations to detect the maximum parallax effect?

Are these dates and coordinates arbitrary?
Mar 21 2022: 06h00m15.205s 23d29'15.155"
Sep 21 2022: 06h00m15.235s 23d29'15.160"

Equatorial coordinates

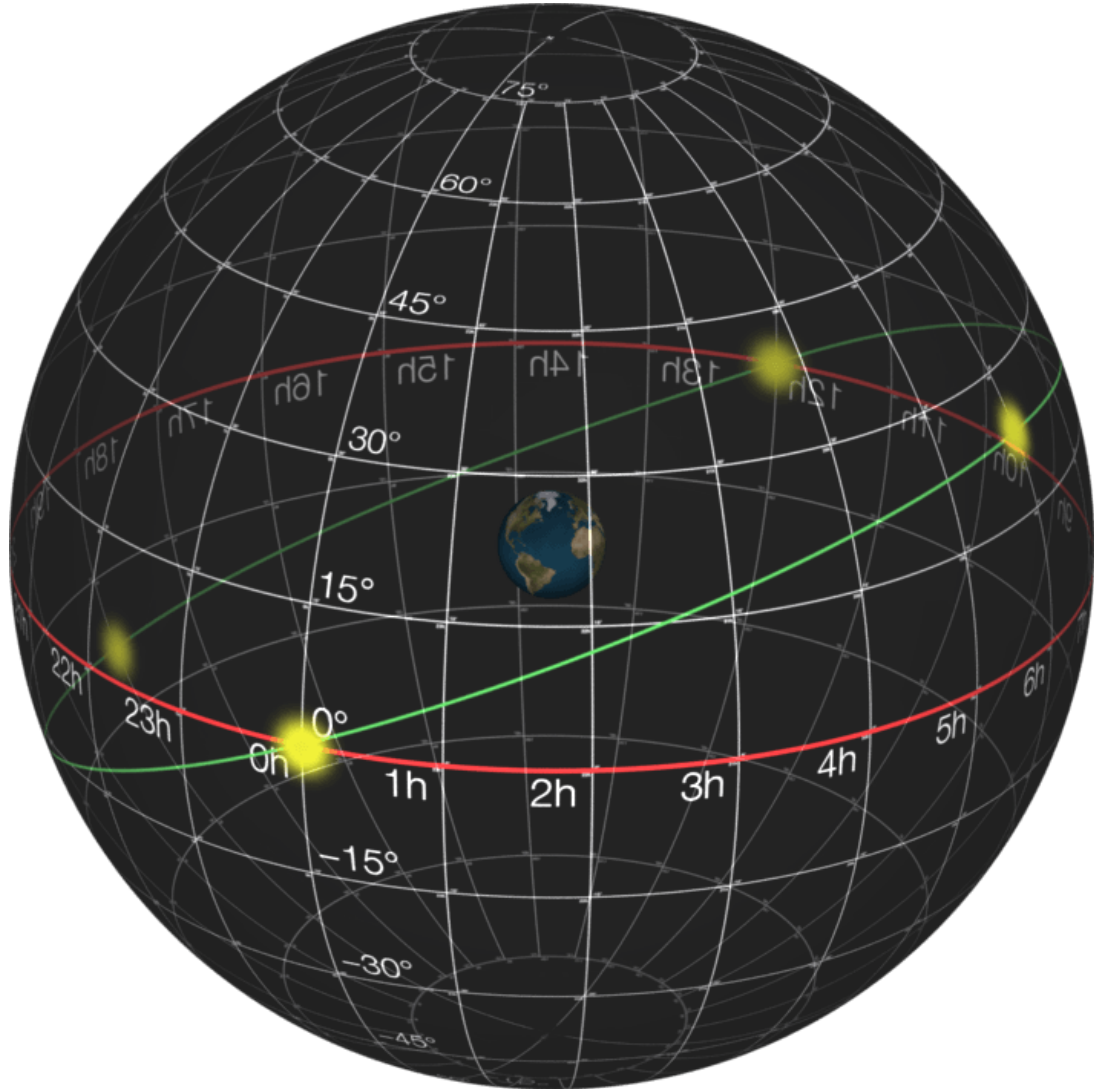
right ascension (RA)
declination (Dec)

Ecliptic coordinates

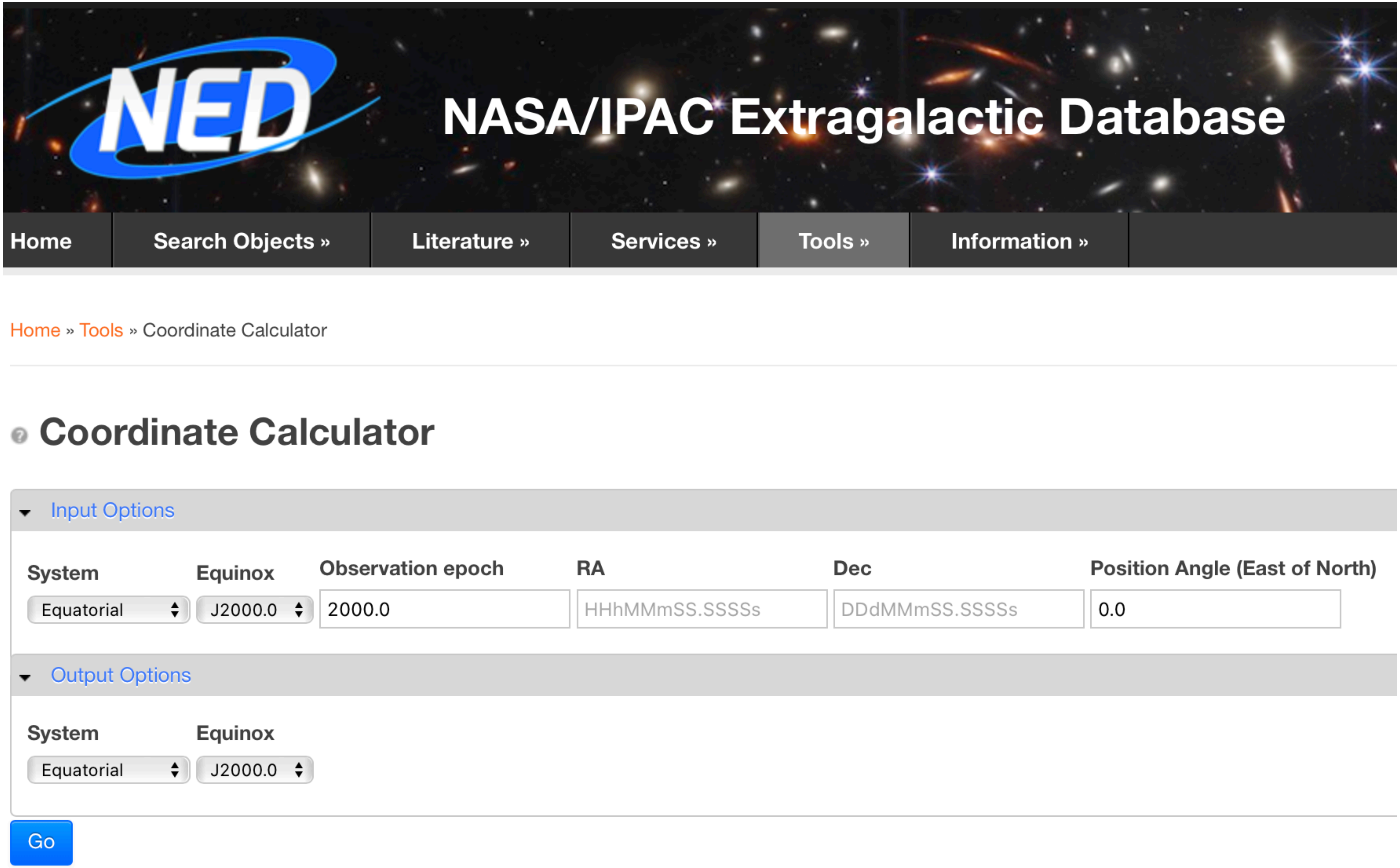
Longitude
Latitude

Ecliptic Longitude ~ RA
~ means "roughly equal"

|Ecliptic Latitude - Dec| < 23.5 degrees



Coordinate Converter: https://ned.ipac.caltech.edu/coordinate_calculator



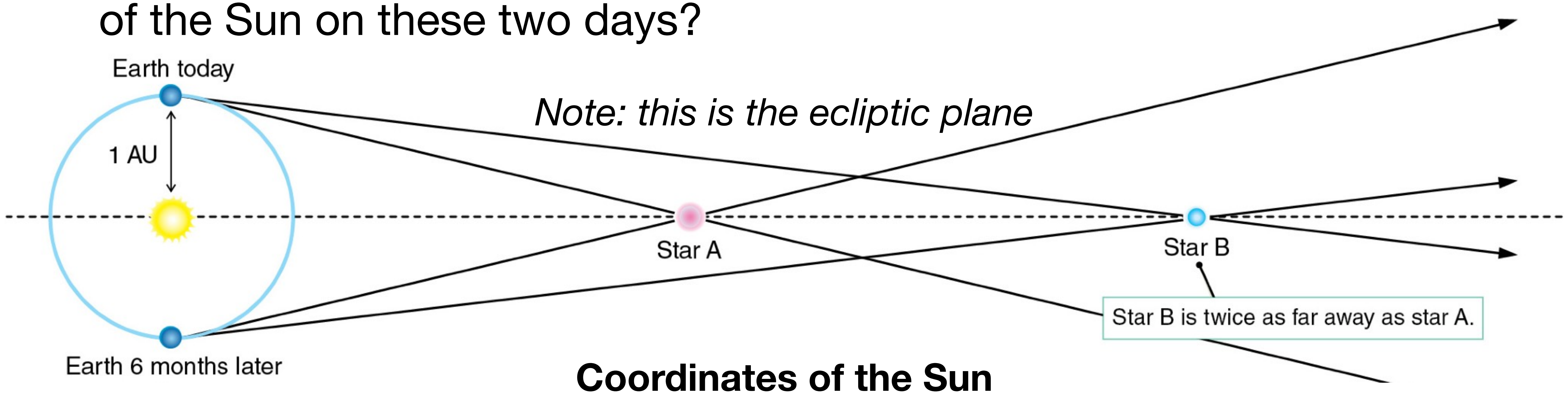
The Equatorial and Ecliptic Coordinates of the Sun

- In the course of a year, the Sun travels on the Ecliptic from Spring Equinox, to Summer Solstice, to Fall Equinox, to Winter Solstice, and back to Spring Equinox

	RA	*Dec*	*Ecliptic Longitude*	*Ecliptic Latitude*	*Notes*
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

Stellar Parallax: Observational Considerations

- On these two days illustrated in the graph below, at what local time do Stars A and B transit the meridian?
- What are the **Ecliptic Longitudes** of Star A and Star B relative to those of the Sun on these two days?

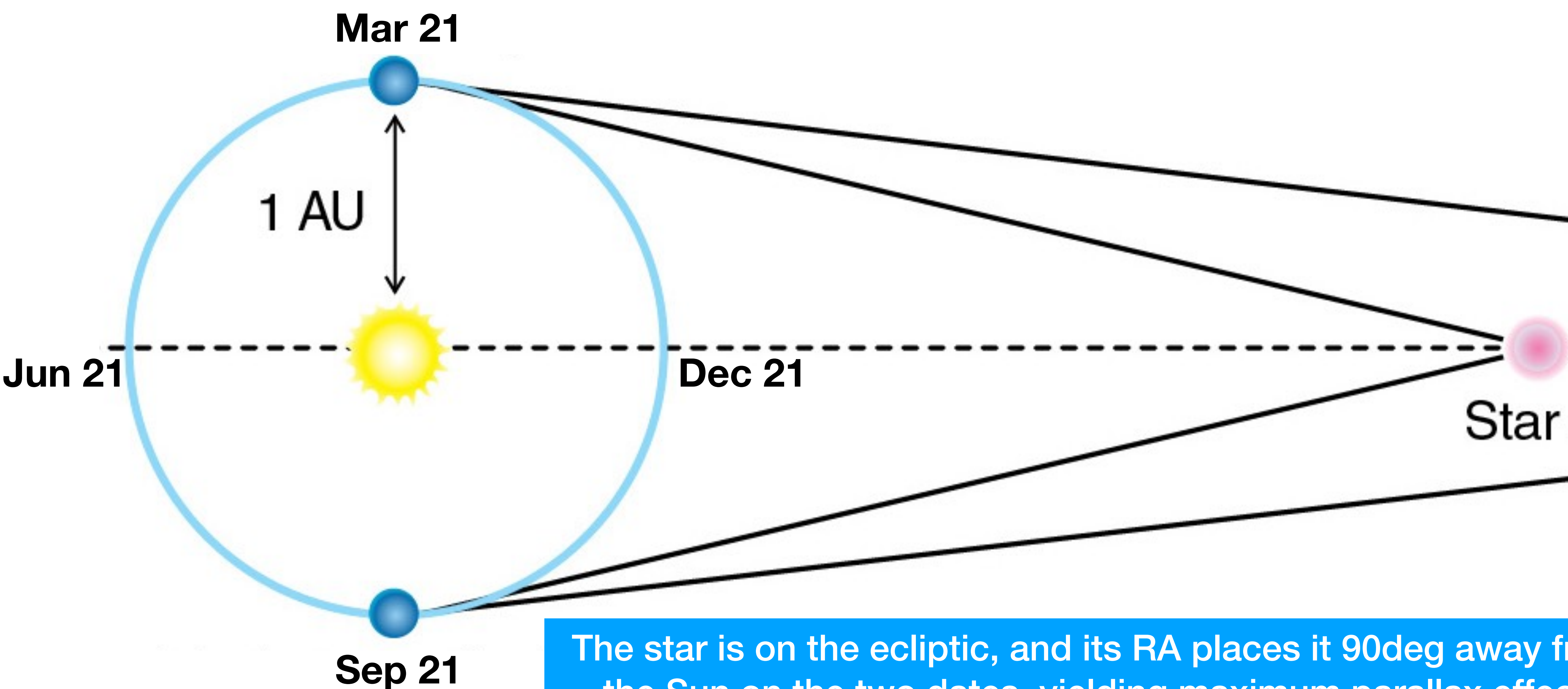


Coordinates of the Sun

	RA	Dec	Ecliptic Longitude	Ecliptic Latitude	Notes
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

Let’s check the coordinates in the practice example

Are these dates and coordinates arbitrary? Should its RA increase?
Why its Dec did NOT change much?
Mar 21 2022: 06h00m15.205s 23d29’15.155”
Sep 21 2022: 06h00m15.235s 23d29’15.160”



The star is on the ecliptic, and its RA places it 90deg away from the Sun on the two dates, yielding maximum parallax effect.

Recap

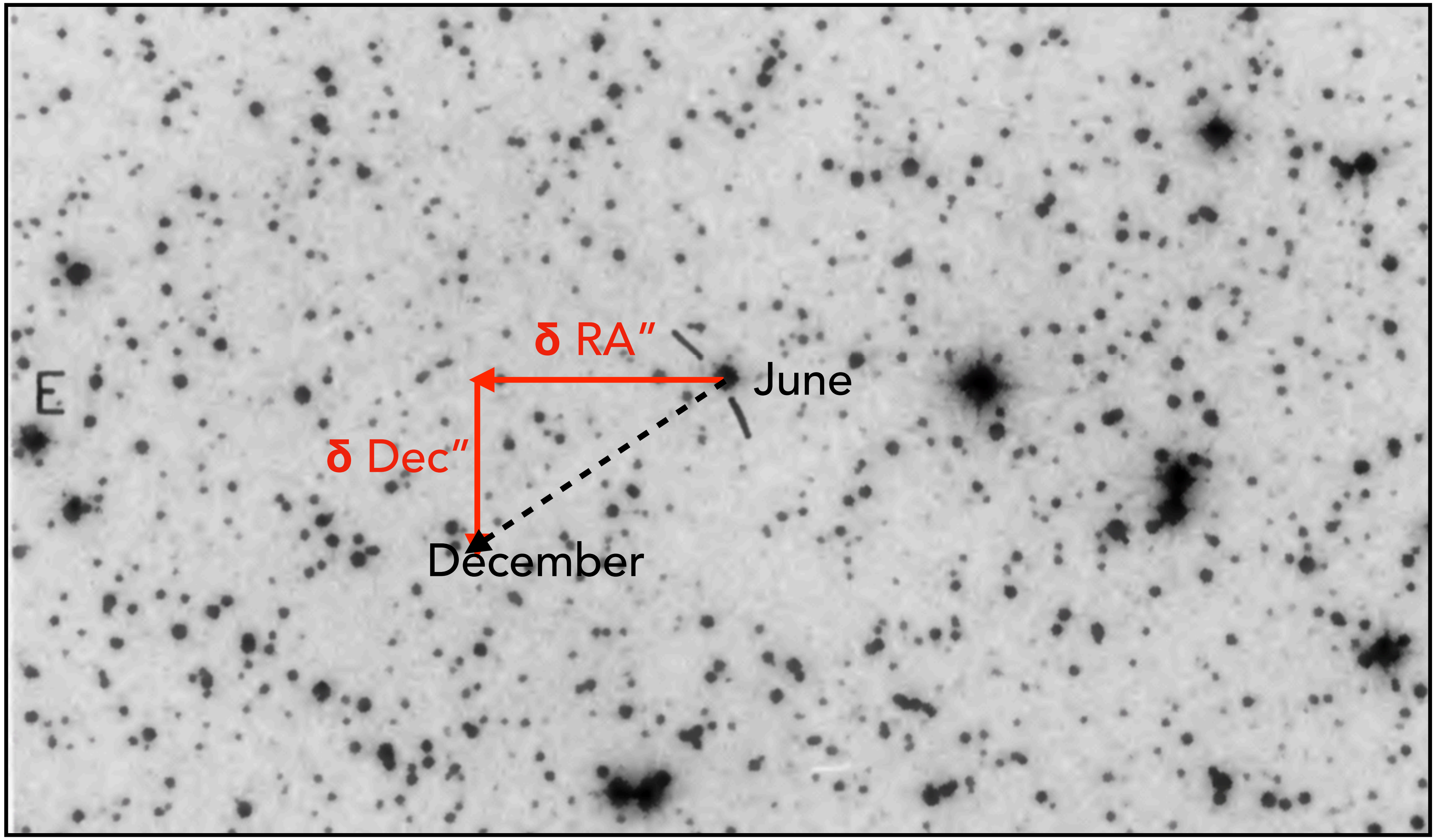
Advanced Topics of Parallax

Calculate angular offset given Equatorial coordinates

$$\Delta'' = \sqrt{\delta\mathrm{RA}''^2 + \delta\mathrm{Dec}''^2}$$

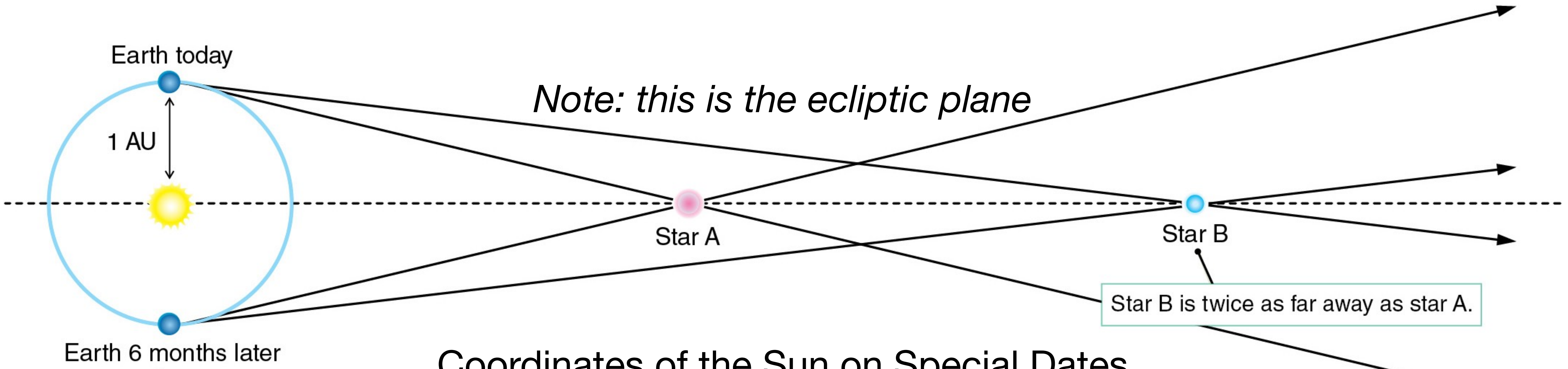
$$\delta\mathrm{RA}'' = (\mathrm{RA}_1^s - \mathrm{RA}_2^s) \cdot \cos(\mathrm{Dec}^\circ) \cdot 15''/\mathrm{s}$$

$$\delta\mathrm{Dec}'' = \mathrm{Dec}_1'' - \mathrm{Dec}_2''$$



Stellar Parallax: Observational Considerations

- To see maximum parallax effect, you must choose two nights when the **Ecliptic Longitudes** of the target is **6 hrs (90 deg)** away from the **Sun**.



Coordinates of the Sun on Special Dates

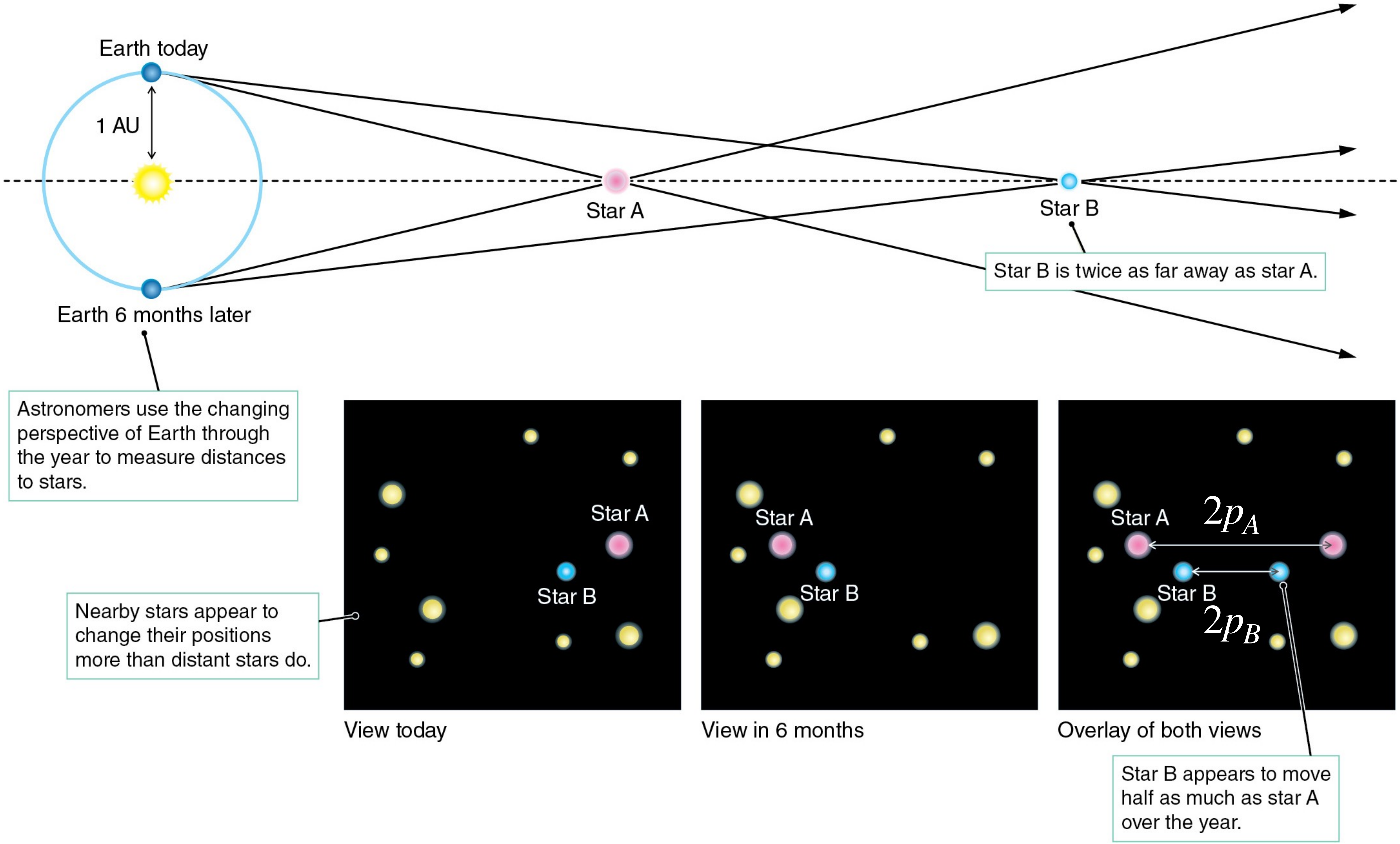
	RA	*Dec*	*Ecliptic Longitude*	*Ecliptic Latitude*	*Notes*
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

Annual Parallax Traces

What kind of pattern does a star draw on the sky due to Earth's annual motion?
We can record this pattern if we continuously monitor its position over a year

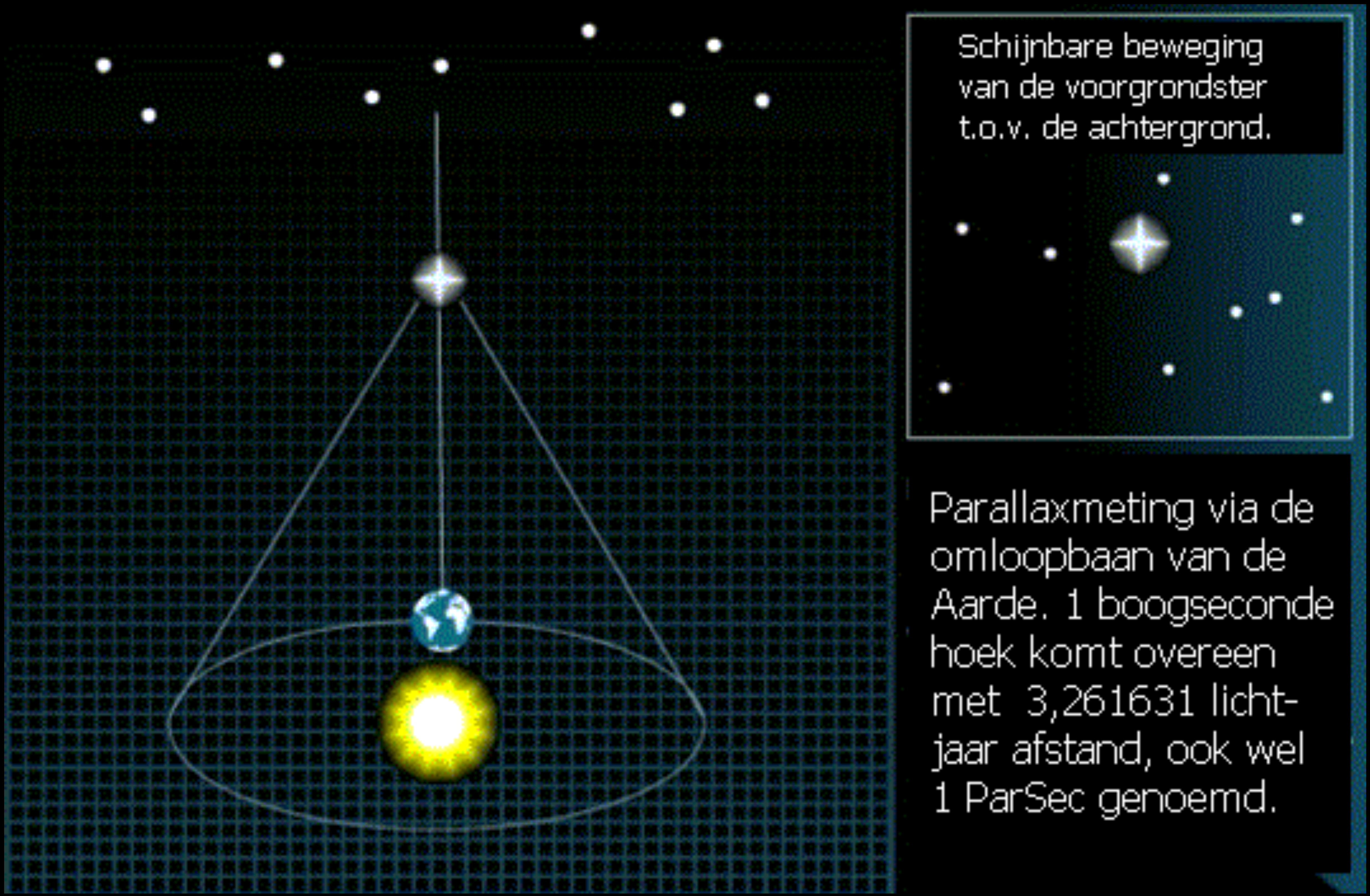
Simplest case: sources on the ecliptic

oscillating along a short line



Simpler case: sources on the ecliptic poles

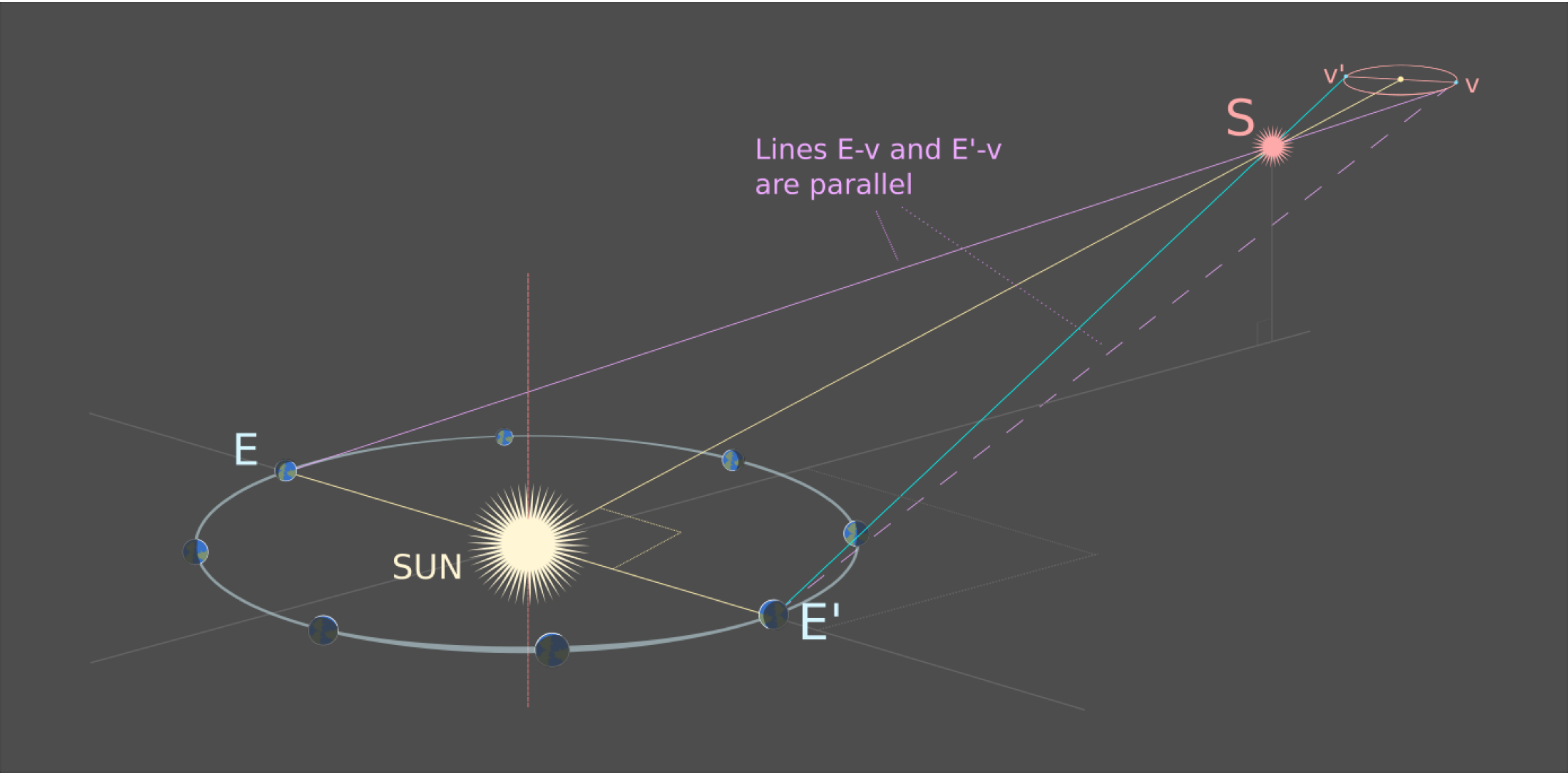
moving along a circle



Weten Schaps

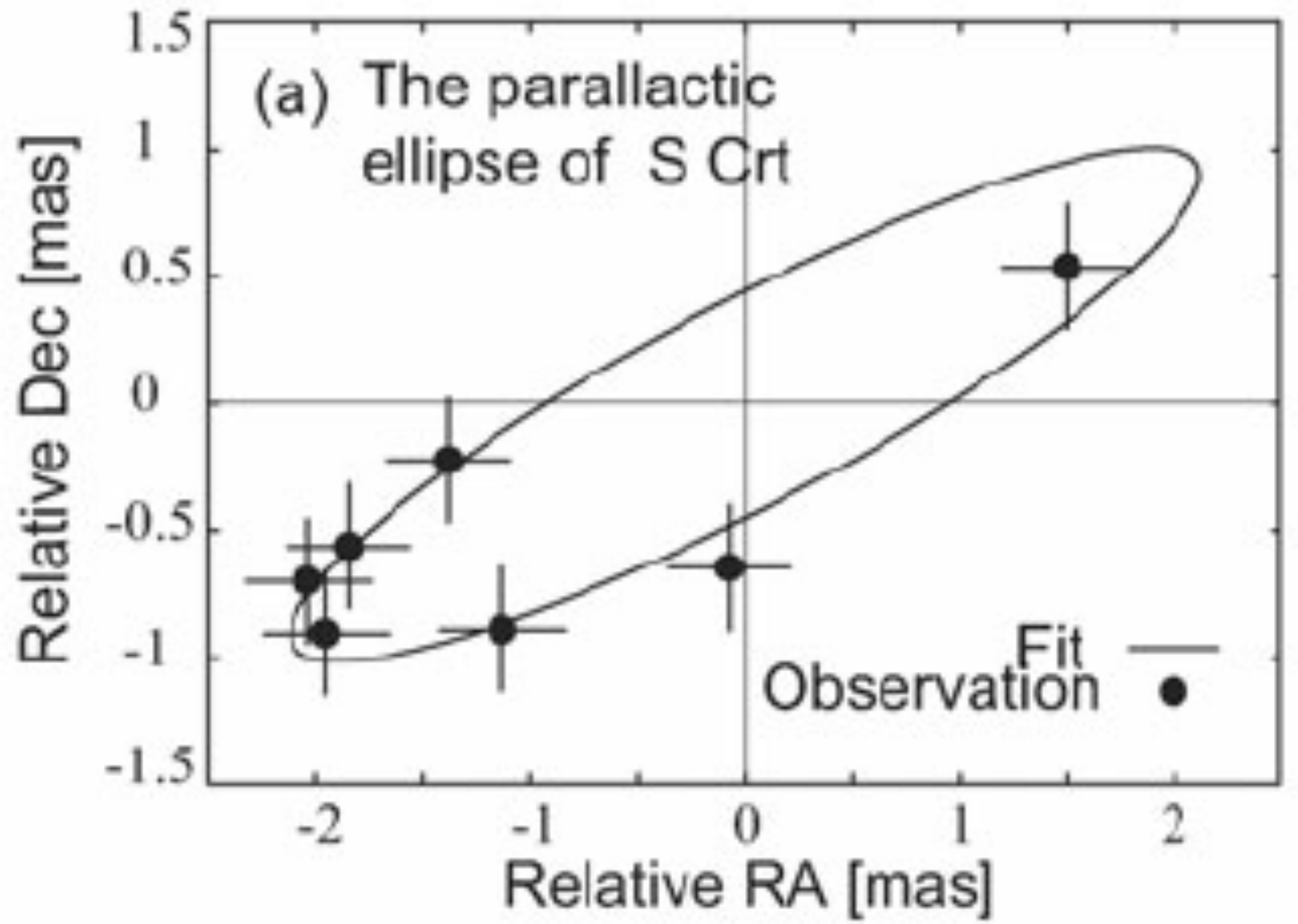
General cases: 0 < ecliptic latitude < 90 deg

moving along an ellipse



Summary: Parallactic Traces & Parallax Measurements

- Sources on the ecliptic oscillate on short lines along the ecliptic; the parallax to measure distance is half of the length of the line.
- Sources on the ecliptic poles draw parallactic circles; the parallax to measure distance is the radius.
- All other sources draw ellipses with major axes parallel to ecliptic; For a parallactic ellipse, what is the parallax to measure distance?



Nakagawa+2008

One more thing - Proper Motion

Unlike a tree or a mountain relative to a geographical surveyor, stars always move relative to the Sun because of their different trajectories in the Milky Way. Such relative motions are called **proper motion.**

2 million stars’ motion 5 million years into the future

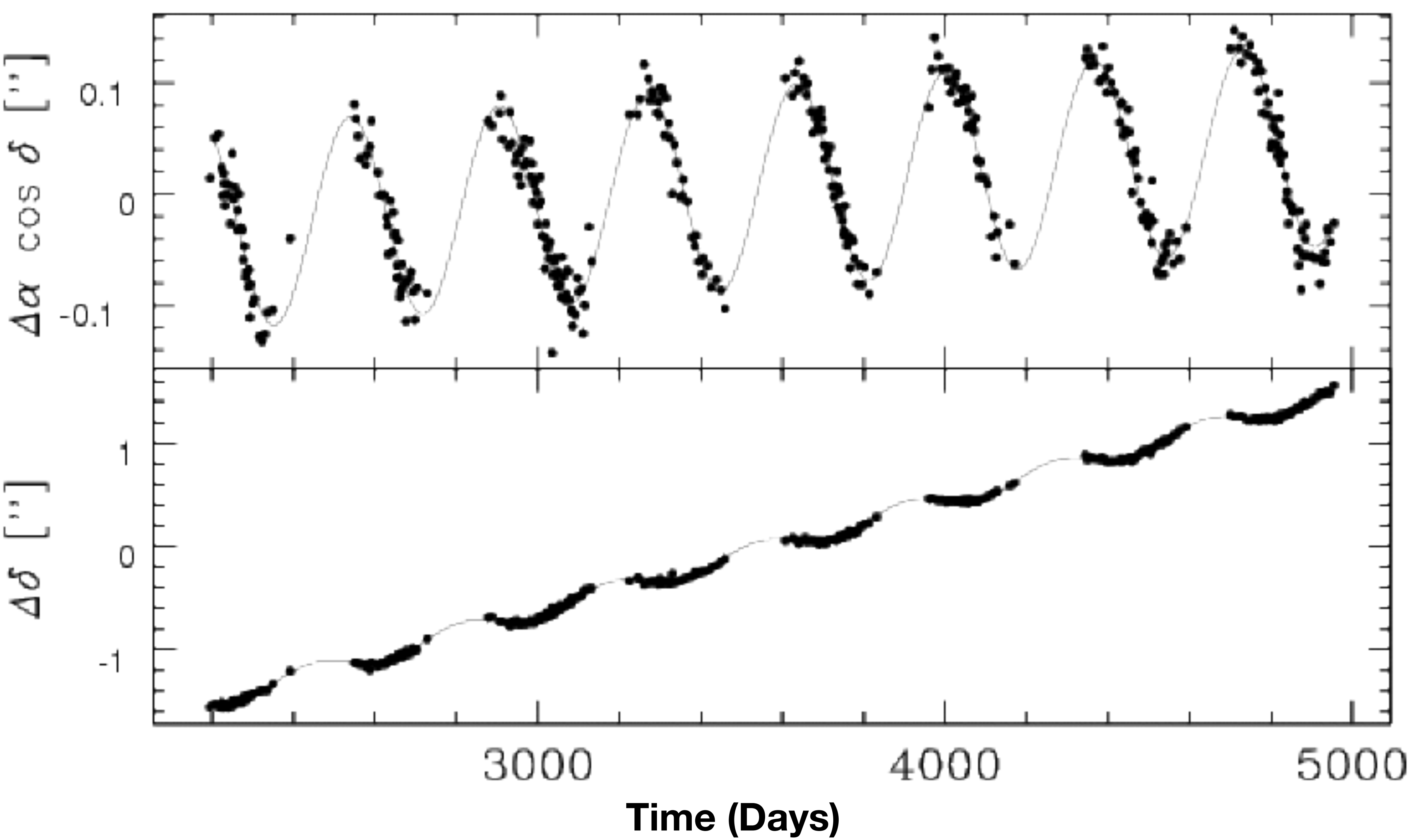
61 Cygni A+B proper motion

So a star's position changes on the sky because of its own motion relative to us (proper motion) and our motion around the Sun (parallax).

What would the combined motion look like on the sky?

RA offset vs. time & Dec offset vs. time

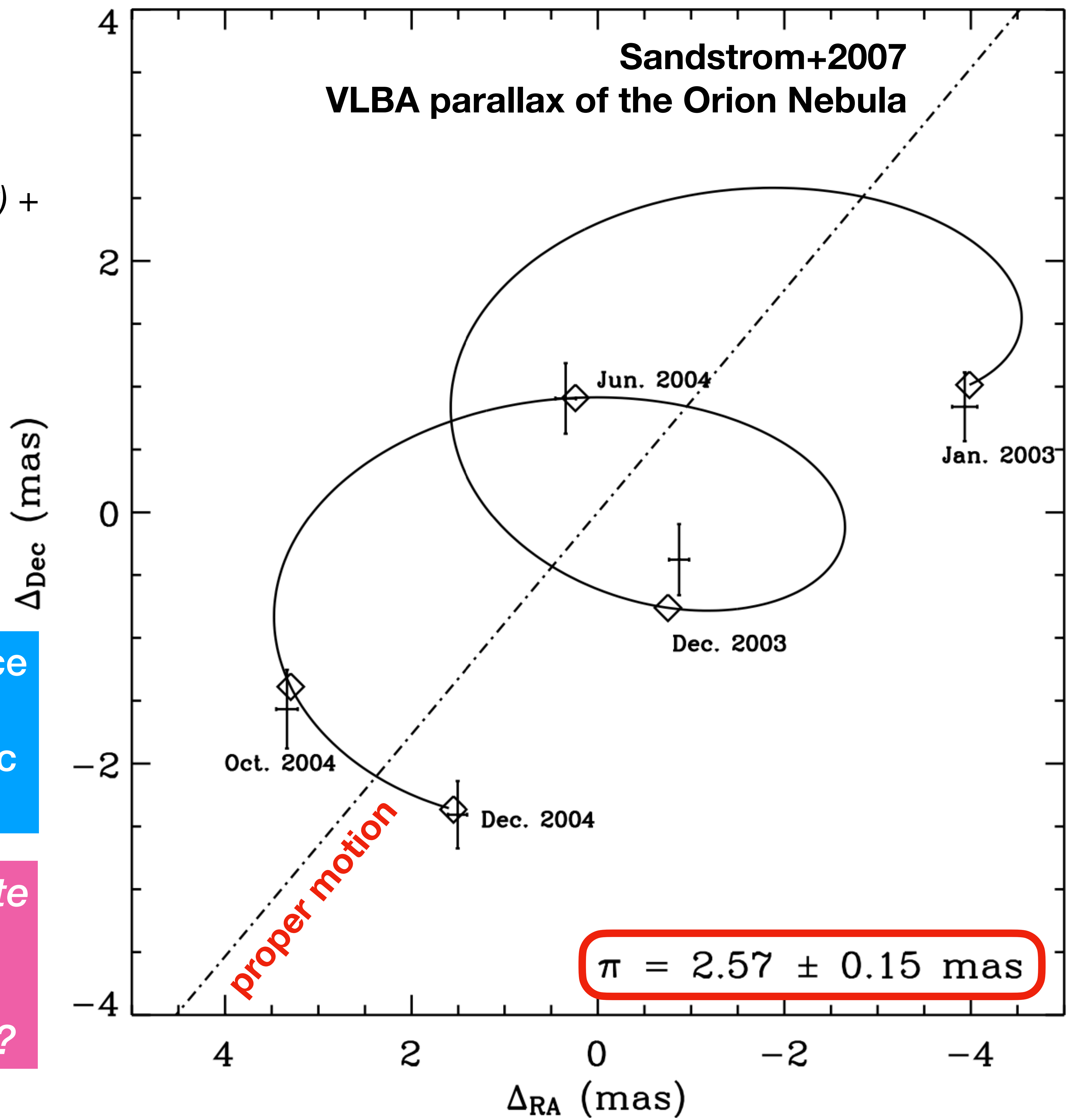
Proper motion (Linear) + Parallax (Periodic)



Poleski *et al.*, 2011, *Acta Astron.,* 61, 199 (arXiv:1110.2178)

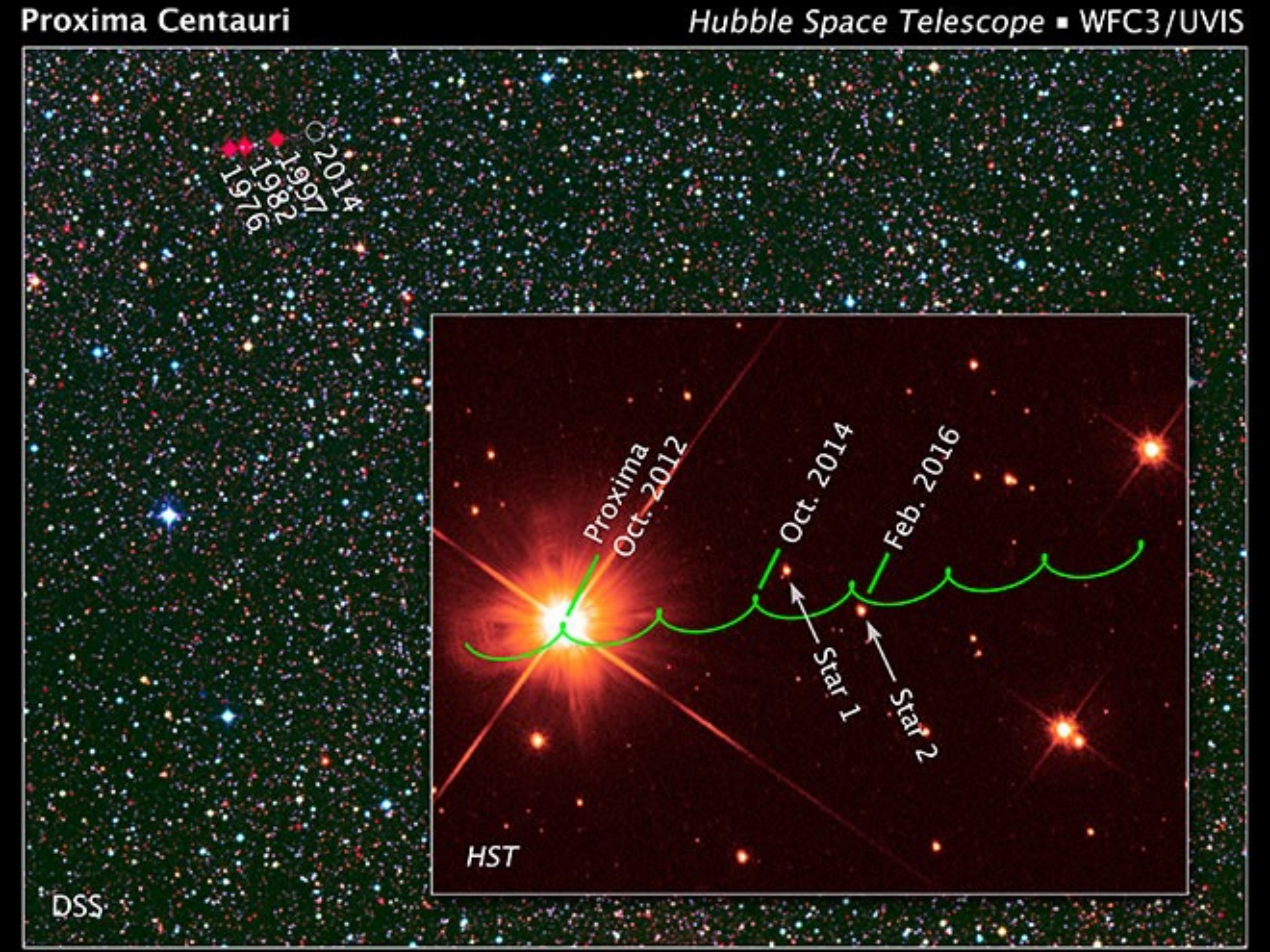
RA offset vs. Dec offset

Proper motion (Linear) + Parallax (Periodic)



What's the distance to the source?
p = 2.57e-3 arcsec
d = ? parsec

Can you propagate the error of parallax to the error of distance?



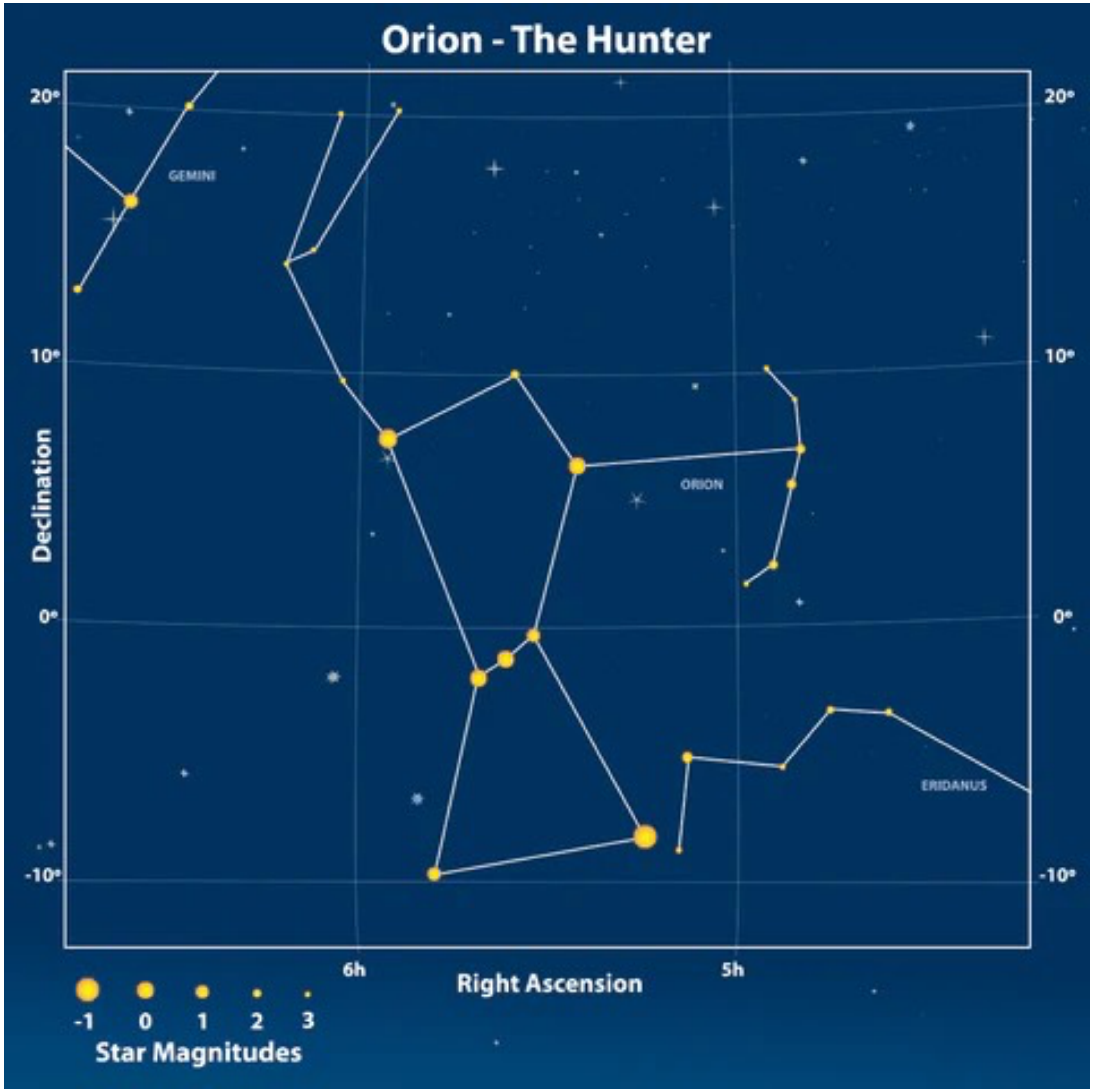
Proxima Centauri
Hubble Space Telescope ▪ WFC3/UVIS
2014
1997
1982
1976
Proxima
Oct. 2012
Oct. 2014
Feb. 2016
Star 1
Star 2
HST
DSS

Brightness Measurements: Apparent Magnitude

Visual classification of brightness: The Greek Magnitude System

Ancient Greeks: “*the stars that appear first after sunset are the 1st magnitude stars, the stars that appear second are the 2nd magnitude stars, and so on ……*”

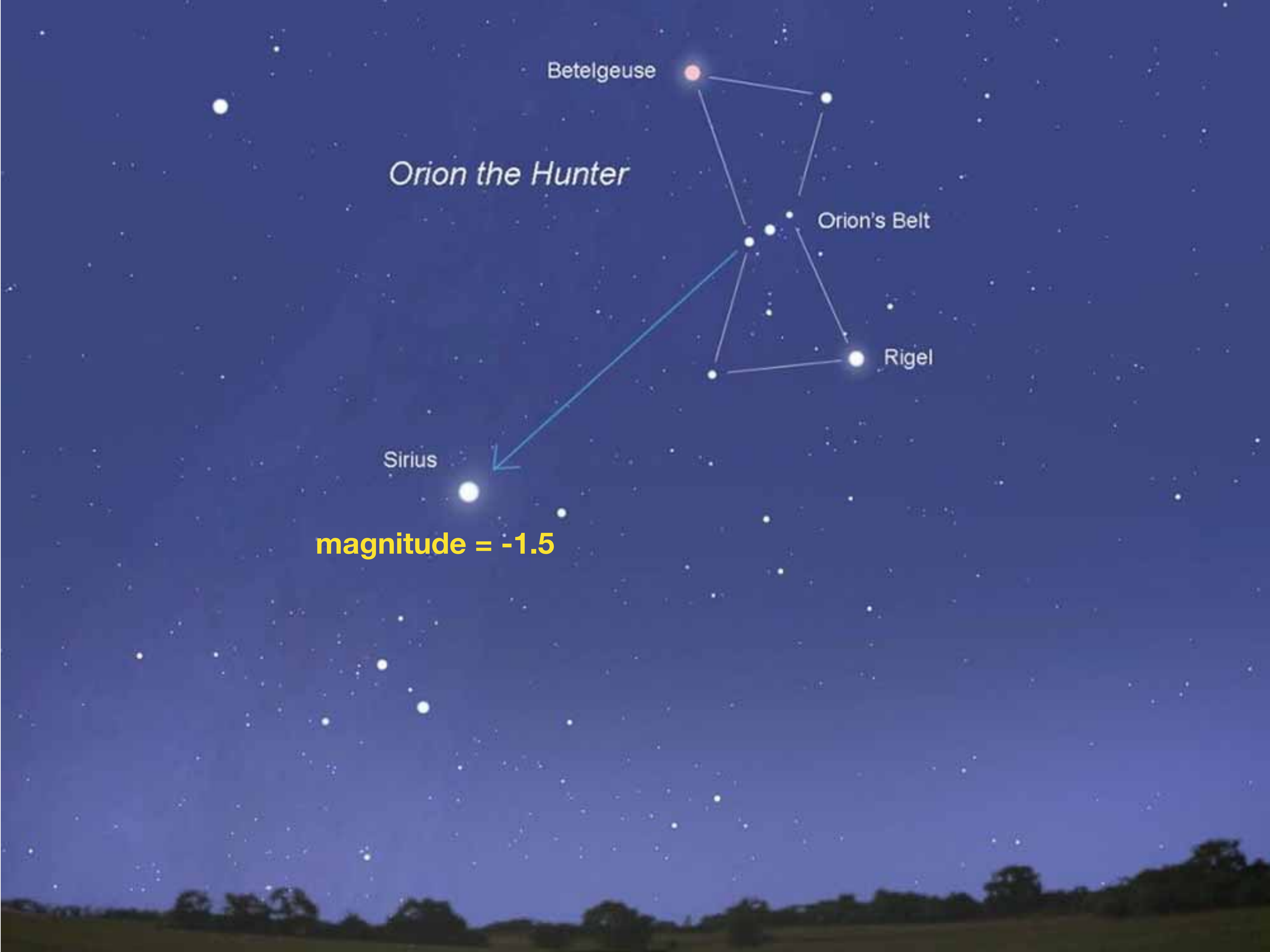
129 BC, first formally introduced by **Hipparchus**, then refined by **Ptolemy** in 150 AD:
visual classification of stars into 6 classes, **brightest as being of 1st magnitude, faintest of 6th magnitude**



MAGNITUDE & ENERGY FLUX

A BRIEF HISTORY

- 129 BC, first Hipparchus, then refined by Ptolemy in 150 AD: visual classification of stars into 6 classes, brightest as being of 1st magnitude, faintest of 6th magnitude
- 1856, Norman Pogson: 5 magnitude difference = 100x in **energy flux**, while preserving historically classified 6th mag stars, some brightest stars have negative magnitudes (e.g., **Sirius**, V-band mag = -1.5)
- 1850s - 1990s: photographic glass plates
- 1940s, photoelectric cells, tubes, photomultipliers
- 1969, Boyle & Smith: CCD detectors (2009 Nobel Prize for Physics). First used in astronomy in 1976 at U. of Arizona

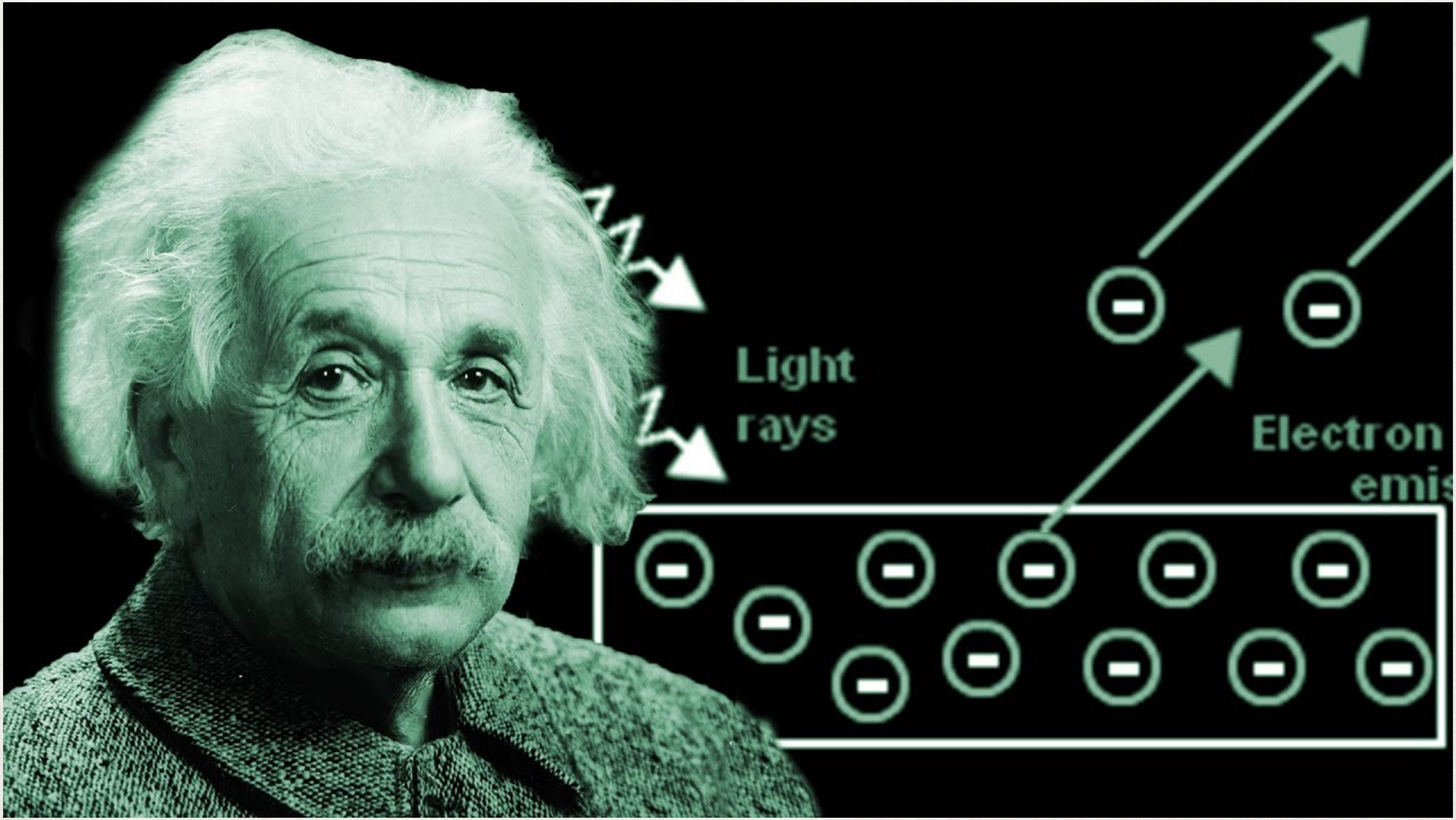


Betelgeuse
Orion the Hunter
Orion's Belt
Rigel
Sirius
magnitude = -1.5

LIGHT CARRIES ENERGY

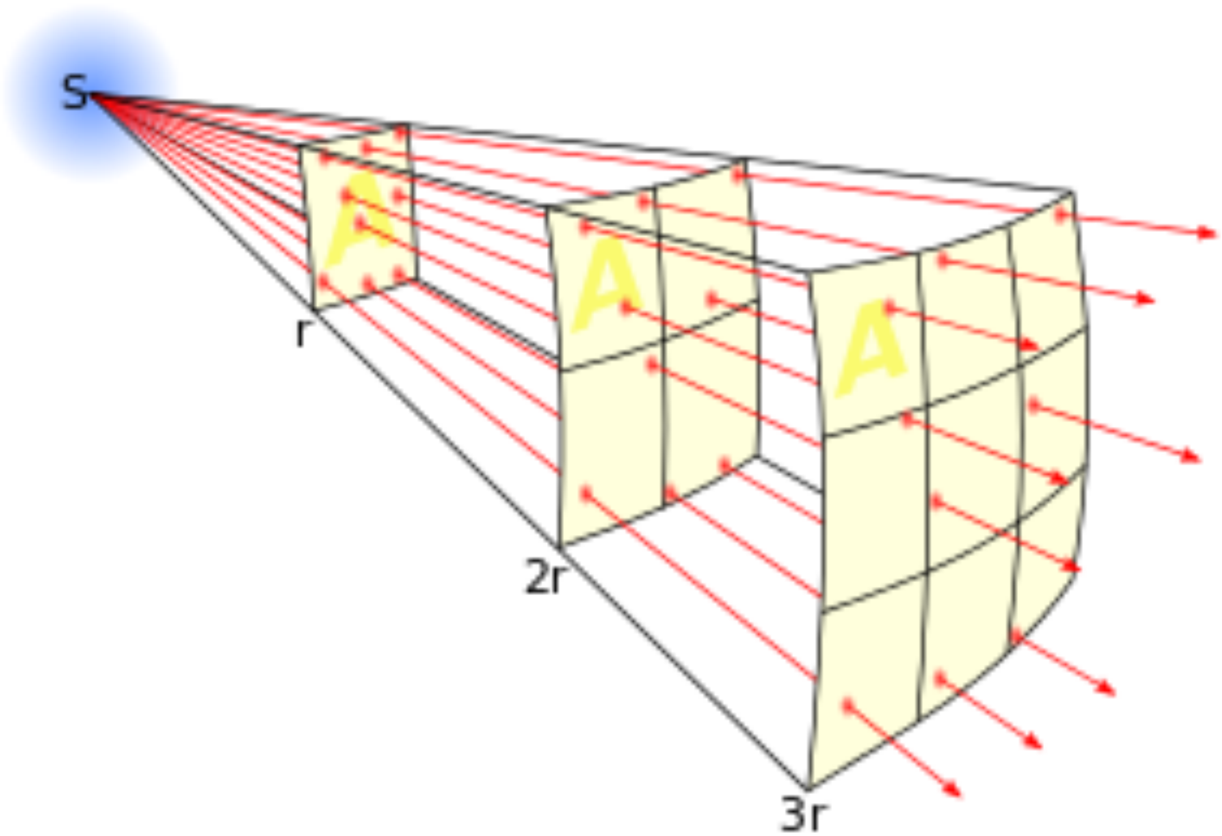
$E = h\nu = hc/\lambda$ where h = 6.6e-34 J/Hz

Einstein's 1922 Nobel Price was awarded "for his discovery of the law of the photoelectric effect"



Inverse Square Law of Flux

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m^2) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law**:



$$F = \frac{L}{4\pi d^2}$$

Observed Brightness of Stars show a HUGE range

- The Sun is the brightest star, which dominates the sky during the day, rendering it impossible to see any other stars
- The faintest star your eye can see is 10^{13} fainter than the Sun
- The faintest star that can be detected by the Hubble space telescope is 10^{20} fainter than the Sun.

- How do we deal with such a large range? We put everything on a logarithmic scale similar to that used by the Greeks, thus preserving the history started from Hipparchus in 129 BC.
- As a result, brighter stars still have lower magnitudes (*a minor annoyance astronomy students have to live with*).

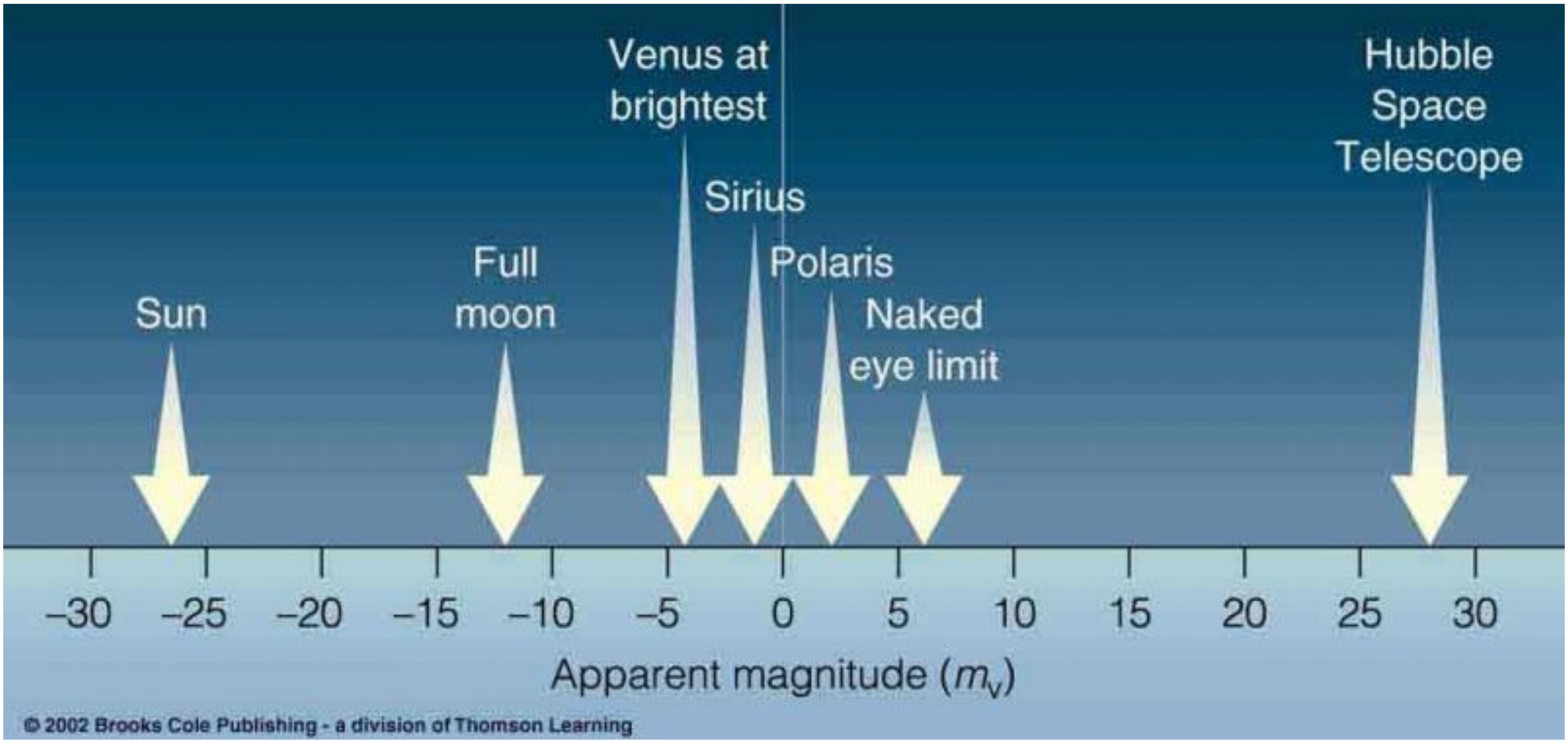
- Mathematically we have the Pogson's ratio:

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda / f_{\lambda,0})$$

where _0 indicate the reference source's magnitude and flux. For example, Vega is usually defined as the reference star and its magnitude is defined as zero.

The magnitude system put everything on a nice logarithmic scale

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda / f_{\lambda,0})$$



Practice: From flux ratio to apparent magnitude

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda/f_{\lambda,0})$$

- Normally in the optical wavelengths, the reference star is Vega.
- For simplicity, Vega's magnitude is set to be zero at all wavelengths

$$\text{For Vega magnitude}: m_\lambda = -2.5\log(f_\lambda/f_{\lambda,\text{Vega}})$$

- What's the magnitude of a star that is 50x fainter than Vega at 500nm?
- What's the magnitude of a star that is 30x fainter than Vega?

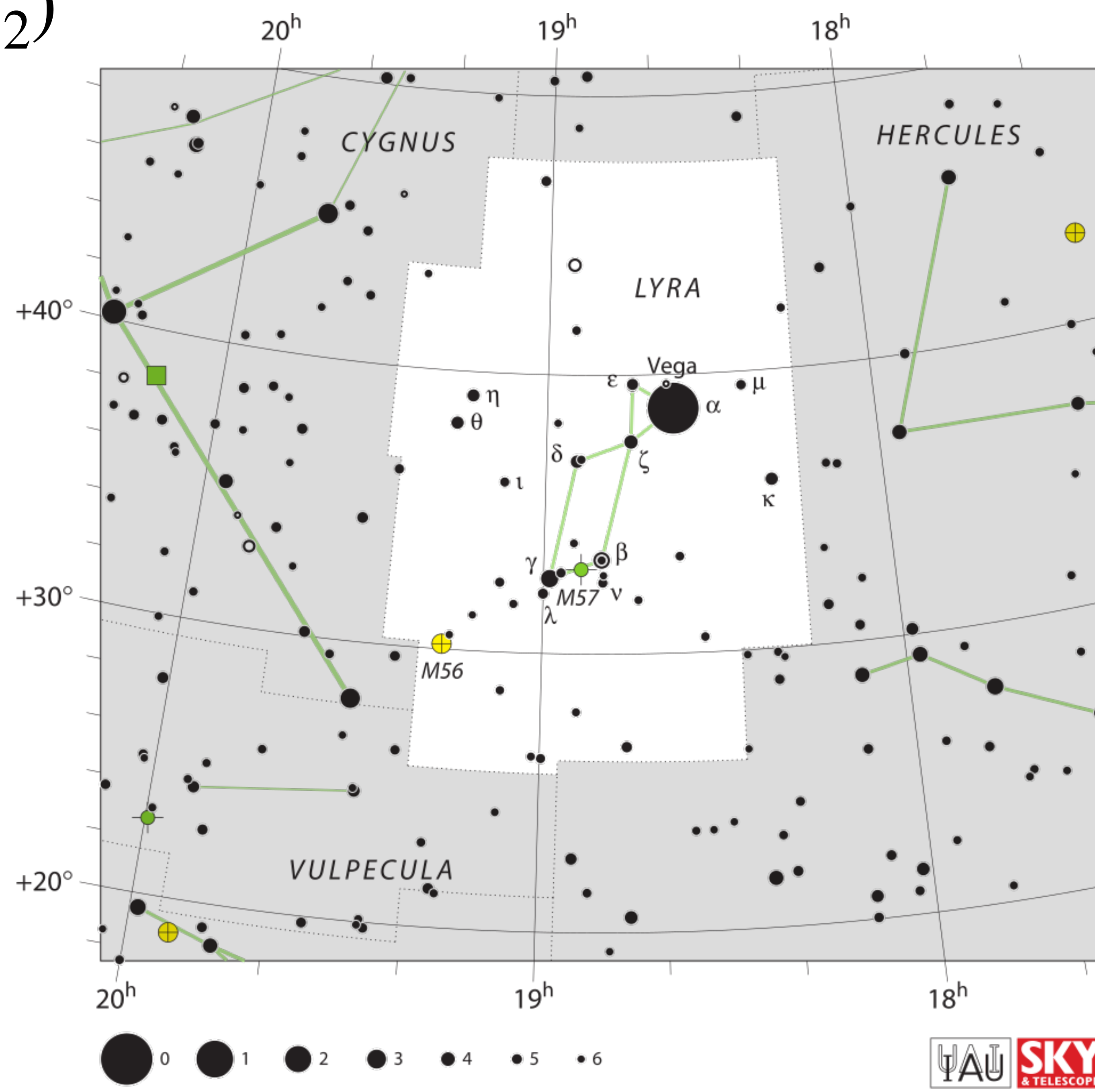
m(50x fainter) = 4.25
m(30x fainter) = 3.69

Practice: From apparent magnitude to flux ratio

$$\text{Pogson's ratio}: m_{\lambda,1} - m_{\lambda,2} = -2.5\log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

$$\Rightarrow \frac{f_{\lambda,1}}{f_{\lambda,2}} = 10^{-0.4(m_{\lambda,1}-m_{\lambda,2})}$$

- δ Lyrae has an apparent magnitude of 4.2 in V-band (551 nm), how many times fainter is it compared to Vega (α Lyrae)?
- 17 Lyrae has an apparent magnitude of 5.2 in V-band, how many times fainter is it compared to δ Lyare?



10^(0.4*4.2) = 47.9

10^(0.4*(5.2-4.2)) = 2.512

Summary: Apparent Magnitude and Flux Ratio

- 100x in flux ratio corresponds to a magnitude difference of 5
- 1 magnitude difference corresponds to 2.514x difference in flux

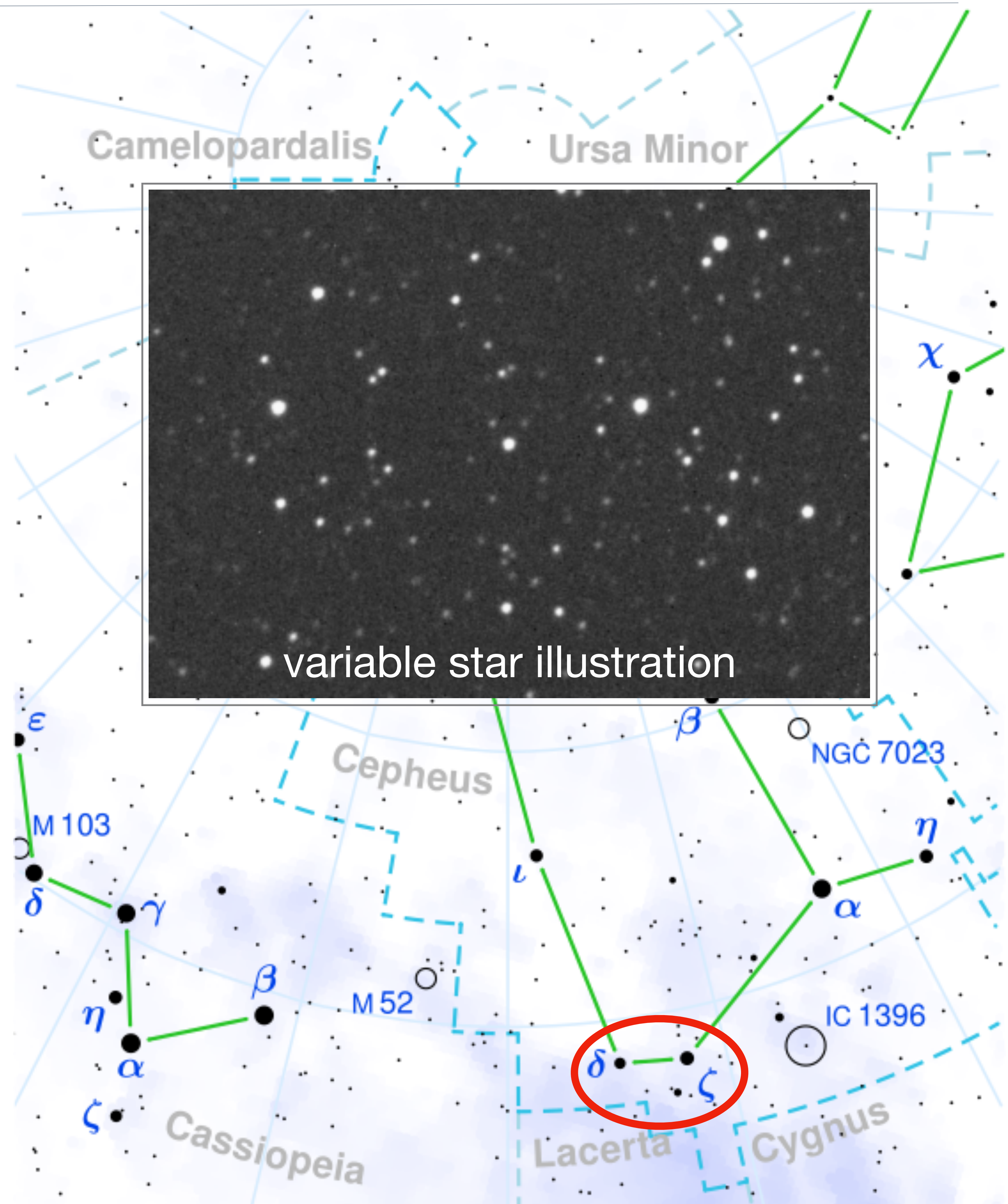
- To determine the magnitude of one source, you must know the magnitude and flux of another source (reference or standard) and compare the fluxes of the two sources

$$m_{\lambda,1} - m_{\lambda,2} = -2.5\log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

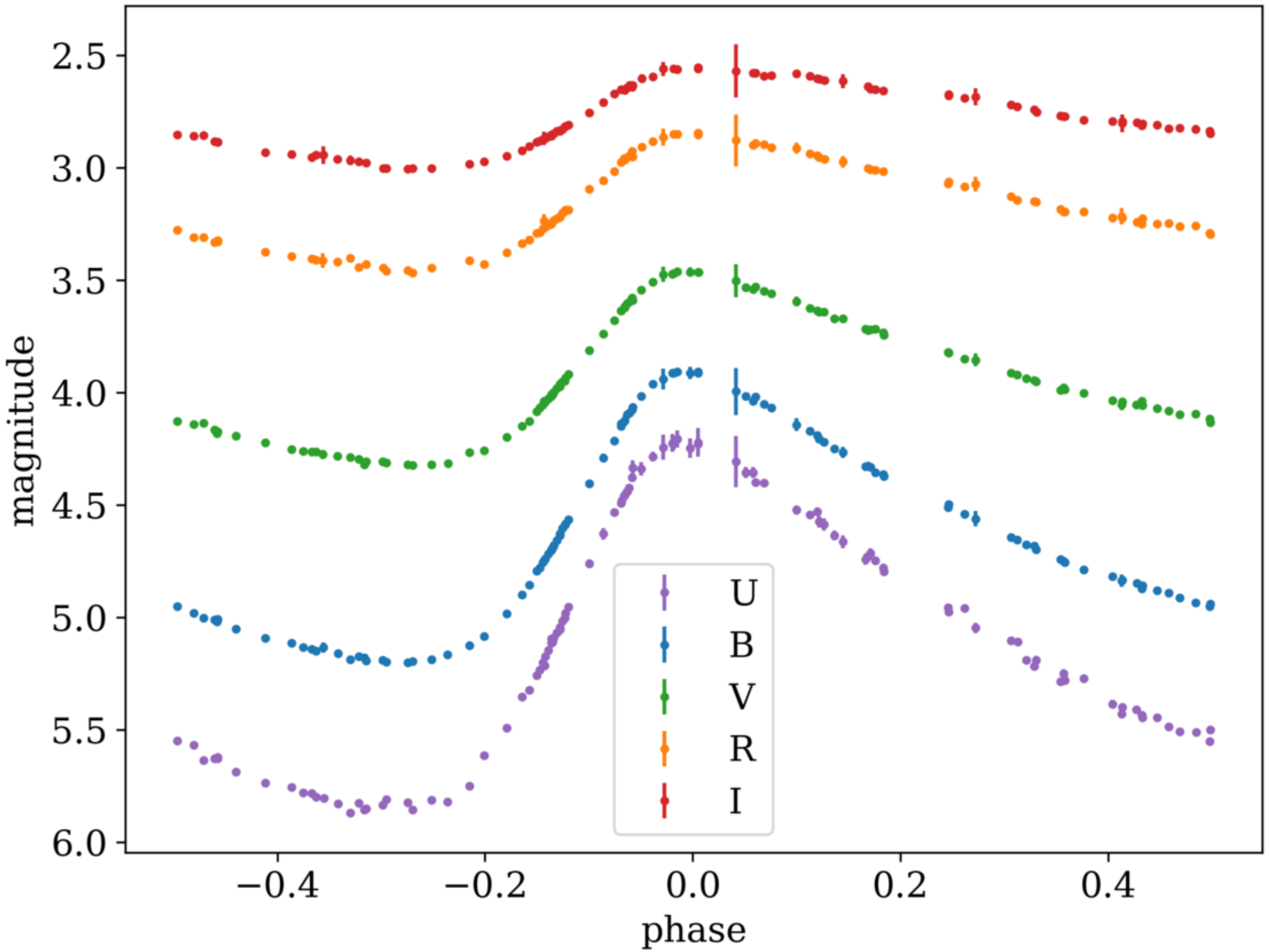
$$\Rightarrow \frac{f_{\lambda,1}}{f_{\lambda,2}} = 10^{-0.4(m_{\lambda,1}-m_{\lambda,2})}$$

Differential Photometry: Compare the count rates between sources

- We can point the same telescope at two difference sources and measure their relative fluxes.
- This approach is easier because all instrumental effects in the two measurements cancel out.
- If we know the magnitude from one of the sources, we can infer the magnitude of the other source using this relative measurement.



Delta Cephei: the prototype Cepheid variable (discovered in 1784)

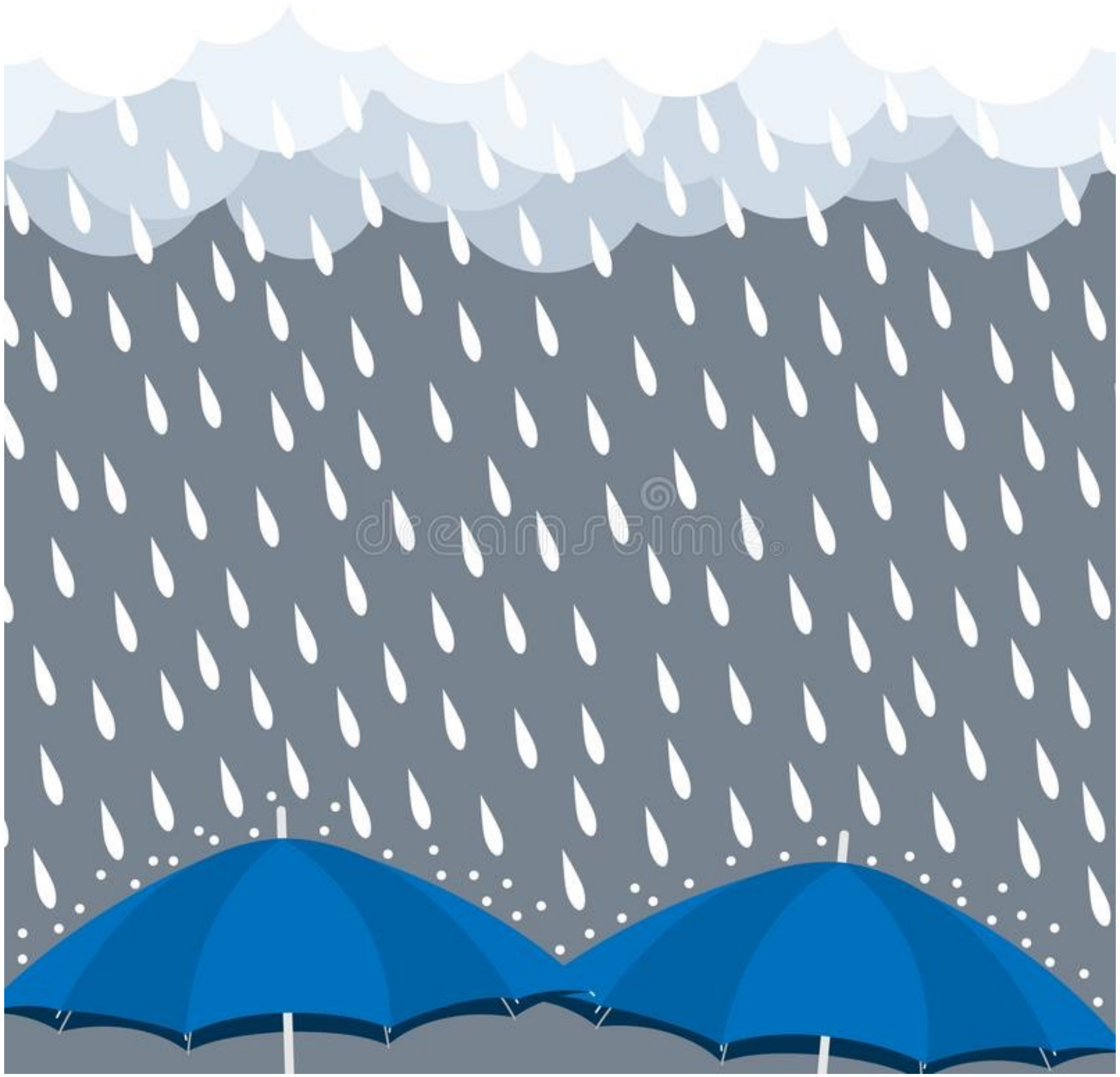


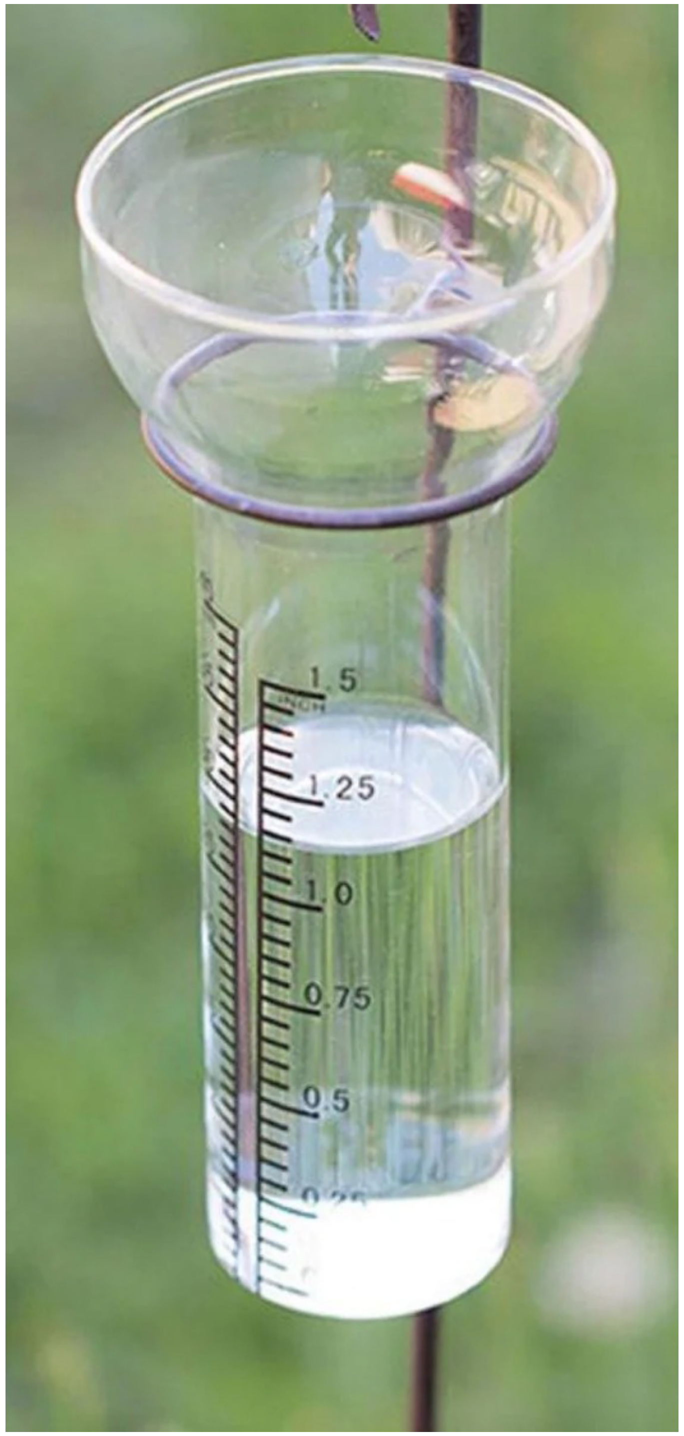
The Modern Technique of Brightness Measurements:

CCD Photometry

Measure the flux from rainfall

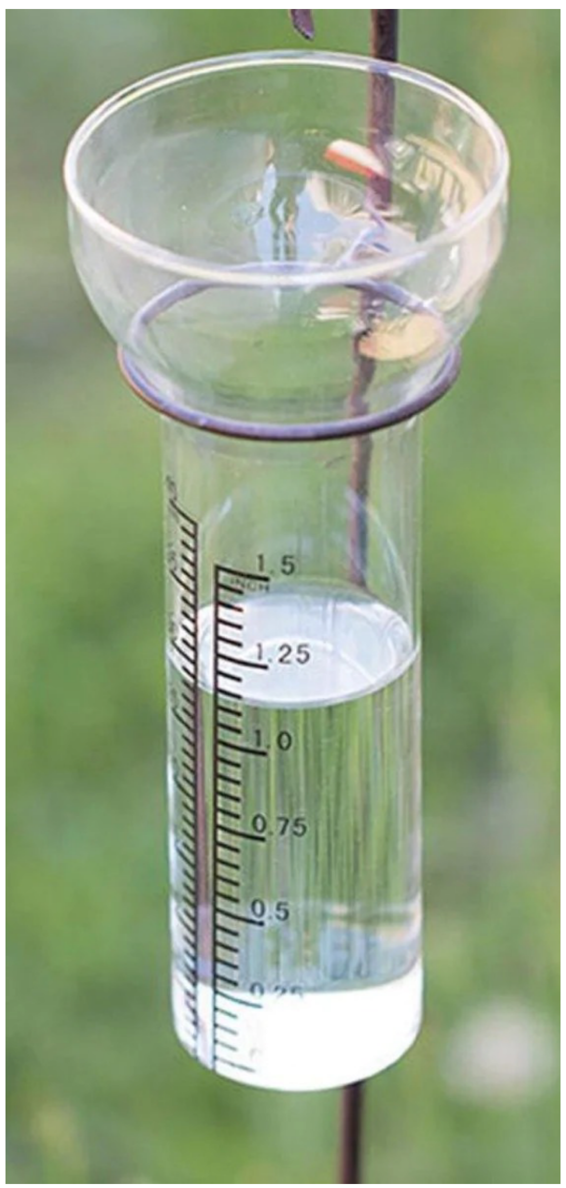
- The level of a rainfall can be measured with a **rain gauge,** you empty it first, let it sit in the rain for an hour, take it back and read off the result from the side: **XX mm/hour**



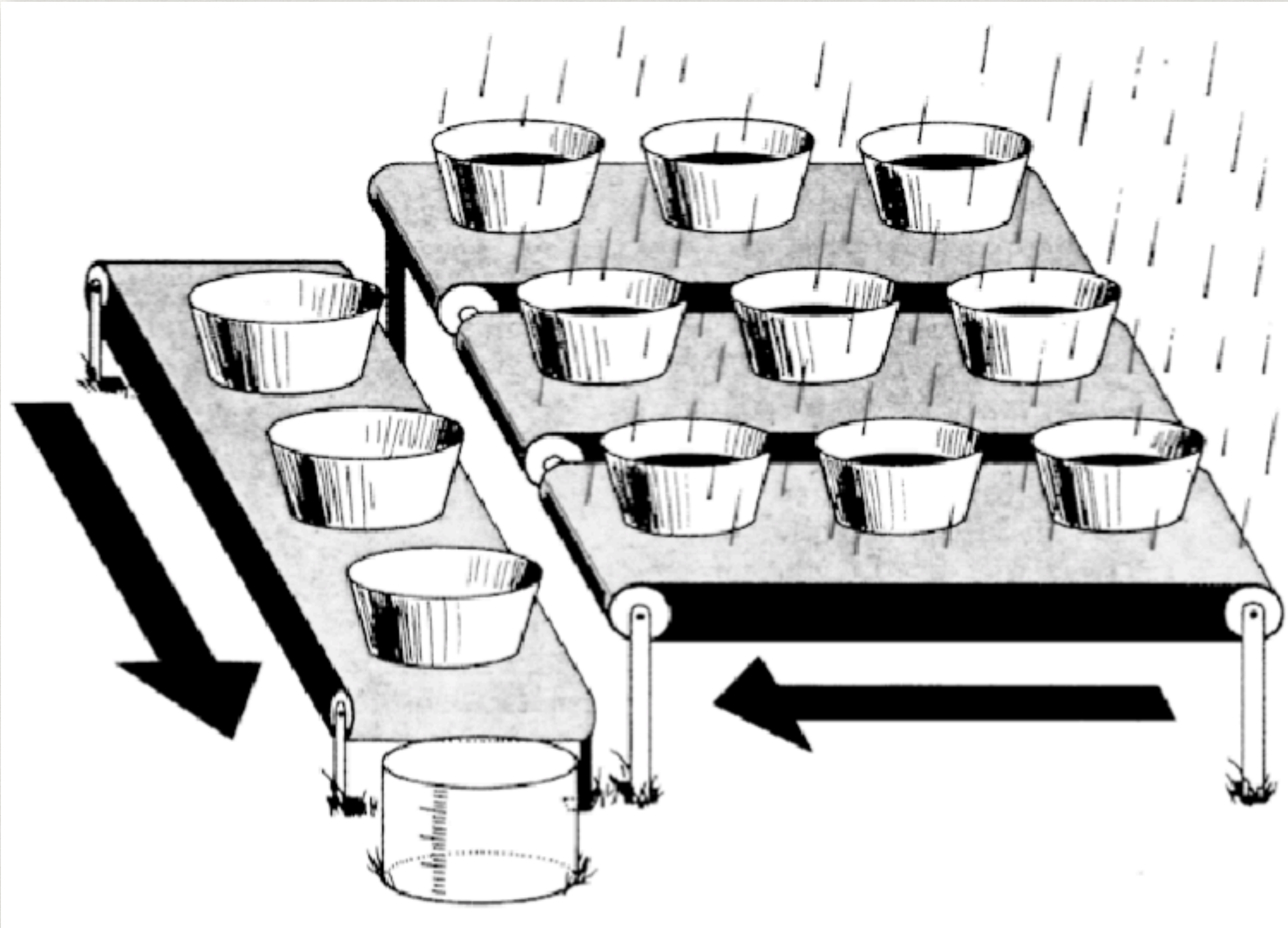


Measure the energy flux from photons

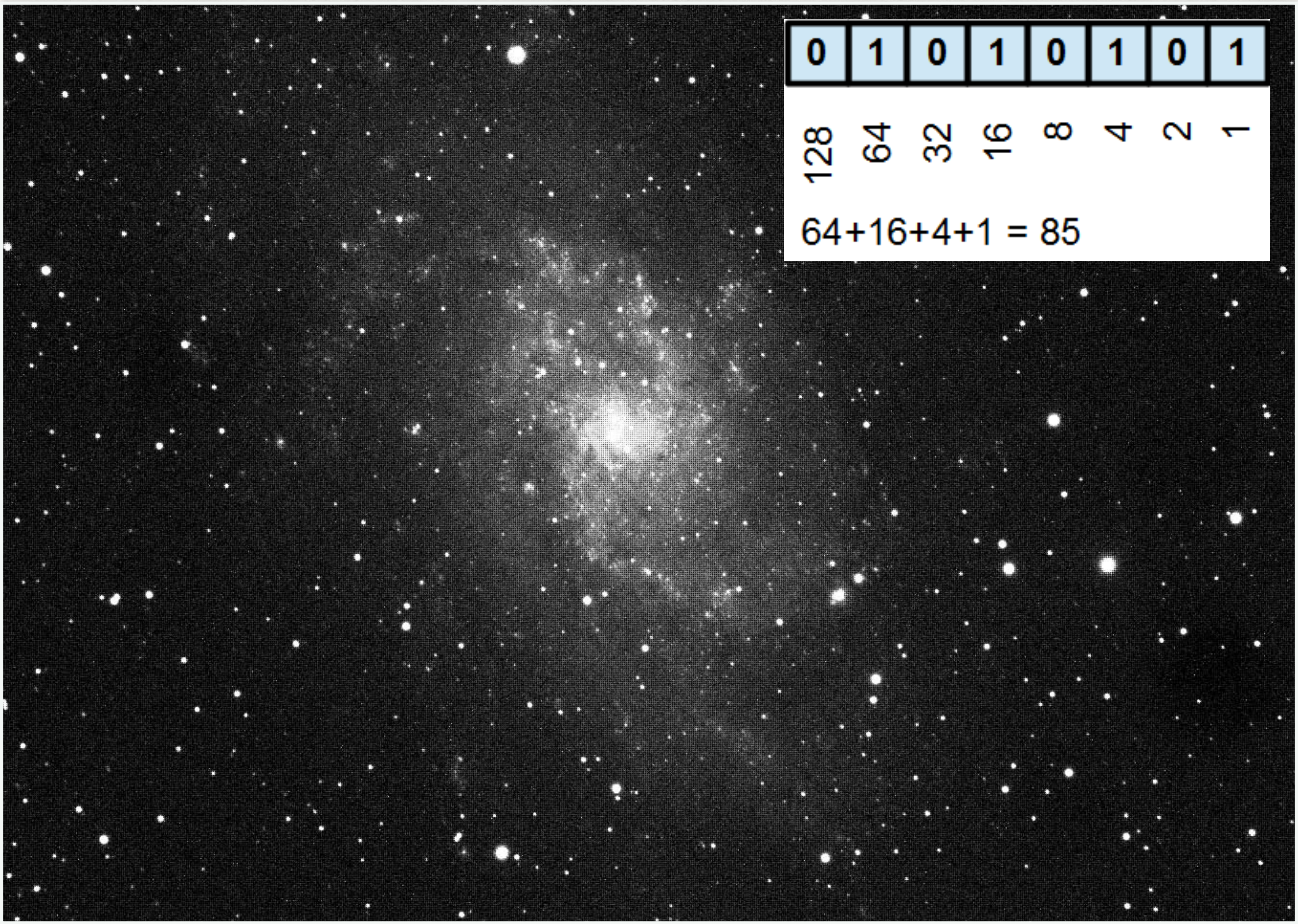
- **Flux** is the amount of arriving **energy per unit time per unit area** (unit: Watt/m^2) at the location of the observer, it can be measured by counting the number of photons **restricted in a wavelength range**
- Just like measuring rainfall with a rain gauge, we need **a device to count the accumulated photons**, and we also need to know (1) **the aperture of the telescope** and (2) **the integration time**



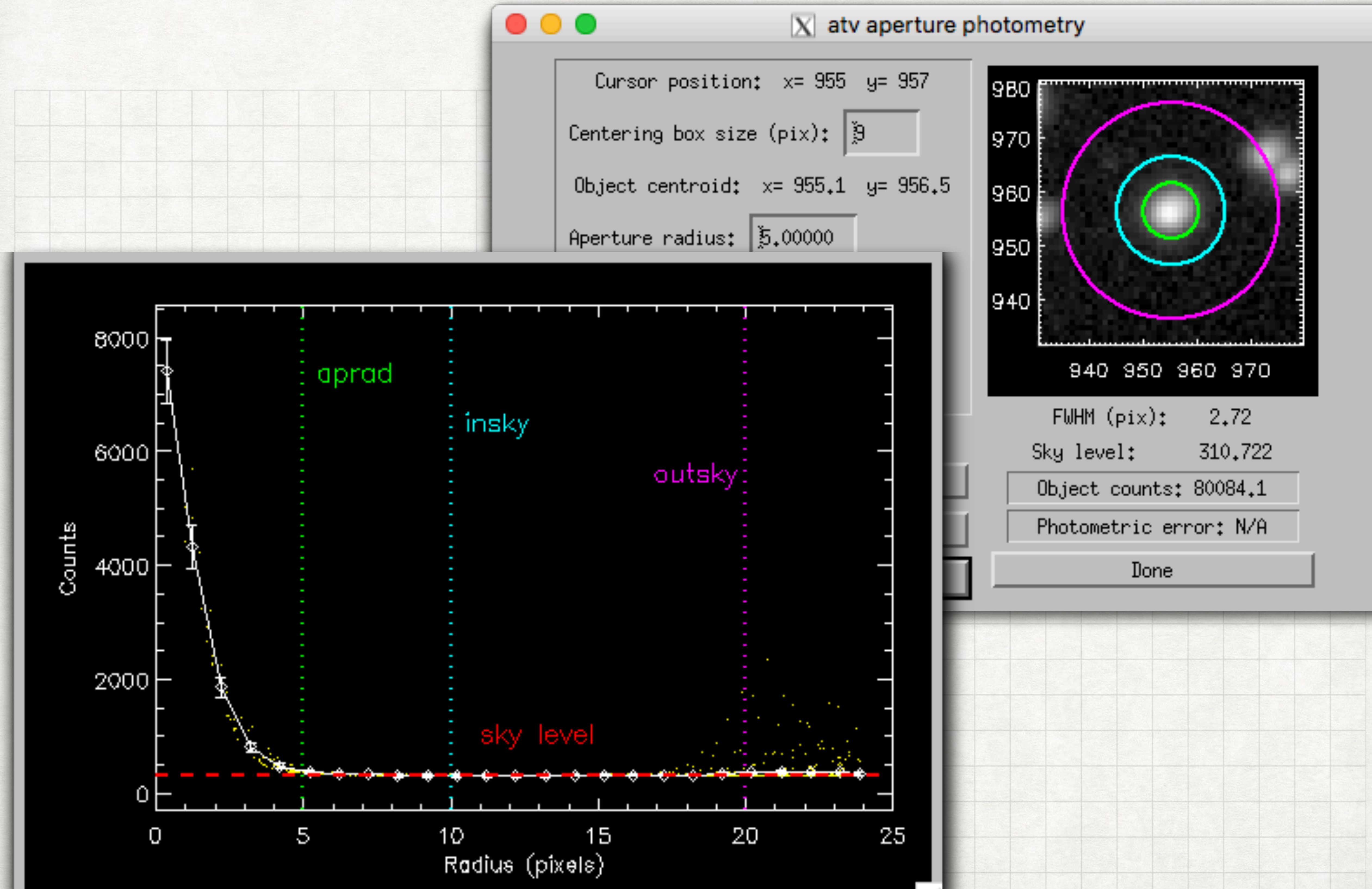
CCD as Light buckets (each bucket is a pixel)



A typical CCD image - data illustrated with DS9: the number of e- collected in each pixel (from 0 to ~65k; 16 bit) is represented by only 256 shades of gray (8 bit)



TO COUNT ELECTRONS FROM A SOURCE, WE USE APERTURES



Definition of Magnitudes is based on Differential Photometry

$$m_\lambda = m_{\lambda \mathrm{ref}} - 2.5 \log(f_\lambda / f_{\lambda,\mathrm{ref}})$$

the canonical reference star is Vega but it is too bright for medium/large telescopes and is not always visible

Count rates to magnitude difference

$$m_a - m_b = -2.5 \log \left(\frac{F_a}{F_b} \right) = -2.5 \log \left(\frac{Q_a/t_a}{Q_b/t_b} \right)$$

where object a is your science target and object b is the standard star with known magnitudes.

Practice: from count rates to magnitude

$$m_a - m_b = -2.5 \log\left(\frac{F_a}{F_b}\right) = -2.5 \log\left(\frac{Q_a/t_a}{Q_b/t_b}\right)$$

where object a is your science target and object b is the standard star with known magnitudes.

Your standard star has a magnitude of 10.5 mag in V-band, you took a CCD image of the standard star with a V-band filter and you got a total of 1500 counts in 10 seconds.

Next, you slew the telescope to take a V-band image of your science target, say a random galaxy far away, and with 30 min exposure, you could barely see it. The total count from the galaxy is 50.

What's the V-band magnitude of the galaxy?

$V_{galaxy} = 10.5 - 2.5 \log((50/1800)/(1500/10)) = 19.83$

The Amazing Design of Charged-Couple Devices (CCDs)

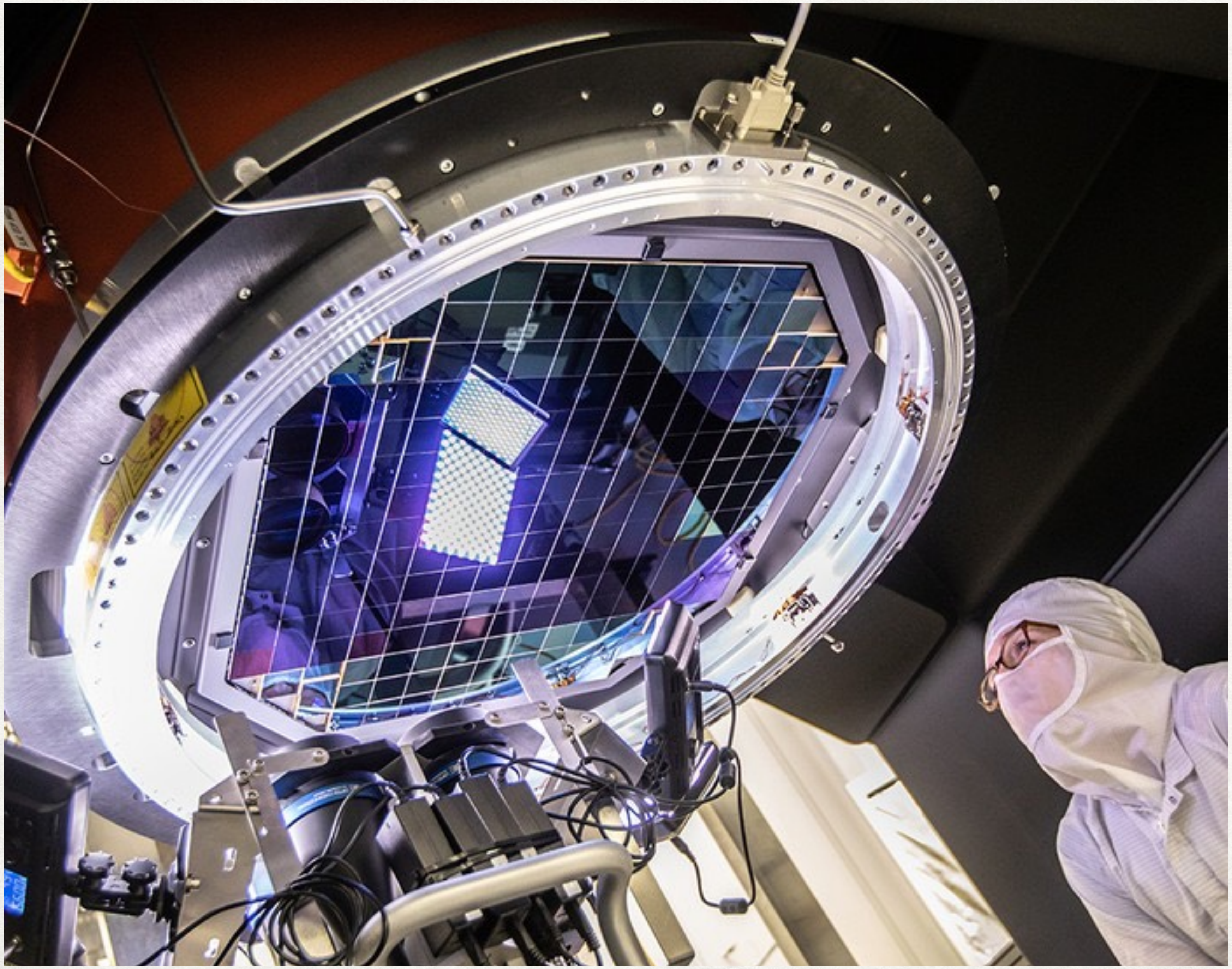
2009 NOBEL PRIZE IN PHYSICS

Willard S Boyle and George E Smith (1969 invention at Bell Labs)

The charge-coupled device (CCD) provided the first way for a light-sensitive silicon chip to store an image and then digitize it, opening the door to the creation of digital images.

CHARGED COUPLE DEVICE (CCD): SEMICONDUCTOR LIGHT BUCKETS

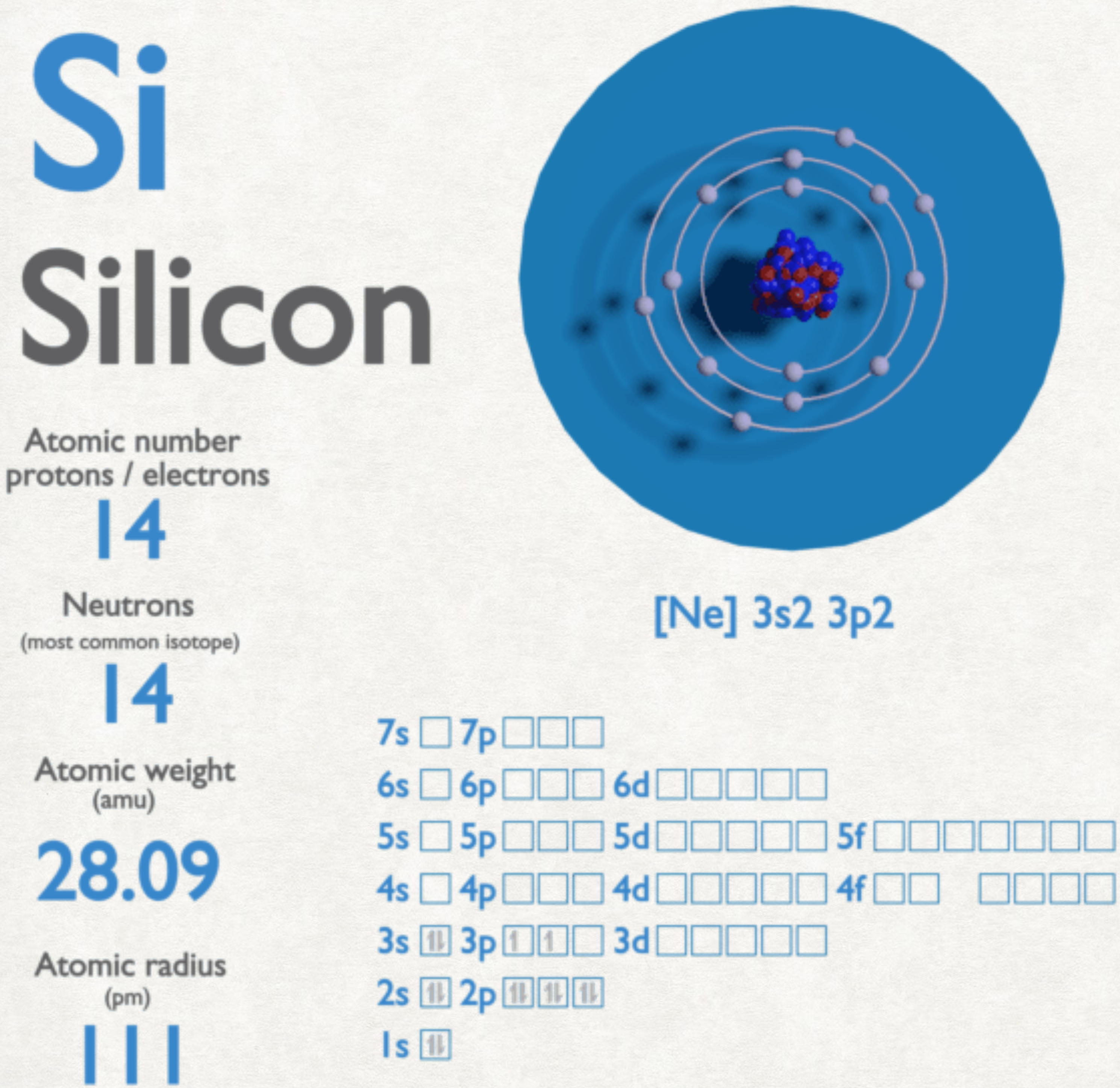
The largest CCD camera today:
189 CCD detectors, each 16 megapixels
Rubin Observatory, 3.2-gigapixel camera

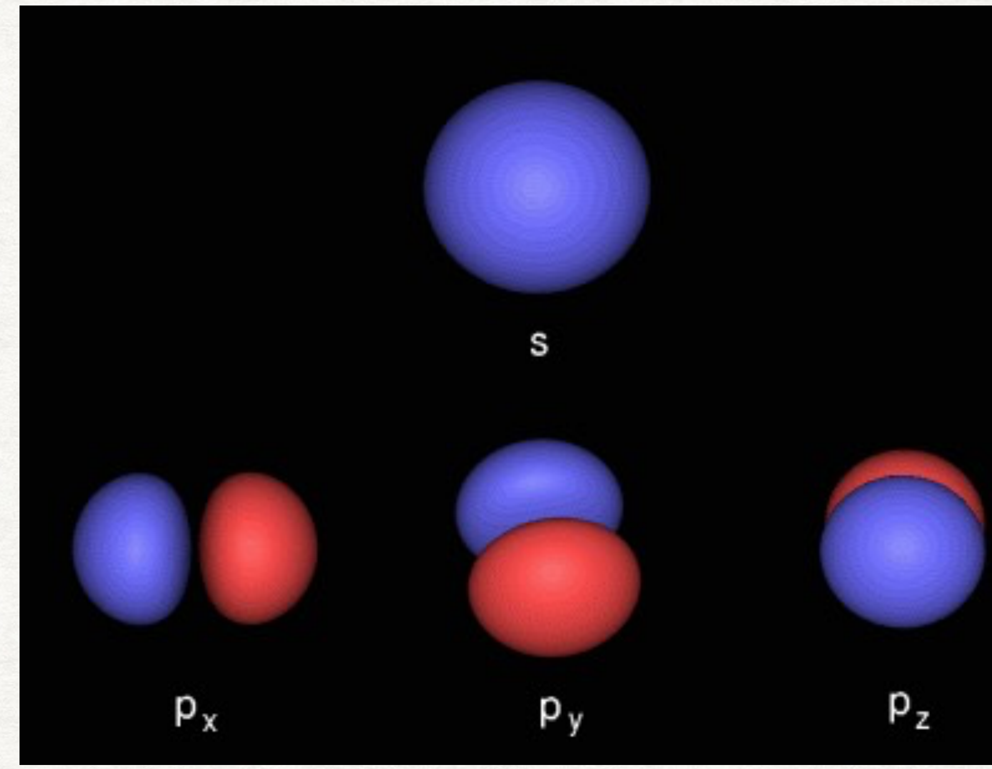


a single-crystal silicon
ingot grown by the
Czochralski method

SILICON: ELECTRONIC CONFIGURATION

Si (Z=14): $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$ (electronic configuration; *l = 0,1,2,3 = s,p,d,f*)



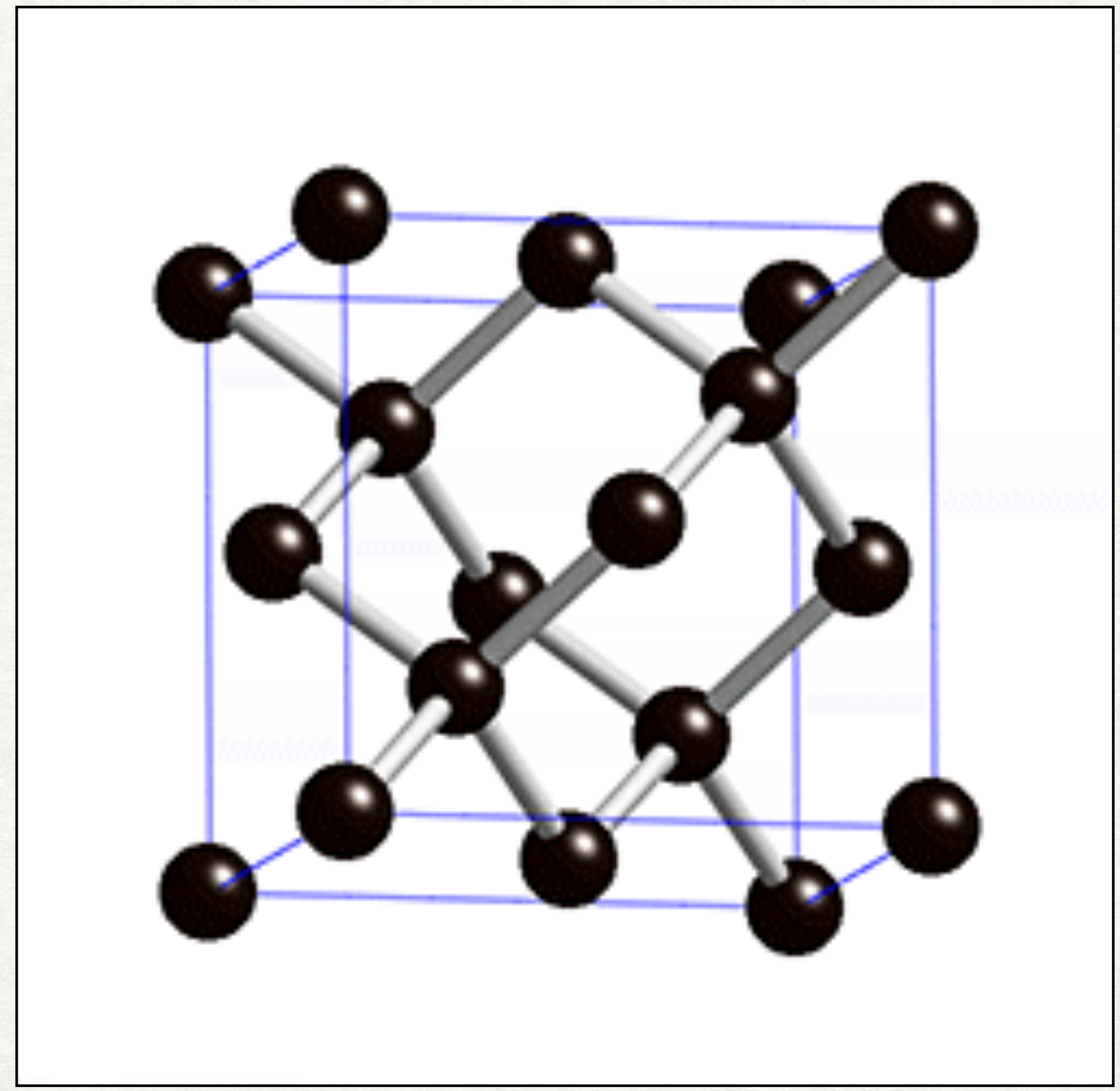


SILICON: CRYSTAL LATTICE STRUCTURE

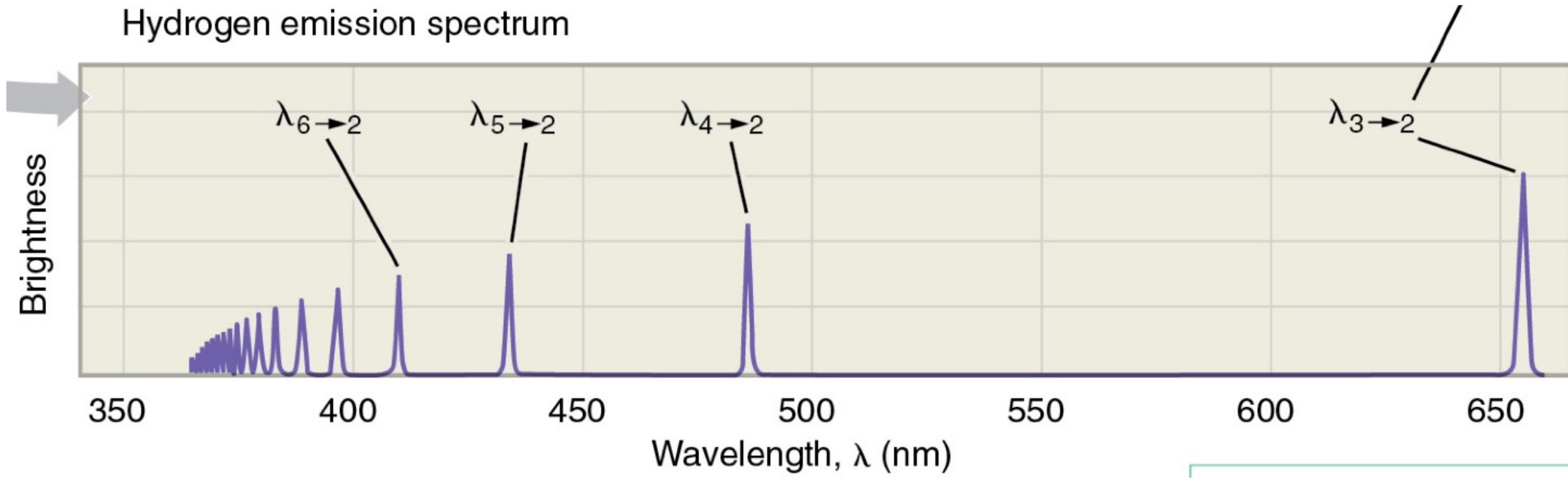
diamond cubic structure with a nearest neighbor interatomic spacing of 235 pm (1 picometer = 1e-12 m),

for comparison, the atomic radius of Silicon is 111 pm

What happens to the e-'s energy levels when we pack atoms so close together?



Isolated Hydrogen Atoms: Energy Levels and Spectral Series

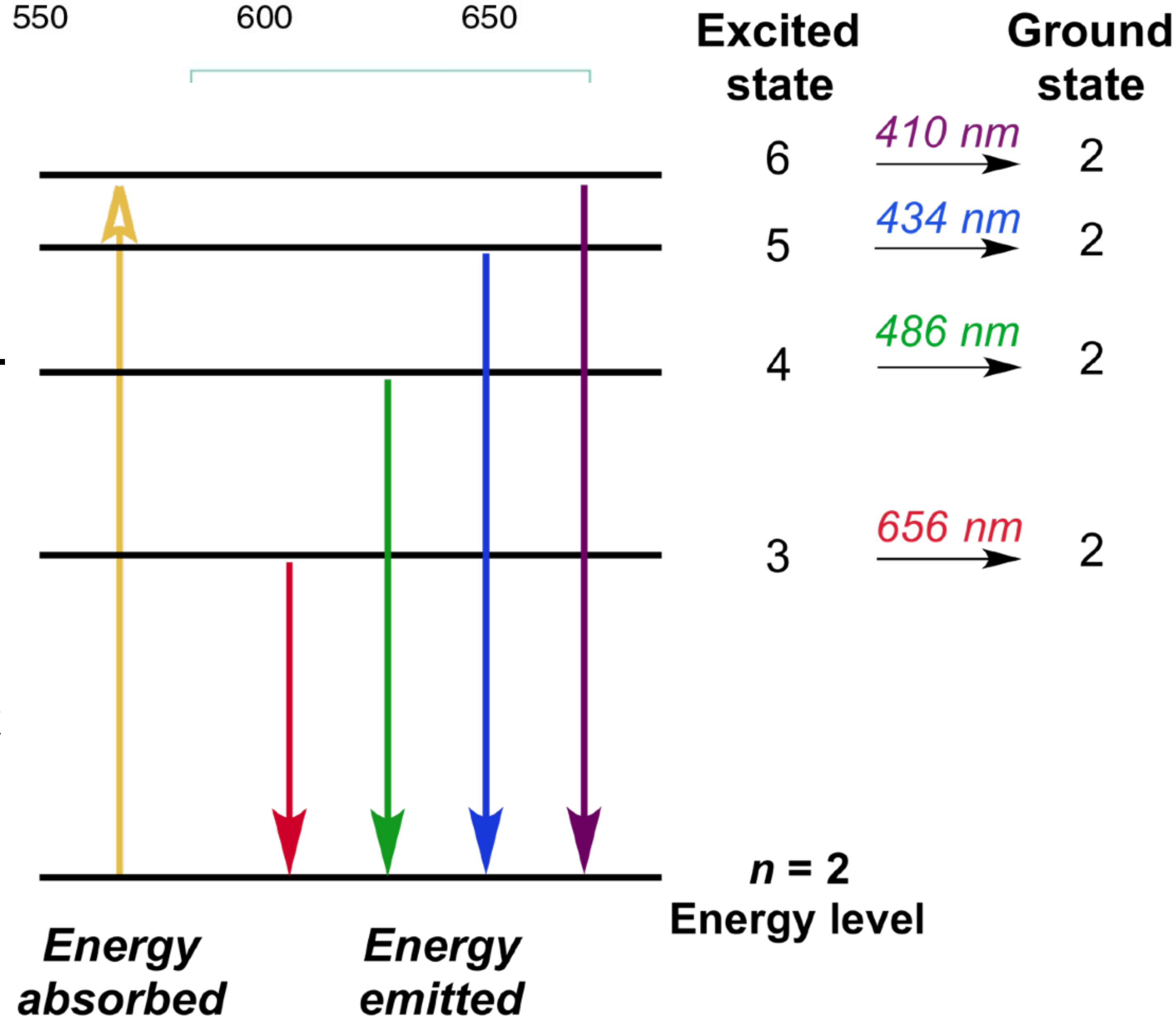


$$\frac{1}{\lambda} = \left(\frac{1}{n_{\mathrm{low}}^2} - \frac{1}{n_{\mathrm{high}}^2}\right)\frac{13.6\mathrm{eV}}{hc}$$

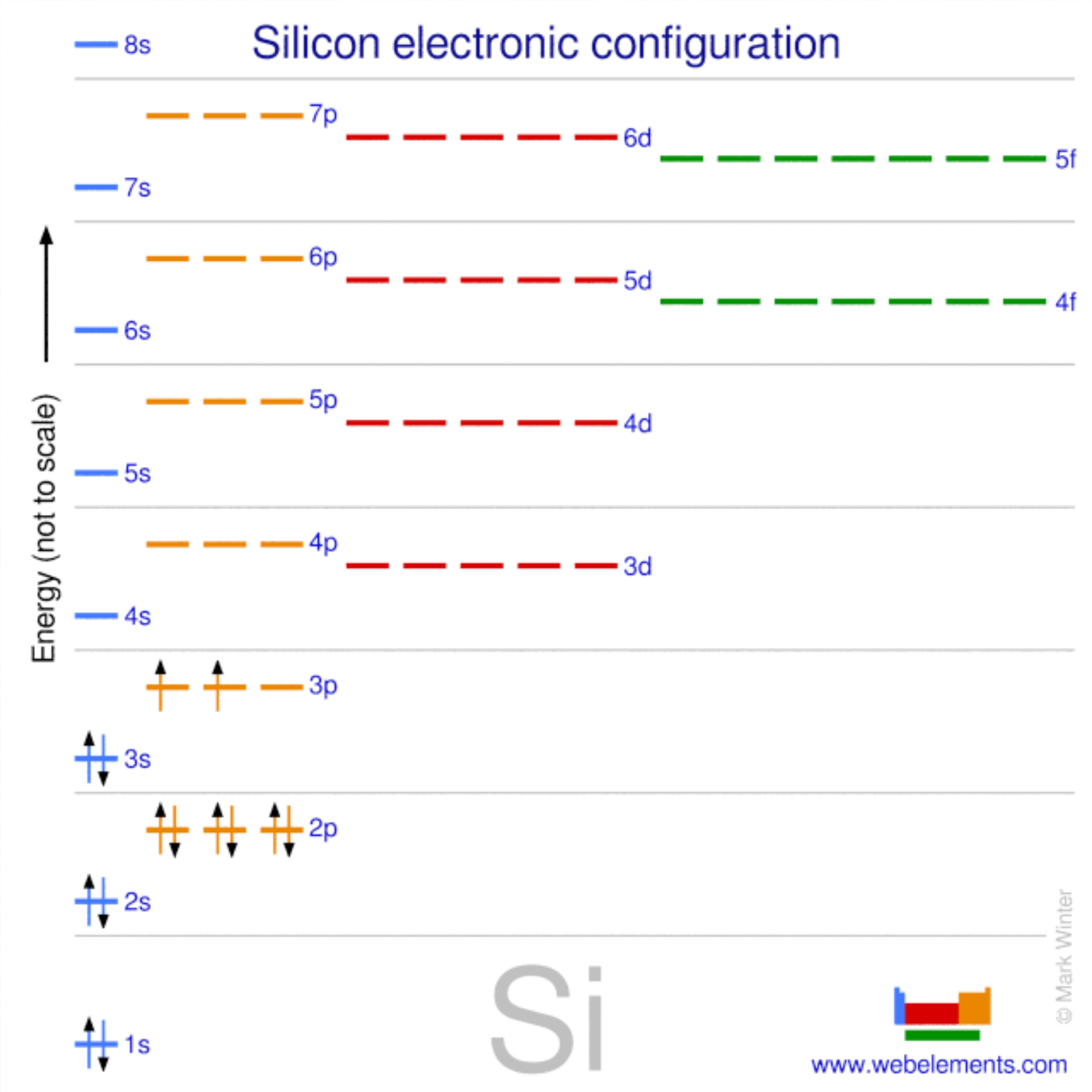
n_{low} = quantum number of lower orbit

n_{high} = quantum number of higher orbit

λ = wavelength of emitted photon



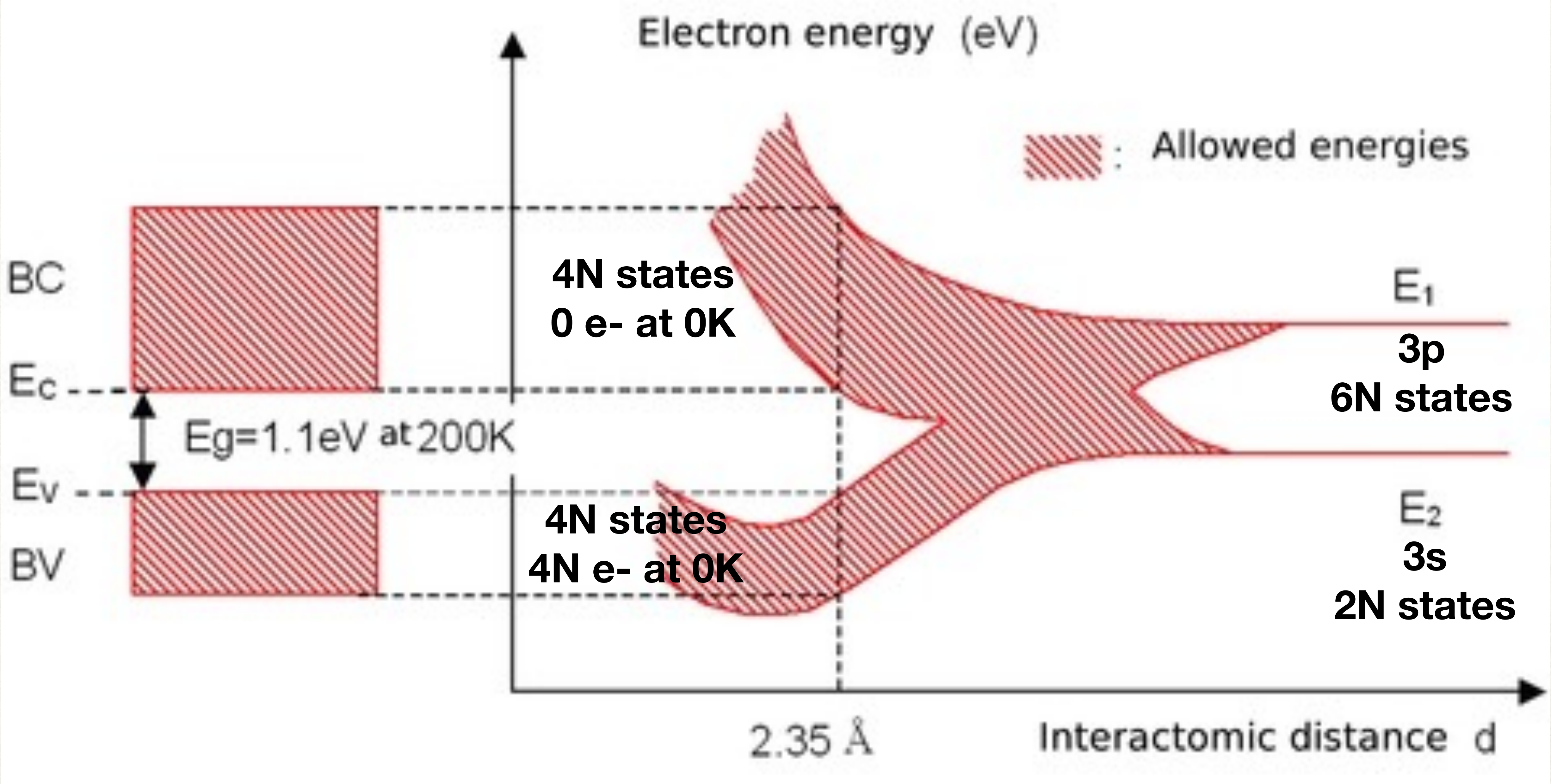
SILICON: ELECTRON ENERGY LEVELS



ENERGY GAP OF SILICON CRYSTALS

Si (Z=14): $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$ (electronic configuration)

At the actual interatomic spacing, silicon crystals develop an inaccessible energy band gap of 1.1 eV between a lower **valence band** and an upper **conduction band** **in the outermost n=3 shell**



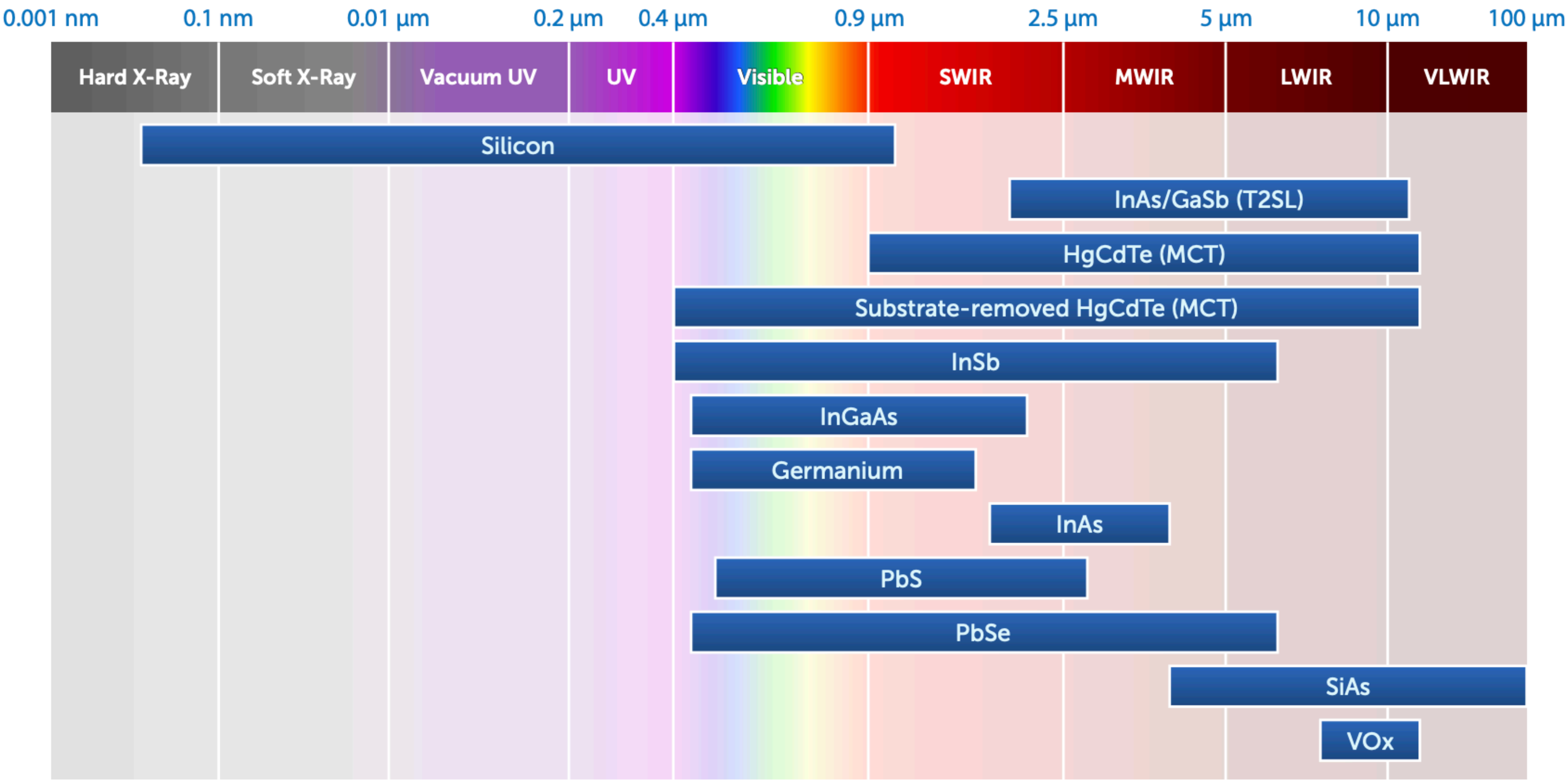
Practice: Energy (eV) - Wavelength (micron) Conversion

- Energy is often given in units of **electron-volt (eV)**, which is the amount of kinetic energy gained by a single electron accelerating through an electric potential difference of one volt
- Wavelength is often given in units of micron (um)
- **1 eV = 1.602e-19 J, h = 6.626e-34 J/Hz, c = 3e8 m/s**, given $E = hc/\lambda$, calculate the wavelength (in micron) of photons with energies of 1 eV.

$$\lambda = 1.24\ \mu\mathrm{m}\ \left(\frac{E}{1\ \mathrm{eV}}\right)$$

Conclusion: In Silicon, electrons can be excited from valance band to conduction band by photons with wavelengths shorter than 1.1 micron, which include UV, optical, and near-IR

Semiconductor Detectors for Astronomy



Practice: Energy (eV) - Temperature (Kelvin) Conversion

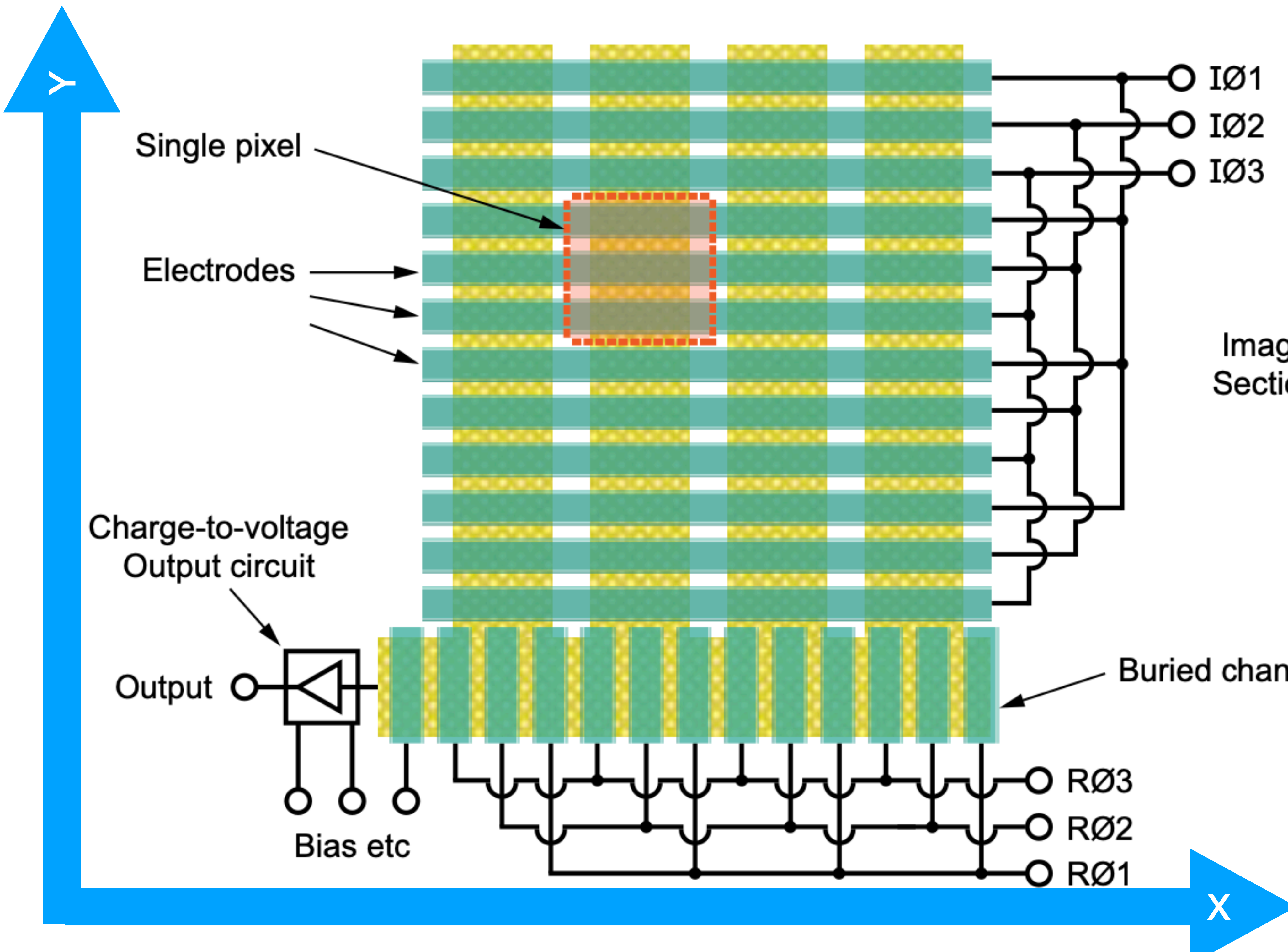
- Energy is often given in units of **electron-volt (eV)**, which is the amount of kinetic energy gained by a single electron accelerating through an electric potential difference of one volt
- Given **1 eV = 1.602e-19 J, k = 1.38e-23 J/K**, given $E = kT$, calculate the temperature (in K) that corresponds to a thermal energy of 1 eV.

$$T = 11604 \text{ K} \left(\frac{E}{1 \text{ eV}} \right)$$

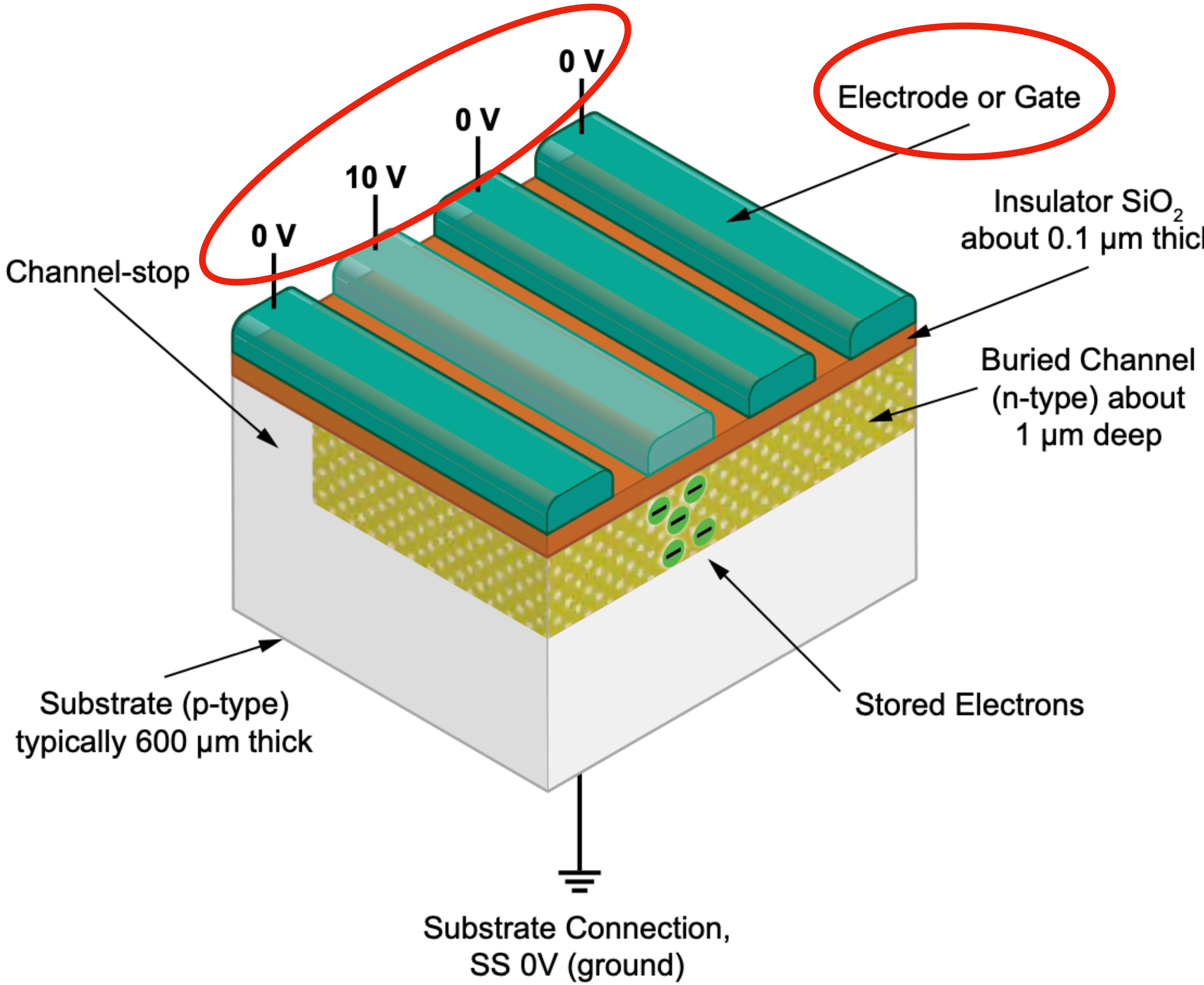
Conclusion: Pure Silicon has very few electrons in the conduction band at room temperature (~300 K), making it a poor conductor (resistivity: $\rho = 2 \times 10^5 \ \Omega \cdot \text{cm}$).

“Pixels” are constructed by channels and electrodes (gates)

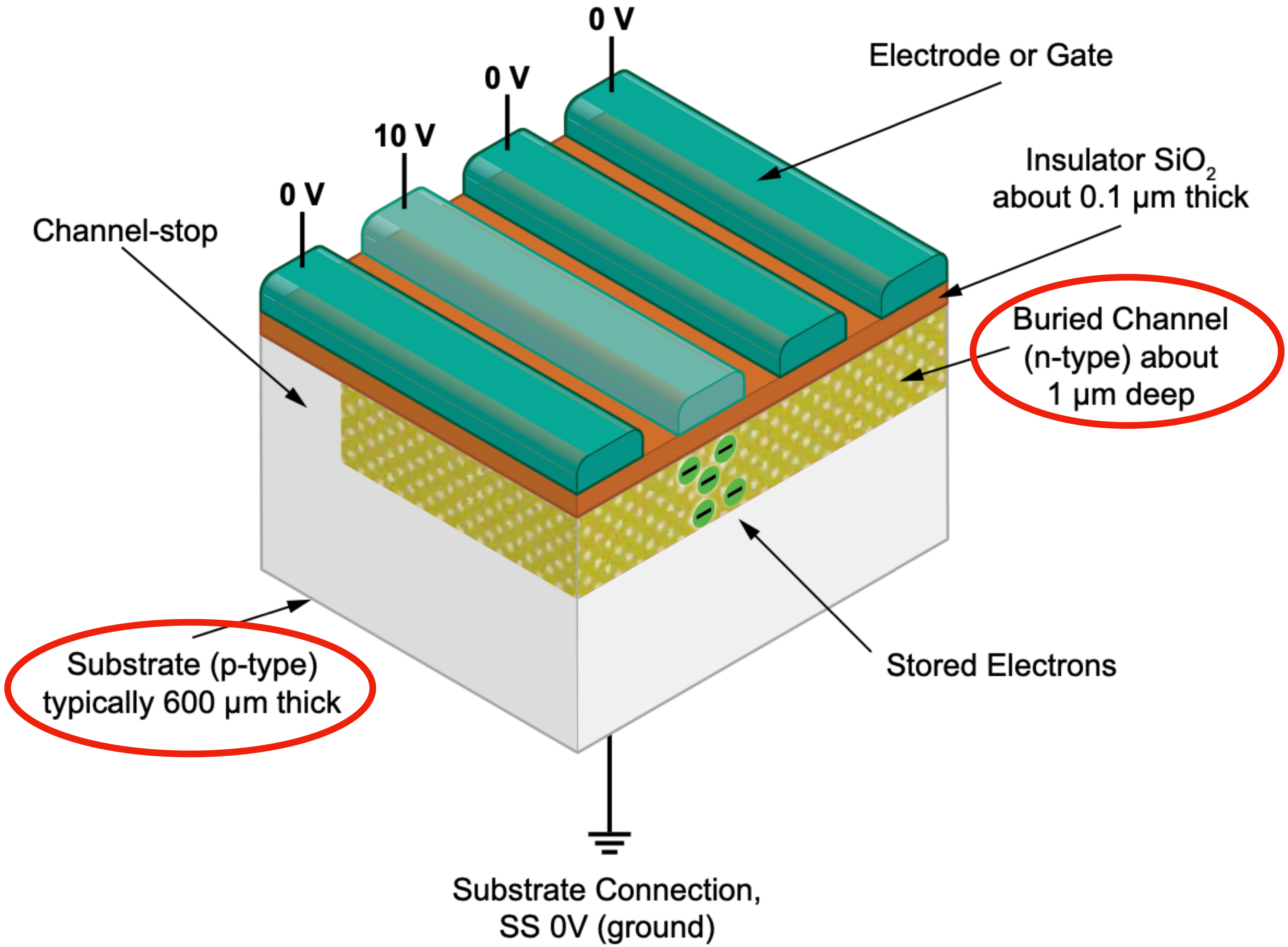
Photon absorption causes electrons in valence band to move to conduction band, our device needs to **hold** these electrons in a **bucket**



To keep electrons at a fixed Y-position, CCDs use electrodes

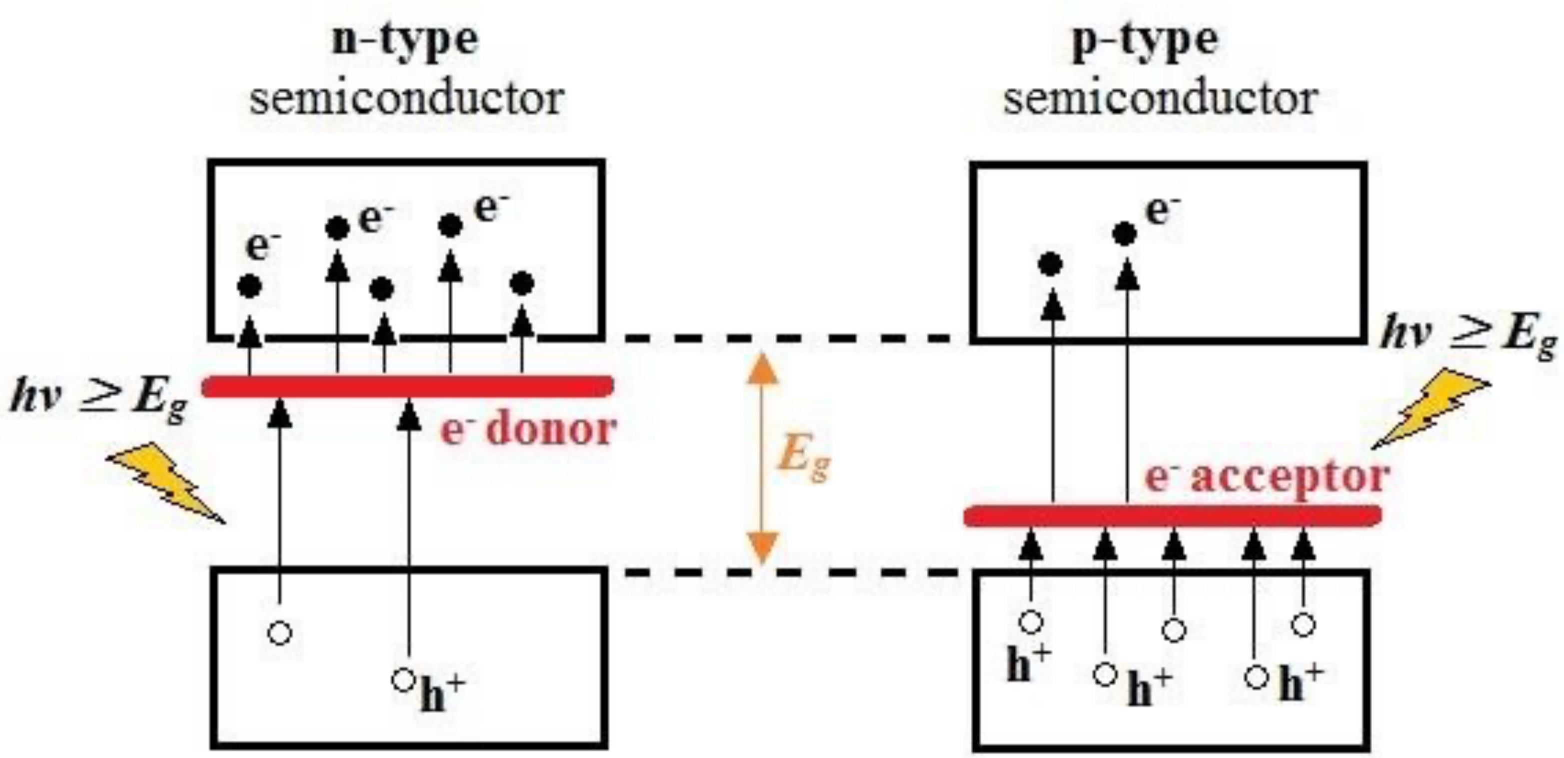


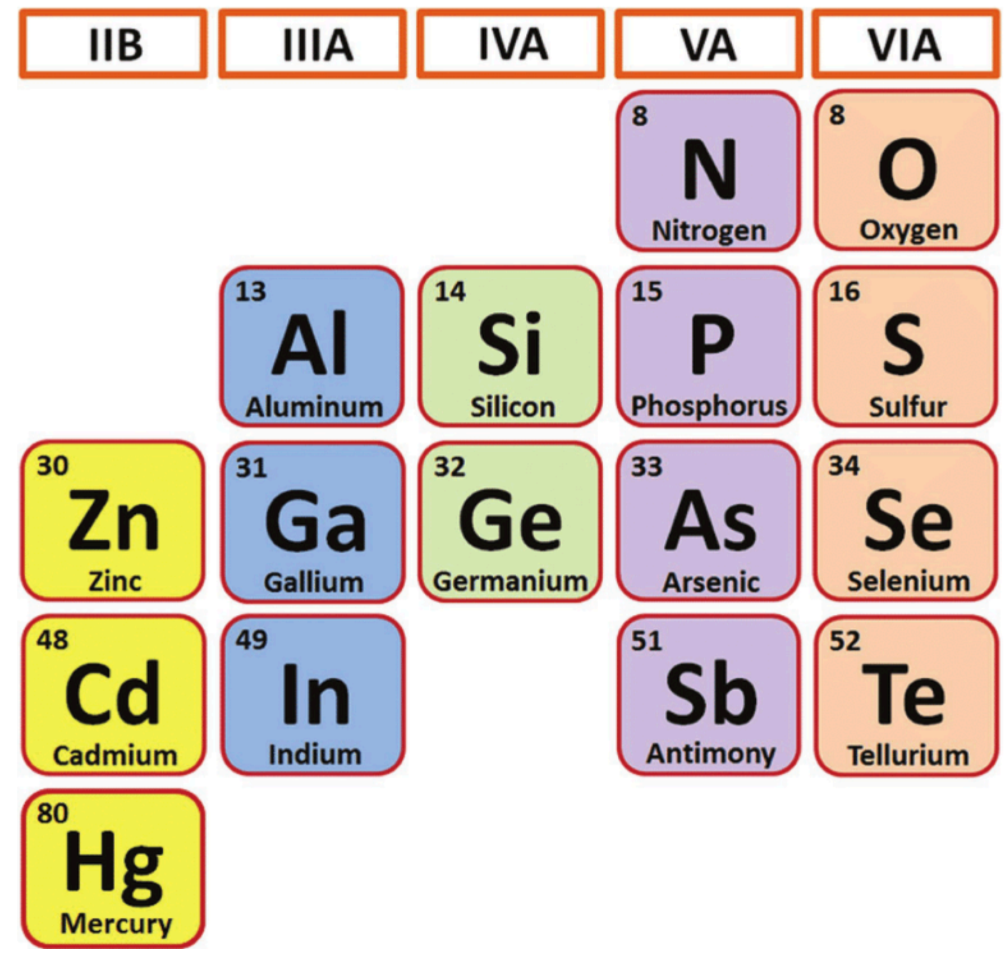
How to keep electrons at a fixed X-position?



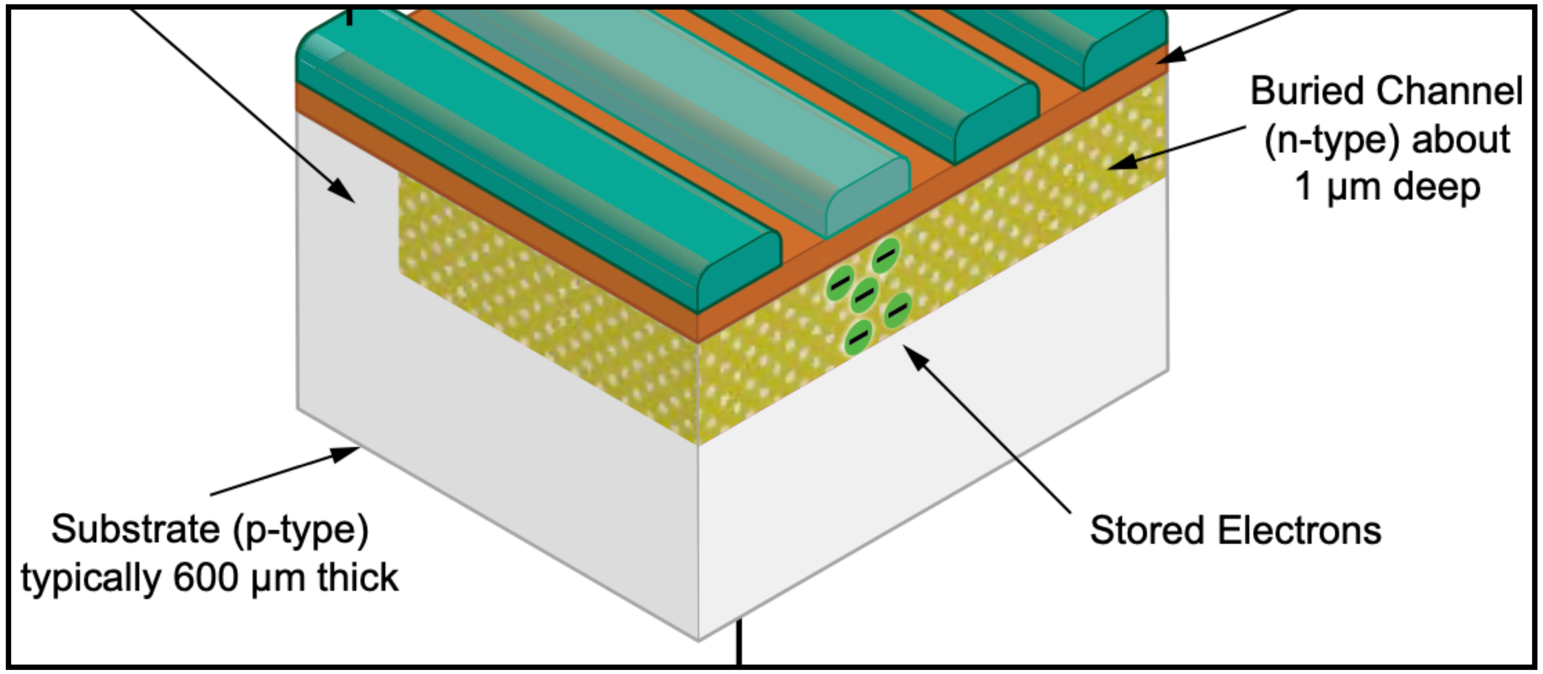
Doping: p-type and n-type semiconductors

- A doping ratio of 2 As (Arsenic, Class V) atoms in 100 million Si atoms would decrease Silicon's resistivity by 40,000 times, making it an **n-type**
- Doping Silicon with class III elements (e.g., Gallium) makes it a **p-type**

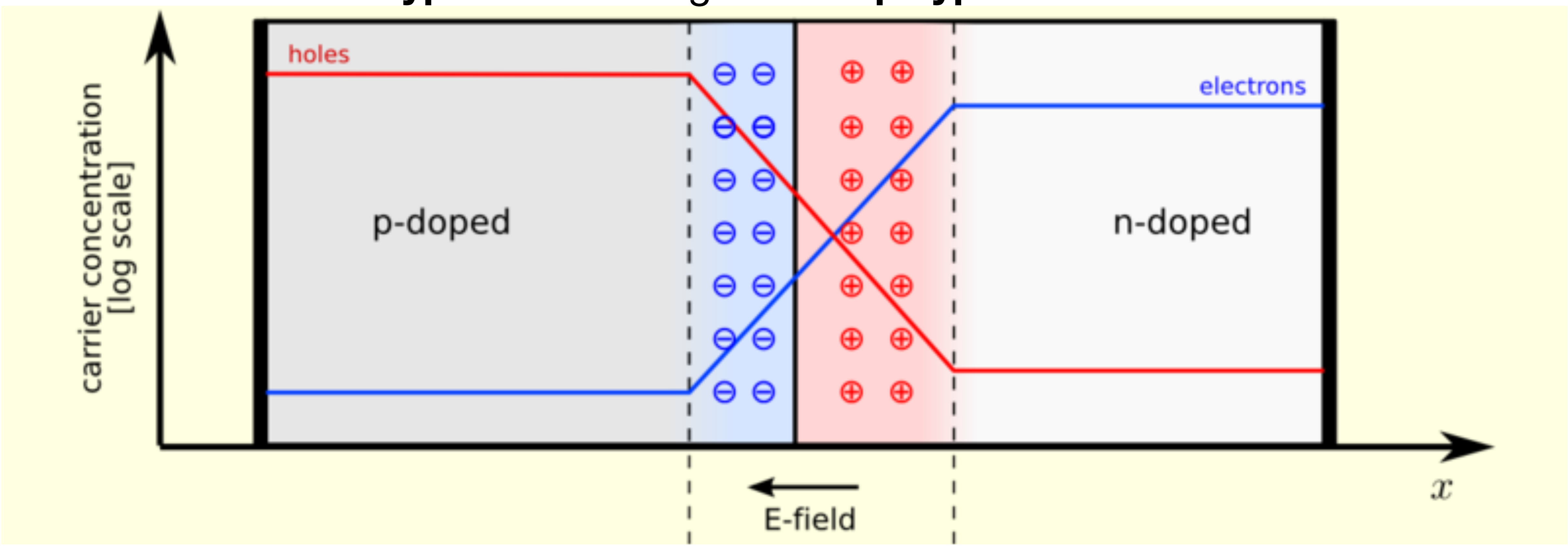




To keep electrons at a fixed X-position, CCD uses p-n interfaces

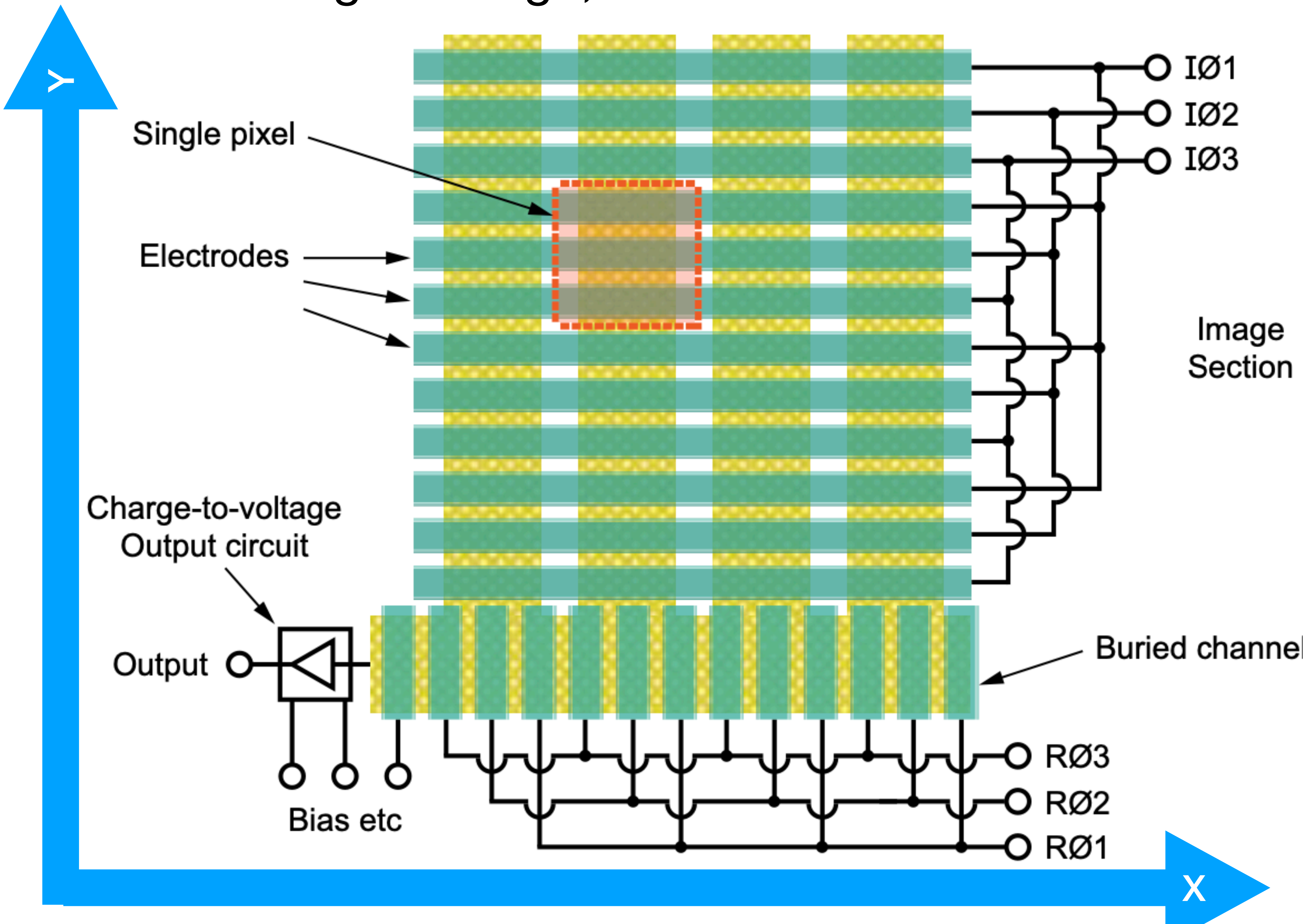


Thermal conduction-band electrons in **n-type** diffuse into **p-type** and combine with the holes in **p-type**; this diffusion forms an electric field near the interface, preventing future electrons in **n-type** from leaking into the **p-type** substrate

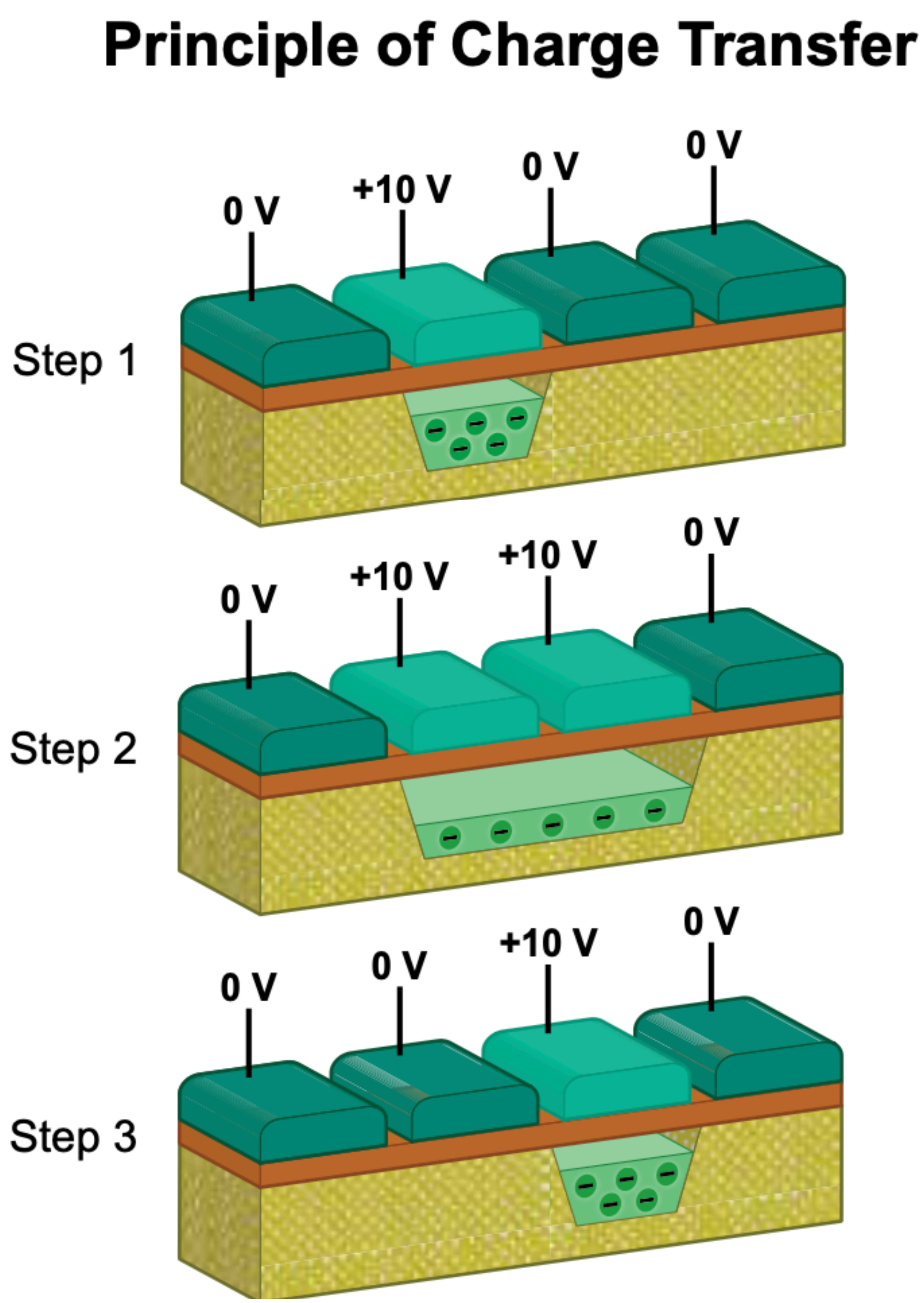


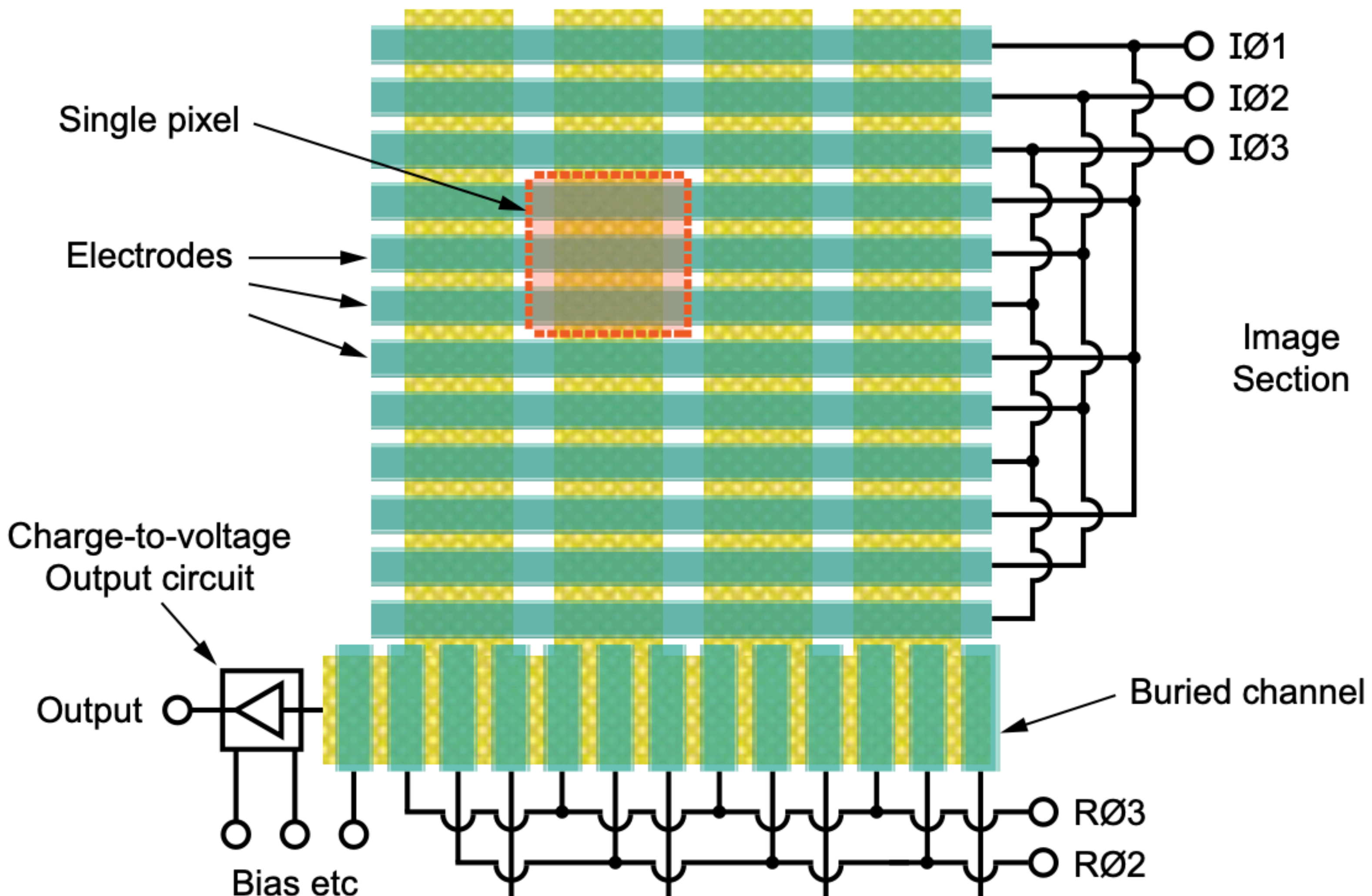
"Pixels" are constructed by vertical channels and horizontal electrodes

Photon absorption causes electrons in valence band to move to conduction band, each pixel is designed to **hold** these electrons. But to obtain a digital image, we need to **read** the electrons

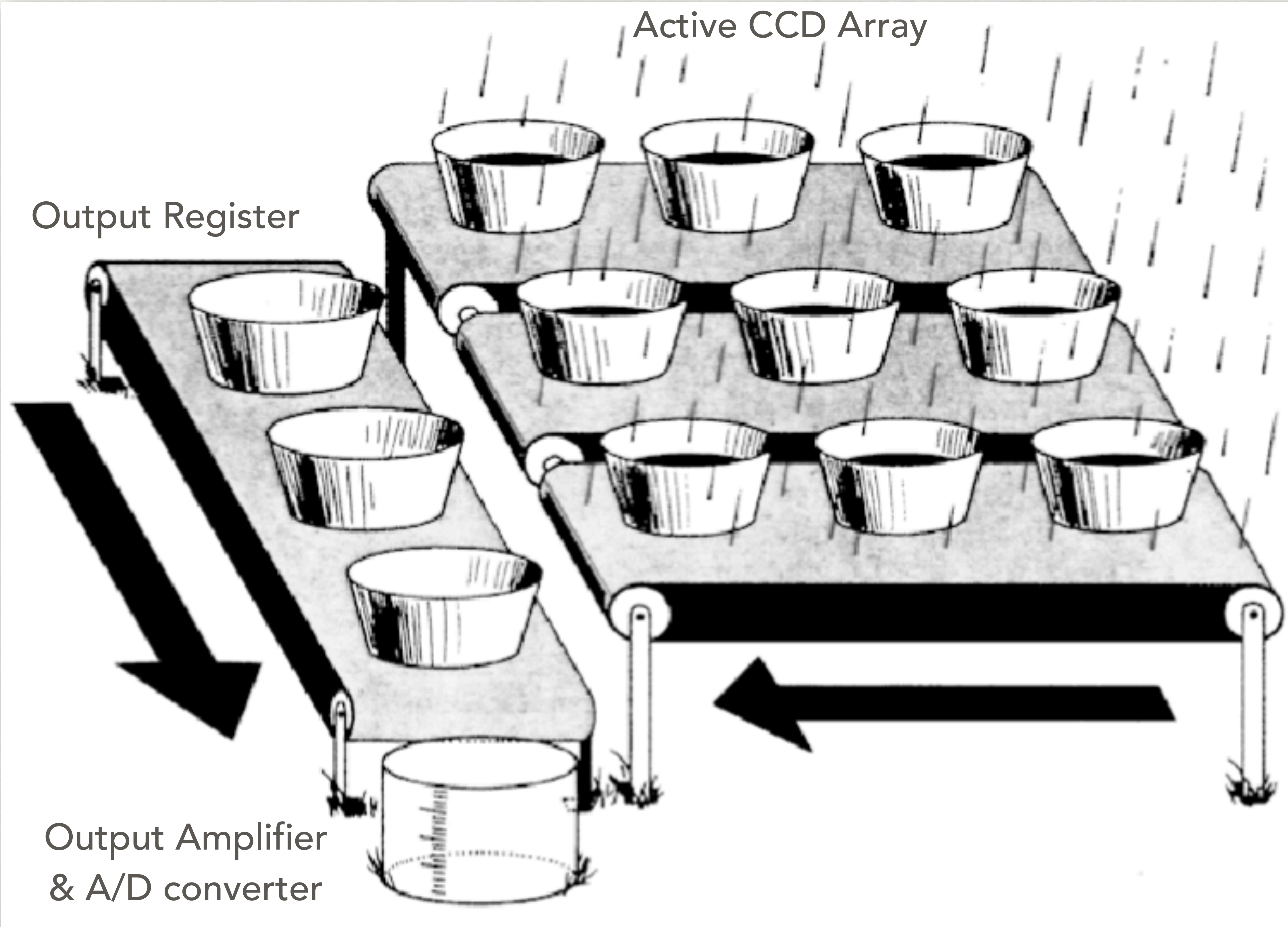


Charge transfer along the columns (Y-direction)





Main functions of CCD: store an image and then digitize it

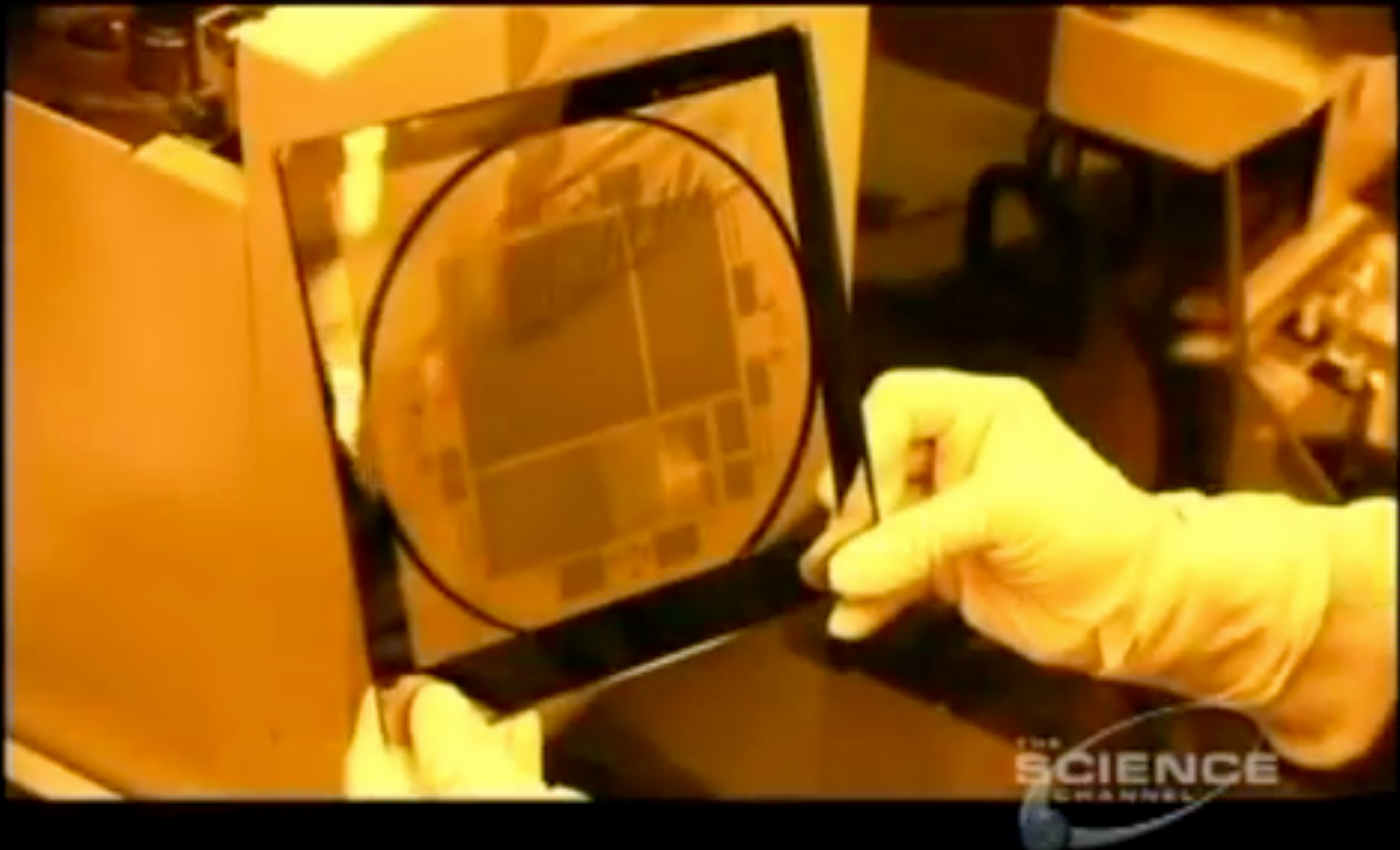


Stuff you should know:
how single-crystal silicon ingots are made?
how CCD detectors are made?

How Single-Crystal Silicon Ingots are made?

https://www.youtube.com/watch?v=13-JmHpCmNA

How CCD detectors are made?



https://www.youtube.com/watch?v=bqJksXwrx7U

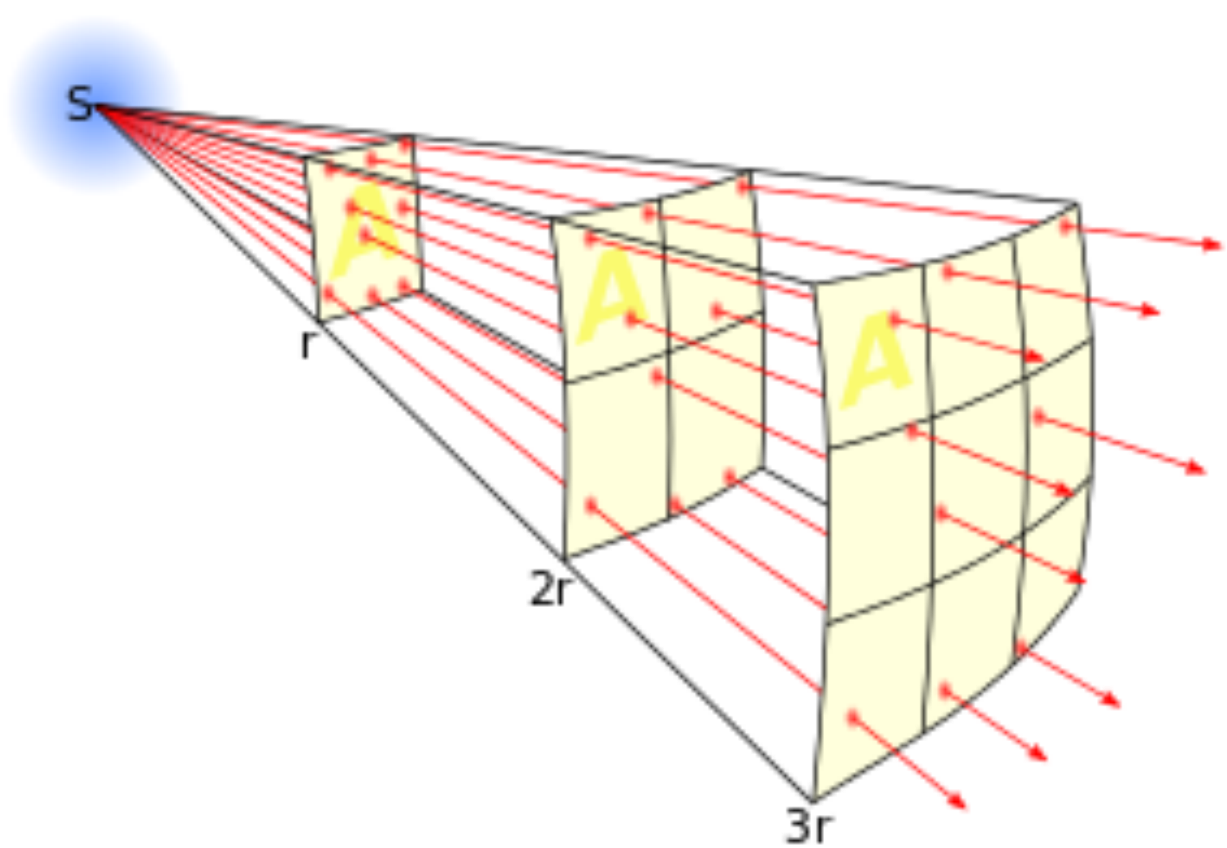
Luminosity Measurements: Absolute Magnitude

(requires Distance & Brightness)

The Inverse Square Law of Flux & the Conservation of Luminosity

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m^2) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law,** which **preserves the luminosity**

$$L = F(d_1)4\pi d_1^2 = F(d_2)4\pi d_2^2$$

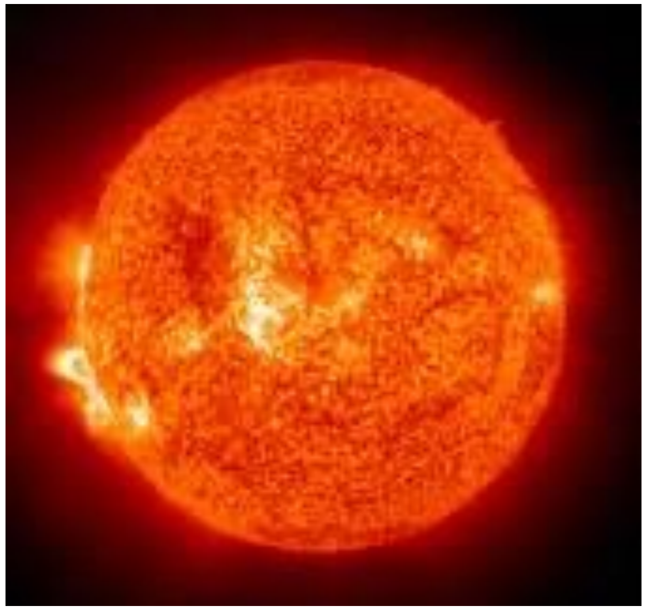


Definition: Absolute Magnitude (M) vs. Apparent Magnitude (m)

- apparent magnitude (m) is the magnitude of the source at its actual distance (d)
- absolute magnitude (M) is defined as the apparent magnitude of the source if it were at a distance of 10 parsec

Practice: Calculate the absolute magnitude of the Sun

- The Sun has an apparent magnitude of -26.74 (d = 1 AU = 1/206265 pc)
- What's its absolute magnitude? Both are in V-band.



Derivation: Absolute Magnitude (M) vs. Apparent Magnitude (m)

- apparent magnitude (m) is the magnitude of the source at its actual distance (d)
- absolute magnitude (M) is defined as the apparent magnitude of the source if it were at a distance of 10 parsec
- because both are measurements of the same source, we can express the same luminosity (L) using its actual flux (f) and its presumed flux (F) at 10 parsec:

$$L_\lambda = 4\pi d^2 f_\lambda = 4\pi (10\ \text{parsec})^2 F_\lambda \quad \Rightarrow \frac{F_\lambda}{f_\lambda} = \frac{d^2}{(10\ \text{parsec})^2}$$

$$m_\lambda - m_{\lambda,0} = -2.5 \log(f_\lambda / f_{\lambda,0})$$

$$M_\lambda - m_{\lambda,0} = -2.5 \log(F_\lambda / f_{\lambda,0})$$

$$m_\lambda - M_\lambda = 2.5 \log\left(\frac{d}{10\ \text{parsec}}\right)^2 = 5\ [\log d(\text{parsec}) - 1]$$

This, **m-M**, is called the **distance modulus**, because it only depends on distance

Practice: What's the absolute magnitude of the Sun?

- distance = 1 AU, V-band magnitude = -26.74
- What's its absolute magnitude in V-band?

M = -26.74-5*(log(1/206265)-1)
= 4.83

$$m_\lambda - M_\lambda = 5\ [\log d(\text{parsec}) - 1]$$

$$\Rightarrow M_\lambda = m_\lambda - 5\ [\log d(\text{parsec}) - 1]$$

Practice: Calculate absolute magnitude from p and m

- Suppose you measured a star's apparent magnitude in V-band (550 nm) to be $m_V = 10.5$
- You also measured its parallax to be $p = 5\ mas$ (milli-arcsec).
- What's its distance in parsec?

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p}\right)$$

- What's its absolute magnitude in V-band (M_V)?

$$m_\lambda - M_\lambda = 5\ [\log d(\text{parsec}) - 1]$$
$$\Rightarrow M_\lambda = m_\lambda - 5\ [\log d(\text{parsec}) - 1]$$

d = 200 parsec
M = 10.5 - 5 * (log(200) - 1) = 4.0

Distance measurement based on the distance modulus:

The Standard Candle Methods

Distance Modulus: the difference between m and M

- The definition of absolute magnitude and the conservation of luminosity for an isotropic emitter gives us this equation:

$$m - M = 5\ [\log d(\mathrm{parsec}) - 1]$$

- The term on the left side, **m-M**, is called the **distance modulus**, because it only depends on distance
- **m-M** offers us a group of methods to measure distances called **the standard candle**

$$d(\mathrm{parsec}) = 10^{1+0.2(m-M)}$$

The Standard Candle Methods

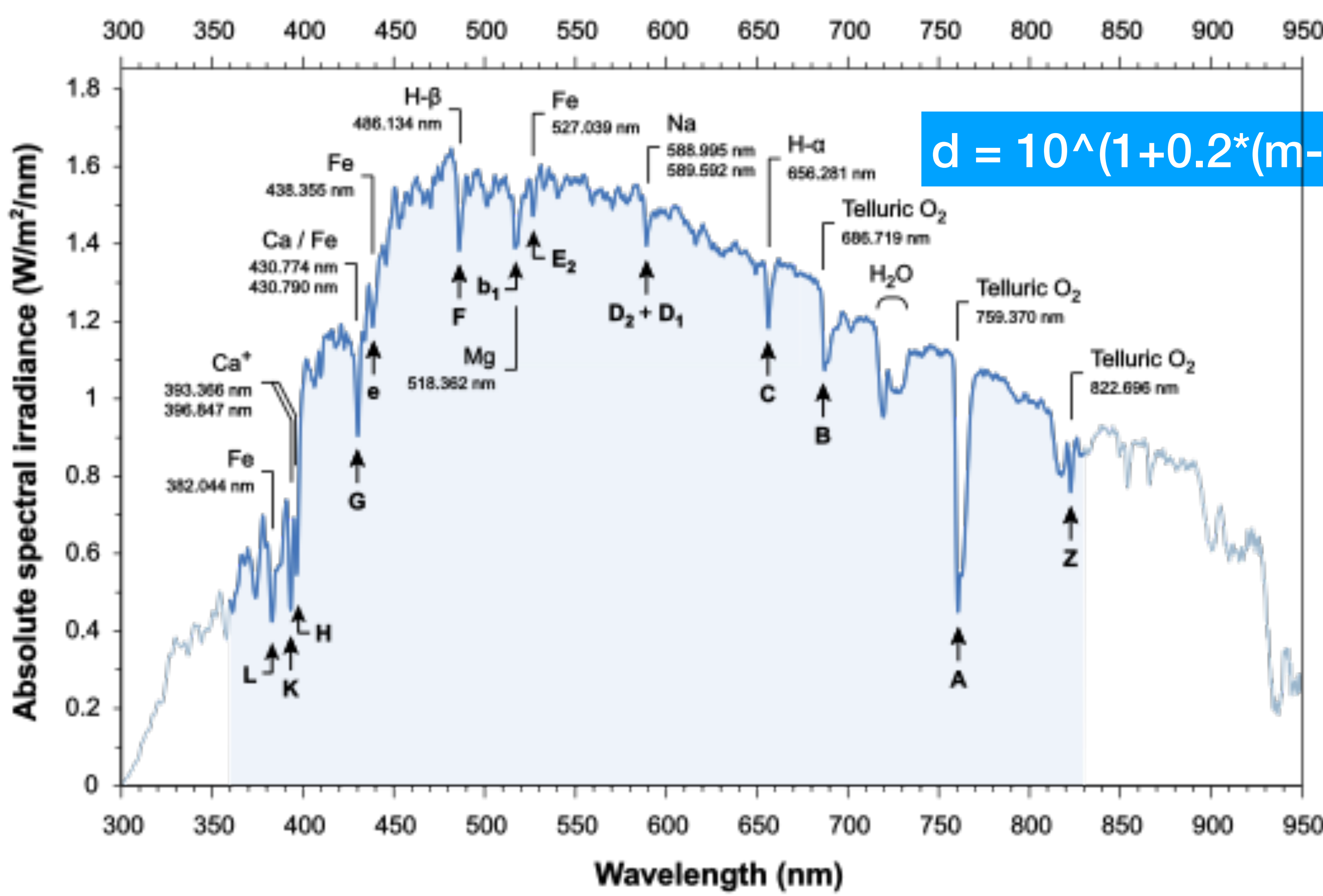
- If we had measured or inferred the absolute magnitude of a class of astrophysical objects, we can get the distance modulus (m-M) from its apparent magnitude.
- The distance modulus then gives us the distance:

$$m - M = 5\ (\log d_{\mathrm{pc}} - 1) \Rightarrow d_{\mathrm{pc}} = 10^{1+0.2(m-M)}$$

Standard Candle Method 1 — Spectroscopic "Parallax"

Suppose we find a solar-type star in the constellation Ursa Major, its spectrum looks just like that of the Sun, so we assume that this star has the same luminosity as the Sun. Given the Sun has **M_V = 4.83** and this star has **m_V = 10.5**, can you estimate its distance?

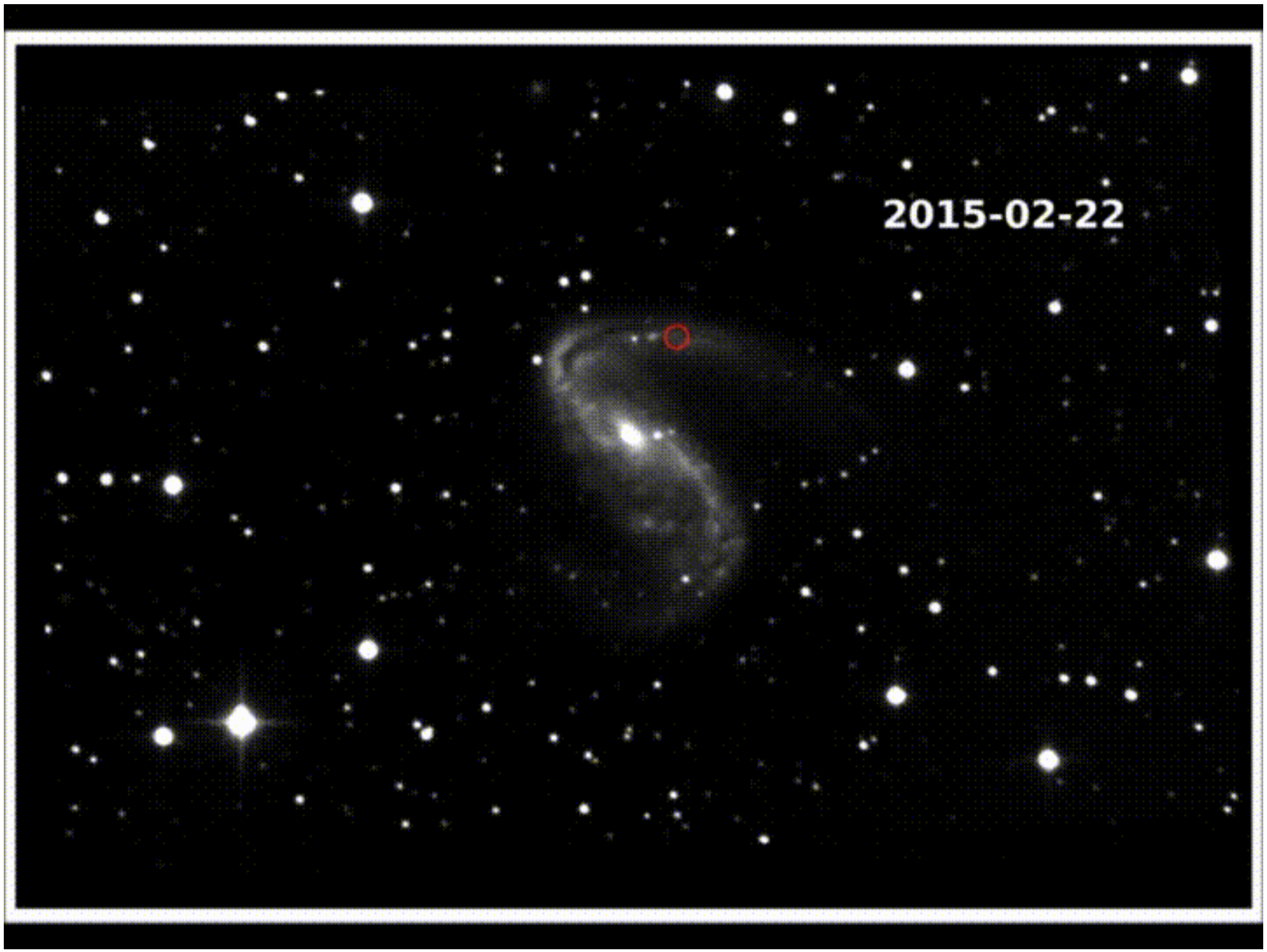
$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$



d = 10^(1+0.2*(m-M)) = 136 parsec

The Standard Candle Method 2 — Type Ia SNe

- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach 1.44 solar mass (the Chandrasekhar mass) to trigger the thermonuclear explosion



Practice: The Standard Candle Method of Distance Measurement

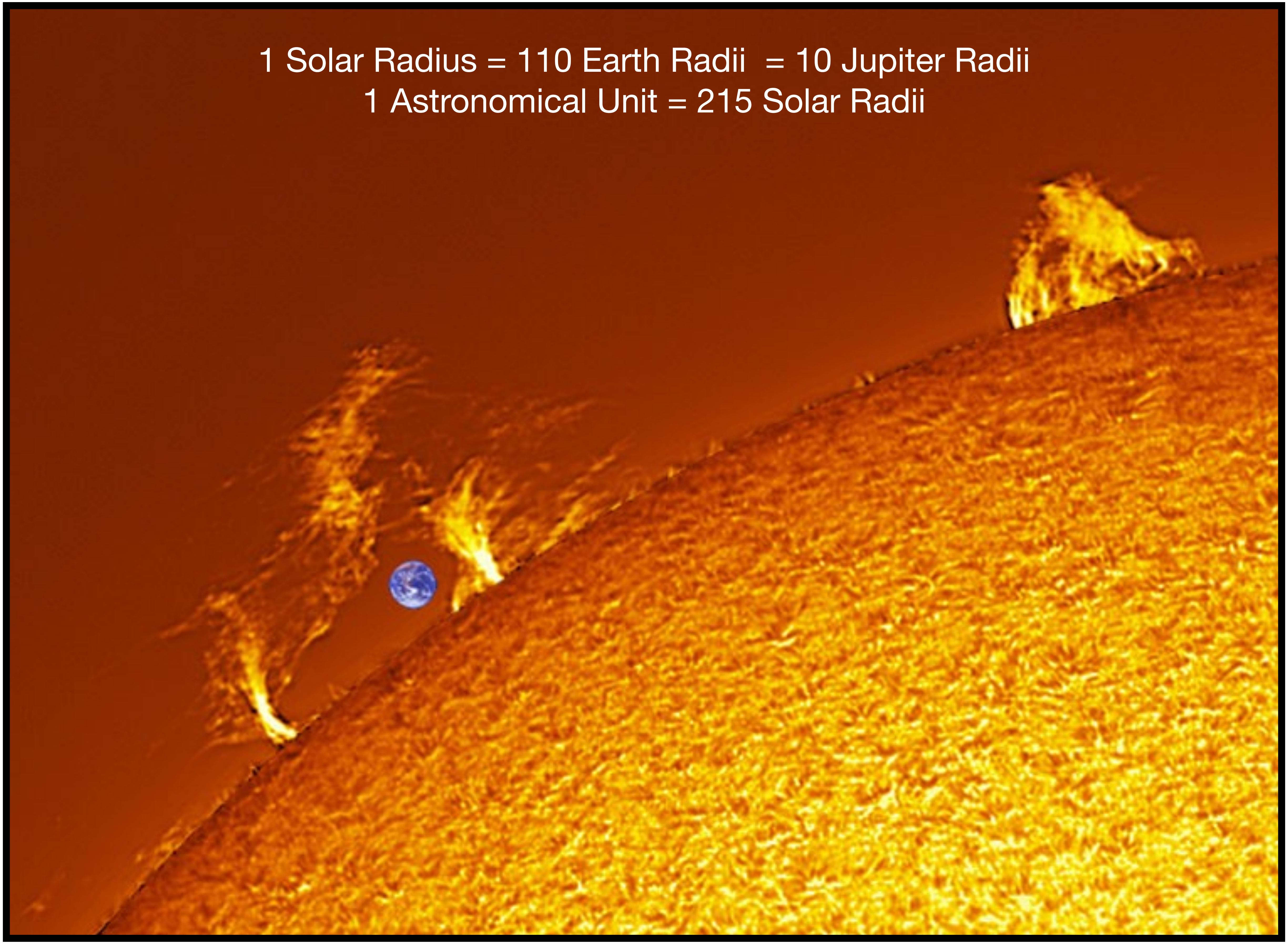
- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach 1.44 solar mass (the Chandrasekhar mass) to trigger the thermonuclear explosion
- At its <u>peak</u>, the absolute magnitude in V-band (550 nm) is $M_V = -19$, and you measured a <u>peak</u> apparent magnitude of $m_V = 10$, what's the distance?

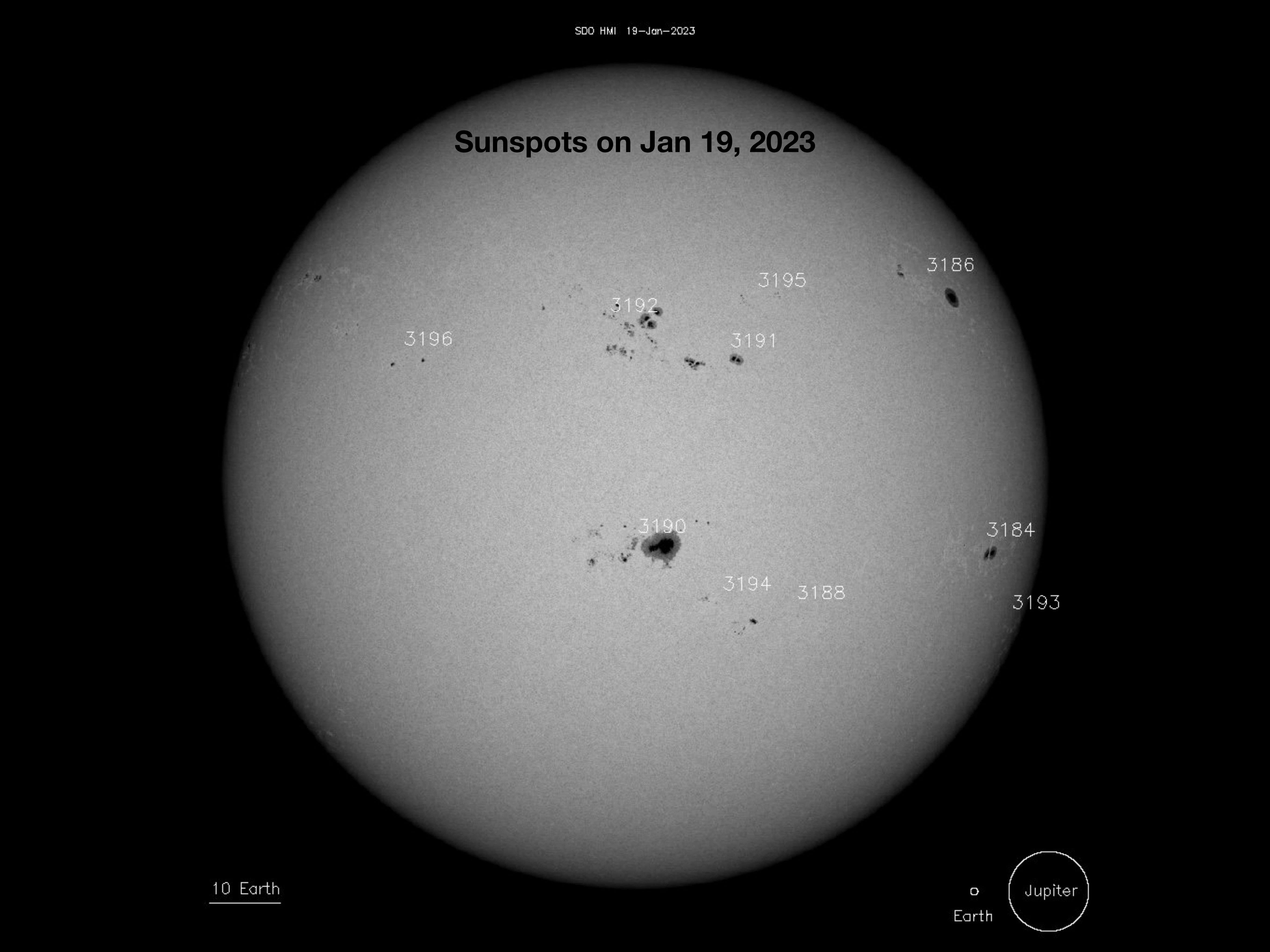
$$m - M = 5\ [\log d(\text{parsec}) - 1]$$

$$\Rightarrow d = 10\ \text{parsec} \cdot 10^{0.2(m-M)}$$

10 parsec * 10^(0.2*(10-(-19))) = 6.3 Mpc

Size comparison: Solar prominence vs. Earth




SDO HMI 19-Jan-2023
Sunspots on Jan 19, 2023
3186
3195
3192
3196
3191
3190
3184
3194
3188
3193
10 Earth
Earth
Jupiter

Distance Modulus: the difference between m and M

- The definition of absolute magnitude and the conservation of luminosity for an isotropic emitter gives us this equation:

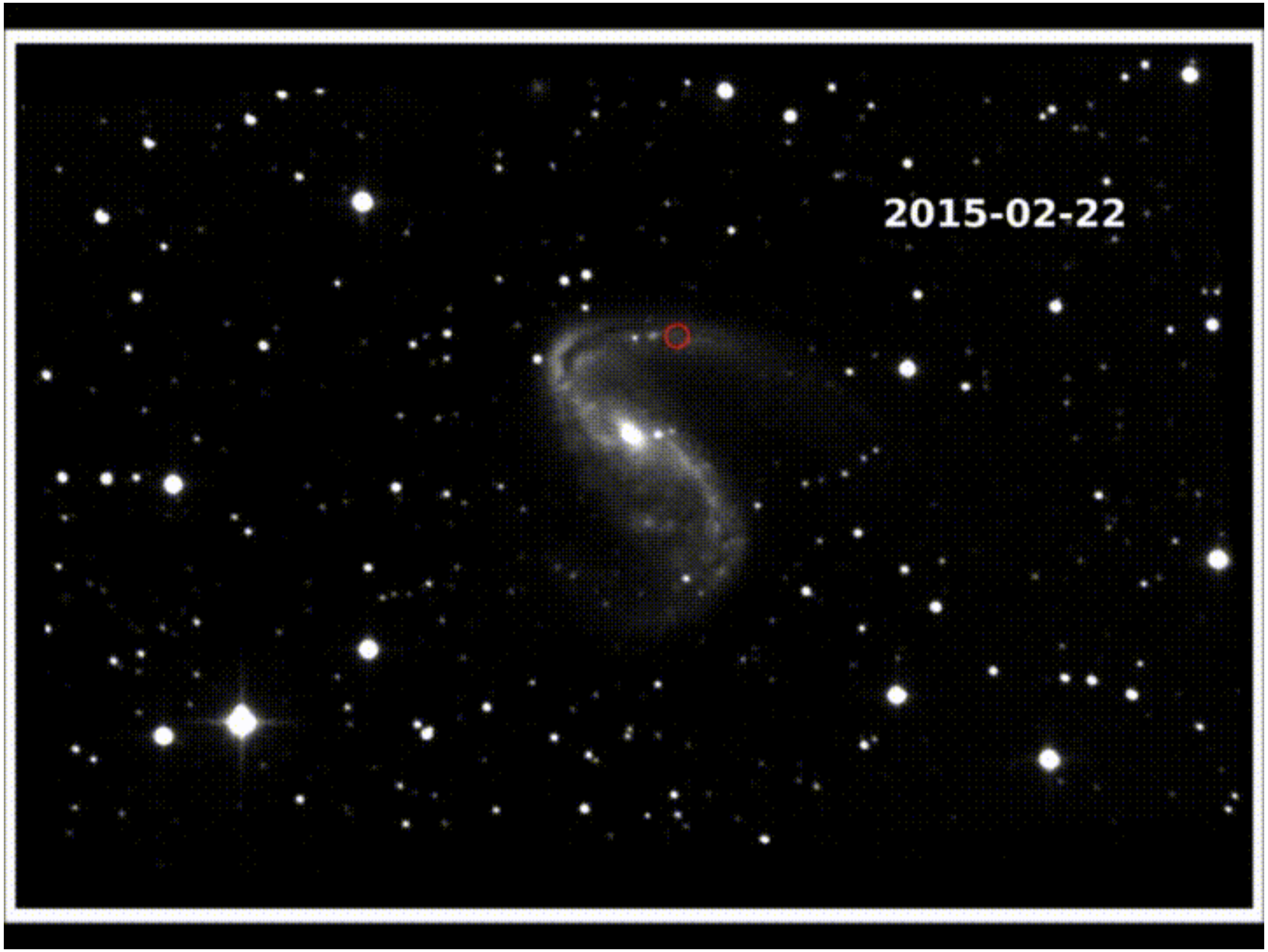
$$m - M = 5\ [\log d(\text{parsec}) - 1]$$

- The term on the left side, **m-M**, is called the **distance modulus**, because it only depends on distance
- **m-M** offers us a group of methods to measure distances called **the standard candle**

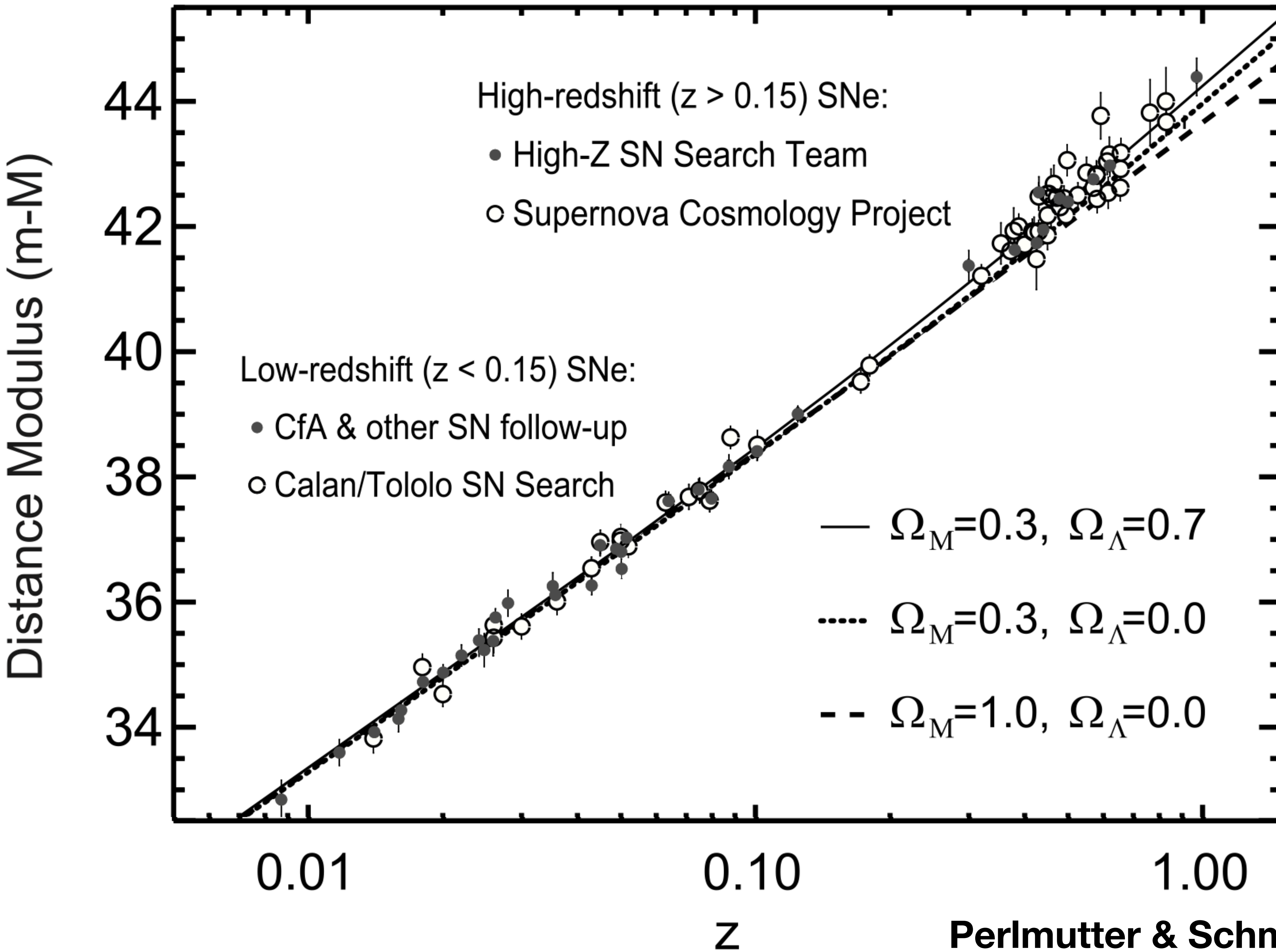
$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

The Standard Candle Method 2 — Type Ia SNe

- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because the white dwarfs have to reach ~1.44 solar masses (the Chandrasekhar mass) to trigger the thermonuclear explosion, reaching a peak absolute magnitude of M_V = -19.



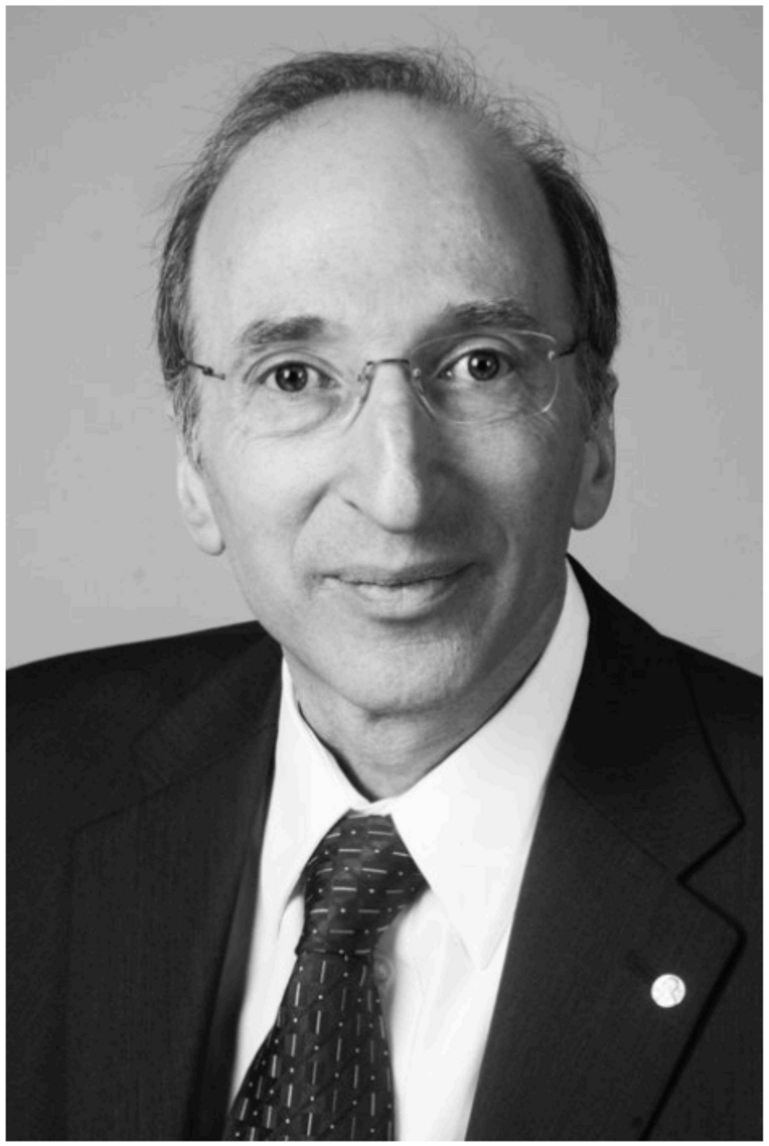
Distance Modulus vs. Cosmological Redshifts (Hubble Diagram)



Perlmutter & Schmidt 2003

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

The Nobel Prize in Physics 2011



Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

Check out Appendix 7: Observing the Sky

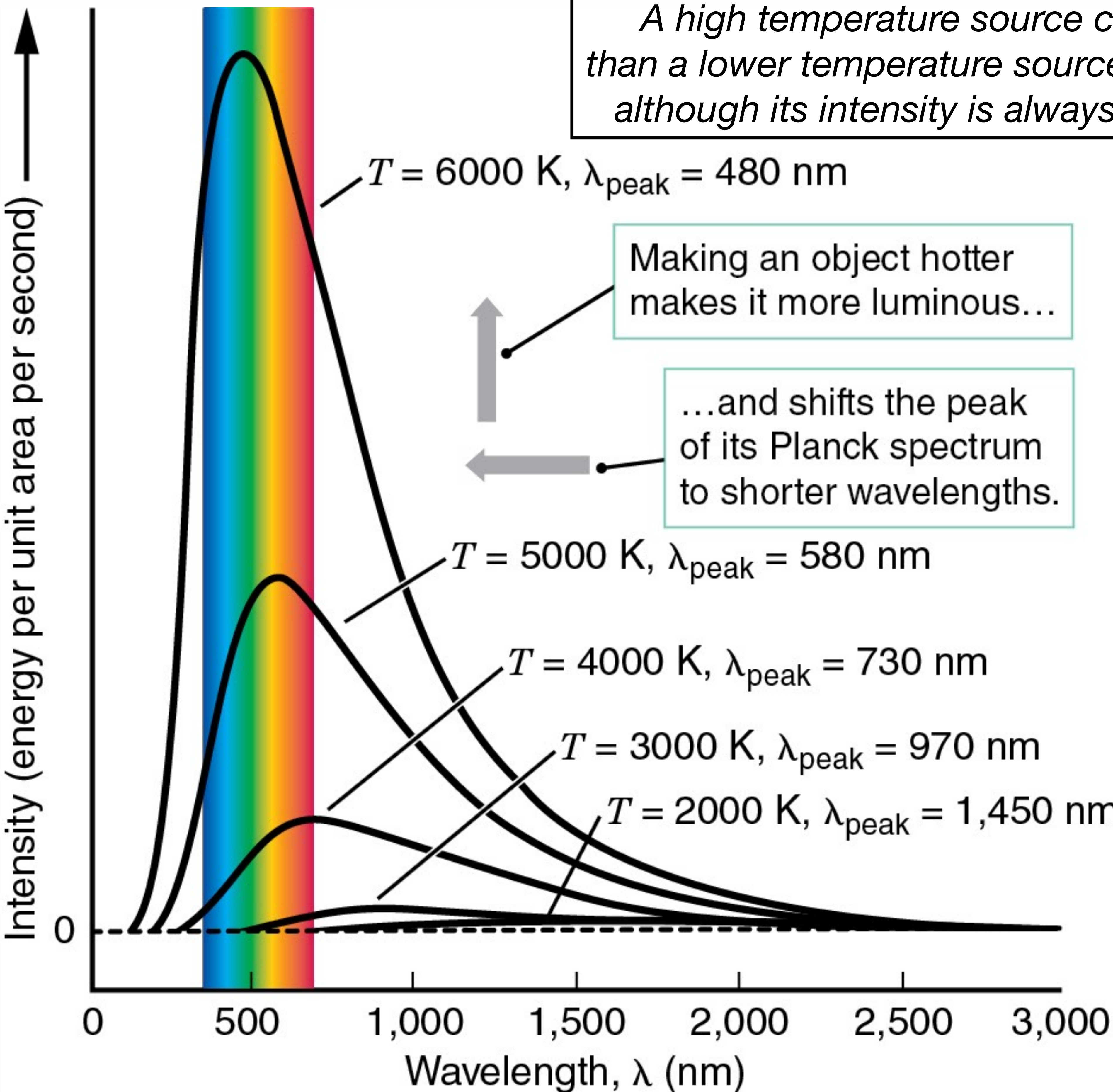
- Celestial Equatorial Coordinates:
  - RA & Dec
- Astronomical Magnitudes:
  - apparent magnitude and brightness
  - absolute magnitude
  - distance modulus
  - color index

Temperature

spectroscopic methods: Wien’s law and spectral classification

Planck Curves at Various T

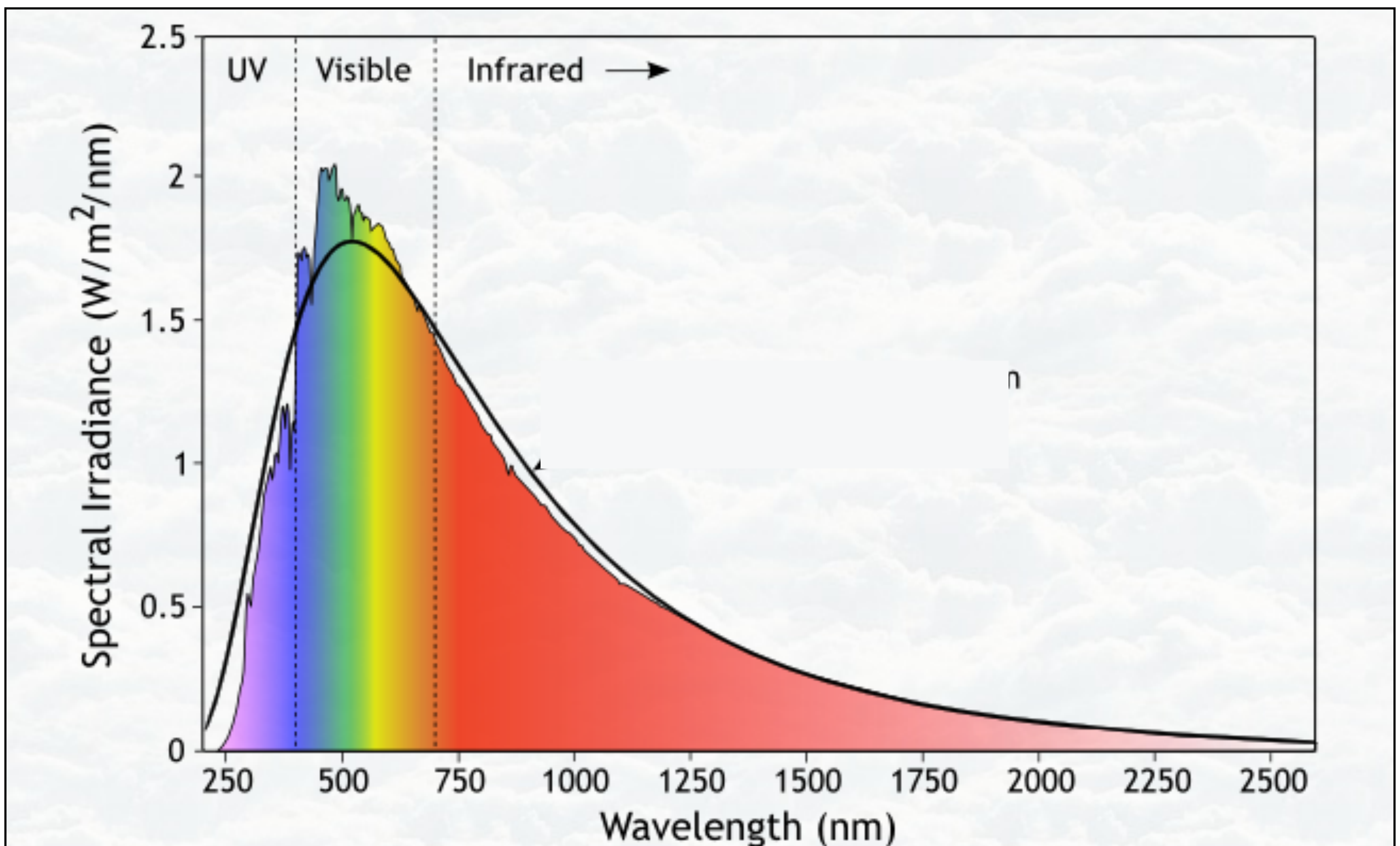
Note that the **Y-axis** is **intensity** not **flux**
Intensity scales with flux per unit angular area
A high temperature source could appear fainter than a lower temperature source because of distance, although its intensity is always higher at all lambda



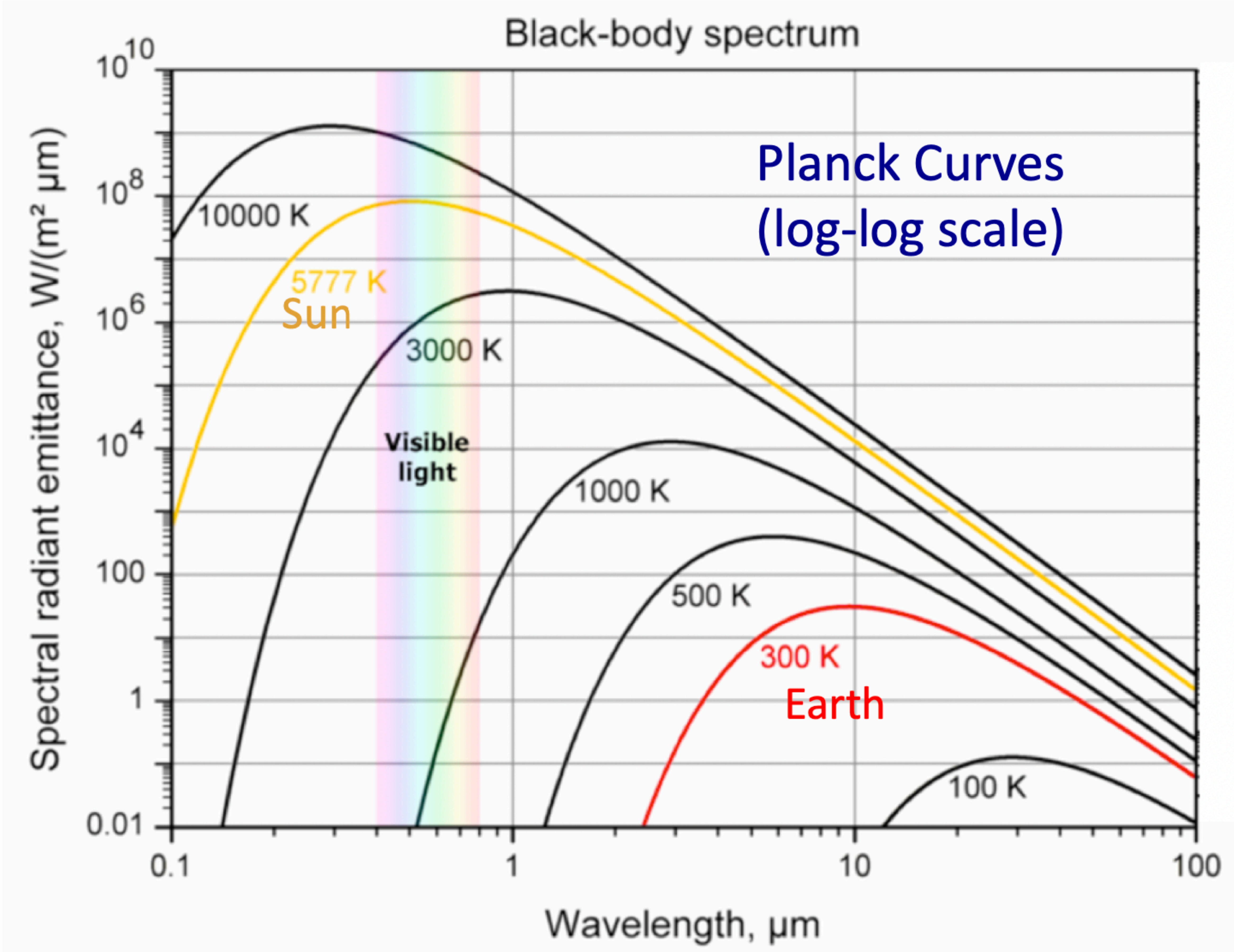
Temperature from Wien's Displacement Law

$$\lambda_{\text{peak}} = \frac{2.9 \text{ mm K}}{T} \Rightarrow T = \frac{2.9 \text{ mm K}}{\lambda_{\text{peak}}}$$

- Given a temperature, calculate the wavelength at which the BB emission's flux density peaks; Or given a peak wavelength, calculate the temperature.

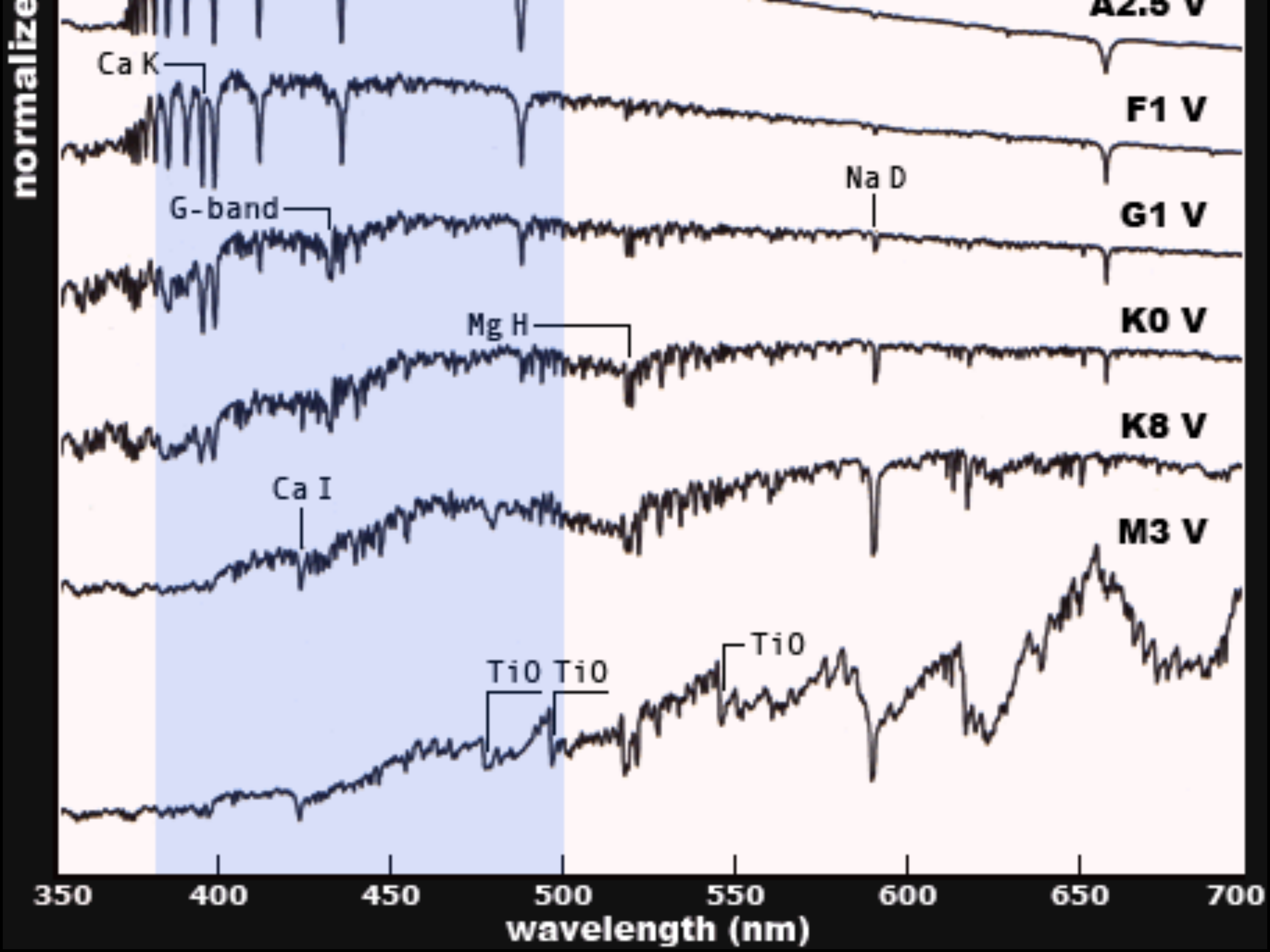


What to do when the peak shifts outside of the visible light window?
e.g. when T > 9000 K or T < 3000 K

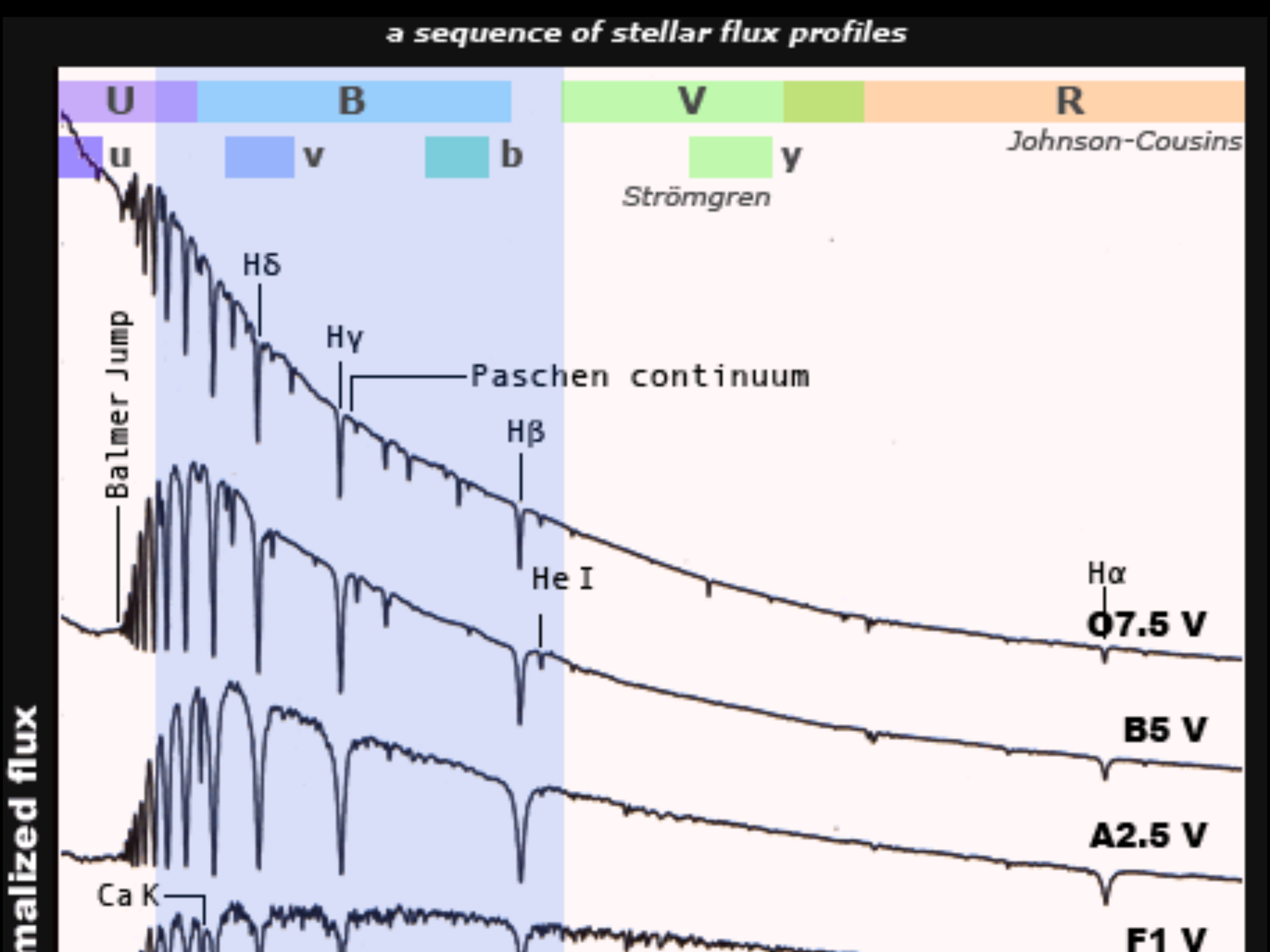


Optical spectral classification of stars

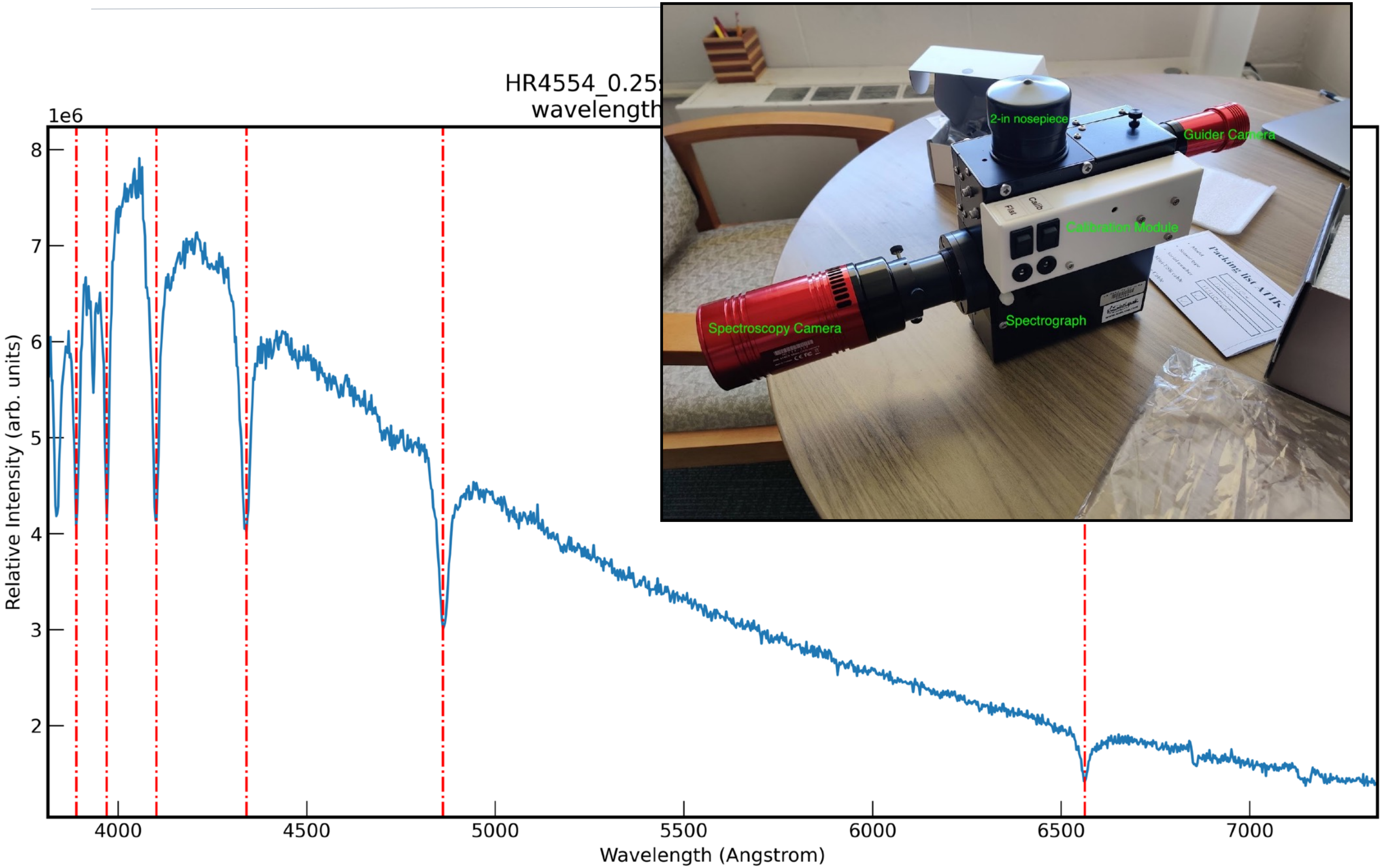
- **The strength of absorption lines from different elements depend mainly on the temperature (because of ionization equilibrium).**
- The current classification scheme was **re-ordered** and **simplified** by **Annie Jump Cannon** (1863–1941) at Harvard College Observatory.
- The full sequence is **O B A F G K M**, which are further subdivided by adding numbers to the letter. The Sun is a **G2** spectral-type star.



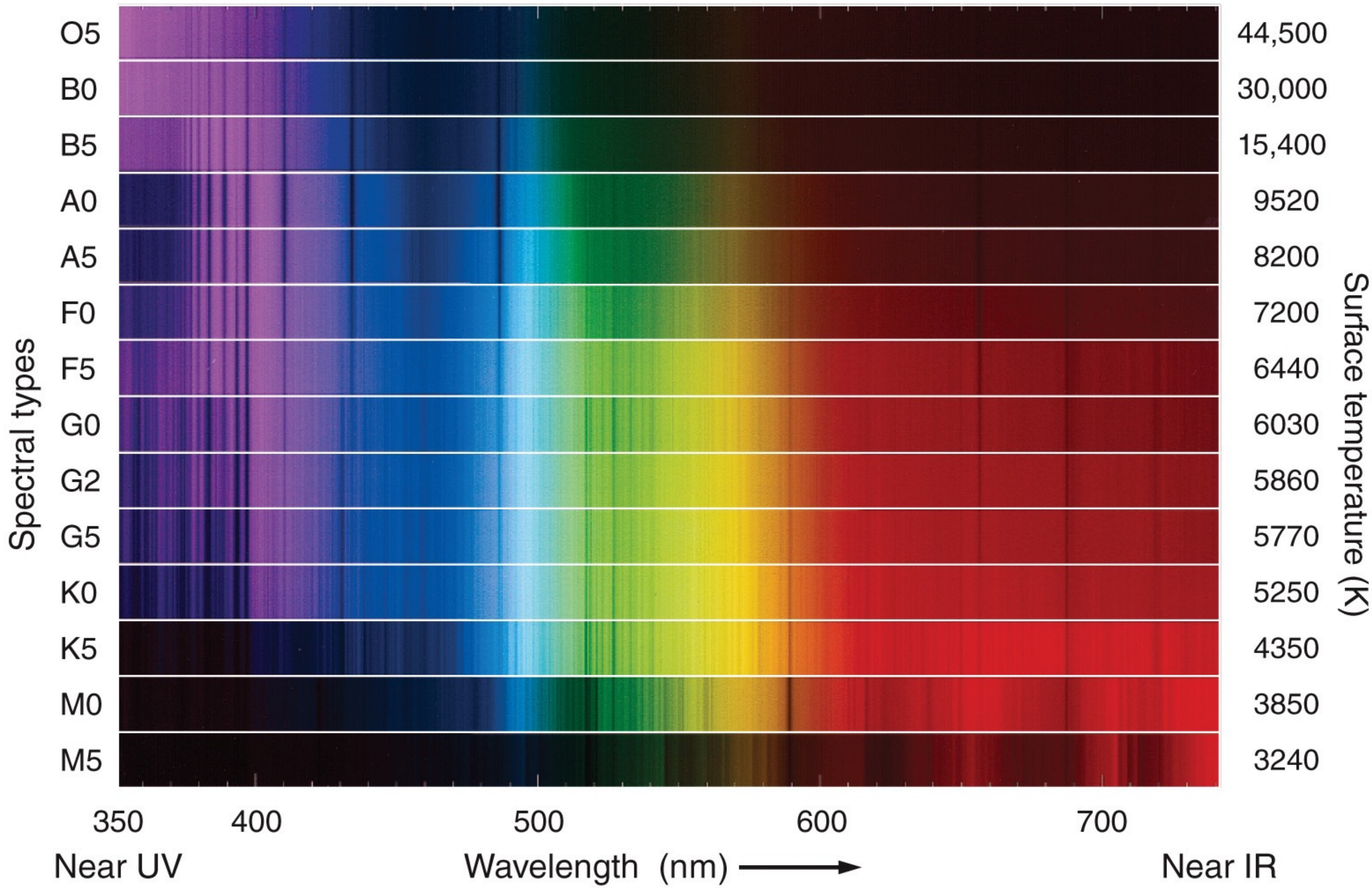
normalize
A2.5 V
Ca K
F1 V
Na D
G-band
G1 V
Mg H
K0 V
K8 V
Ca I
M3 V
TiO
TiO TiO
350
400
450
500
550
600
650
700
wavelength (nm)


a sequence of stellar flux profiles
U
B
V
R
Johnson-Cousins
u
v
b
y
Strömgren
Hδ
Hγ
Paschen continuum
Hβ
Balmer Jump
He I
Hα
O7.5 V
B5 V
A2.5 V
Ca K
F1 V
malized flux

An A-type star's spectrum taken by the Van Allen Observatory



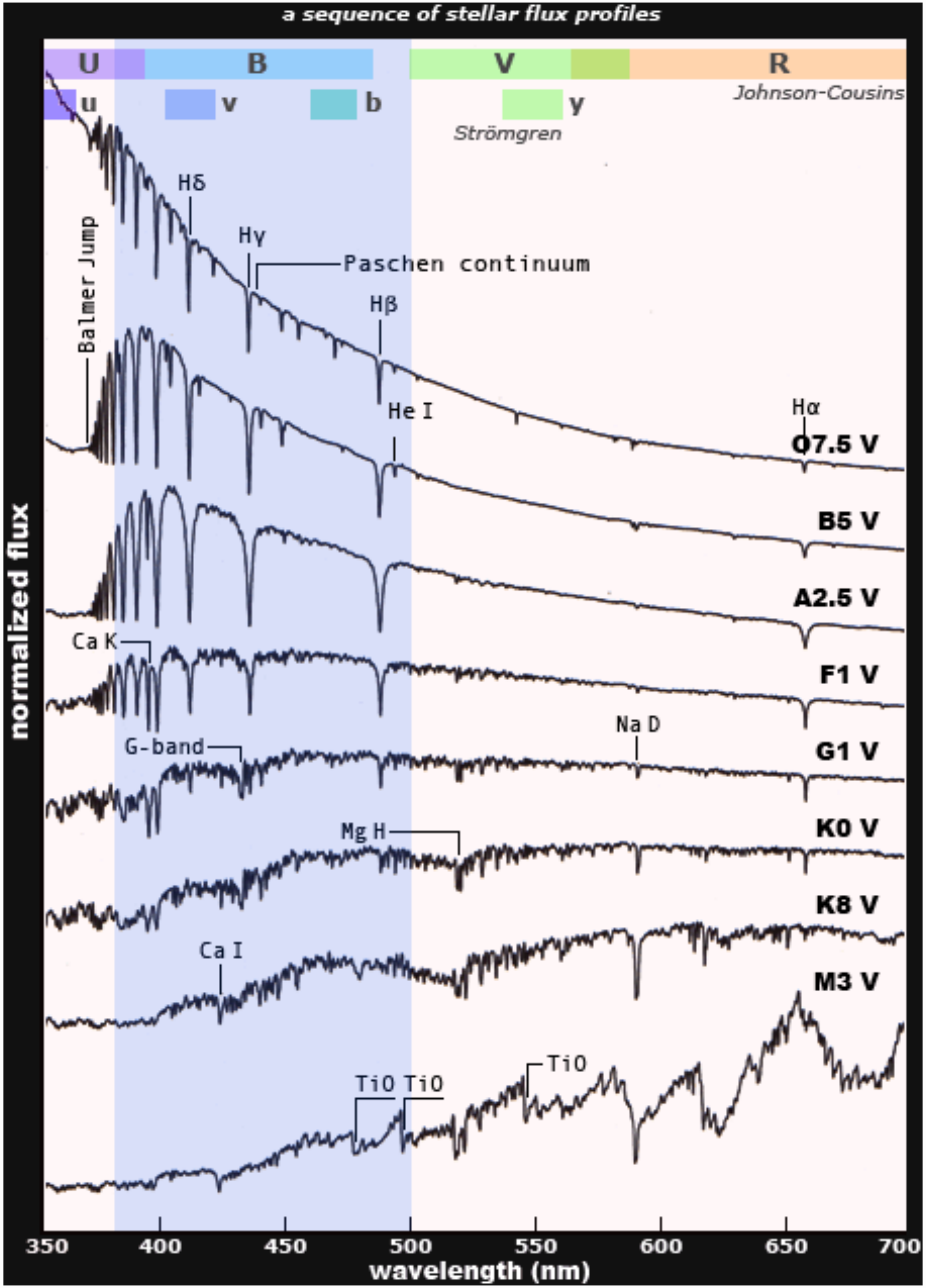
Temperature from Spectral Classes



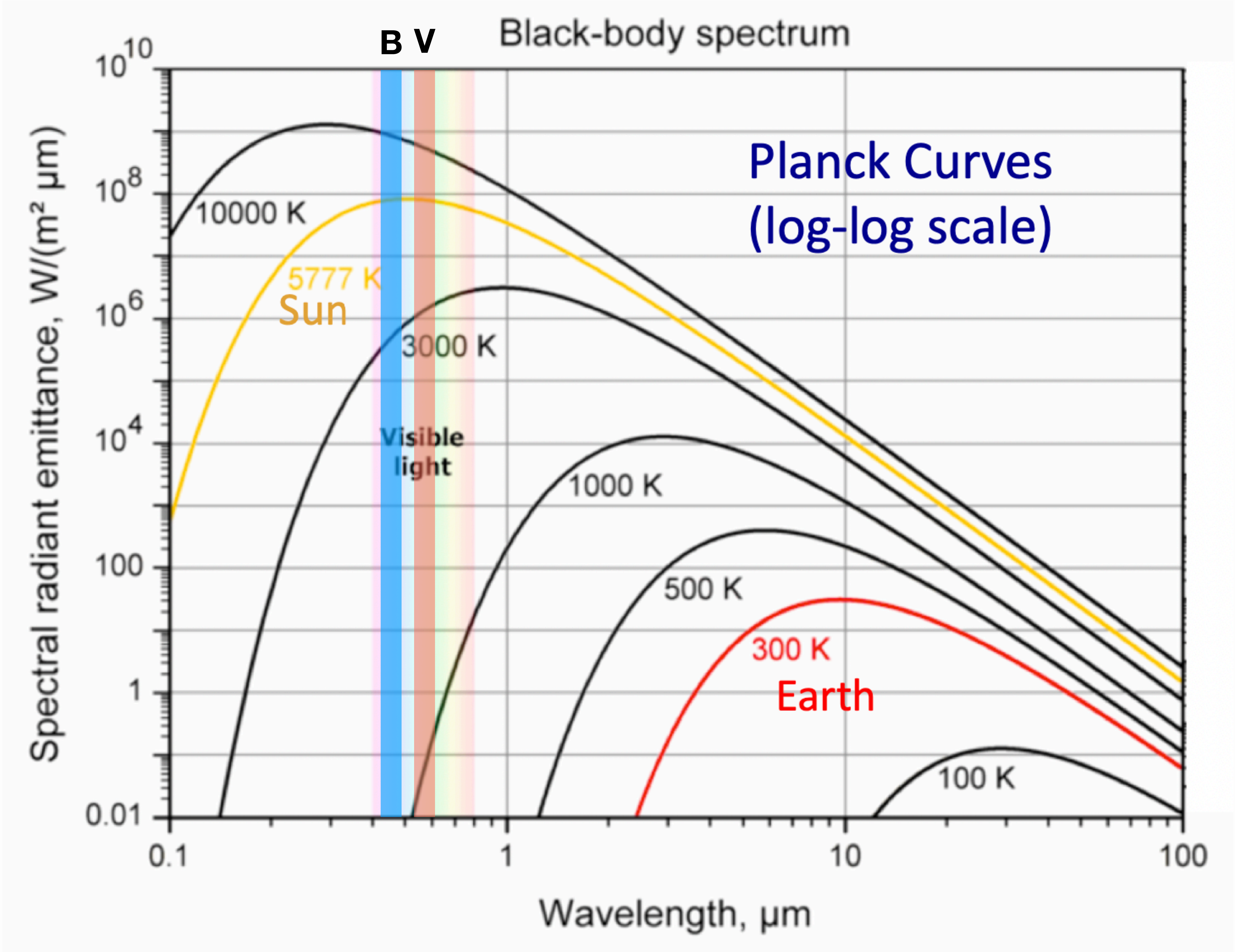
Temperature

photometric method: color index

Spectroscopy takes longer time to acquire, because each star would require its own spectroscopic observations with a traditional longslit spectrograph



Two-band photometry offers a much simpler way to estimate temperature



Temperature from Color Index

- **Color index** is defined as the magnitude difference of the same object at two different wavelengths.

- According to **Pogson**, the magnitude difference corresponds to a flux ratio at two different wavelengths:

$$m_B - m_V = -2.5\log(f_B/f_V)$$

or simply

$$B - V = -2.5\log(f_B/f_V)$$

- Typically, we subtract a bluer magnitude (e.g., B) to a redder magnitude (e.g., V), so that **the higher the value of the color index, the redder the object appears** (i.e., the object appears much fainter in B-band than in V-band)

Practice: From flux ratio to color index

$$m_{\lambda 1} - m_{\lambda 2} = -\,2.5\log(f_{\lambda 1}/f_{\lambda 2})$$

$$\Rightarrow m_B - m_V = -\,2.5\log(f_B/f_V)$$

- Vega is the usual reference star that sets the zero point of the apparent magnitude system. Its surface temperature is at 9600 K, much hotter than that of the Sun (5800 K).
- Consider a star that is 100x fainter than Vega at 440nm (B-band) and also 100x fainter than Vega at 550nm (V-band), what are the magnitudes of the star in B and V? What is the color index? What is its surface temperature?

Practice: From flux ratio to color index

$$m_{\lambda 1} - m_{\lambda 2} = -2.5 \log(f_{\lambda 1}/f_{\lambda 2})$$

$$m_B - m_V = -2.5 \log(f_B/f_V)$$

- Vega is the usual reference star that sets the zero point of the apparent magnitude system. Its surface temperature is at 9600 K, much hotter than that of the Sun (5800 K).

- Consider another star that is 100x fainter than Vega at 440nm but 200x fainter than Vega at 550nm (V-band), what are the B and V magnitudes? What is the color index? Is this star hotter or cooler than Vega?

B = 5, V = 5.75; B-V = -0.75

Temperature vs. Color Index vs. Apparent Color

Spec Type	Surface Temperature	Color Index (Vega Sys)	Apparent Color
O	≥ 33,000 K	blue (B-V < 0)	blue
B	10,000–30,000 K	blue to blue white	blue white
A	7,500–10,000 K	white (B-V ~ 0)	white to blue white
F	6,000–7,500 K	yellowish white	white
G	5,200–6,000 K	yellow (B-V > 0)	yellowish white
K	3,700–5,200 K	orange	yellow orange
M	≤ 3,700 K	red	orange red

A table that gives the color indices at a range of temperatures

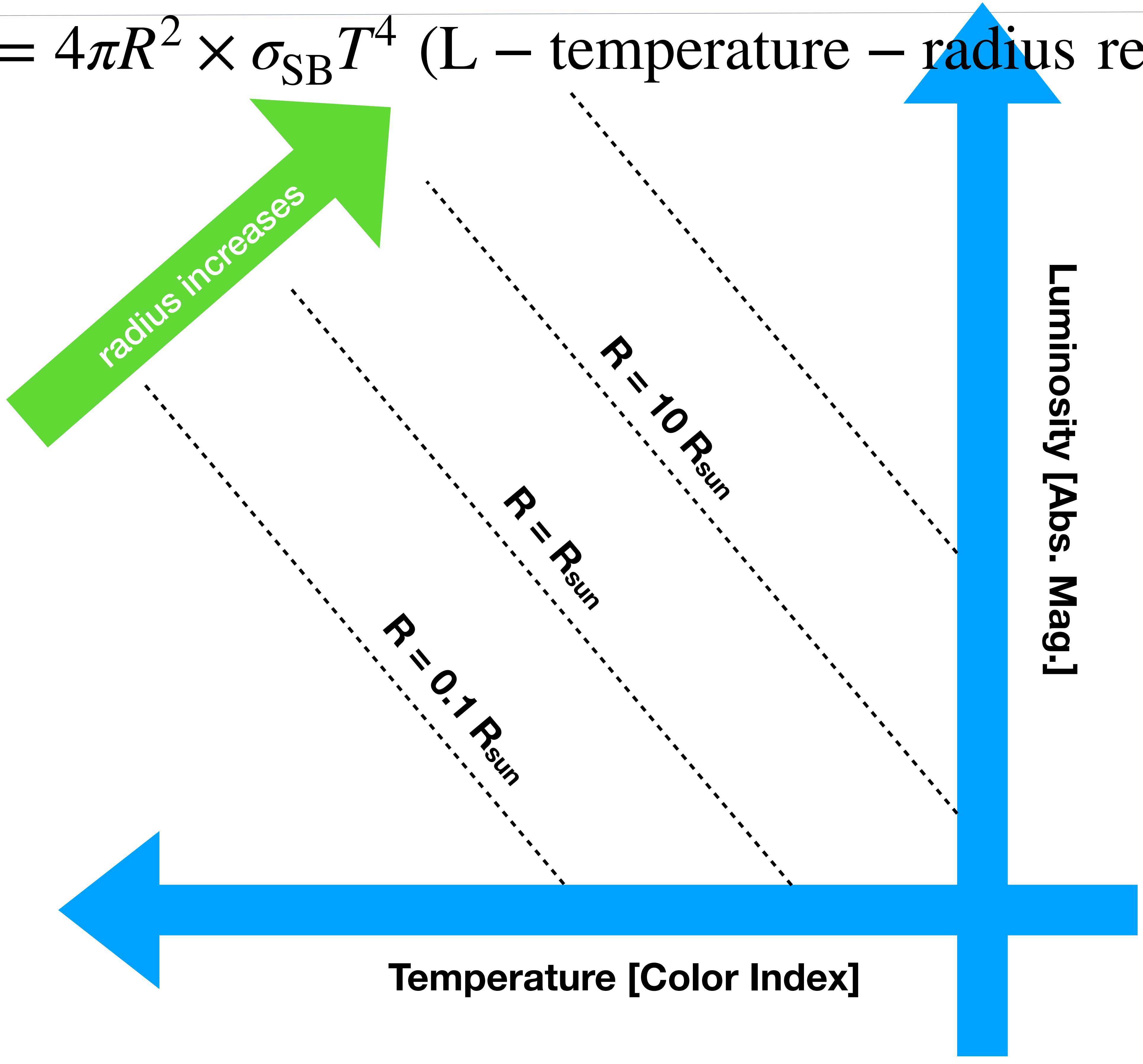
Sample calibration colors[1]

Class	B−V	U−B	V−R	R−I	T_{eff} (K)
O5V	−0.33	−1.19	−0.15	−0.32	42,000
B0V	−0.30	−1.08	−0.13	−0.29	30,000
A0V	−0.02	−0.02	0.02	−0.02	9,790
F0V	0.30	0.03	0.30	0.17	7,300
G0V	0.58	0.06	0.50	0.31	5,940
K0V	0.81	0.45	0.64	0.42	5,150
M0V	1.40	1.22	1.28	0.91	3,840

The Hertzsprung-Russell Diagram: M vs. color index (or, Luminosity vs. Temperature)

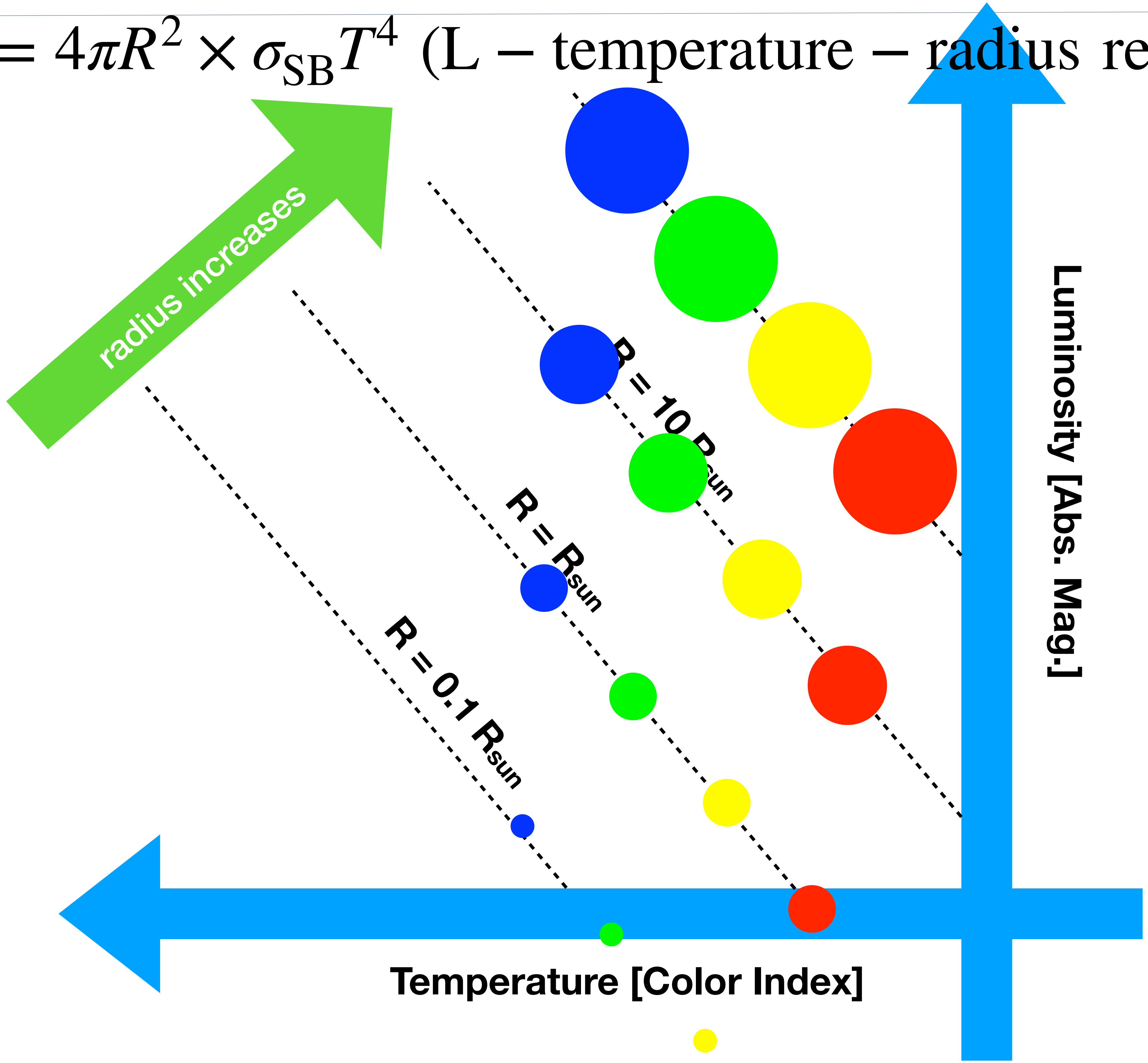
What if we plot Abs. Mag. against Color Index?

$$L = 4\pi R^2 \times \sigma_{\mathrm{SB}} T^4 \ (\mathrm{L - temperature - radius\ \ relation})$$

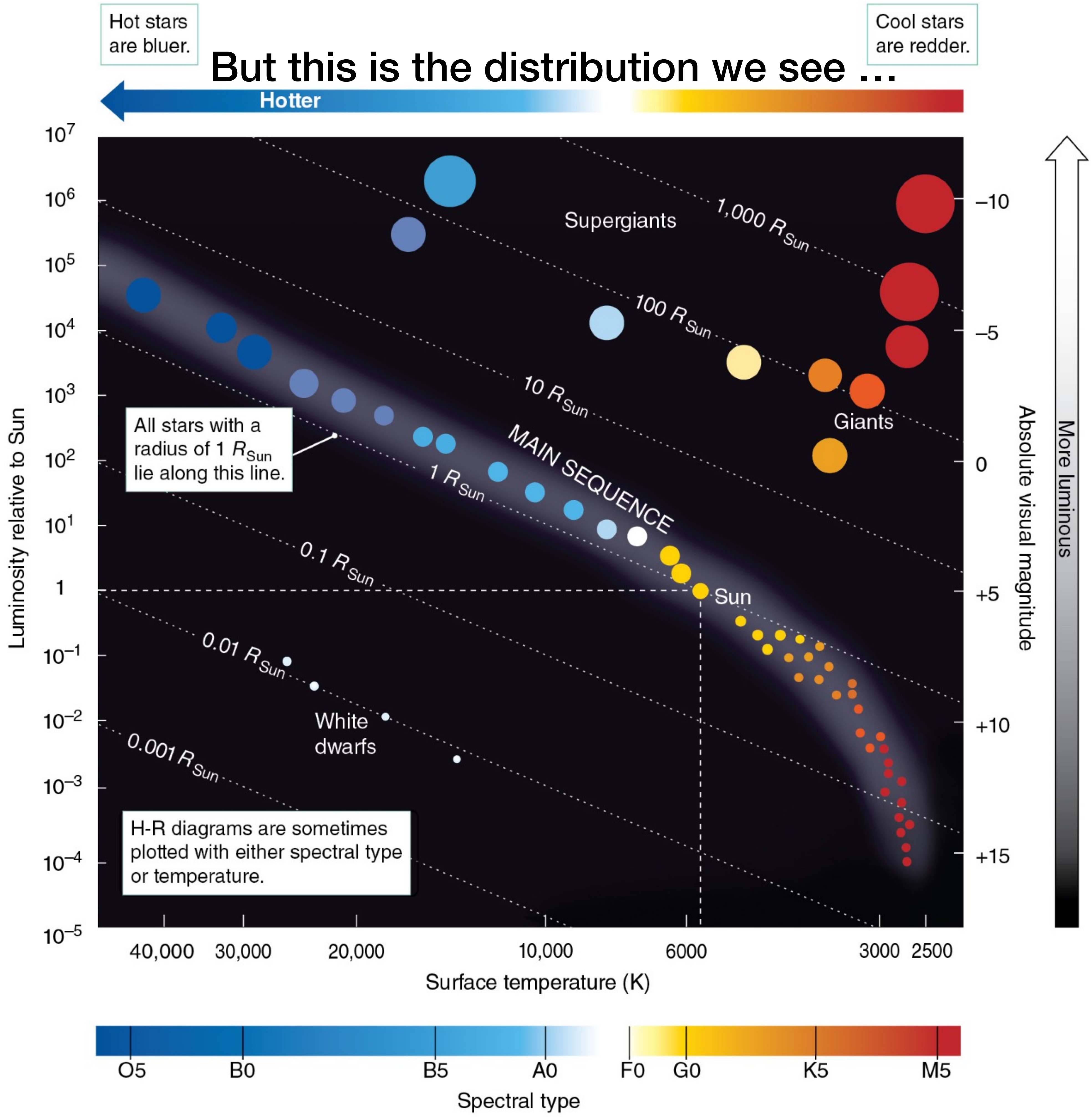


The distribution of stars on this plot could be completely random

$$L = 4\pi R^2 \times \sigma_{SB} T^4 \ \text{(L – temperature – radius relation)}$$

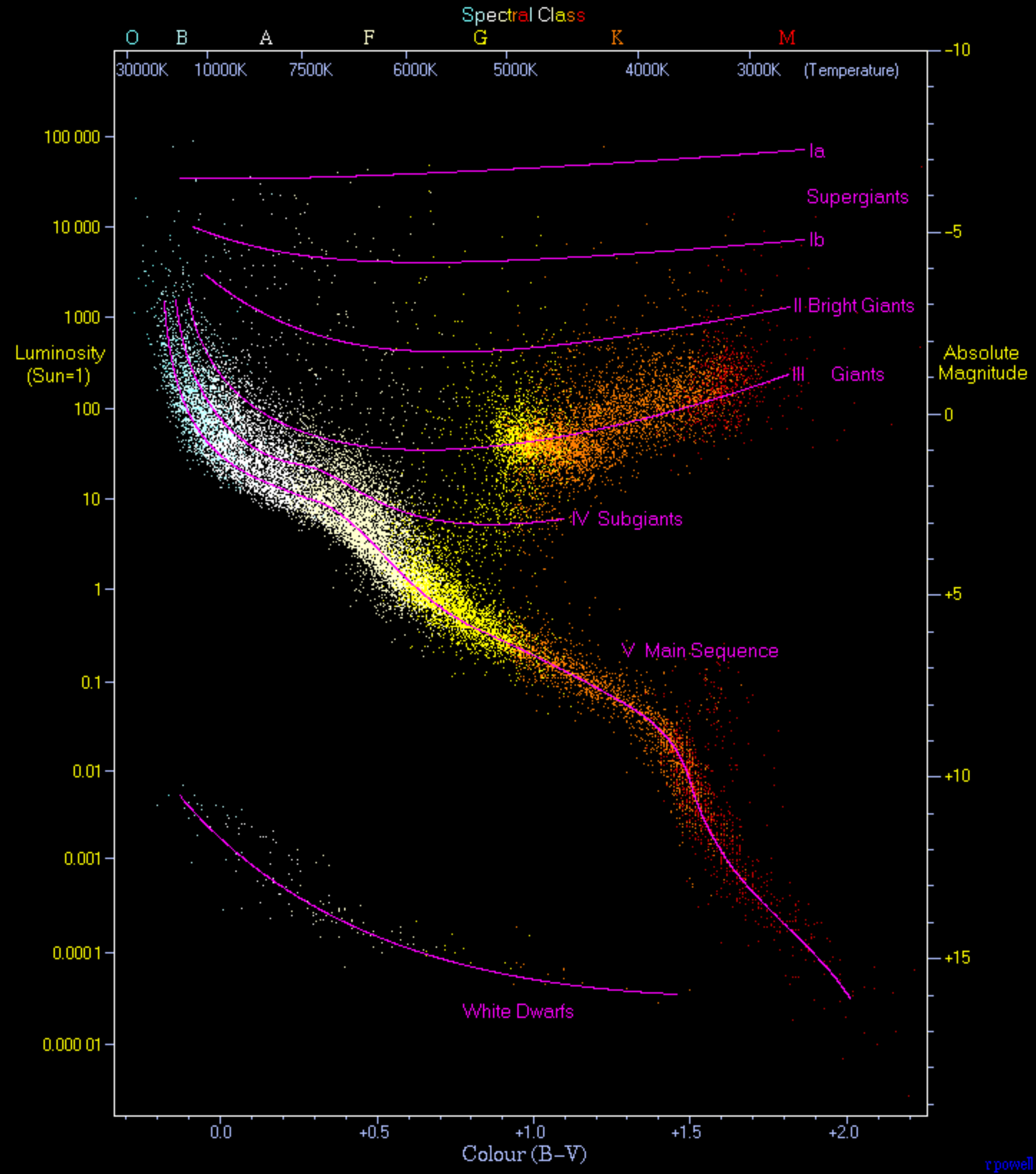


But this is the distribution we see …



The H-R Diagram

- The Hertzsprung-Russell Diagram is a plot of M vs. color index.
- The H-R diagram is the most important graph in stellar astronomy.
- The H-R diagram is key to unraveling stellar evolution (Chaps 16 & 17)



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The AI-powered chatbot did better than expected on a Wharton exam. That's something to get excited about, says the professor behind the experiment.

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ChatGPT could hypothetically get hired as an entry level coder if it interviewed at Google, internal document reportedly says

Sawdah Bhaimiya Feb 1, 2023, 10:38 AM

When ChatGPT tries to solve Prof. Fu's homework problems ...

Now given a parallax measurement of 5 milliarcsec, what is the distance?

The distance can be calculated using the formula:

distance (in parsecs) = 1 / (parallax in arcseconds)

So, for a parallax measurement of 5 milliarcseconds (5 x 10^-3 arcseconds), the distance would be:

distance (in parsecs) = 1 / (5 x 10^-3 arcseconds) = 1 / 0.005 = 200 parsecs

What is the angular distance between the following two celestial coordinates? Coordinates 1: 21h06m53.940s +38d44′57.90″, Coordinates 2: 21h06m53.990s +38d44′57.84″.

In this case, the difference in right ascension is 21h06m53.990s - 21h06m53.940s = 0h00m00.050s, which is equivalent to a difference of 0.050/3600 = 0.0000139 radians. The difference in declination is 38d44′57.84″ - 38d44′57.90″ = -0.06″, which is equivalent to a difference of -0.06/3600 = -0.0000167 radians.

The angular distance between the two coordinates can then be calculated as the Euclidean distance in three-dimensional space, using the following formula:

d = sqrt(delta_ra^2 + delta_dec^2)

where delta_ra is the difference in right ascension and delta_dec is the difference in declination.

In this case, the angular distance would be:

d = sqrt(0.0000139^2 + (-0.0000167)^2) = 0.0000185 radians

This angular distance is very small, and corresponds to an angular separation of approximately 0.01 arcseconds on the sky, which is much smaller than the resolution of typical telescopes.

For a star at right ascension of 6 hours and declination of 23 degrees, which dates of the year should we plan the parallax observations?

Parallax observations for a star are best made at different times of the year when the Earth is at opposite sides of its orbit around the Sun. This allows for a baseline change in the observer's position, which can then be used to calculate the star's distance using the parallax angle.

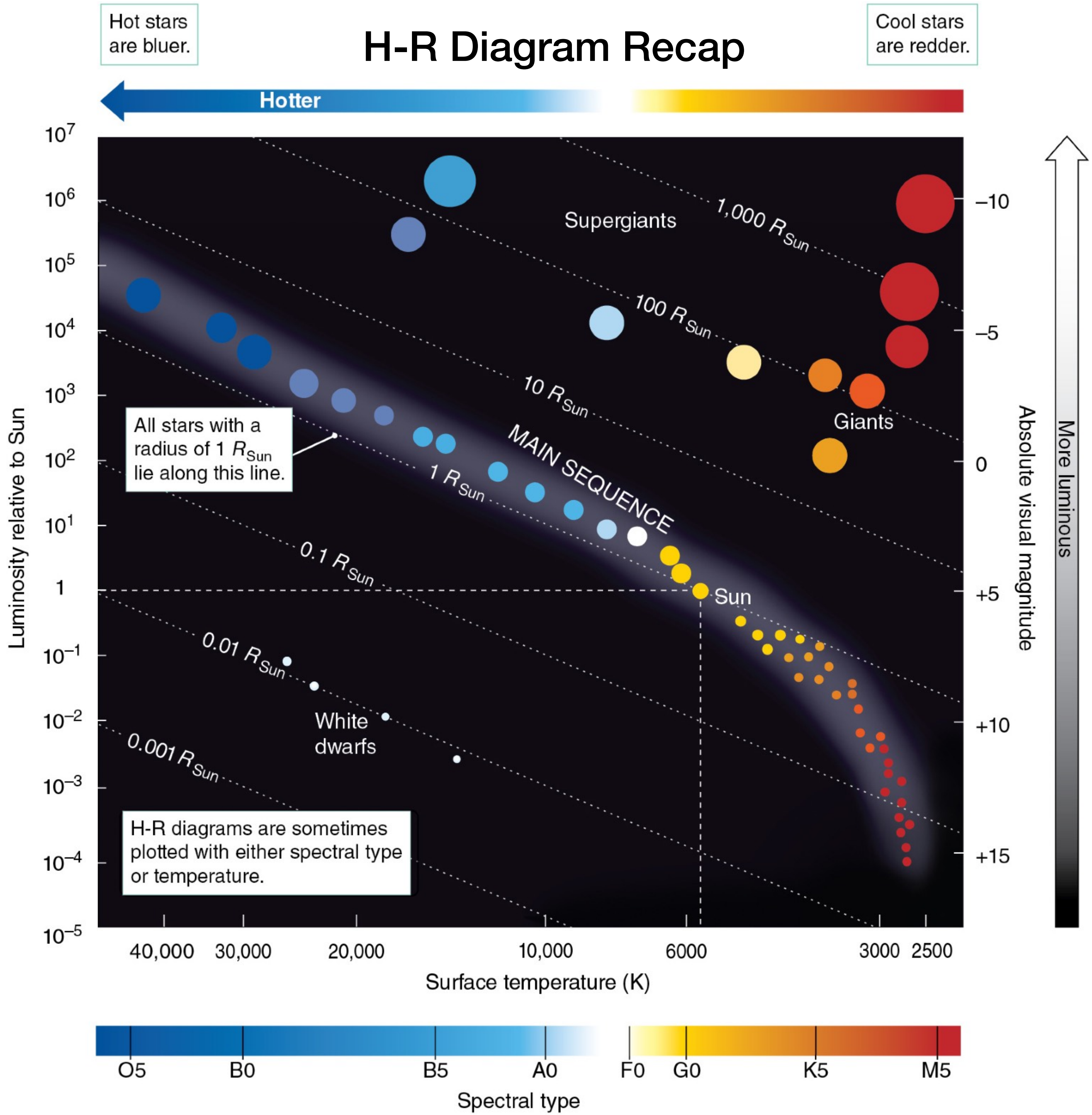
For a star with a right ascension of 6 hours and a declination of 23 degrees, the best time to observe the star would be in late December and late June, when the Earth is at opposite sides of its orbit and the star is at its highest point in the sky at around midnight.

Bonus Points Before Spring Break

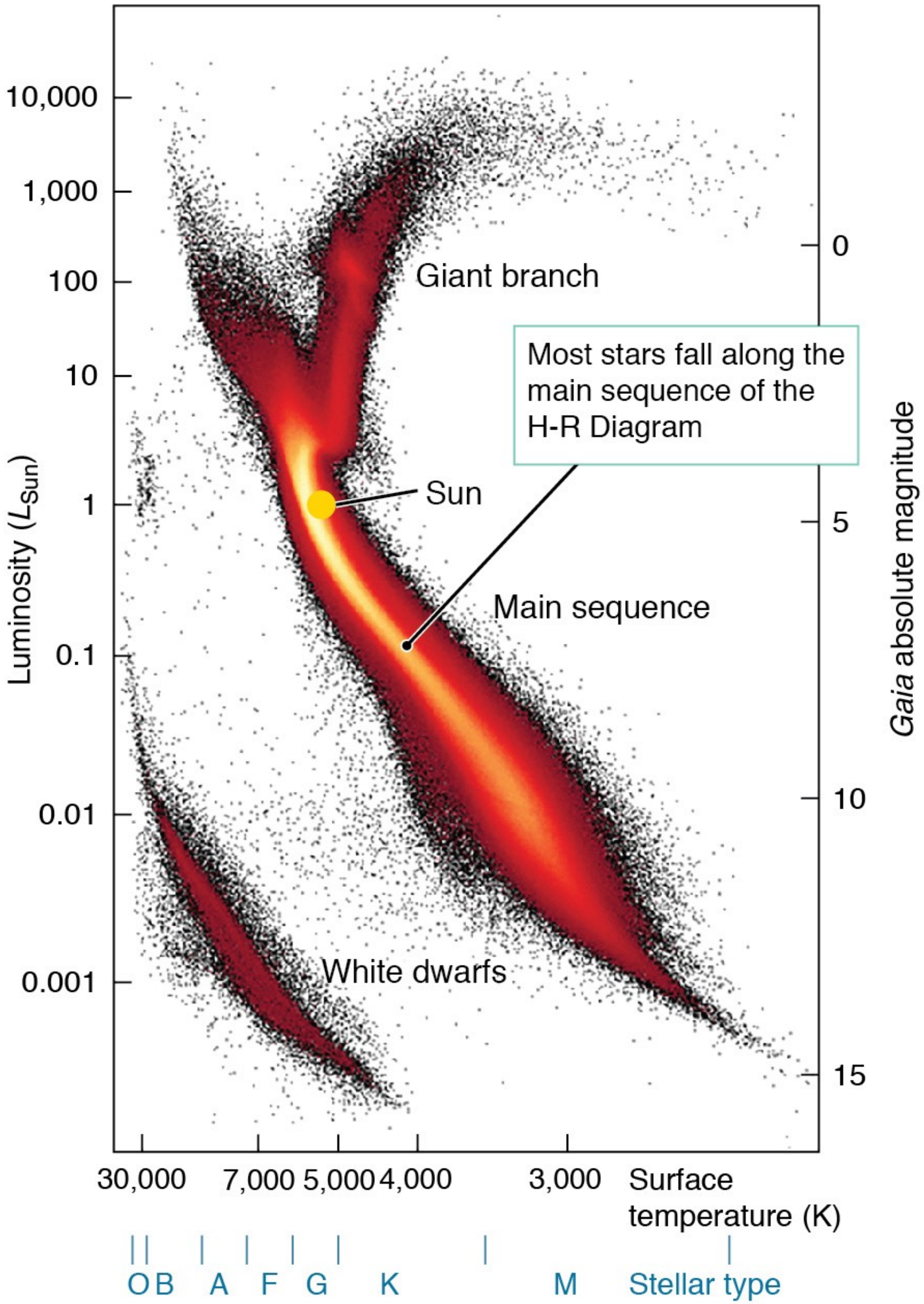
Activity	*Additional weight*
Teach ChatGPT to solve a homework problem and prove it has learned how to solve it	**1.5%**
Visit one of my office hours and ask questions about astronomy in this session	**0.5%**
Attend a Department Colloquium	**1.0%**
Attend a Department Seminar	**1.0%**

For comparison,
the weight of one homework assignment is ~2.3%,
and the weight of one lab session is ~1.9%

H-R Diagram Recap

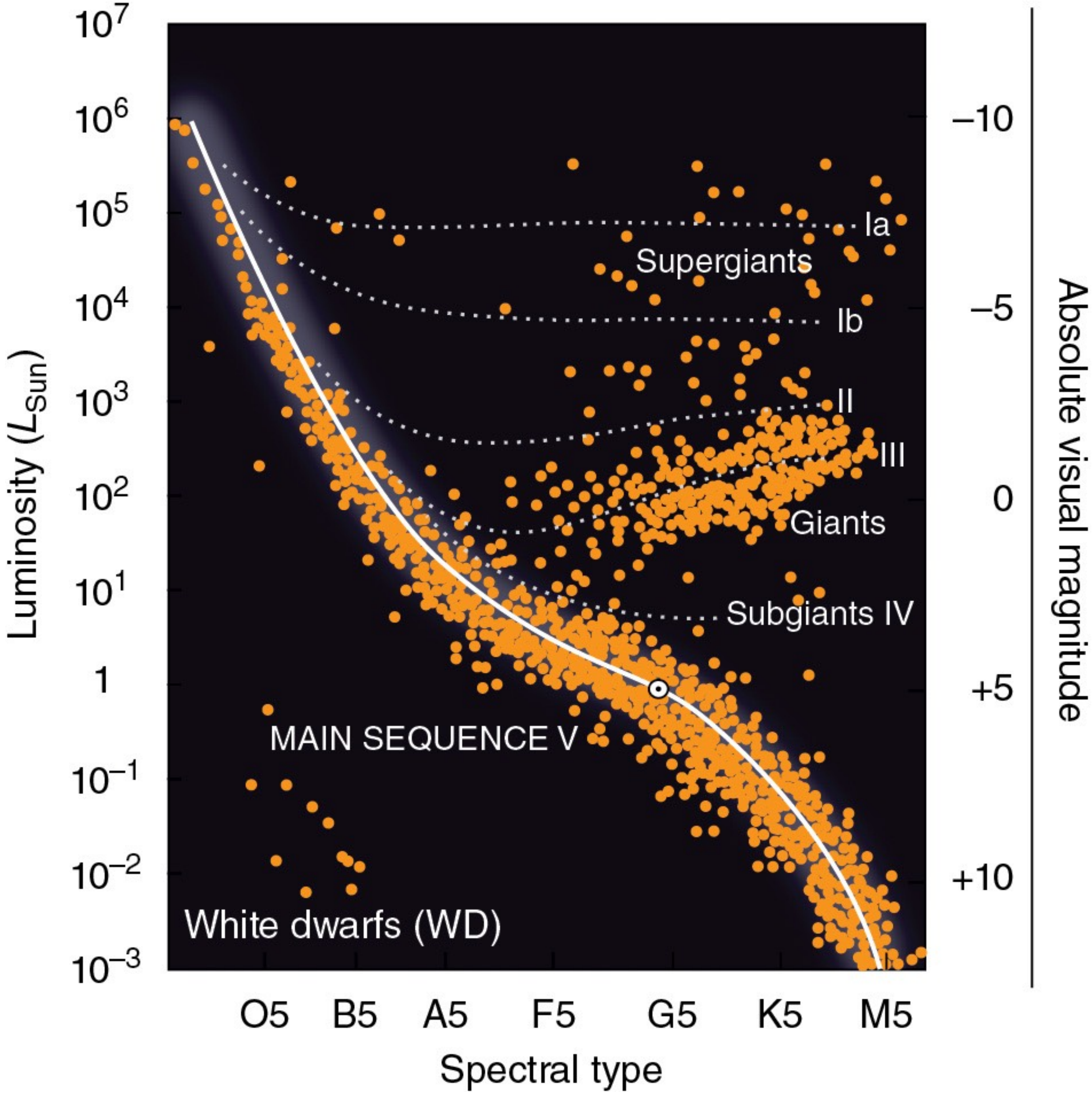


Main Features: Main Sequence, Giant Branch, White Dwarfs



- Most stars exist on the **main sequence**.
- It runs from luminous/hot in upper left corner to low-luminosity/cool in lower right corner.
- Massive main sequence stars are large, luminous, and hot.
- Stars are on the main sequence as long as they burn hydrogen to helium in the core.
- The Sun is on the main sequence.

Giant Stars and Dwarf Stars: the Luminosity Classes



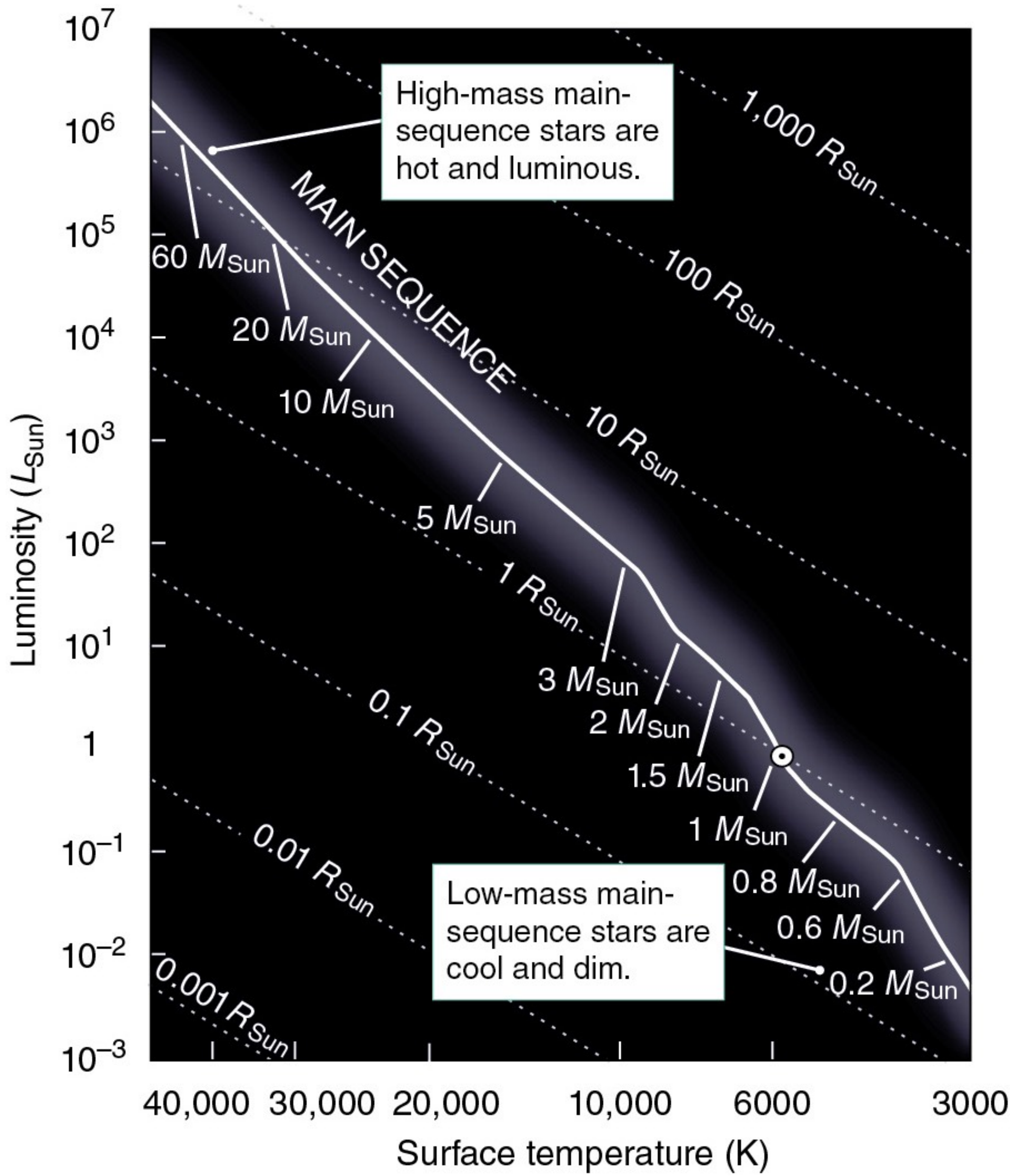
- Not all stars are on the main sequence.
- There are different **luminosity classes**.

- The Sun is a **G2V** star:
  G2 - spectral type
  V - luminosity class
- Betelgeuse is a **M1Ia**:
  M1 - spectral type
  Ia - luminosity class

How did we know that the main sequence stars cover a range of masses?

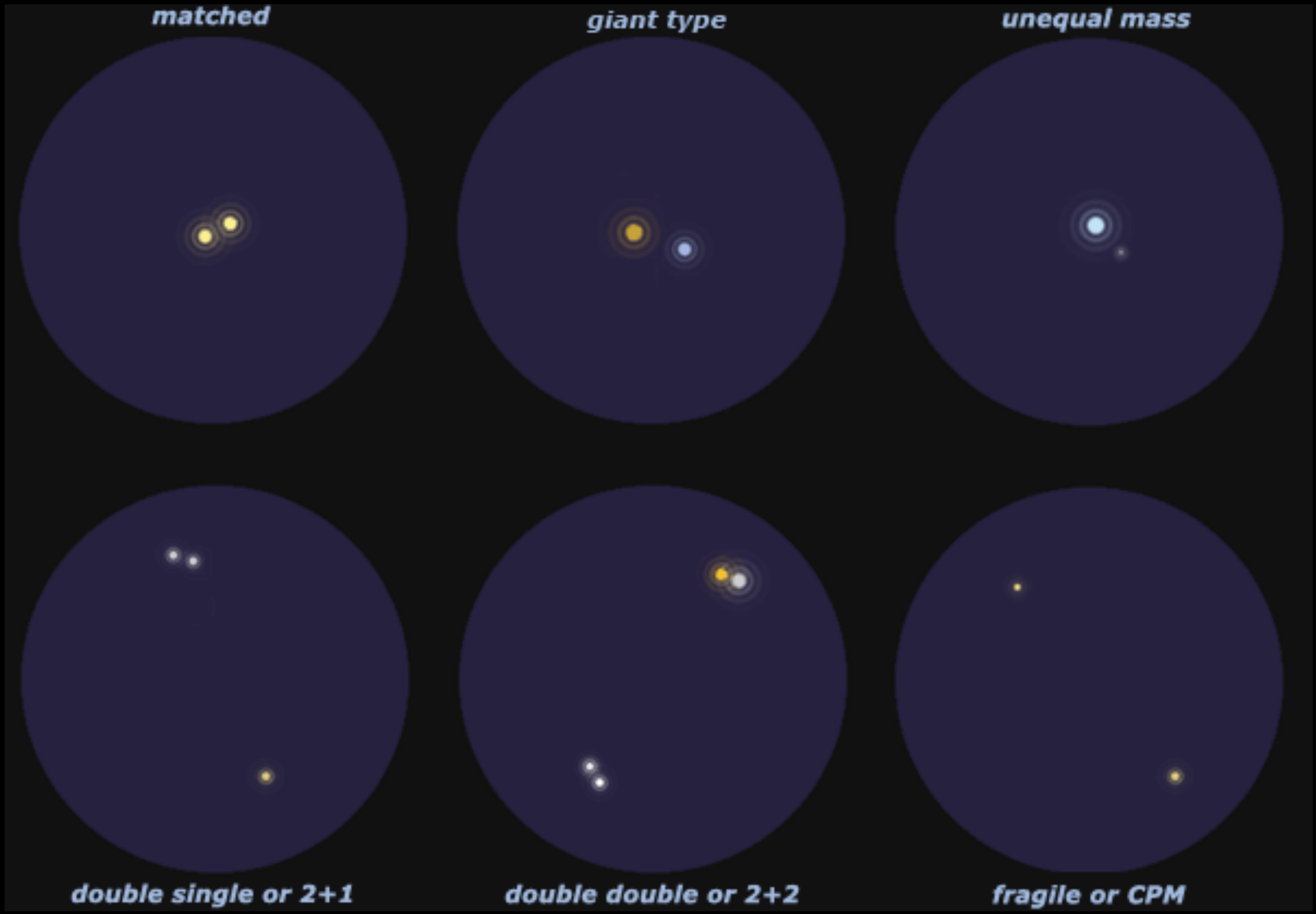
TABLE 13.2 Properties of Main-Sequence Stars

Spectral Type	Temperature (K)	Mass (M_{Sun})	Radius (R_{Sun})	Luminosity (L_{Sun})
O5	42,000	60	13	500,000
B0	30,000	17.5	6.7	32,500
B5	15,200	5.9	3.2	480
A0	9800	2.9	2.0	39
A5	8200	2.0	1.8	12.3
F0	7300	1.6	1.4	5.2
F5	6650	1.4	1.2	2.6
G0	5940	1.05	1.06	1.25
G2 (Sun)	5780	1.00	1.00	1.0
G5	5560	0.92	0.93	0.8
K0	5150	0.79	0.93	0.55
K5	4410	0.67	0.80	0.32
M0	3840	0.51	0.63	0.08
M5	3170	0.21	0.29	0.008



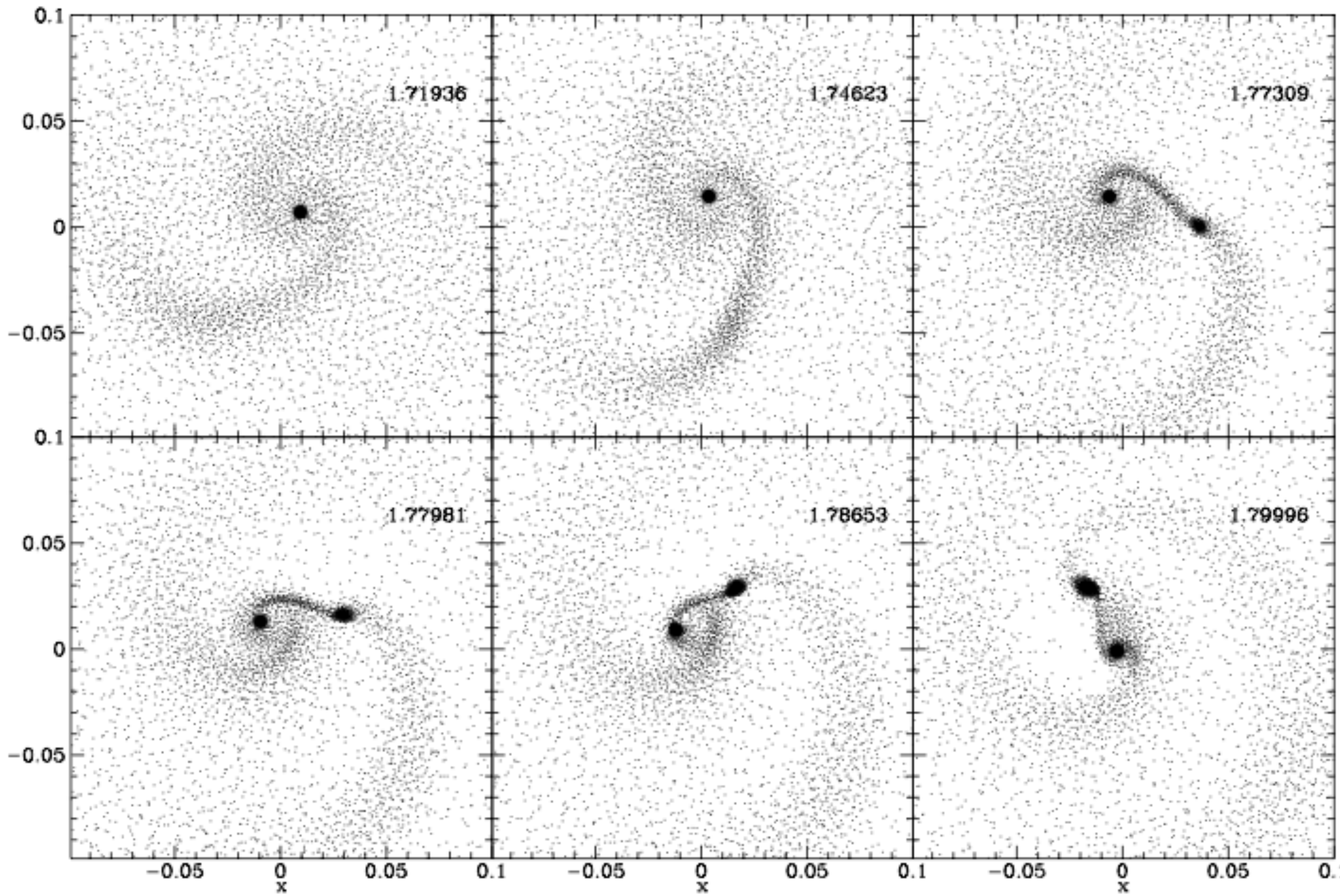
How to measure mass?
Binary stars and Kepler’s Laws

The various configurations of visual binaries and multiples



B. MacEvoy 2012

Binary Star Formation: Accretion Disk Fragmentation



Fragmentation of the protostar accretion disk is believed to be a frequent if not the most common path to binary formation at distances of around 40 AU (Type 4) ... a massive spiral arm forces the protostar off the center of mass to produce a binary structure; the spiral arms draw more mass into the accretion disk while reducing the binary orbital momentum via gravitational (and possibly magnetic) torque (Source: Bonnell & Bate, 1994 [a])

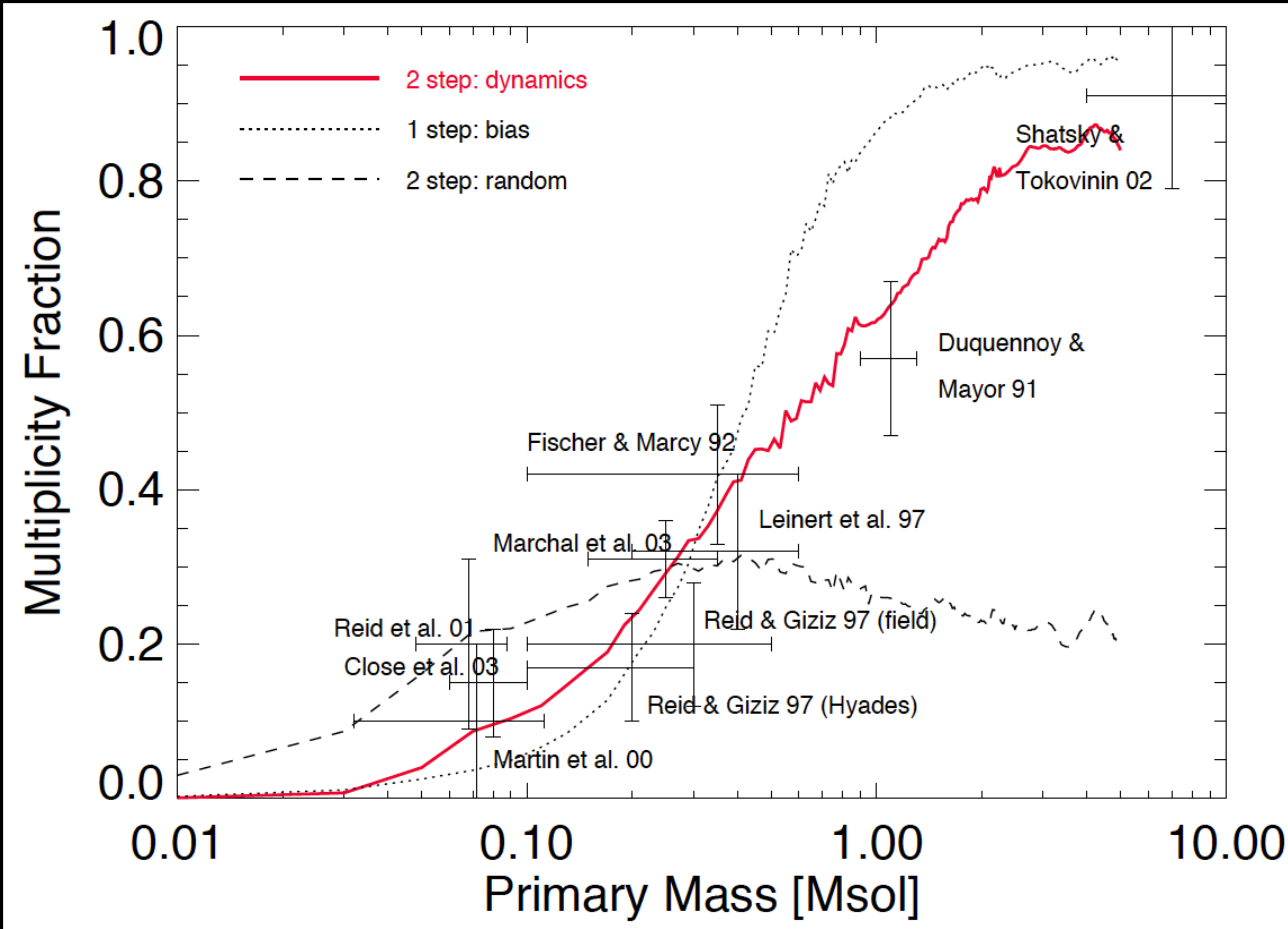
What fraction of stars are binaries?

For solar-type stars, ~60% of star systems are single stars, and ~60% of the stars are components of binary or multiple star systems

	Kuiper (1942)	***Heintz (1969)***	***Abt & Levy (1976)****	***Duquennoy & Mayor (1991)***	***Nordström et al. (2004)***	***Raghavan et al. (2010)***
Systems (N)	274	*n.a.*	123	164	16682	454
Stars as Singles	70%	30%	45%	**57%**	66%	**56%**
Binary	25%	47%	46%	38%	34%	33%
3	4%	16%	8%	4%	.	8%
4+	1%	7%	1%	1%	.	3%
All Double Star Systems	30%	70%	55%	43%	34%	44%
Median R		50 AU		35 AU		40 AU
Stars in Doubles	52%	85%	73%	**62%**	51%	**65%**

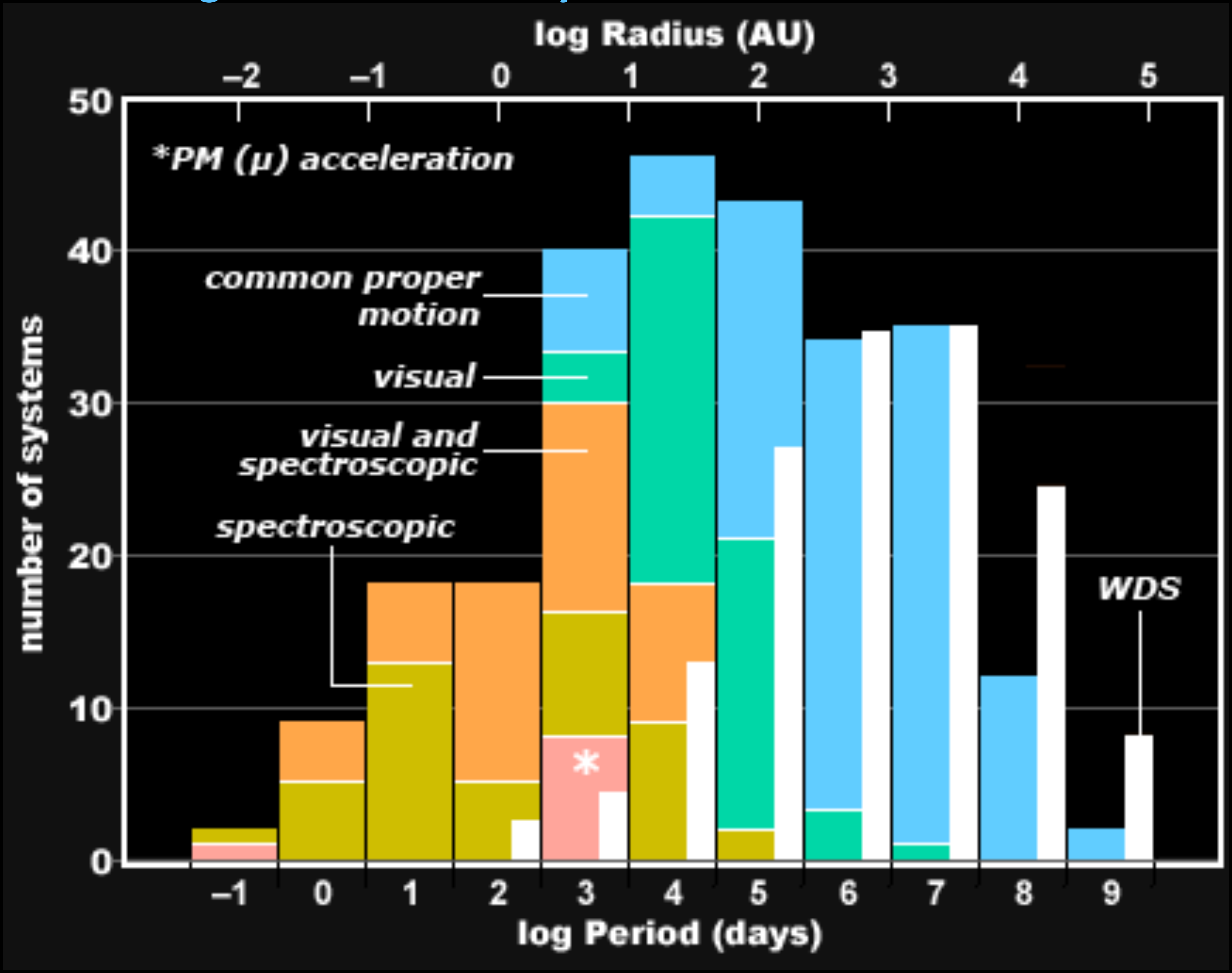
B. MacEvoy 2012

The multiplicity fraction increases with the mass of the primary



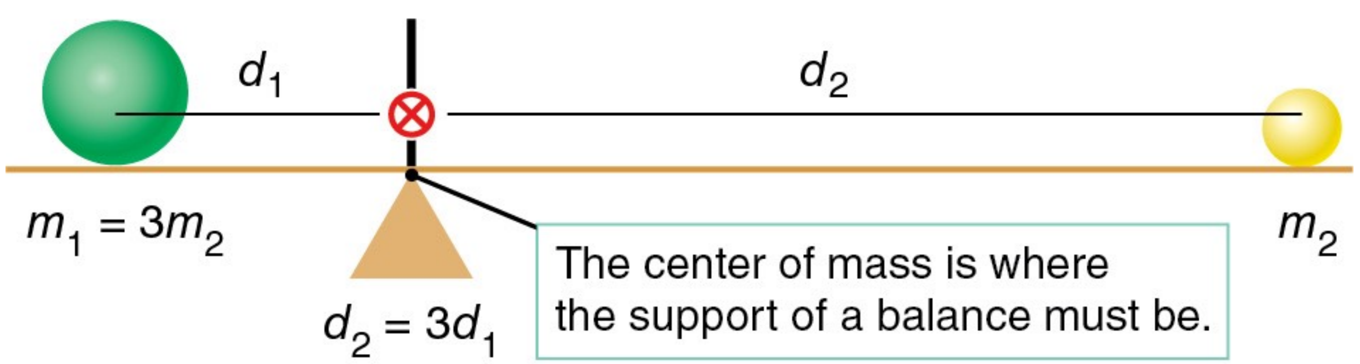
B. MacEvoy 2012

The logarithmic of Binary Periods follow a "Bell" curve

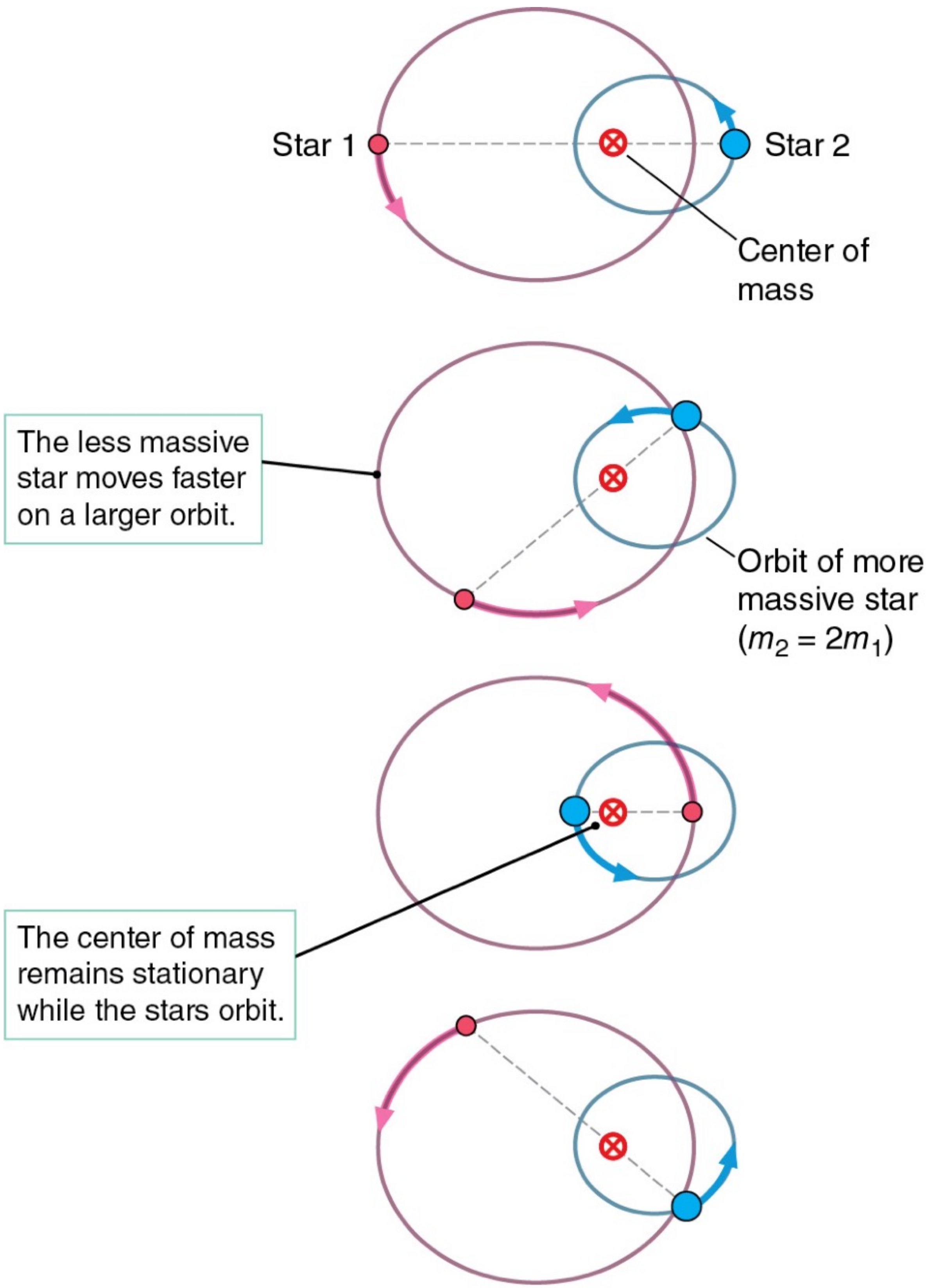


Binary Star - Center of Mass

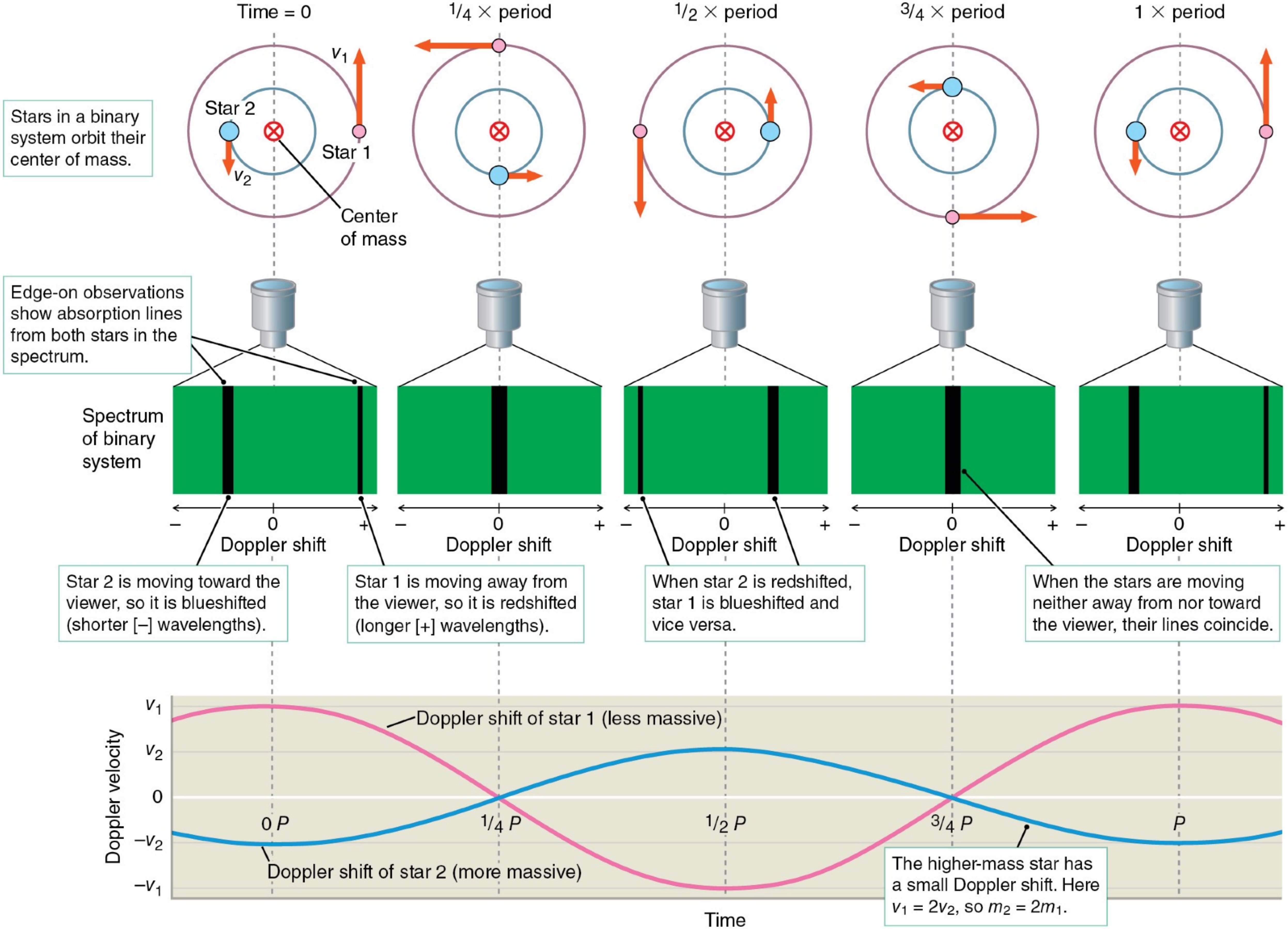
- To measure mass, we must look for the effects of gravity.
- Many stars are **binary stars** orbiting a common **center of mass**.
- A less massive star moves faster on a larger orbit.



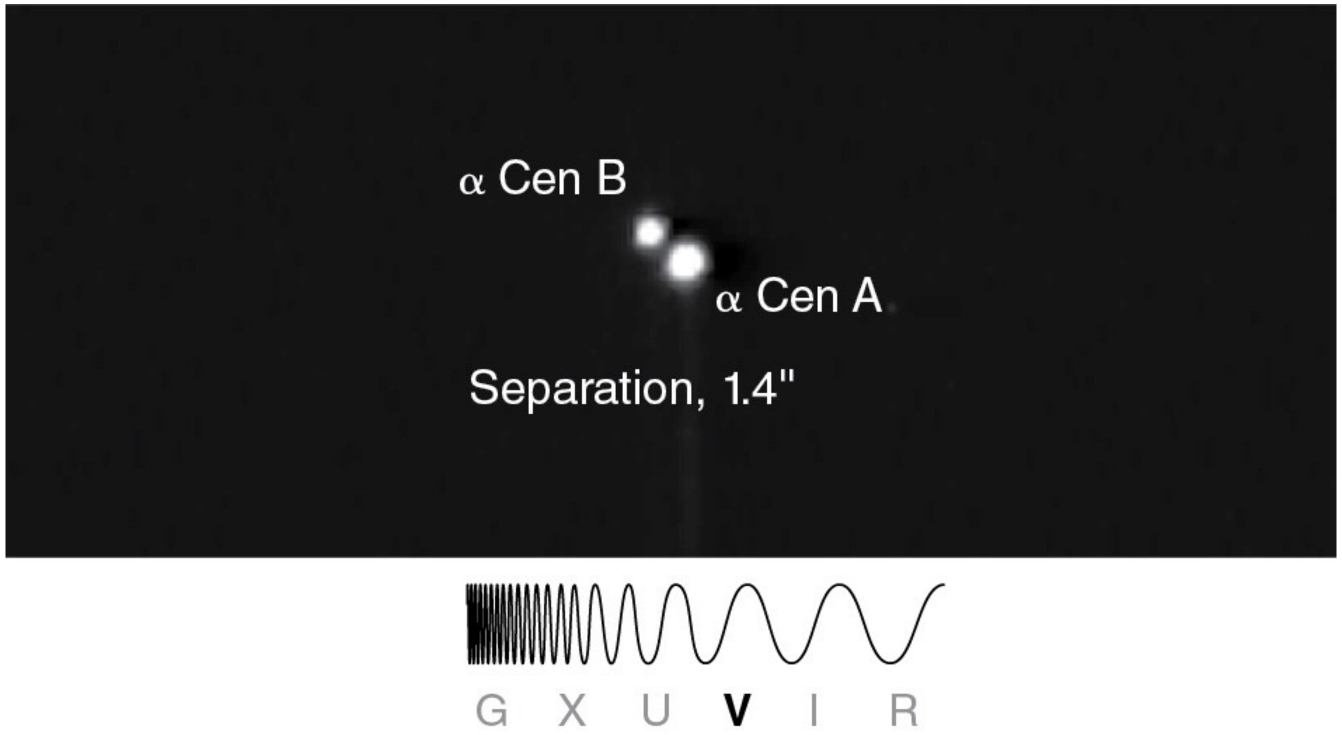
Center of mass "seesaw" equation:
$m_1 d_1 = m_2 d_2$



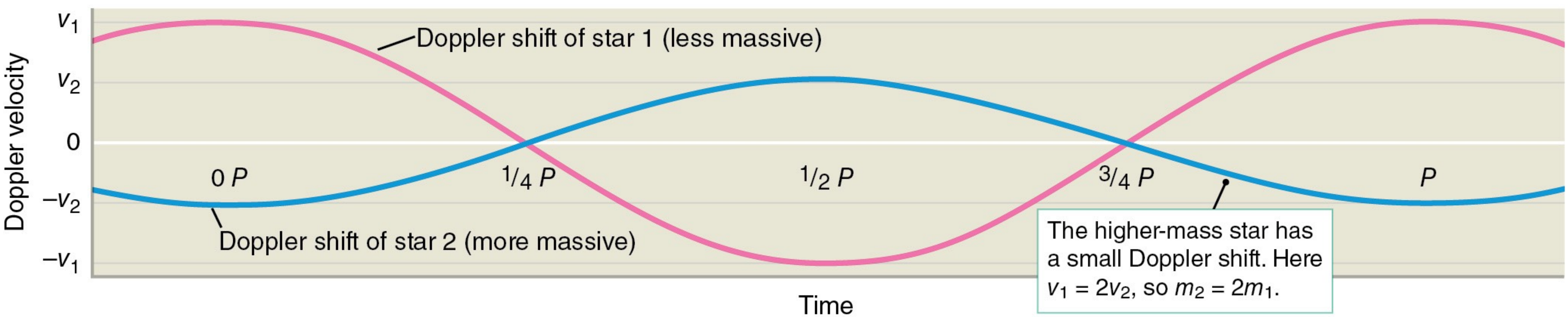
Binary Star - Doppler Shift Measurements vs. Time



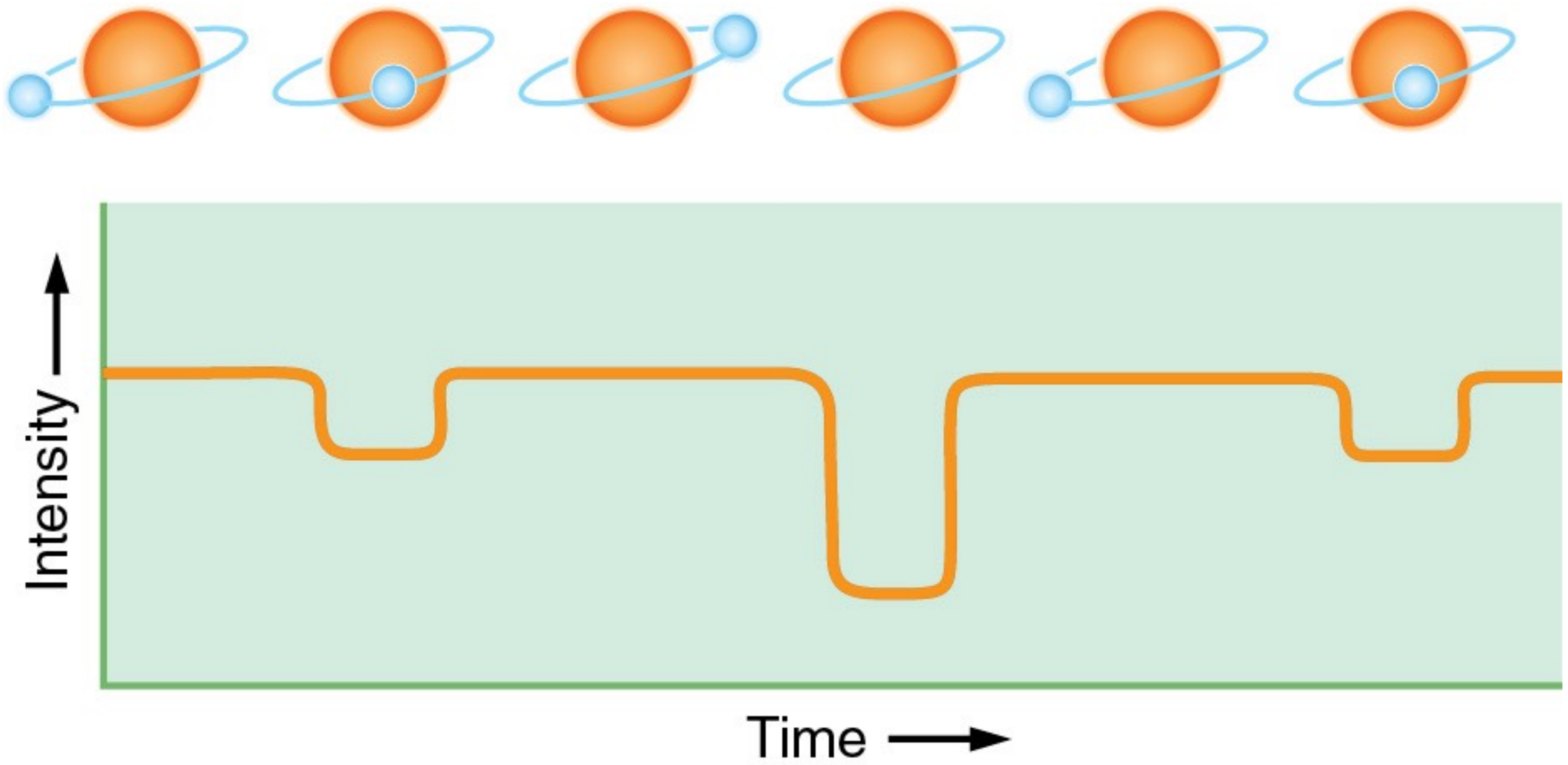
Binary Stars: Doppler shift curves from spectroscopy



- A **visual binary** system is one in which both stars are distinguished visually.
- In a **spectroscopic binary** system, stars are too far away to distinguish; pairs of Doppler-shifted lines trade places.

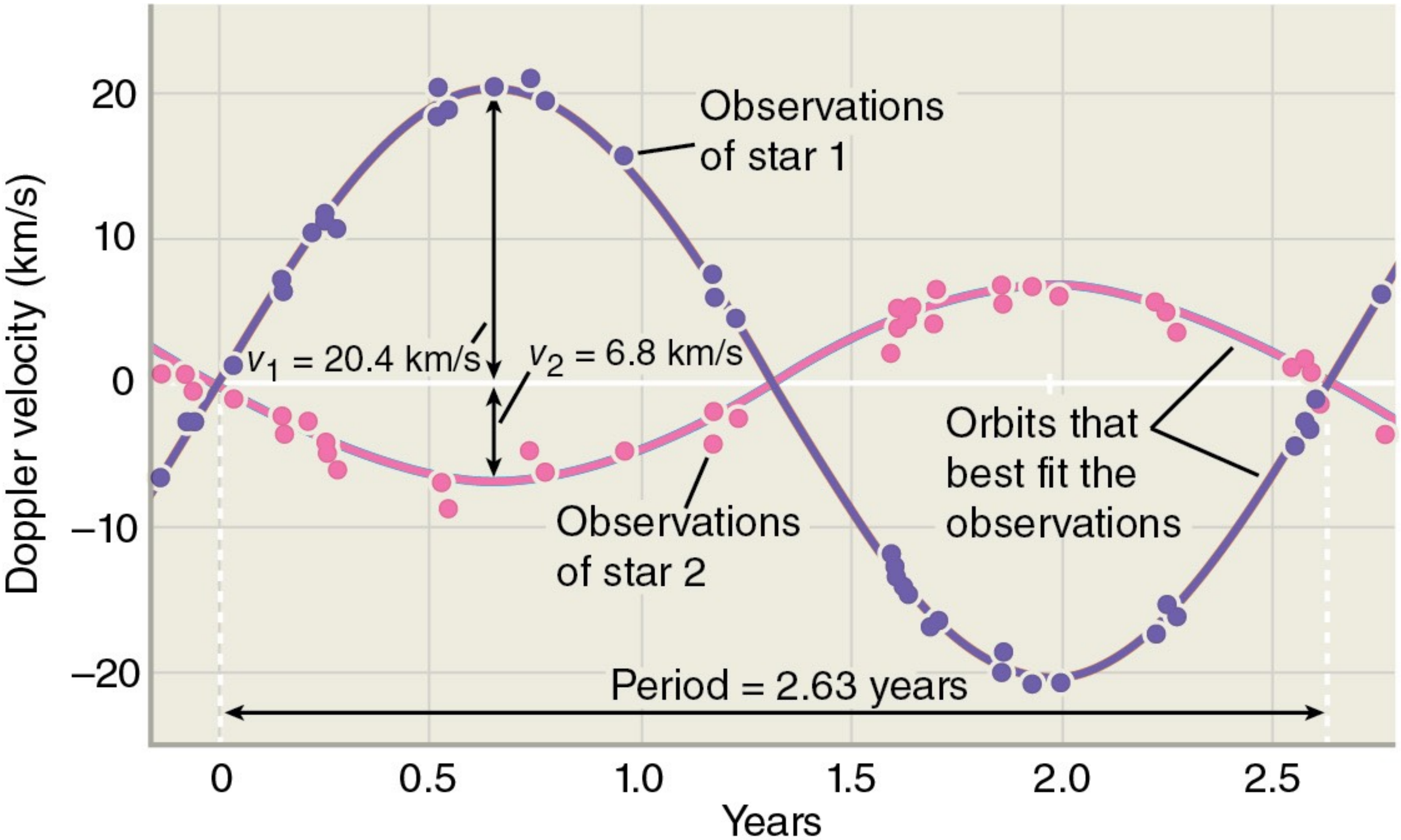


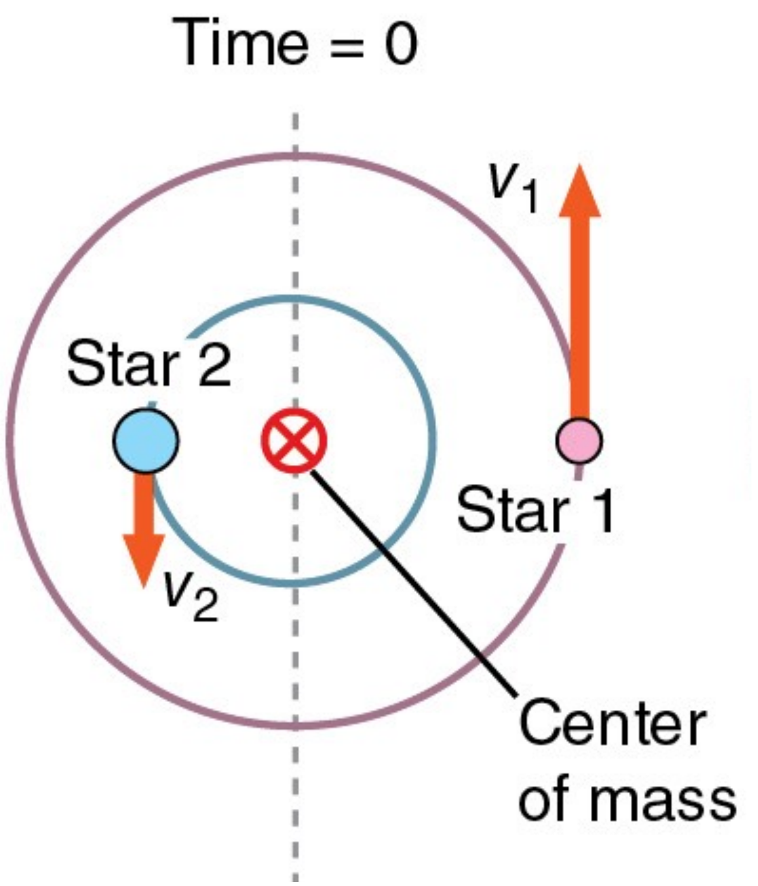
Eclipsing Binary Stars - Light curve from photometry



- In an **eclipsing binary** system, the total light coming from the star system decreases when one star passes in front of the other.
- We also can measure the radii of the stars in these systems.

WIO 13.4: Measuring the Masses of Stars in an Eclipsing Binary Pair





- Being an eclipsing binary implies that their orbits are viewed edge-on
- The doppler shift results shown above give key parameters:
  - The period of the binary (***P***)
  - The orbital velocities of star 1 and star 2 ($\boldsymbol{V_1}$ and $\boldsymbol{V_2}$)
- What are the circumferences and radii of the two orbits?

$$C_1 = V_1 \times P = 2\pi a_1$$

$$C_2 = V_2 \times P = 2\pi a_2$$

From Chap 4: Equations of Kepler's 3rd Law

3rd Law:
period-distance
relation

$$\frac{a^3}{P^2} = \frac{GM}{4\pi^2}$$

$$\frac{a_{AU}^3}{P_{year}^2} = M_{solar-mass}$$

But there are two masses (m1 and m2), and two semimajor axes (r1 & r2), how should we use the Kepler's 3rd law to estimate mass?

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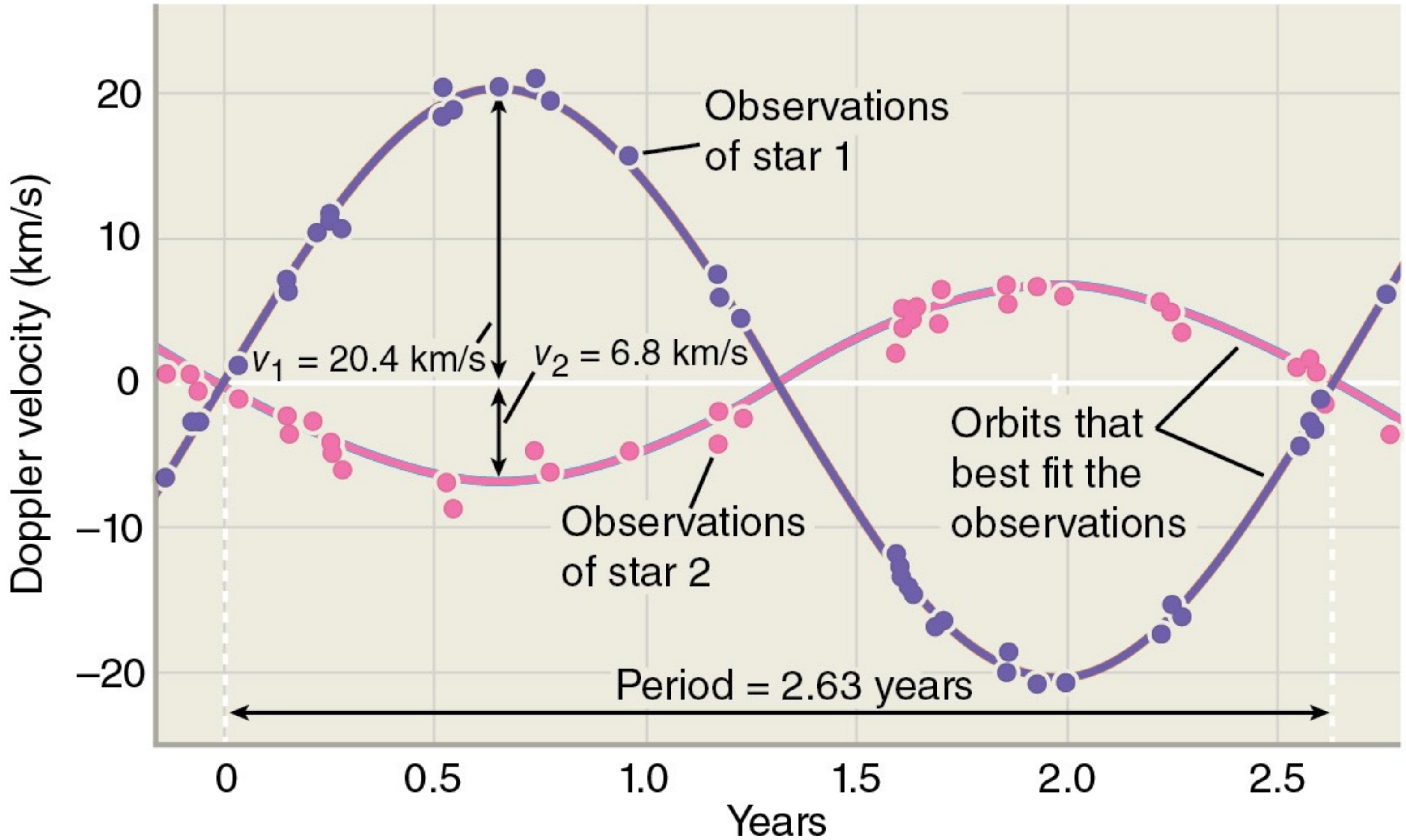
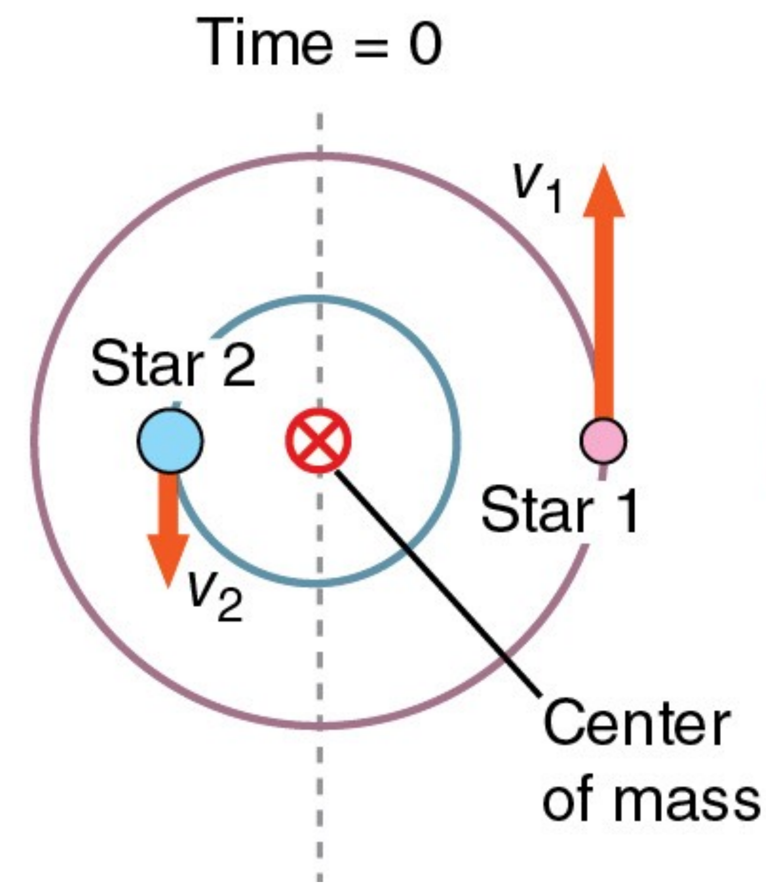




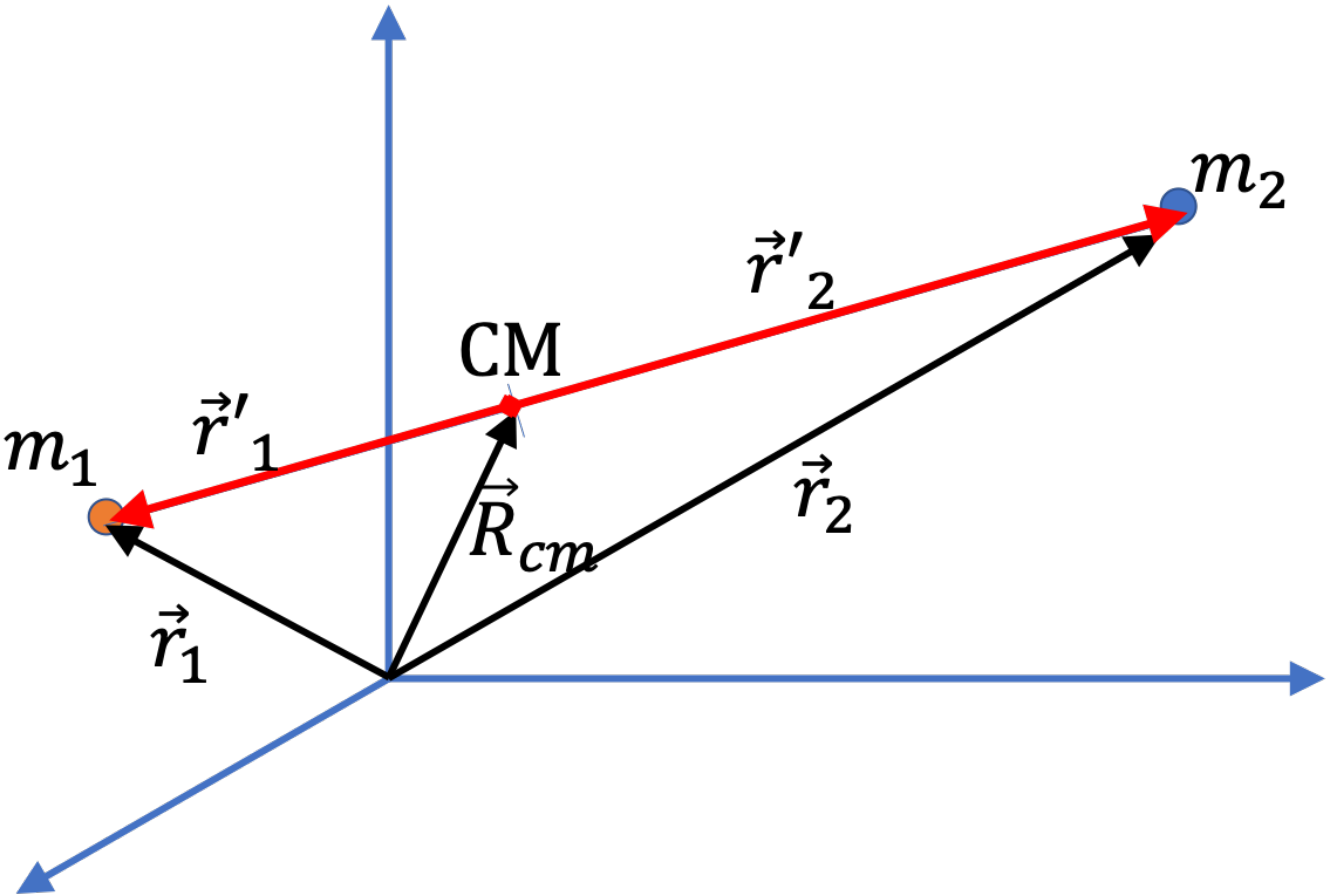
- Next, we can calculate the total mass using Kepler's third law:

$$\frac{M_1 + M_2}{1\ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

- Finally, we obtain the individual masses based on the velocity ratio:

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{V_2}{V_1}$$

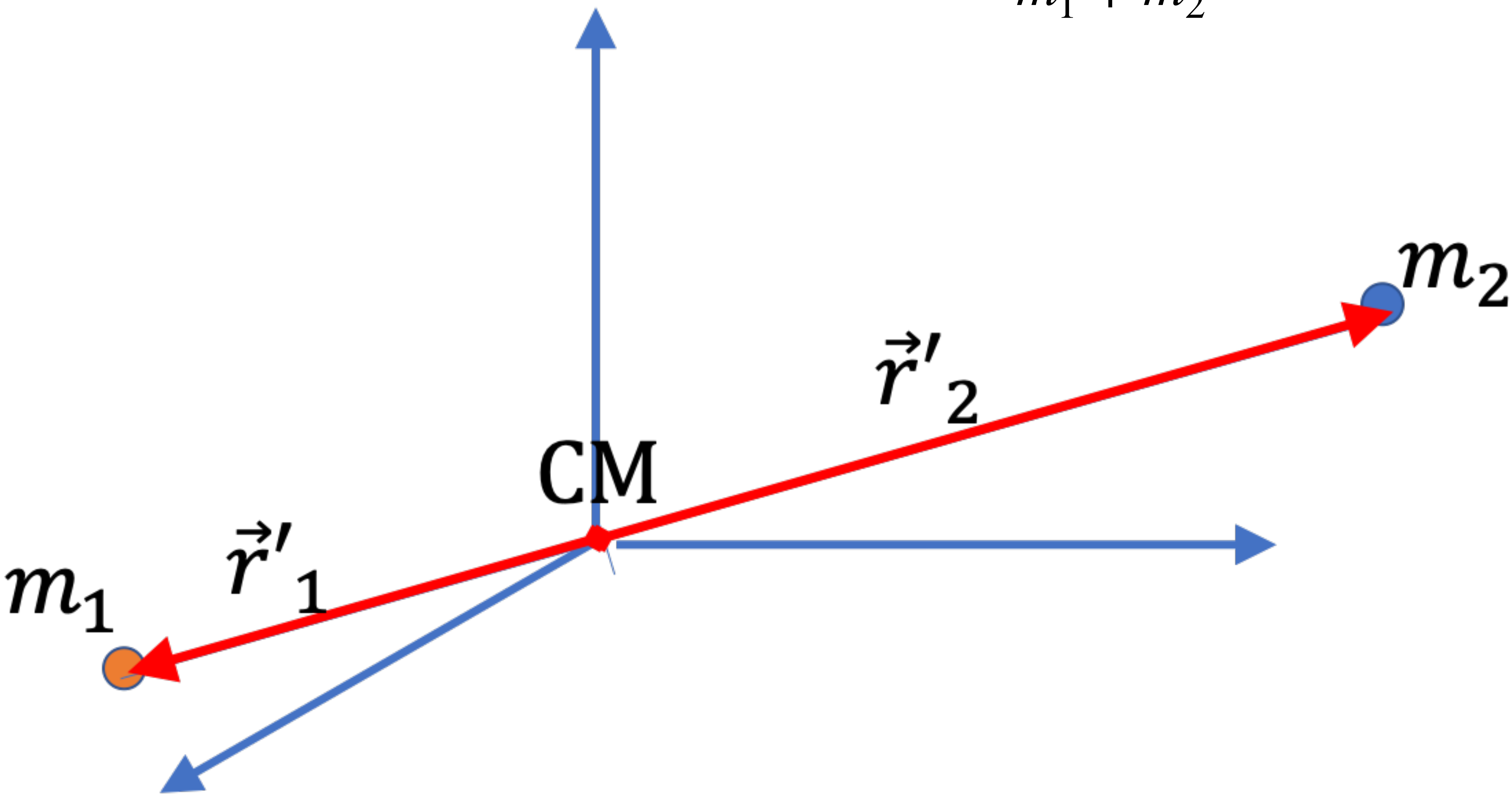
Two-body Problem - General Reference Frame



Two-body Problem - The Center-of-Mass Reference Frame

$$m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} = 0$$
$$\overrightarrow{r_2} - \overrightarrow{r_1} = \vec{r}$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r}$$
$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r}$$



Two-Body Problem reduced to One-Body Problem

define reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r} = -\frac{\mu}{m_1}\vec{r}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r} = \frac{\mu}{m_2}\vec{r}$$

$$\vec{v}_1 = -\frac{\mu}{m_1}\vec{v}$$

$$\vec{v}_2 = \frac{\mu}{m_2}\vec{v}$$

Then write down the total kinetic and gravitational potential energy

$$E = \frac{1}{2}m_1|\vec{v}_1|^2 + \frac{1}{2}m_2|\vec{v}_2|^2 - G\frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|}$$

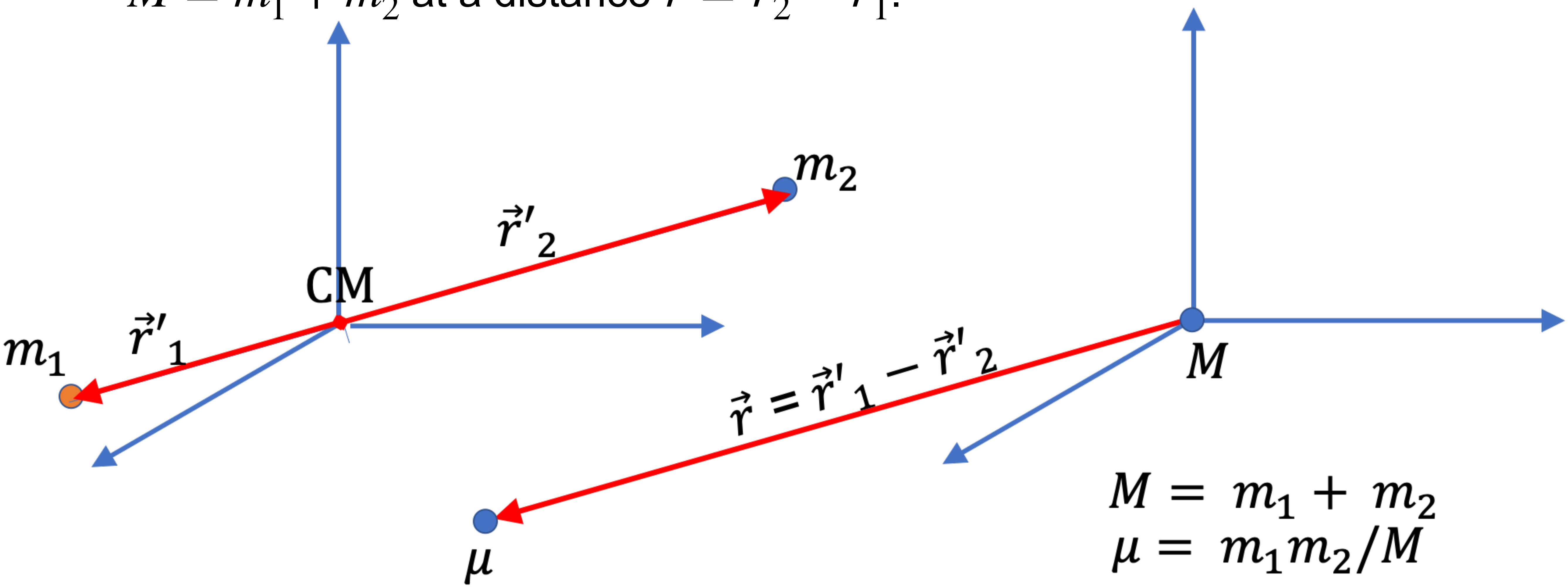
$$= \frac{1}{2}m_1\left(\frac{\mu}{m_1}\right)^2 v^2 + \frac{1}{2}m_2\left(\frac{\mu}{m_2}\right)^2 v^2 - G\frac{(m_1 + m_2)\cdot m_1 m_2/(m_1 + m_2)}{r}$$

$$= \frac{1}{2}\mu\left(\frac{\mu}{m_1} + \frac{\mu}{m_2}\right)v^2 - G\frac{M\mu}{r} \Rightarrow E = \frac{1}{2}\mu v^2 - G\frac{M\mu}{r}$$

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a fixed total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.

Kepler's 3rd Law for Binary Stars (Two-body Problem)

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a fixed total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.



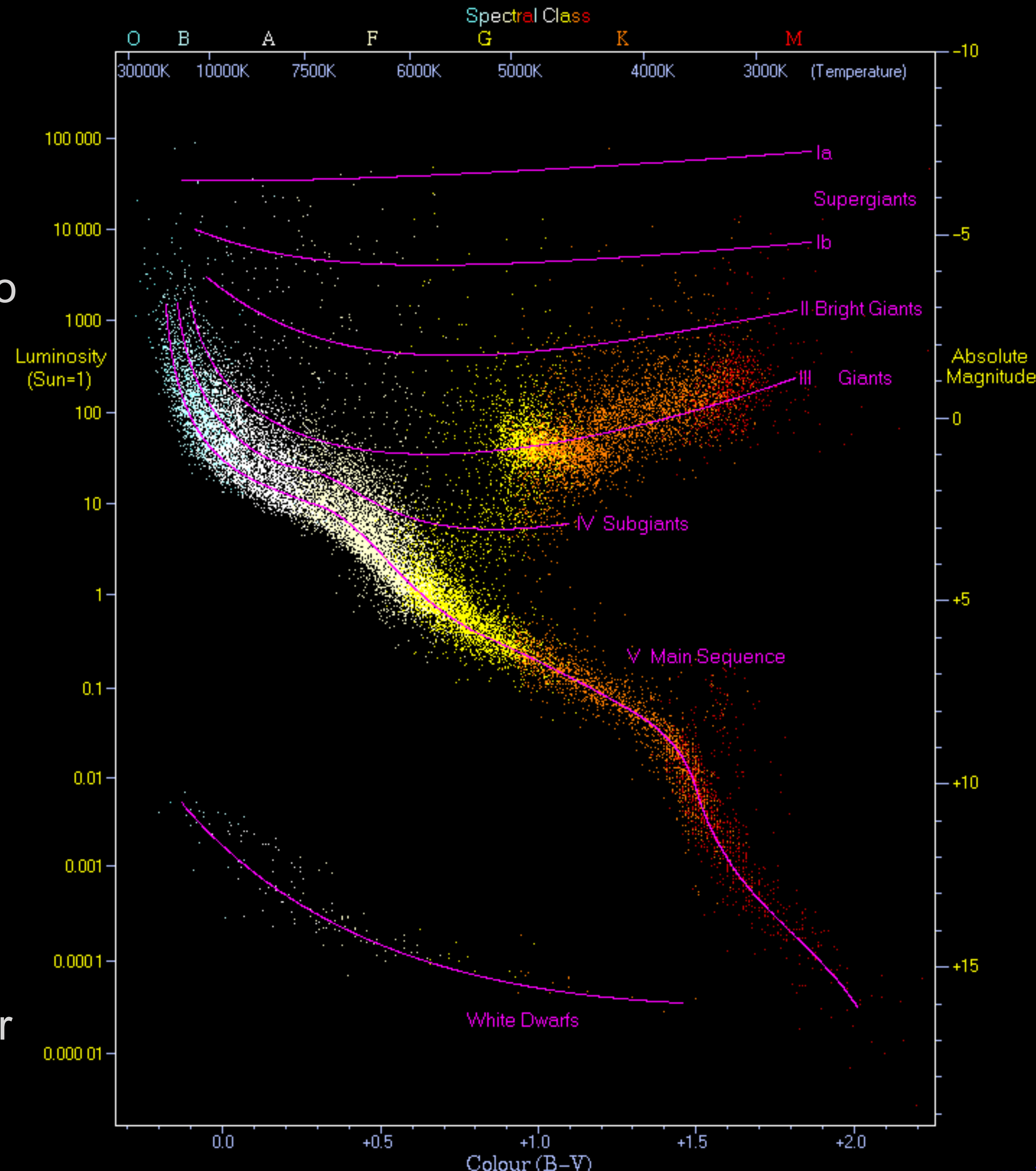
$$M = m_1 + m_2$$
$$\mu = m_1 m_2 / M$$

One-body problem: $$\frac{m}{1\ M_{\text{sun}}} = \left(\frac{a}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Two-body problem: $$\frac{m_1 + m_2}{1\ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Chap 13: Key Concepts

- stellar parallax
- Unit parsec defined by AU
- Pogson's ratio:
  apparent magnitude and flux ratio
- CCD photometry: count rate to magnitude
- absolute magnitude
- distance modulus (m-M)
- standard candle methods
    - spectroscopic parallax
    - type Ia supernovae
- color index and temperature
- luminosity-temperature-size relation
- HR diagram: the main sequence
- spectroscopic binaries and stellar masses



Chap 13: Key Equations

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p} \right)$$

$$m_{\lambda,2} - m_{\lambda,1} = -2.5 \log(f_{\lambda,2}/f_{\lambda,1})$$

$$m_\lambda - M_\lambda = 2.5 \log \left(\frac{d}{10 \text{ parsec}} \right)^2 = 5 \; [\log d(\text{parsec}) - 1]$$

$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

$$\lambda_{\text{peak}} = 2.9 \text{ mm} \frac{1 \text{ K}}{T}$$

$$L = 4\pi R^2 \times \sigma_{\text{SB}} T^4 \qquad \frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}} \cdot \left(\frac{T}{T_\odot} \right)^{-2}$$