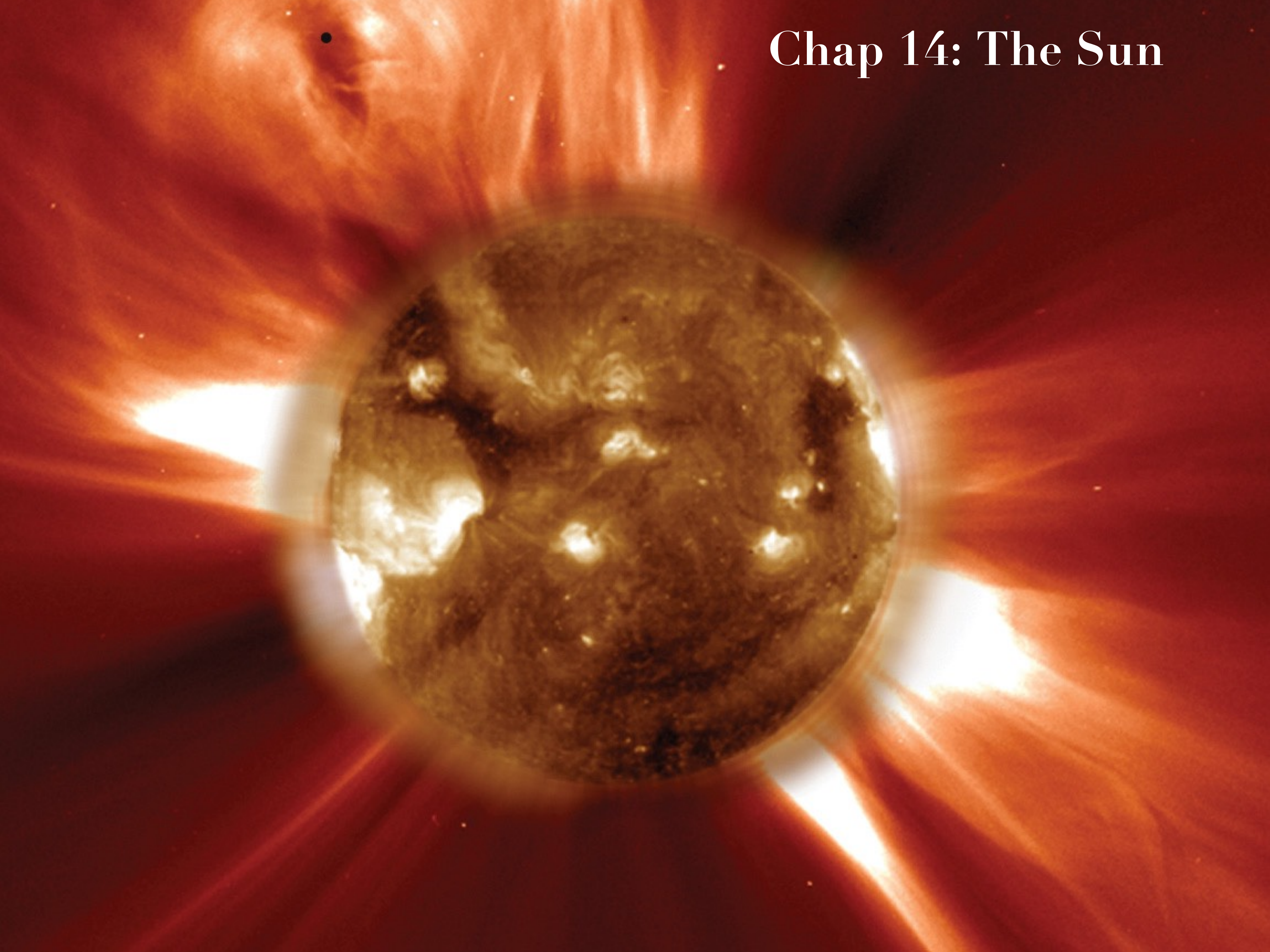


Chap 14: The Sun

Chap 14: The Sun - Key Topics

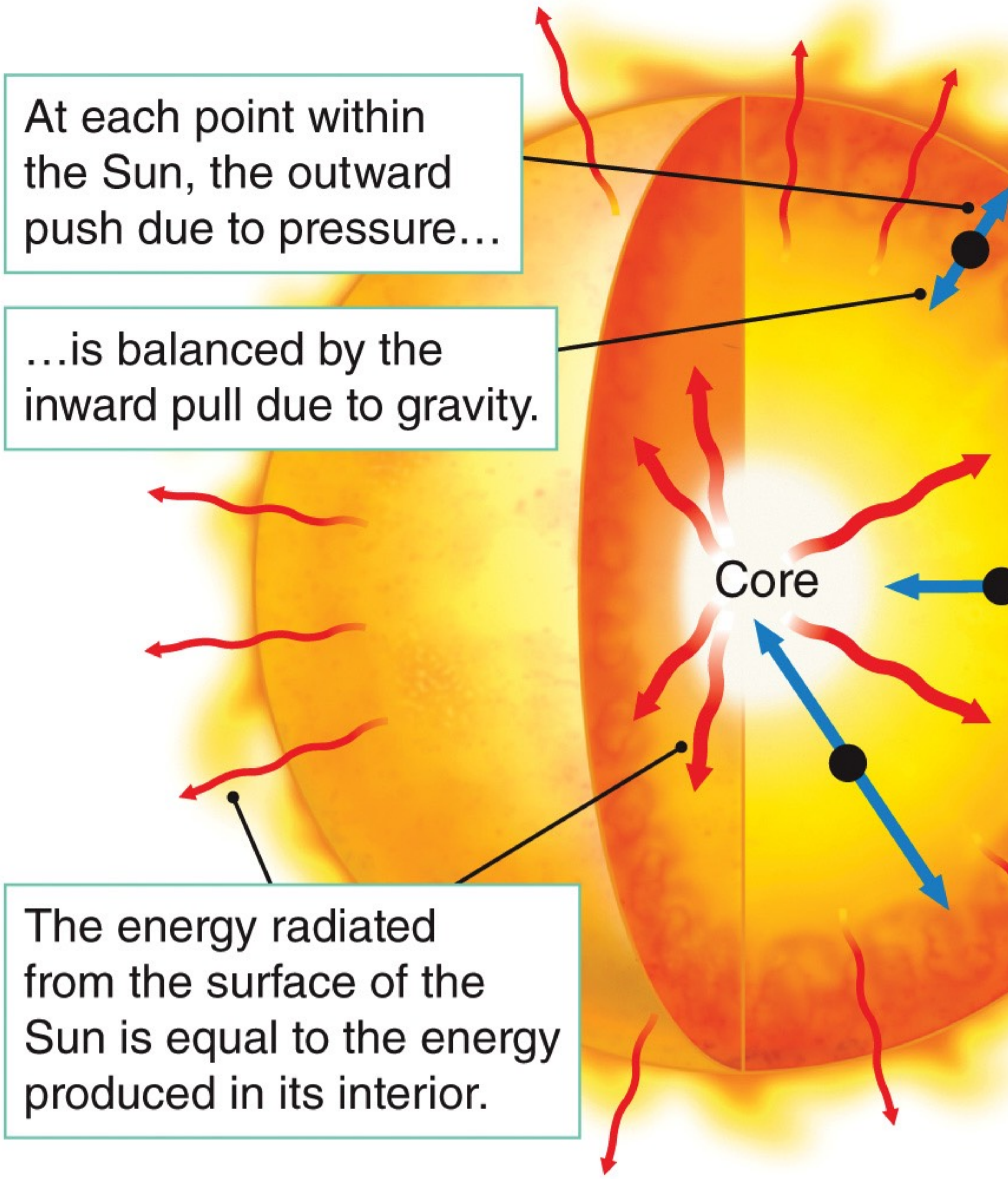
- hydrostatic equilibrium: $dP(r)/dr = -\rho(r)\ g(r)$
- nuclear fusion of Hydrogen as the powering mechanism
    - main sequence lifetime (lifetime ~ mass/luminosity)
    - fusion theory prediction vs. observation (solar neutrinos)
- energy transport from core to the surface
    - radiative random walk & convection
- atmospheres of the Sun: photosphere, chromosphere, corona
    - solar activities and magnetic fields

How a star stay stable?

Hydrostatic Equilibrium

The Sun is in Hydrostatic Equilibrium

- The internal pressure of the Sun pushes against its self-gravity.
- This occurs at each point in the Sun and is called **Hydrostatic equilibrium.**
  - outward pressure = inward force of gravity
  - Hydrostatic equilibrium prevents the Sun from collapsing.

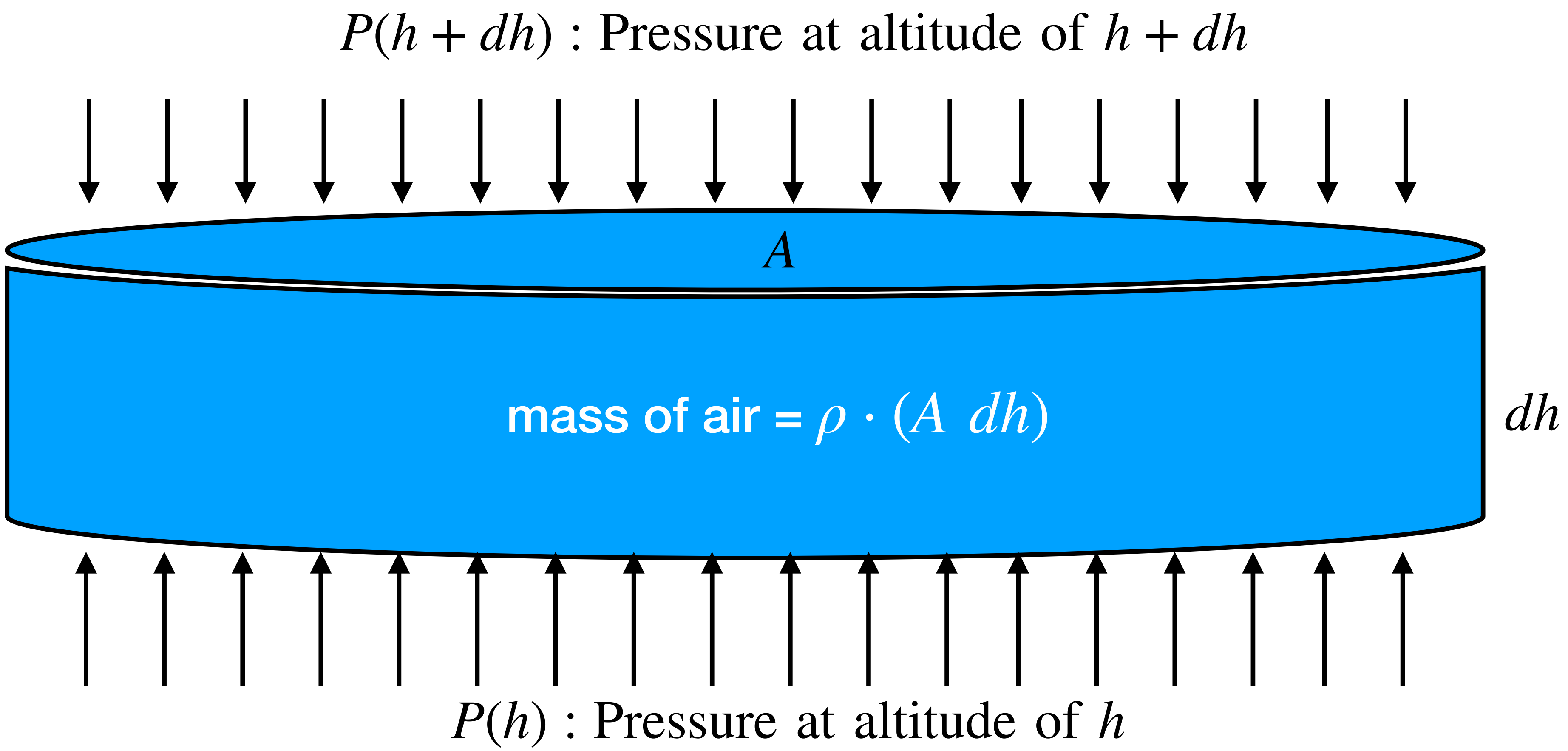


At each point within the Sun, the outward push due to pressure…

…is balanced by the inward pull due to gravity.

The energy radiated from the surface of the Sun is equal to the energy produced in its interior.

Derivation: Hydrostatic Equilibrium Equation



- consider the force balance in a packet of air at an altitude of **h**. The packet has a cylindrical shape with an area of **A** and an infinitesimal height of **dh**
- this packet of air can stay stationary because of a force balance
  - upward force from pressure = $[P(h) - P(h+dh)] \cdot A = -\,dP \cdot A$
  - downward gravitational force = $\rho \cdot (A\ dh) \cdot g$

Derivation: Hydrostatic Equilibrium Equation

- consider the force balance in a packet of air at an altitude of **h**. The packet has a cylindrical shape with an area of **A** and an infinitesimal height of **dh**
- this packet of air can stay stationary because of a **force balance**
  - upward force from pressure = $[P(h) - P(h + dh)] \cdot A = -dP \cdot A$
  - downward gravitational force = $\rho \cdot (A \ dh) \cdot g$
- Equating the two forces means **hydrostatic equilibrium**:

$$-dP = \rho \ g \ dh \Rightarrow$$

$$-d(nkT) = (\mu m_H n) \ g \ dh$$

where we have applied the **ideal gas law** and expressed mass density as number density. If the temperature is constant, i.e., the **isothermal condition**, we can take *kT* out of the differentiation, and we can also move n to the left side (assuming also **thin atmosphere** so that *g* is constant):

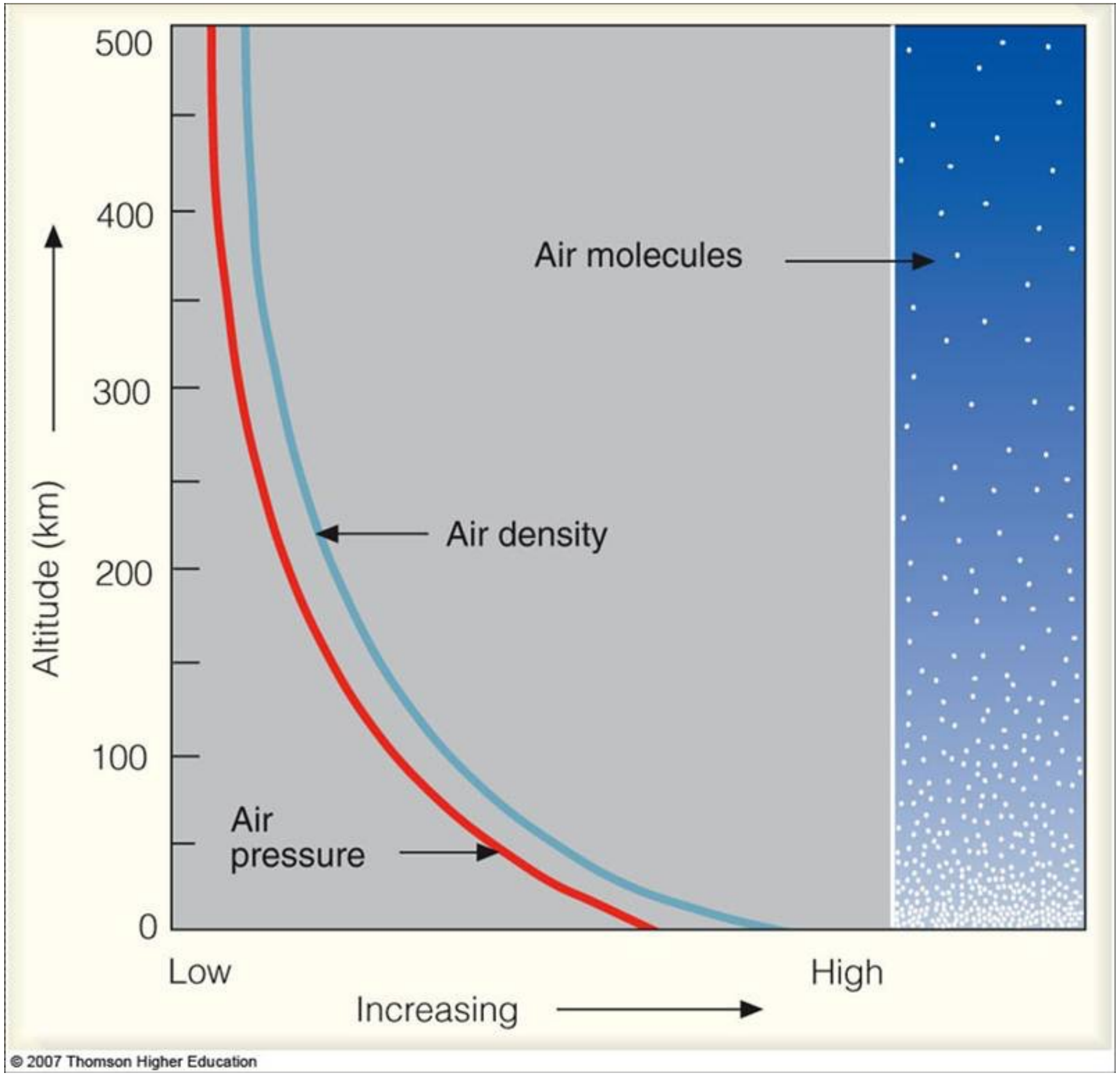
$$\frac{dn}{n} = -\frac{\mu m_H g}{kT} \ dh$$

- Integrating both side from altitude of *0* to altitude of *h*, we have a solution:

$n(h) = n_0 \exp\left(-\frac{h}{h_S}\right)$ where $h_S = \frac{kT}{\mu m_H g}$ is the **scale height**.

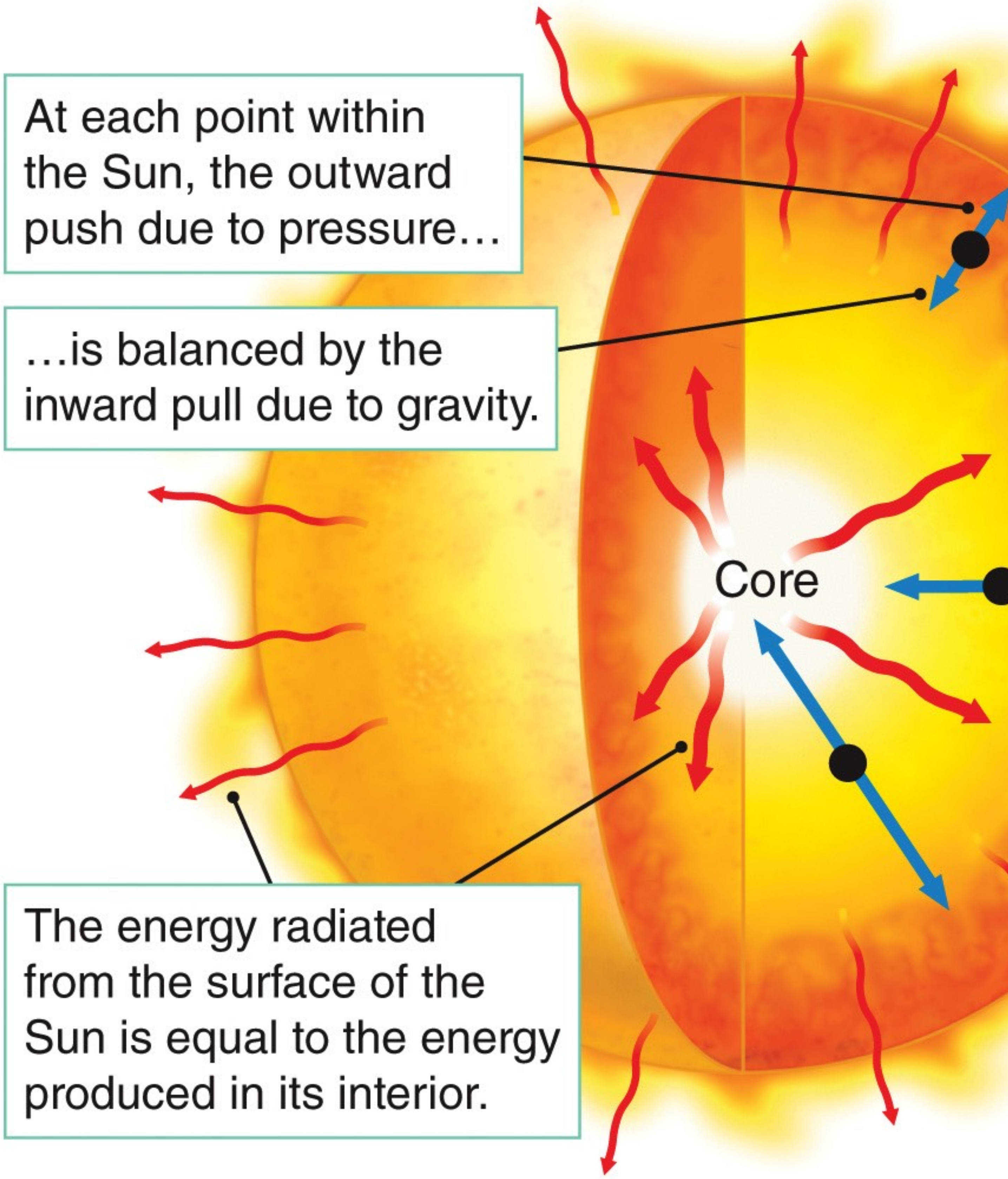
Pressure/Density Profile in Isothermal Condition

$n(h) = n_0 \exp\left(-\frac{h}{h_S}\right)$ where $h_S = \frac{kT}{\mu m_H g}$ is the **scale height**.



Increasing Pressure and Temperature Deeper into the Sun

- Deeper into the Sun, more material is pushing down, which means the force of gravity increases.
- The pressure must also increase for the Sun to remain in balance.
- Pressure is proportional to temperature, so increasing pressure will lead to increasing temperature.



At each point within the Sun, the outward push due to pressure…

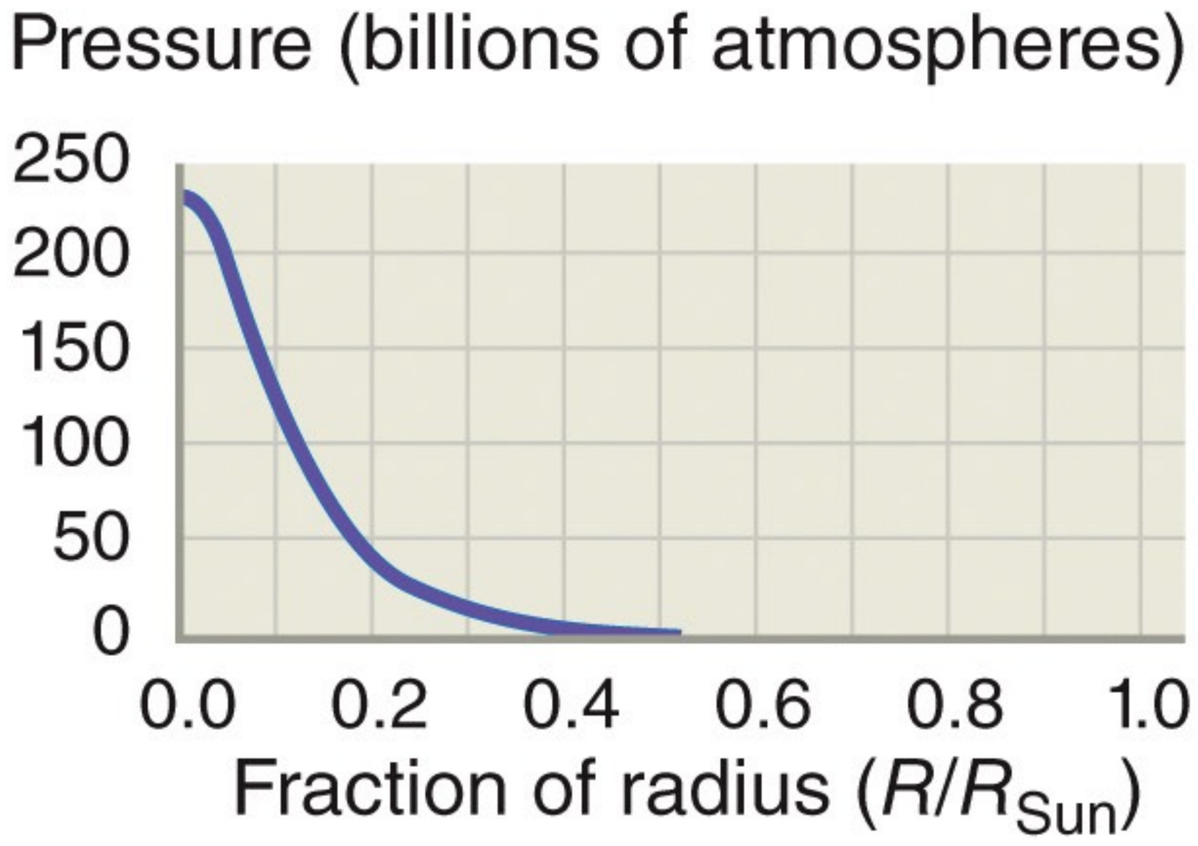
…is balanced by the inward pull due to gravity.

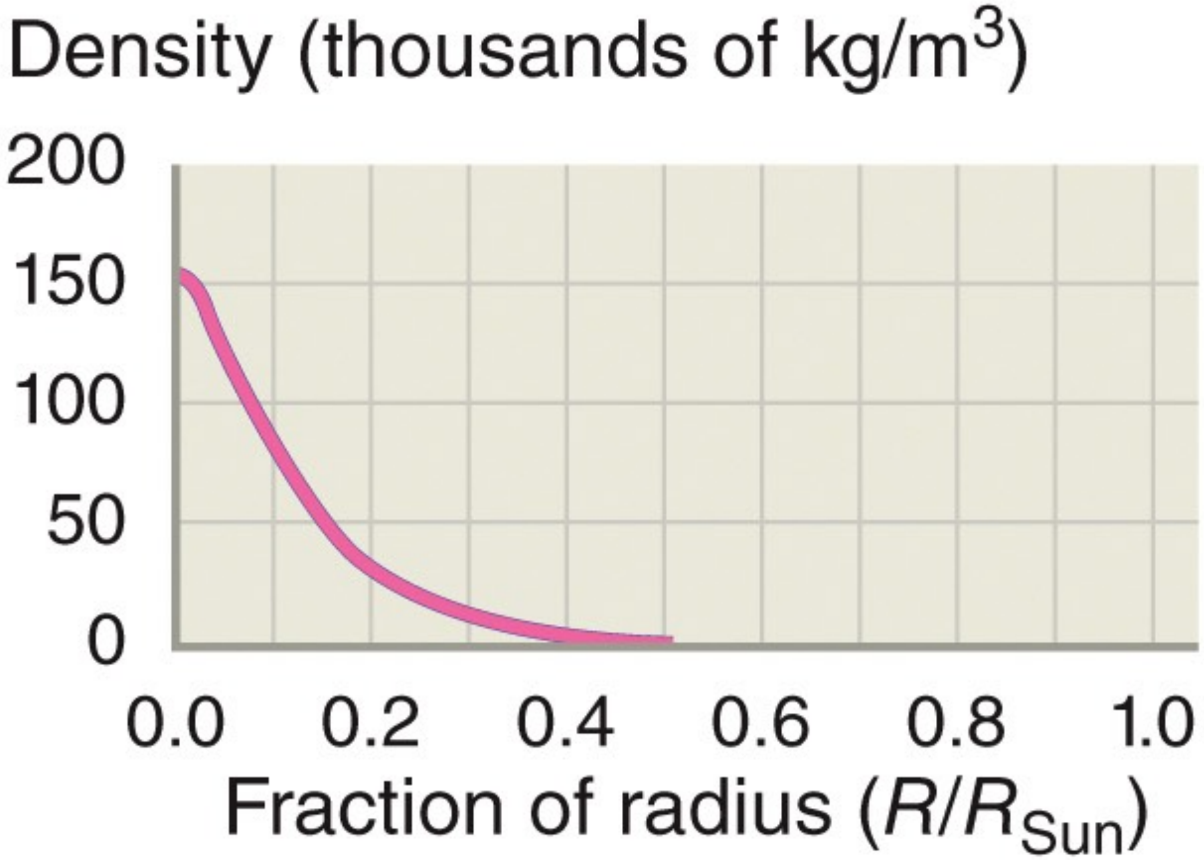
The energy radiated from the surface of the Sun is equal to the energy produced in its interior.

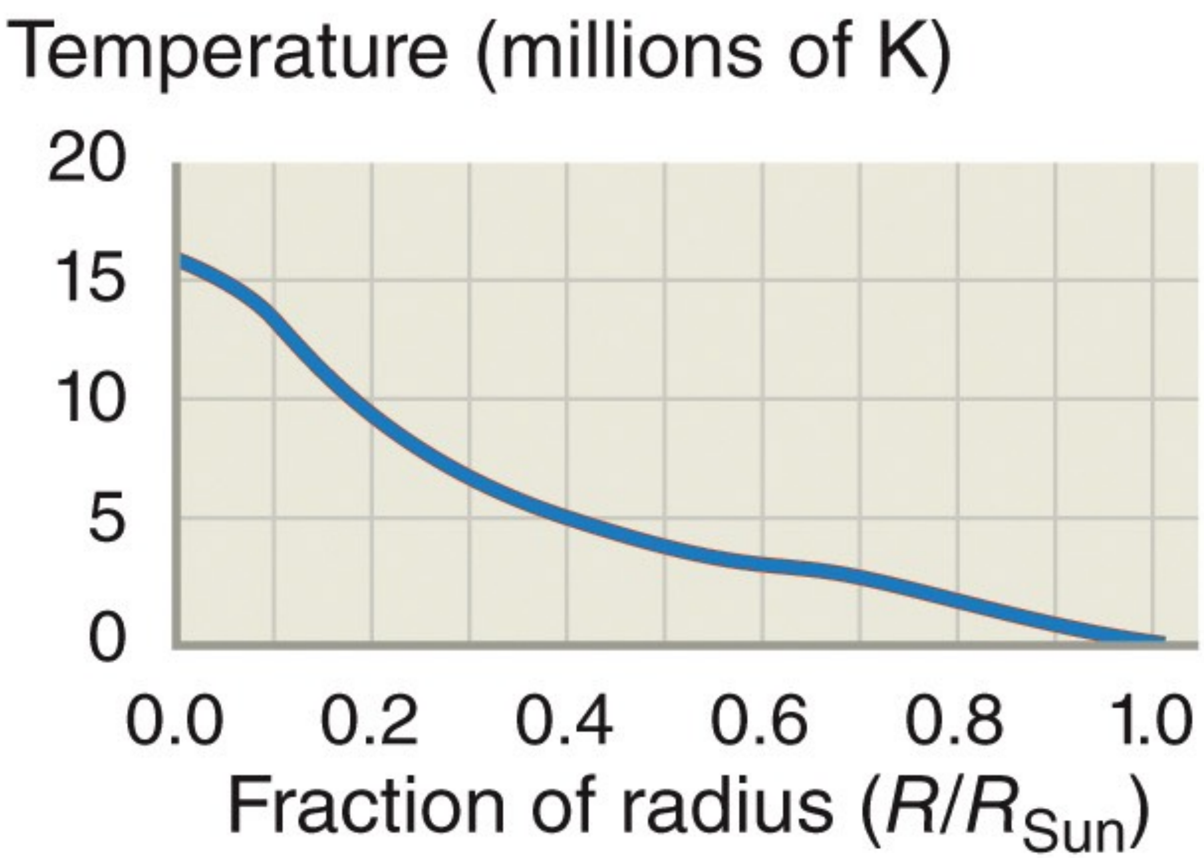
Increasing Pressure and Temperature as You Are Buried Deeper

Conditions in the Center of the Sun

- Density, temperature, and pressure increase toward the center of the Sun, thus creating the necessary conditions for **nuclear fusion**.
- 15 million K
- 150 metric ton/m^3


Pressure (billions of atmospheres)


Density (thousands of kg/m^3)

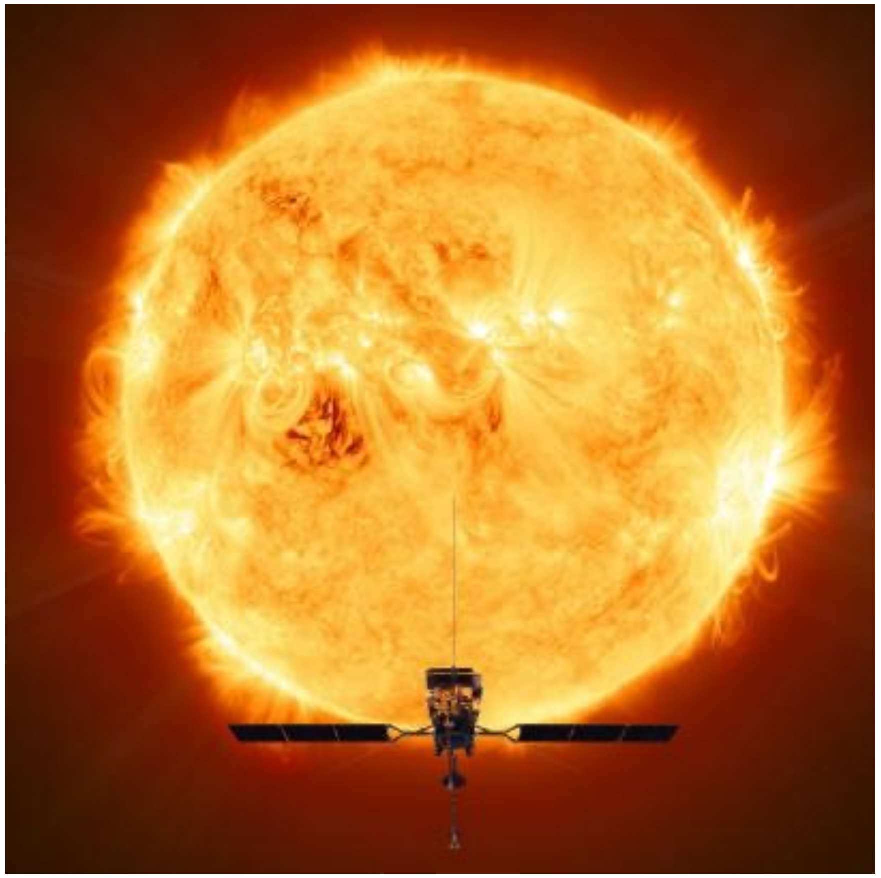

Temperature (millions of K)

How the Sun generates its energy?

Nuclear fusion

Energy Balance: Energy Generation Rate = Energy Loss Rate

- As measured by radiometric dating of meteorites, the Sun has existed for at least 4.6 billion years;
- The Sun releases 3.8e26 Joules per second through electromagnetic radiation; For 4.6 billion years, this amounts to a total energy of 5.5e43 Joules.



- For comparison, the total gravitational potential energy lost during the gravitational collapse of the solar nebula is only:

  $$E_G = 0 - (-GM^2/R) = 3.8 \times 10^{41} J$$

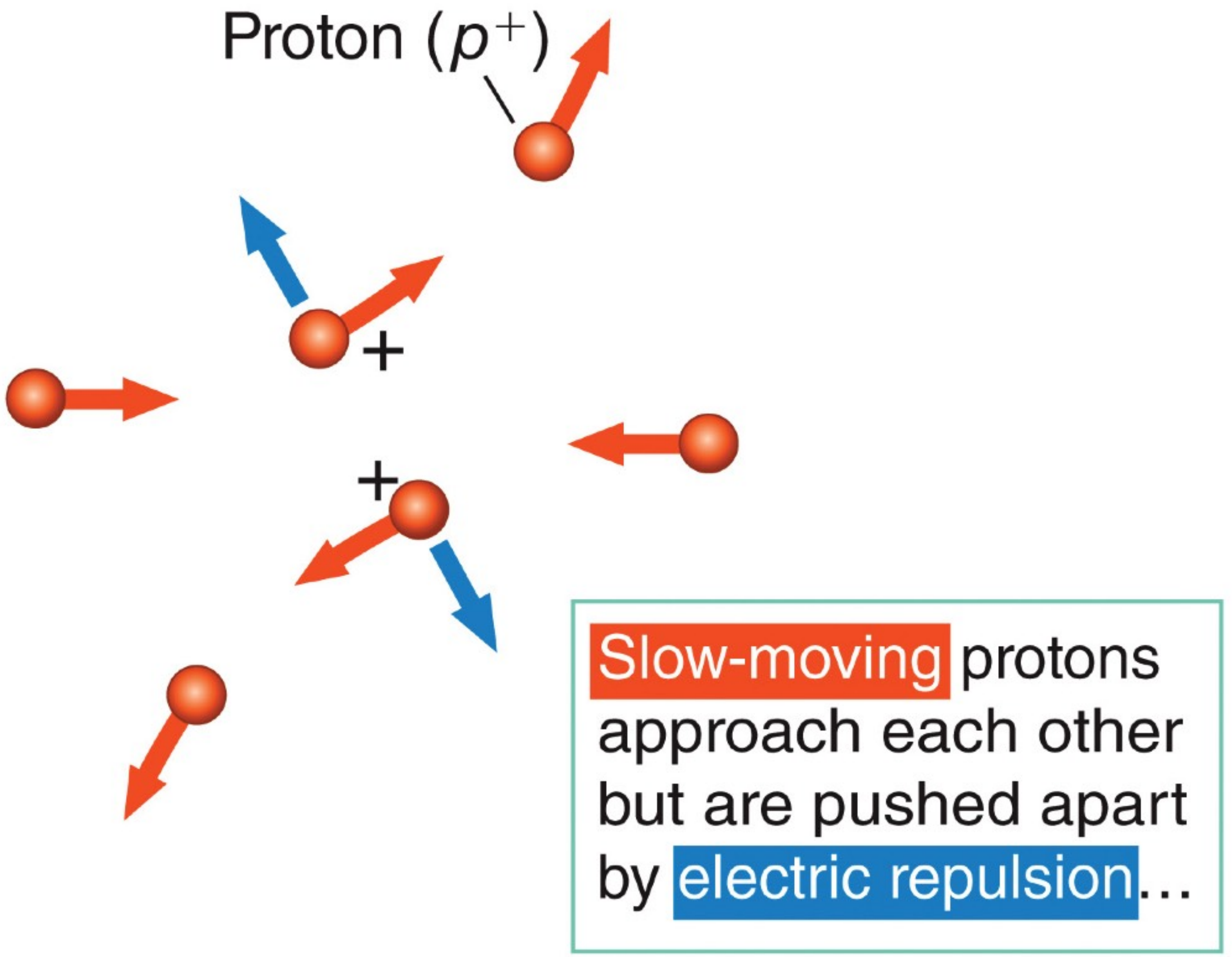
  which could only last the Sun for 32 Myrs.
- Nuclear fusion is the only viable source of energy capable of powering the Sun for this long. Fusion creates more massive nuclei from less massive ones but loses total mass in the process.

Nuclear Fusion: the Coulomb barrier

- **Nuclear fusion** involves the fusing of atomic nuclei to form heavier elements.
- Nuclei consist of protons (positively charged) and neutrons (no charge). So electrostatic force push nuclei apart following Coulomb's law:

$$F = K\frac{q_1 q_2}{r^2}$$

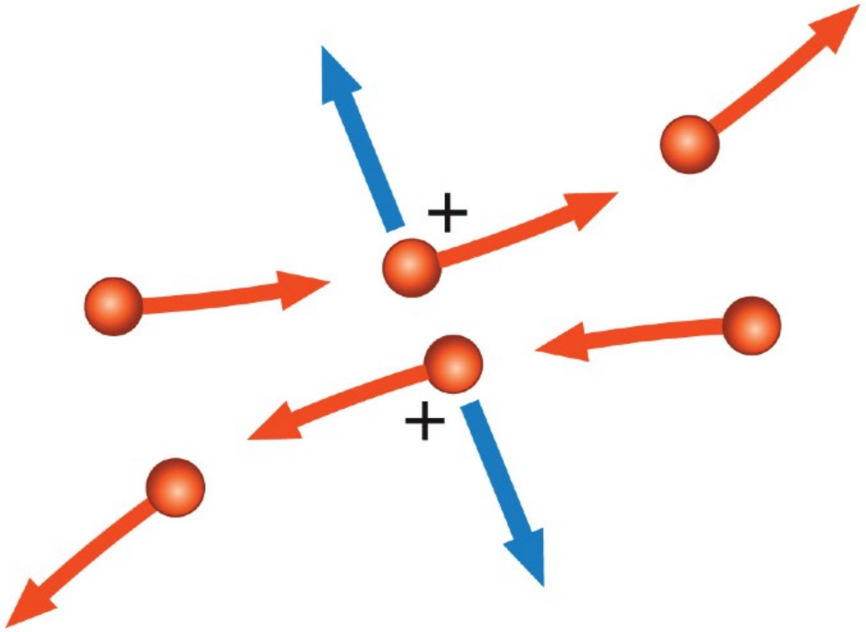
- Eventually, the strong nuclear force can overcome the push of the electrostatic force and bind protons together. But to enable fusion, this Coulomb barrier must be overcome first so that the nuclei are brought into extreme proximity.



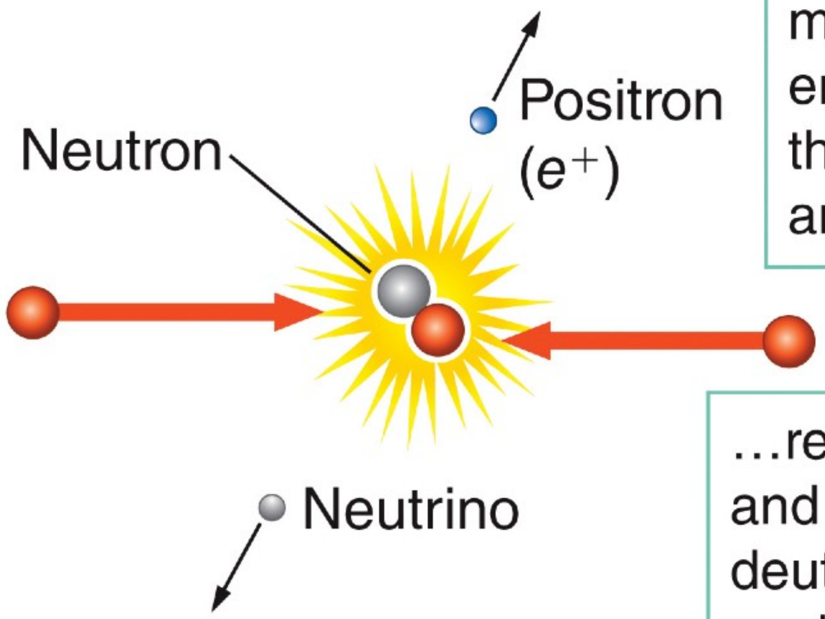
Slow-moving protons approach each other but are pushed apart by electric repulsion…

Nuclear Fusion: Required Conditions

- Fusion requires slamming protons together at high speed (i.e., at high temperature) to overcome the Coulomb barrier.
- Sufficient frequency of collisions can be sustained only when the nuclei are densely packed together (i.e., at high density).



…but the faster they are going, the closer together they can get.

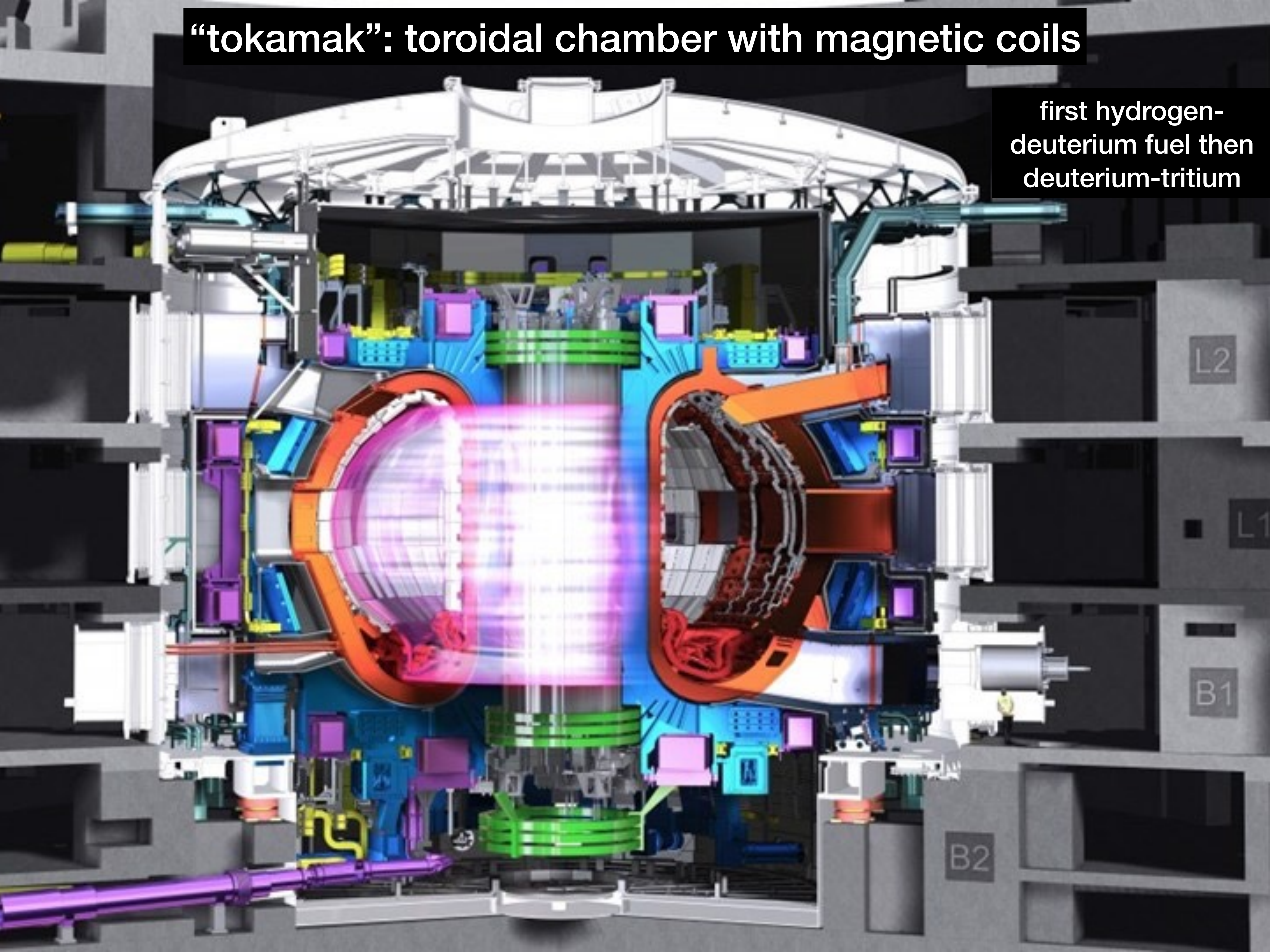


At high enough temperatures, protons may move rapidly enough to overcome the electric repulsion and to fuse…

…releasing energy and leaving behind a deuterium nucleus, a positron, and a neutrino.

The largest fusion reactor ever built:
ITER - International Thermonuclear Experimental Reactor

“tokamak”: toroidal chamber with magnetic coils



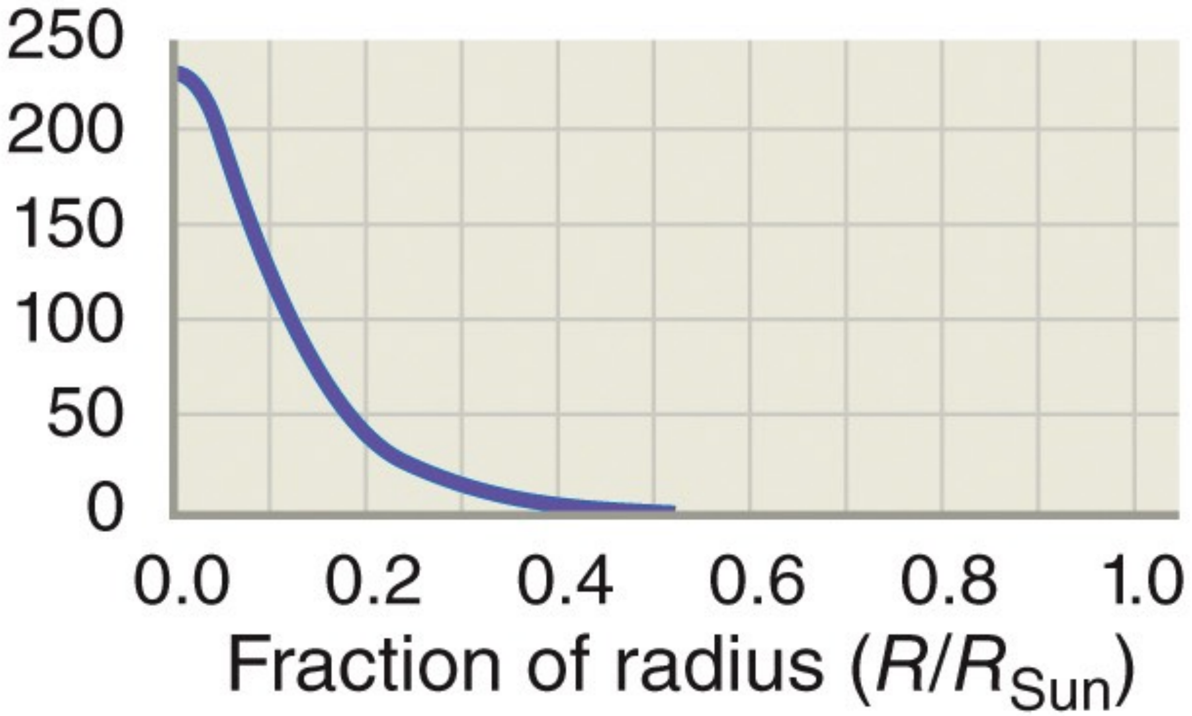
first hydrogen-deuterium fuel then deuterium-tritium

Nuclear scientist explains the Fusion Ignition Breakthrough at LLNL
deuterium-tritium fuel (Dec 13, 2022)

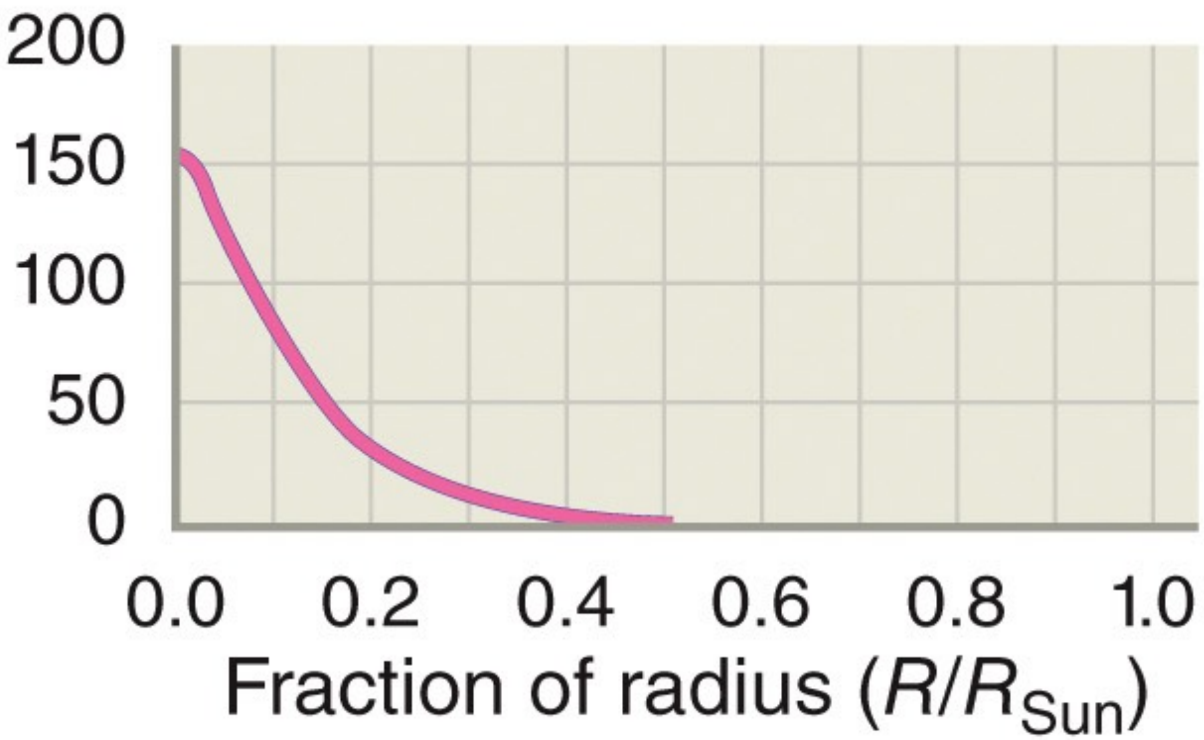
Conditions in the Center of the Sun

- Density, temperature, and pressure increase toward the center of the Sun, thus creating the necessary conditions for **nuclear fusion**.
- 15 million K
- 150 metric ton/m^3

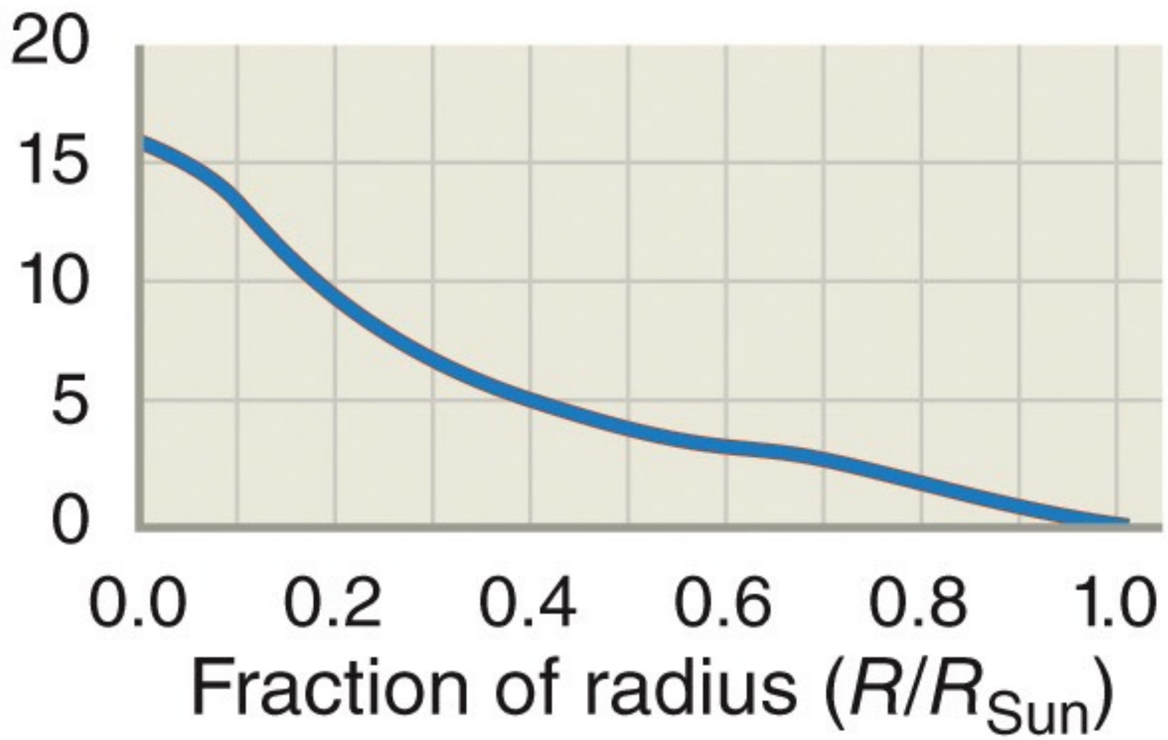
Pressure (billions of atmospheres)



Density (thousands of kg/m^3)



Temperature (millions of K)

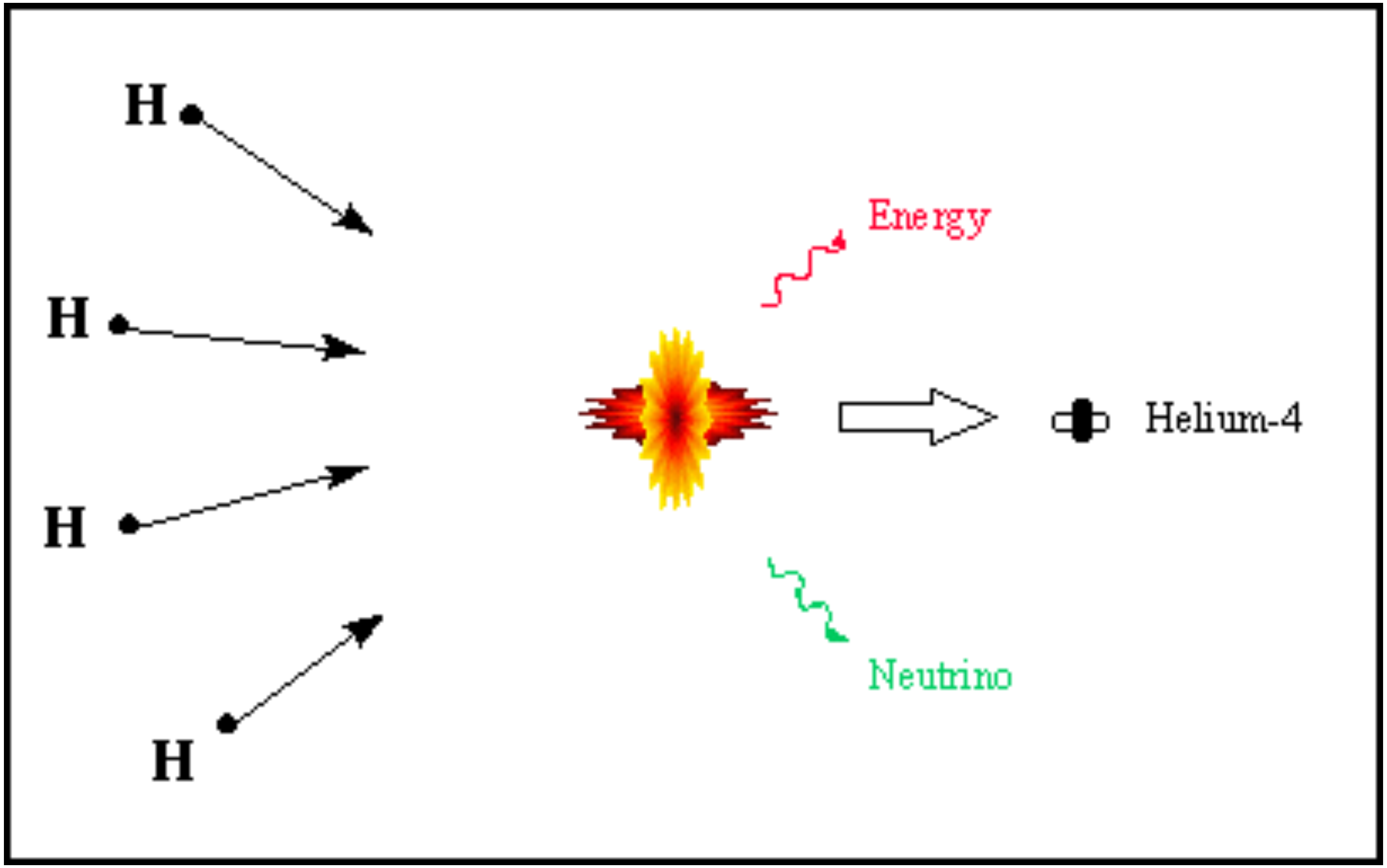


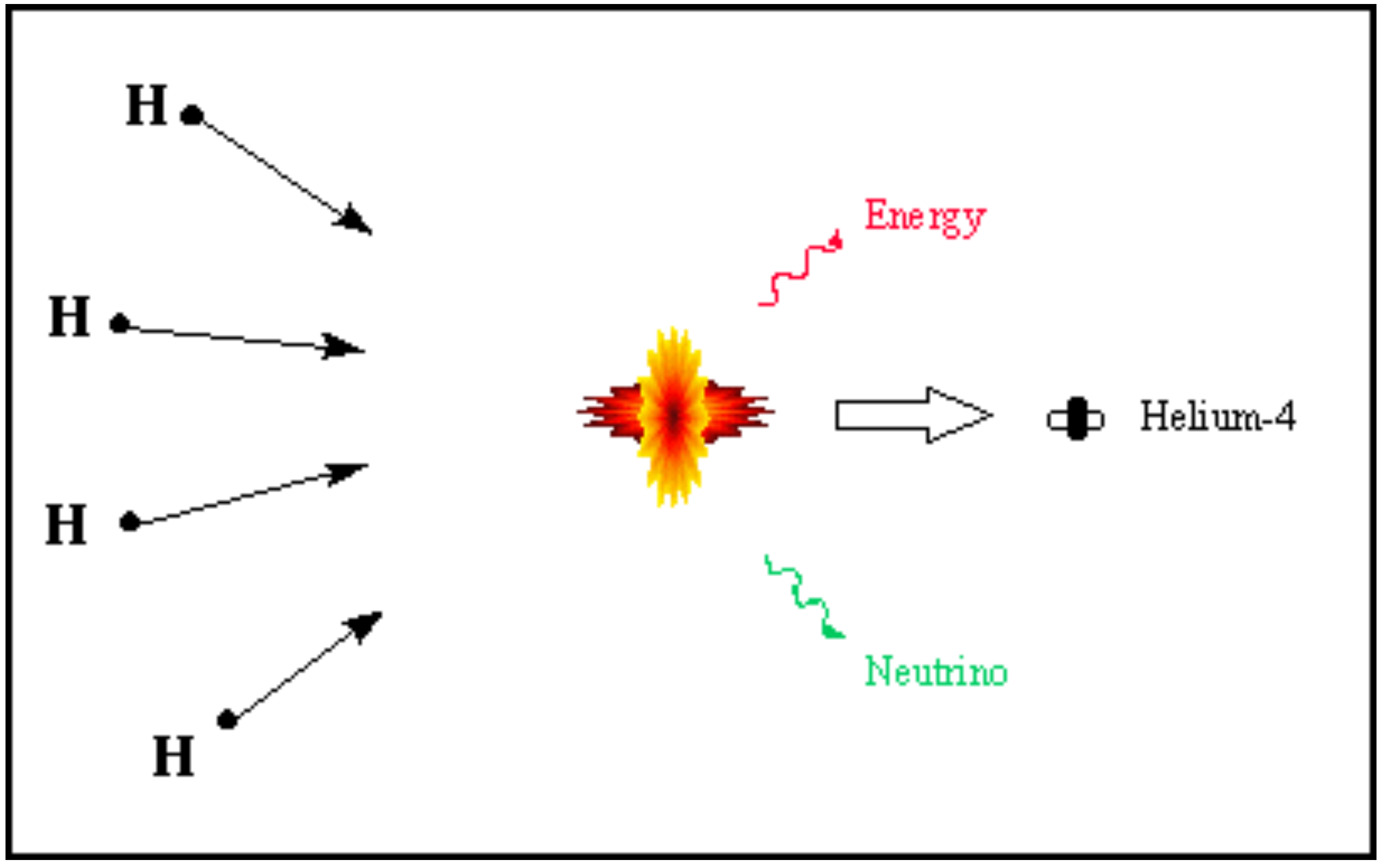
How the Sun generates its energy?

The proton-proton chain

Hydrogen Burning in the Solar Core

- Hydrogen is the most abundant element in the Universe (~75% of the baryonic mass).
- Fusing Hydrogen to the next element (Helium) requires four Hydrogen atoms
- How does this process actually happen? Can we simply slam 4 protons?





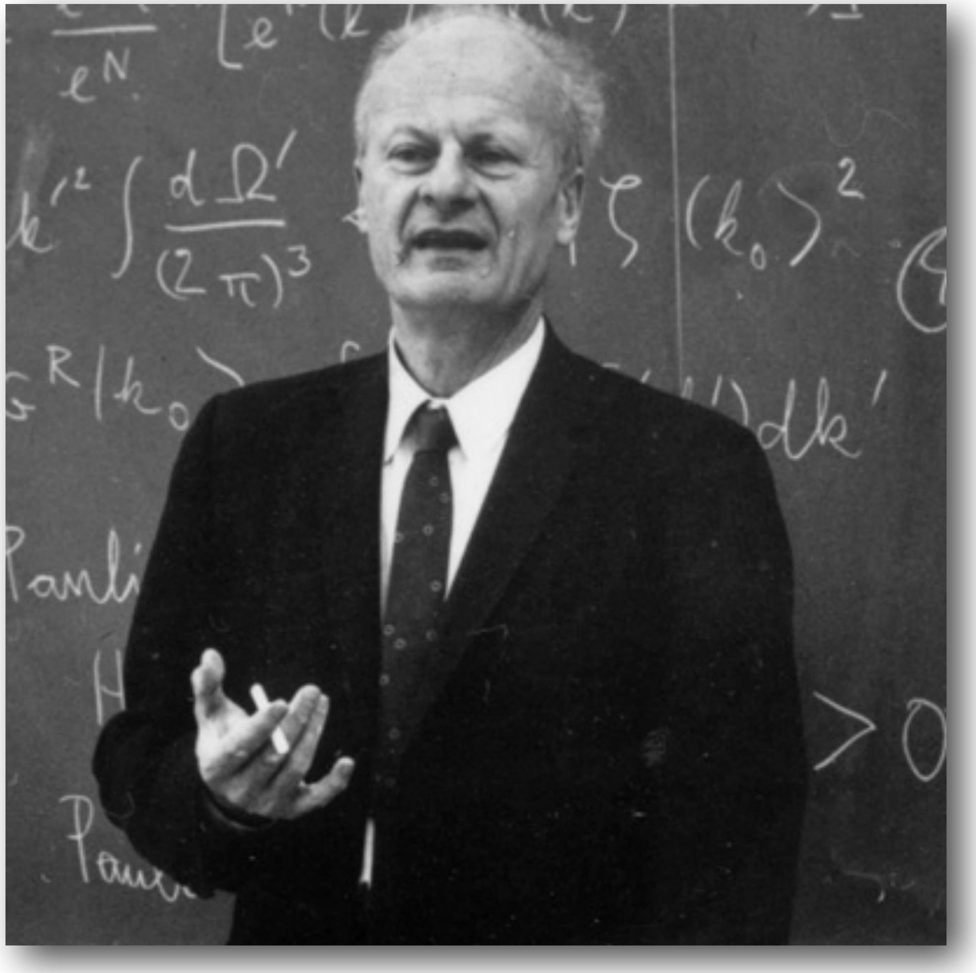
But the probability of 4 hydrogen atoms all colliding at the same time is extremely small.

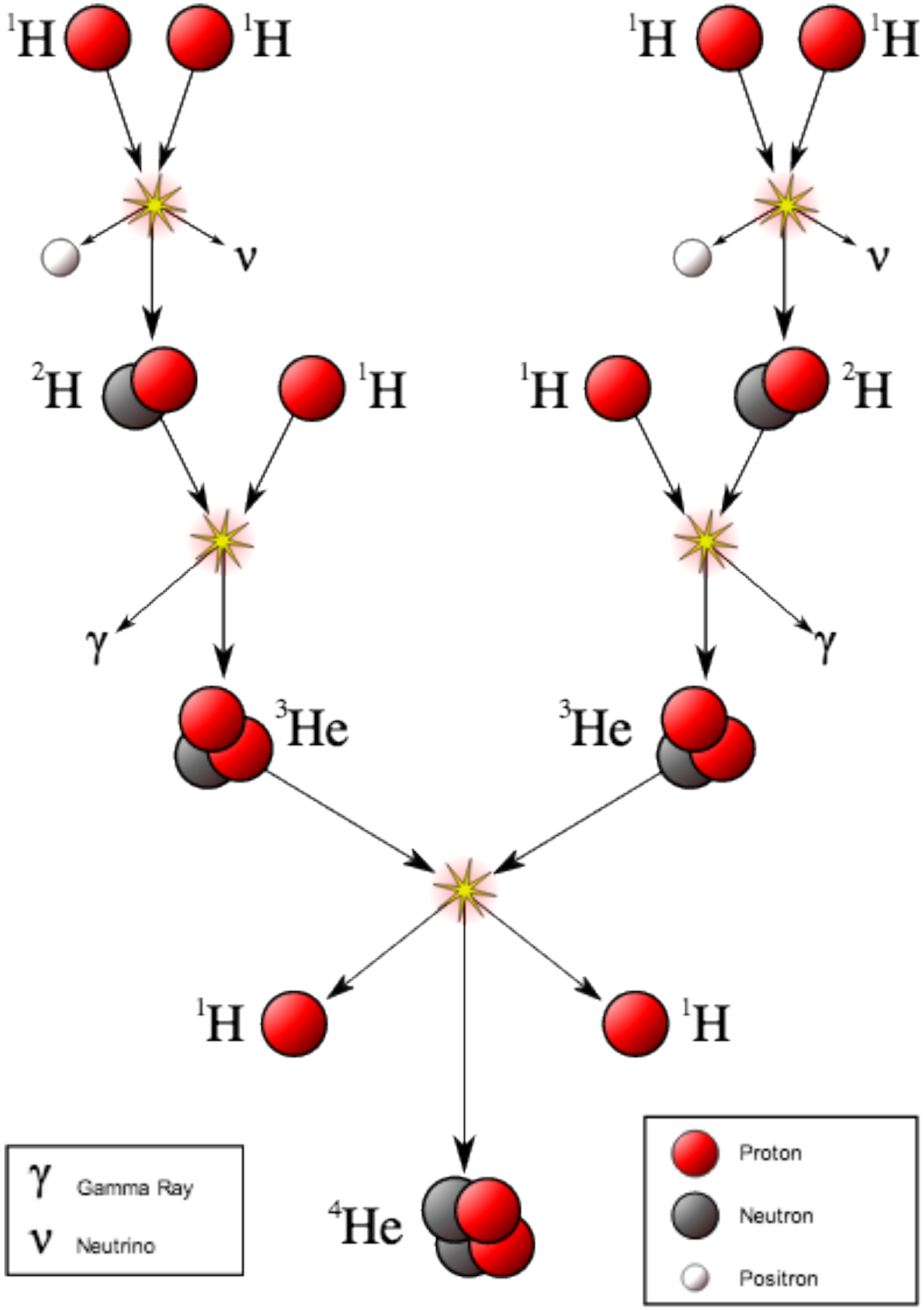
Think about four protons trying to set up a committee meeting …

So what actually happens?

p-p chain: a more detailed description (binary collisions)

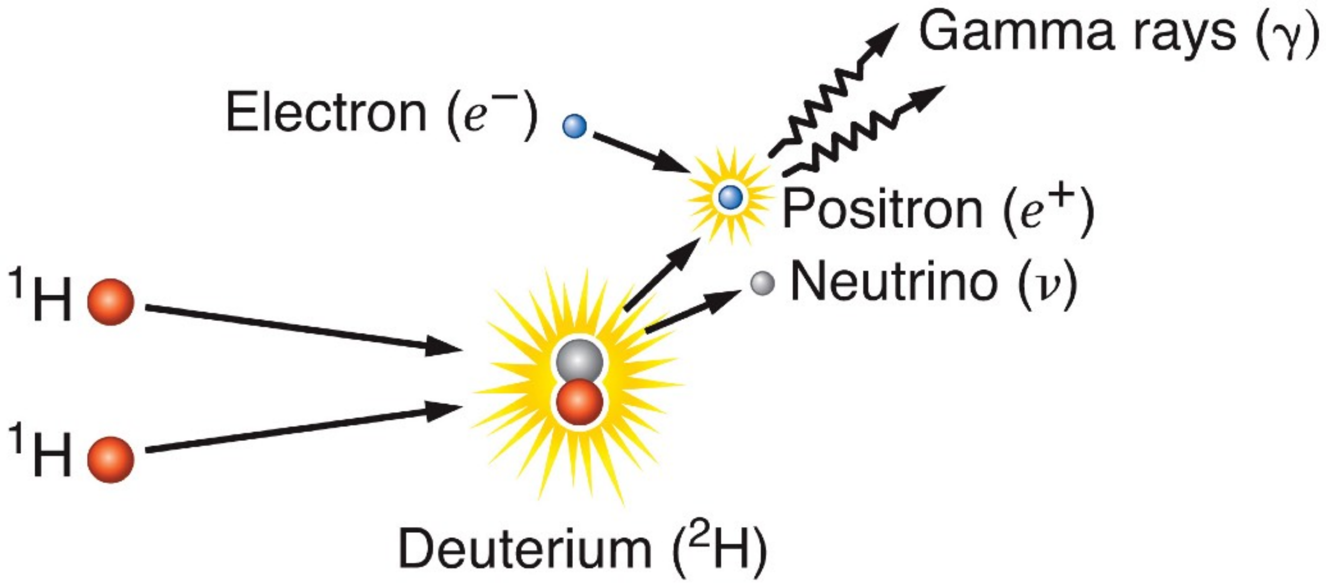
first worked out by
Hans Bethe in 1938





Proton-Proton Chain, Step 1

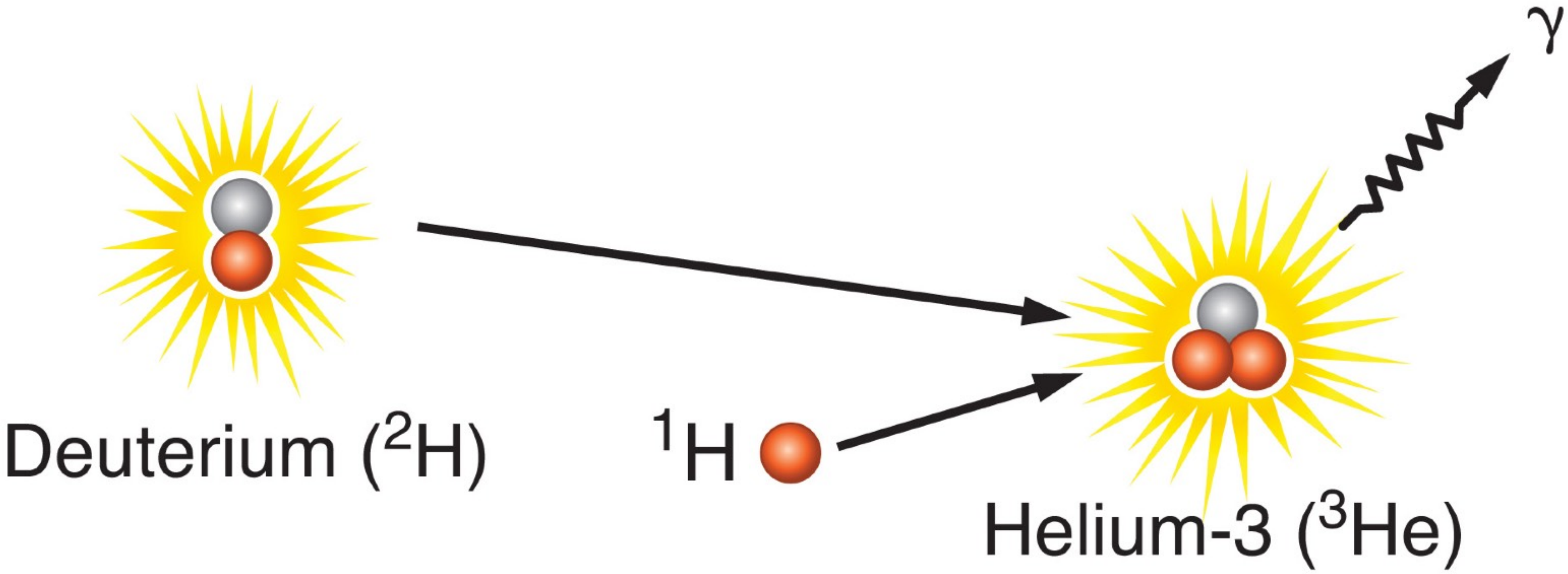
- Hydrogen nuclei are really just single protons, so the hydrogen fusion process is called the **proton-proton chain**.
- Step 1: Two hydrogen nuclei (protons) fuse to make a deuterium nucleus (^{2}H).



- Two protons collide. One of them emits a **positron** and **neutrino**, which makes it become a neutron.
- A **positron** is the **antimatter** counterpart of an electron. The positron meets an electron and they annihilate each other.
- The mass of both is converted into energy in the form of gamma-ray photons.
- This step must happen twice in order to make one helium nucleus (^{4}He).

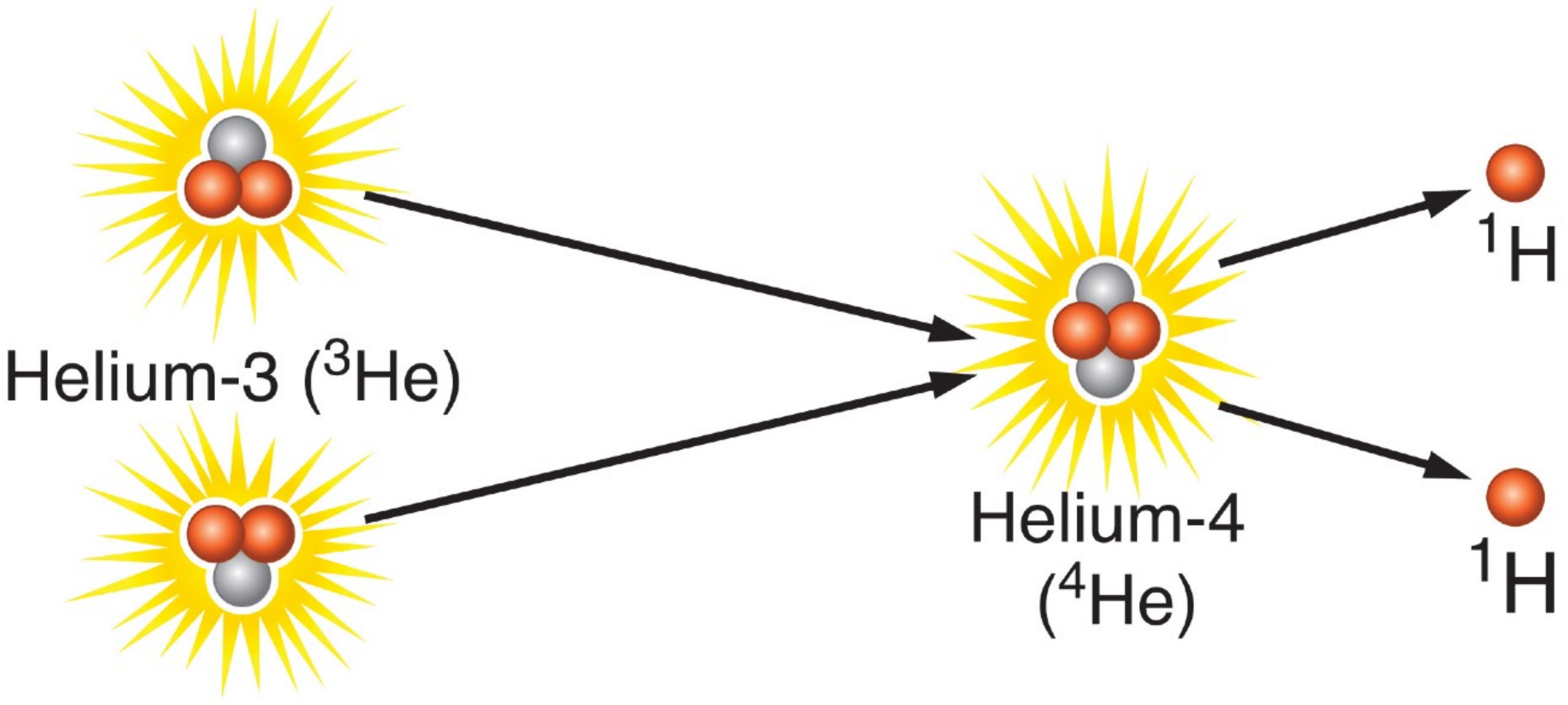
Proton-Proton Chain, Step 2

- Step 2: The deuterium nucleus collides with another proton, producing a helium-3 nucleus (^{3}He).
  - ^{3}He is an isotope of helium. It has the same number of protons but has one fewer neutron than a normal helium nucleus (^{4}He).
  - This fusion reaction directly produces another gamma-ray photon.
- These gamma-ray photons and the ones produced in Step 1 leave the core and eventually make their way to the surface.
- This step must also happen twice to produce one helium nucleus (^{4}He).

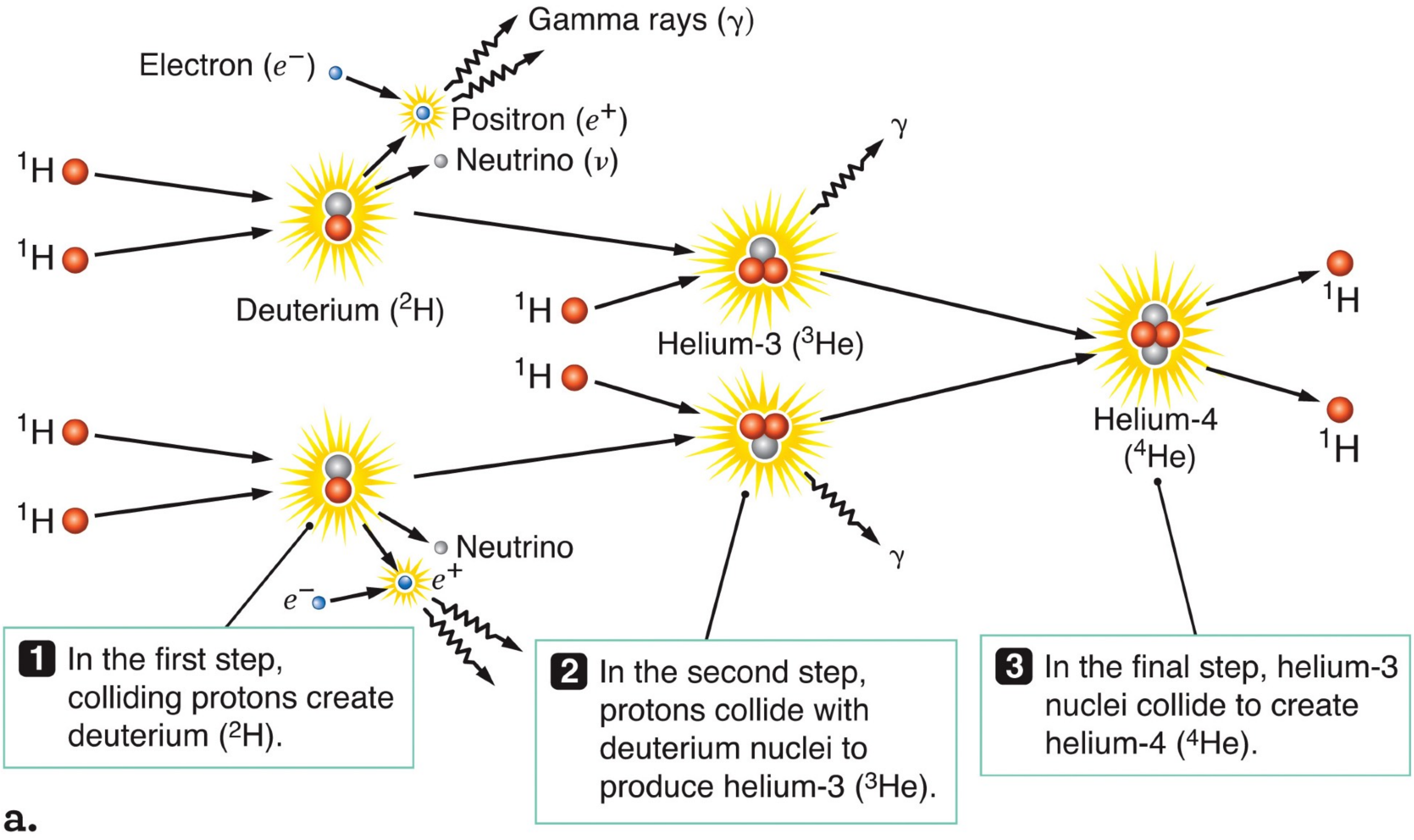


Proton-Proton Chain, Step 3

- Step 3: Two helium-3 nuclei fuse to create one normal helium-4 nucleus (^{4}He).
  - Two protons (H nuclei) are ejected during the collision.
  - The energy produced in this step makes the ^{4}He and protons move faster than they were before, ensuring more collisions.
- The process is now complete. It started with four protons (H nuclei) and results in one helium nucleus (^{4}He) and a lot of energy released.



proton-proton chain - detailed procedure of binary collisions

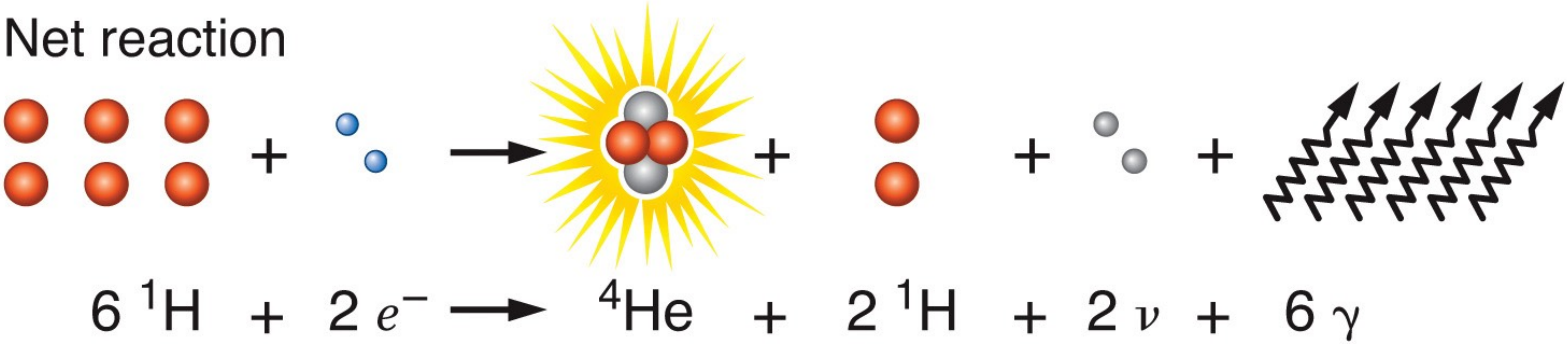


1 In the first step, colliding protons create deuterium (^{2}H).

2 In the second step, protons collide with deuterium nuclei to produce helium-3 (^{3}He).

3 In the final step, helium-3 nuclei collide to create helium-4 (^{4}He).

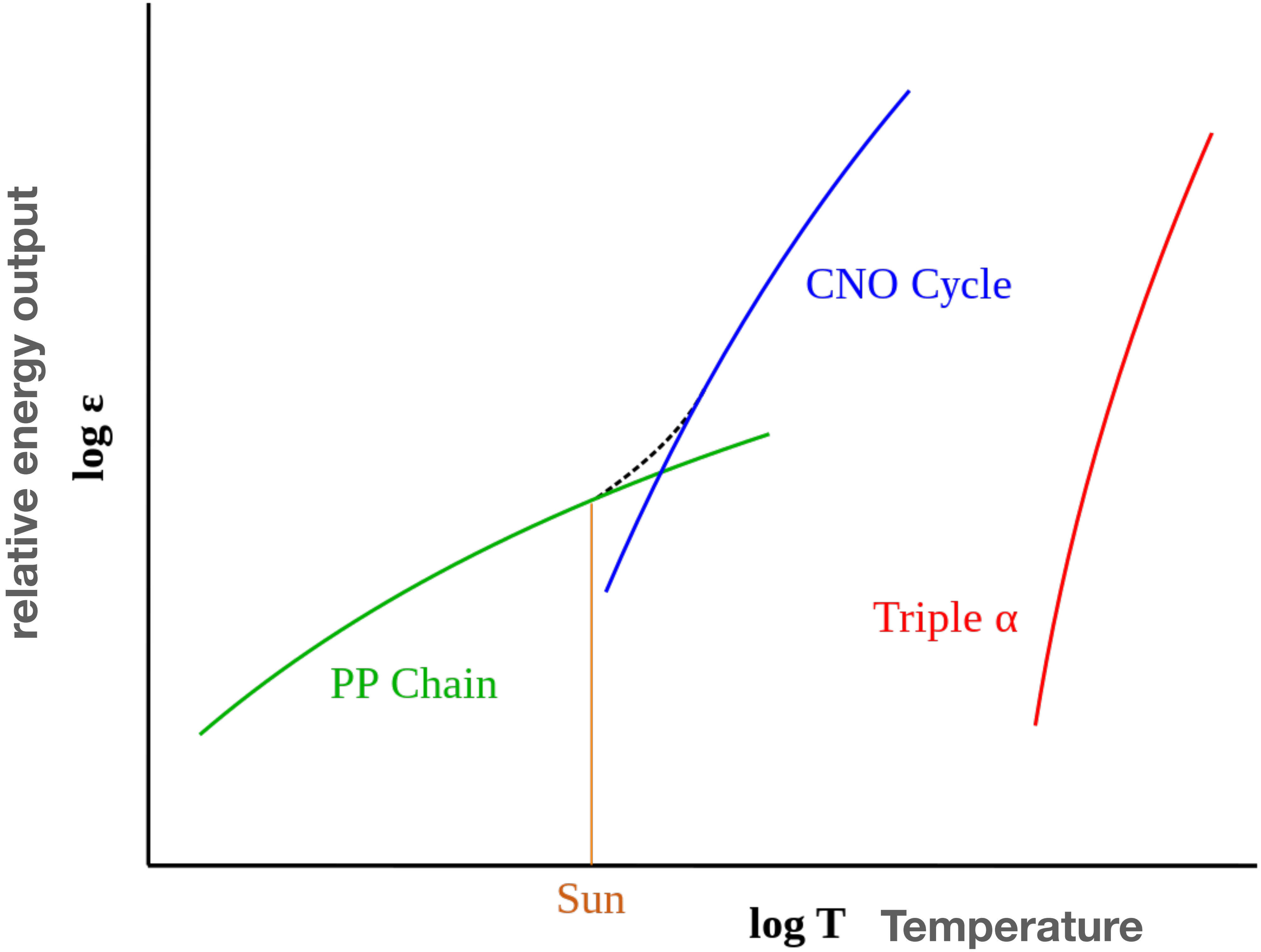
proton-proton chain: net reaction



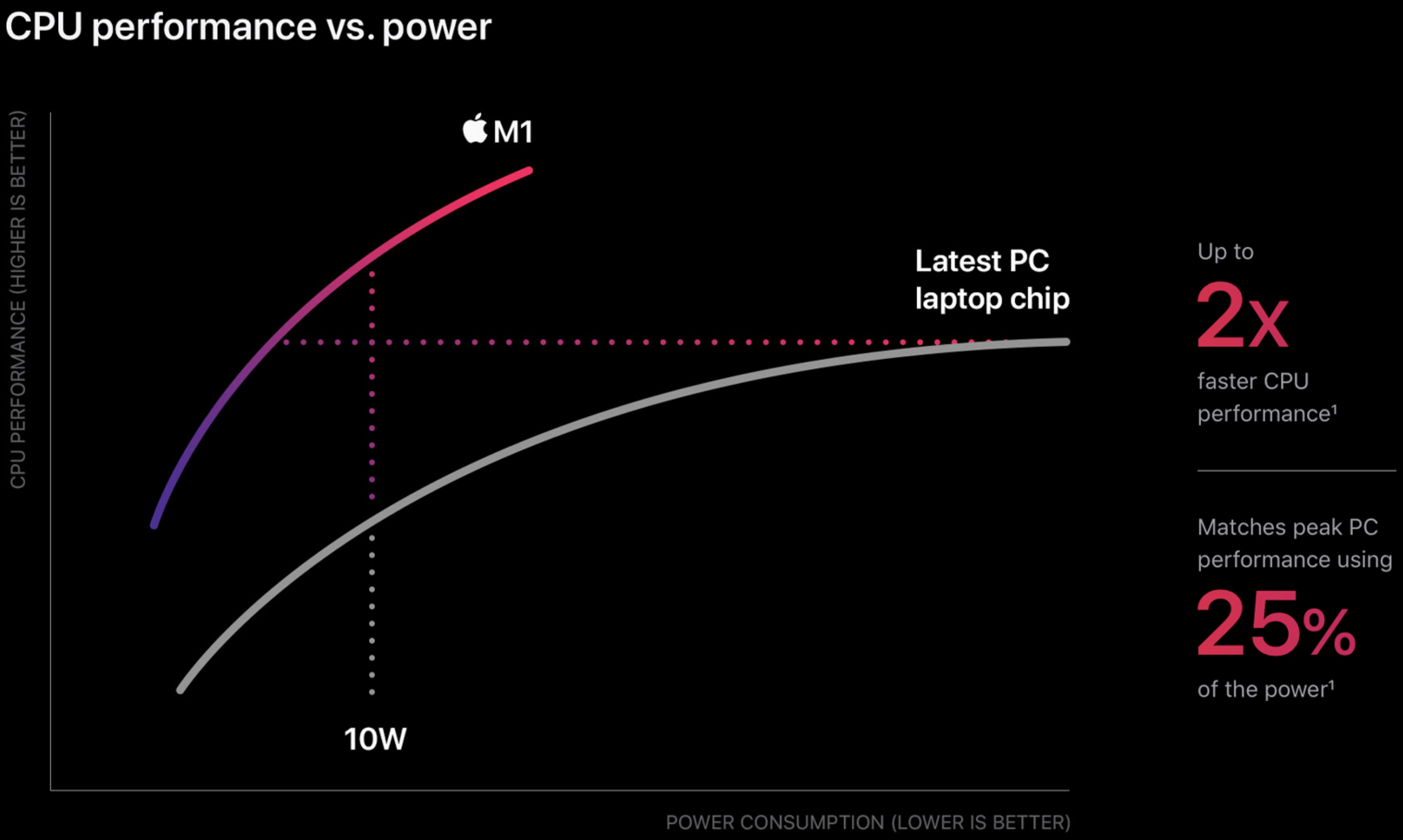
$$6\ {}^{1}\mathrm{H} + 2\,e^{-} \longrightarrow {}^{4}\mathrm{He} + 2\ {}^{1}\mathrm{H} + 2\,\nu + 6\,\gamma$$

Hydrogen Fusion in Stellar Cores

- The rate of fusion is sensitive to temperature and density of the gas, and there are three main fusion processes in stellar cores.



The previous plot looks like an Apple PR



CPU performance vs. power

Up to 2x faster CPU performance[1]

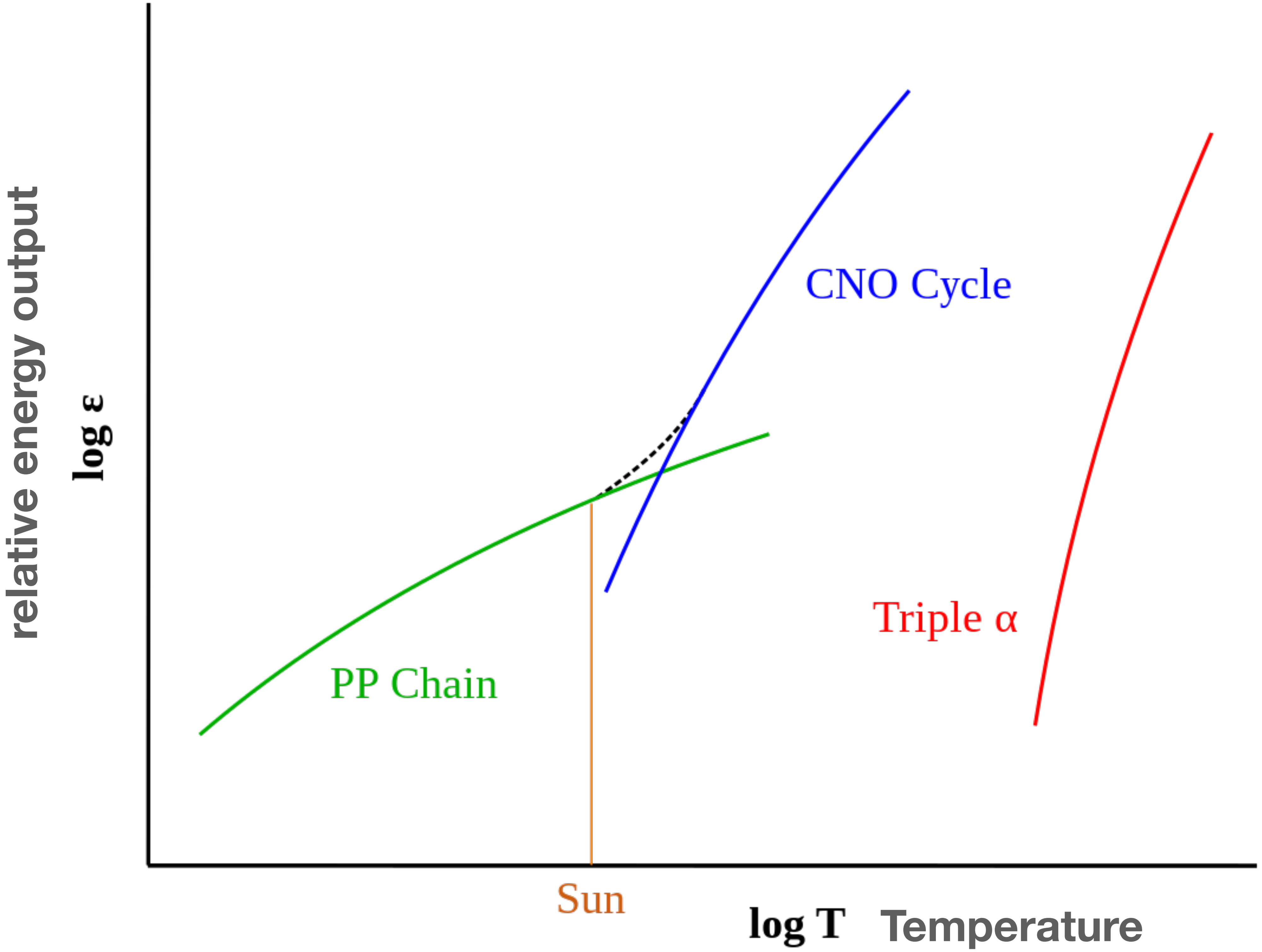
Matches peak PC performance using 25% of the power[1]

How does the Sun maintain a constant energy output?

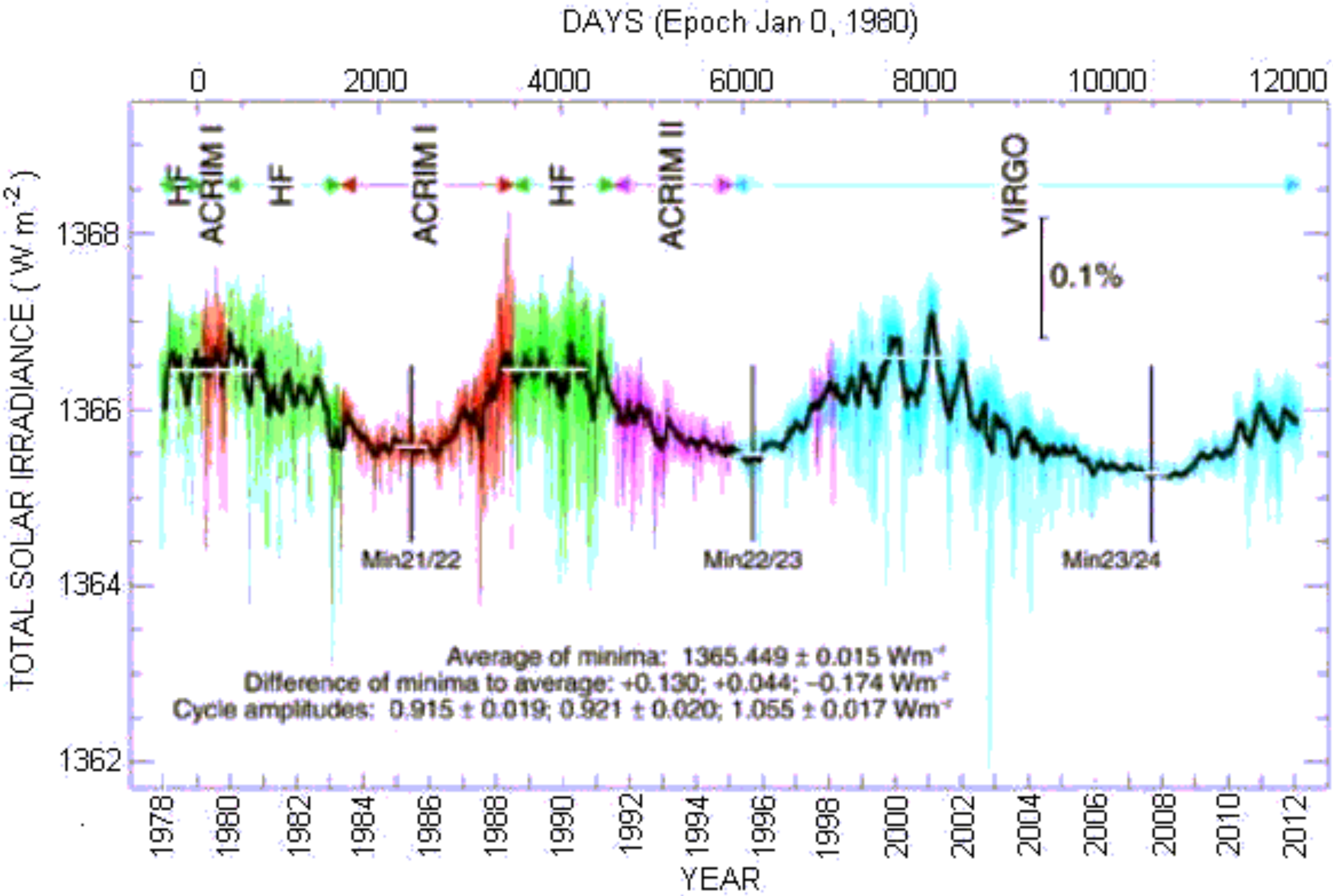
gravitational thermostat

Hydrogen Fusion's Strong Temperature Dependency

- The rate of fusion is **sensitive to temperature:**
  - For PP chain, reaction rate ~ T^4; for CNO cycle, rate ~ T^{20}



Solar “Constant” vs. Time (corrected for orbit ellipticity)



Average of minima: 1365.449 ± 0.015 Wm^{-2}
Difference of minima to average: +0.130; +0.044; −0.174 Wm^{-2}
Cycle amplitudes: 0.915 ± 0.019; 0.921 ± 0.020; 1.055 ± 0.017 Wm^{-2}

Steady Fusion in the Cores of MS stars:

A gravitational ``thermostat’’

H fusion rate increases

T & P increases

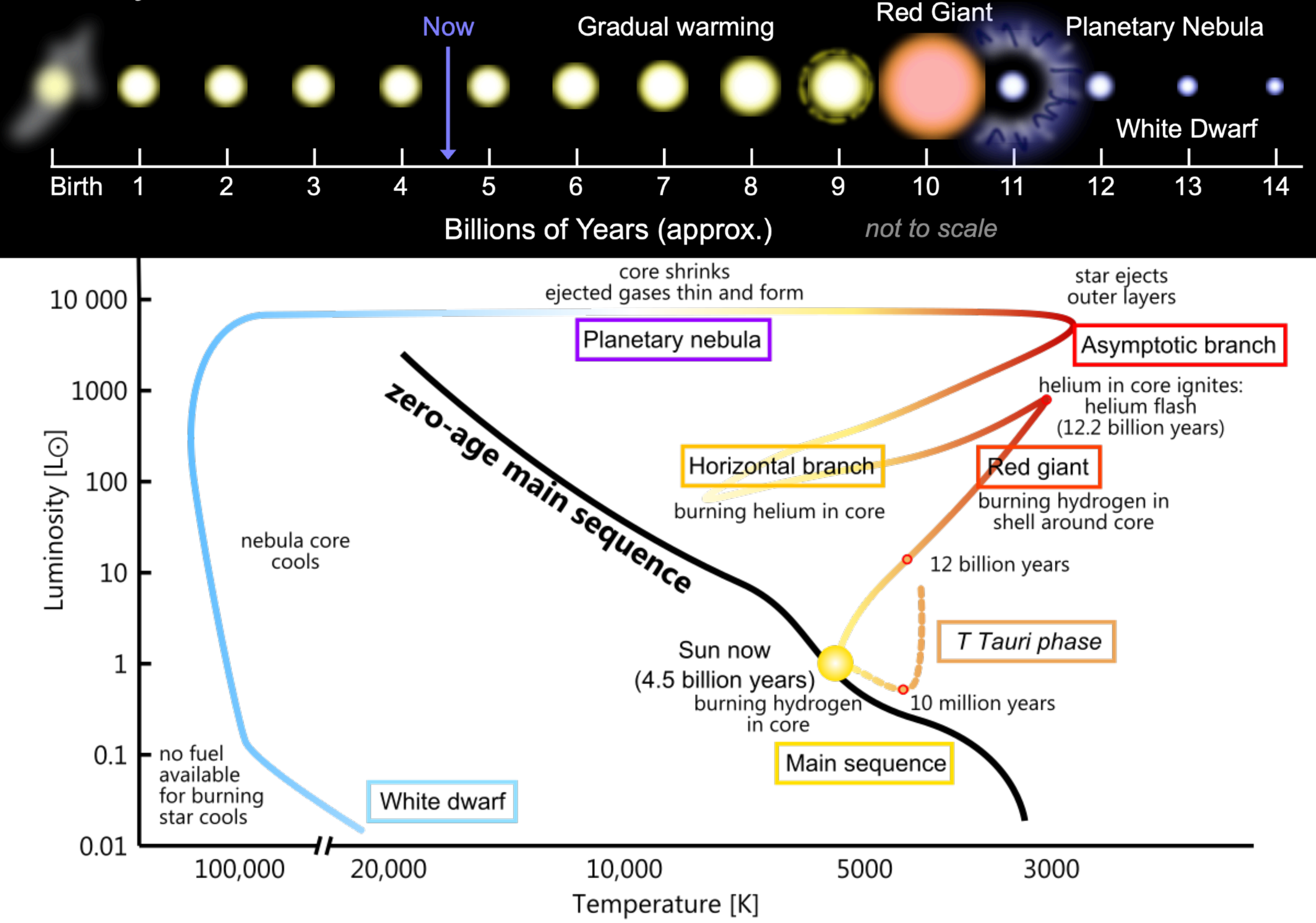
Core expands,
work against Gravity

T decreases

H fusion rate decreases

Steady H Burning

Life Cycle of the Sun



How long can our Sun last?

main-sequence lifetime

Working It Out 14.1: Energy Generation Rate & Fuel Consumption Rate

- The Sun has a limited amount of hydrogen available to fuse into helium—this determines the lifetime of the Sun.
- The mass of four hydrogen nuclei ($4 \times 1.6726 \times 10^{-27}\text{kg}$) is larger than one helium nucleus ($6.6447 \times 10^{-27}\text{kg}$) by $4.57 \times 10^{-29}\text{kg}$.
- The energy associated with that mass loss is:

$$E = mc^2 = \left(4.57 \times 10^{-29}\text{kg}\right) \times (3.00 \times 10^8 \text{m/s})^2 = 4.11 \times 10^{-12} J$$

- Each single PP-chain reaction produces $4.11 \times 10^{-12} J$.

- The Sun emits energy at a rate of 3.8e26 Watt (=J/s), how much hydrogen fuel does it burn every second (in kg of hydrogen)?

620 billion kilogram per second

Working It Out 14.1: The Lifetime of the Sun

- The Sun consumes hydrogen at a rate of 620 billion kilograms per second, so each year the Sun consumes:

$$M_{\text{year}} = \left(6.2 \times 10^{11}\ \text{kg/s}\right) \times \left(3.16 \times 10^{7}\ \text{s/yr}\right) \approx 2 \times 10^{19}\ \text{kg/yr}$$

- The mass of the Sun is $2 \times 10^{30}\ \text{kg}$, but only 10% is hot and dense enough for fusion to occur:

$$0.1 \times \left(2 \times 10^{30}\right)\ \text{kg} = 2 \times 10^{29}\ \text{kg}.$$

- If we know how much fuel the Sun has and how much fuel the Sun fuses in one year, we can find the lifetime of the Sun:

$$\text{Lifetime} = \frac{M_{\text{fuel}}}{M_{\text{year}}} = \frac{2 \times 10^{29}\ \text{kg}}{2 \times 10^{19}\ \text{kg/yr}} = 10^{10}\ \text{yr}$$

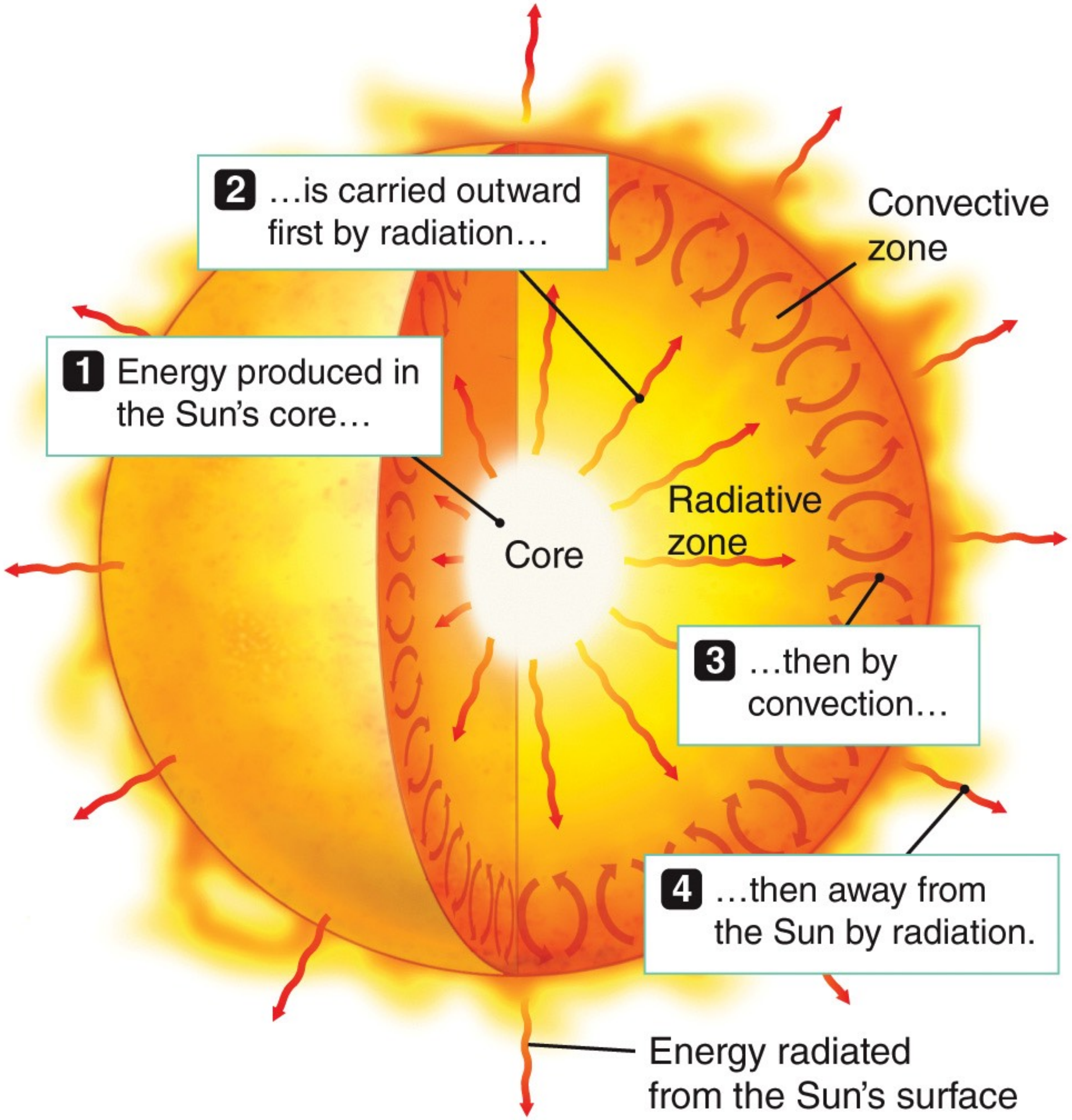
- The Sun has a lifetime of 10 billion years!

How does energy gets out?

radiative random walk,
then convection

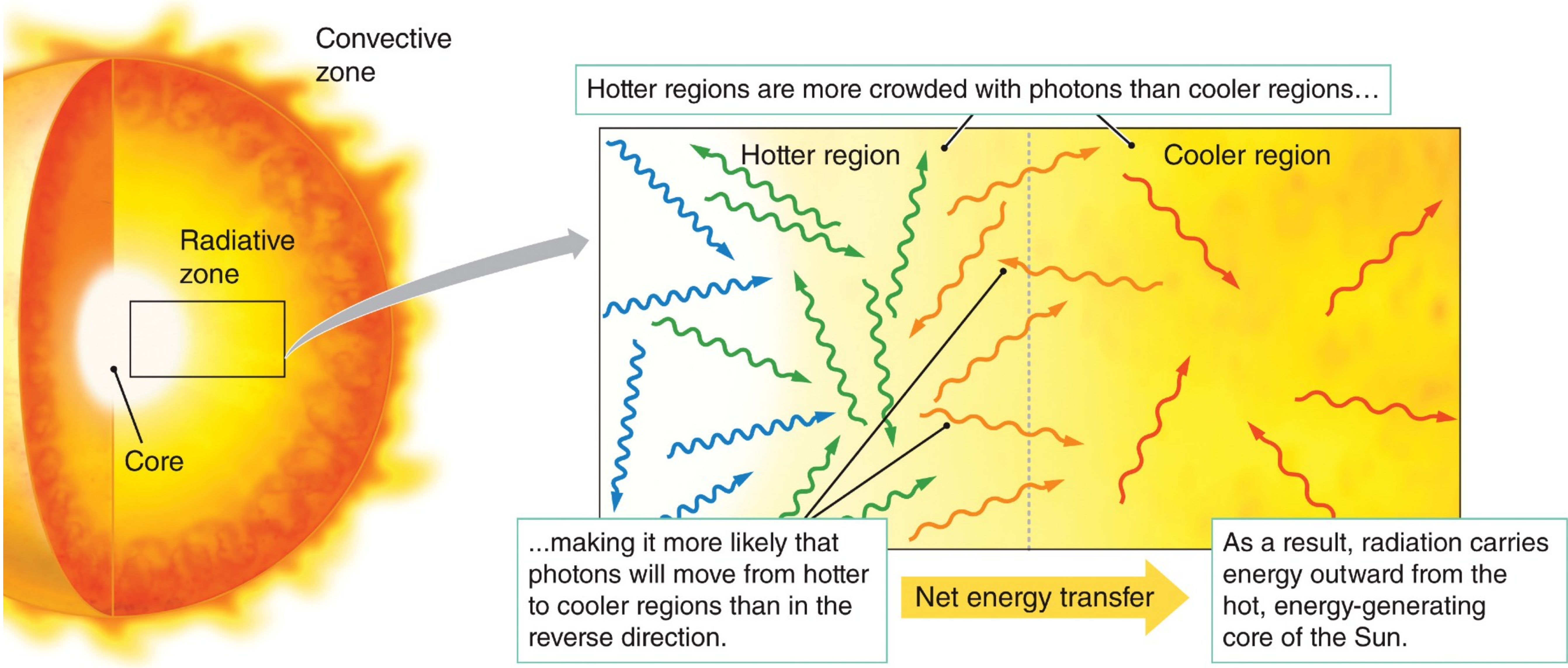
Energy Transport in the Sun: First Radiation then Convection

- Energy produced in the core must get out.
- Escaping energy passes through two different layers, defined by their temperature and density.
- In the inner layer, **radiation** transfers energy via photons.
- In the outer layer, **convection** carries energy by moving hot gas up and cool gas down.



Radiative Zone: Lower 70% of the Sun

- **Radiative transfer**: Energy escapes in the form of high energy photons, radiating outward from the core. **The radiative zone extends to about 70 percent of the way to the surface.**
- Because temperature decreases from the core to the surface, photons tend to diffuse outwards, as illustrated below.



Hotter regions are more crowded with photons than cooler regions…

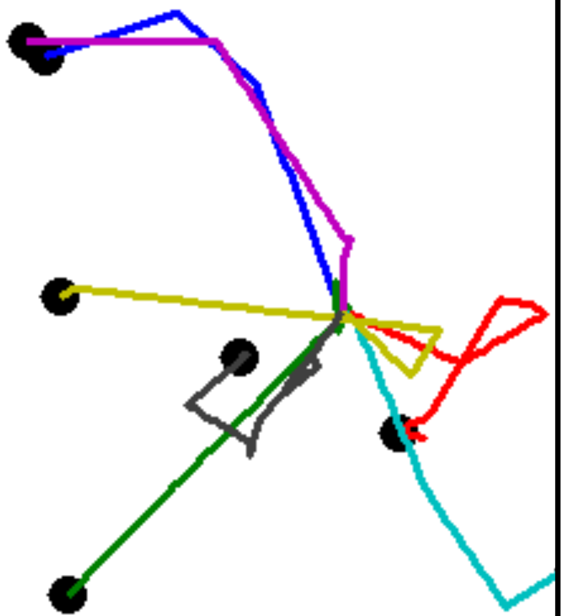
...making it more likely that photons will move from hotter to cooler regions than in the reverse direction.

Net energy transfer

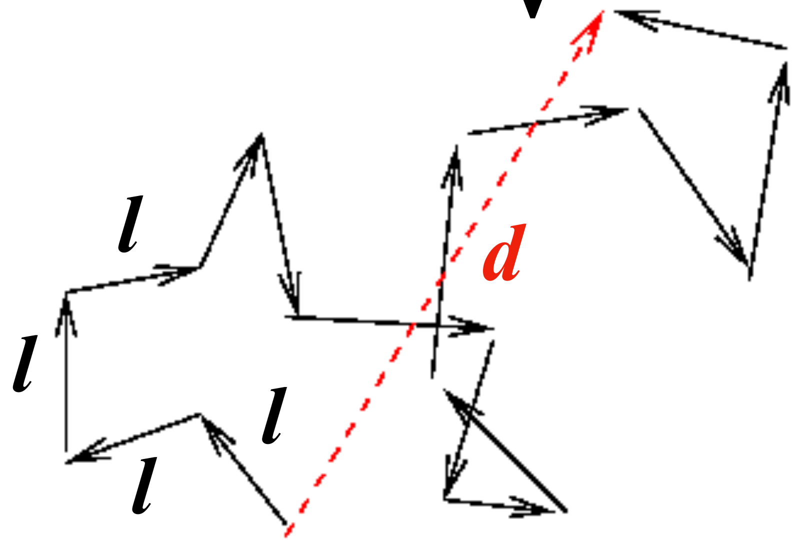
As a result, radiation carries energy outward from the hot, energy-generating core of the Sun.

Radiative Zone: Random Walk of Photons

- Because of the **opacity of the gas**, each photon can only travel a short distance before it is either (a) absorbed and re-emitted or (b) scattered.
- As a result, photons take random walks to slowly diffuse from the core to the surface.
- The random-walk process forces the photons to travel much greater distances than one solar radius, delaying their escape by thousands of years.



$$d = l\sqrt{N}$$



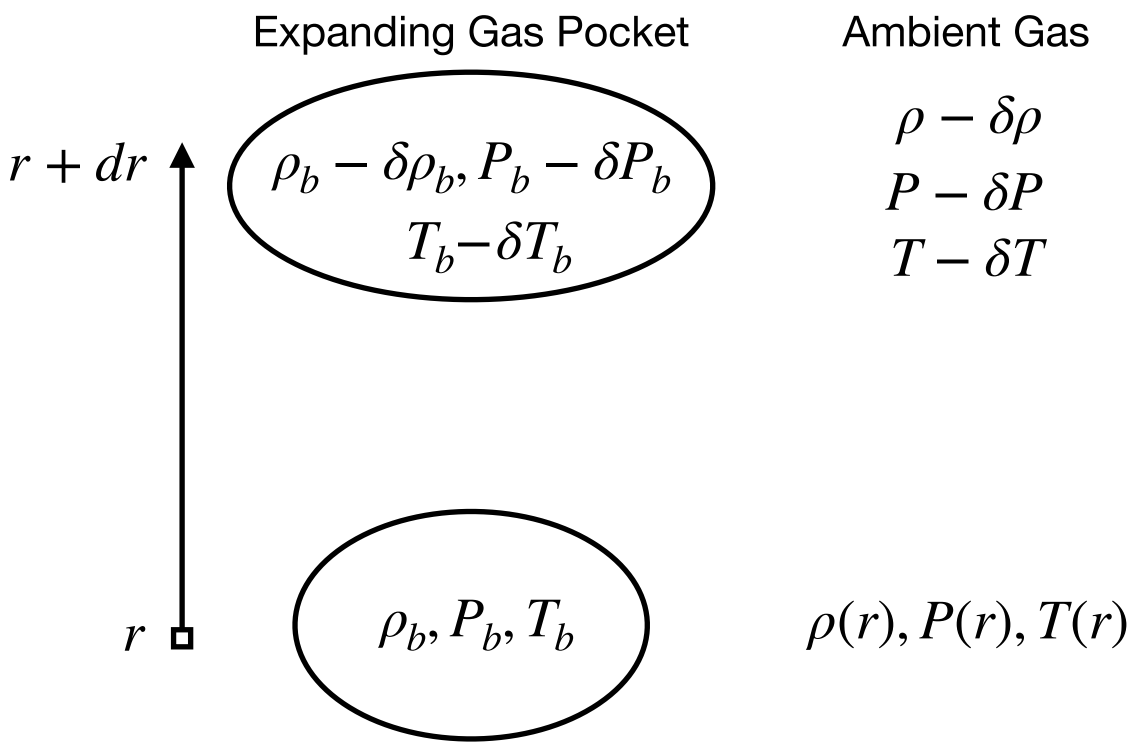
where d is the overall displacement, l is the mean free path, and N is the number of steps

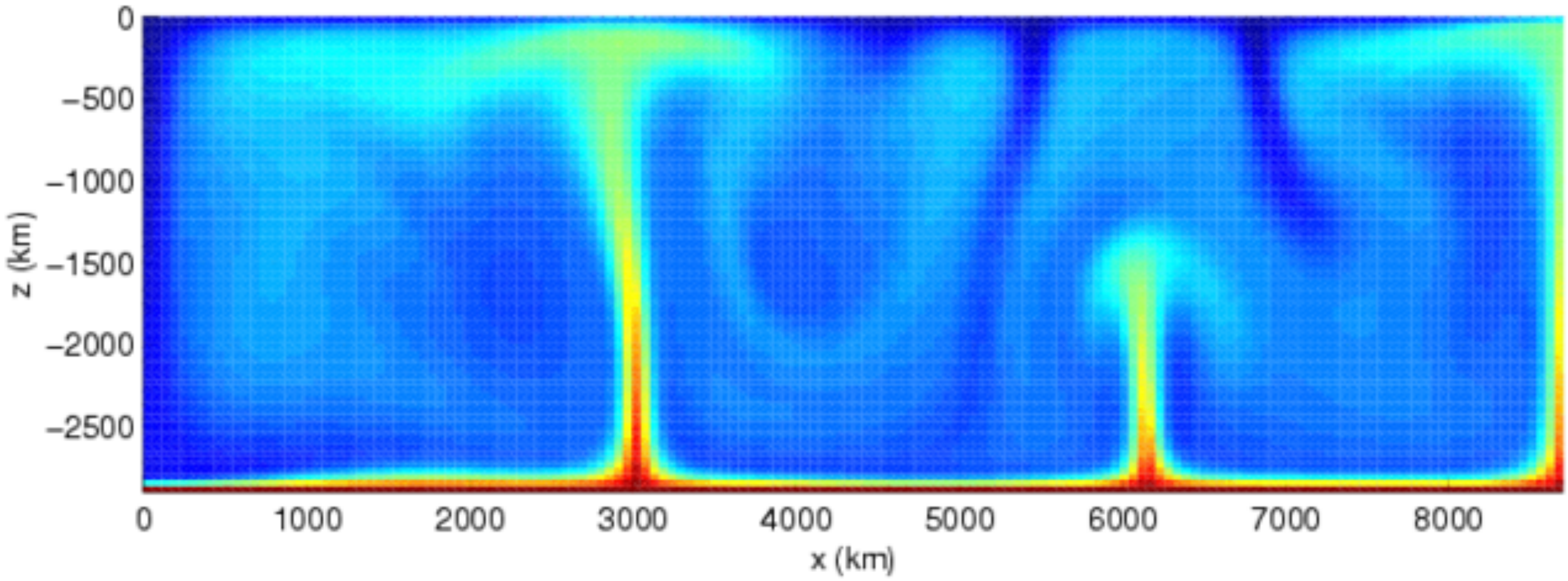
Physical Conditions for Convection to Happen

- When adiabatic expansion of a gas pocket causes its temperature to drop *less* than that of the ambient gas, **convection** ensues:

$$-(1-\frac{1}{\gamma})\frac{T}{P}\frac{dP}{dr} < -\frac{dT}{dr}$$

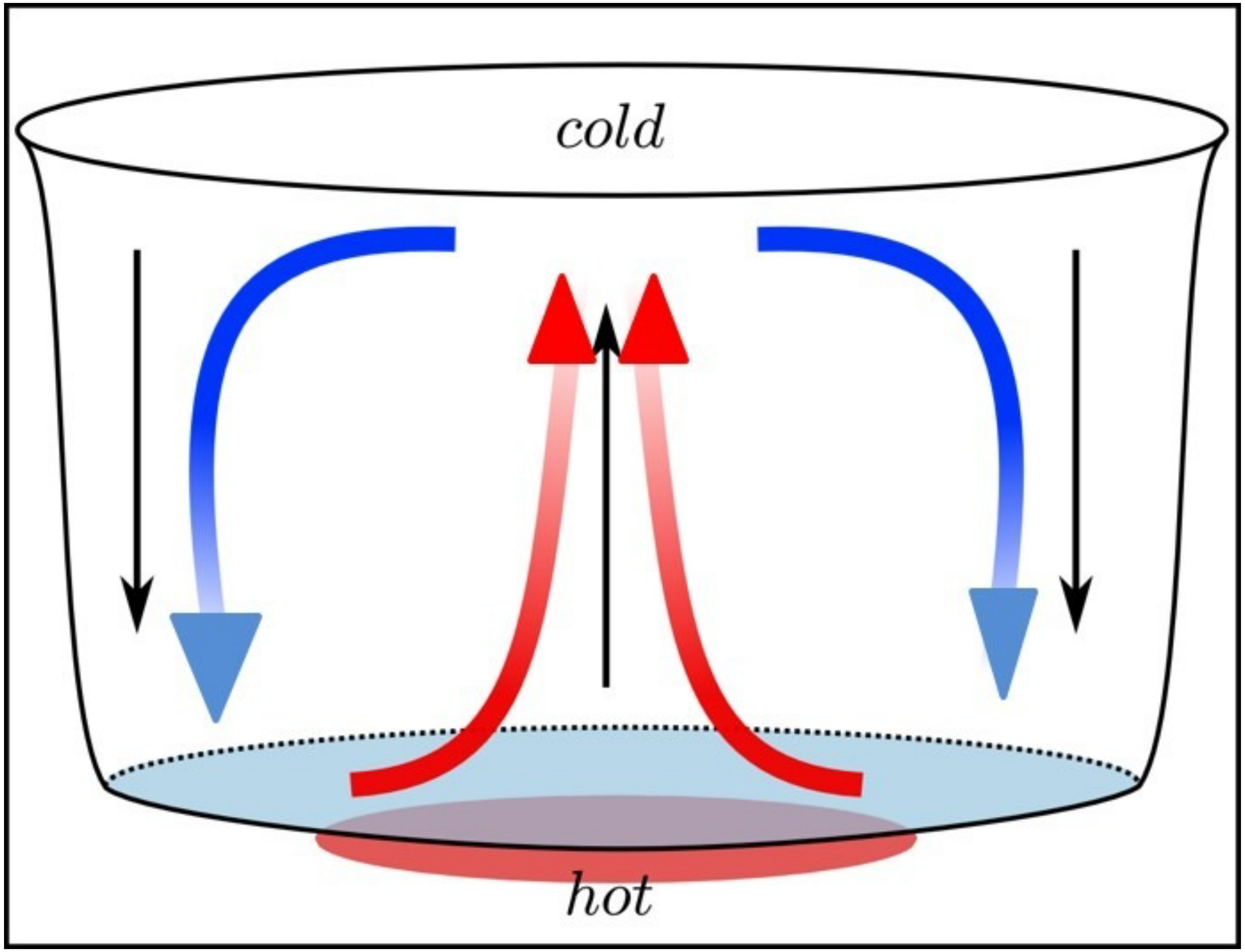
- **Why**? $P = nkT$, warmer gas at the same pressure as colder gas will have lower density. So the pocket will continue to rise due to buoyancy





Convective Zone: Outer 30% of the Sun

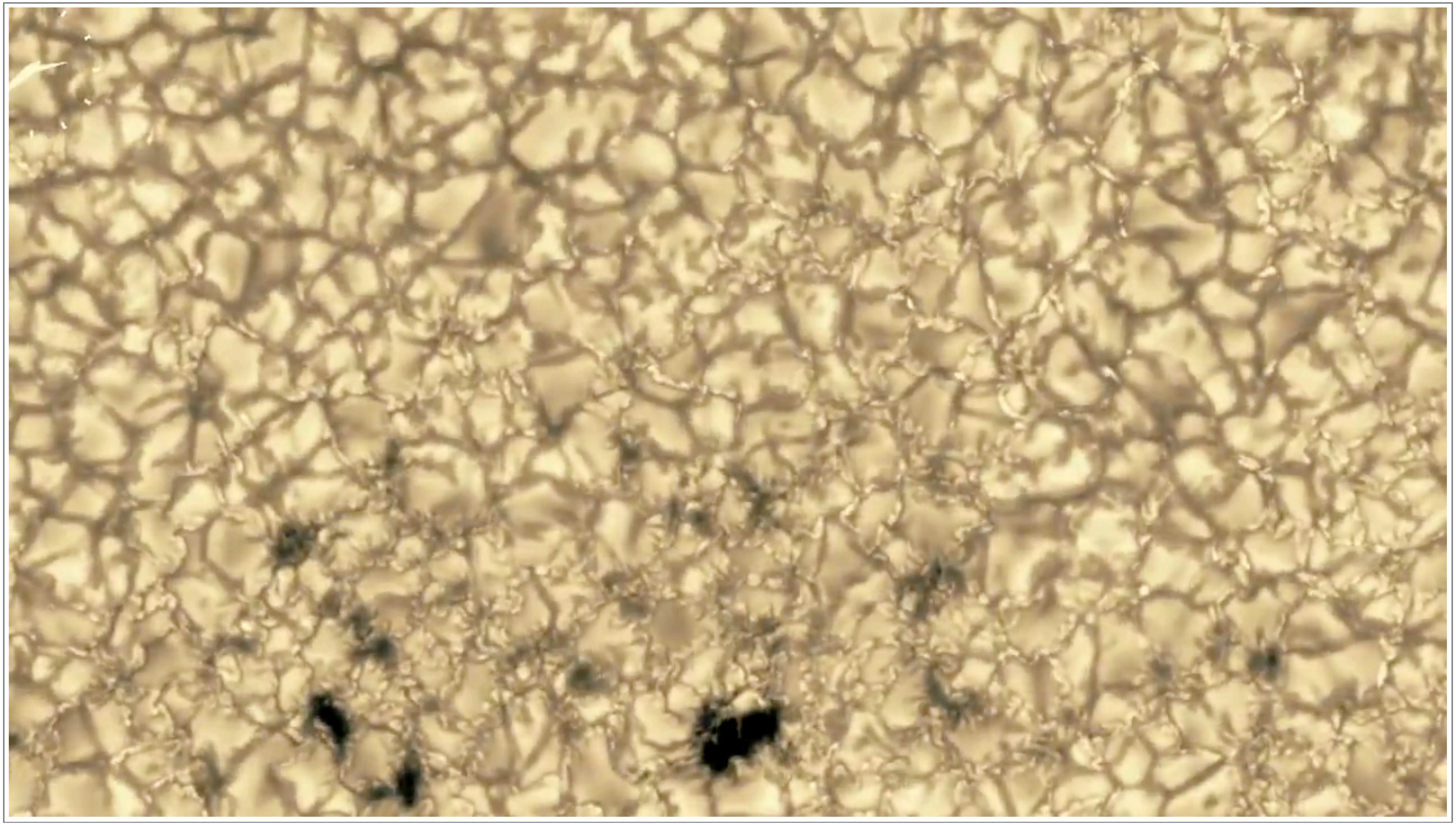
- The hot gas rises, and when it reaches the top of the convective zone (the Sun's surface), it releases its energy as photons that radiate into space.
- The gas at the surface is now cold and it sinks, which allows another bubble of hot gas to rise up in its place.



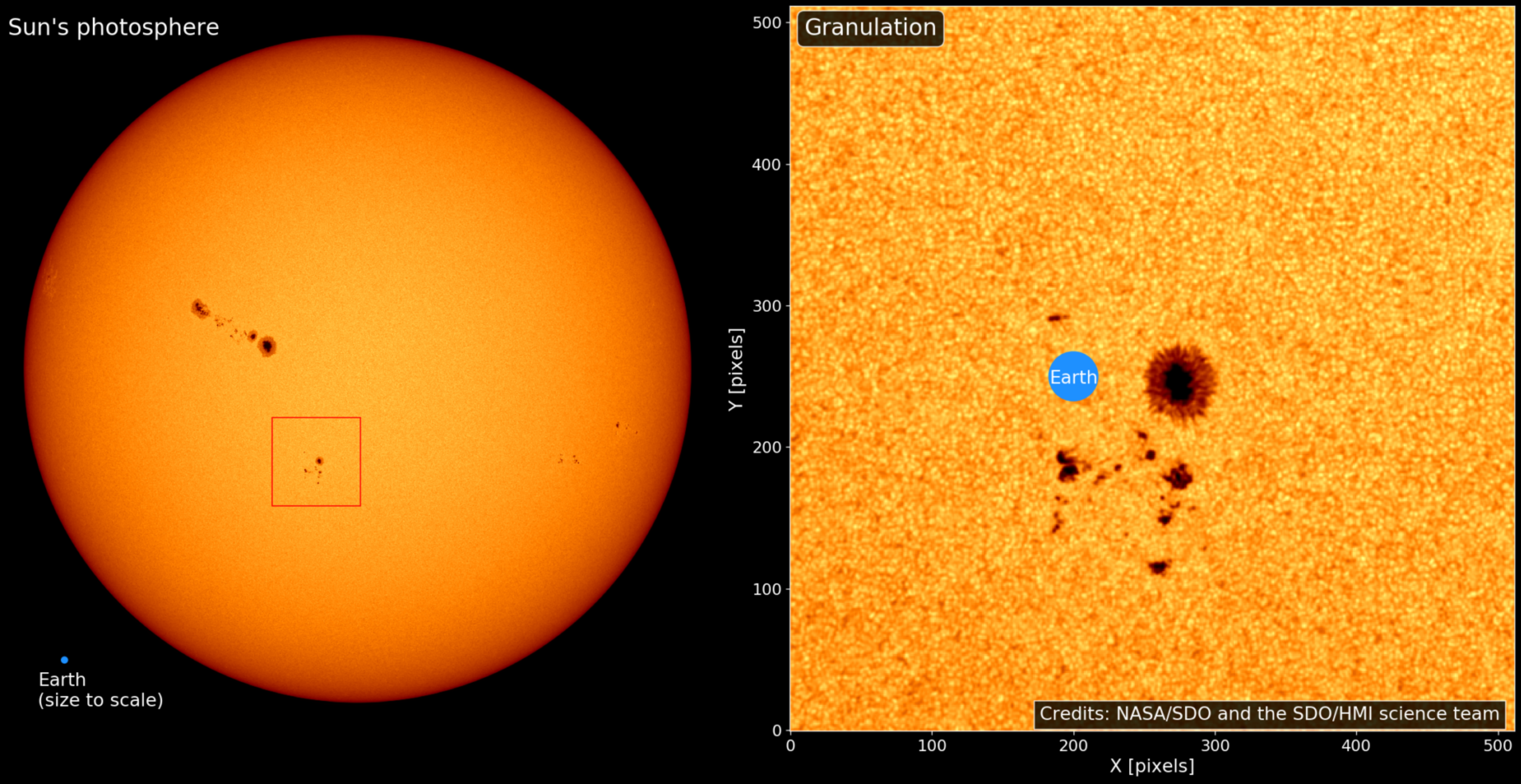
How do we know convection is important in the Sun?

How do we know convection is important in the Sun?

- We observe convection cells called "granulation"

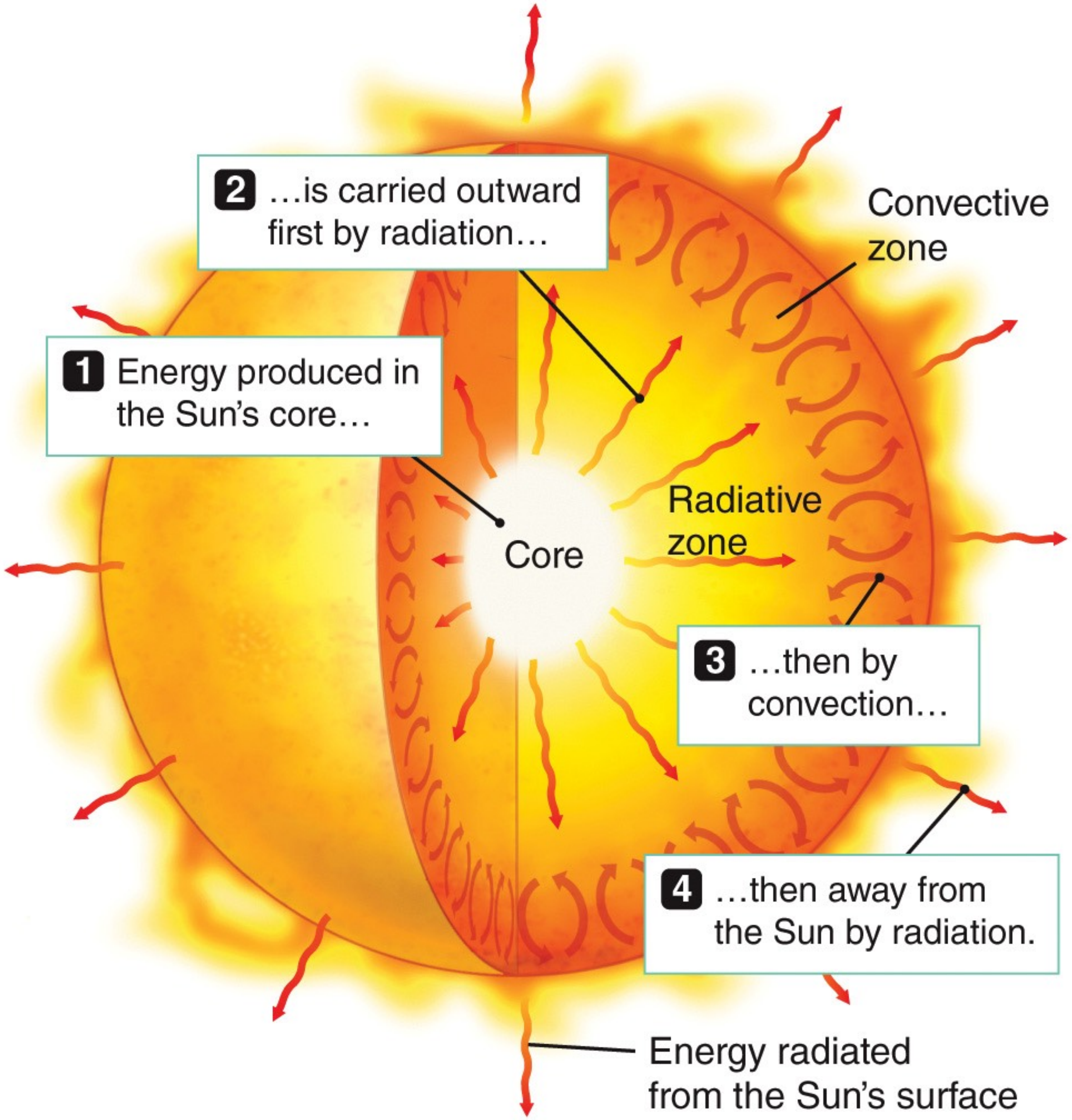


Size Comparison: Solar Granulation vs. Earth



Energy Generation and Transport in the Sun

- Energy produced in the core must get out. **The rate of energy loss must equal the rate of energy gain** to maintain stability.
- The core maintains a constant fusion rate by using the weight of its envelop as a **gravitational thermostat**.
- In the inner layer, **radiation** transfers energy via photons.
- In the outer layer, **convection** carries energy by moving hot gas up and cool gas down.

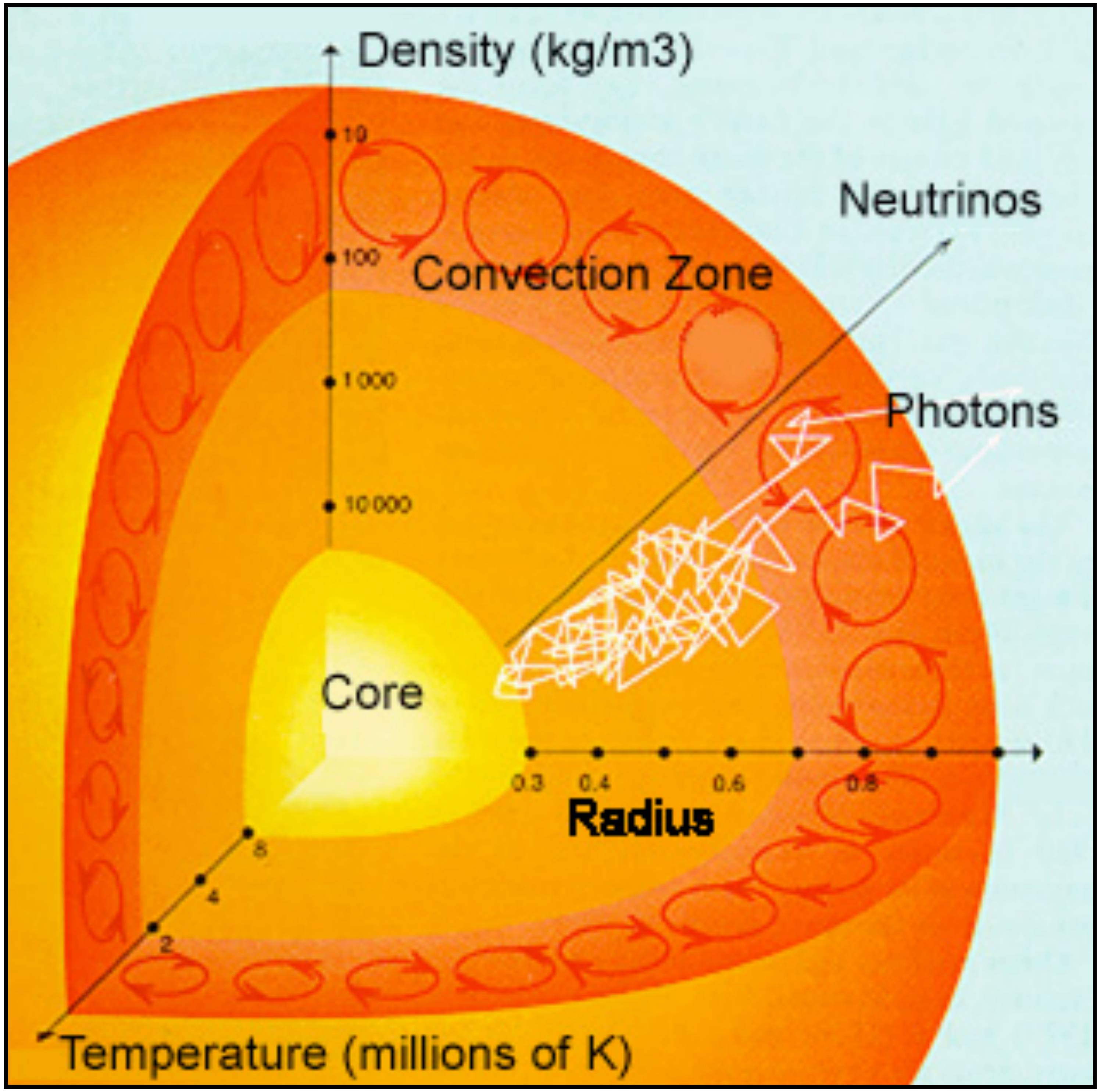


How do we test the hypothesis that nuclear fusion powers the Sun?

measure solar neutrino flux

Testing the fusion model: Solar neutrino flux

- Hydrogen fusion emits neutrinos.
- *Neutrinos*: weakly interacting particles, little mass, no charge.
- *Very* weak interactions with matter.
- Should escape the core freely.



Estimate neutrino flux from Solar EM flux

How many neutrinos pass through a m² area on Earth per second?

Solar luminosity = Solar constant $\times 4\pi(1\ \mathrm{AU})^2$

Neutrino luminosity = Solar luminosity $/E \times n_\nu$

Neutrino luminosity = Neutrino flux $\times 4\pi(1\ \mathrm{AU})^2$

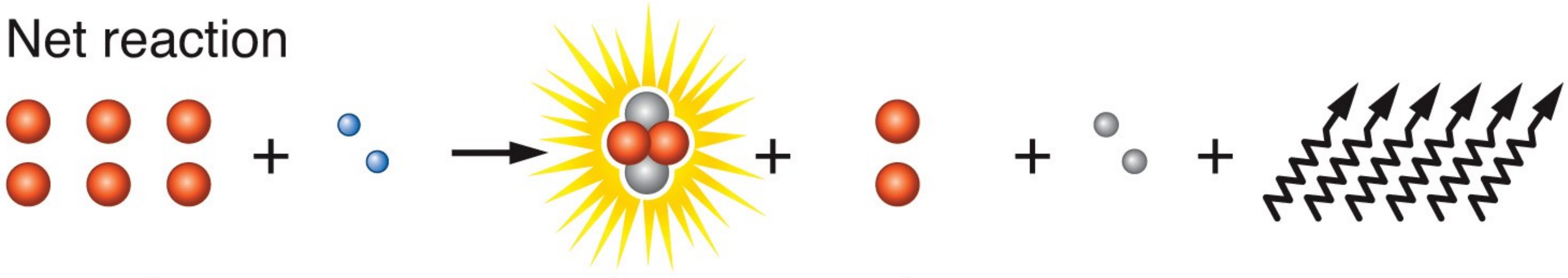
$E = \Delta m \cdot c^2$ = 4 x 10^{-12} J/fusion

$n_\nu = 2$ neutrino/fusion

It is evident from the above equations that:

Neutrino flux = Solar constant $/E \times n_\nu$ **= 6.4x10^{14} neutrino/s/m²**

Net reaction



$$6\,{}^{1}\mathrm{H} + 2\,e^{-} \longrightarrow {}^{4}\mathrm{He} + 2\,{}^{1}\mathrm{H} + 2\,\nu + 6\,\gamma$$

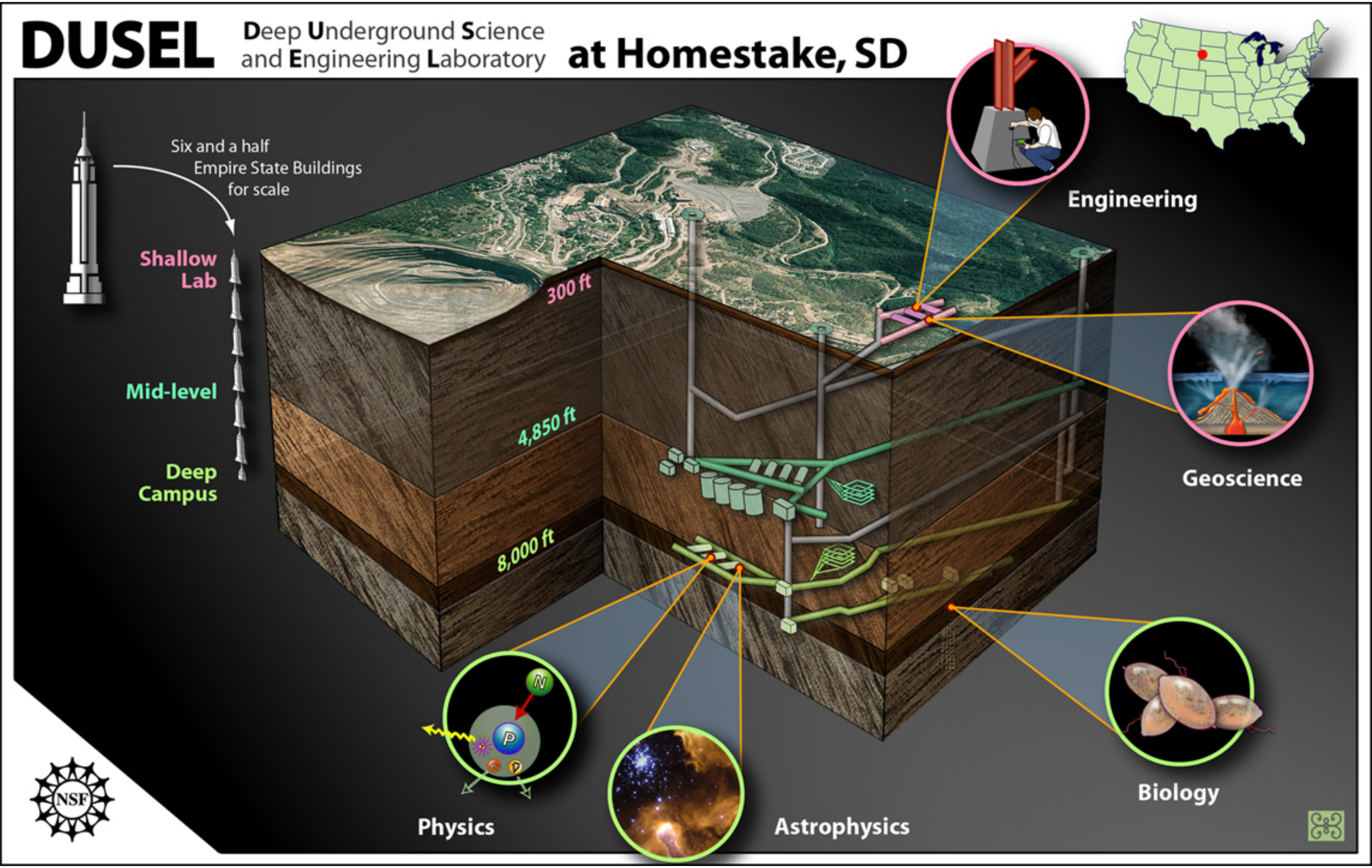
First Neutrino Detection Experiments based on Chlorine (^{37}Cl), 1965-1967


Homestake, South Dakota

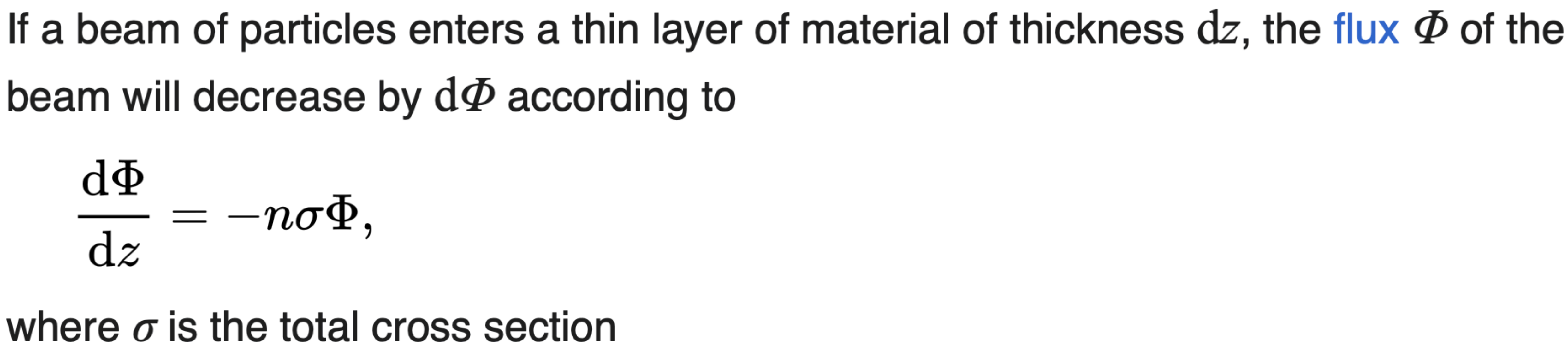
- Although only a tiny fraction of neutrinos interact with matter, there is an enormous flux of solar neutrinos!
- Need large volume detectors to increase detection probability. To fill such large volumes, the detector material better be cheap!
- The Homestake experiment in 1960s used perc (C_2Cl_4), a common dry-cleaning fluid rich in **Chlorine**

$$\nu_e + {}^{37}\mathrm{Cl} \mapsto {}^{37}\mathrm{Ar} + e^-$$

This is an *inverse* beta-decay reaction requires neutrino energy > 0.814 MeV



Interaction Cross Section: Neutrino-Chlorine Interaction



If a beam of particles enters a thin layer of material of thickness dz, the flux Φ of the beam will decrease by dΦ according to

$$\frac{\mathrm{d}\Phi}{\mathrm{d}z} = -n\sigma\Phi,$$

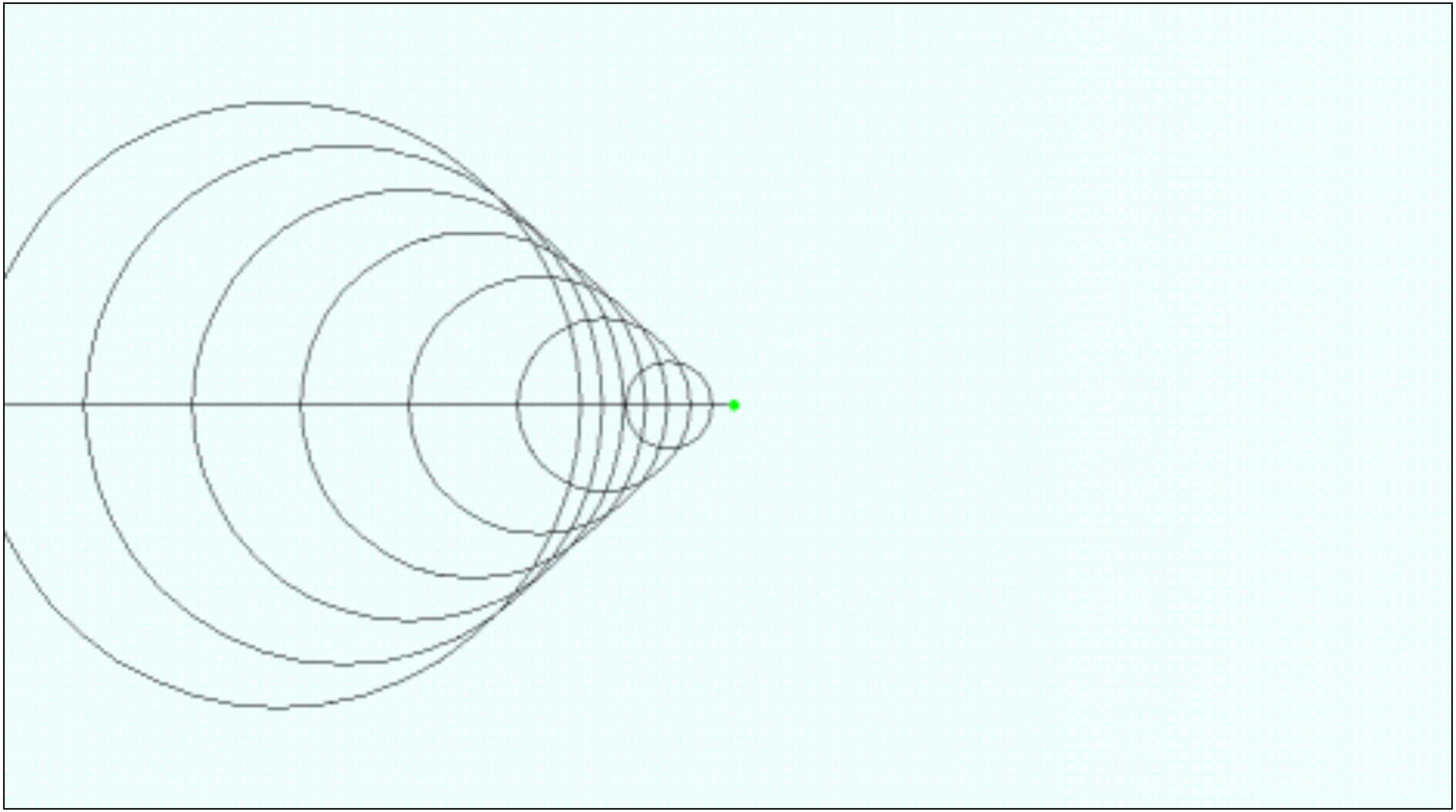
where σ is the total cross section

- The Homestake tank is filled with 615 metric tons of C_2Cl_4 (molar mass: 165.8 g/mol, mass density 1.62 g/cm^3). Given 1 mole = 6e23 entities, what's the volume density of Chlorine (n in cm^{-3})?
- Given a neutrino cross section of 10^{-38} cm^2, and the neutrino flux of 6×10^{14} per s per square meter, calculate the amount of attenuated neutrino flux over a 15 m long tank (dz = 1500 cm).

n = mass density / molar mass x 6e23 x 4 = 2.3e22 Cl/cm^3
d Phi/Phi = n sigma dz = 2.3e22 x 1e-38 x 1500 = 3.45e-13
=> the attenuated flux is d Phi = 200 neutrinos/s/m^2

New Generation Cherenkov Neutrino Detectors: the Principle

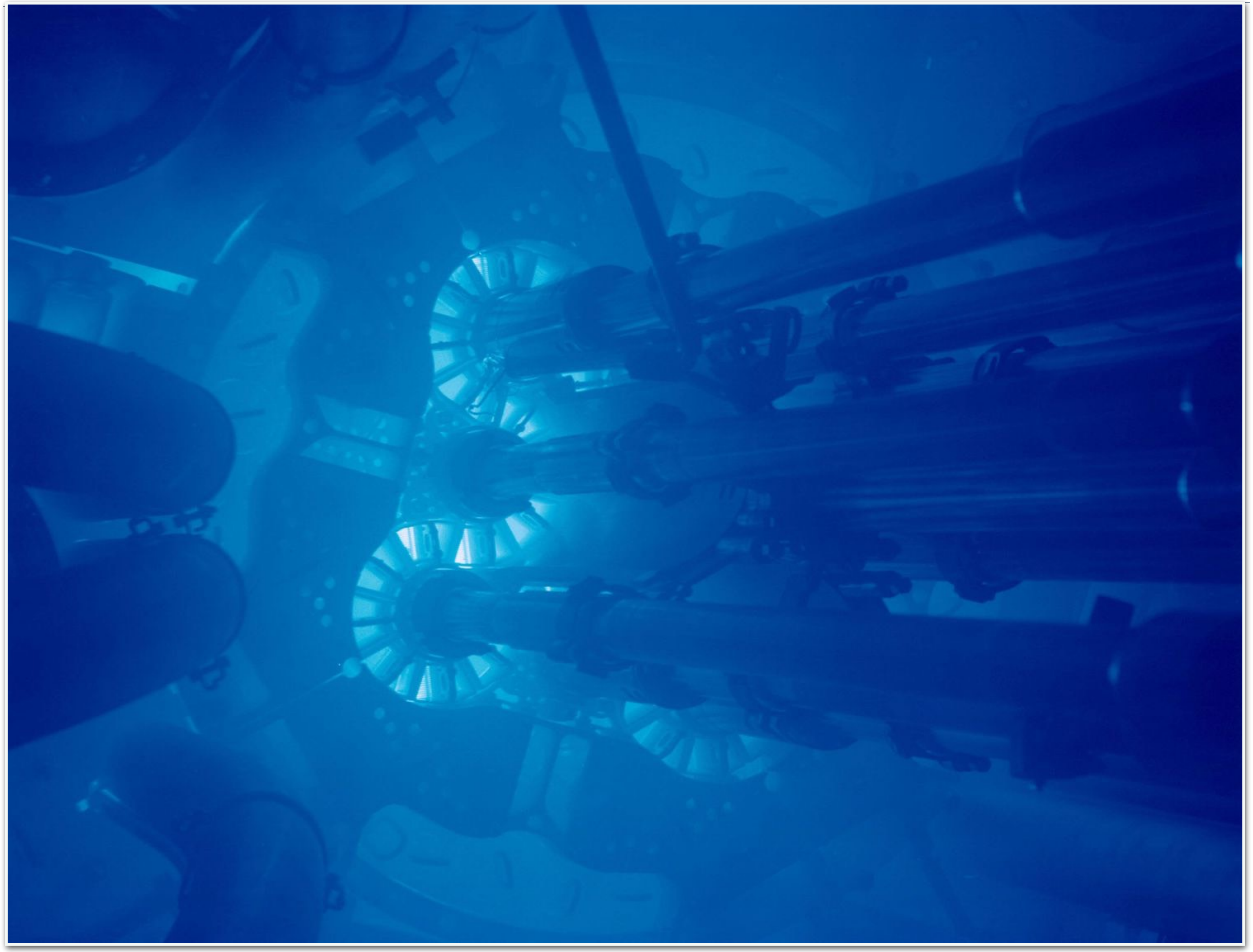
- A neutrino interaction with water produce a charged particle (electron or position) that travels faster than the speed of light in water, creating Cherenkov radiation, similar to a sonic boom



Sonic boom from a volcanic explosion

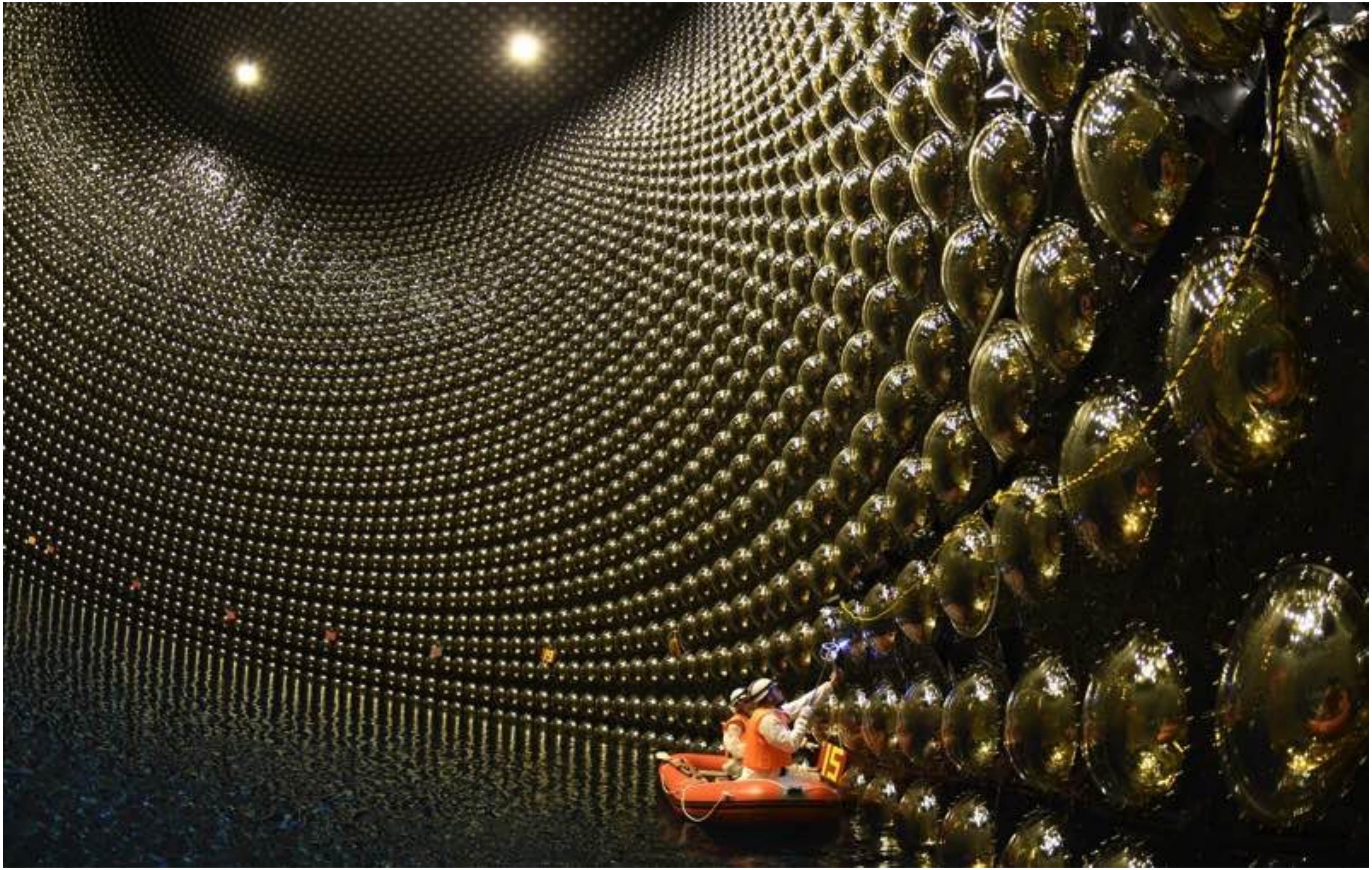
Sonic boom from a supersonic fighter jet

Cherenkov radiation: blue glow of water in a nuclear reactor

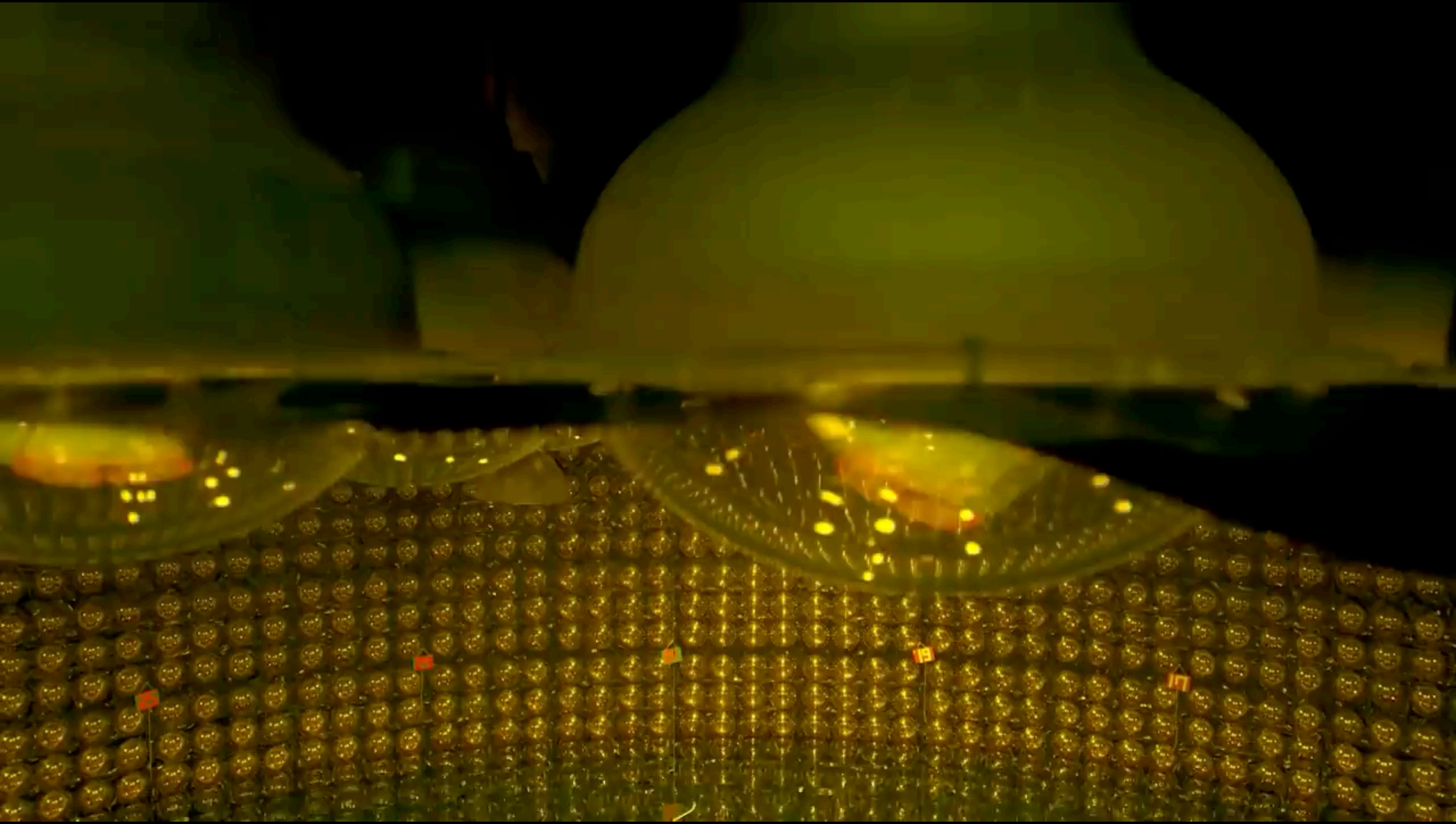


Cherenkov Neutrino Detectors: Super-Kamiokande

The detector is a cylindrical tank holding **50,000 tons of pure water**. The inside of the wall is mounted with 13,000 **photomultipliers** that can detect **Cherenkov radiation**

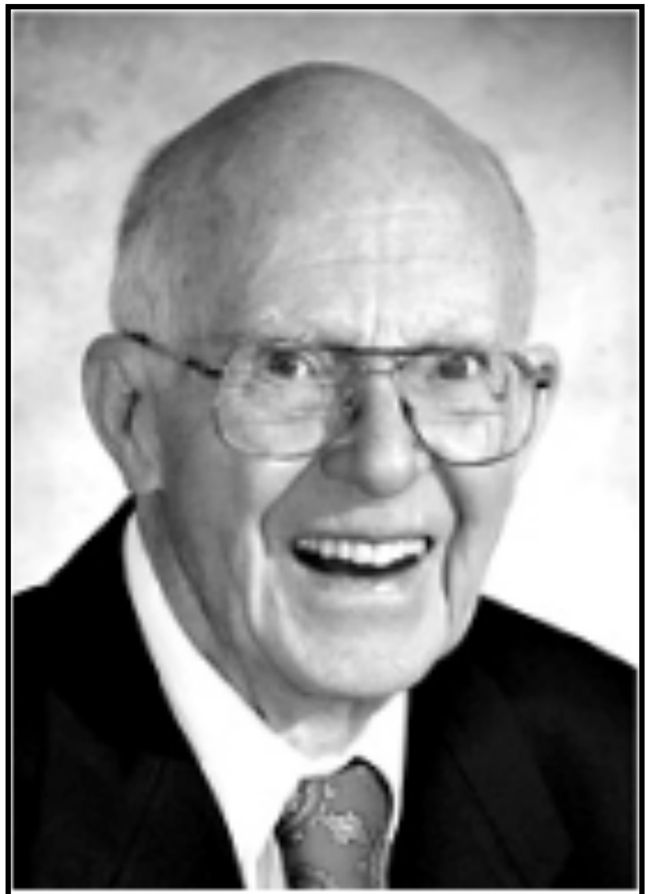


Cherenkov Neutrino Detectors: Super-Kamiokande



Two Nobel Prizes in Physics awarded to Solar Neutrino Experiments

- 2002 Nobel Prize in Physics: Davis & Koshiba for the detection of solar neutrinos (confirming the fusion model)
- 2015 Nobel Prize in Physics: Kajita & McDonald for the discovery of neutrino oscillation (solving the missing neutrino problem)


Raymond Davis Jr.


Masatoshi Koshiba

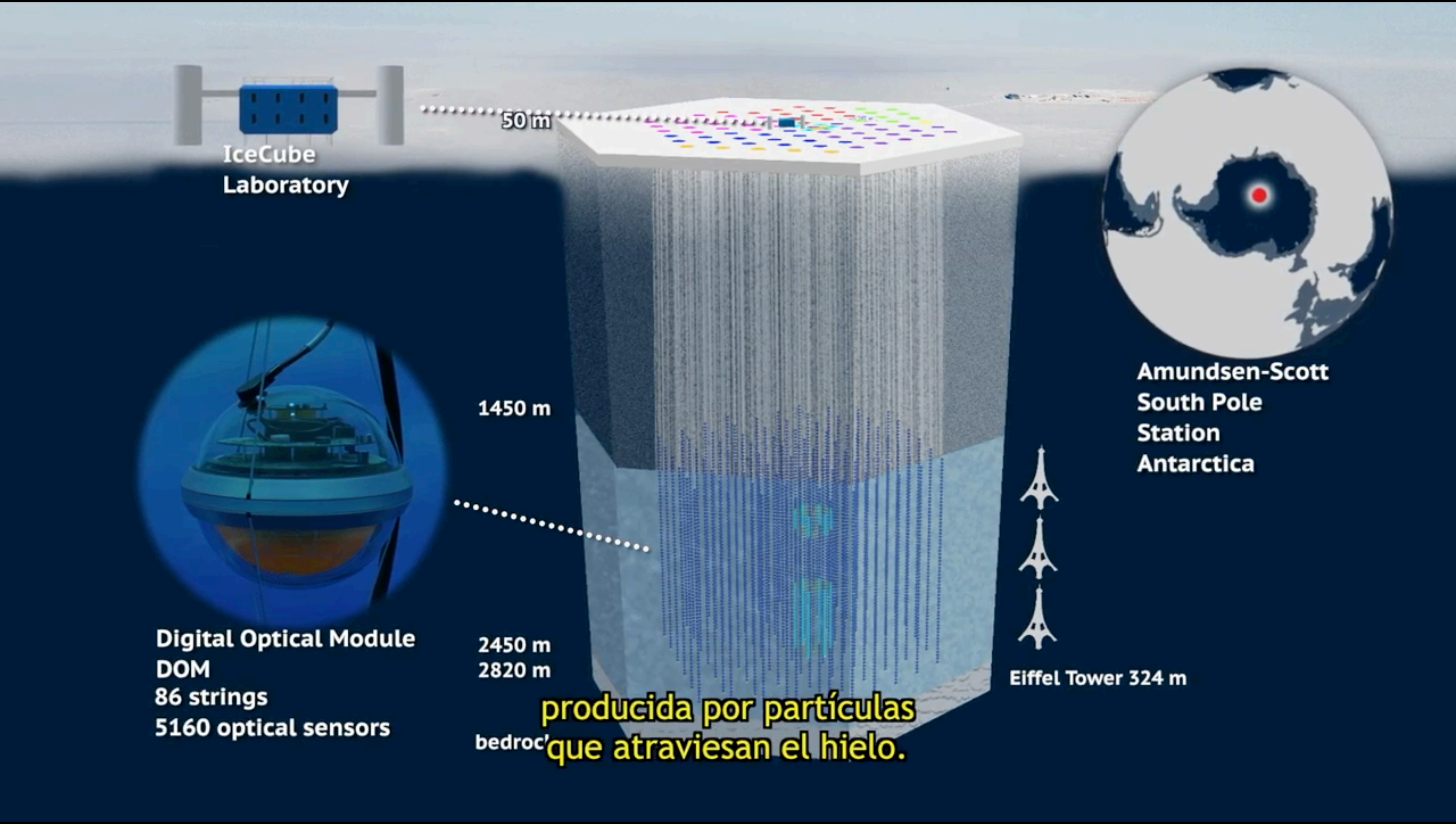

© Nobel Media AB. Photo: A. Mahmoud

Takaaki Kajita


© Nobel Media AB. Photo: A. Mahmoud

Arthur B. McDonald

What about neutrinos from sources other than the Sun? We not only need to detect but also to trace the direction of the neutrinos



Interaction Cross Section
(a revisit)

The Attenuation Equation, where we first introduced cross section

If a beam of particles enters a thin layer of material of thickness $\mathrm{d}z$, the flux Φ of the beam will decrease by $\mathrm{d}\Phi$ according to

$$\frac{\mathrm{d}\Phi}{\mathrm{d}z} = -n\sigma\Phi,$$

where σ is the total cross section

- The Homestake tank is filled with 615 metric tons of C_2Cl_4 (molar mass: 165.8 g/mol, mass density 1.62 g/cm^3). Given 1 mole = 6e23 entities, what's the volume density of Chlorine (n in cm^{-3})?
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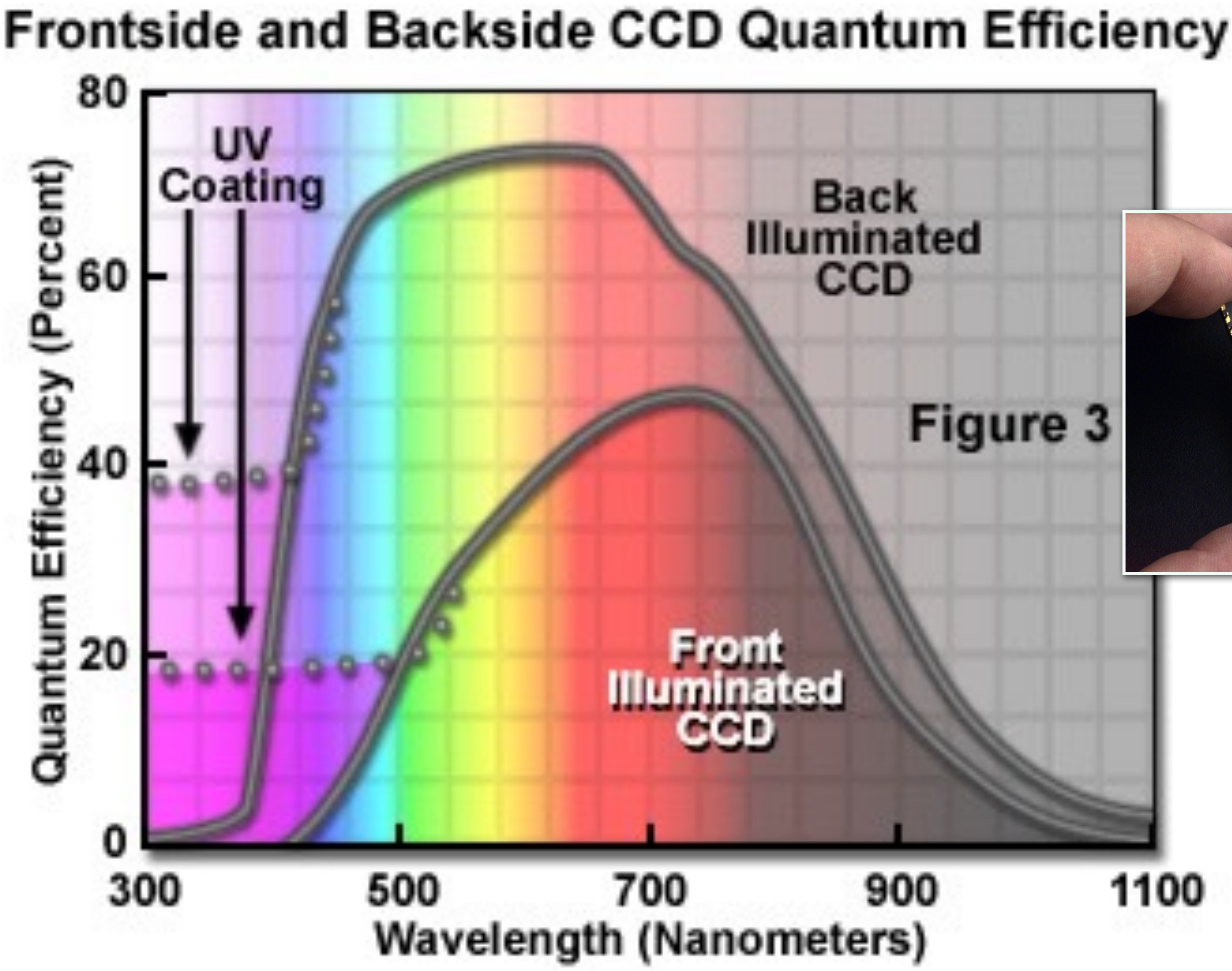
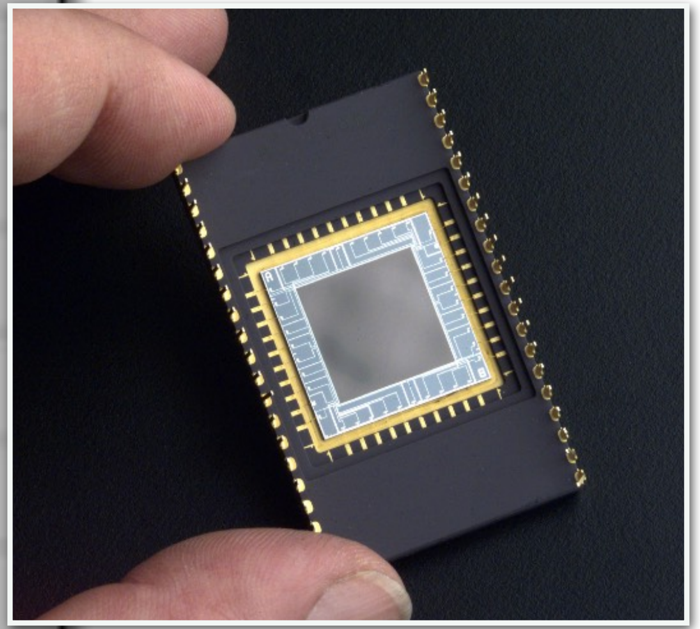
n = mass density / molar mass x 6e23 x 4 = 2.3e22 Cl/cm^3
d Phi/Phi = n sigma dz = 2.3e22 x 1e-38 x 1500 = 3.45e-13
=> the attenuated flux is
d Phi = 200 neutrinos/s/m^2

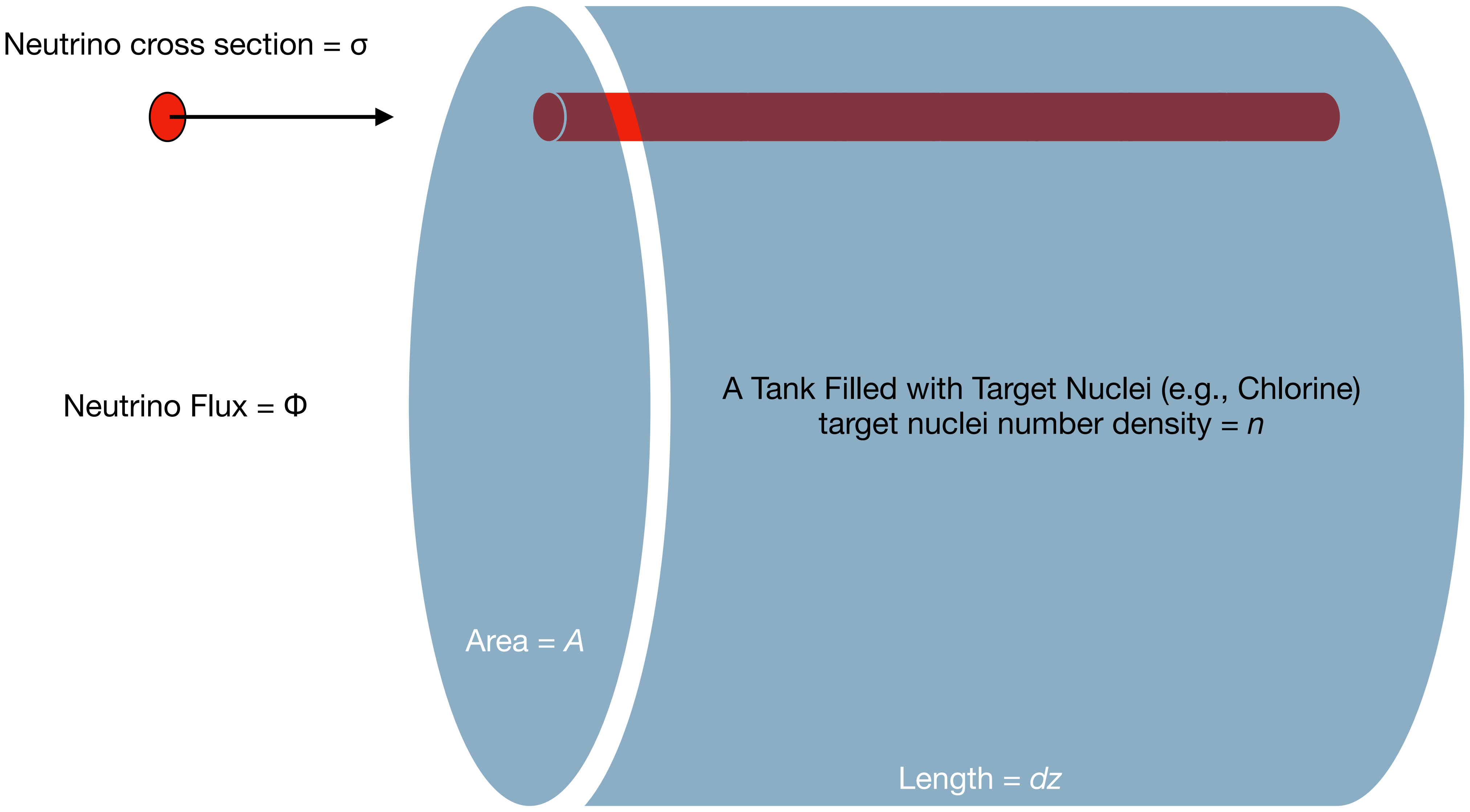
The attenuation of neutrino detector vs. light detector (CCD)





Understanding cross section and the attenuation equation

- When a neutrino travels through the tank, how many target nuclei would have interacted with it? That would equal to the number of target nuclei in the volume carved through by the neutrino's cross section.
- How many neutrinos pass through the tank per unit time?
- What's the total number of reactions that could happen in the tank per unit time?



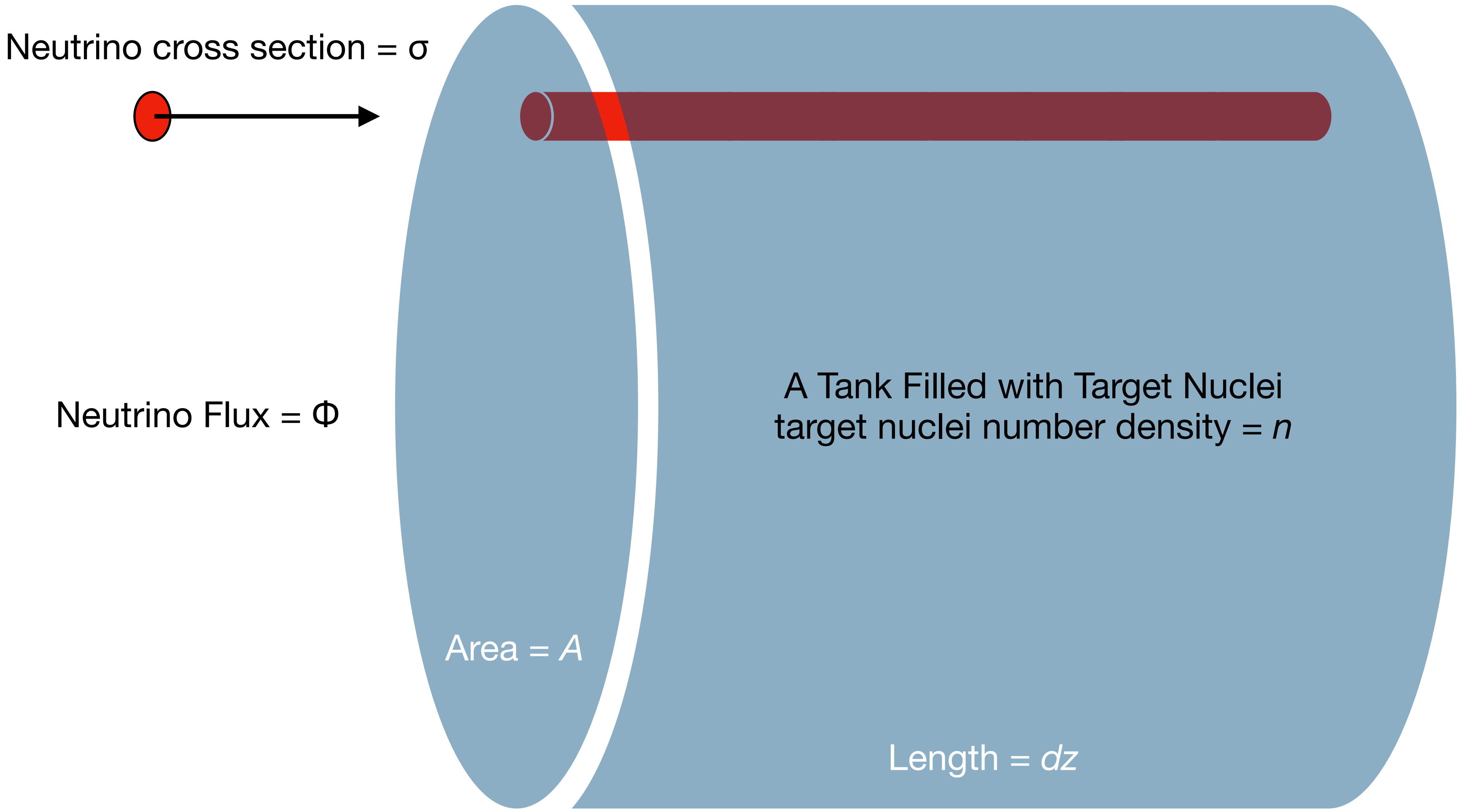
Cross Section and Mean Free Path

- **Mean free path** is defined as the average distance over which a particle will interact with another particle (e.g., a photon's mean free path is the distance it can travel before its absorption/scatter).
- From the previous slide, we know that for one incoming particle, **the number of interactions that will happen along its path is:**

$$n_{\text{interaction}} = n\sigma dz$$

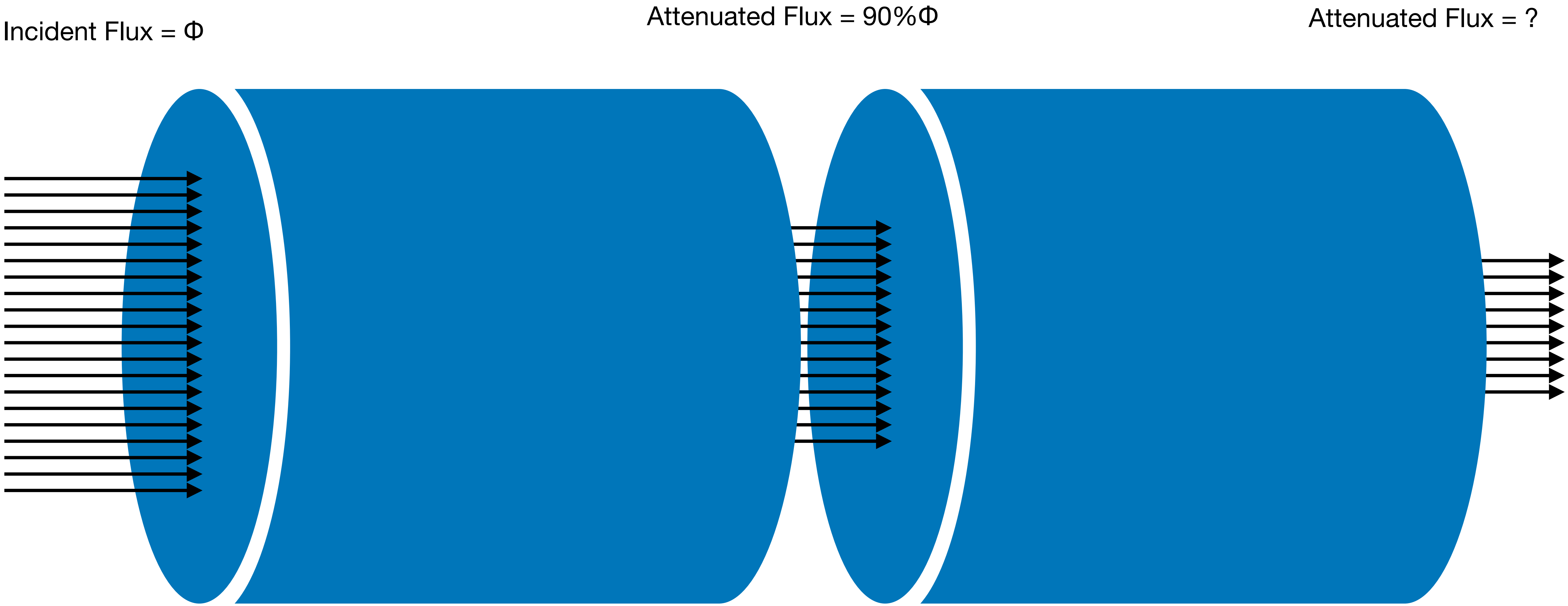
- when we set $n_{\text{interaction}} = 1$ according to the definition of the mean free path, we can solve for the dz that corresponds to the mean free path (λ):

$$l_{\text{mfp}} = \frac{1}{n\sigma}$$



Utilizing the attenuation equation

- Suppose the particle flux is reduced to 90% of the incident flux by a tank that is 1 km long, what would be the particle flux if the beam passes through a tank that is 2 km long?



Solution of the attenuation equation

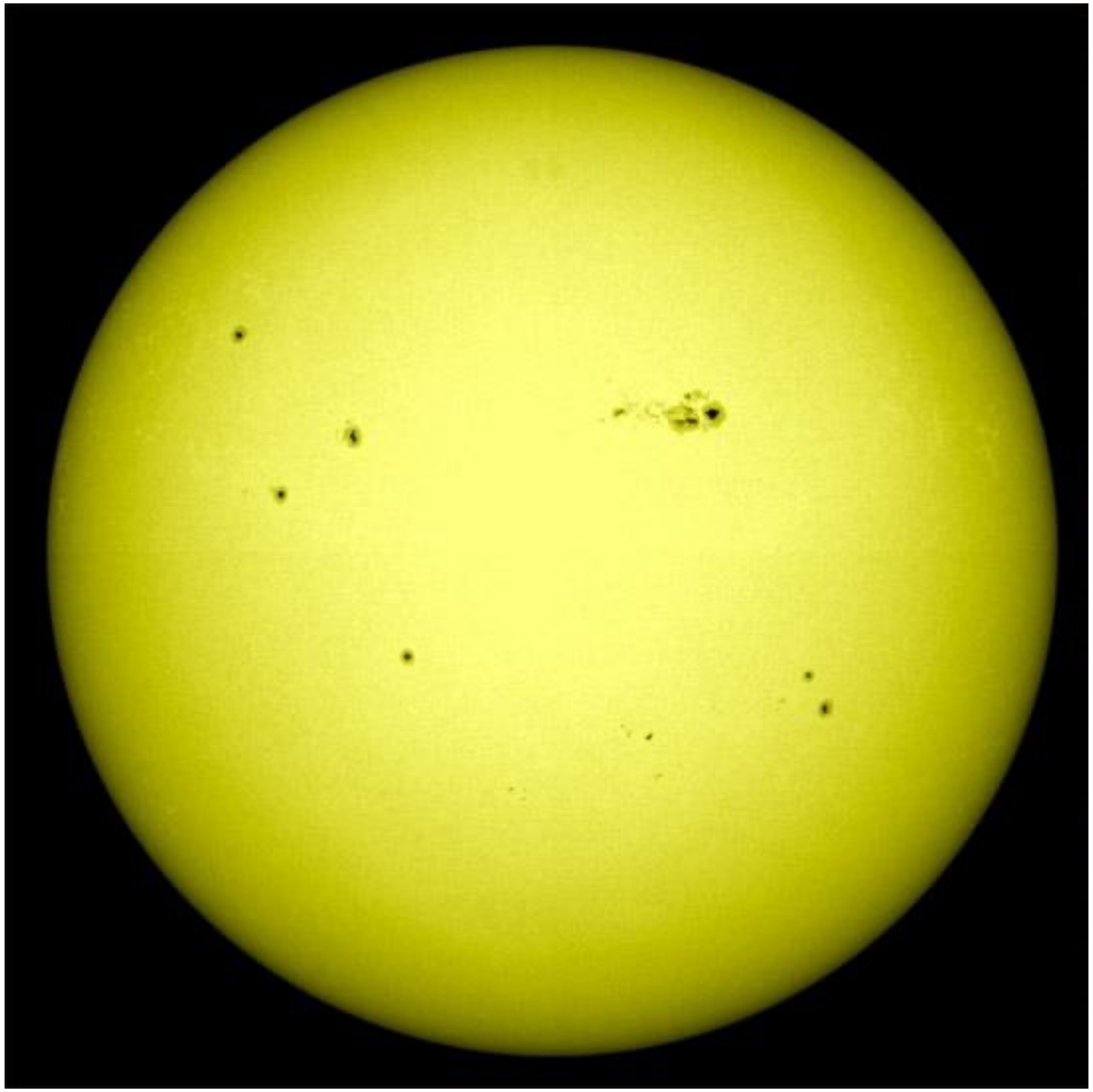
By integrating the attenuation equation: $$\frac{d\Phi}{\Phi} = -n\sigma dz$$

We obtain the solution of the diff. equation: $$\Phi = \Phi_0 \exp(-n\sigma z) = \Phi_0 \exp\left(-\frac{z}{l_{\text{mfp}}}\right)$$

The Atmospheres of the Sun

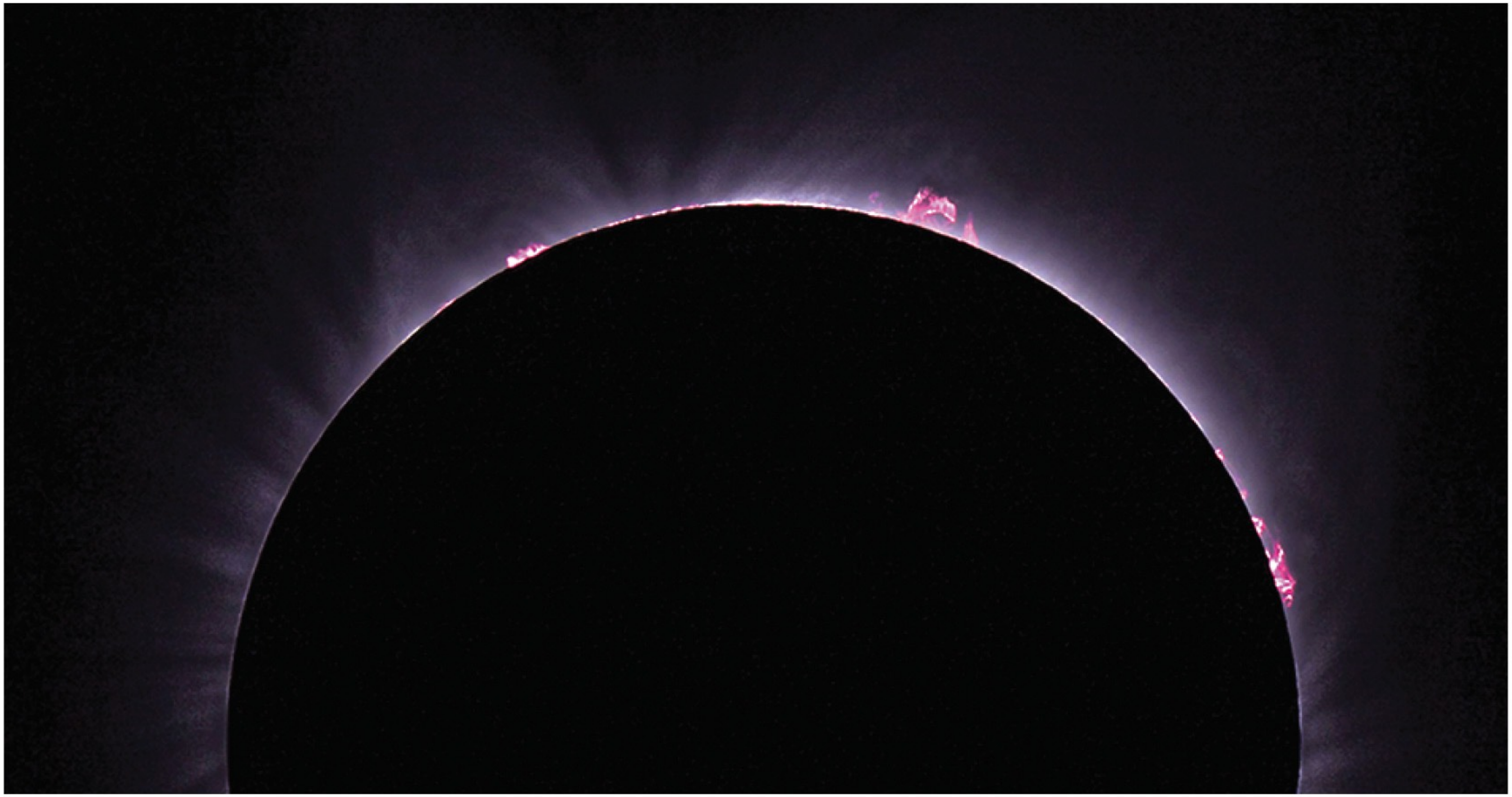
Solar Atmosphere: Photosphere

- **Photosphere**: layer where light is emitted (the apparent surface)
- **Effective temperature**: 5780 K
- **Thickness**: ~500 km
- Temperature decreases outward in the photosphere.
- Atmosphere density drops rapidly with increasing altitude, like all the atmosphere layers



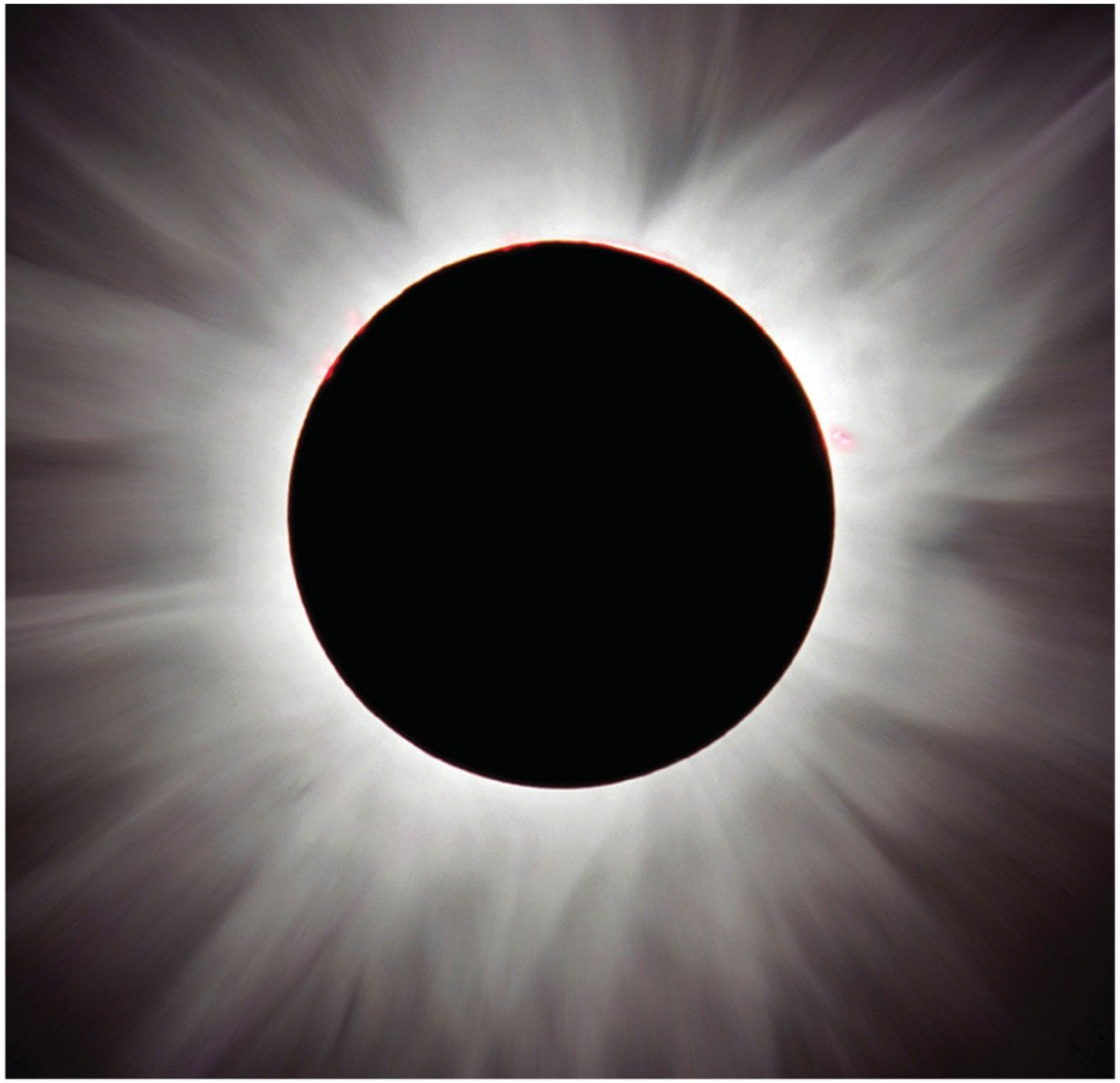
Solar Atmosphere: Chromosphere

- **Chromosphere**: above the photosphere
- Higher temperature than the photosphere
- It gives off a **reddish emission-line spectrum from hydrogen**.
- The red color is what gives the chromosphere its name, because "chromosphere" means the "**place where the color comes from**."



Solar Atmosphere: Corona

- **Corona**: above the chromosphere
- Temperature inversion: Very hot — $T = 1$ to 2 million K (emits X-rays as well as visible light).
- It can extend for several solar radii (8 million km above surface).
- The emission is so diffuse that it is not visible unless there is a solar eclipse.



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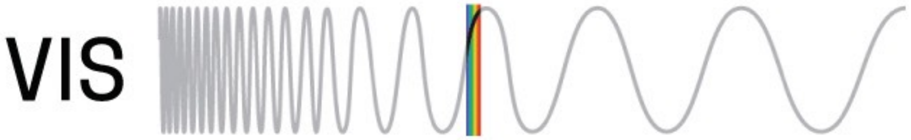
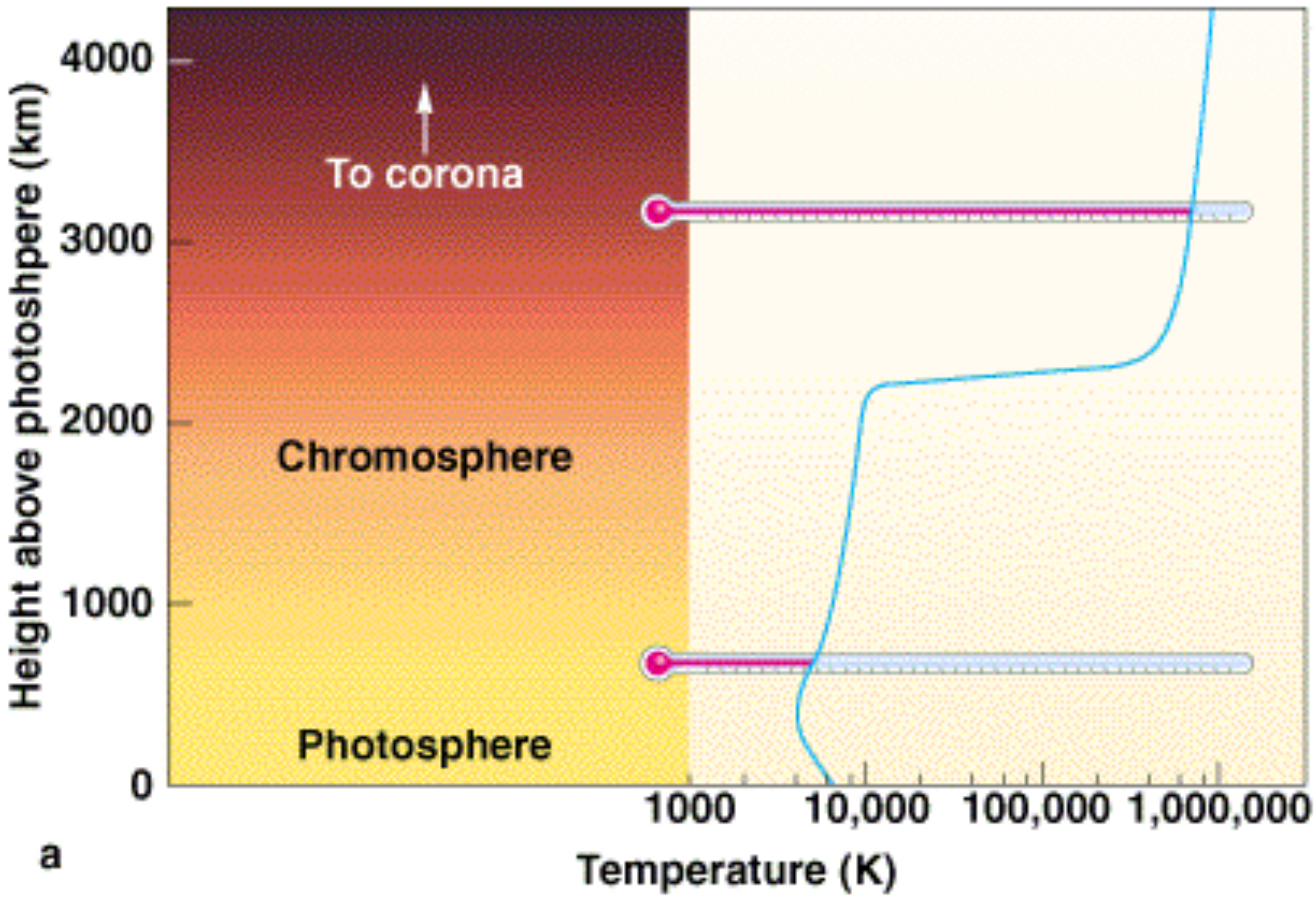


Solar Atmosphere: Temperature & Density vs. Height

- The density consistently decreases with increasing distance from the Sun.
- The temperature decreases in the photosphere, but rises in the chromosphere, potentially due to magnetic field energy.



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Solar Atmosphere: Temperature & Density vs. Height

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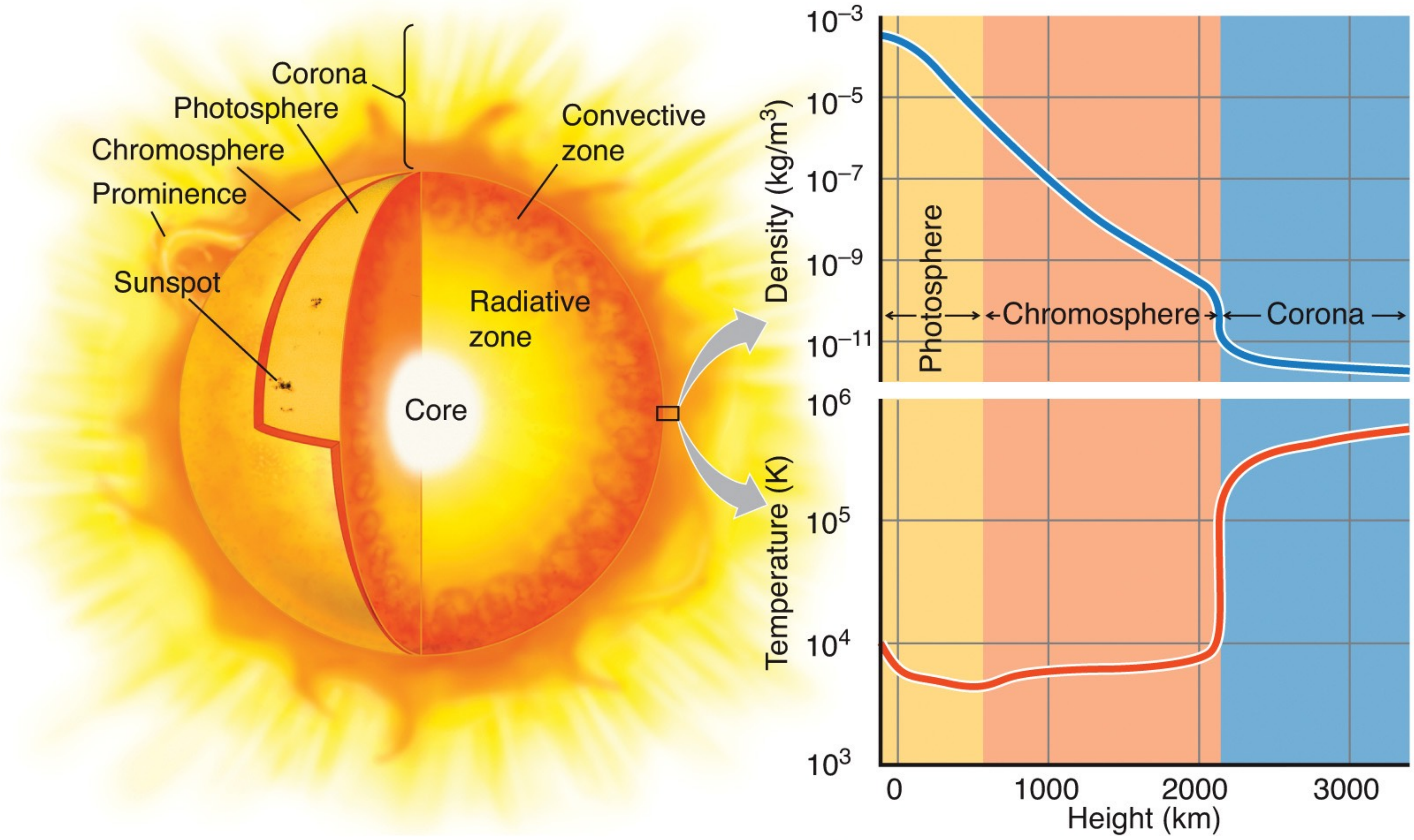
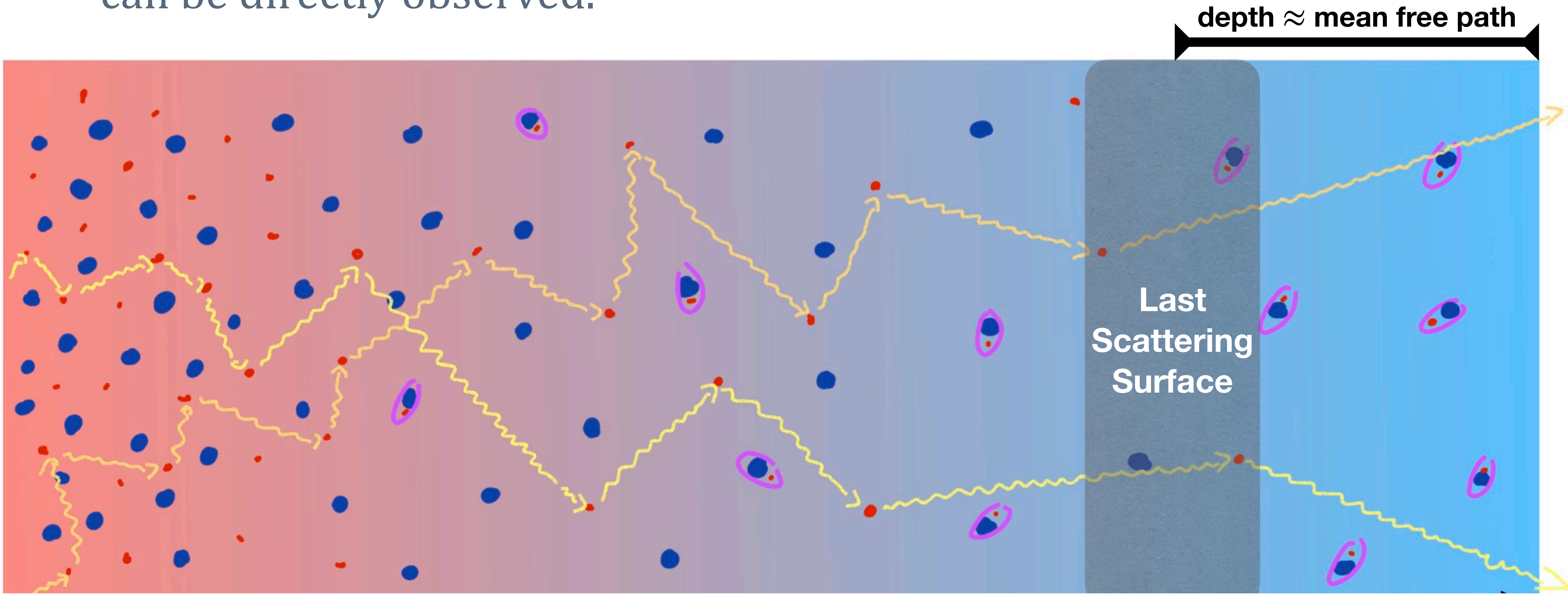


Limb Darkening and Spectral Line Formation

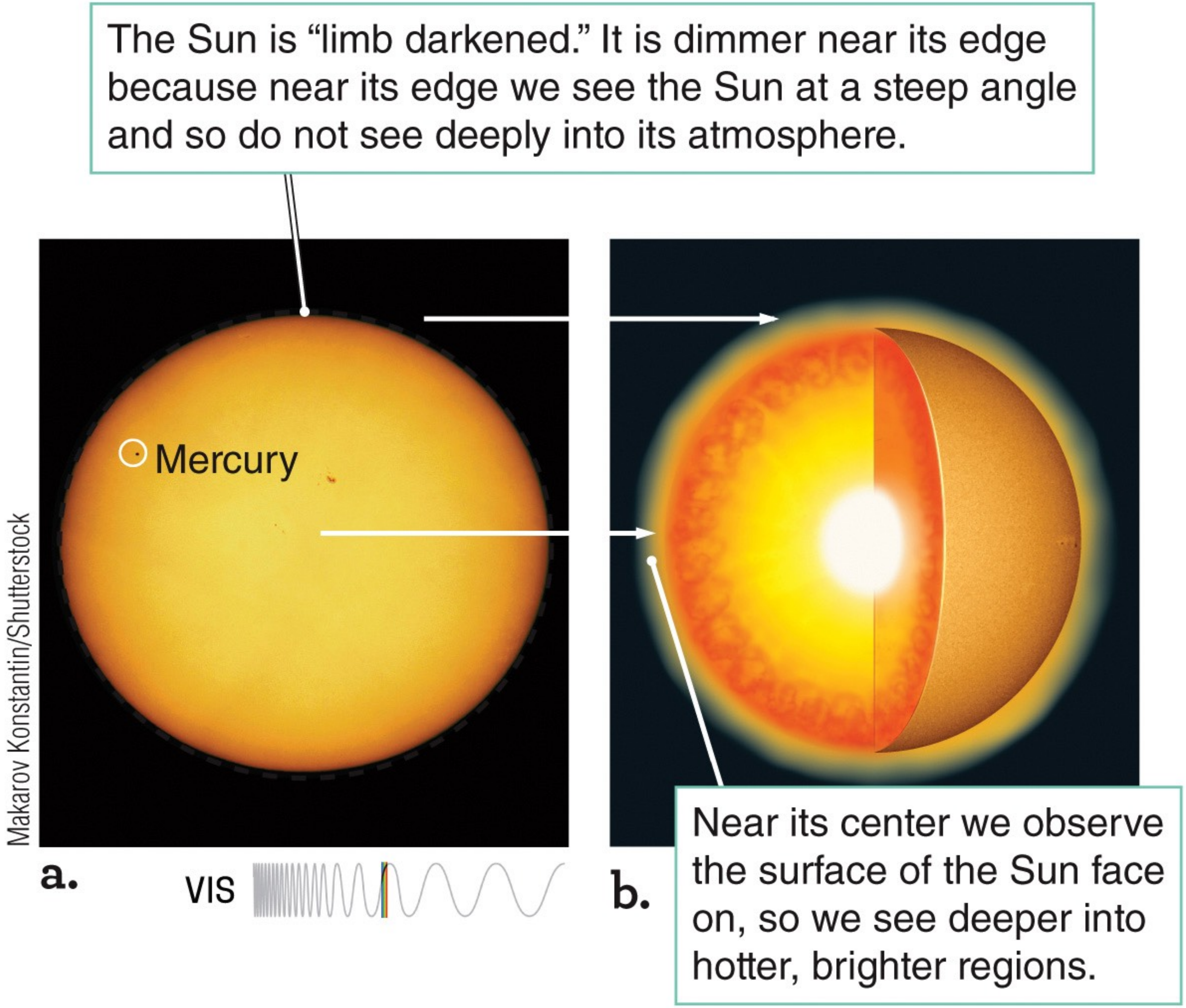
Solar Atmosphere: Last Scattering Surface

- The Sun has no solid surface, but the apparent surface of the Sun is the surface at which light can directly escape into space.
- Let's call this surface the **last scattering surface** (a concept also used in cosmology). Note that its depth depends on **(1) the angle we look into the Sun and (2) the wavelength of the photons**
- The layers above this point are known as the **atmosphere**, which can be directly observed.



Solar Atmosphere: Limb Darkening of Photosphere

The Sun appears **darker near its edge** because our sightline penetrates less depth at a steeper angle, the **last scattering surface** is at a lower temperature.



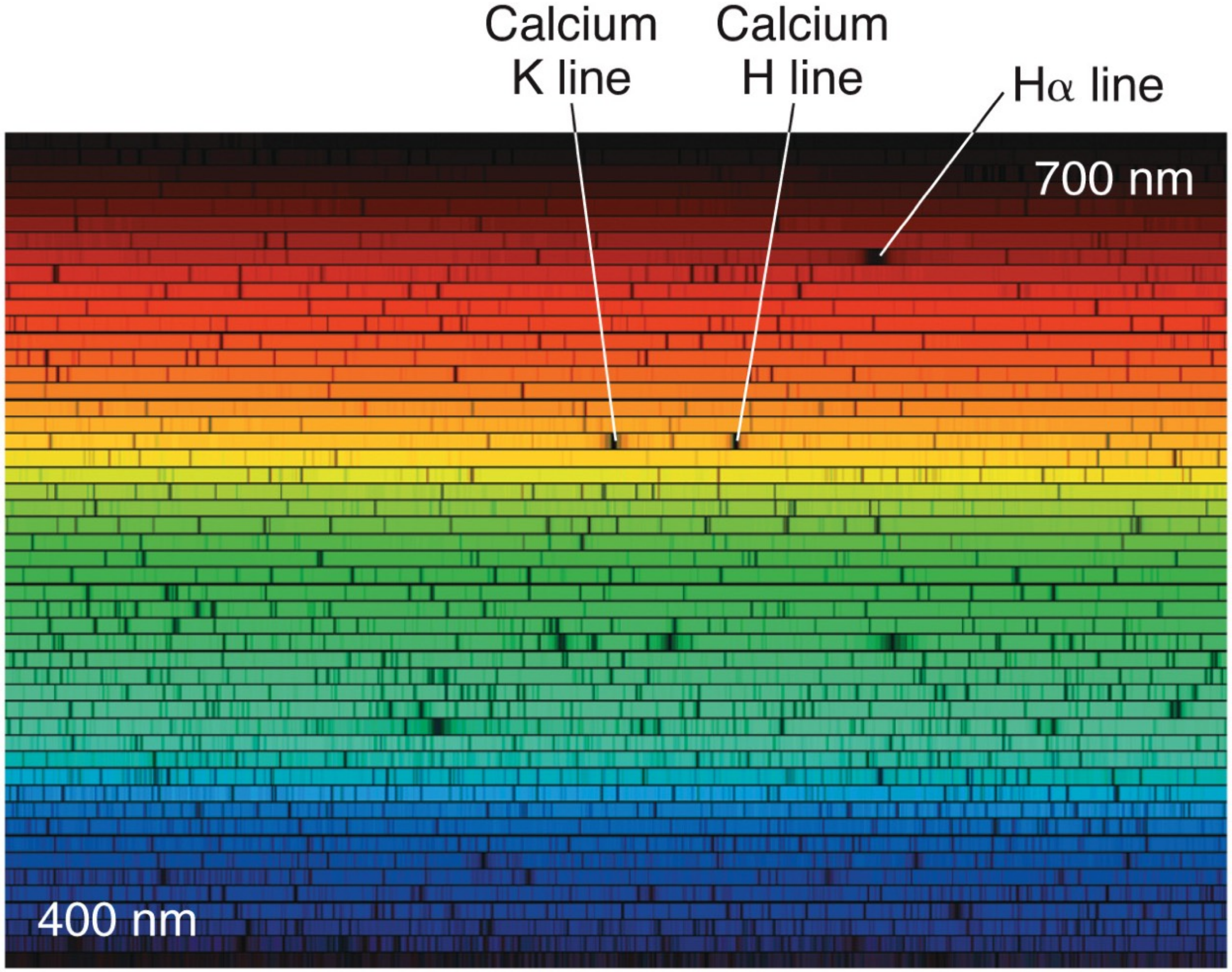
The Sun is “limb darkened.” It is dimmer near its edge because near its edge we see the Sun at a steep angle and so do not see deeply into its atmosphere.

Near its center we observe the surface of the Sun face on, so we see deeper into hotter, brighter regions.

Makarov Konstantin/Shutterstock

Solar Atmosphere: Understanding the Absorption Line Spectrum

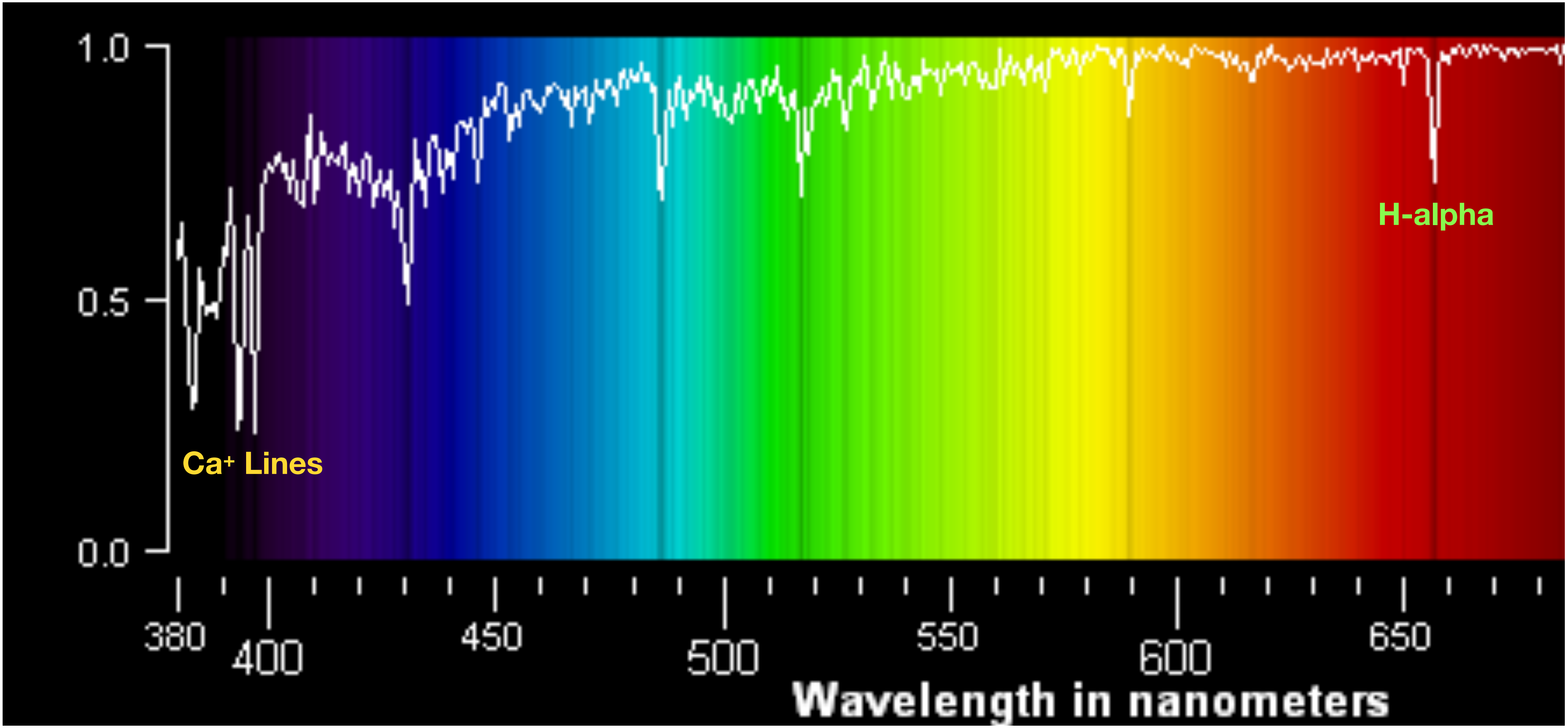
- **The depth of the last scattering surface depends on wavelength**. The depth decreases at wavelengths where there are corresponding atomic or ionic transitions, because the opacity of the gas increases (i.e., cross section increases, thus decreasing the mean free path).
- The lines appear darker because the last scattering surface is at shallower depth compared to other wavelengths, and **shallower depth means lower temperature**.



Nigel Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF, https://noirlab.edu/public/images/noao-sun/, https://creativecommons.org/licenses/by/4.0/

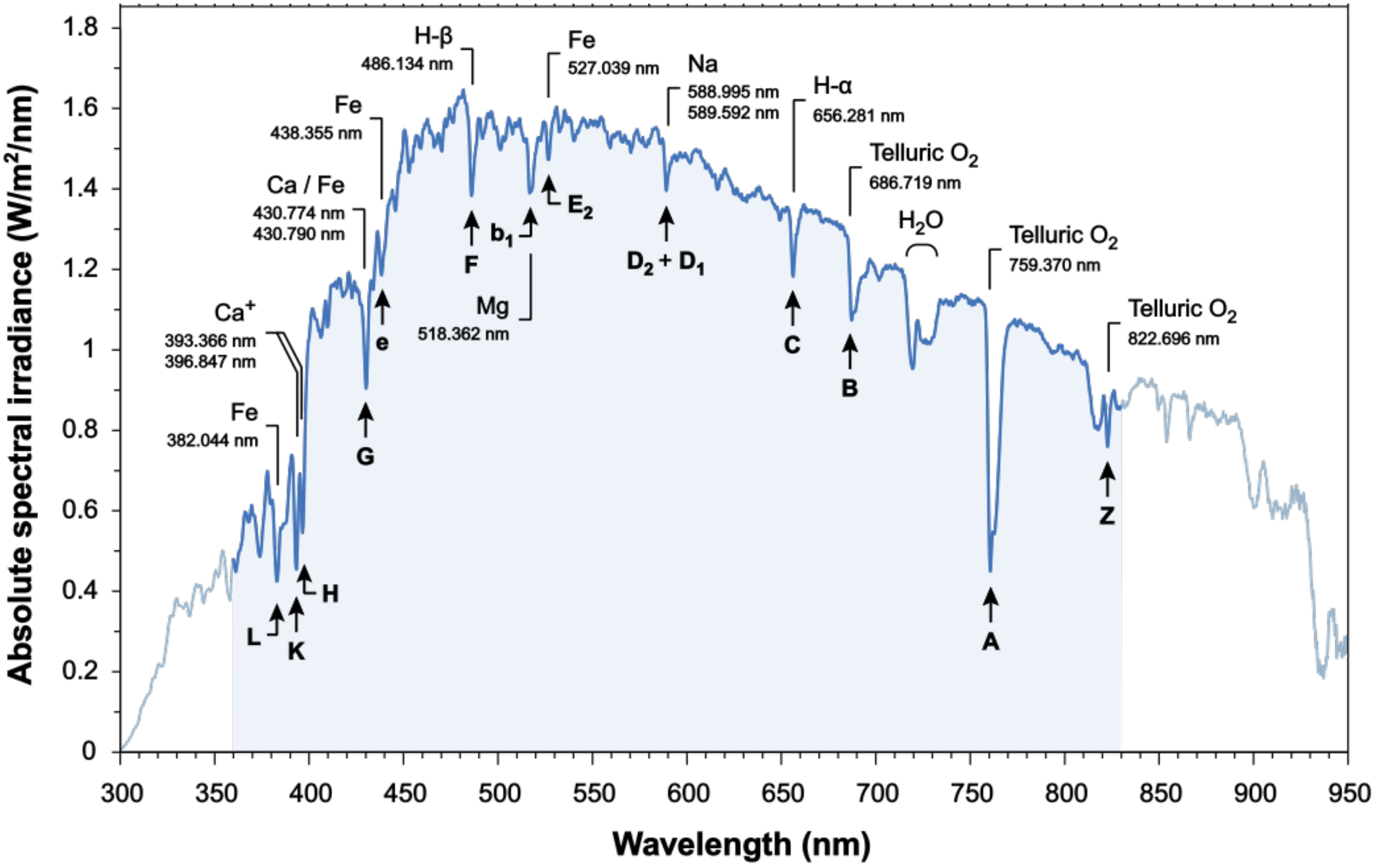
Solar Atmosphere: Understanding the Absorption Line Spectrum

- The Sun displays a complex absorption spectrum from the presence of over 70 elements.
- The strongest lines are from singly-ionized Calcium ions at 3968.5 and 3933.7 Angstroms. This seems quite strange given that Calcium is much more rare than Hydrogen.

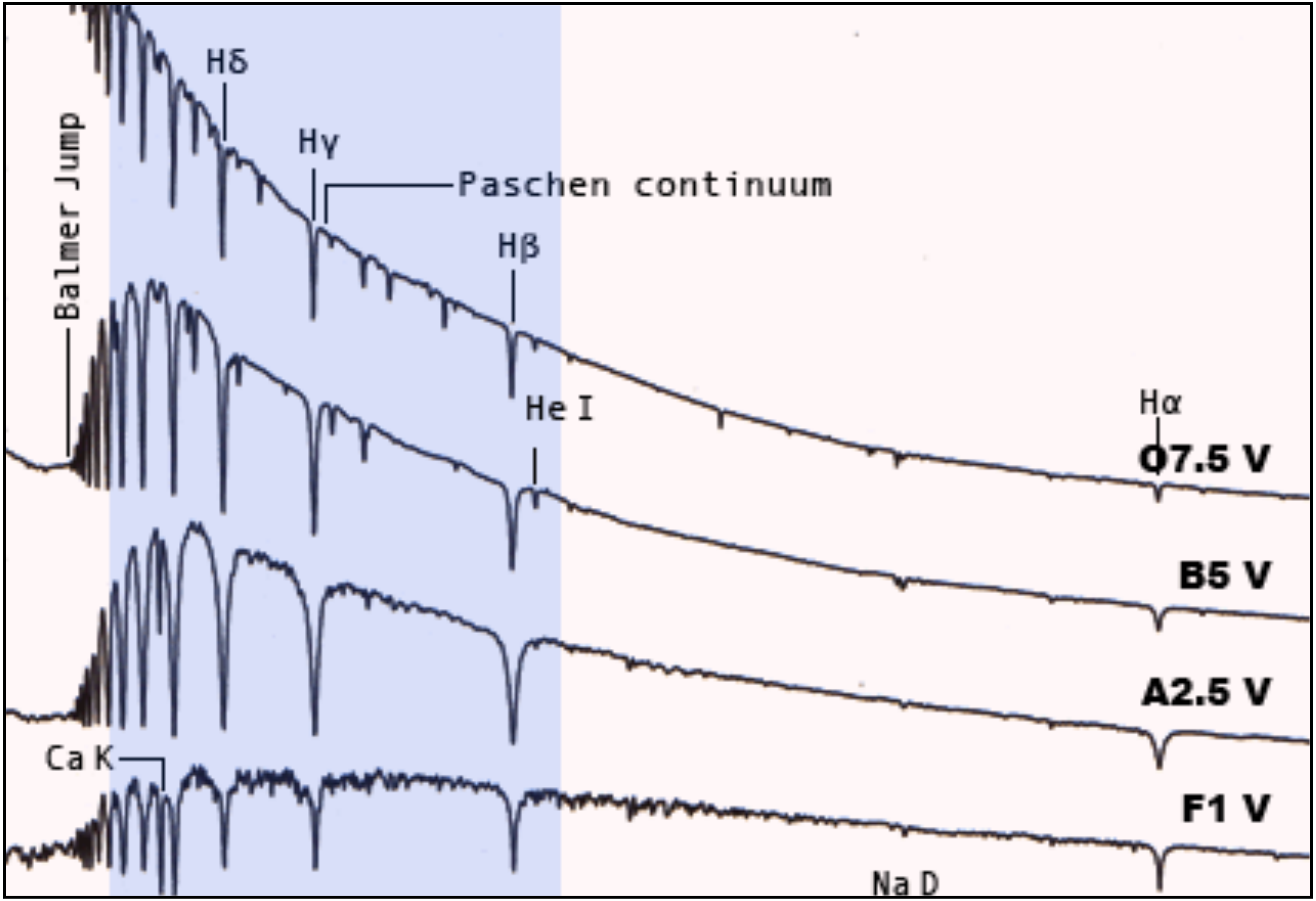


Solar Atmosphere: Understanding the Absorption Line Spectrum

- Why the strongest lines in the Solar spectrum are from singly-ionized Calcium ions in their ground states? Given that for every 1 Ca atom there are 500,000 H atoms.

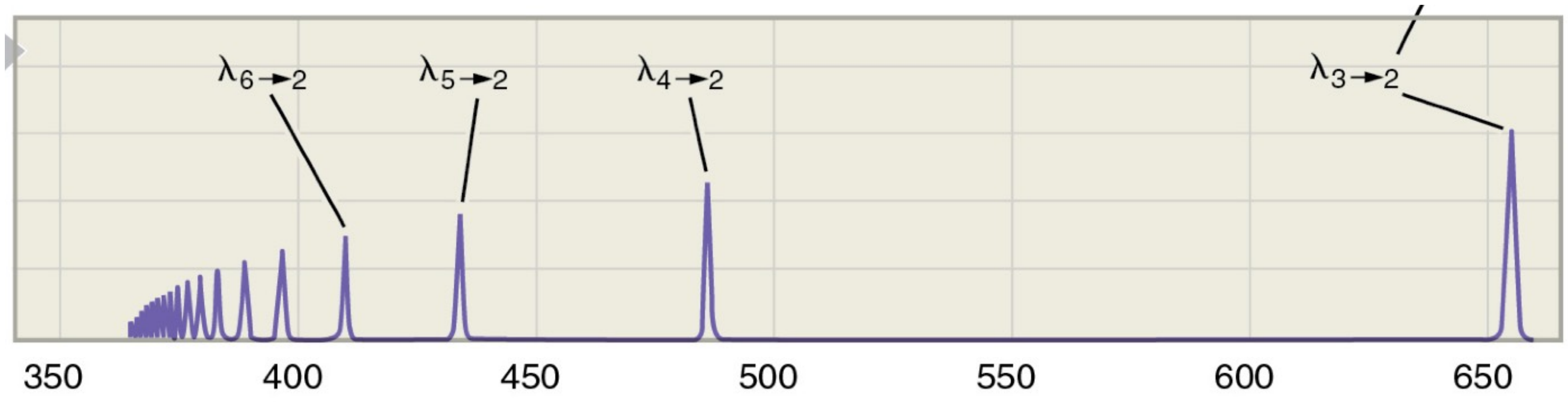


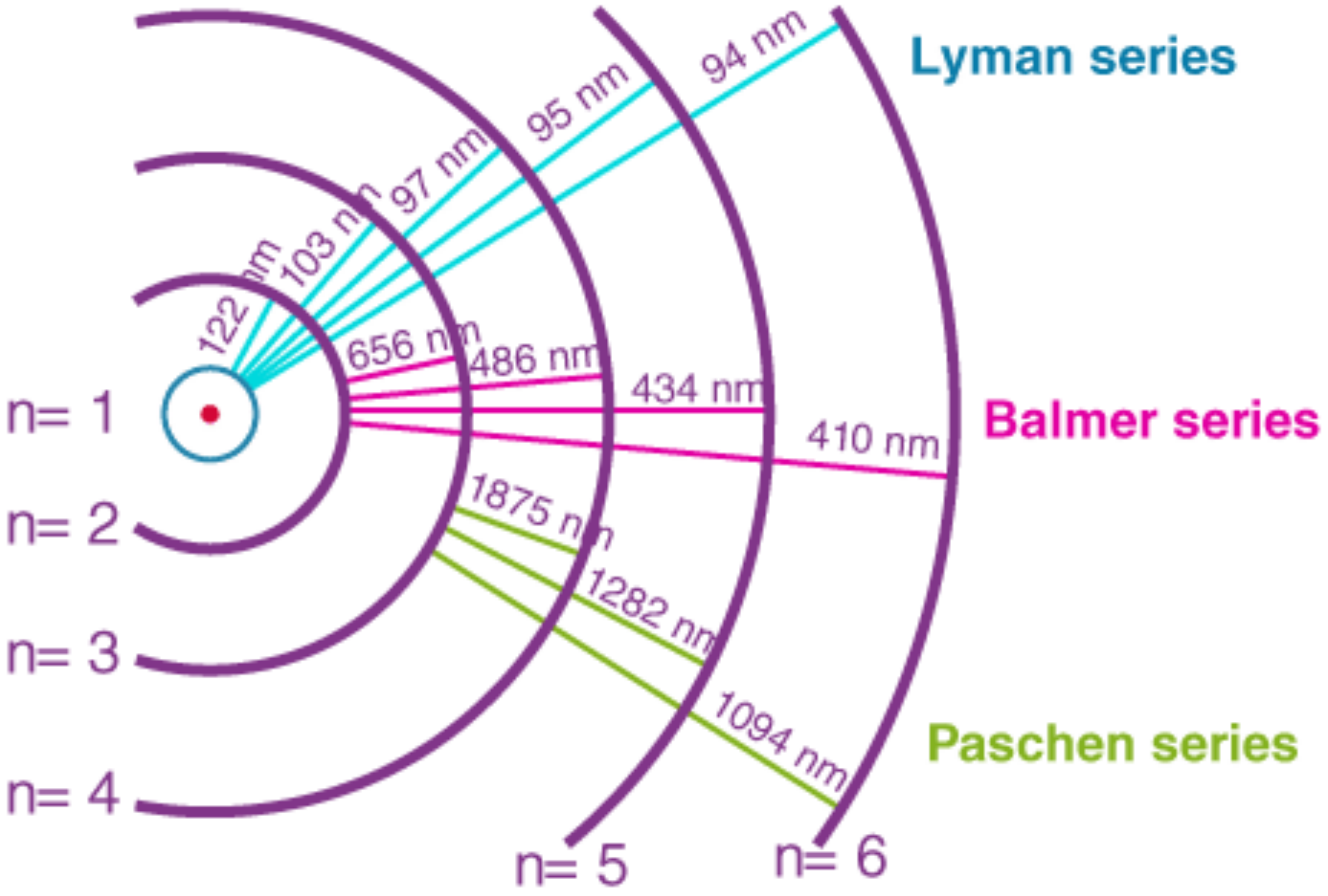
Why Hydrogen Balmer lines are strongest in A-type stars? (H-alpha, H-beta, H-gamma, H-delta, …)



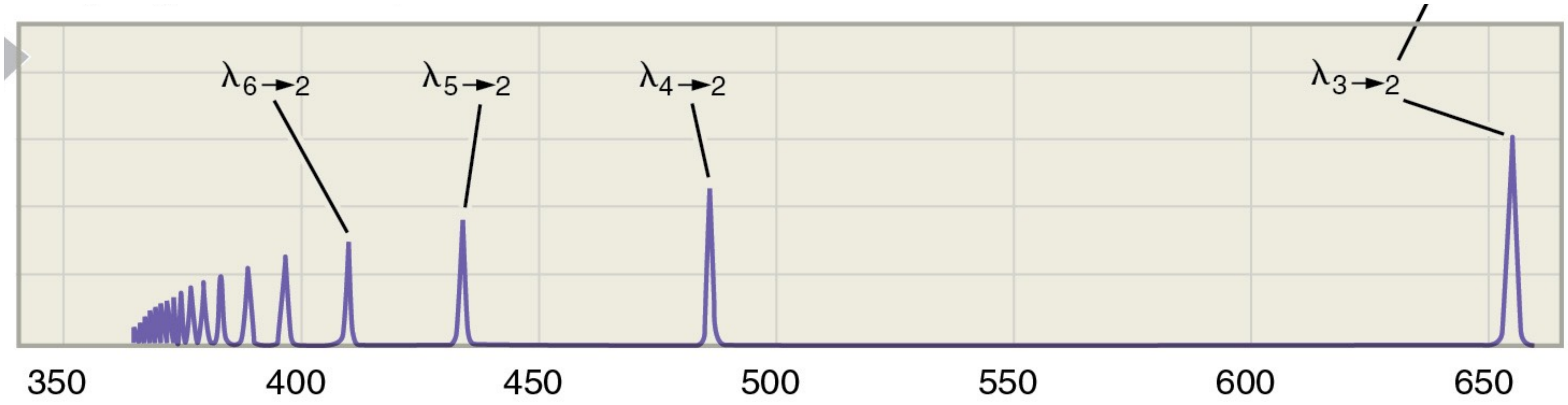
Why Hydrogen Balmer lines are strongest in A-type stars?

- What are needed to produce high opacity in the Hydrogen Balmer lines?
  - **Neutral hydrogen atoms excited to the first excited state (n=2)**





Calculating the Wavelengths of Hydrogen Emission Lines



$$\frac{1}{\lambda} = \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right)\frac{13.6\text{eV}}{hc}$$

n_{low} = quantum number of lower orbit

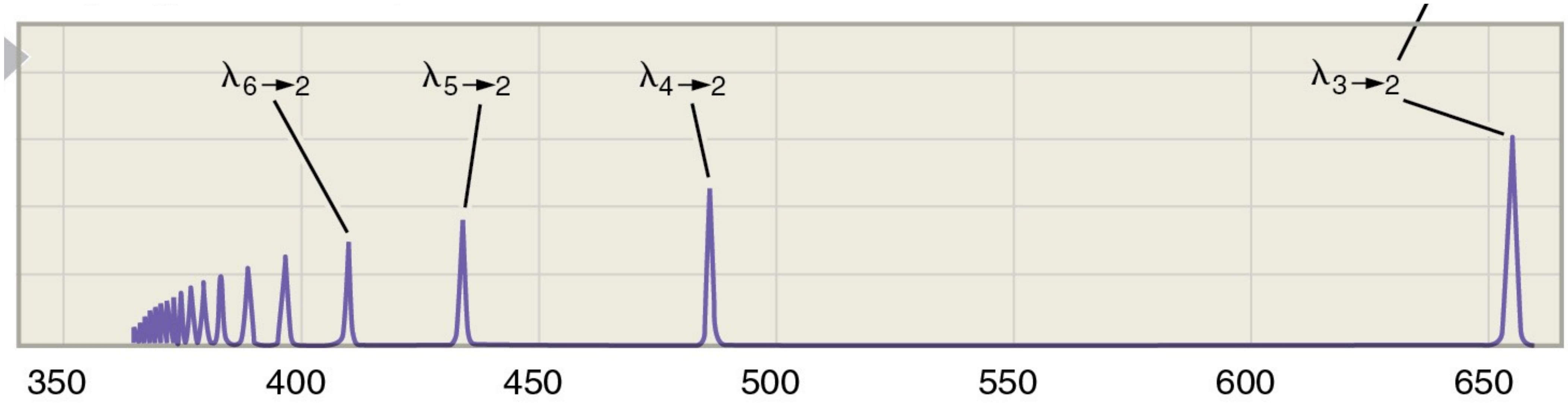
n_{high} = quantum number of higher orbit

λ = wavelength of emitted photon

- Electron-volt (eV) is the amount of kinetic energy gained by a single e- accelerating through an electric potential difference of one volt.

$$T = 11604\ \text{K}\left(\frac{E}{1\ \text{eV}}\right) \qquad \lambda = 1.24\ \mu\text{m}\left(\frac{E}{1\ \text{eV}}\right)$$

Ionization Energy of Hydrogen: 13.6 eV



$$\Delta E = 13.6\ \text{eV}\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right)$$

n_{low} = quantum number of lower orbit

n_{high} = quantum number of higher orbit

ΔE = energy required for the transition

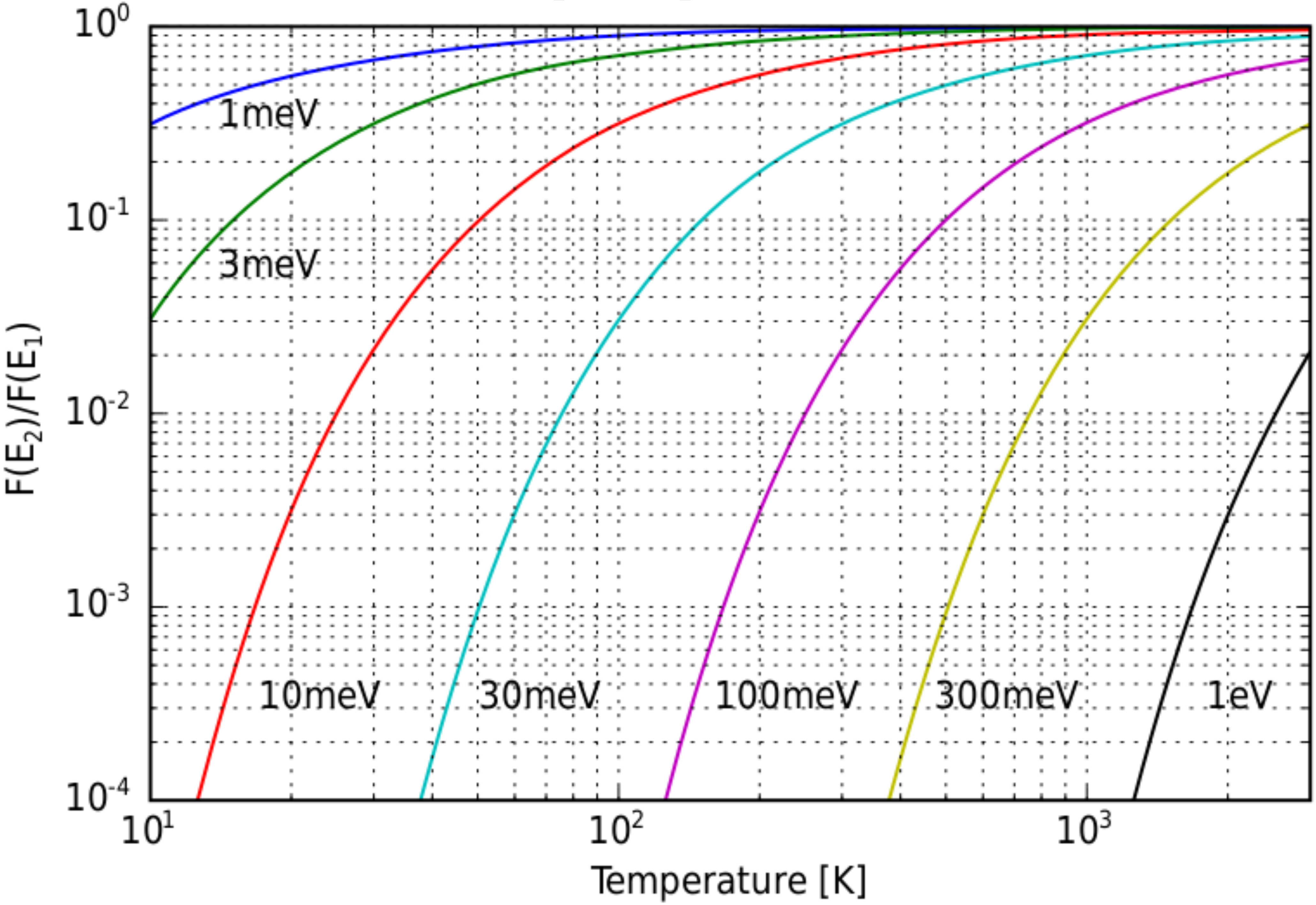
- To calculate the ionization energy, we can plug in $n_{\text{low}} = 1$ and n_{high} = infinity to the equation. What temperature does 13.6 eV correspond to?

$$T = 11604\ \text{K}\left(\frac{E}{1\ \text{eV}}\right)$$

The Excitation of Neutral Hydrogen to Higher Energy Levels

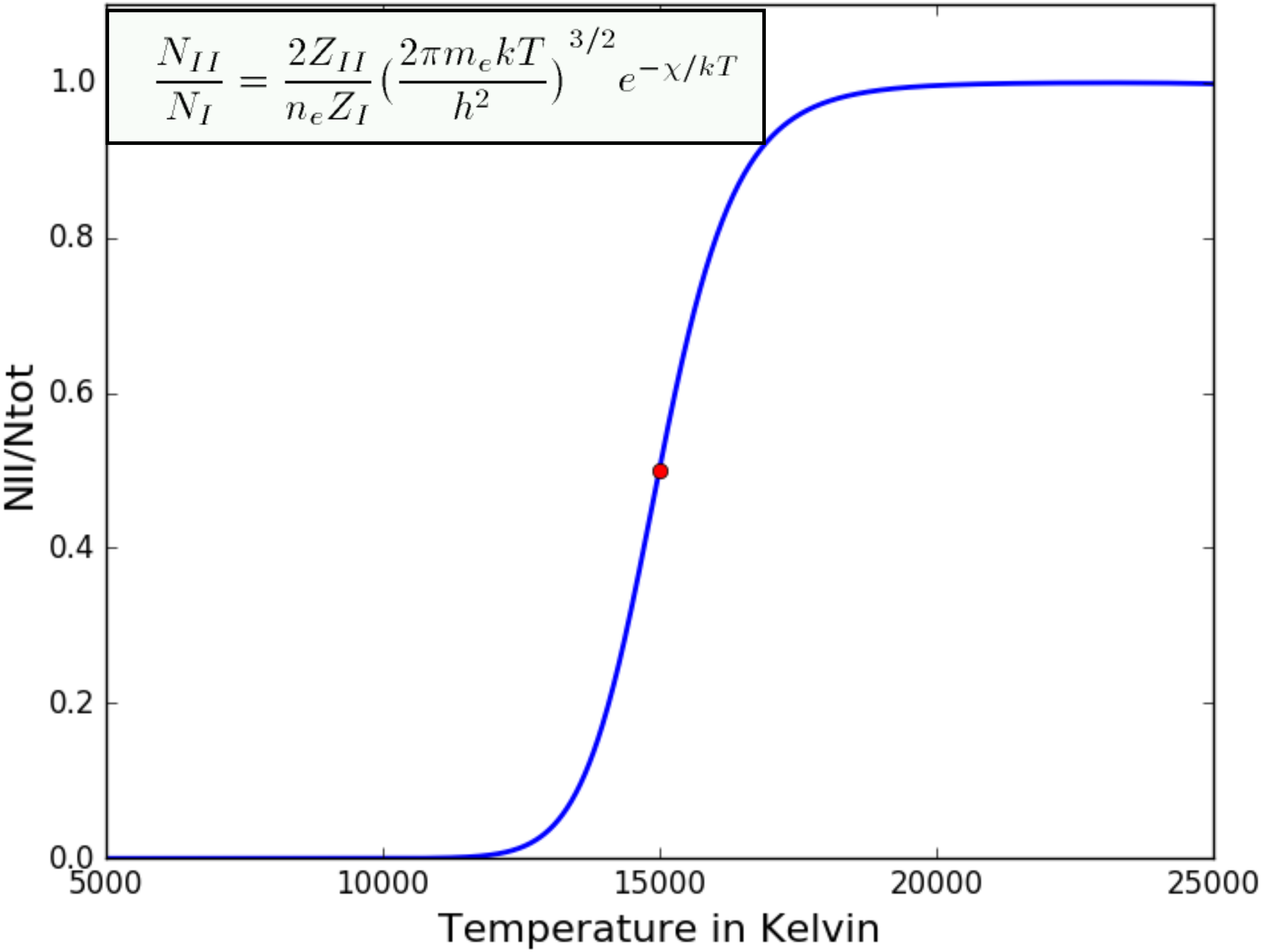
- As temperature increases, more and more remaining neutral hydrogen are excited to the n=2 state (**Excitation Ratio: Boltzmann Distribution**)

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT}$$



Ionization Equilibrium: Balance between ionization and recombination

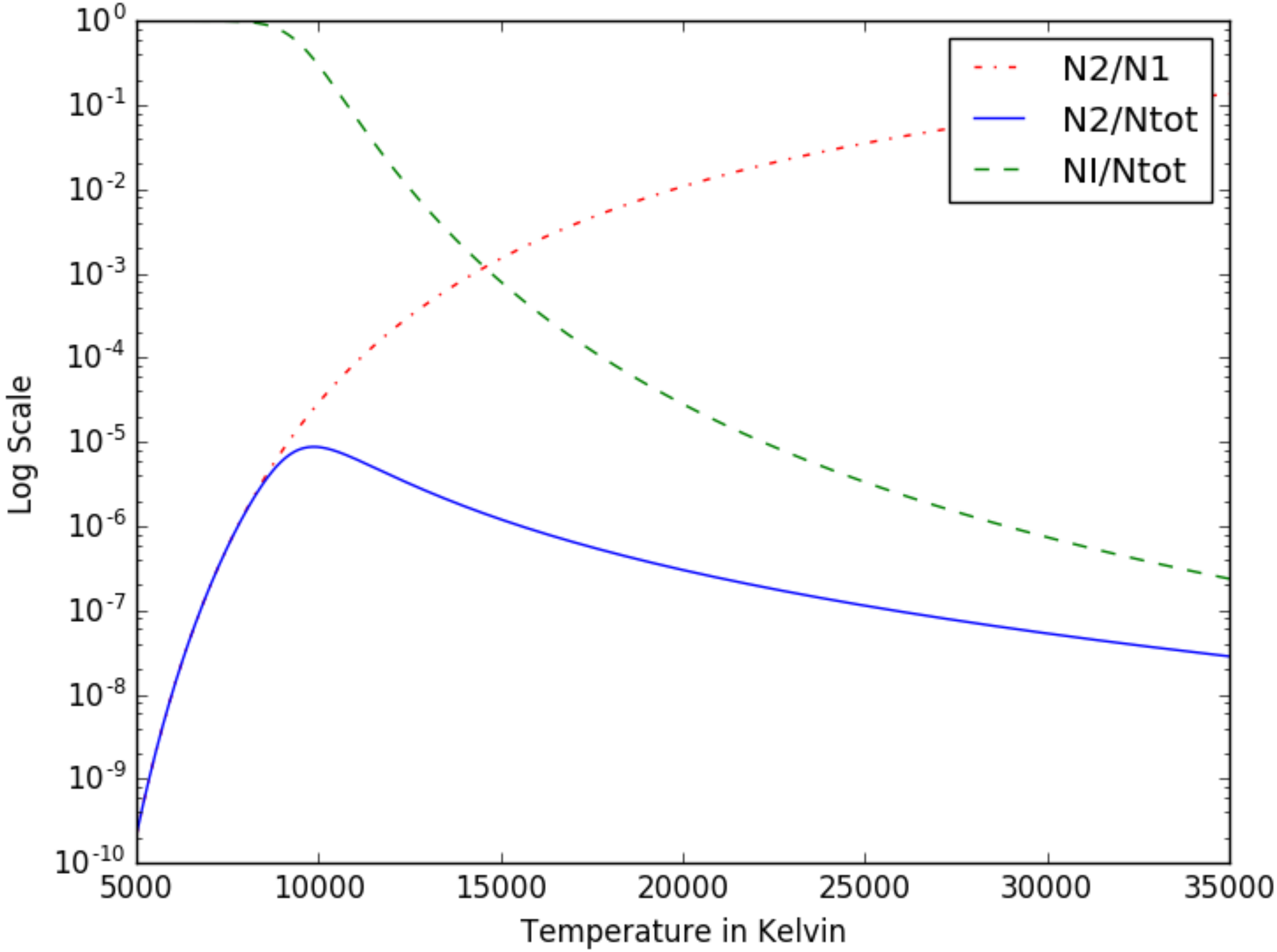
- As temperature increases, more and more hydrogen become ionized, so we are losing neutral hydrogen (Equilibrium calculated with the **Saha Equation**)
- But 15000 K is only 1.3 eV, which is 10x smaller than the ionization energy (13.6 eV)



$$\frac{N_{II}}{N_I} = \frac{2Z_{II}}{n_e Z_I} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi/kT}$$

The Combined Effect of Ionization and Excitation

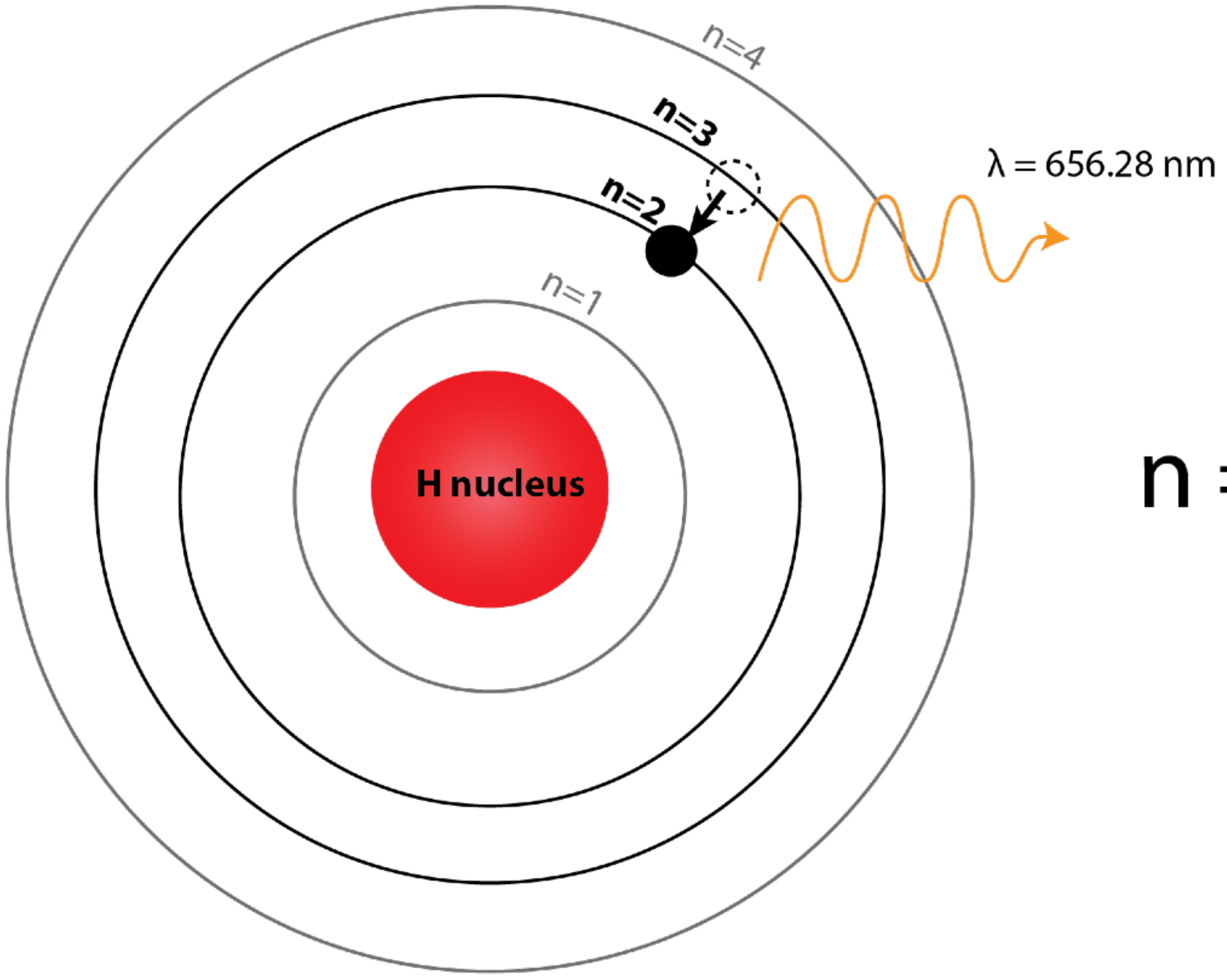
- Causes a peak in the number of desired species that creates the H Balmer lines: neutral hydrogen (H I) at the first excited state (n=2)



Solar Atmosphere: Understanding the Absorption Line Spectrum

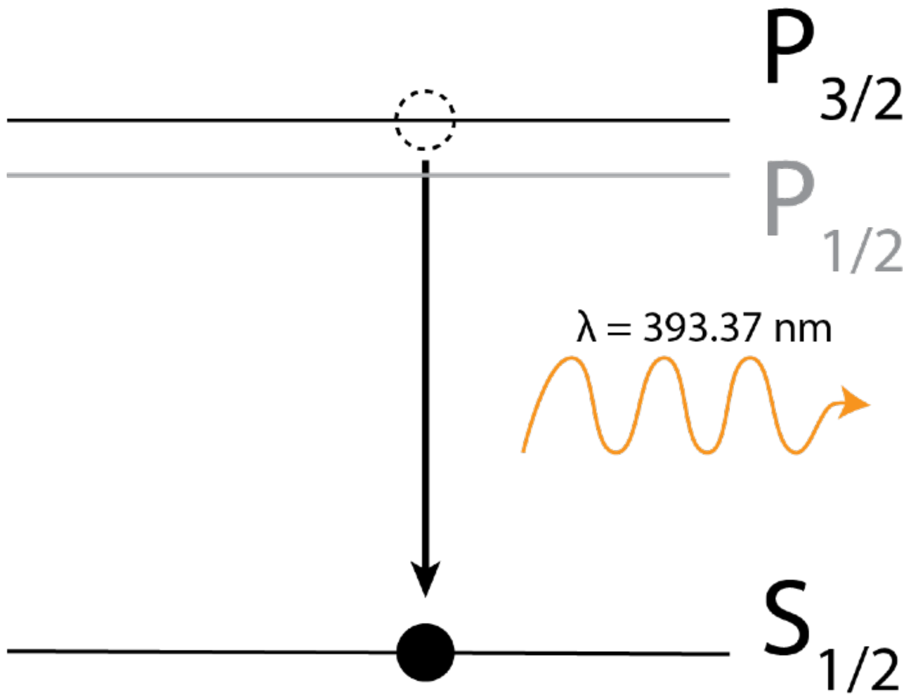
- Why the strongest lines in the Solar spectrum are from singly-ionized Calcium ions in their ground states? Given that for every 1 Ca atom there are 500,000 H atoms.

H atom



n = 4

Ca$^+$ ion



Calcimu II K

Hydrogen-alpha
Naming: Balmer lines

Dependence of spectral line strengths on temperature

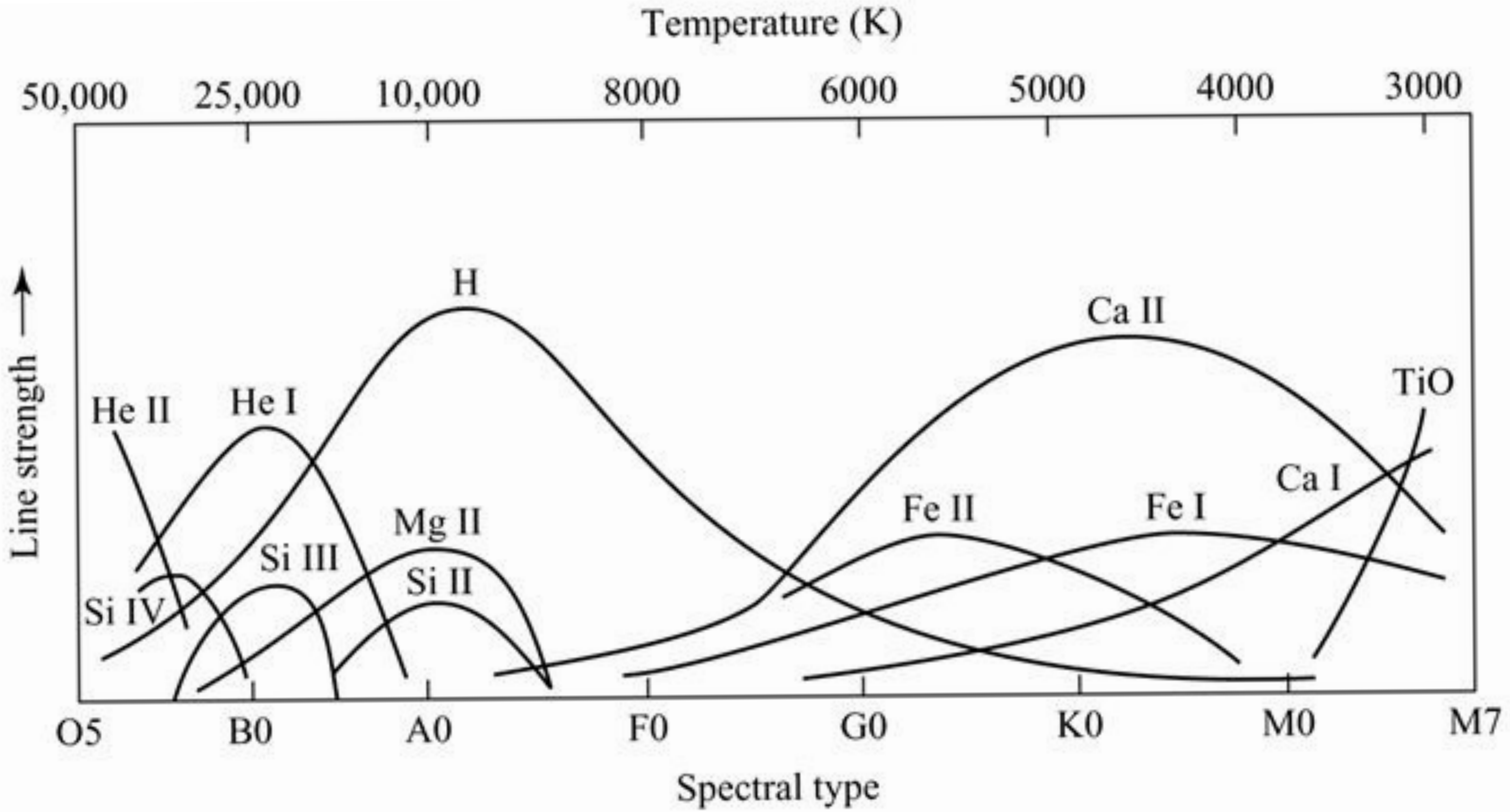
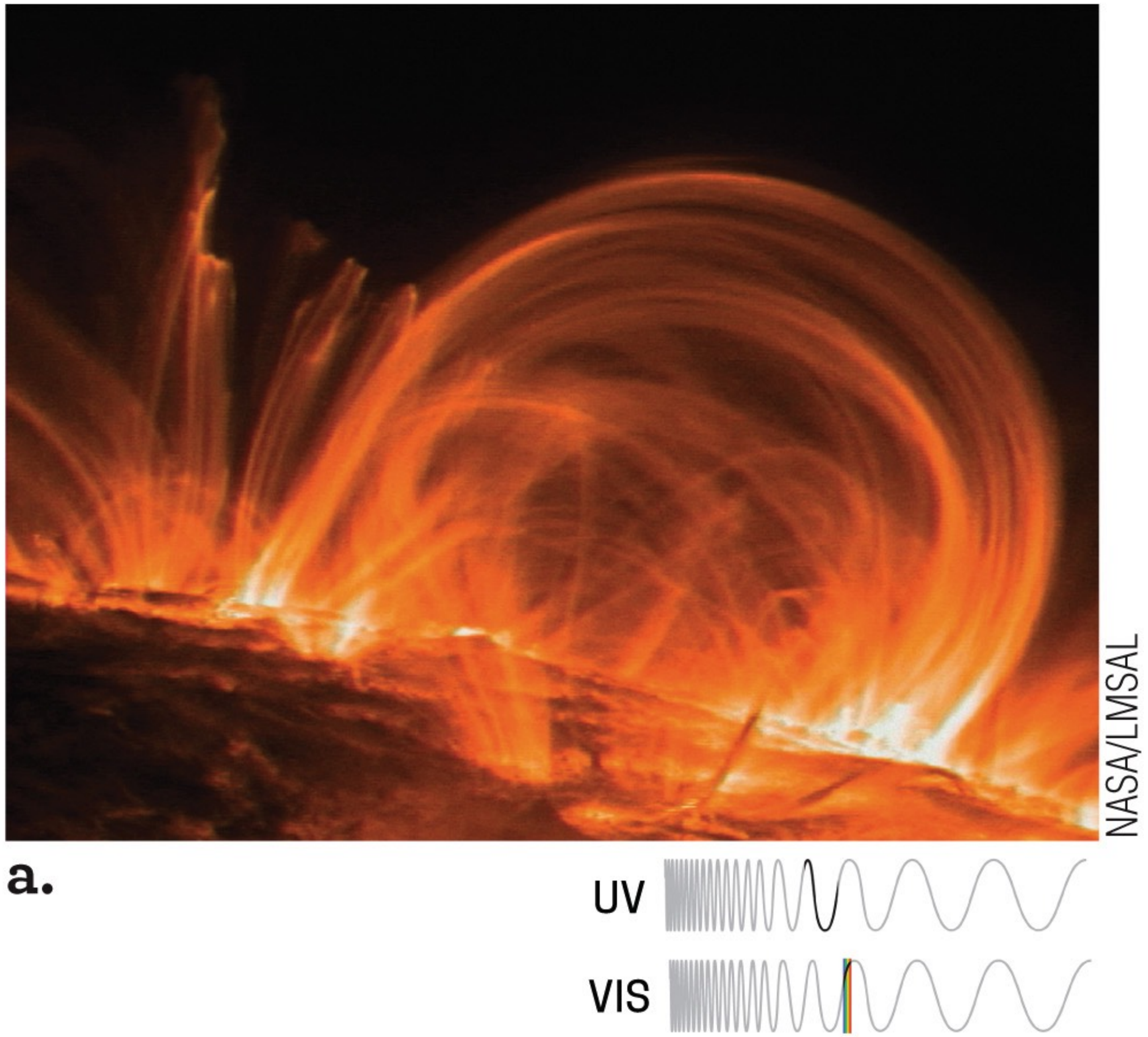


Figure 8.11 Carroll & Ostlie

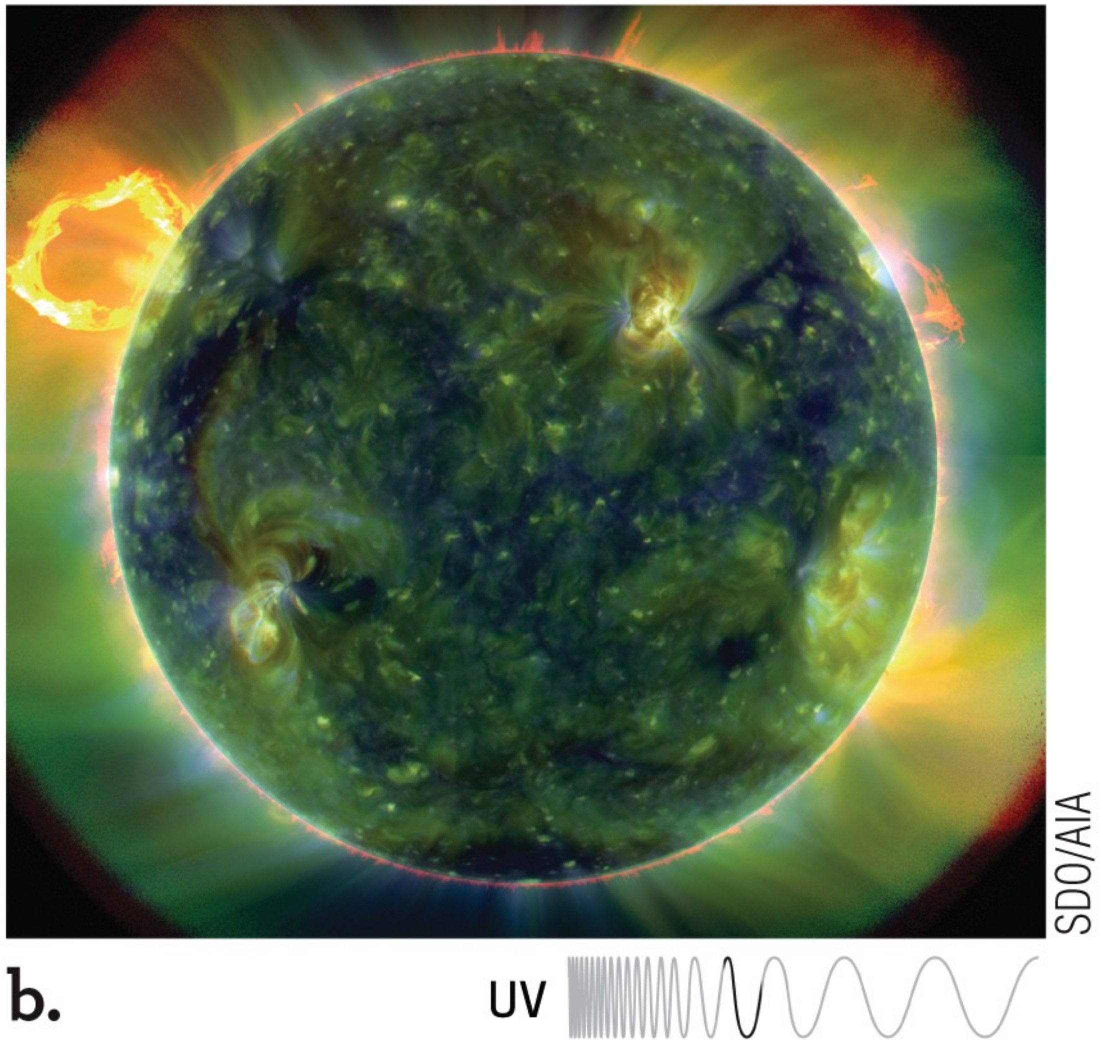
The Atmospheres of the Sun is Very Active

Solar Activity

- The Sun has **active regions** where loops of material and explosions fling particles into the Solar System. Active regions are associated with the Sun’s magnetic field.


a.

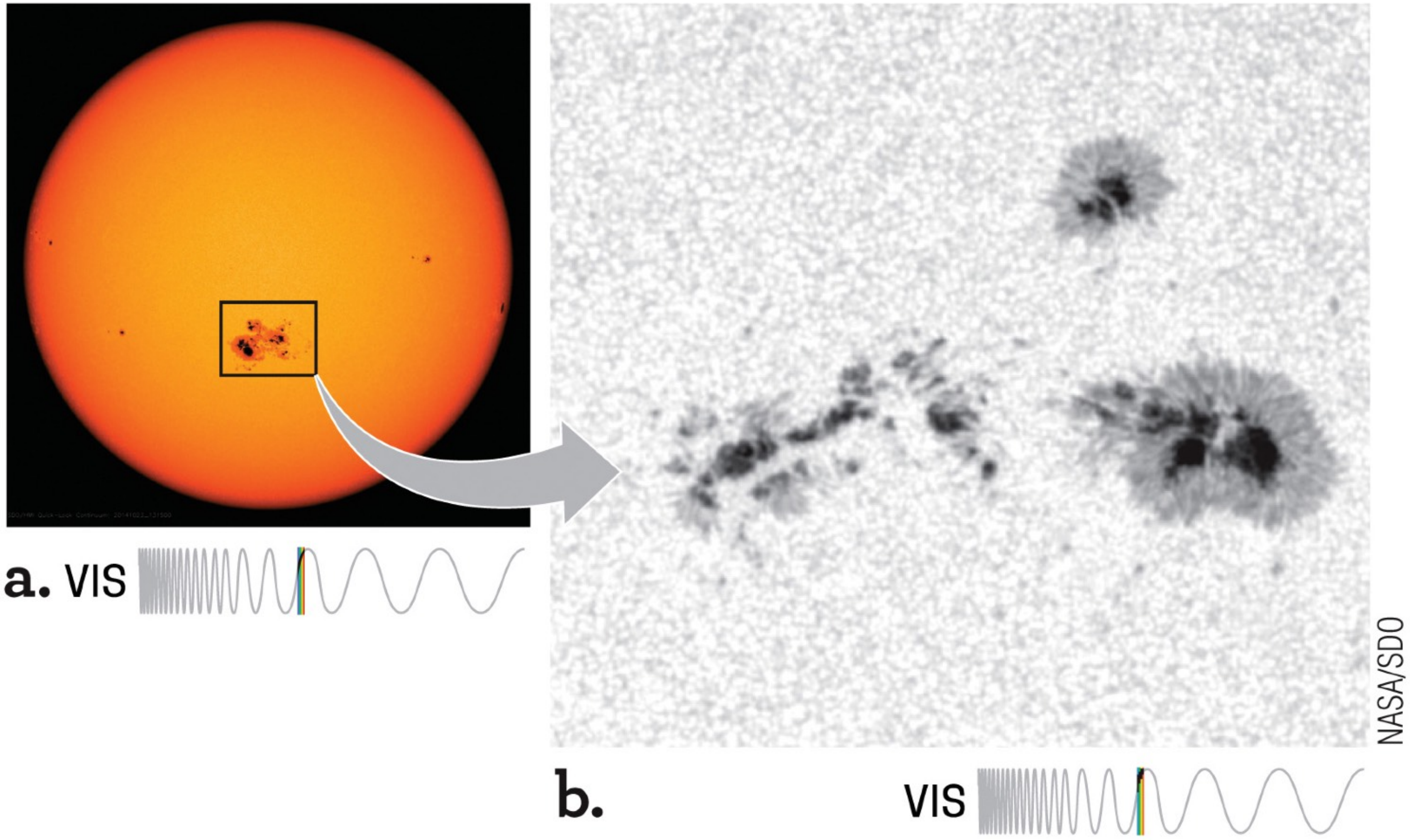
NASA/LMSAL


b.

SDO/AIA

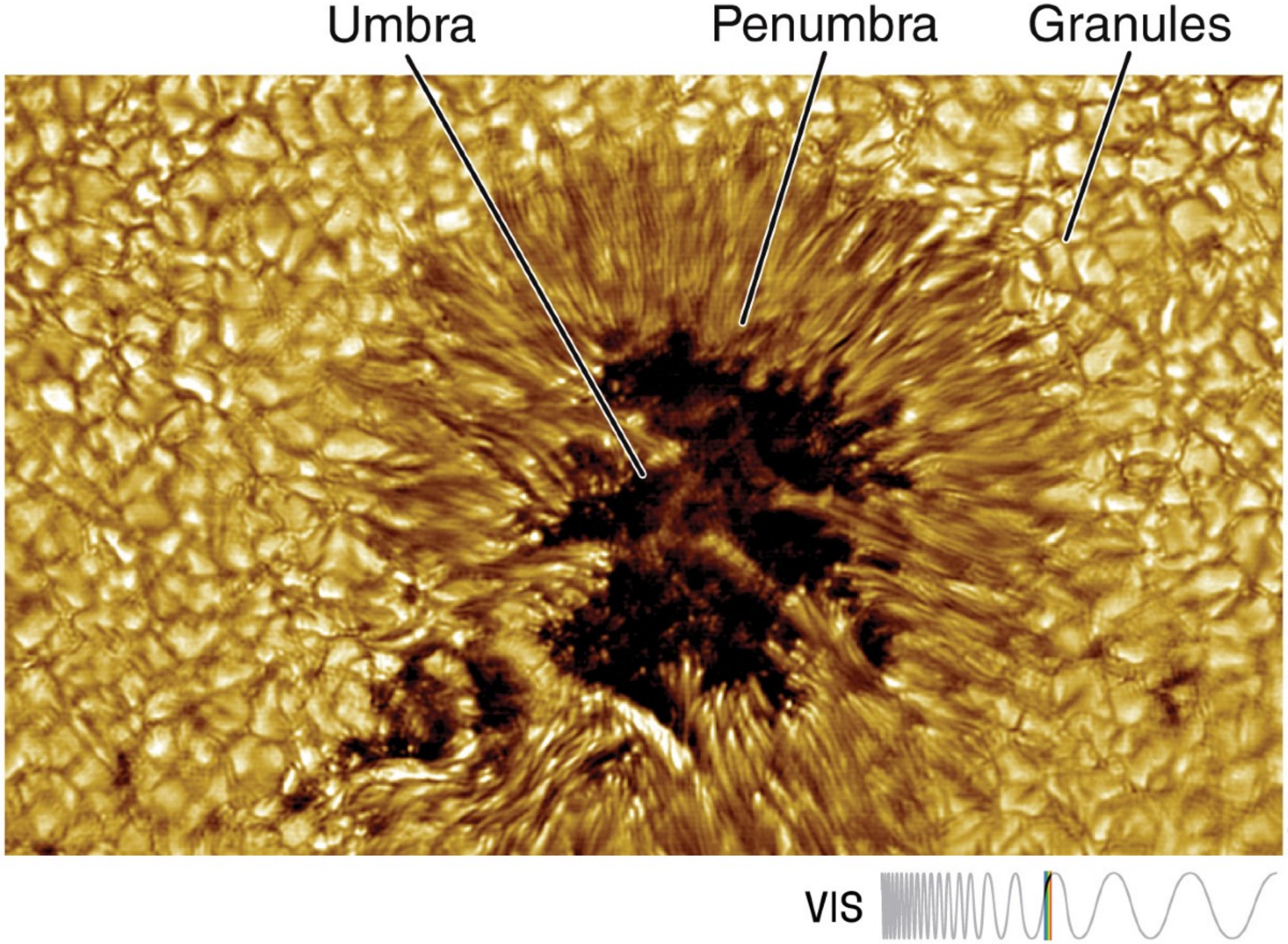
Solar Activity: Sunspots

- The Sun is made of gas, so it undergoes **differential rotation**: It rotates faster at the equator than at the poles.
- The magnetic field goes through this material, so because of differential rotation, it gets tangled.
- The areas where the magnetic field gets knotted up are **sunspots**.



Anatomy of a Sunspot

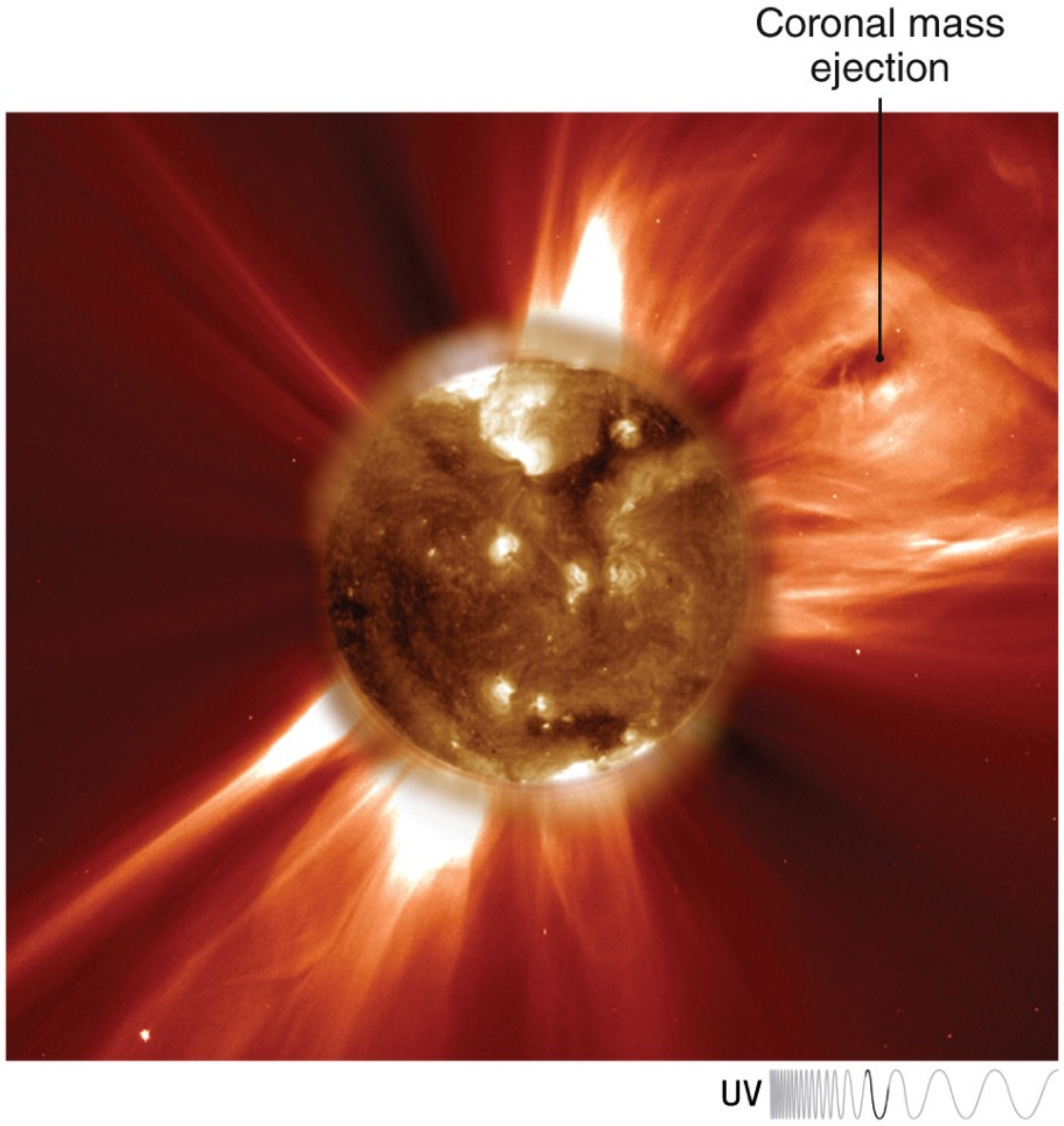
- **Sunspots**: cooler areas in the photosphere
- Sunspot structure: dark inner **umbra** with surrounding **penumbra**
- Sunspots are caused by tangled magnetic fields that trap gas at the surface, prohibiting them from sinking and warming (**impeding convection**).
- Sunspots occur in pairs connected by a magnetic loop.
- Sunspots last approximately 2–11 days.



NOIRLab. https://noirlab.edu/public/images/noao9808a/. https://creativecommons.org/licenses/by/4.0/

Flares and Prominences

- **Solar prominences**: hot rising gas in the chromosphere constrained by magnetic fields
- **Solar flares** and **coronal mass ejections** are highly energetic, violent bursts and eruptions.
- They correlate with sunspot positions.



SOHO Consortium, ESA, NASA

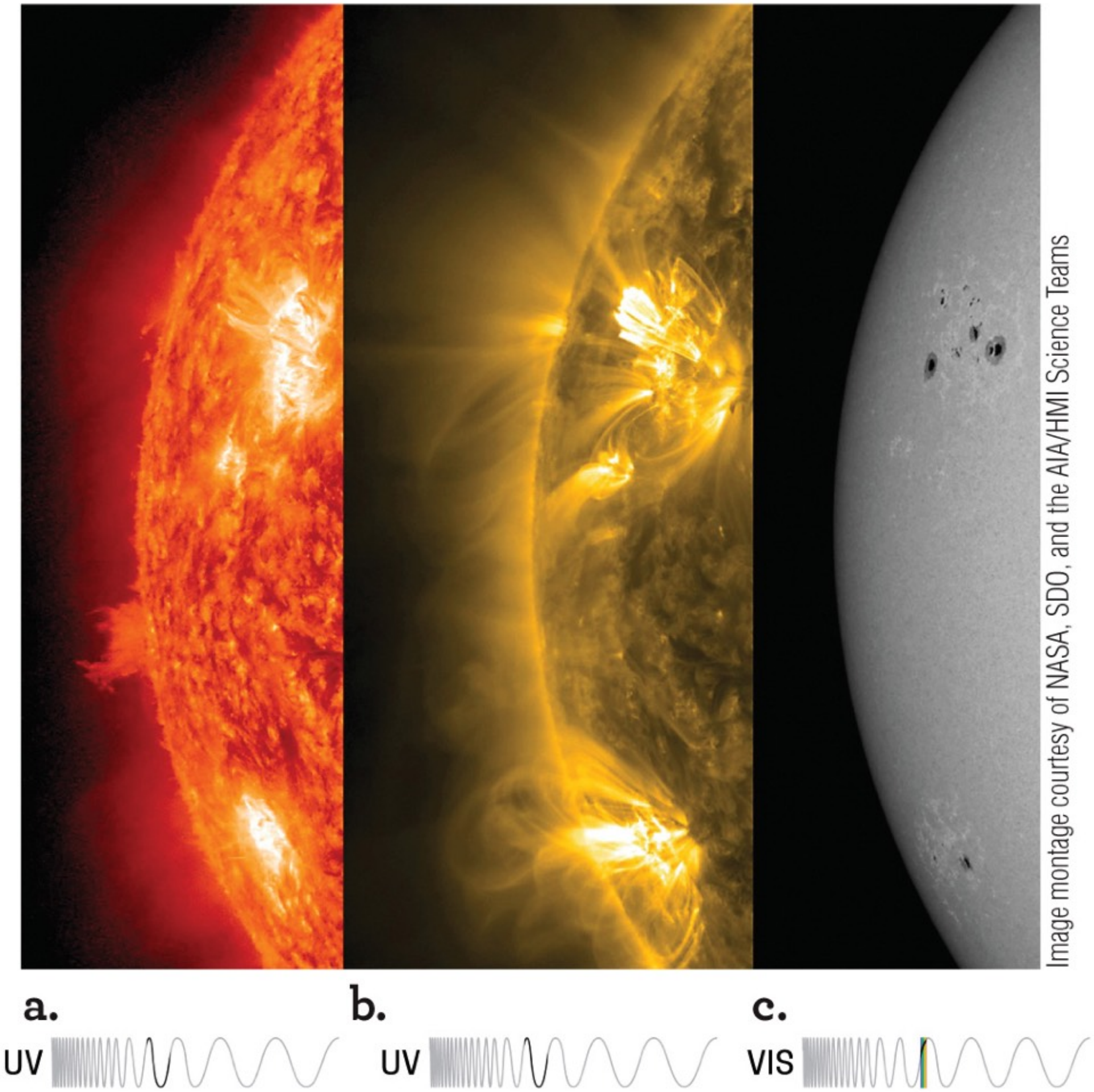
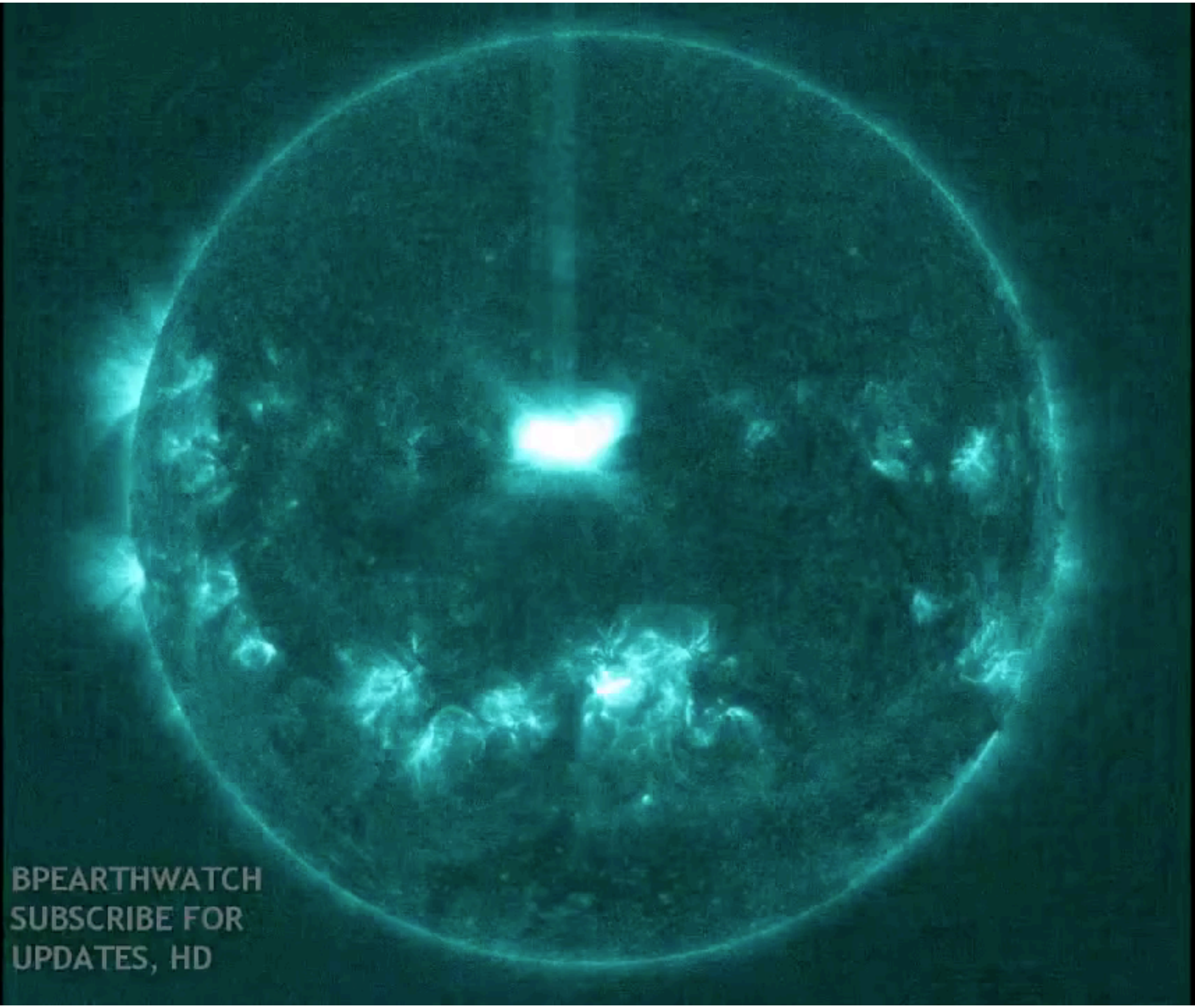
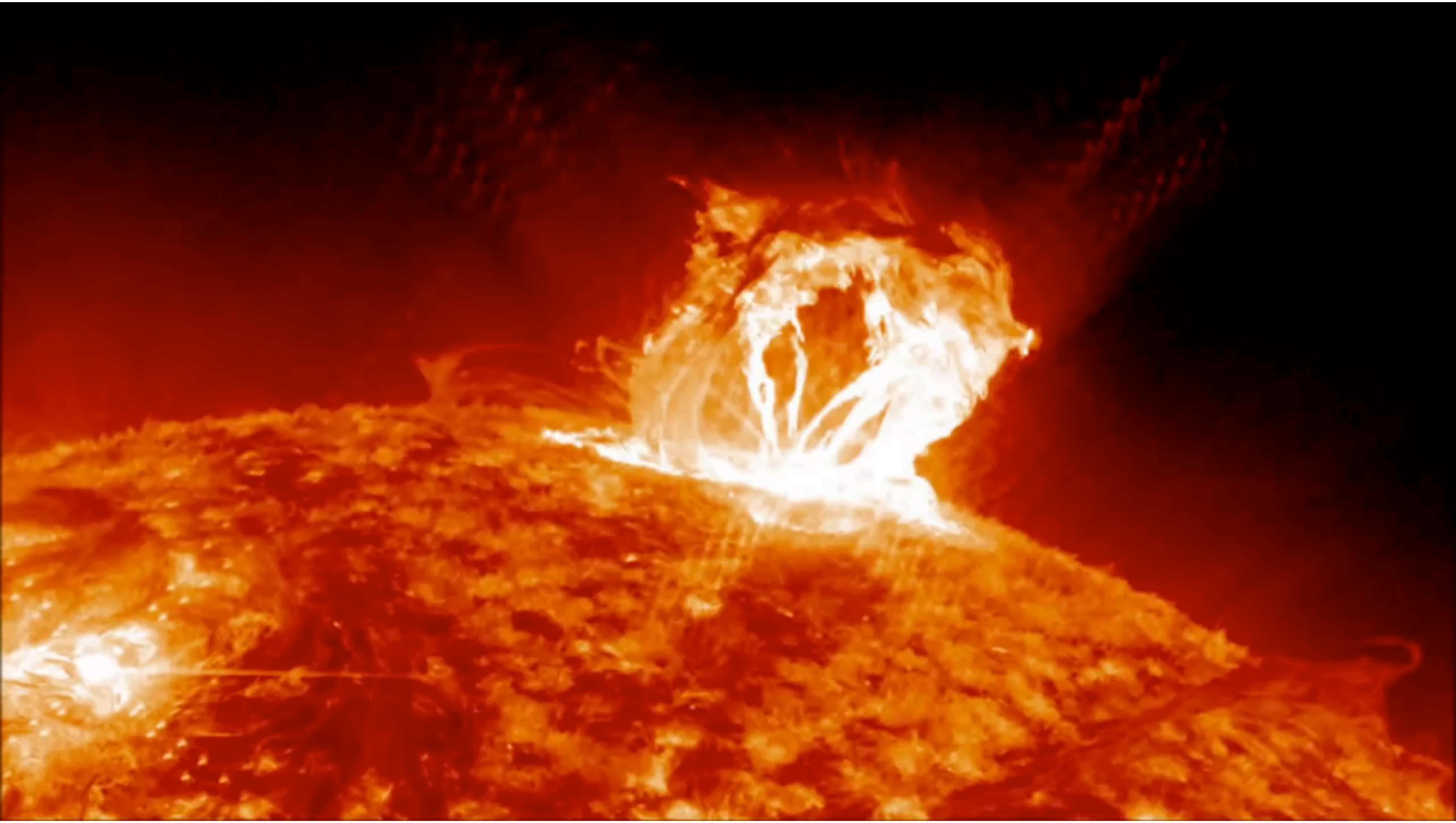


Image montage courtesy of NASA, SDO, and the AIA/HMI Science Teams

- **Solar flares:** powered by the sudden release of magnetic energy stored in the corona. Last only **a few minutes to a few hours**

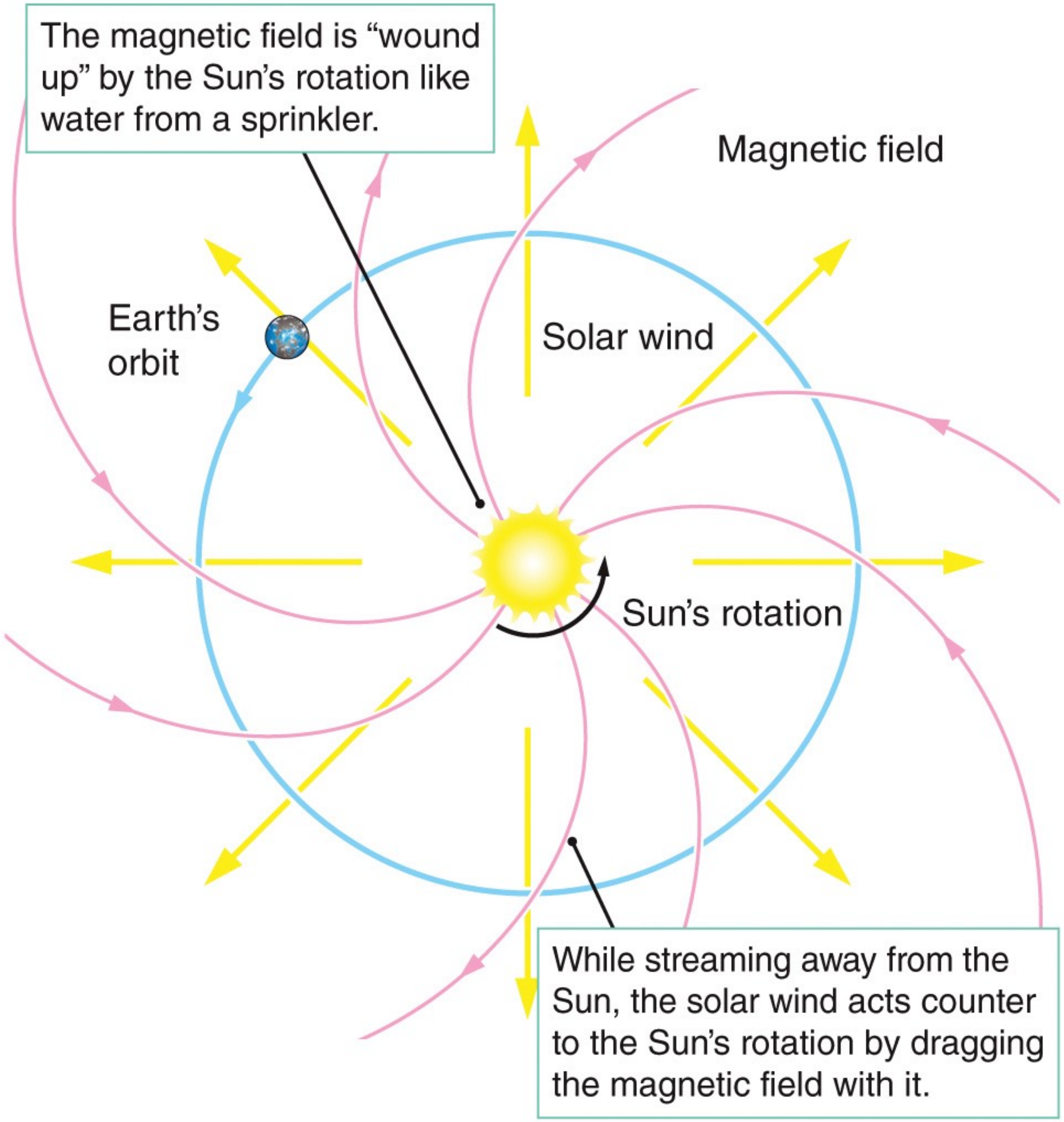


- **Solar flares** are often followed by **coronal mass ejections (CME)**
- Hot plasma ejected at speeds up to 1,500 km/s
- Powerful bursts of energetic particles



Solar Activity Is Caused by Magnetic Effects

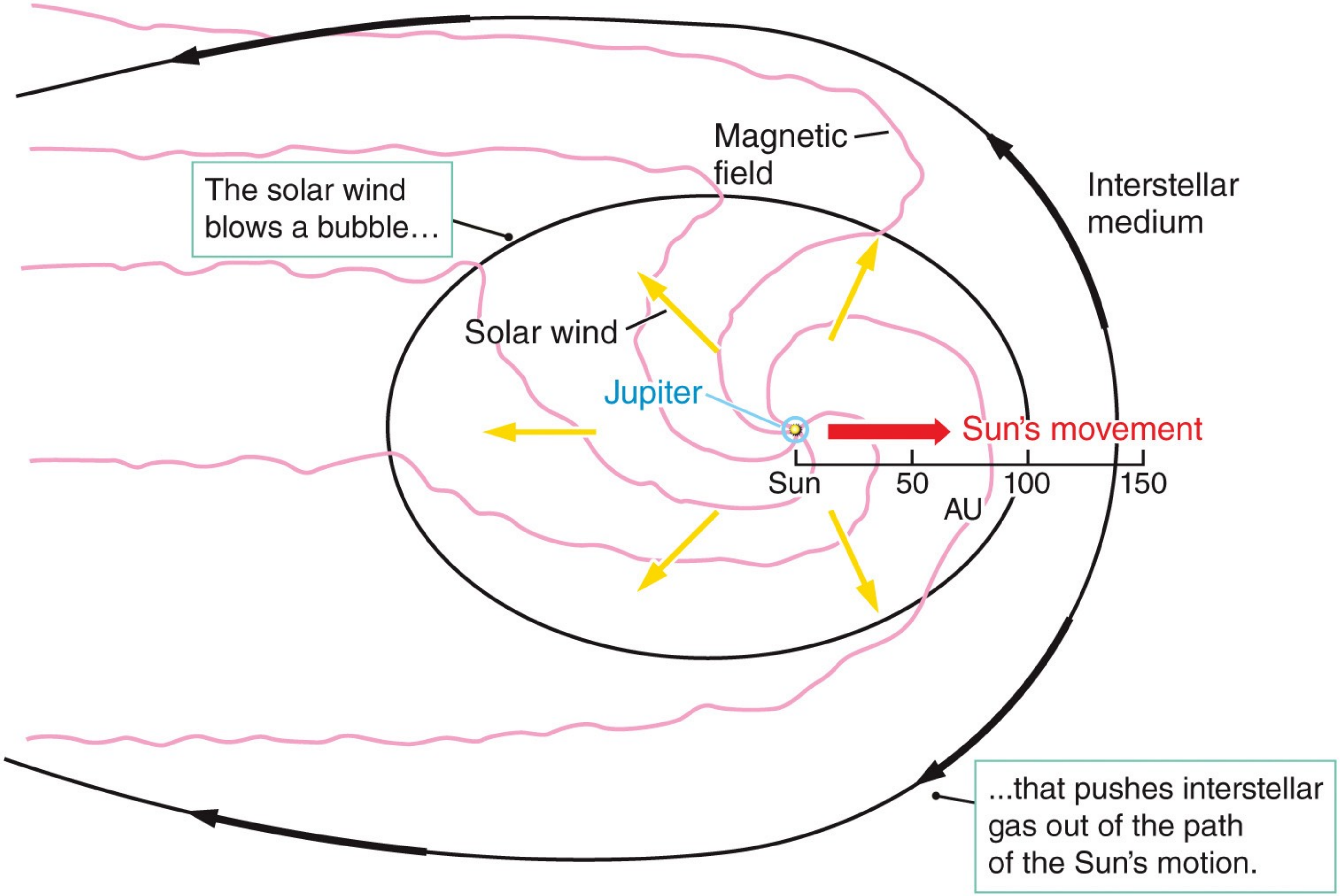
- The Sun's magnetic field structures the atmosphere and causes solar activity.
- The lower atmosphere is made of coronal loops resulting from magnetic flux tubes.
- **Coronal holes** are where magnetic field lines extend away from the Sun.
- The **solar wind** is made up of charged particles that flow away from the Sun through coronal holes.


The magnetic field is "wound up" by the Sun's rotation like water from a sprinkler.

While streaming away from the Sun, the solar wind acts counter to the Sun's rotation by dragging the magnetic field with it.

Solar Wind

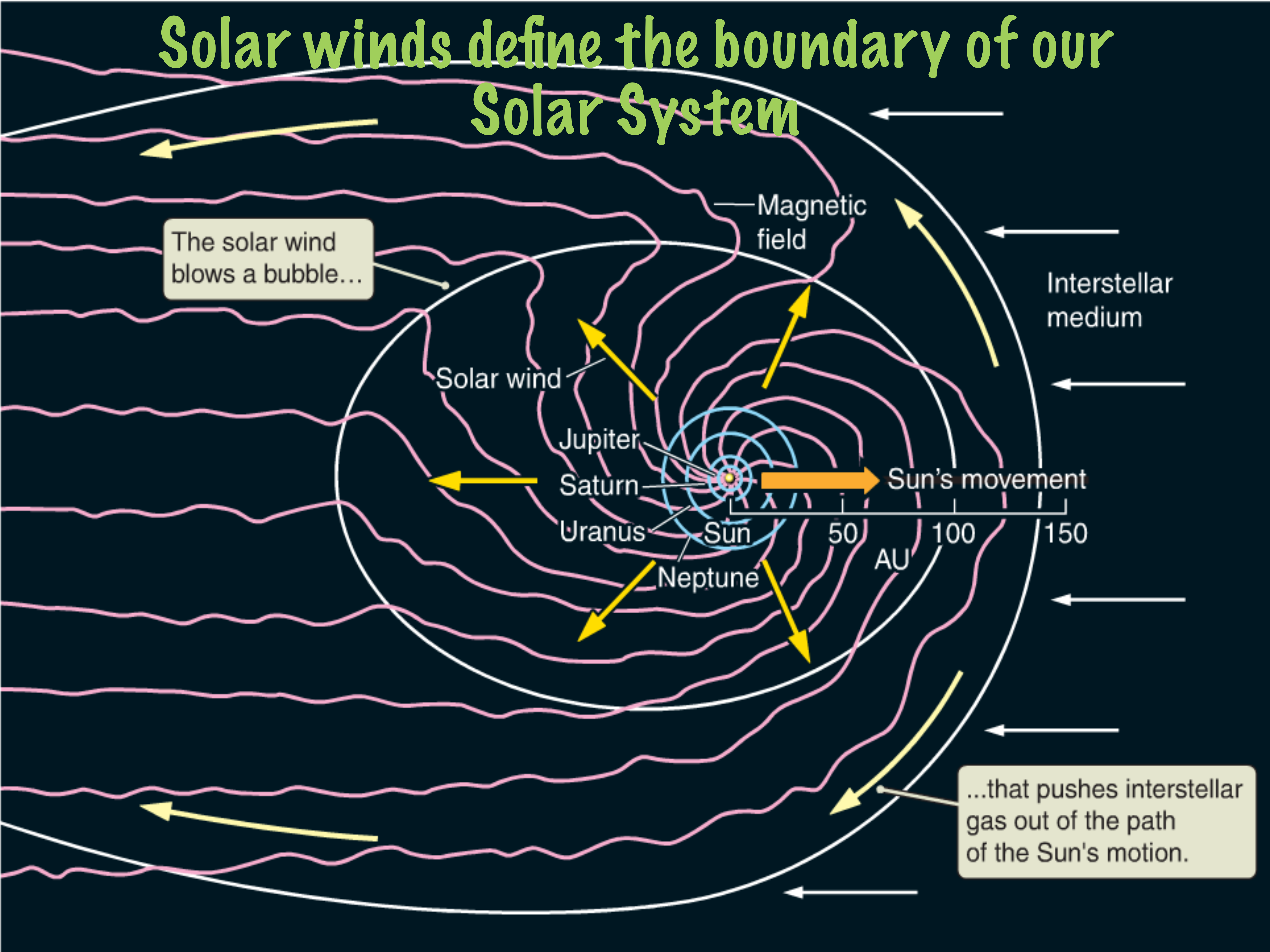
- The solar wind blows the tails of comets away from the Sun.
- The solar wind powers auroral displays on planets.
- The solar wind interacts with the **interstellar medium**, pushing it out of the way.



The solar wind blows a bubble…

...that pushes interstellar gas out of the path of the Sun's motion.

Solar winds define the boundary of our Solar System



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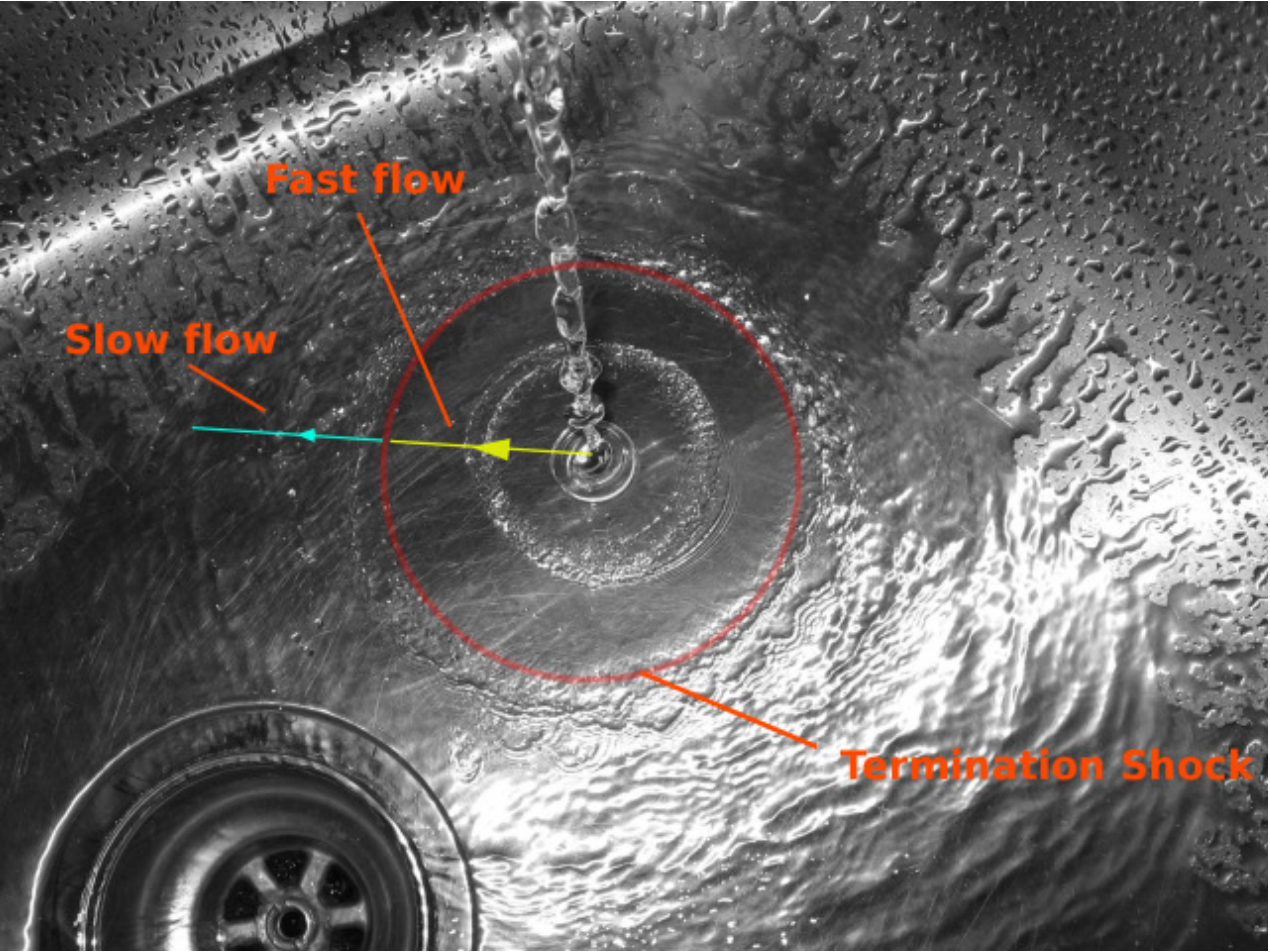
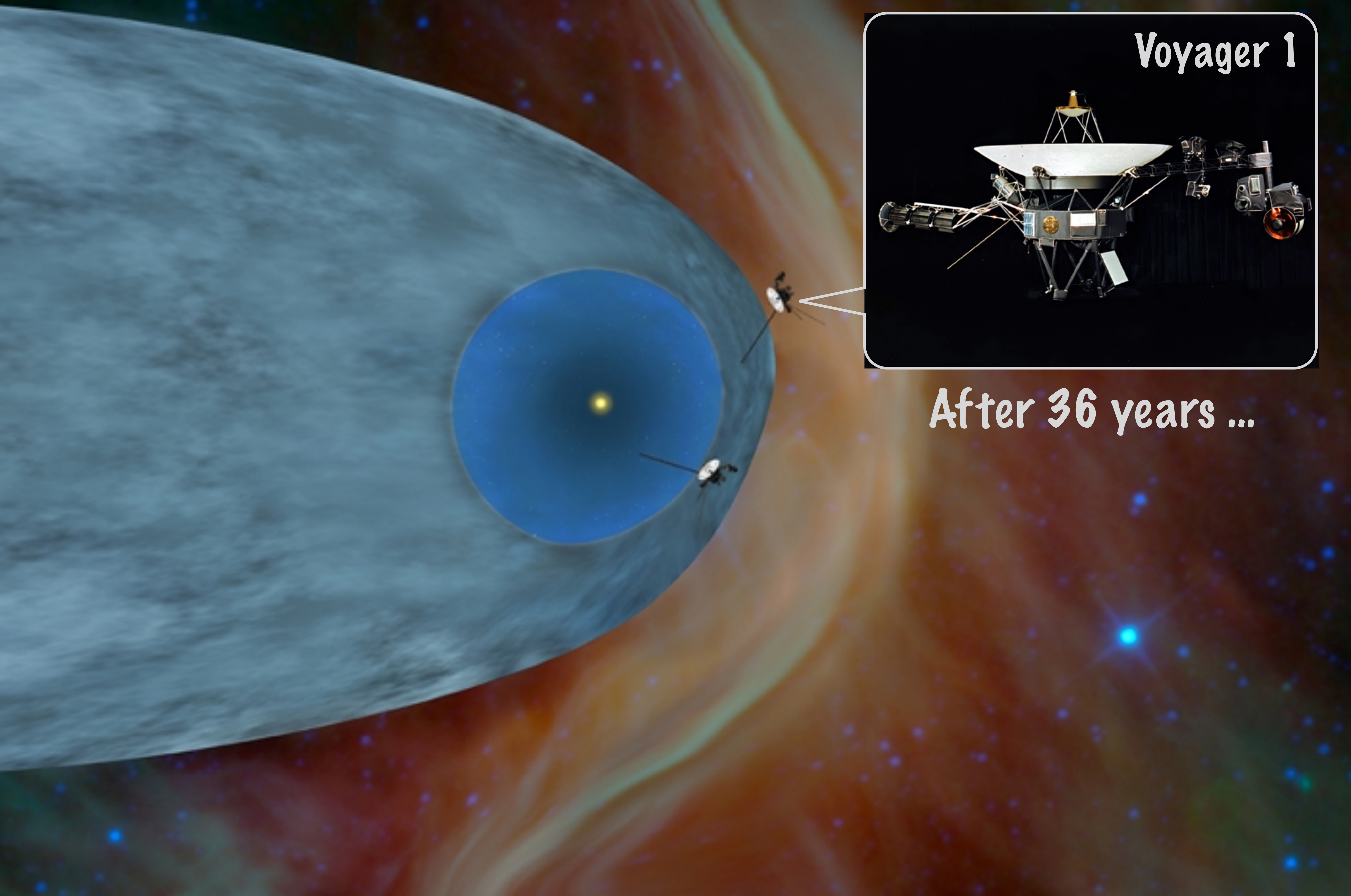

Fast flow
Slow flow
Termination Shock

Voyagers Crossing the Boundary

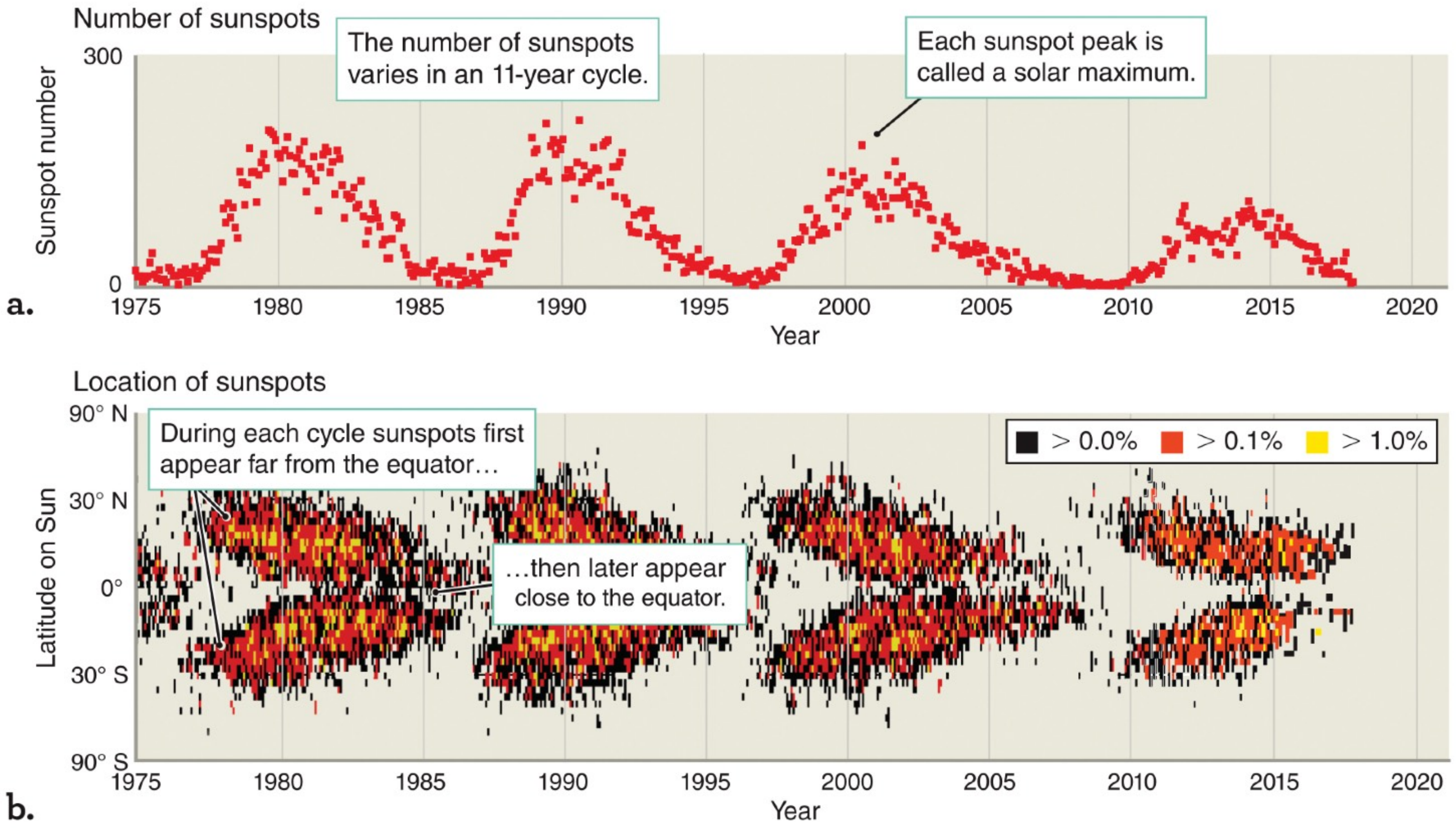


After 36 years ...

Sunspot Cycles

The Sunspot Cycle

- The Sun shows an approximate 11-year **sunspot cycle** (part of a 22-year magnetic cycle).
- **Solar maxima**: most sunspots and activity. **On Earth, the intensity of sunlight increases by 0.1% during solar maxima when compared to minima.**
- The **Maunder minimum** showed a distinct lack of sunspots between 1645 and 1715.



The number of sunspots varies in an 11-year cycle.

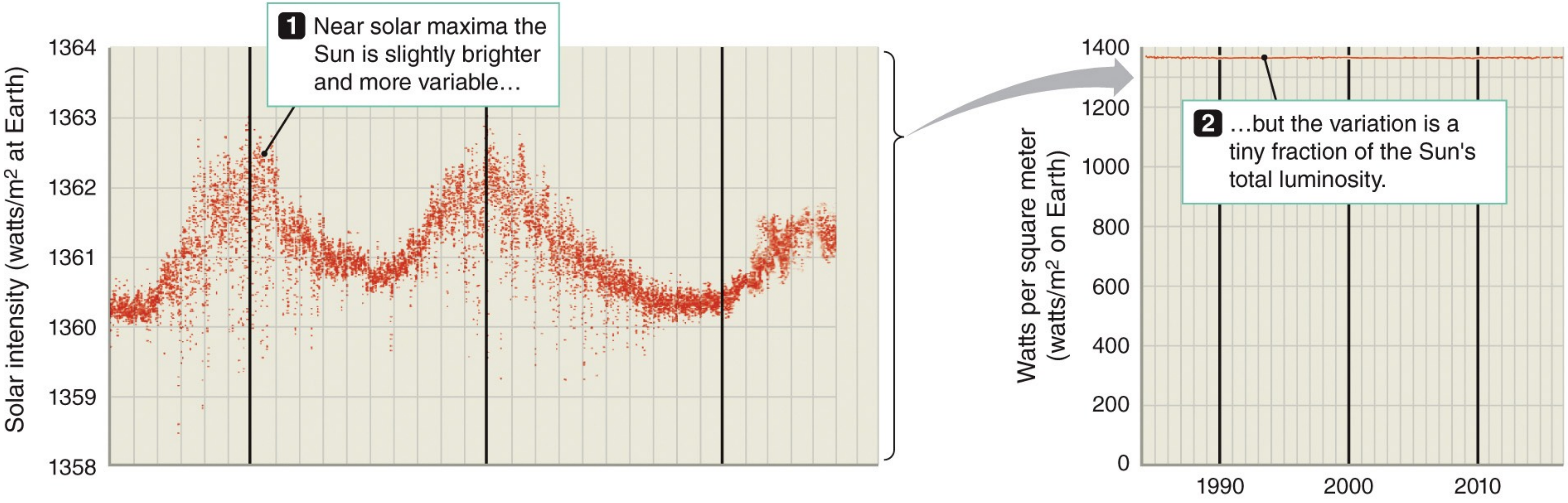
Each sunspot peak is called a solar maximum.

During each cycle sunspots first appear far from the equator…

…then later appear close to the equator.

The Sunspot Cycle

- **Solar maxima**: most sunspots and activity. On Earth, the intensity of sunlight **increases by 0.1%** during solar maxima compared to solar minima.

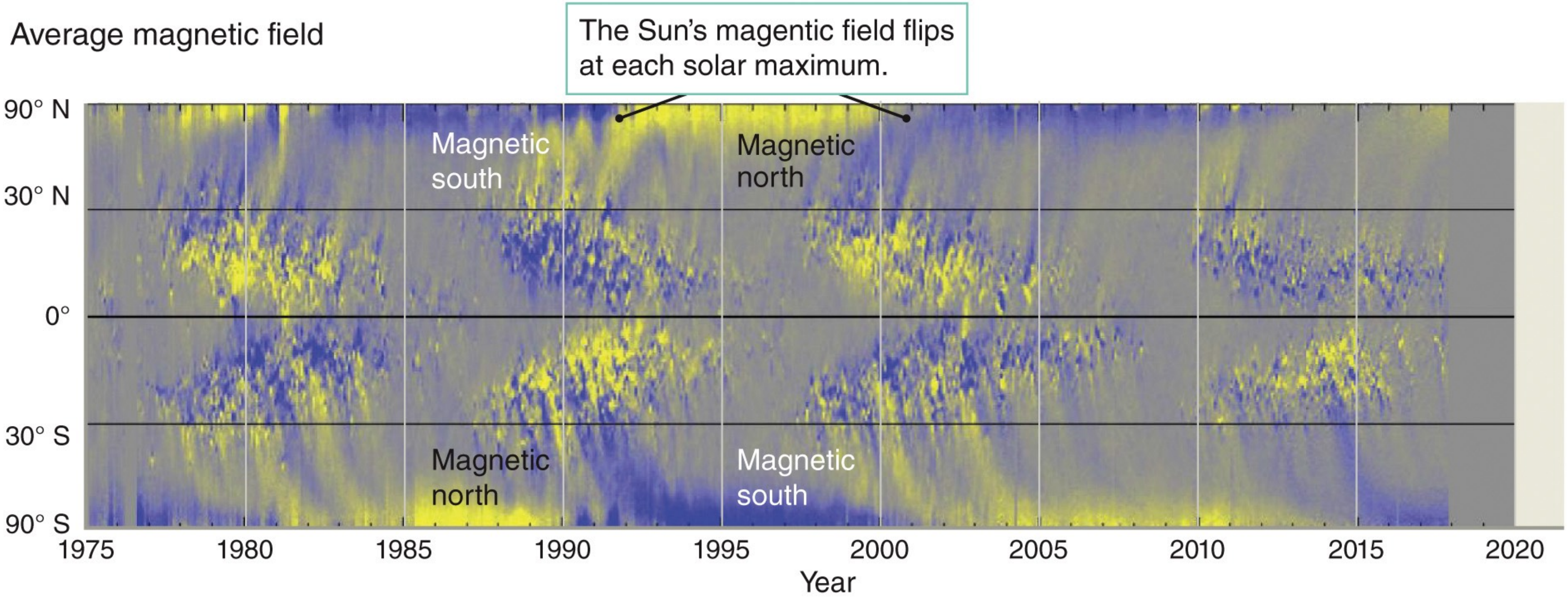


1 Near solar maxima the Sun is slightly brighter and more variable…

2 …but the variation is a tiny fraction of the Sun's total luminosity.

Sunspot Cycles Caused by Global Magnetic Field Flips

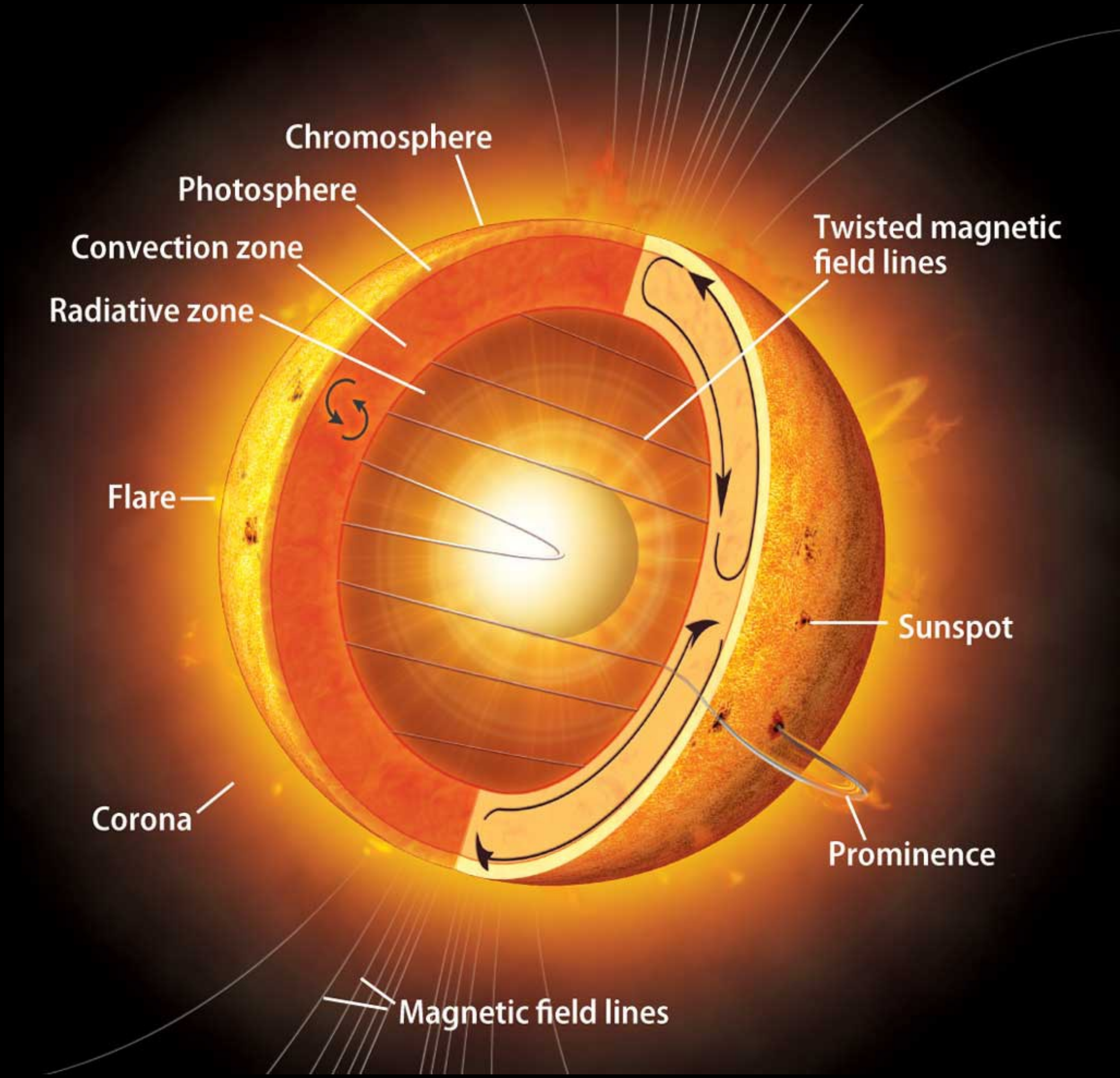
- The Sun’s magnetic field flips every 11 years, during the maximum of the sunspot cycle.
- Sunspots come in pairs. During one cycle, the south magnetic pole sunspot will lead, but during the next cycle, the north magnetic pole sunspot will lead.



The Sun’s magentic field flips at each solar maximum.

Chap 14: Our Star - The Sun: Key Concepts

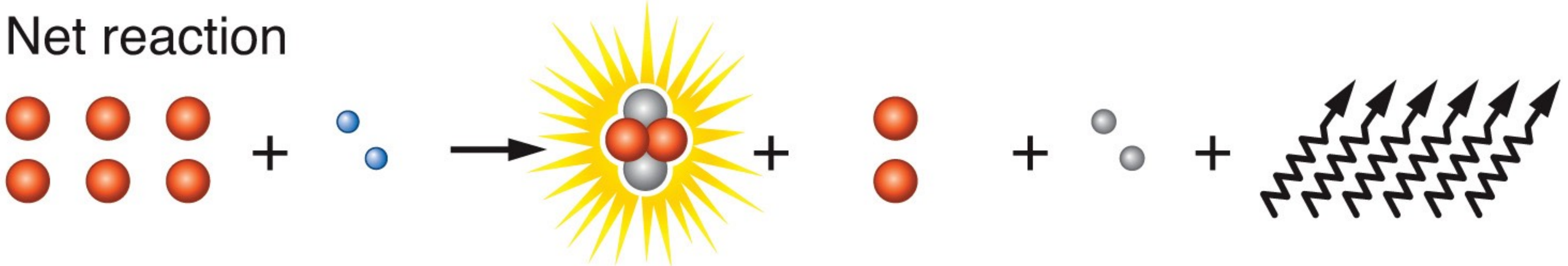
- The sheer mass of the Sun and hydrostatic equilibrium creates the necessary conditions for fusion: dense and hot gas
- Fusion can maintain Solar luminosity over billions of years
- How energy is transported out?
- Fusion model can be tested by neutrino detectors
- Interaction cross section and mean free path
- Last scattering surface
- How limb darkening & absorption lines are produced?
- How temperature determines line strength?
- Solar activities



Chap 14: Our Star - The Sun: Key Equations

$$\frac{dP(r)}{dr} = -\rho(r)\ g(r)$$

$$h_S = \frac{kT}{\mu m_H g}$$



$$6\,{}^{1}\mathrm{H} + 2\,e^{-} \longrightarrow {}^{4}\mathrm{He} + 2\,{}^{1}\mathrm{H} + 2\,\nu + 6\,\gamma$$

$$E = \Delta m c^2$$

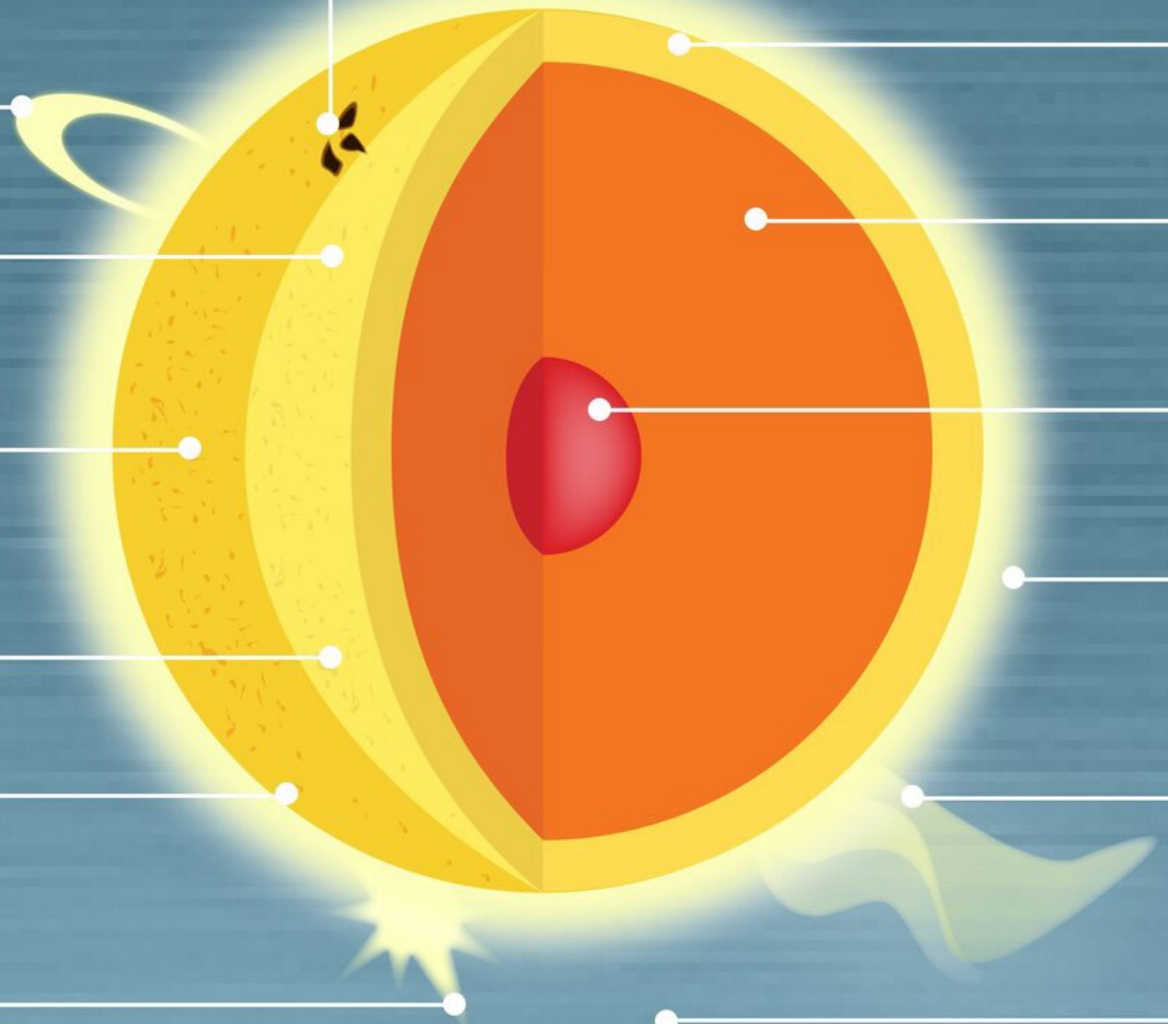
$$l_{\mathrm{mfp}} = \frac{1}{n\sigma} = \frac{\mu m_H}{\rho\sigma} = \frac{1}{\rho\kappa}$$

$$d = l\sqrt{N}$$

$$\frac{d\Phi}{\Phi} = -n\sigma dz$$

$$\Phi = \Phi_0 \exp\left(-\frac{z}{l_{\mathrm{mfp}}}\right)$$

ANATOMY OF THE SUN



Sunspots
Darker, cooler areas on the photosphere with concentrations of magnetic field

Prominence
Large structure, often many thousands of kilometres in extent

Granulation
Small, short-lived grainy features that cover the Sun, caused by thermal currents rising from below

Chromosphere
Layer above the photosphere, where the density of plasma drops dramatically

Photosphere
The visible 'surface' of the Sun

Transition region
Thin, irregular layer that separates the relatively cool chromosphere from the much hotter corona

Flare
Sudden release of energy in the form of radiation

Convective zone
Rapid heating of plasma creates currents of heated and cooled gas

Radiative zone
Energy created in the core diffuses slowly through the plasma

Core
Where the Sun generates its energy via thermonuclear reactions

Corona
The Sun's outer atmosphere, which extends millions of kilometres into outer space

Coronal mass ejection
Vast eruption of billions of tonnes of plasma and accompanying magnetic fields from the Sun's corona

Solar wind
A continuous stream of charged particles released from the corona

#SolarOrbiter #WeAreAllSolarOrbiters