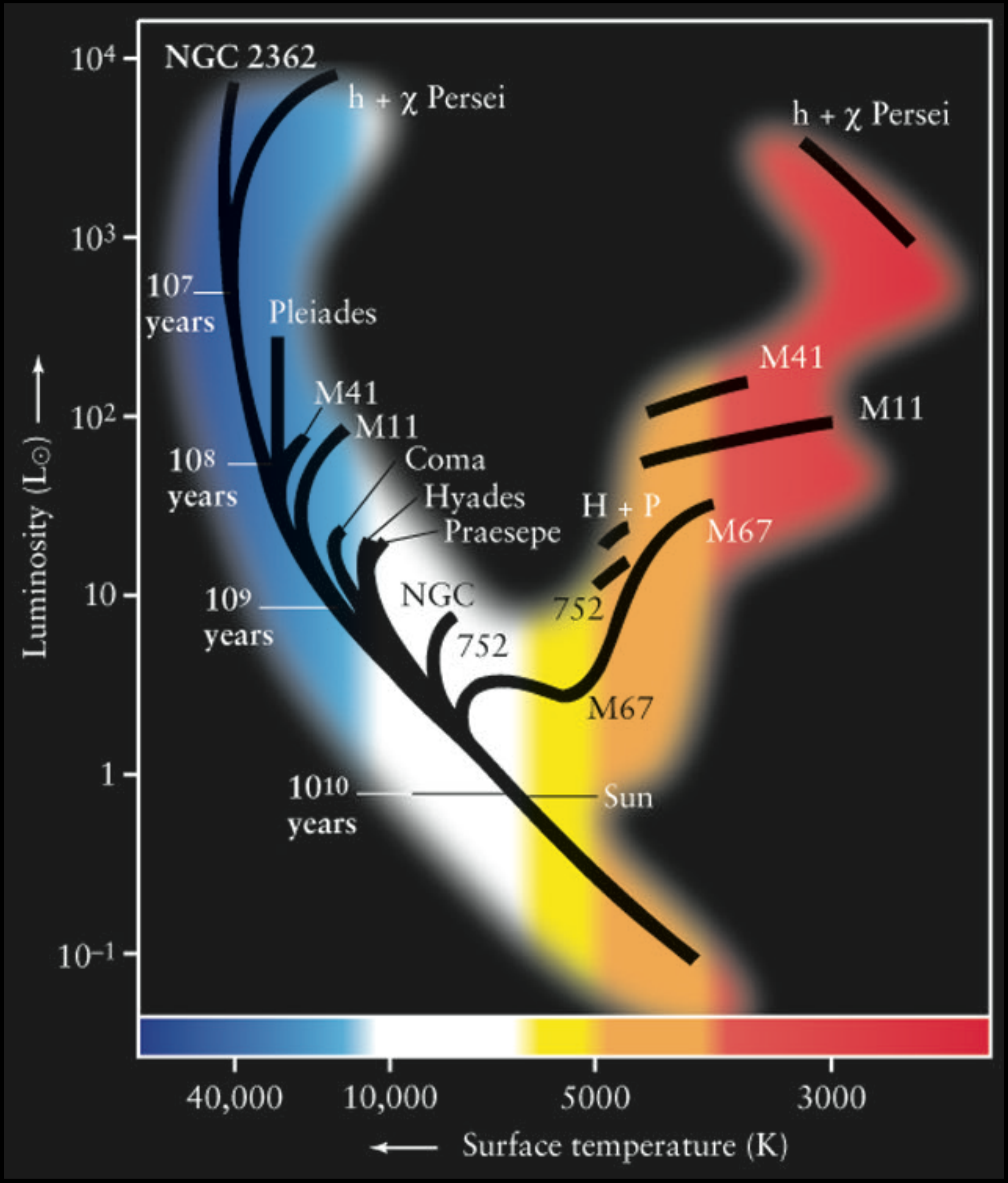
Chap 16: The Evolution of Low-Mass Stars

Chap 16: Key Concepts

- Observations
  - Nothing last forever, even stars
  - H-R diagram of star clusters
- Numerical Models
  - Equations of stellar structure and evolution
  - Stellar evolutionary tracks
- Fine-Tune Models
  - Isochrones (equal-age lines)
  - Fitting cluster H-R diagrams
  - Cluster age estimates
- Model Inferences
  - Main stages and rough lifetimes
  - Changes in the interiors of the stars: e- degenerate core + fusion shells
- Mass-Transfer Binaries
  - Roche Lobe, Lagrange Points
  - Novae, Type Ia SNe, Blue Stragglers

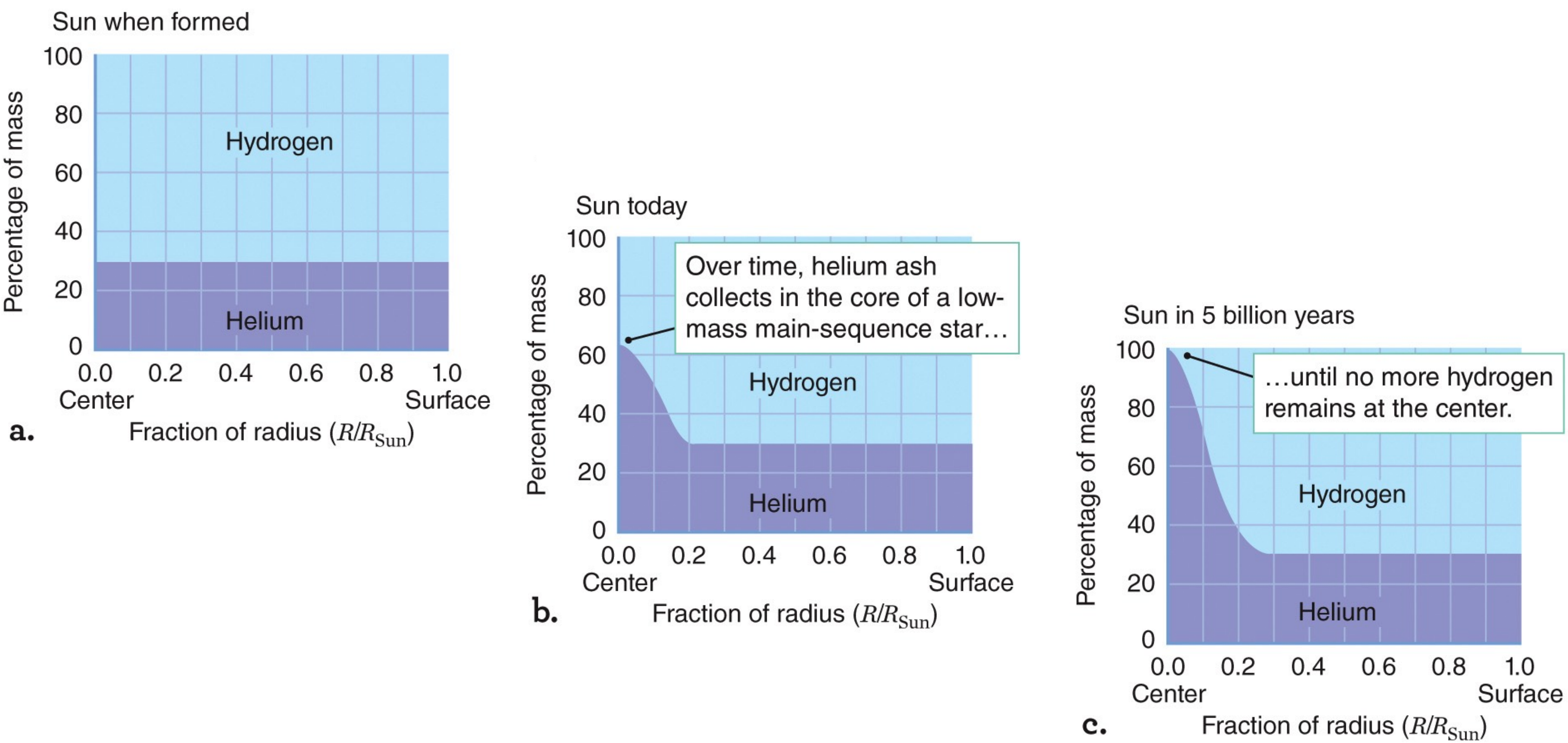


Why do we think stars must evolve?

Logical deduction from our understanding of the Sun

Changes on the Main Sequence due to Fuel Exhaustion

- The chemical composition inside a star changes over time as hydrogen is fused into helium.
- The Sun started with 70 percent hydrogen by mass, but now contains only 35 percent hydrogen in the core.
- What will happen when the hydrogen is exhausted in the core?



Main-Sequence Lifetime of a Star Depends on Its Initial Mass

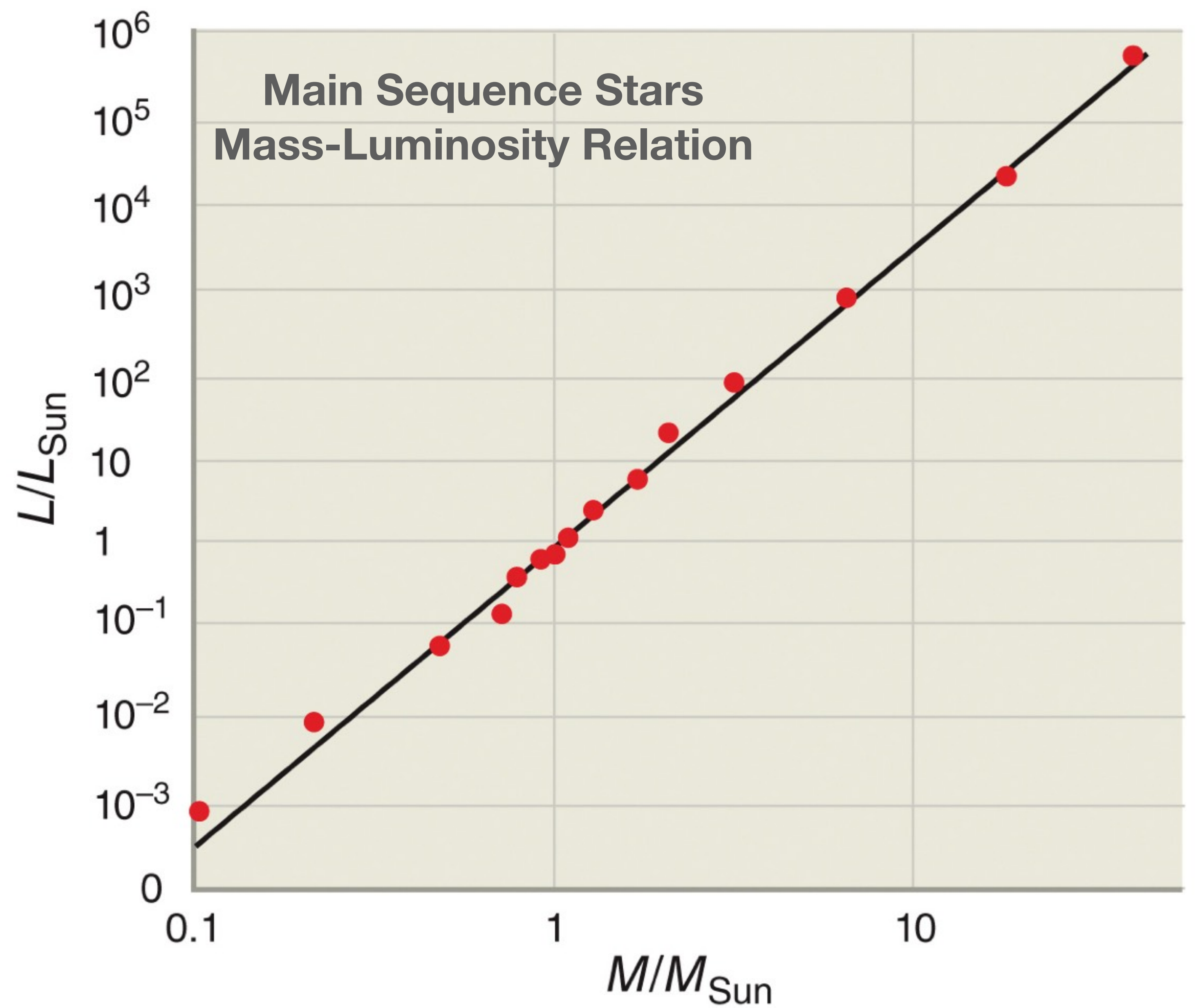
- The main-sequence lifetime of a star is the amount of time that it spends fusing hydrogen as its primary source of energy.

$$\text{Lifetime of star} = \frac{\text{Amount of fuel } (\propto \text{ mass of star})}{\text{Rate fuel is used } (\propto \text{ luminosity of star})}$$

- Stars with high masses have shorter lifetimes.
- Higher-mass stars have more fuel, but they use it more quickly:

$$L \propto M^{3.5}$$

=> MS lifetime ~ $M^{-2.5}$



Calculating the Main-Sequence Lifetime

- In our homework, we have used the proportionalities to get:

$$\text{Lifetime}_{\text{MS}} = 10^{10} \times \frac{M_{\text{MS}}/M_{\text{Sun}}}{\left(M_{\text{MS}}/M_{\text{Sun}}\right)^{3.5}}\ \text{yr} = 10^{10} \times \left(\frac{M_{\text{MS}}}{M_{\text{Sun}}}\right)^{-2.5}\ \text{yr}$$

- Let's compare the lifetime of a K5 star with the Sun. A K5 star has a mass of 0.67 M_{Sun}:

$$\text{Lifetime}_{\text{K5}} = 10^{10} \times (0.67)^{-2.5}\ \text{yr} = 2.7 \times 10^{10}\text{yr}$$

- A **K5** star has a lifetime of **27 billion years**, which is 2.7 times longer than the Sun's lifetime!

- A **O5** main-sequence star has a mass of 60 M_{Sun}, its lifetime is only **360,000 years**! BTW, homo sapiens first evolved in Africa about 300,000 years ago.

Stars Must Change and The Rate of Change Depends on Initial Mass

- A star's life depends on mass and composition because the rates and types of fusion depend on the star's mass.
- Stars of different masses evolve differently. There are three categories of stars:
  - **low-mass stars** (Mass < 3 M_{Sun})
  - **intermediate-mass stars** (Mass between 3 M_{Sun} and 8 M_{Sun})
  - **high-mass stars** (Mass > 8 M_{Sun})

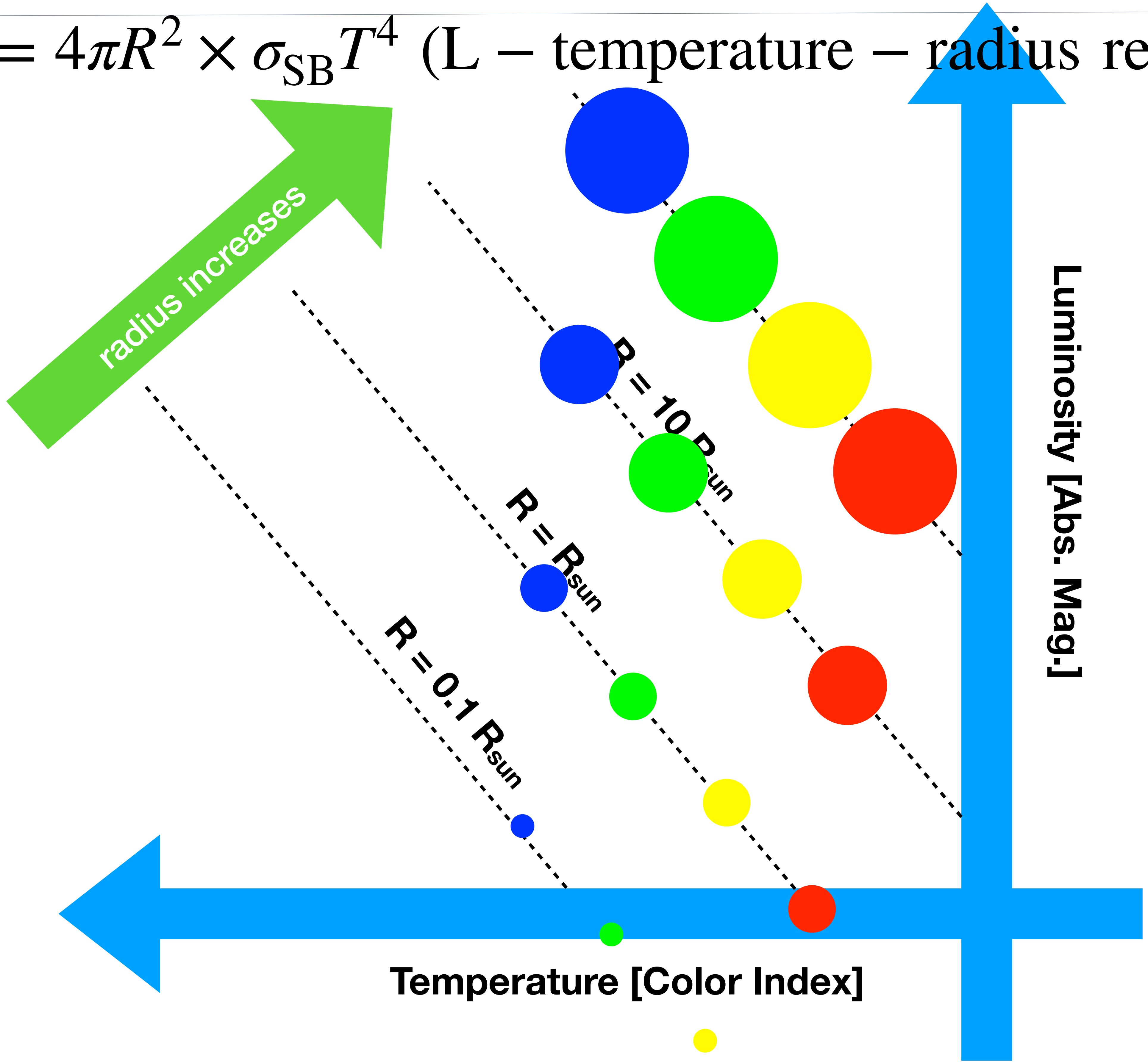
Name	High-mass stars	Medium-mass stars	Low-mass stars	Very low-mass stars	Brown dwarfs
Spectral type	O, B	B	A, F, G, K	M	M, L, T, Y
Minimum mass	8 M_{Sun}	3 M_{Sun}	0.5 M_{Sun}	0.08 M_{Sun}	~0.01 M_{Sun} (~13 $M_{Jupiter}$)

How do we know stars evolve?

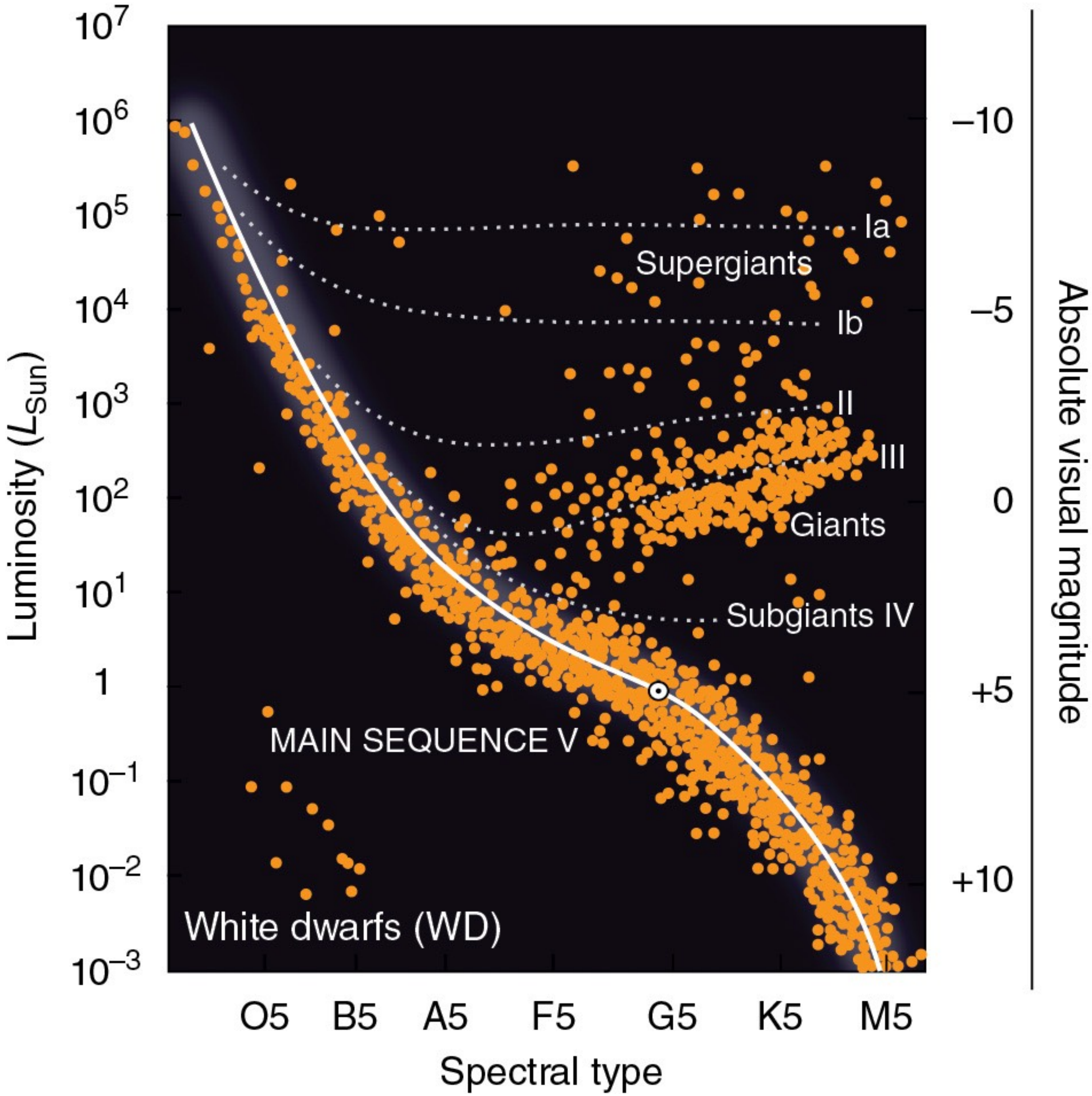
H-R diagram of star clusters

The position of a star on the H-R diagram tells its *L, T,* and *R*

$$L = 4\pi R^2 \times \sigma_{\rm SB} T^4 \ \text{(L − temperature − radius relation)}$$



Main Features: Main Sequence, Giant Branch, White Dwarfs



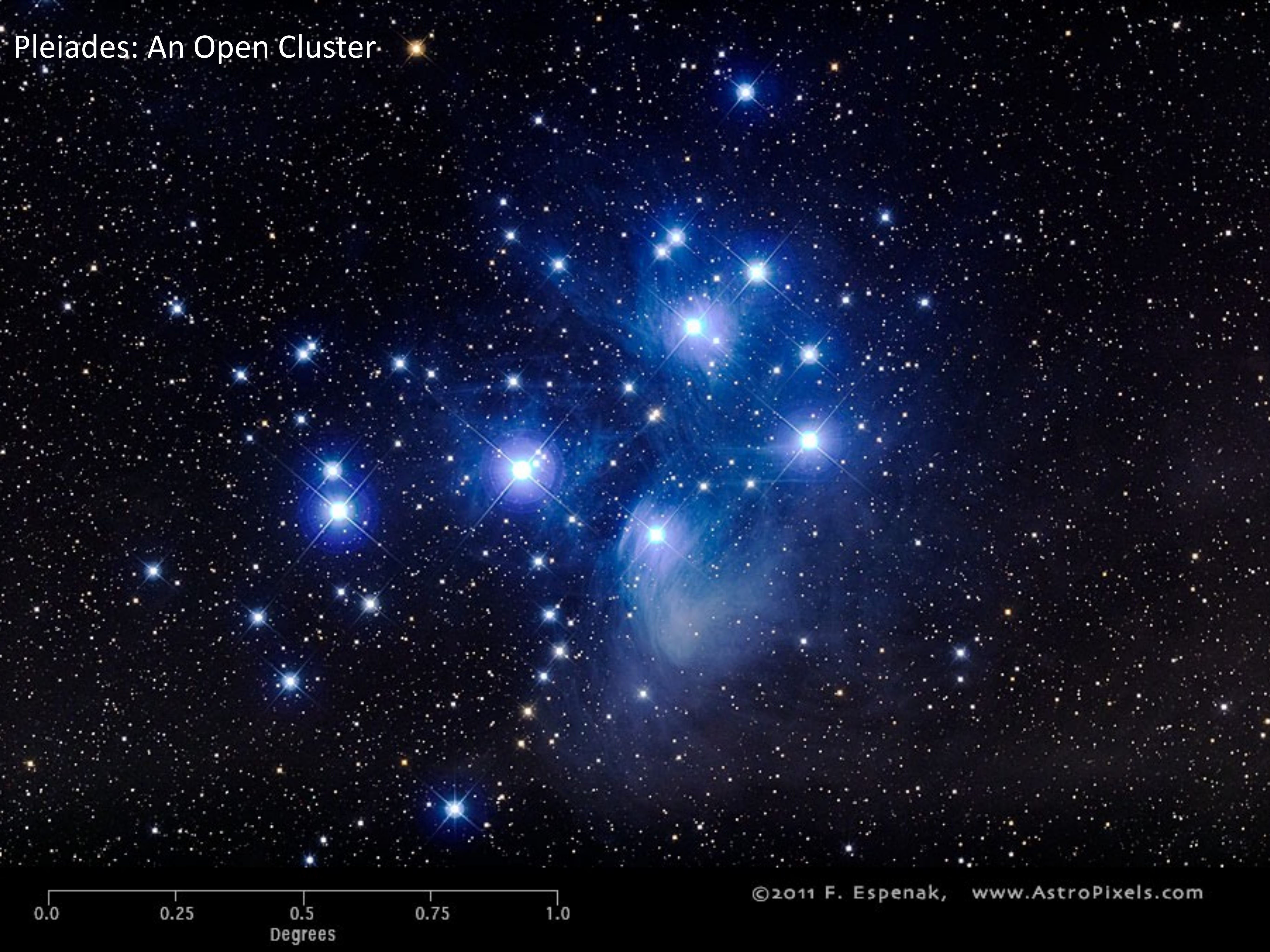
- Most stars exist on the **Main Sequence**.
- Not all stars are on the main sequence: **Giant Branch** and **White Dwarfs**
- There are six **luminosity classes** in Roman numerals
- The Sun is a **G2 V** star:
  G2 - spectral type
  V - luminosity class
- Betelgeuse is a **M1 Ia**:
  M1 - spectral type
  Ia - luminosity class

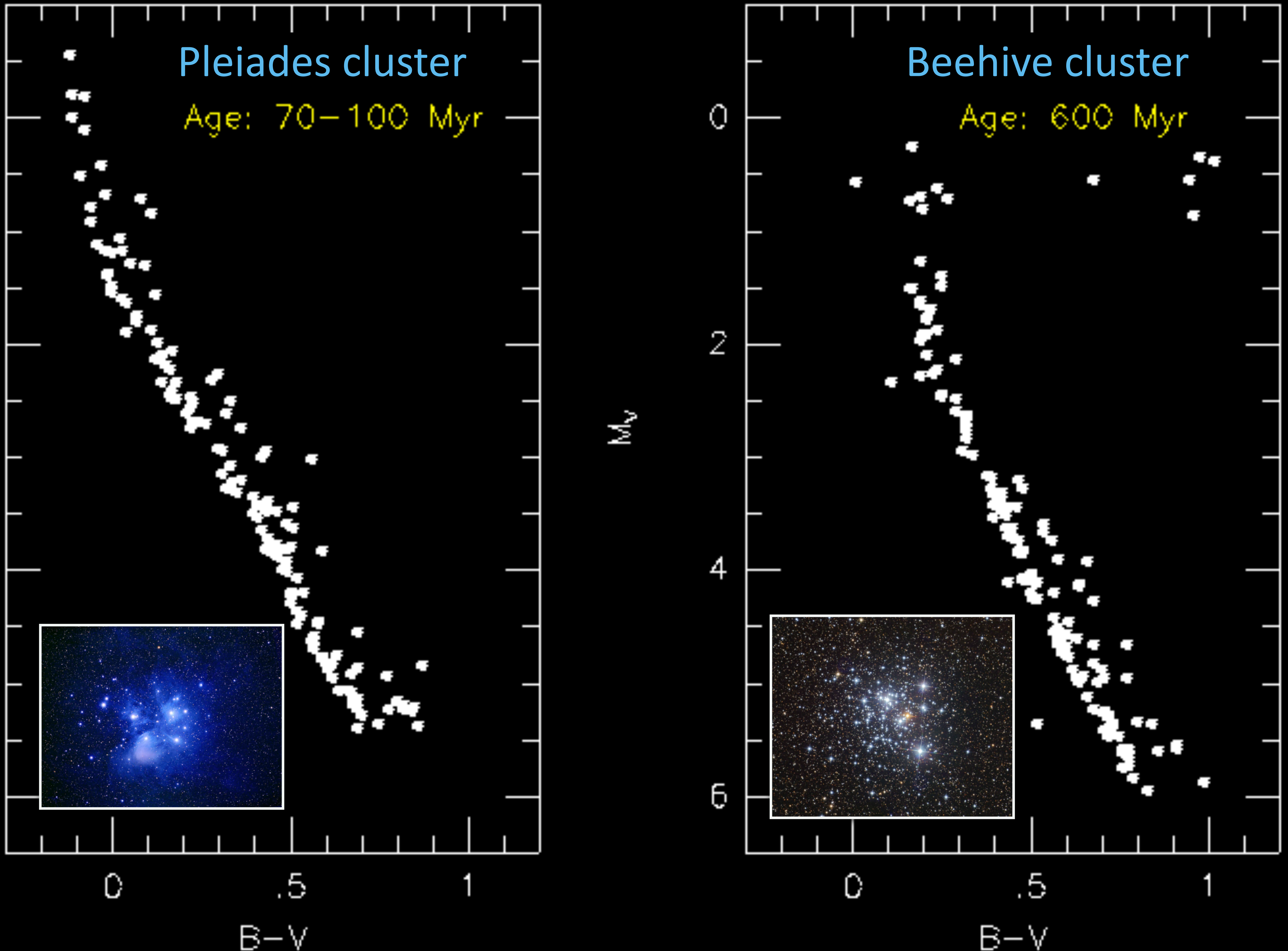
Star clusters are ideal laboratories to study stellar evolution

- Star clusters are bound groups of stars, all made at the same time.
- Each star evolves at a rate set by its mass.
- High-mass stars evolve more quickly along their evolutionary tracks.

M53: a Globular Cluster

Pleiades: An Open Cluster




Pleiades cluster
Age: 70–100 Myr
0
.5
1
B–V
M_V
0
2
4
6
Beehive cluster
Age: 600 Myr
0
.5
1
B–V

H-R diagram of open clusters in the MW galaxy

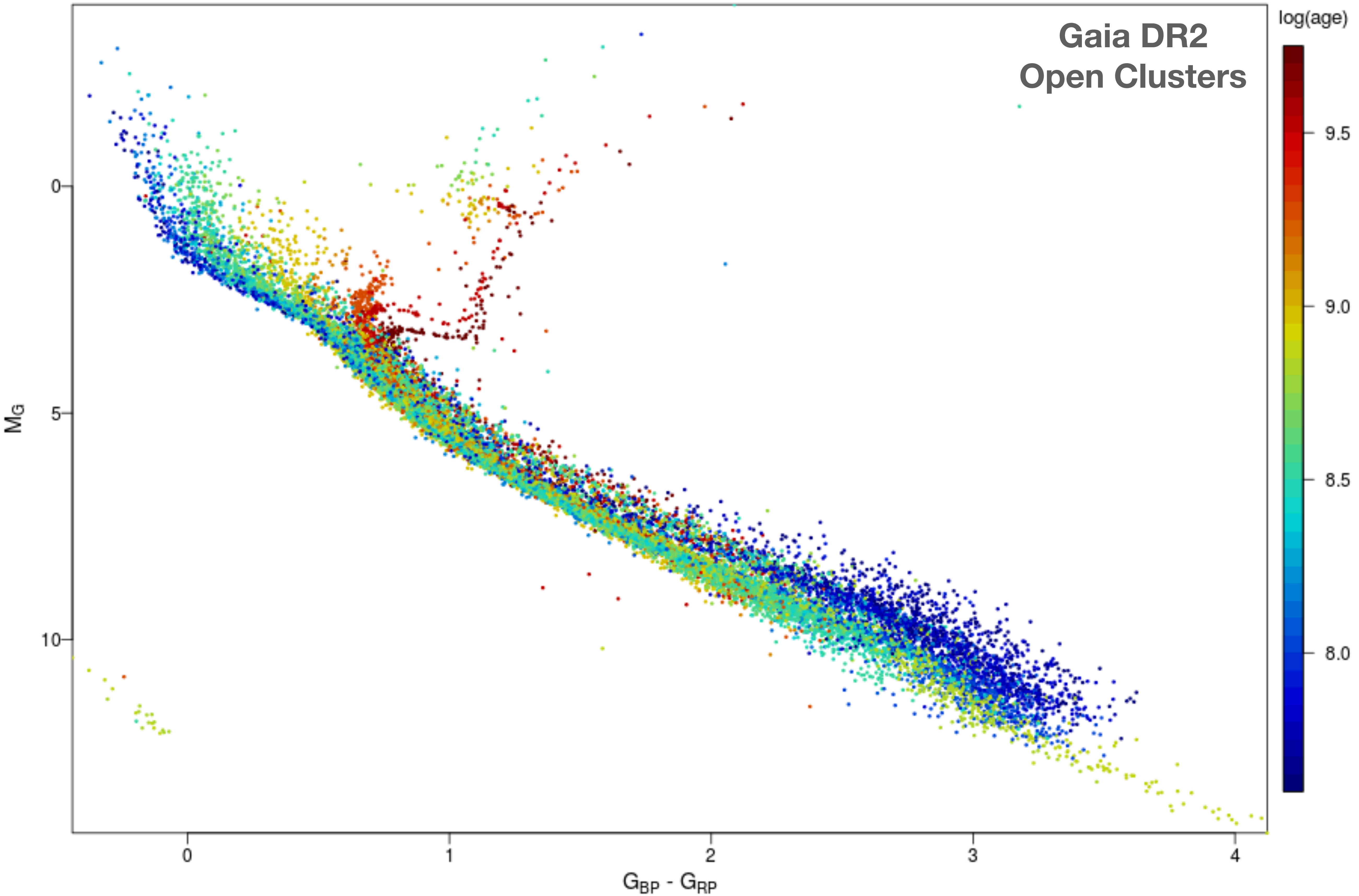


Table 2. Overview of reference values used in constructing the composite HRD for open clusters (Fig. 2).

Cluster	DM	log(age)	[Fe/H]	$E(B-V)$	Memb
Hyades	3.389	8.90	0.13	0.001	480
Coma Ber	4.669	8.81	0.00	0.000	127
Pleiades	5.667	8.04	−0.01	0.045	1059
IC 2391	5.908	7.70	−0.01	0.030	254
IC 2602	5.914	7.60	−0.02	0.031	391
α Per	6.214	7.85	0.14	0.090	598
Praesepe	6.350	8.85	0.16	0.027	771
NGC 2451A	6.433	7.78	−0.08	0.000	311
Blanco 1	6.876	8.06	0.03	0.010	361
NGC 6475	7.234	8.54	0.02	0.049	874
NGC 7092	7.390	8.54	0.00	0.010	248

H-R diagram of globular clusters in the MW galaxy

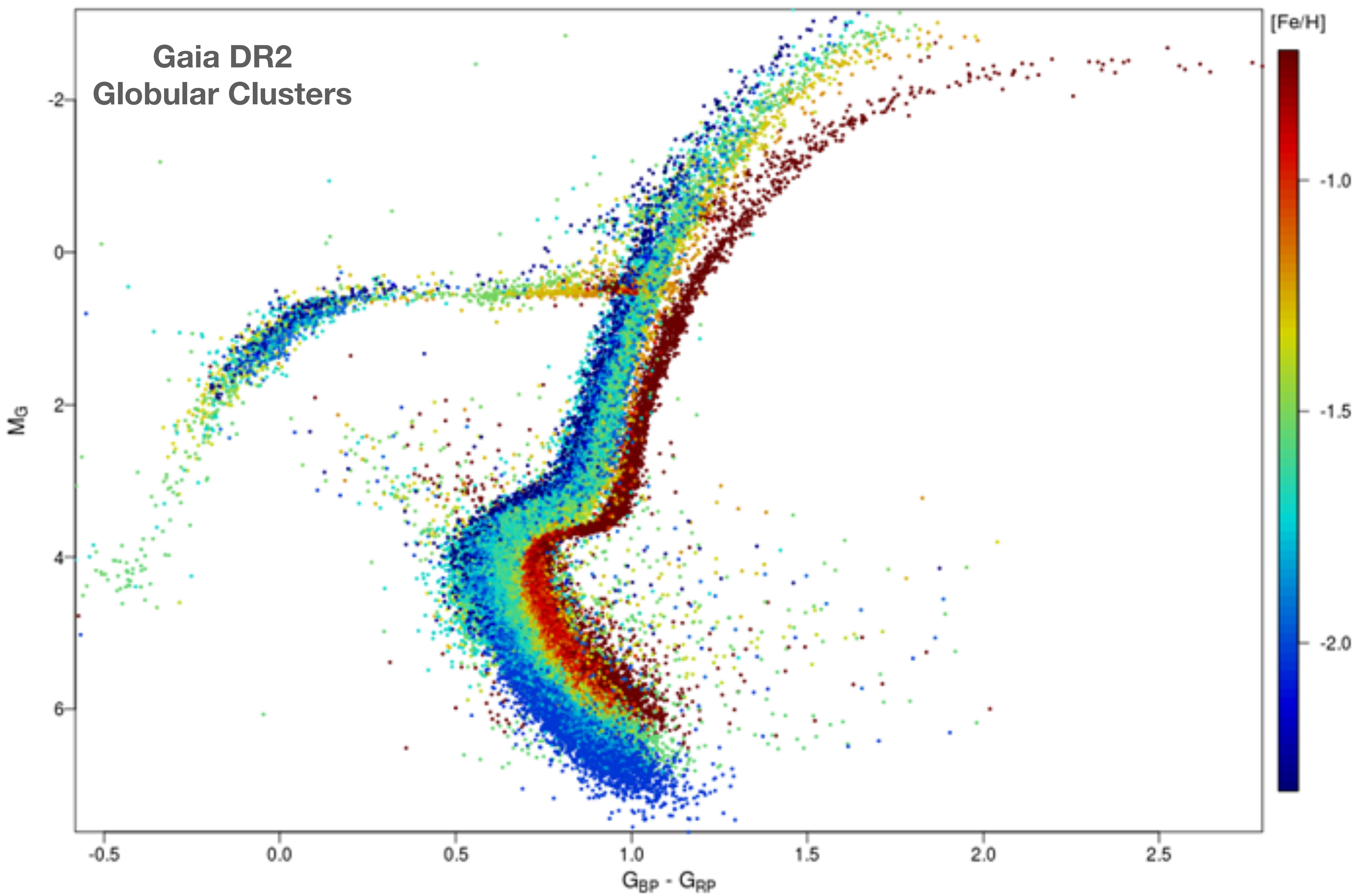
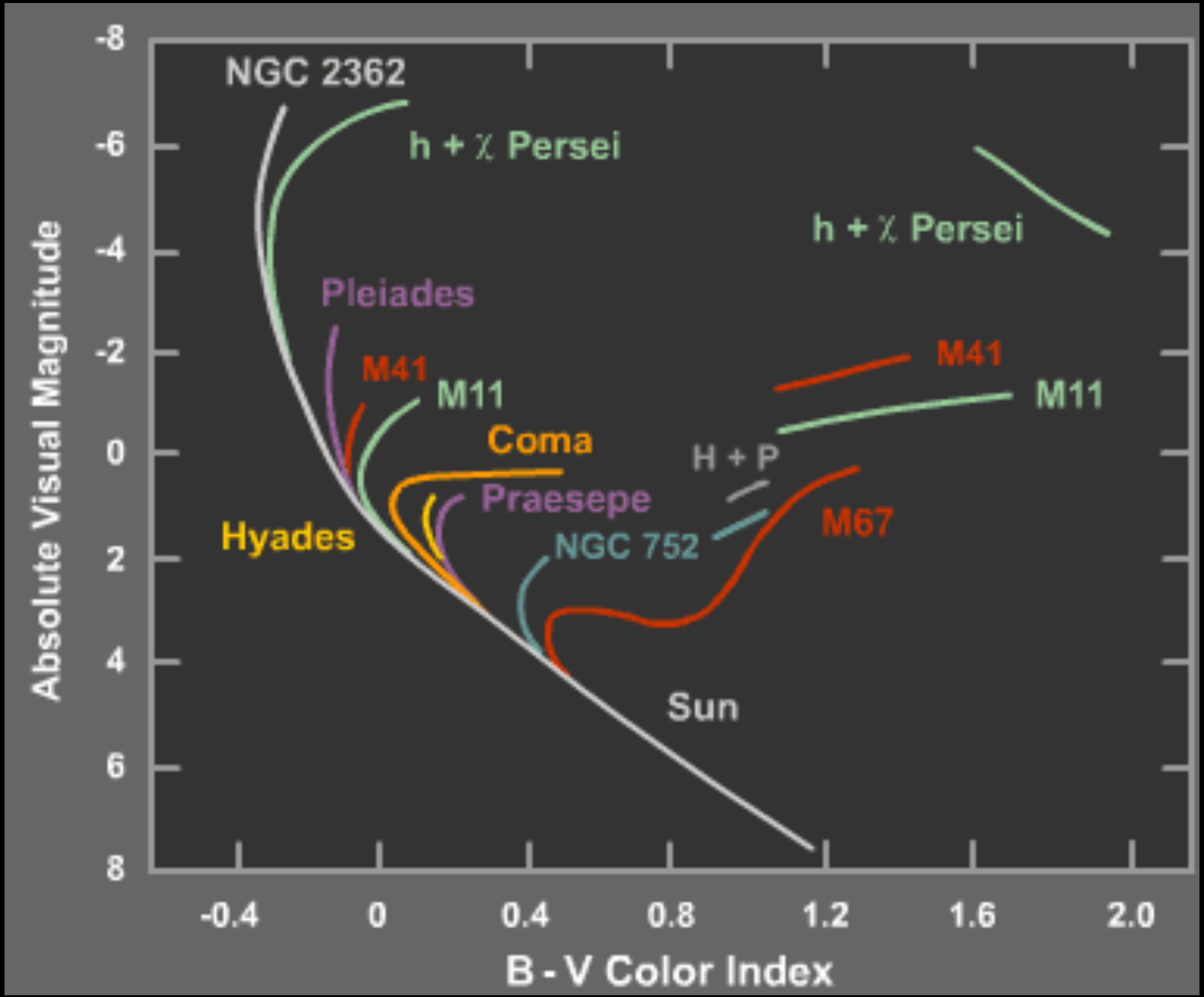


Table 3. Reference data for 14 globular clusters used in the construction of the combined HRD (Fig. 3).

NGC	DM	Age (Gyr)	[Fe/H]	$E(B-V)$	Memb
104	13.266	12.75^a	–0.72	0.04	21580
288	14.747	12.50^a	–1.31	0.03	1953
362	14.672	11.50^a	–1.26	0.05	1737
1851	15.414	13.30^c	–1.18	0.02	744
5272	15.043	12.60^b	–1.50	0.01	9086
5904	14.375	12.25^a	–1.29	0.03	3476
6205	14.256	13.00^a	–1.53	0.02	10311
6218	13.406	13.25^a	–1.37	0.19	3127
6341	14.595	13.25^a	–2.31	0.02	1432
6397	11.920	13.50^a	–2.02	0.18	10055
6656	12.526	12.86^c	–1.70	0.35	9542
6752	13.010	12.50^a	–1.54	0.04	10779
6809	13.662	13.50^a	–1.94	0.08	8073
7099	14.542	13.25^a	–2.27	0.03	1016

Simplified H-R diagram of open clusters in the MW galaxy

Can we build a single model to explain all of the star clusters?
If so, what parameter makes different clusters look different on the HRD?



How do we model stellar evolution?

Computational code of stellar evolution

Stellar Structure Models - Basic Equations

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$ HYDROSTATIC EQUILIBRIUM

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$ MASS CONSERVATION

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$ ENERGY EQUATION

$$\left.\frac{dT}{dr}\right|_{rad} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$ RADIATIVE TRANSPORT

$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$ ADIABATIC CONVECTION

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CONSTITUTIVE RELATIONS (CR)

$$P = \frac{\rho k T}{\mu m_{\mathrm{H}}} + \frac{1}{3} a T^4$$

$$\overline{\kappa} = \left\{ \begin{array}{l} \overline{\kappa}_{bf} = \text{bound-free} \\ \overline{\kappa}_{ff} = \text{free-free} \\ \overline{\kappa}_{es} = \text{electron scattering} \end{array} \right\}$$

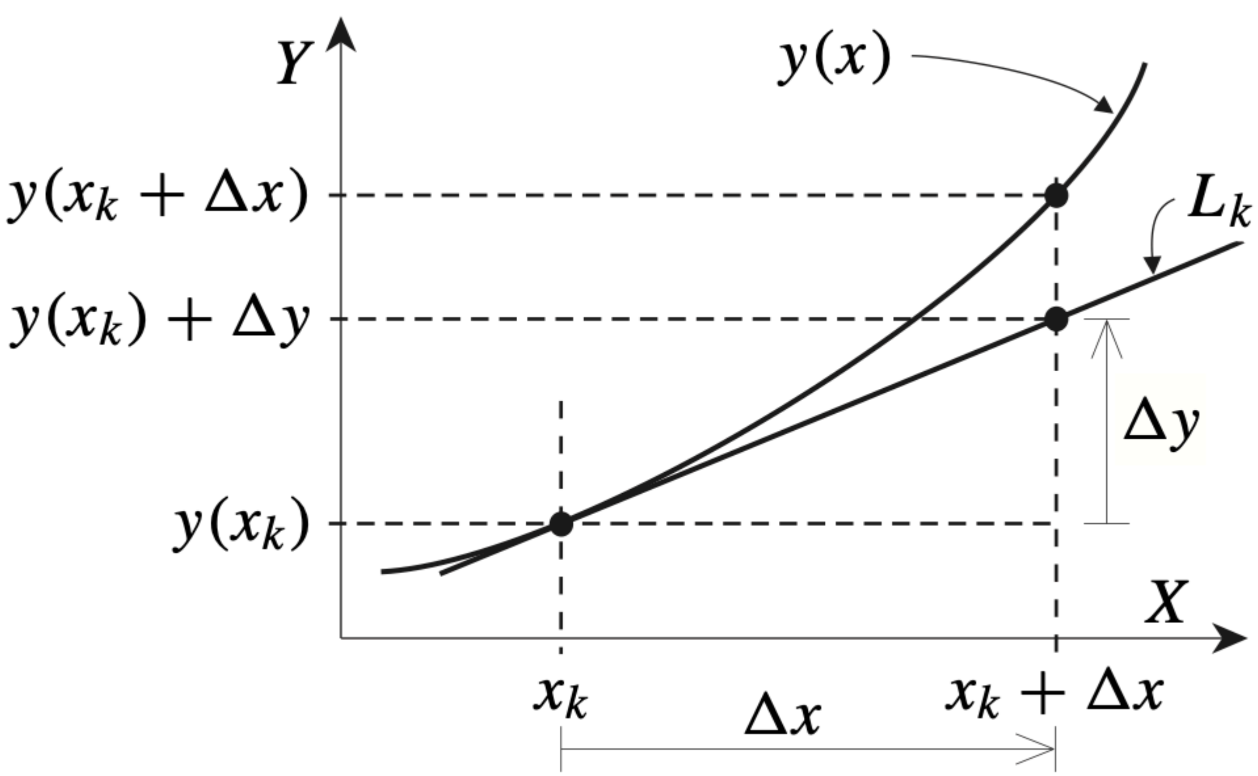
FROM TABLES OR FITTED TO A FUNCTION

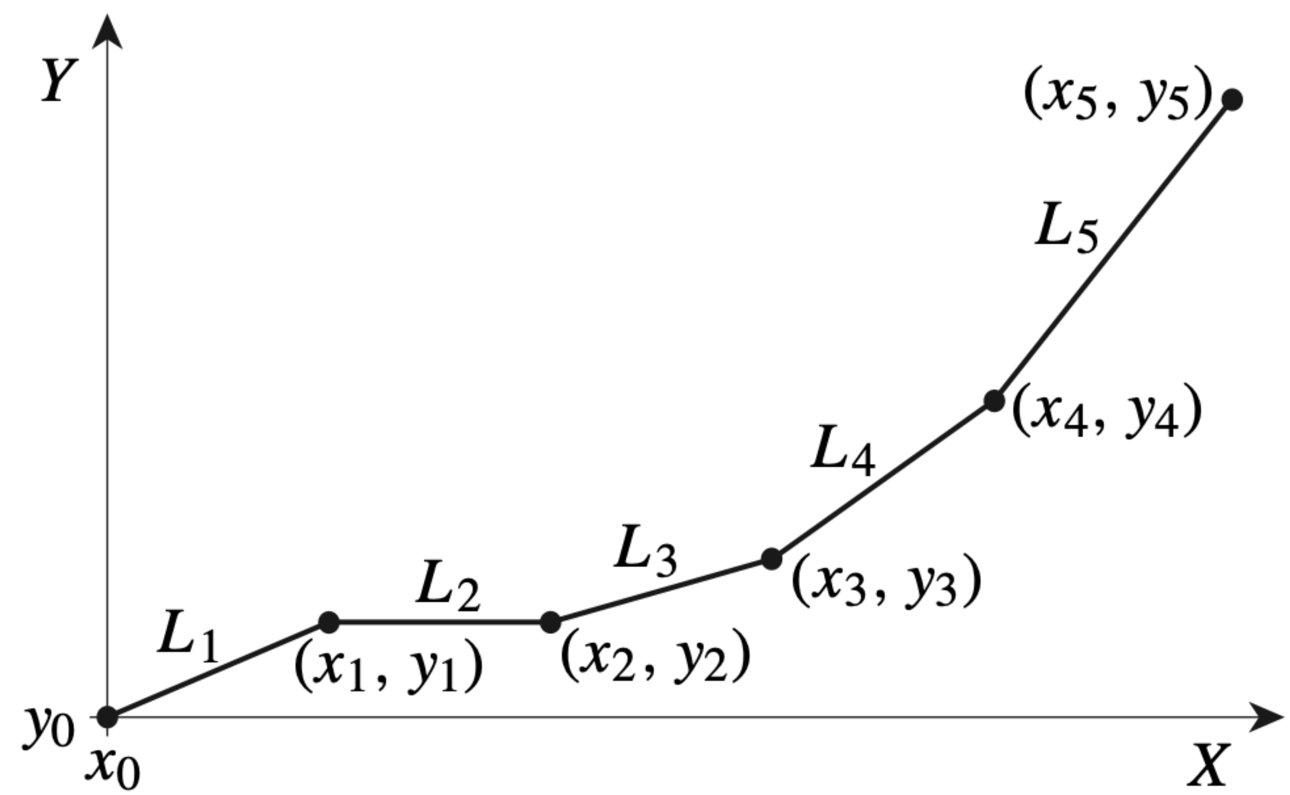
$$\epsilon = \left\{ \begin{array}{l} \epsilon_{\text{pp-chain}} \\ \epsilon_{\text{CNO cycle}} \\ \epsilon_{3\alpha} \end{array} \right.$$

Euler Method: a numerical procedure to solve differential equations

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_k + \Delta x) \approx y_k + \Delta x \cdot f(x_k, y_k)$$





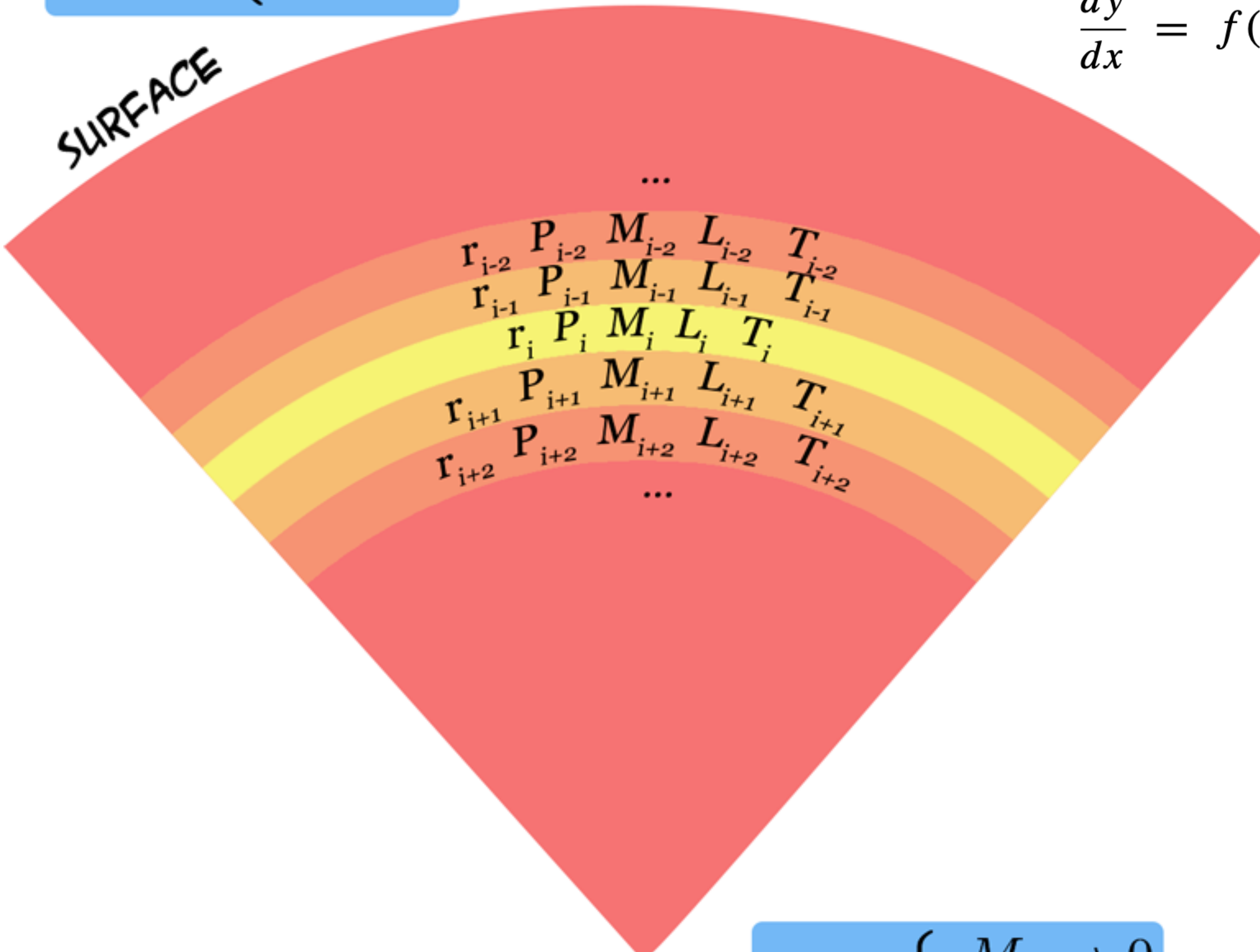
https://www.uah.edu/images/people/faculty/howellkb/DEText-Ch9.pdf

$$r = R^* \begin{cases} T \to 0 \\ P \to 0 \\ \rho \to 0 \end{cases}$$

Euler Method: a numerical procedure to solve differential equations

$$y(x_k + \Delta x) \approx y_k + \Delta x \cdot f(x_k, y_k)$$

$$\frac{dy}{dx} = f(x, y)$$



$$r = 0 \begin{cases} M_r \to 0 \\ L_r \to 0 \end{cases}$$

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Modules for Experiments in Stellar Astrophysics (MESA)

https://docs.mesastar.org/en/release-r22.11.1/index.html

Motivation

Stellar evolution calculations (i.e., stellar evolution tracks and detailed information about the evolution of internal and global properties) are a basic tool that enable a broad range of research in astrophysics. Areas that critically depend on high-fidelity and modern stellar evolution include asteroseismology, nuclear astrophysics, stellar populations, chemical evolution and population synthesis, astrobiology, binary stars, variable stars, supernovae, novae, compact objects, tidal disruption events, stellar hydrodynamics, and stellar activity.

New observational capabilities are emerging in these fields that place a high demand on exploration of stellar dependencies on mass, metallicity and age. So, even though one dimensional stellar evolution is a mature discipline, we continue to ask new questions of stars. Some important aspects of stars are truly three-dimensional, such as convection, rotation, and magnetism. These aspects remain in the realm of research frontiers with evolving understanding and insights, quite often profound. However, much remains to be gained scientifically (and pedagogically) by accurate one-dimensional calculations, and this is the focus of MESA.

MESA-Web Calculation Submission

To submit a *MESA-Web* calculation, simply enter your email address in the *Email Address* field at the bottom of the form below, and then click the *Submit* button.

The default parameters have been chosen to evolve a 1 $M_\odot$ model from pre main-sequence to white dwarf in less than 2 hours of wall time. To obtain more-Detailed information about each parameter, click on the name of the parameter to visit the corresponding entry on the the *MESA-Web* Input page.

After a calculation completes, you will receive an email with link to a Zip archive that contains the output from *MESA-Web* (note that the link expire after one day). For information on the contents of this archive, see the *MESA-Web* Output page.

Initial Properties

Mass: 1.0 $M_\odot$

Metallicity: 0.02

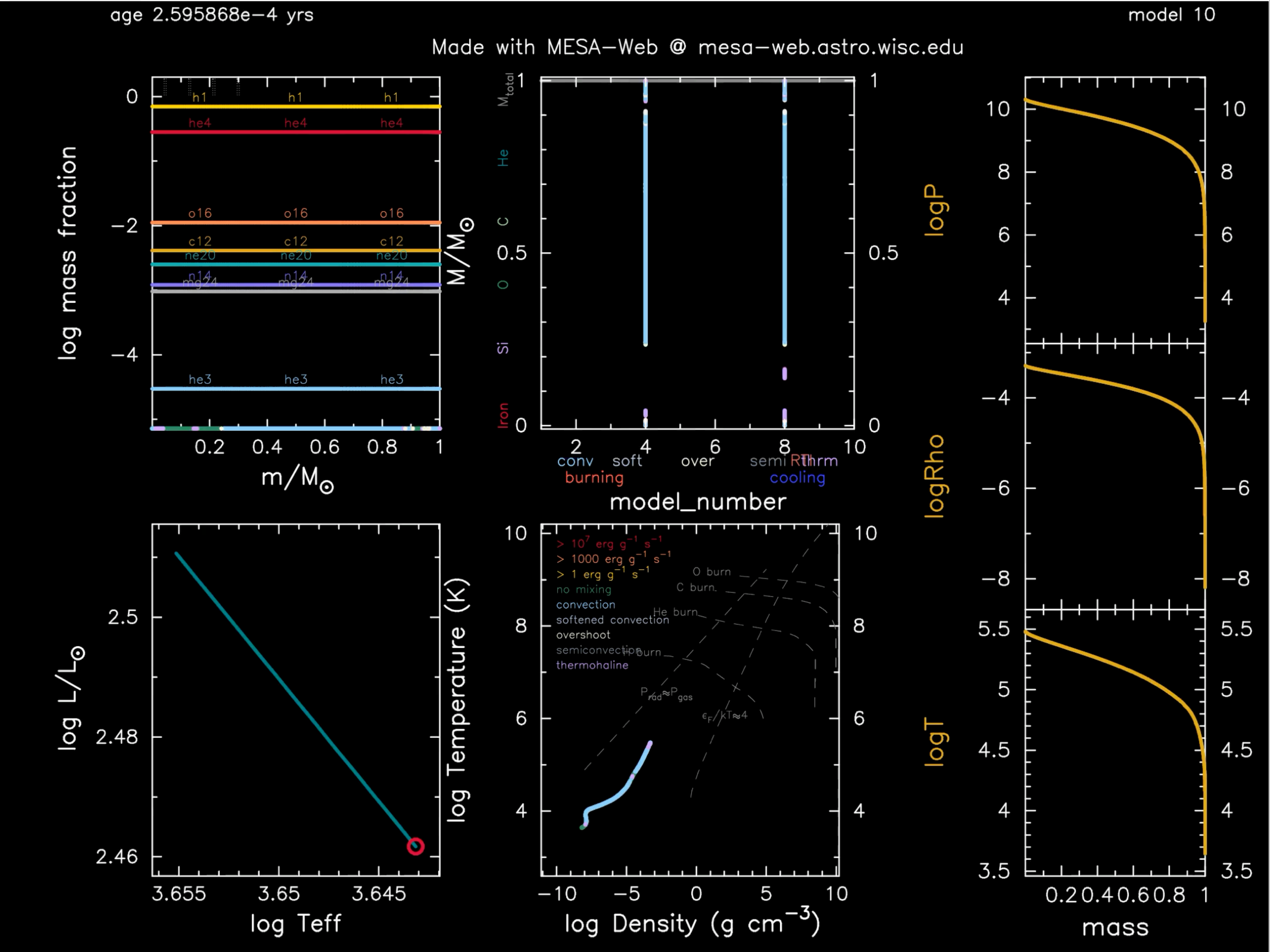
Rotation Rate ($\Omega_{ZAMS}/\Omega_{crit}$): 0.0

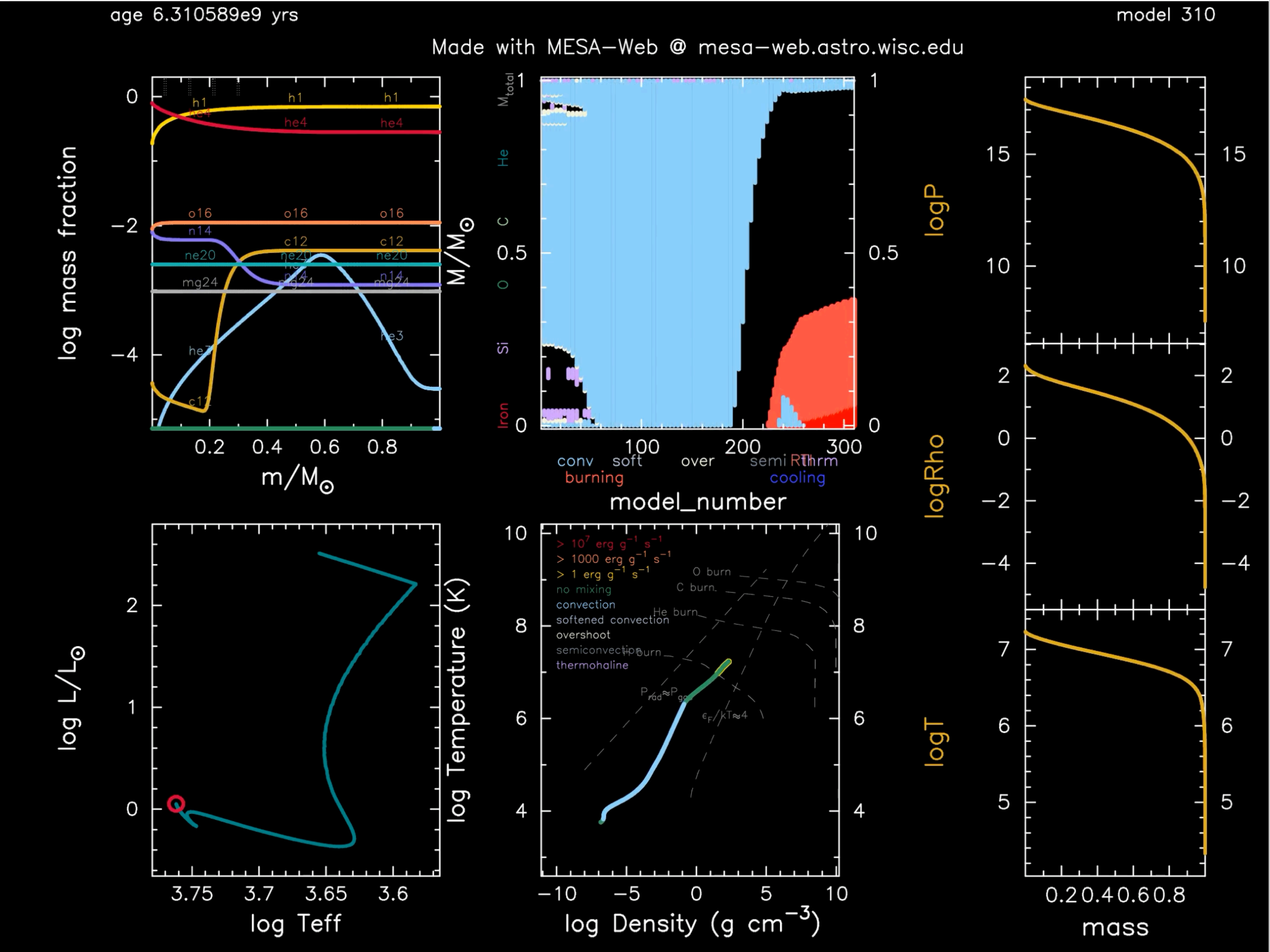
Nuclear Reactions

Network: basic

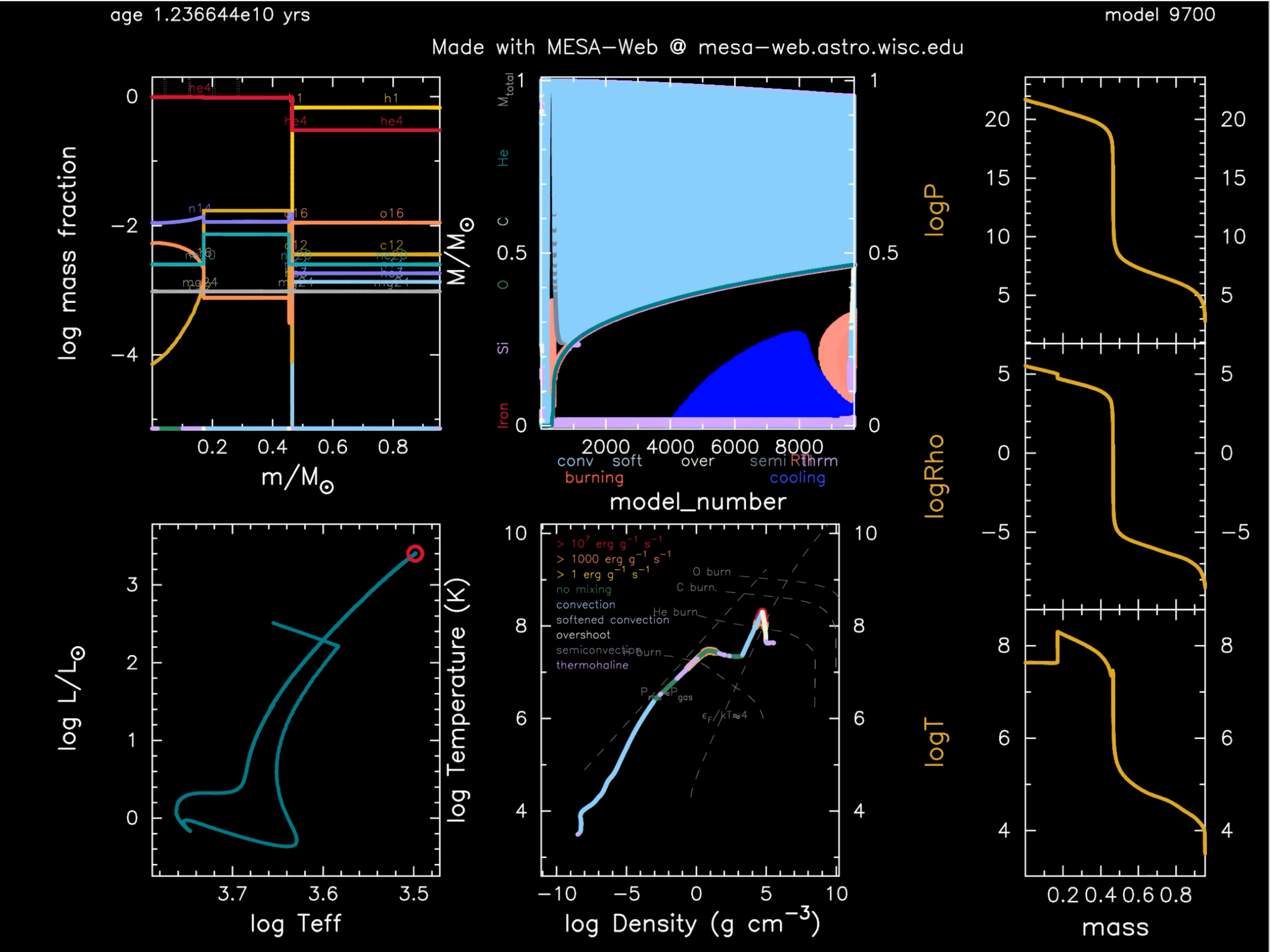
Custom Nuclear Reaction Rate: none

Upload Rate: Choose File no file selected


age 2.595868e−4 yrs
model 10
Made with MESA−Web @ mesa−web.astro.wisc.edu
log mass fraction
h1
he4
o16
c12
ne20
n14
mg24
he3
m/M⊙
M/M⊙
M_total
He
C
O
Si
Iron
conv
soft
over
semi
RTI
thrm
burning
cooling
model_number
logP
logRho
logT
mass
log L/L⊙
log Teff
log Temperature (K)
log Density (g cm−3)
> 10^7 erg g−1 s−1
> 1000 erg g−1 s−1
> 1 erg g−1 s−1
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn
Prad≈Pgas
εF/kT≈4



age 6.310589e9 yrs
model 310
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
m/M☉
M/M☉
model_number
conv soft over semi Rthrm
burning cooling
logP
logRho
logT
mass
log L/L☉
log Teff
log Temperature (K)
log Density (g cm⁻³)
> 10⁷ erg g⁻¹ s⁻¹
> 1000 erg g⁻¹ s⁻¹
> 1 erg g⁻¹ s⁻¹
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn



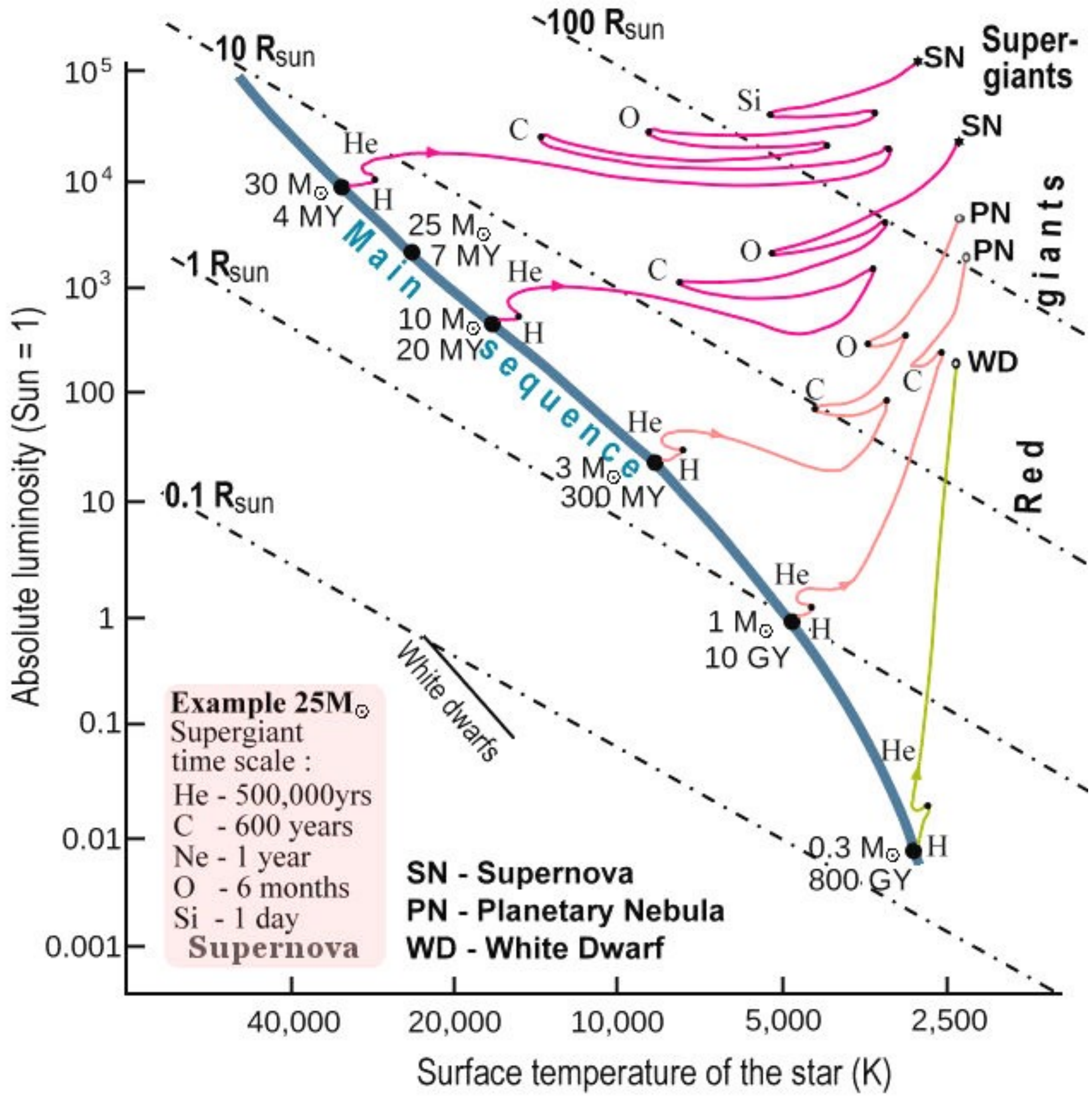
age 1.236644e10 yrs
model 9700
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
m/M⊙
M/M⊙
model_number
conv soft over semi Rti thrm
burning cooling
logP
logRho
logT
mass
log L/L⊙
log Teff
log Temperature (K)
log Density (g cm⁻³)
> 10⁷ erg g⁻¹ s⁻¹
> 1000 erg g⁻¹ s⁻¹
> 1 erg g⁻¹ s⁻¹
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn

How do we check stellar evolution models?

Model Isochrones vs. Cluster H-R Diagrams

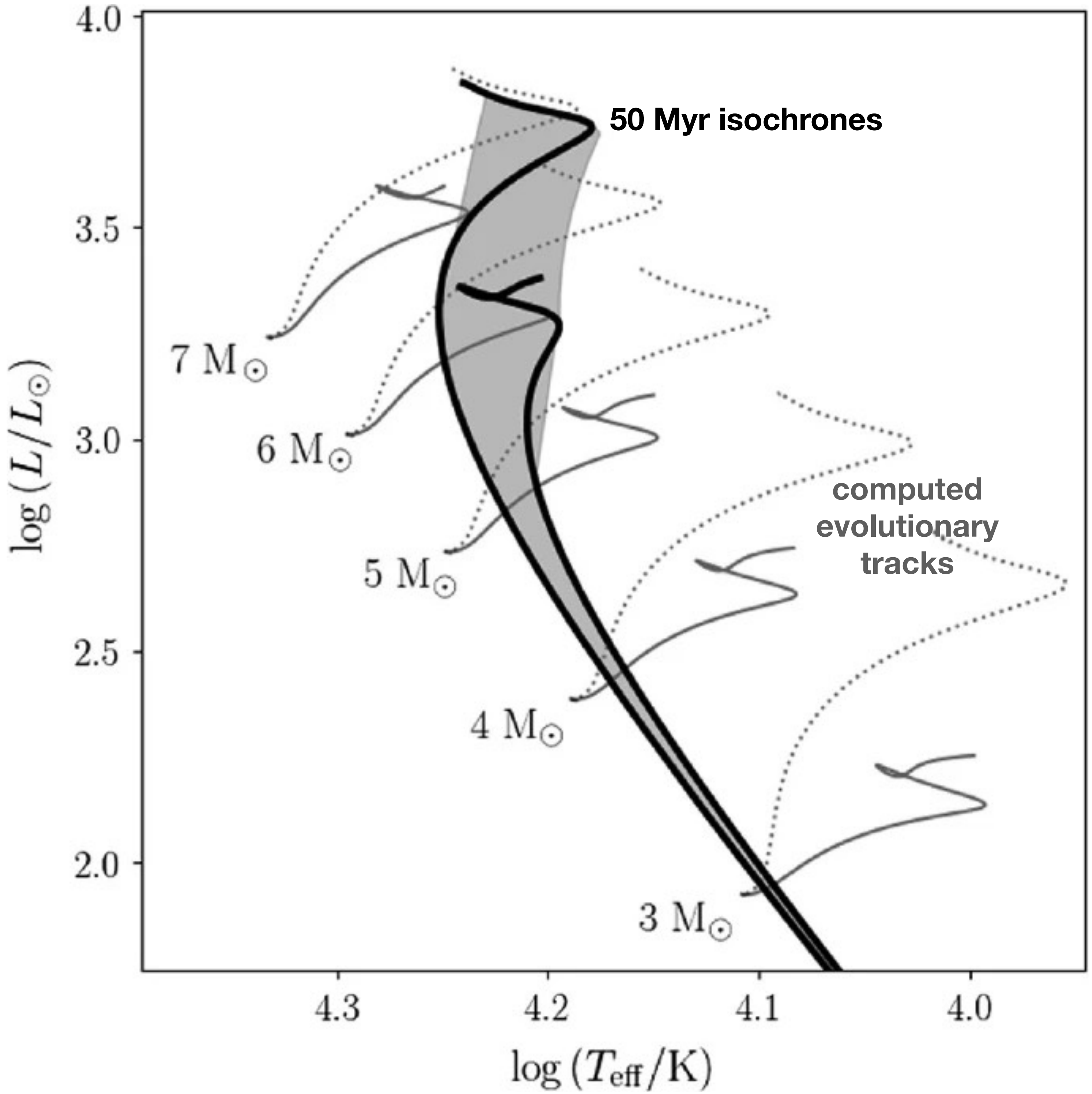
Computed evolutionary tracks of stars with different *initial* masses

- An **evolutionary track** is a computed trajectory of a star on the H-R diagram as it ages over time.

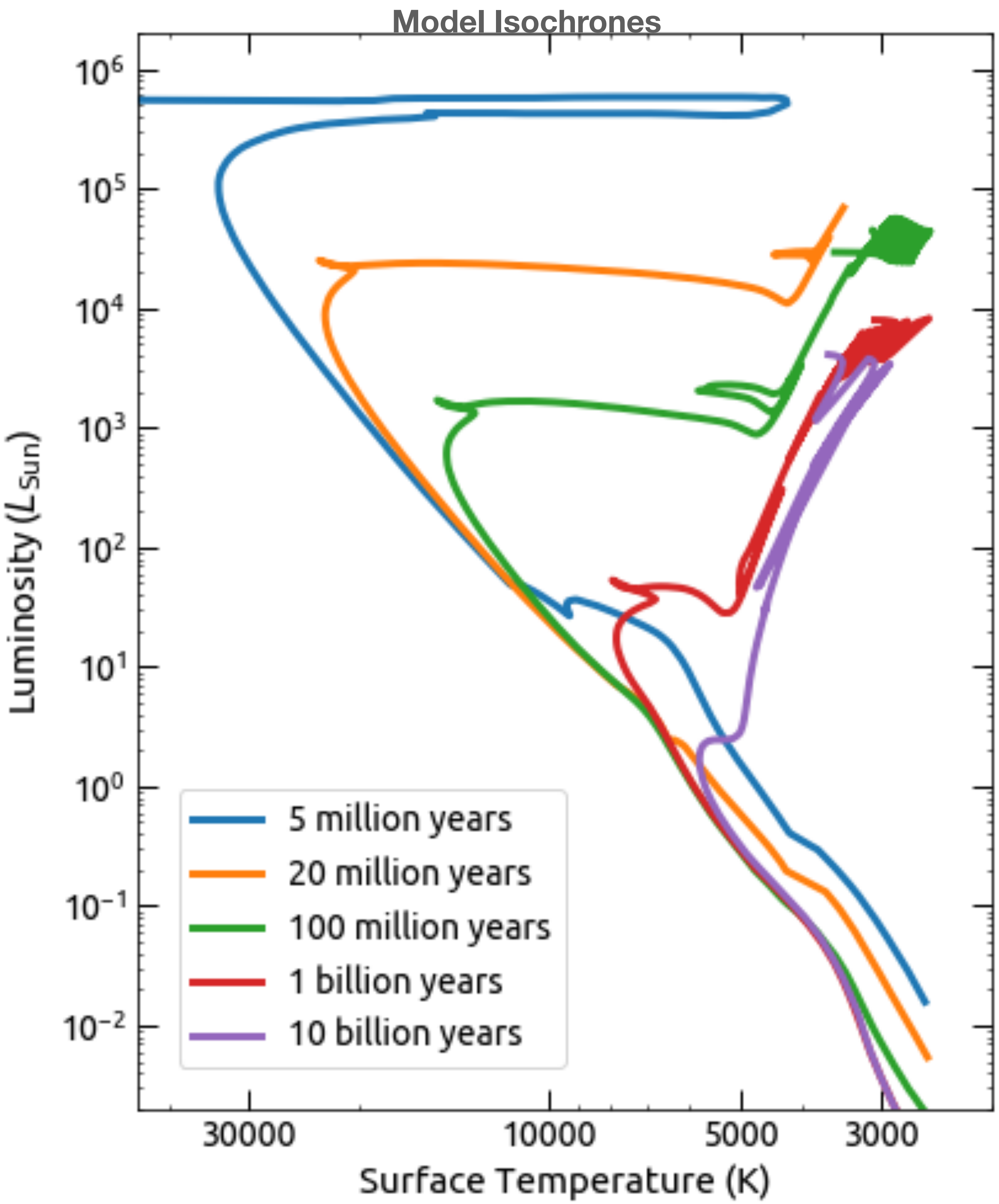


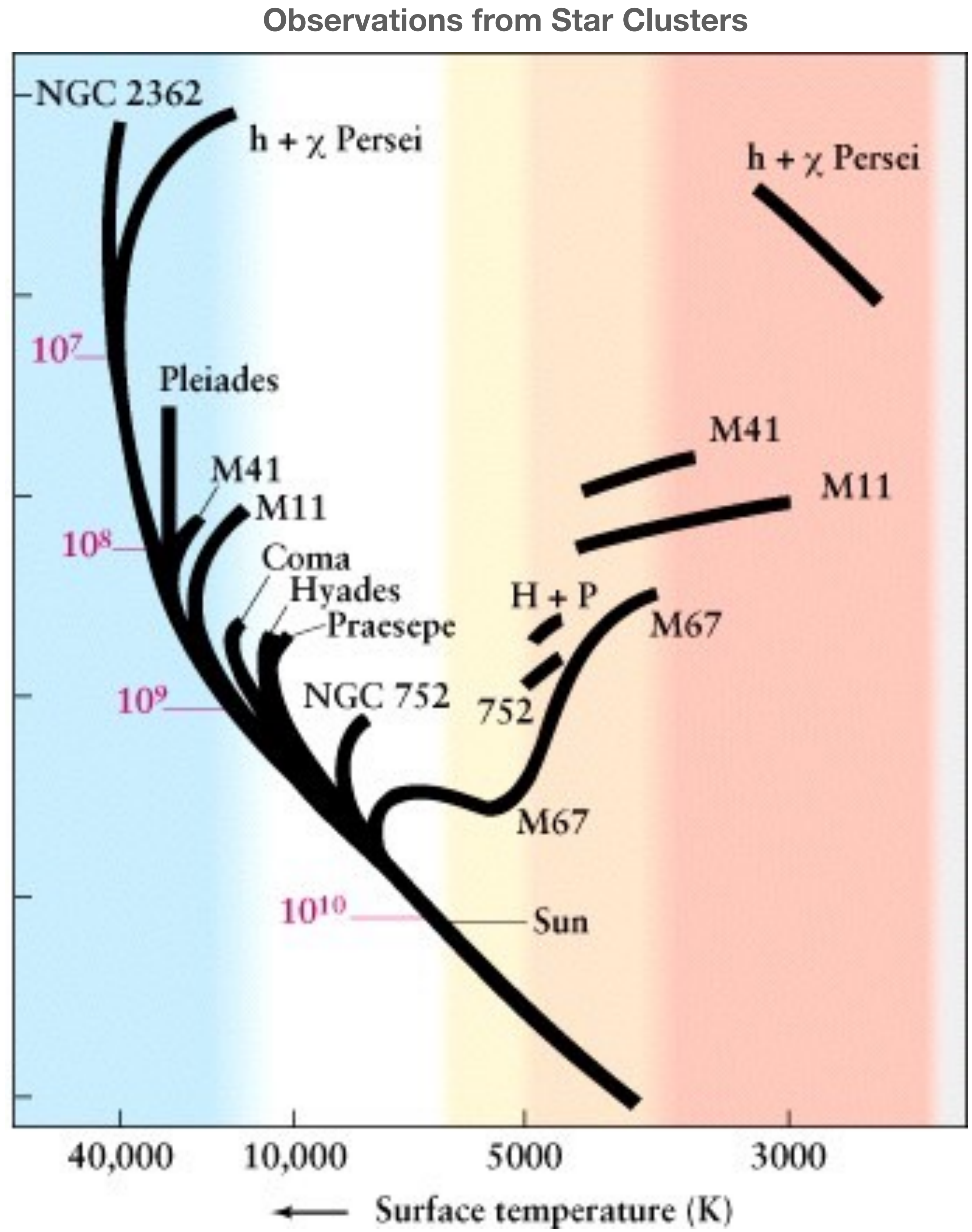
Building isochrones from evolutionary tracks

- An **isochrone** (iso = equal, chrone = time) is a line drawn on the H-R diagram connecting stars of the same age



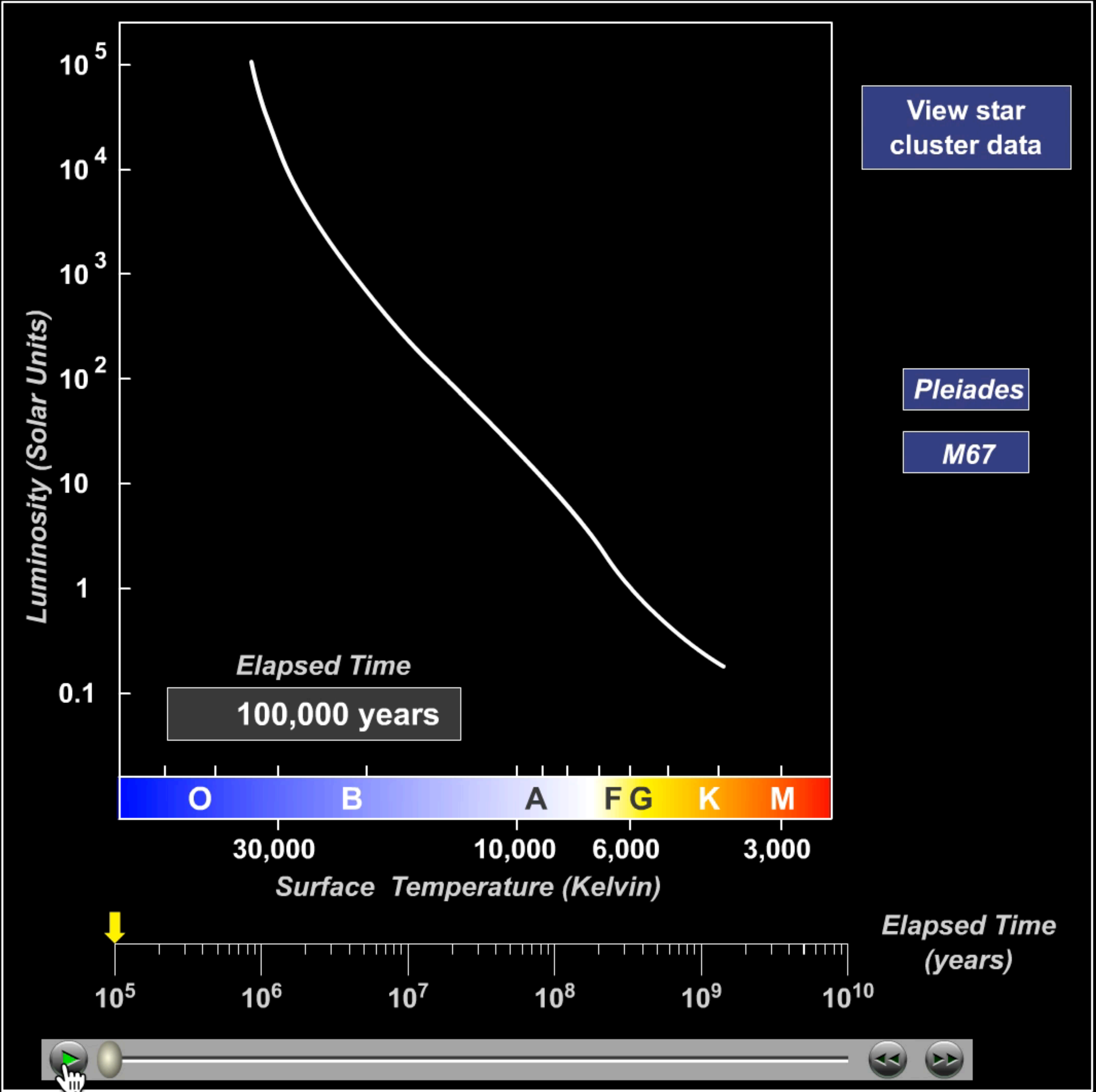
Isochrones change with age: this is how we explain the various different observed H-R diagrams of star clusters



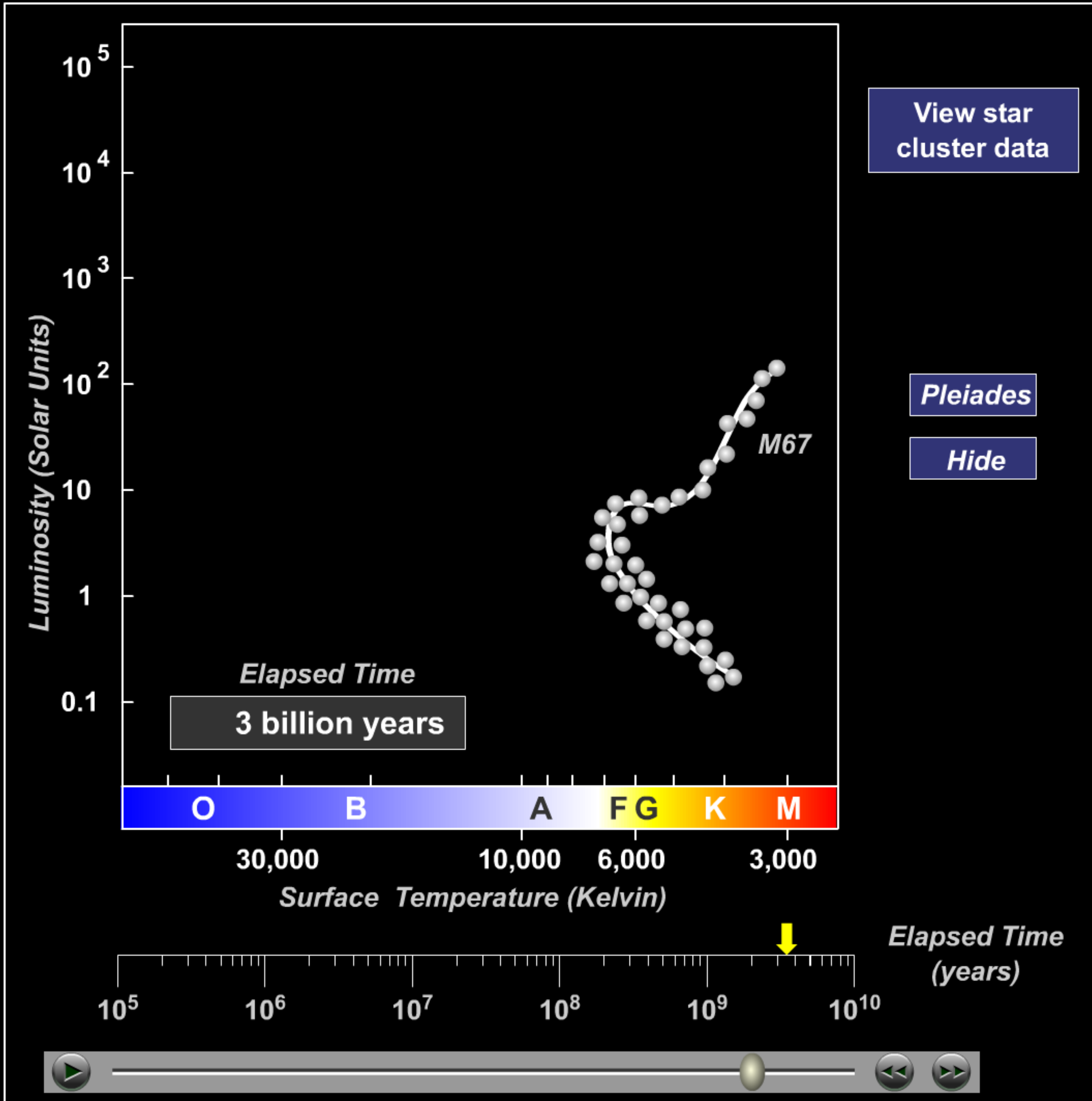


How can you tell if a curve on a HR diagram is an isochrone instead of an evolutionary track?

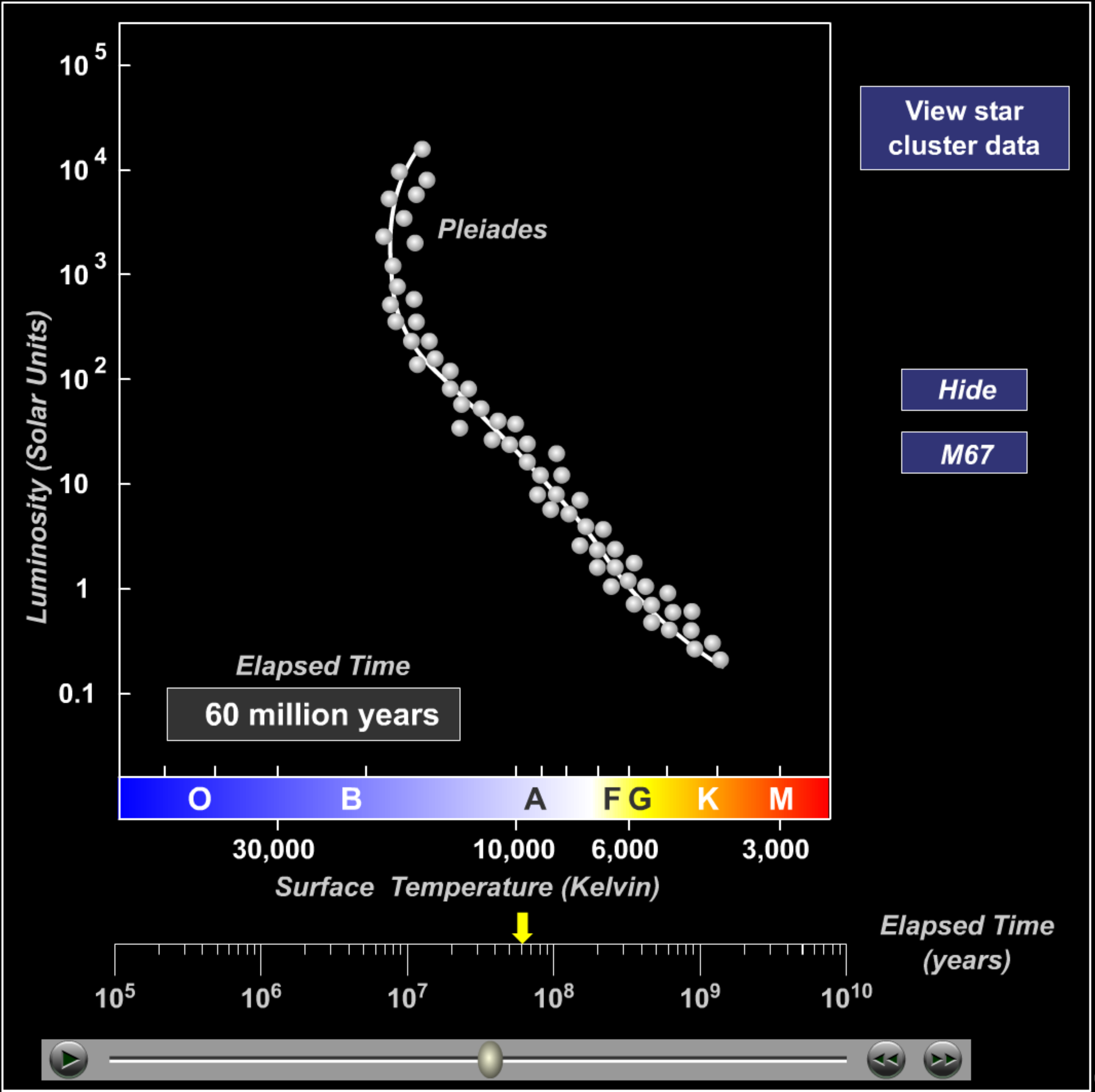
Using the H-R Diagram to Determine the Age of a Star Cluster



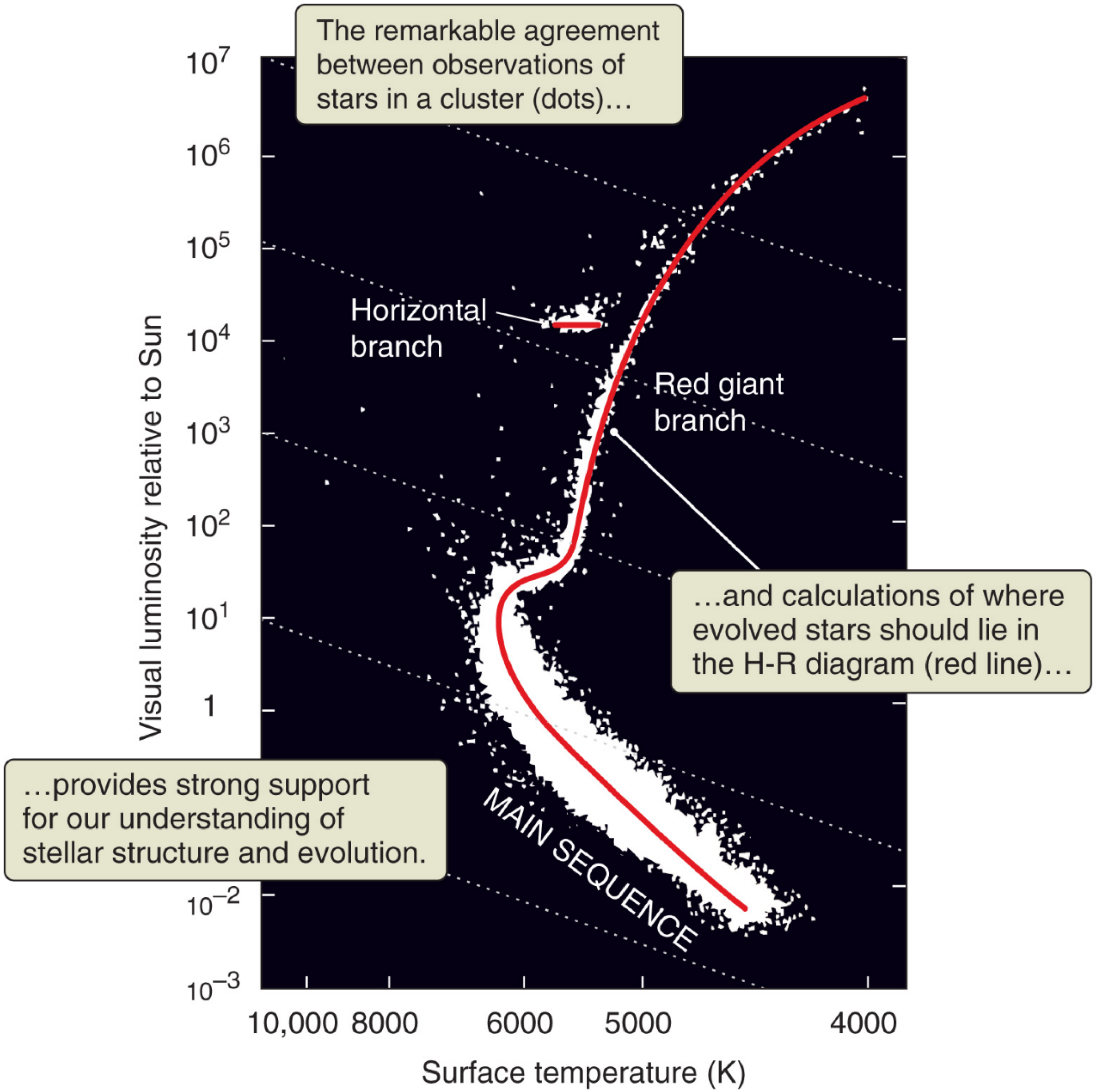
Using the H-R Diagram to Determine the Age of a Star Cluster



Using the H-R Diagram to Determine the Age of a Star Cluster



Summary: Model Isochrones vs. Cluster Data



By comparing the distribution of cluster stars on the HR diagram and model isochrones, we can (1) fine-tune stellar evolution models, and (2) estimate age and chemical composition of clusters

Introducing complex stellar systems

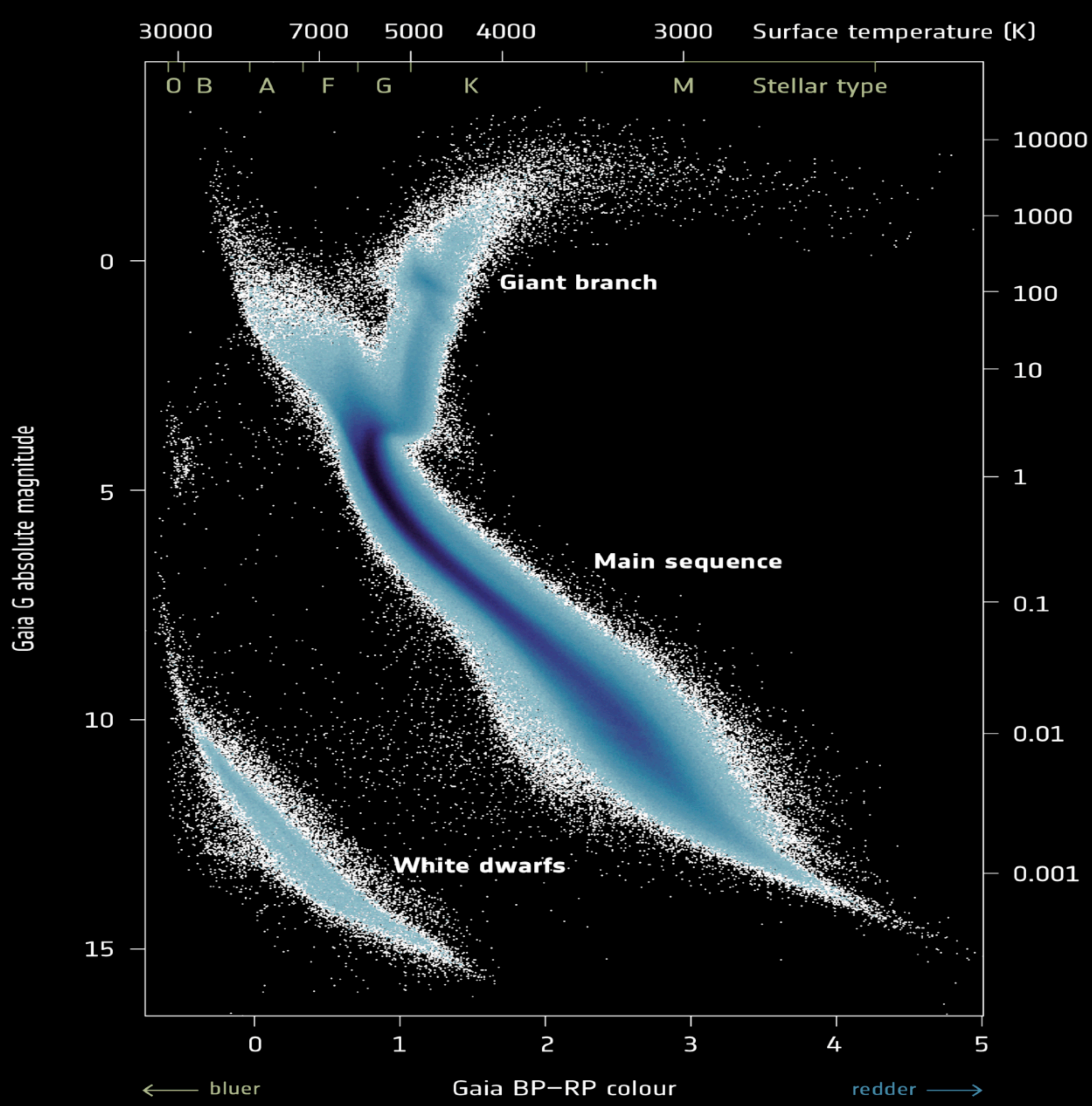
Solar neighborhood stars' H-R diagram

Practice - Draw the HR diagram of solar neighborhood stars

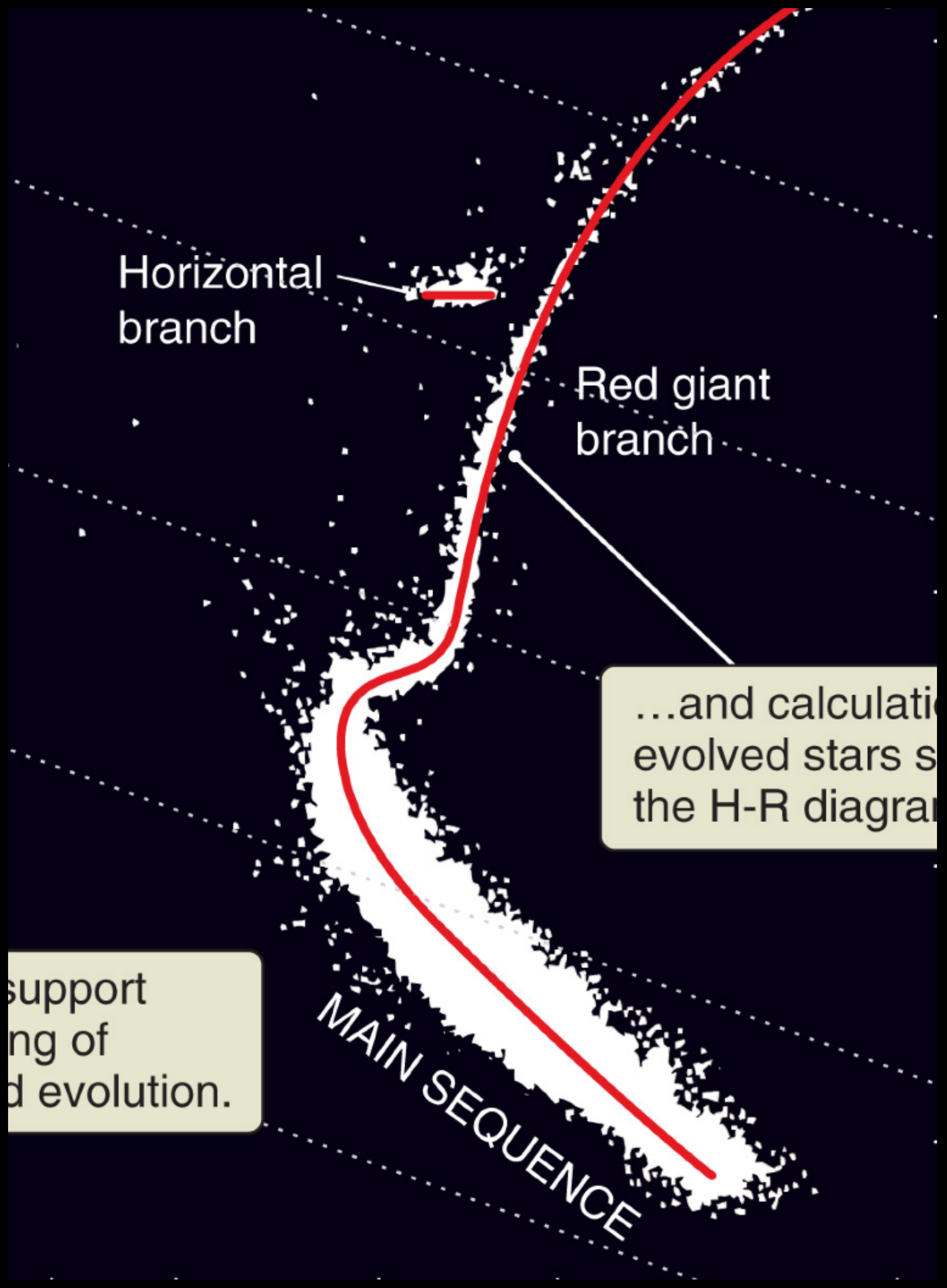
- draw and label the axes
- Indicate the direction in which each property increases
- draw the lines connecting stars of the same radii
- draw the three main features where the distribution of stars concentrate
- label the three main features

What does the HR diagram of Solar Neighbors tell us?

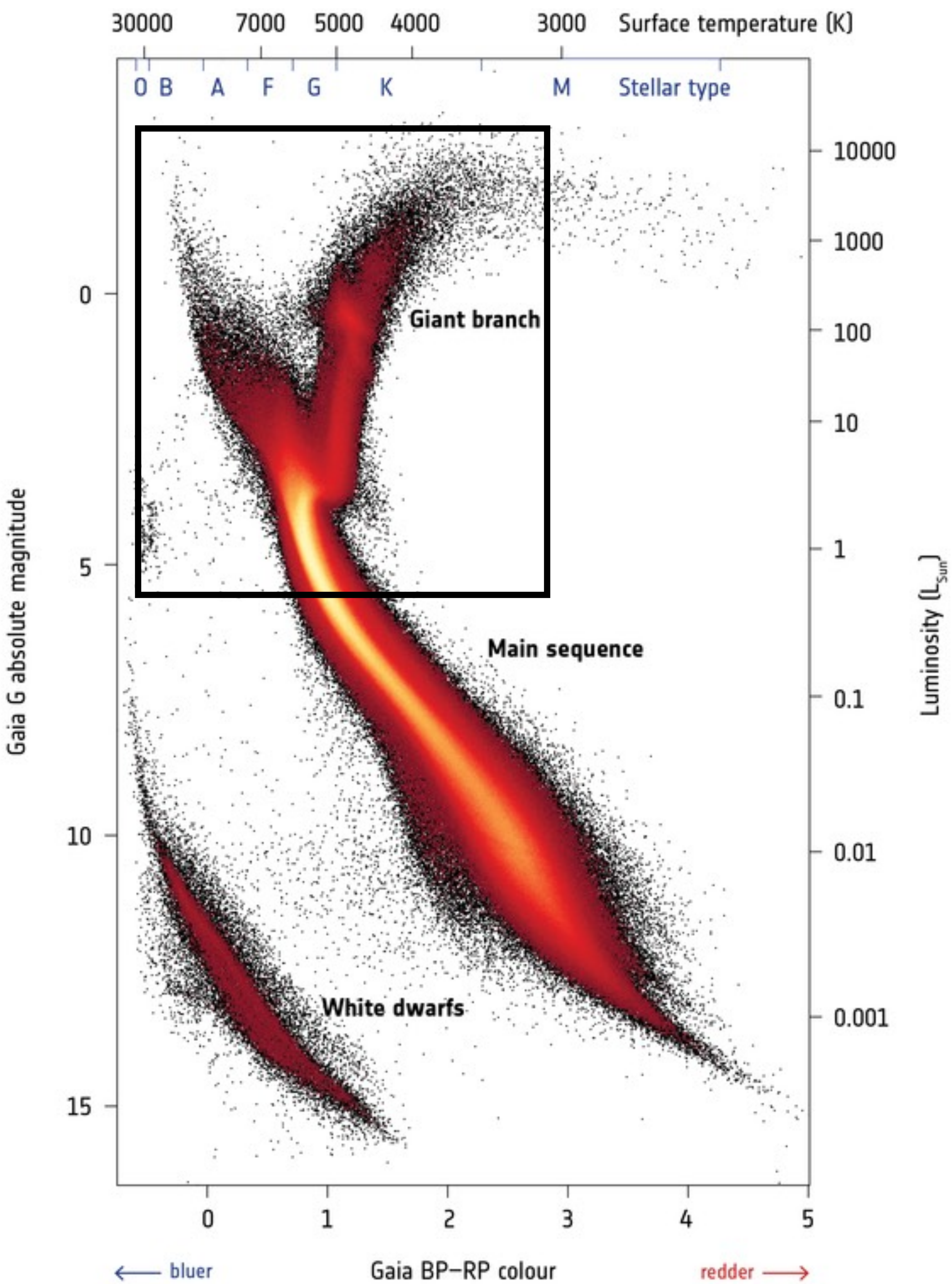
Solar neighborhood HR diagram

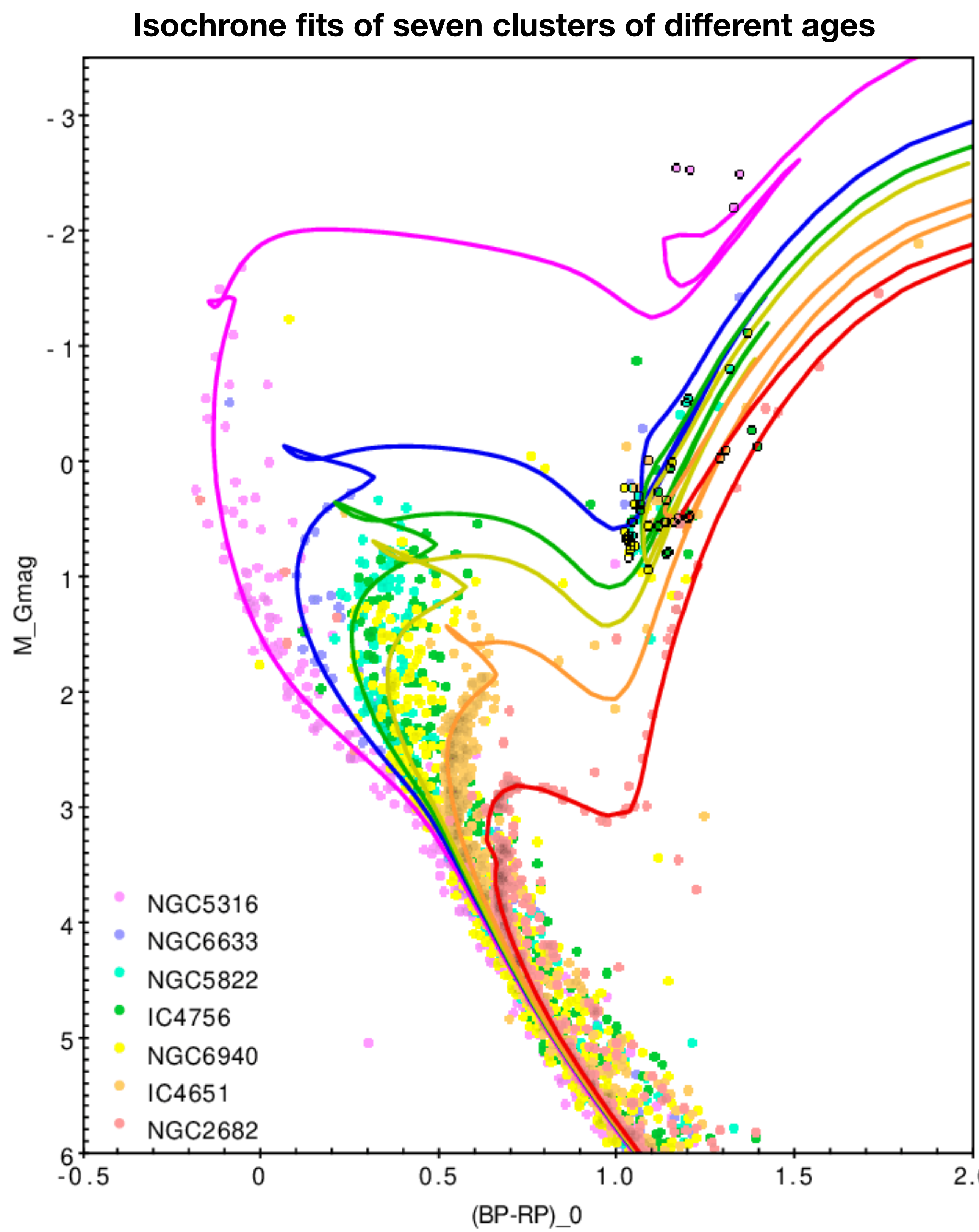


Cluster HR diagram



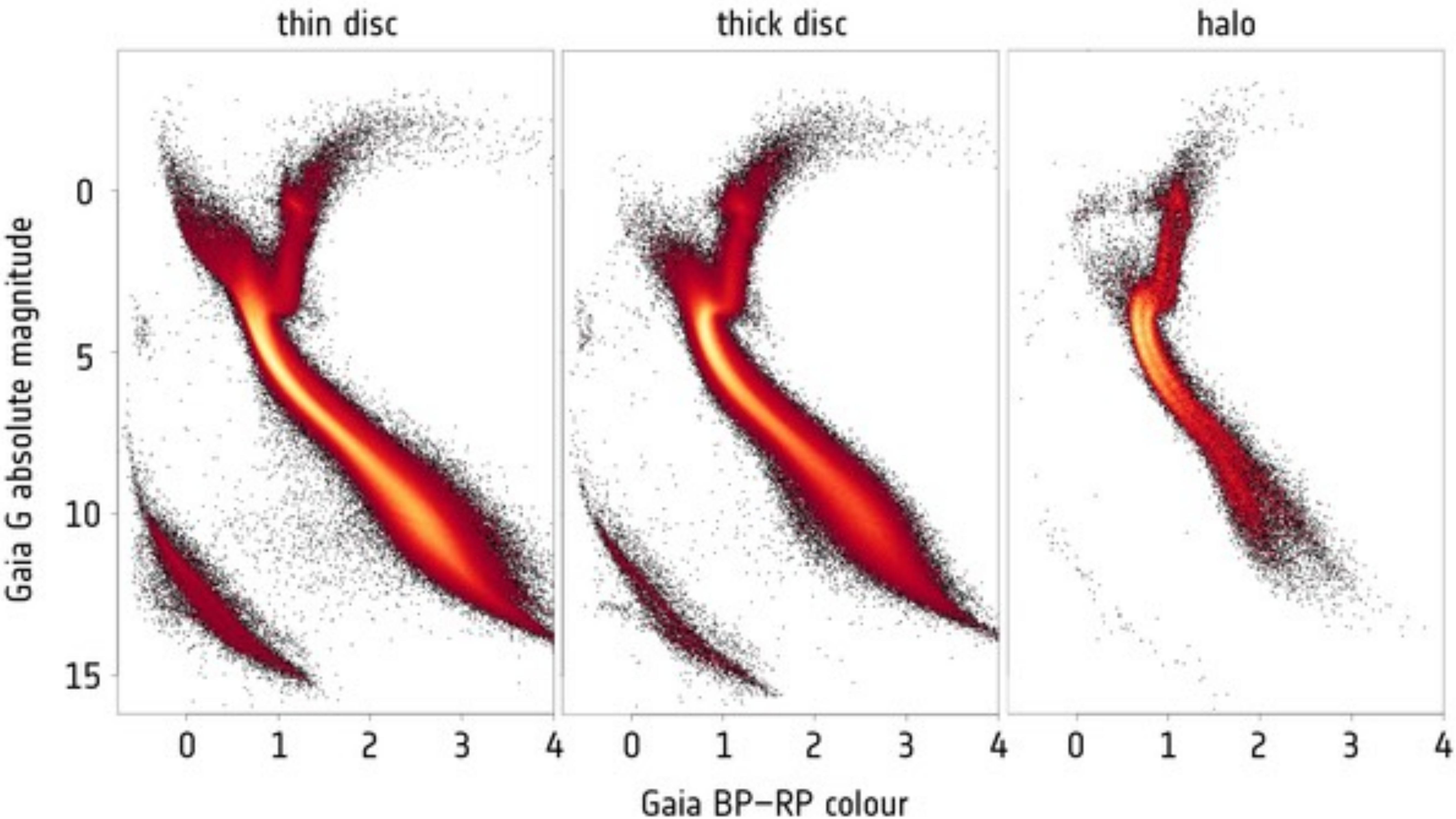
The Solar neighborhood stars are a mixed stellar population, and its HR diagram can be understood as a combination of multiple isochrones



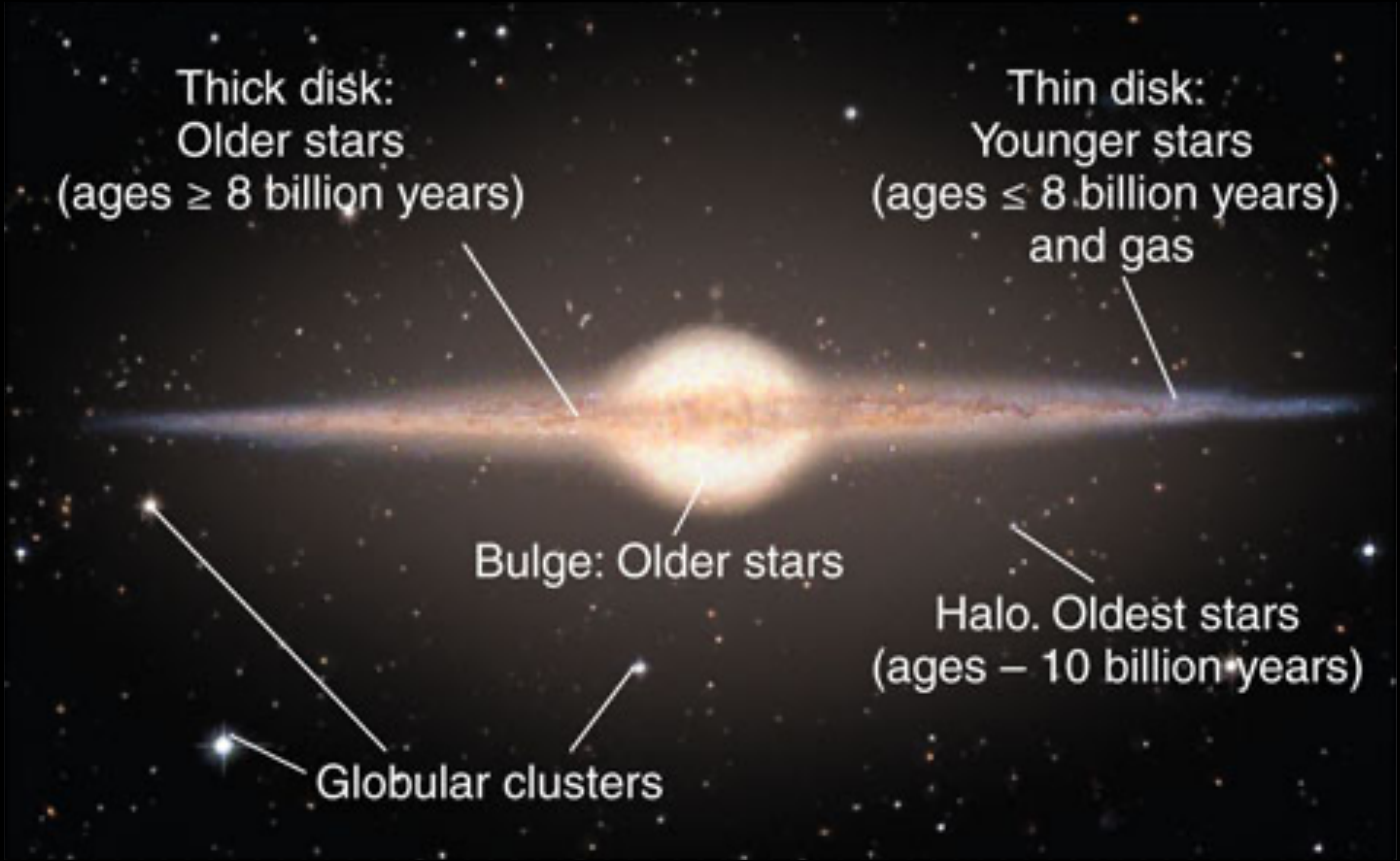


Solar neighborhood stars tell us the formation history of the Galaxy

HR diagram breakdown based on kinematics



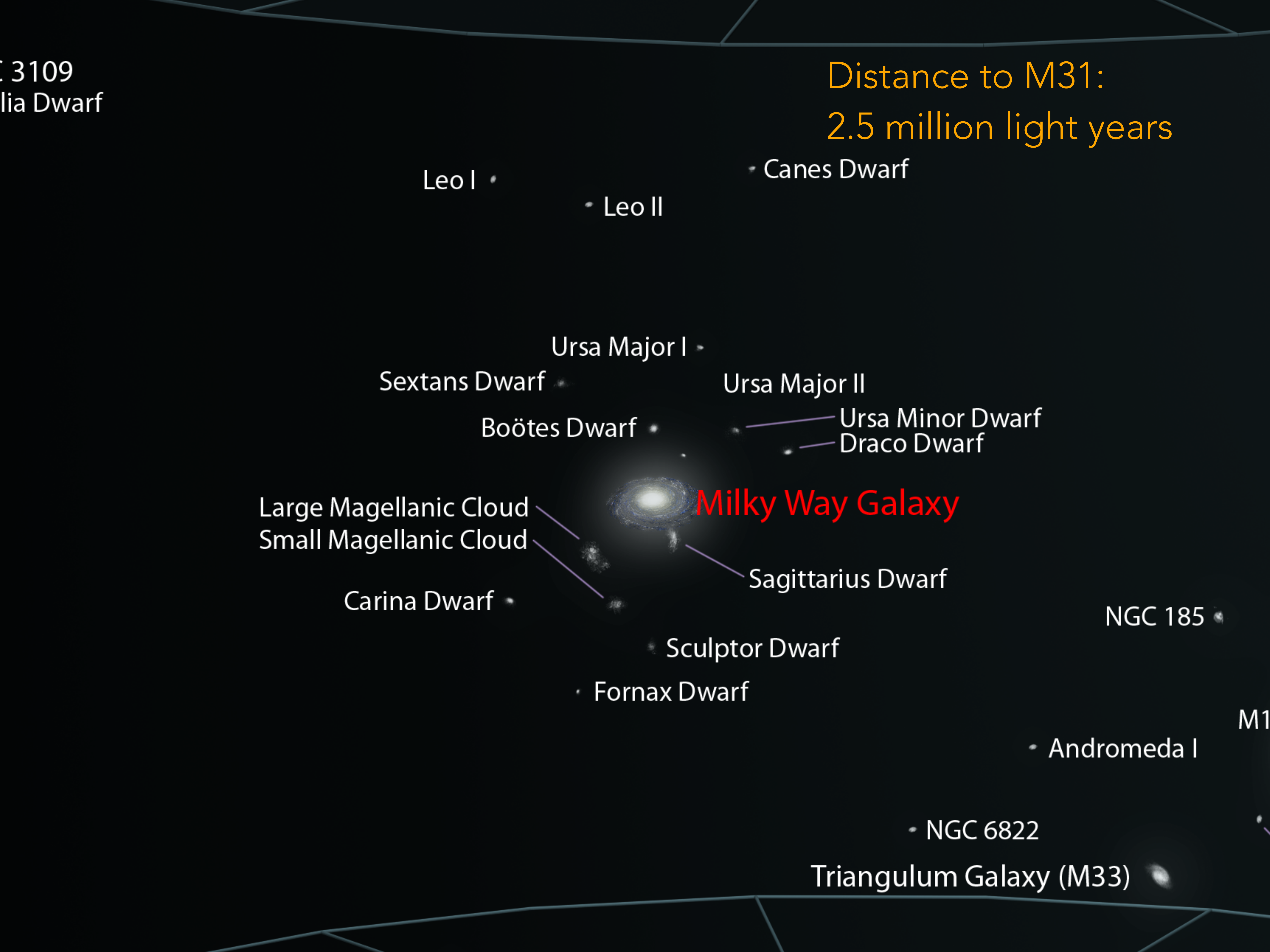
The study of Solar neighbors' HR diagram made us realize that ``Rome wasn't built in a day"



Note: a significant fraction of stars in the Milky Way actually formed in other galaxies

Small galaxies get shredded by large galaxies

Inside the Milky Way's Halo: Tidally Stripped Stars from Dwarf Galaxies



Distance to M31:
2.5 million light years
: 3109
lia Dwarf
Leo I
Leo II
Canes Dwarf
Ursa Major I
Sextans Dwarf
Ursa Major II
Ursa Minor Dwarf
Draco Dwarf
Boötes Dwarf
Milky Way Galaxy
Large Magellanic Cloud
Small Magellanic Cloud
Sagittarius Dwarf
Carina Dwarf
NGC 185
Sculptor Dwarf
Fornax Dwarf
M1
Andromeda I
NGC 6822
Triangulum Galaxy (M33)

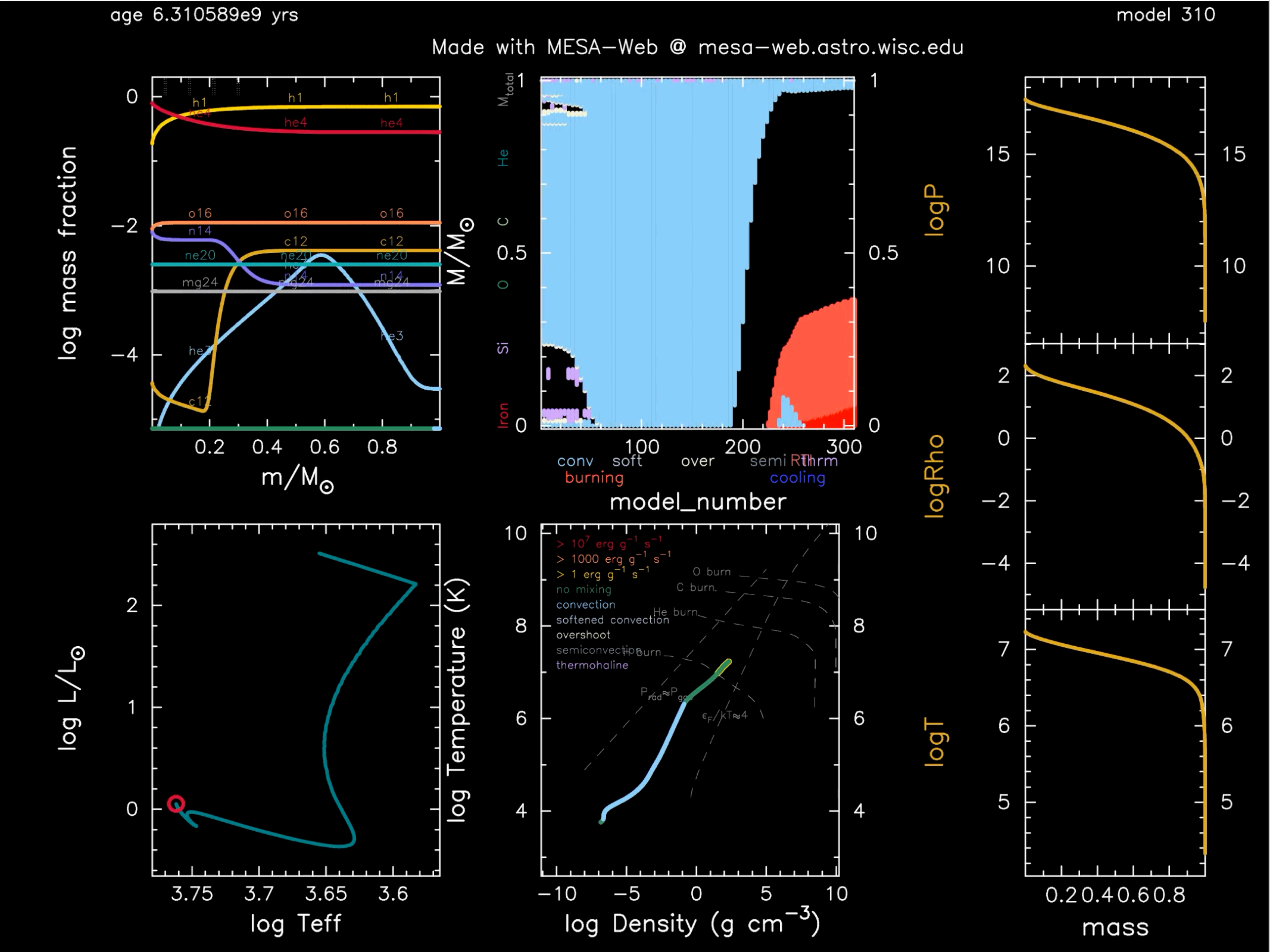
The Milky Way and its neighbour Andromeda
are destined to merge within the next 5 billion years

...or so.

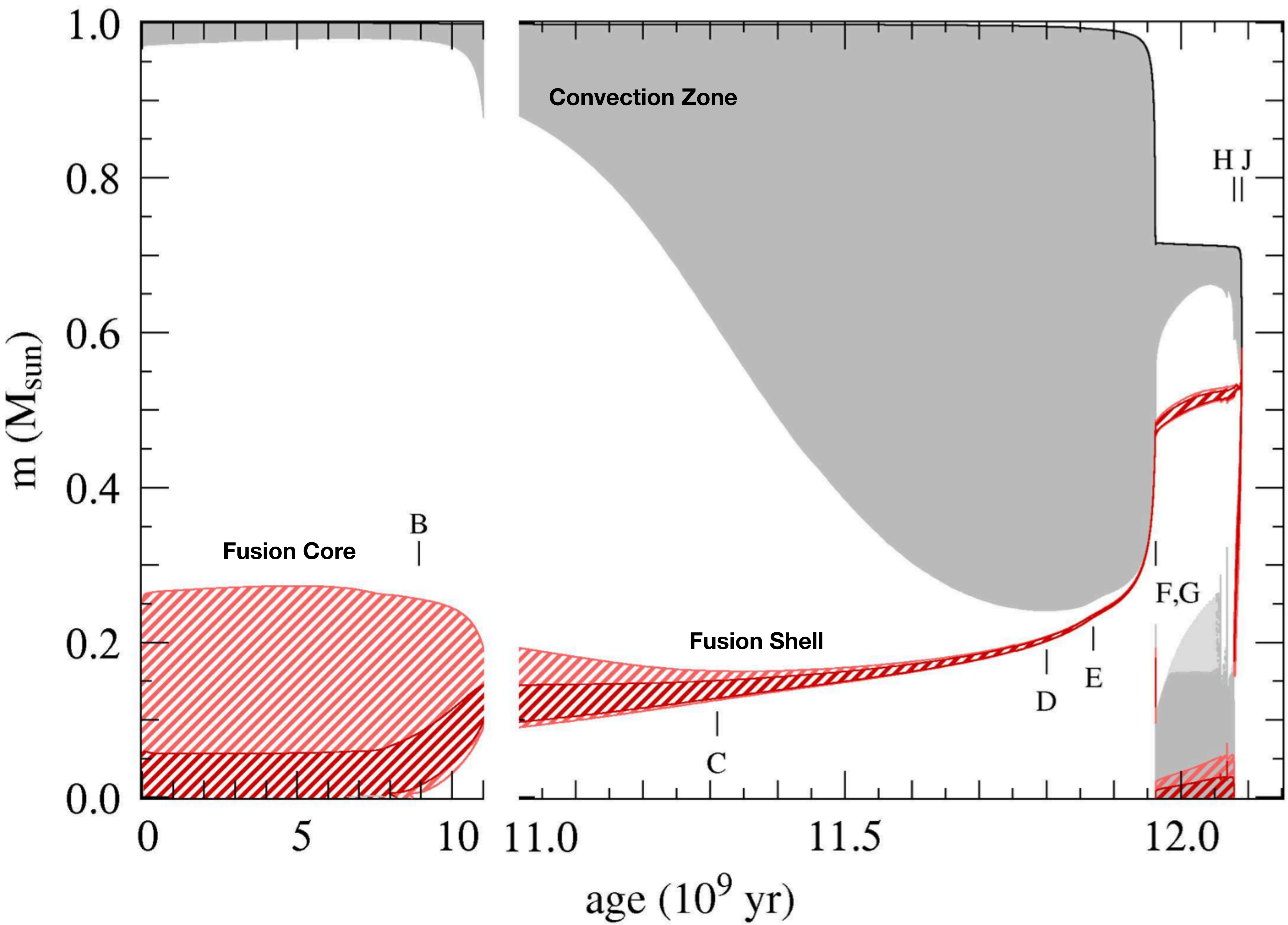
This simulation shows what might happen
to the gas (shades of blue) and newly formed stars (red)
when the two galaxies come together.

The evolution of stars described by the best-fit numerical models

A Summary of Model Inferences

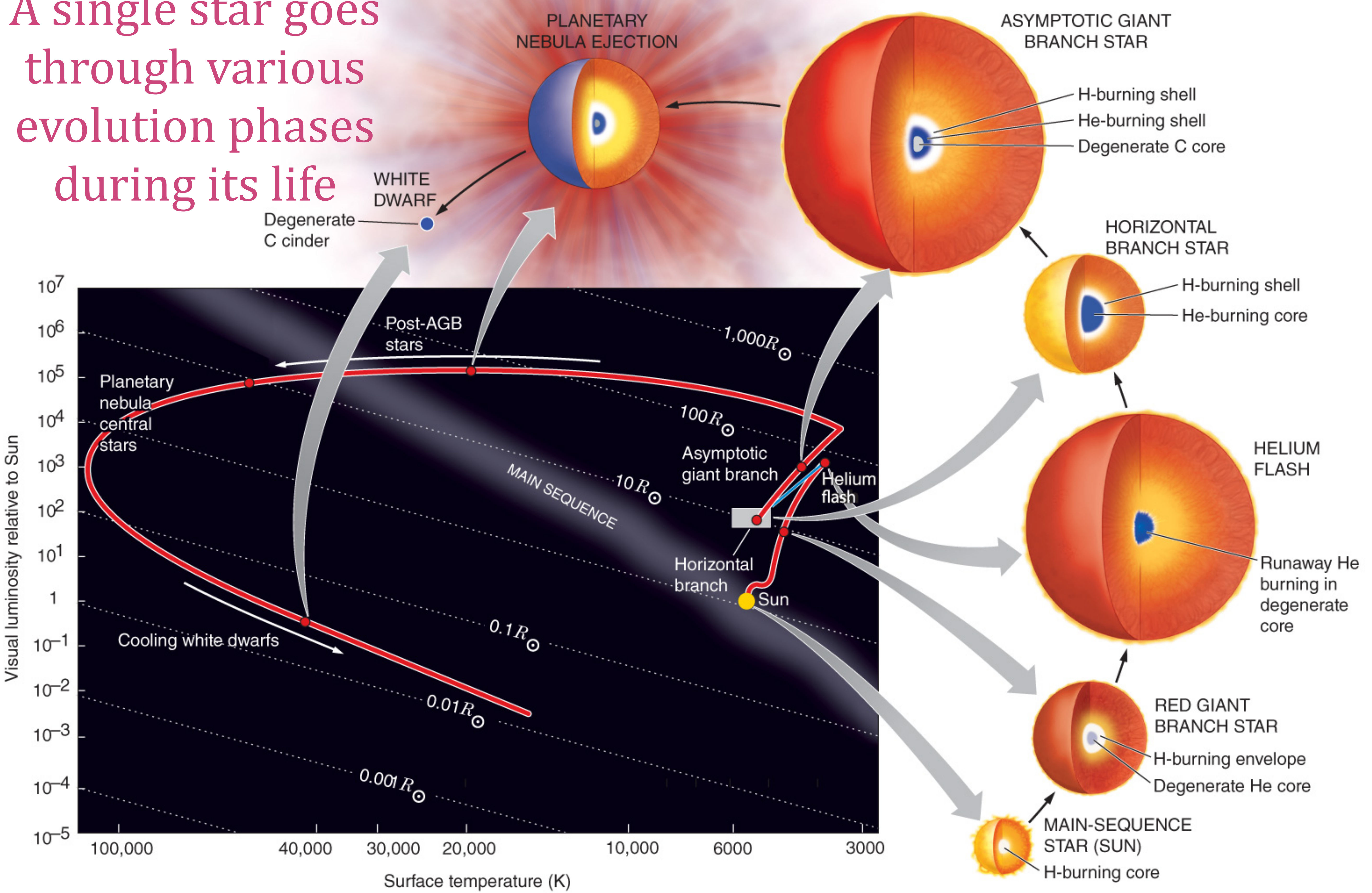

age 6.310589e9 yrs
model 310
Made with MESA-Web @ mesa-web.astro.wisc.edu
log mass fraction
m/M☉
h1
he4
o16
n14
c12
ne20
mg24
he3
model_number
M/M☉
conv
soft
over
semi
thrm
burning
cooling
logP
logRho
logT
mass
log L/L☉
log Teff
log Temperature (K)
log Density (g cm⁻³)
no mixing
convection
softened convection
overshoot
semiconvection
thermohaline
O burn
C burn
He burn
H burn

Kippenhahn Diagram of a Star with an Initial Mass of 1.0 Solar Mass



The evolutionary track of a 1 Solar mass star

A single star goes through various evolution phases during its life

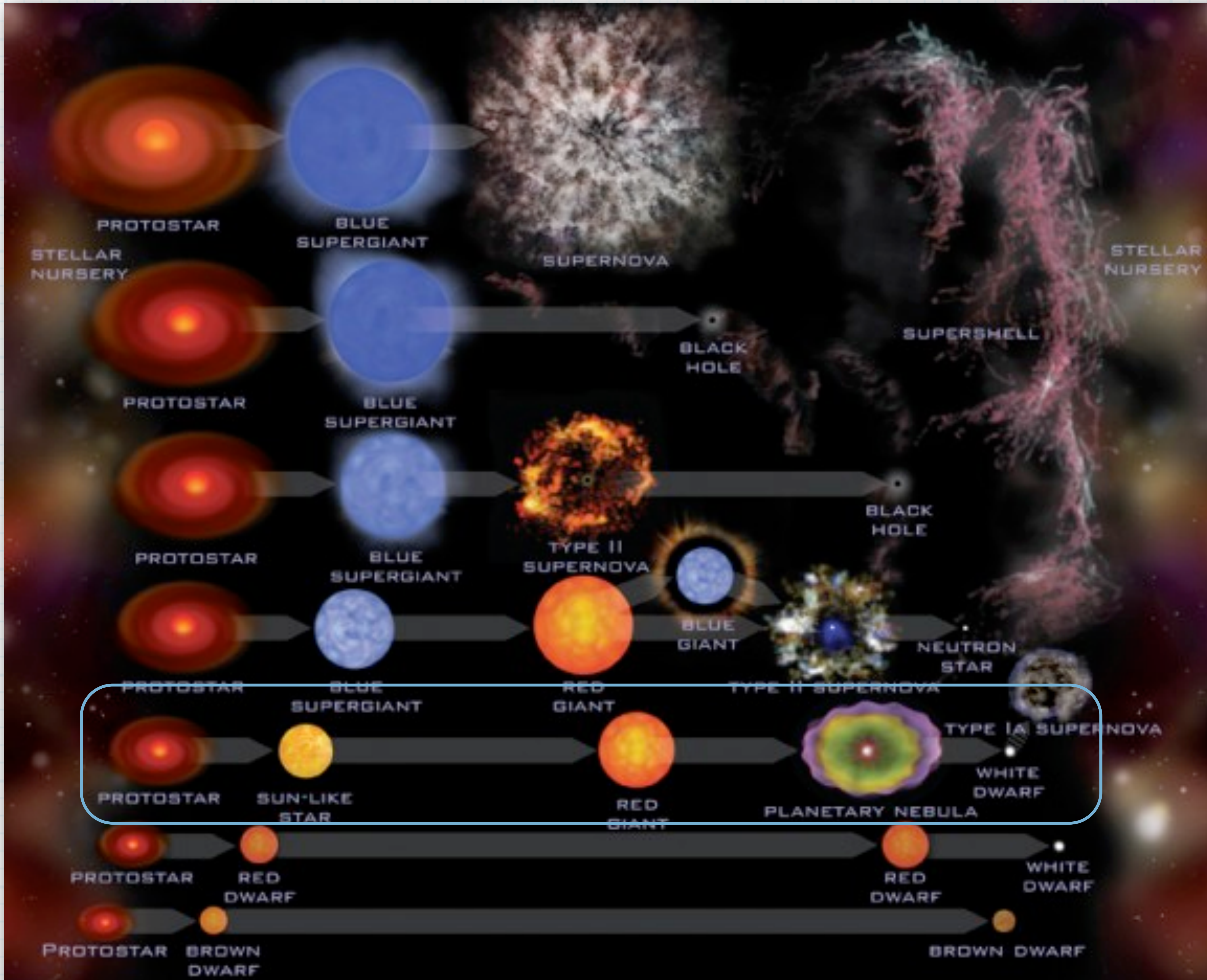


Stellar Evolution: Big Picture

high mass stars

low mass stars

Substellar objects



Six main stages of the evolution of low-mass stars (M < 8 M_{sun})

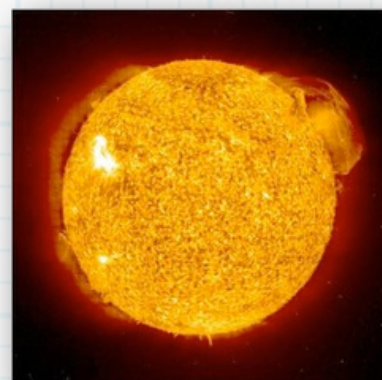
time quoted for a star as massive as the Sun

Star formation region

~10 Myr

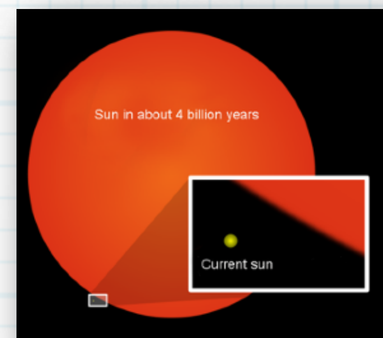
Protostar

~10 Myr



Main sequence

~10 Gyr

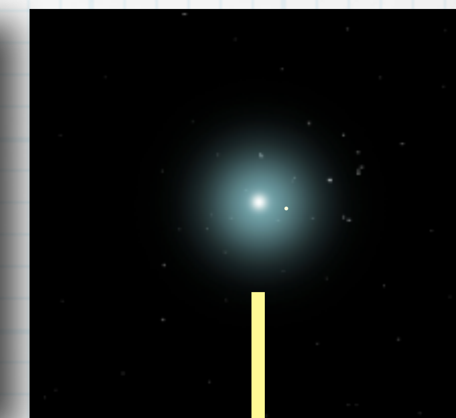


Red Giant

~1 Gyr

Planetary nebula

~10 Kyr



White dwarf

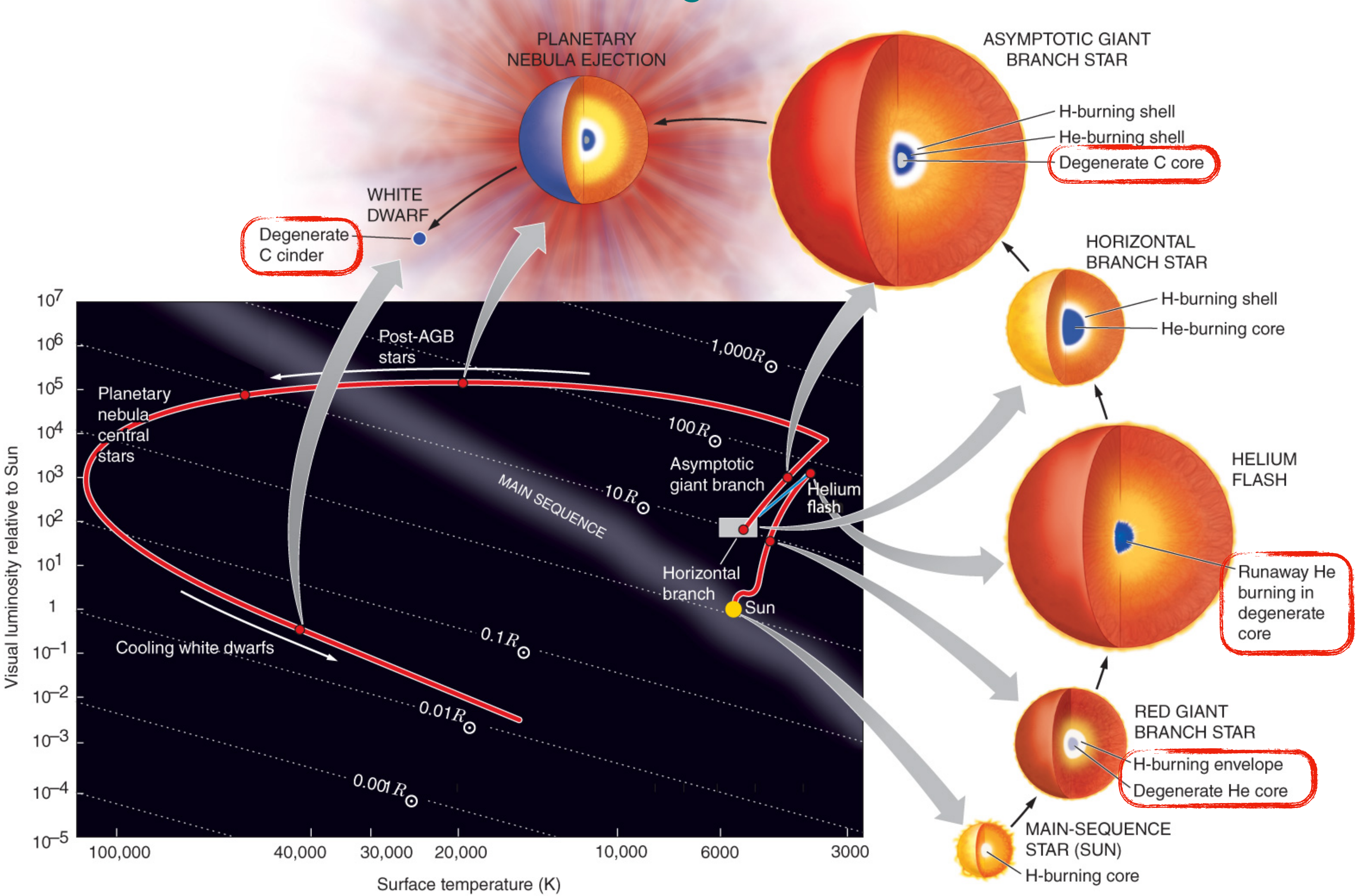
eternal

Sunrise in 7 Billion Years

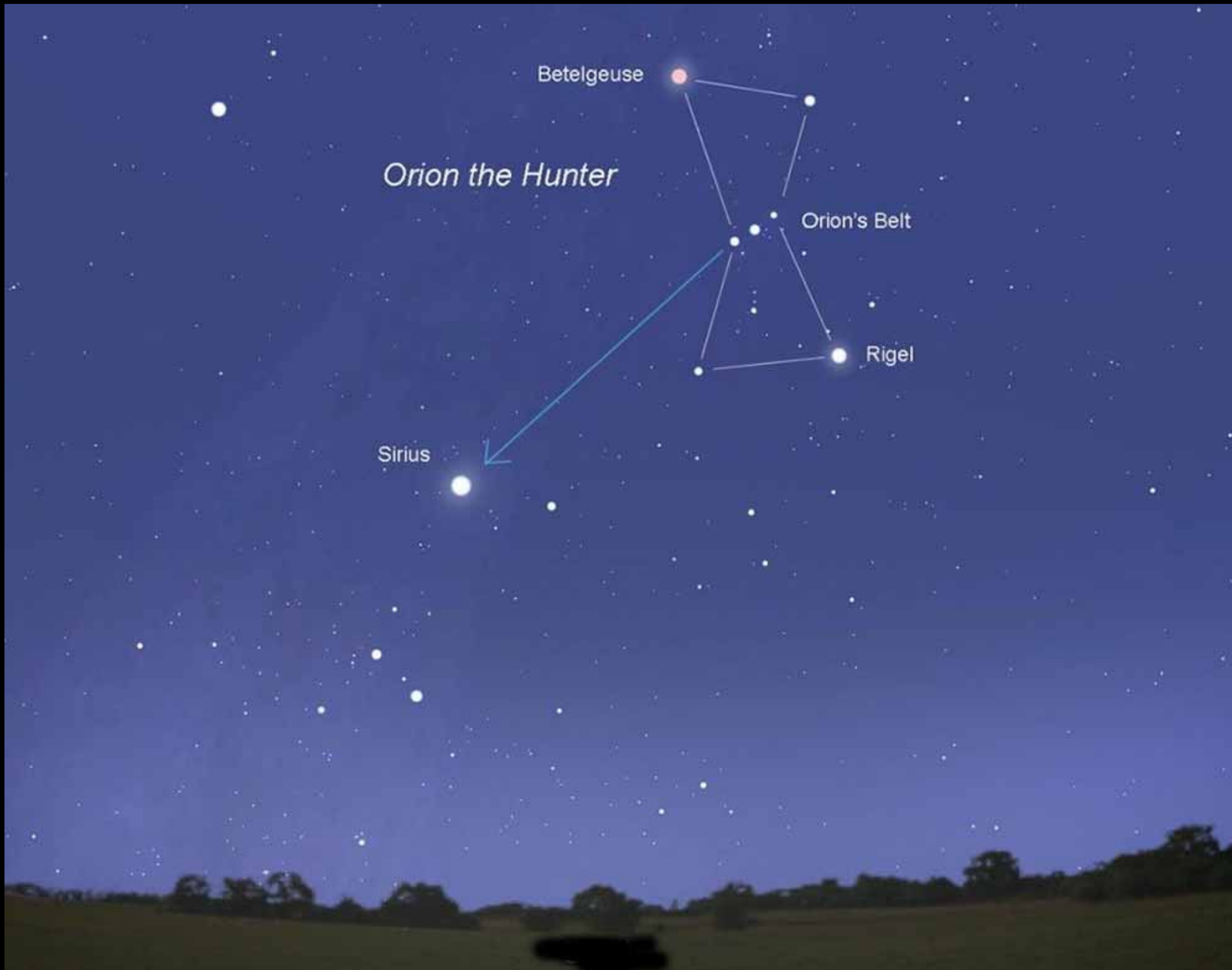
A key prediction of the model is a degenerate core buried at the center of a post-MS star

What is degeneracy? Why is it important?

But white dwarfs are the end state of low-mass star evolution, how did the stellar core first become degenerate?

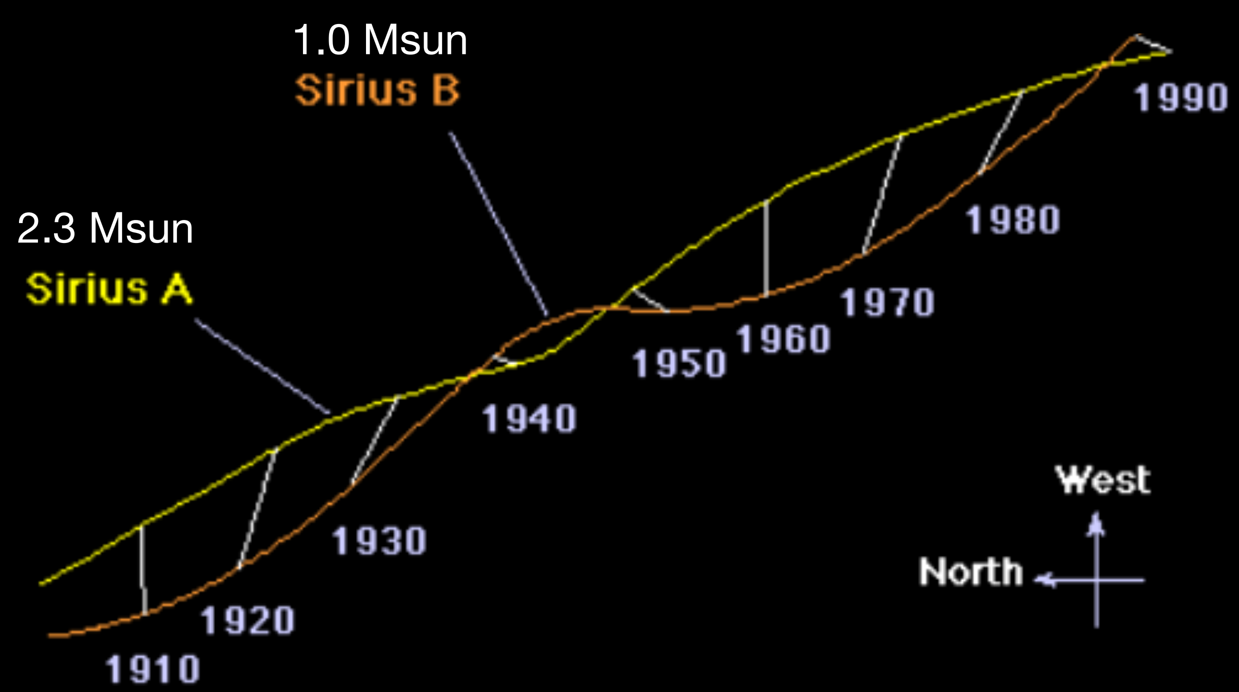


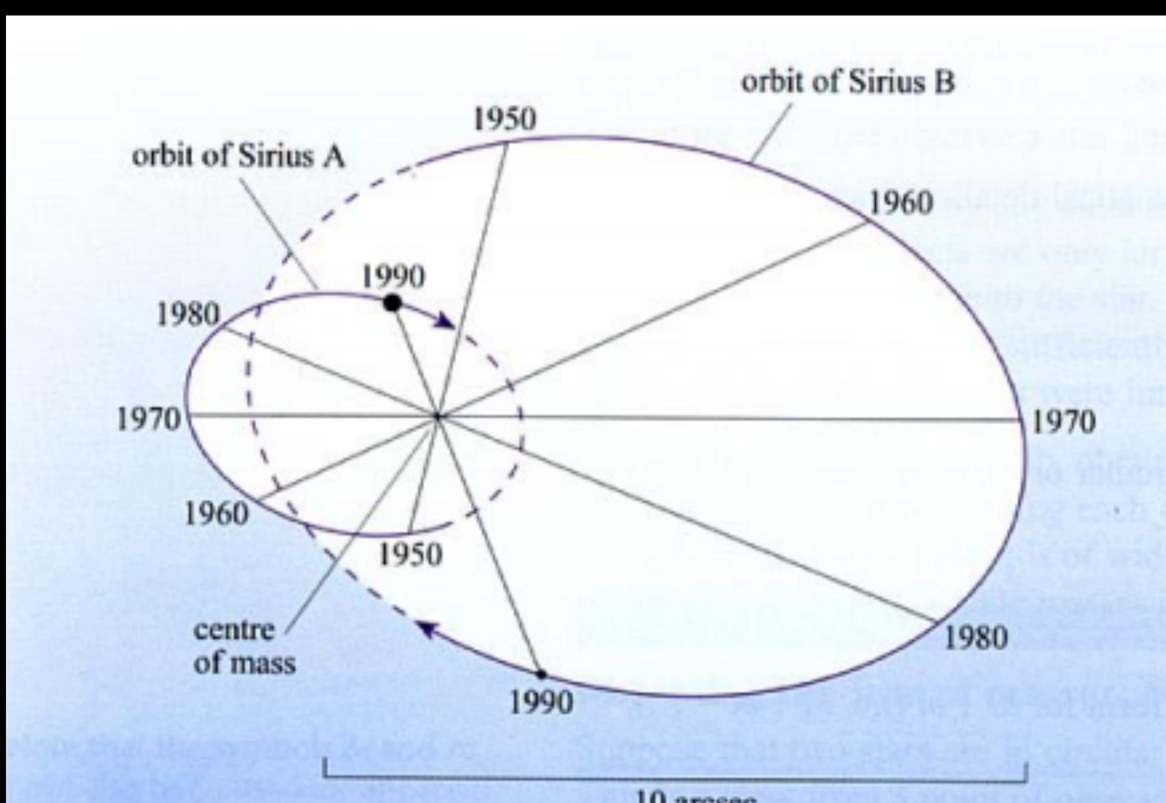
The story of degenerate gas started with the study of Sirius, the brightest star in the night sky



Sirius B - the weird companion of the Dog Star

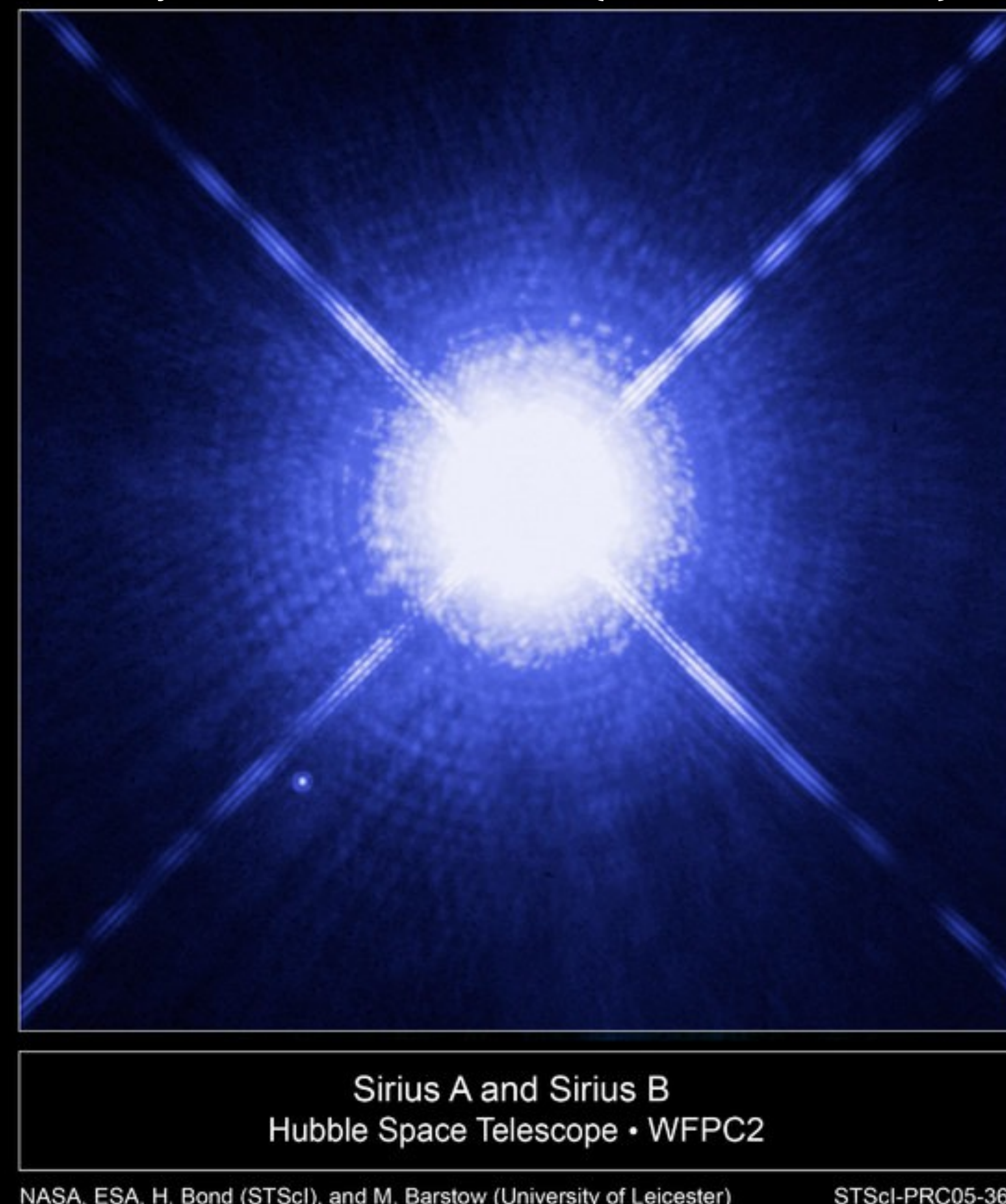
50 year orbit of the binary first inferred by Bessel in 1844
same as in the first stellar parallax measurement of 61 Cygni



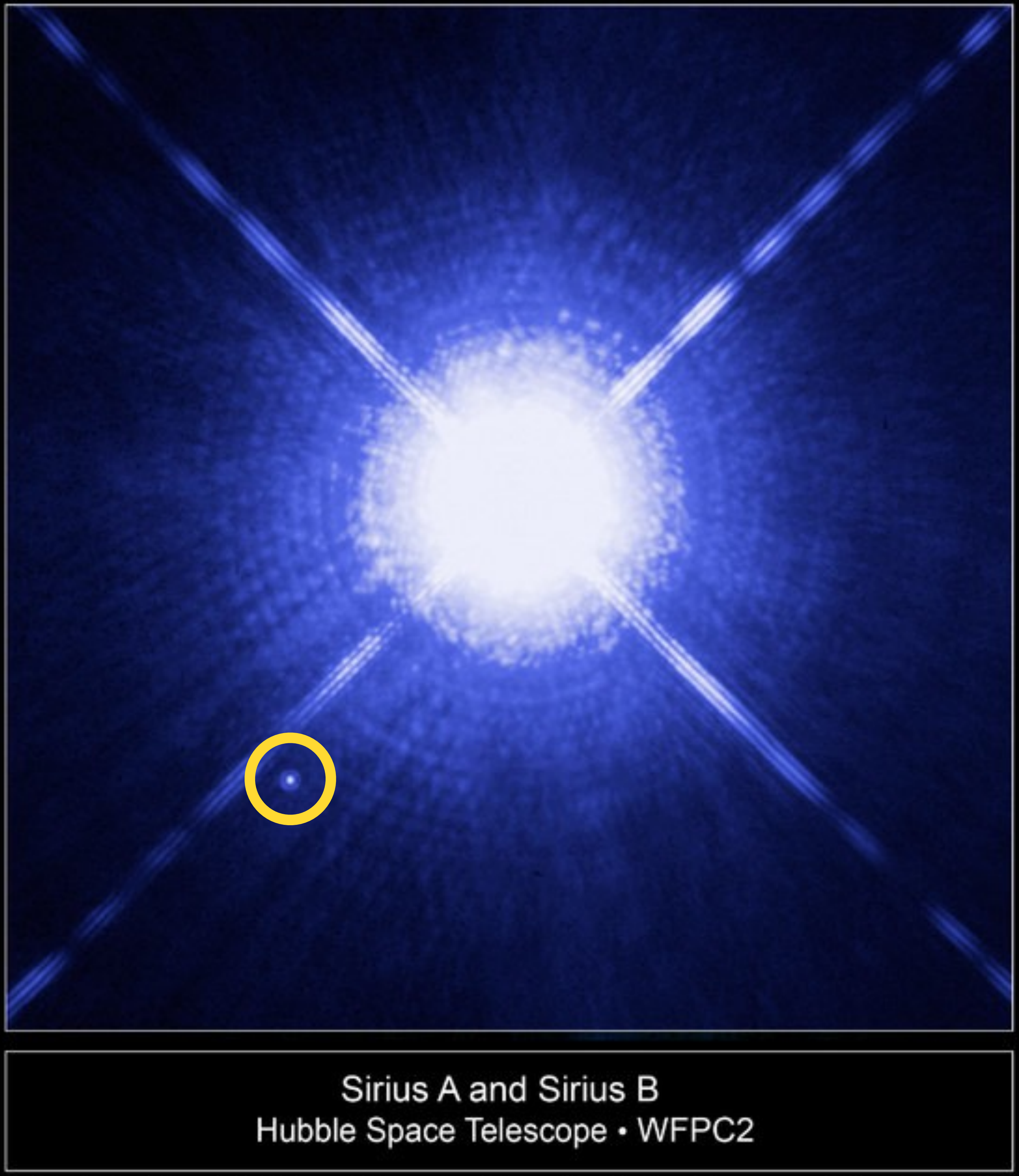


From the orbital motion, it was estimated that
A has 2.3 solar mass and B has 1.0 solar mass

Sirius B is much fainter than Sirius A, is it surprising that if
I tell you that it's much hotter (27000 K vs. 9900 K)?



Sirius B - the weird companion of the Dog Star

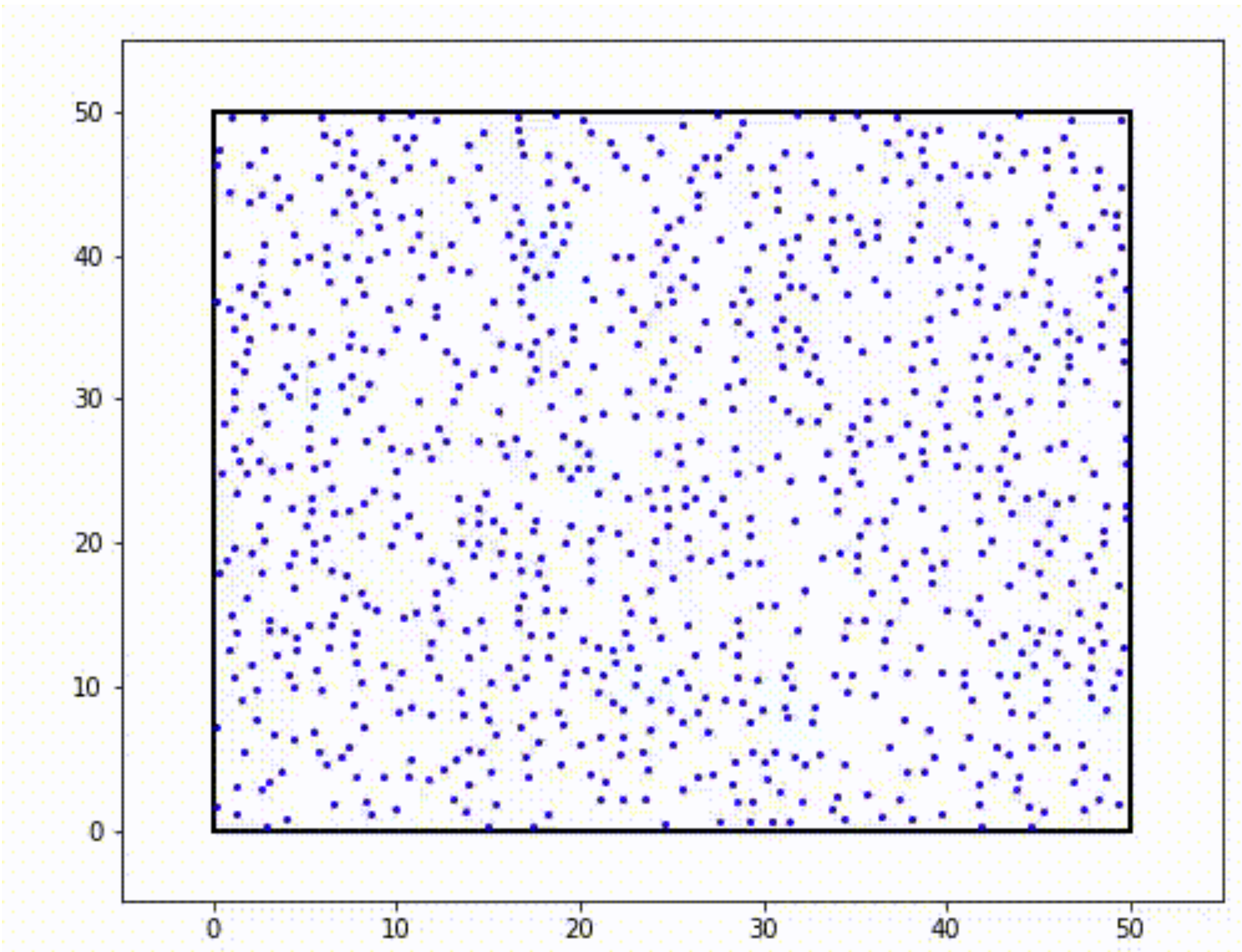


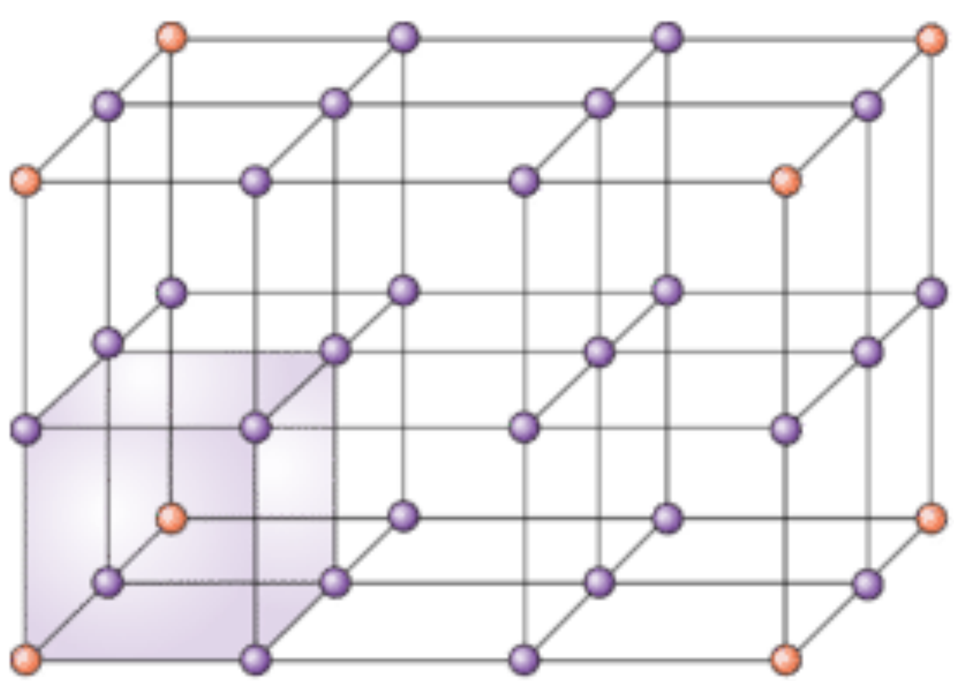
NASA, ESA, H. Bond (STScI), and M. Barstow (University of Leicester) STScI-PRC05-36a

- Inferred properties of Sirius B:
  - 1 Solar Mass
  - 0.03 Solar Luminosity
  - 27,000 K surface temperature
  - 5500 km radius (Earth-size)
- Sirius B represent a class of objects called **White Dwarfs (WDs)**

- The physical conditions of WDs are extreme:
  - extreme density ($\rho \approx$ 3e9 kg/m^3) ($n_e \sim$ 1e36 /m^3)
  - extreme surface gravity (HW)
  - extreme pressure at the center

Given a number density, how to estimate the average distance between particles?





- Extremely high density of WDs:
  - mass density $\rho \approx$ 3e9 kg/m³
  - number density of electrons: $n_e \sim$ 1e36 /m³
- What is the average distance between electrons? How does it compare with the size of an atom (~0.1 nm)?
  - average distance: $\delta x = n_e^{-1/3} = 10^{-12} m$ (<< 0.1 nm, size of atom)
  - Quantum Mechanics must be important in white dwarfs.

Deriving degenerate pressure using uncertainty principle

- **Quantum mechanics** start to become important when e- and ions are packed in a very smaller volume
- **Pauli exclusion principle** (1925): no more than one fermion (leptons and baryons) can occupy the same quantum state. So the uncertainty of the fermion's position cannot be larger than their actual separation:
$$\Delta x \lesssim n^{-1/3}$$
- **Heisenberg's uncertainty principle** (1927):
$$\Delta x \Delta p \approx h/2\pi$$
the smaller the uncertainty in position, the larger the uncertainty in momentum.
- Combining the two and approximating $p \approx \min(\Delta p)$ (ground state):
$$p \approx h n^{1/3}/2\pi$$
- Just like ideal gas, the pressure from degenerate gas is 2/3 the kinetic energy density:
$$P_{\rm degen} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

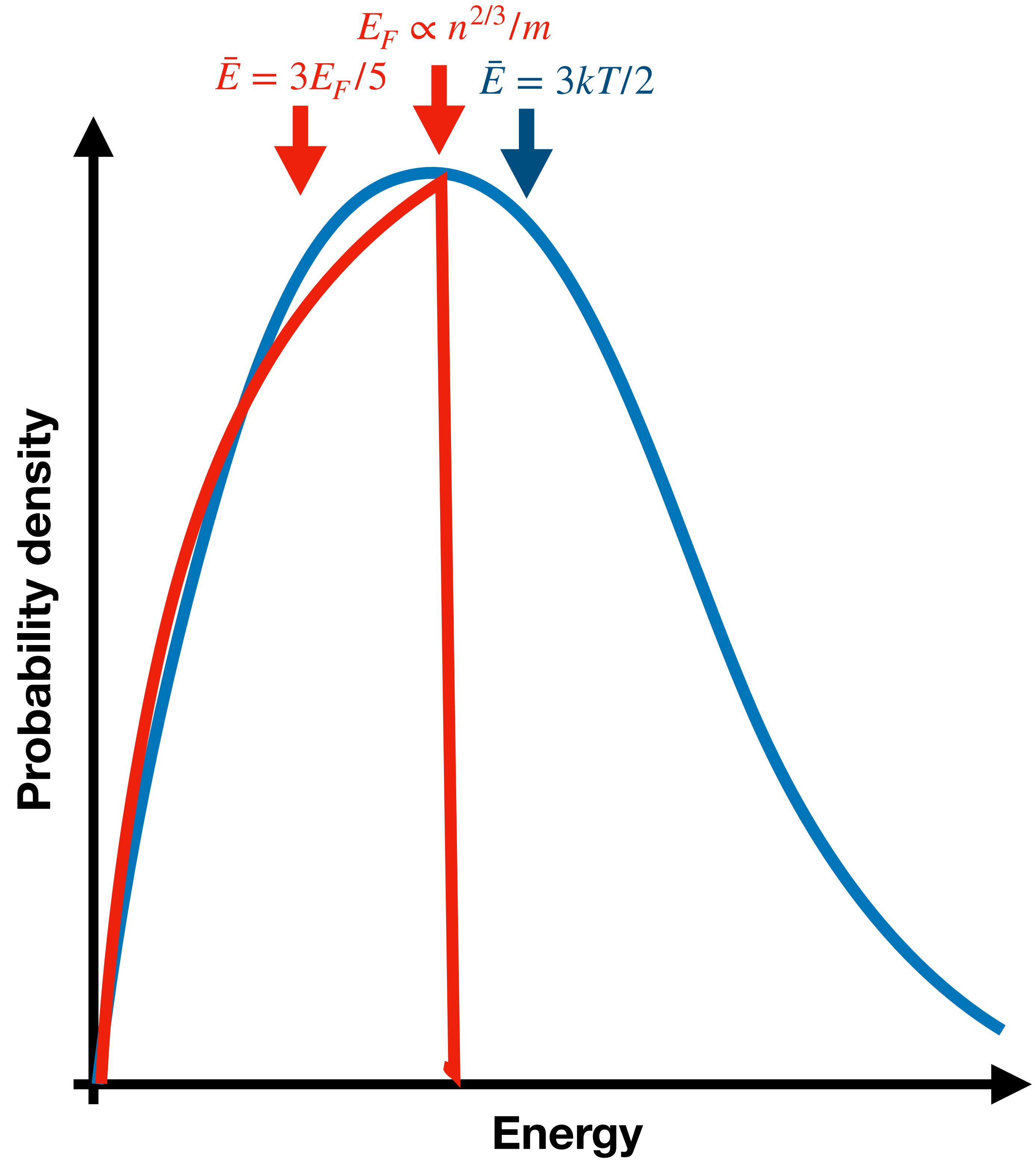
Understanding degenerate pressure using Fermi-Dirac Distribution

- Fermions (half-integer spins; e.g., e-, p+, n, nu) follow **Fermi-Dirac distribution**, where the probability of a particle having an energy between *E* and *E+dE* is:

$$f_E dE \propto \frac{E^{1/2} dE}{e^{(E-E_F)/kT} + 1}$$

- when $kT >> E_F$, it is indistinguishable from the classic Maxwell-Boltzmann distribution: $f_E dE \propto \dfrac{E^{1/2} dE}{e^{E/kT}}$
  The *mean kinetic energy* is $3kT/2$, as a result, pressure depends on both density and temperature

- when $kT << E_F$, all energy states greater than the Fermi energy become unoccupied.
  The *mean kinetic energy* is $3E_F/5$, as a result, pressure depends only on density



The Condition for Degeneracy: Pressure Comparison

- The pressure from non-relativistic degenerate gas is:

$$P_{\text{degen}} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

- The pressure from ideal gas is:

$$P_{\text{ideal}} = \frac{2}{3} n \left(\frac{3}{2} kT \right) = nkT$$

- The condition for degeneracy is simply:

$$P_{\text{degen}} > P_{\text{ideal}}$$

which can be simplified as:

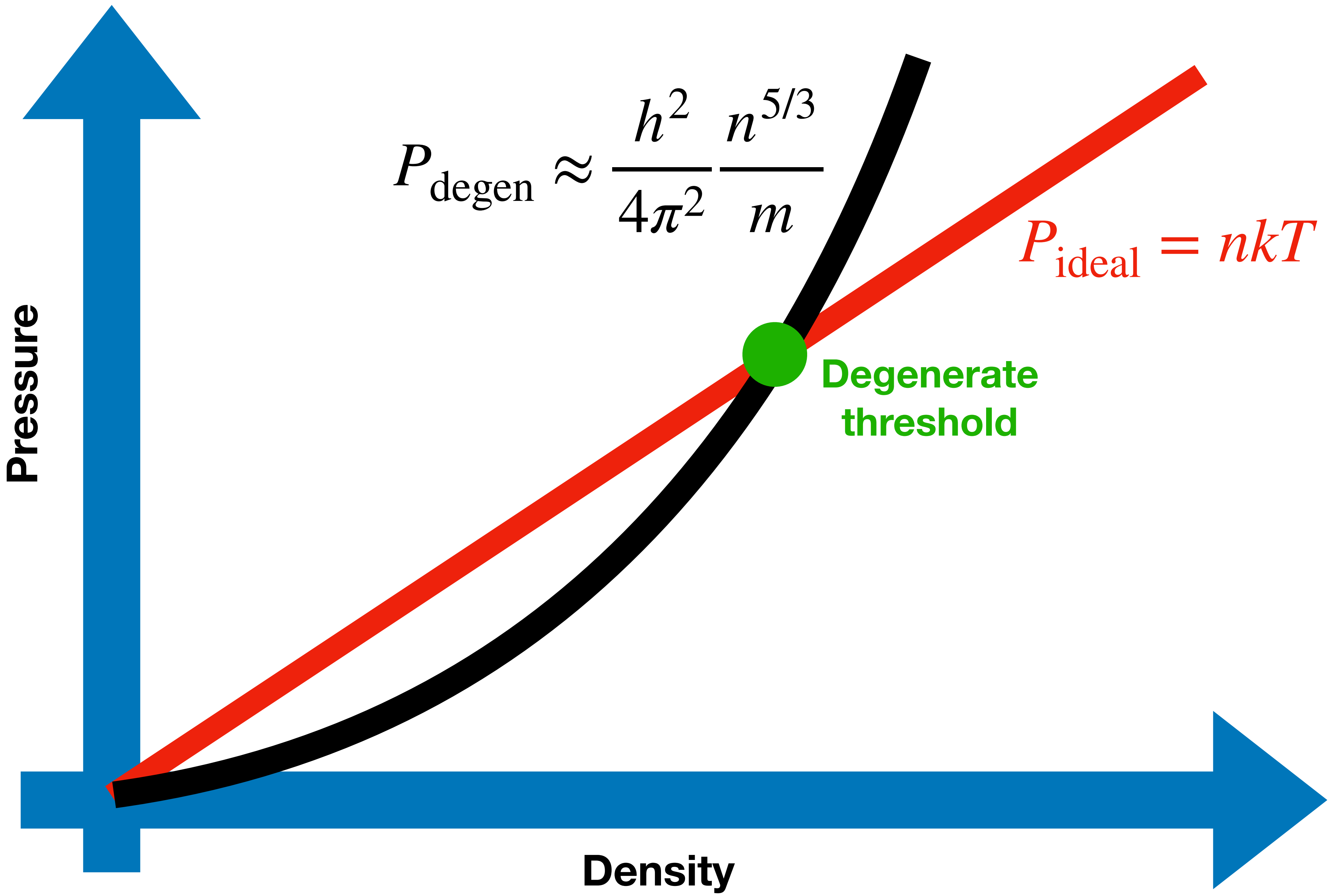
$$\frac{h^2}{4\pi^2} \frac{n^{2/3}}{m} > kT$$

or equivalently (expressed using Fermi energy):

$$kT \lesssim 0.2 \; E_{\text{Fermi}}$$

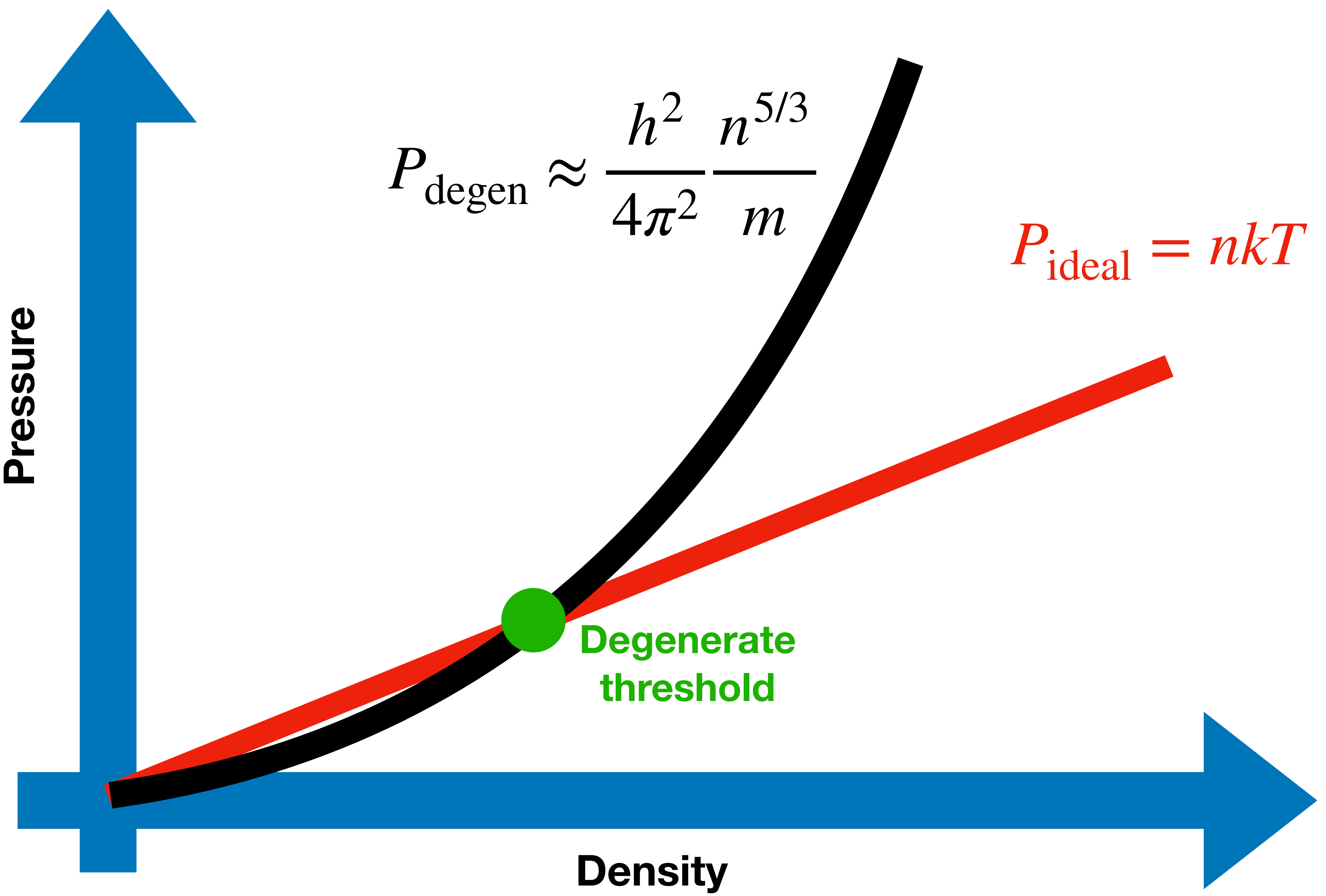
What is degenerate pressure? How to understand it intuitively?

- The contraction of non-fusing cores packs a large amount of mass into a small volume. Each electron finds its position well constrained, which leads to large momentum and kinetic energy due to the **uncertainty principle**. **Pressure**, as the **density** of **kinetic energy**, increases rapidly as a result of increased **(1) number density** and **(2) kinetic energy per particle**. This pressure due to quantum mechanics is called **degenerate pressure**.



What is degenerate pressure? How to understand it intuitively?

- The contraction of non-fusing cores packs a large amount of mass into a small volume. Each electron finds its position well constrained, which leads to large momentum and kinetic energy due to the **uncertainty principle**. **Pressure**, as the **density** of **kinetic energy**, increases rapidly as a result of increased **(1) number density** and **(2) kinetic energy per particle**. This pressure due to quantum mechanics is called **degenerate pressure**.



The Condition for Degeneracy: e- vs. ions

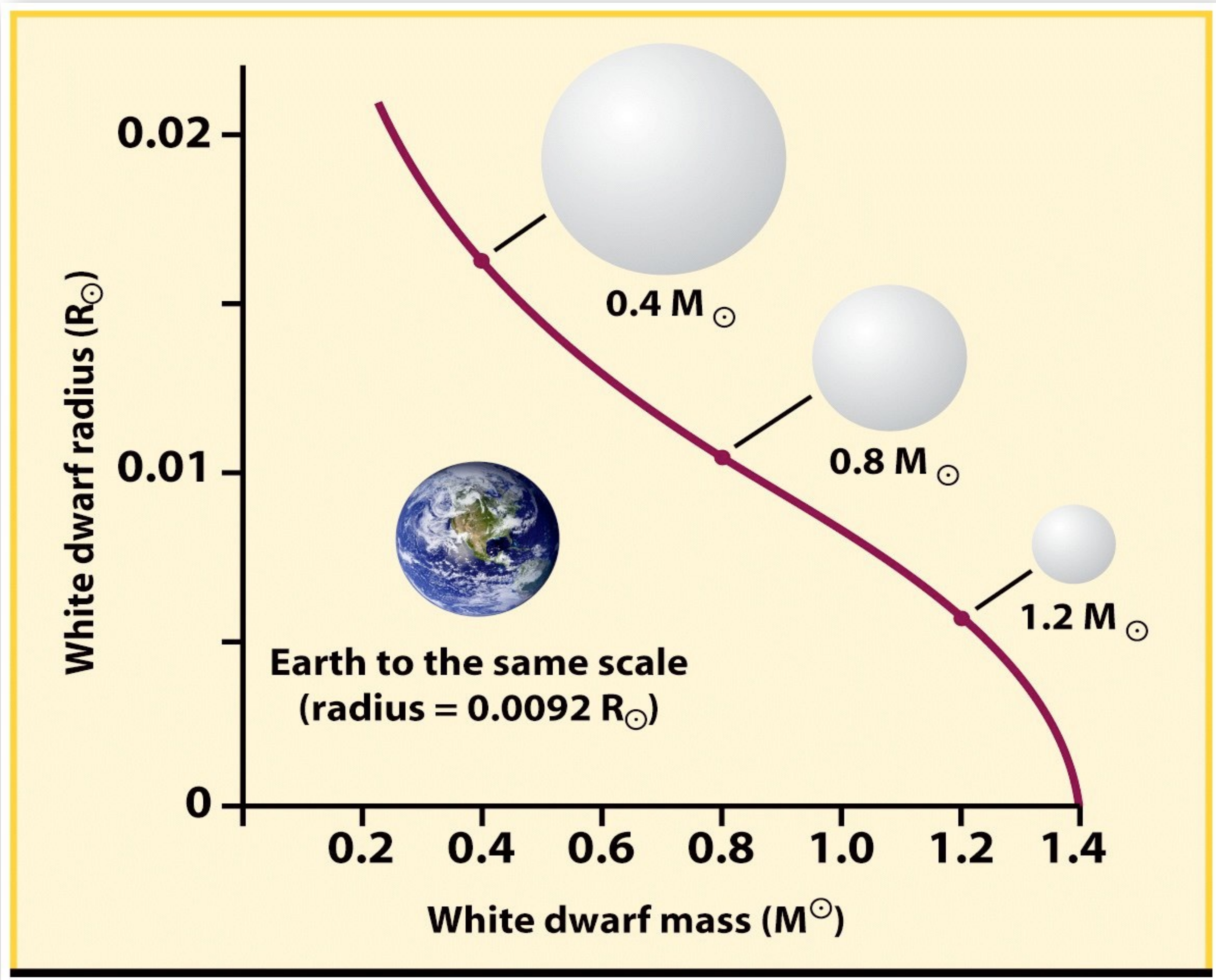
- In the previous slide, we derived the condition for degeneracy:

$$\frac{h^2}{4\pi^2}\frac{n^{2/3}}{m} > kT$$

- The above inequation can be satisfied when:
  - **the temperature is very low**, and/or
  - **the density is very high**
- In the non-fusing core of a star, the density is extremely high, reaching degeneracy condition.
- If ions and electrons share the same temperature in the core, which component will reach degeneracy first?

Mass-Size Relation of White Dwarfs

- White dwarfs show a mass-size relation of $\mathbf{R \sim M^{-1/3}}$, which is in contrast to that of main sequence stars, where $\mathbf{R \sim M^{0.8}}$



Understanding the Mass-Size relation of white dwarfs with degenerate pressure and hydrostatic equilibrium

- The pressure from non-relativistic degenerate gas is:

$$P_{\text{degen}} = \frac{h^2}{4\pi^2}\frac{n^{5/3}}{m} \propto \frac{(M/\mu m_H)^{5/3}}{R^5}$$

- Hydrostatic equilibrium provides an estimate of the central pressure:

$$P_{\text{c}} \propto G\frac{M^2}{R^4}$$

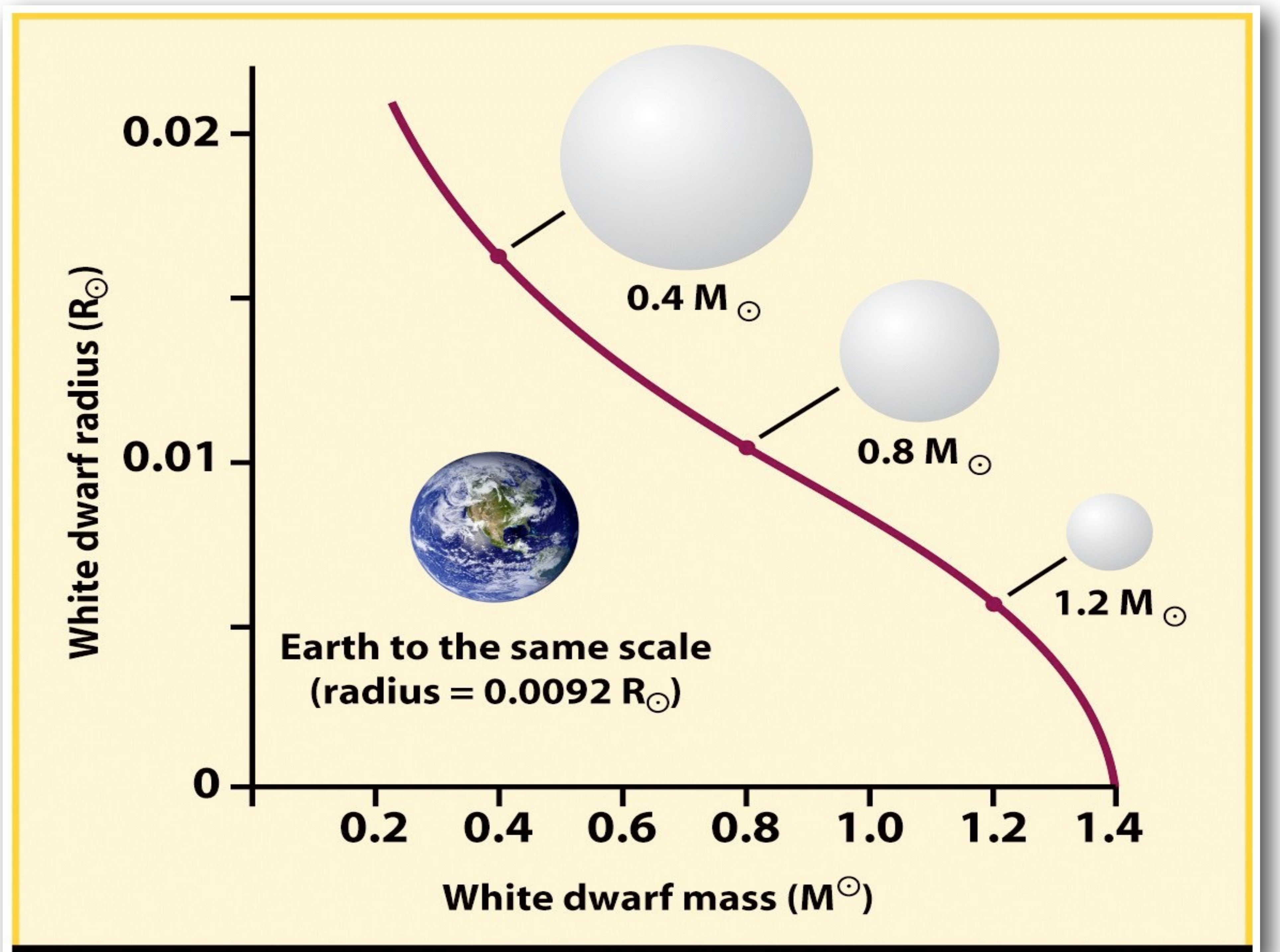
- If degenerate gas provide the central pressure, we can equate the two and solve for the Mass-Radius relation:

$$\frac{M^{5/3}}{R^5} \propto \frac{M^2}{R^4} \Rightarrow R \propto M^{-1/3}$$

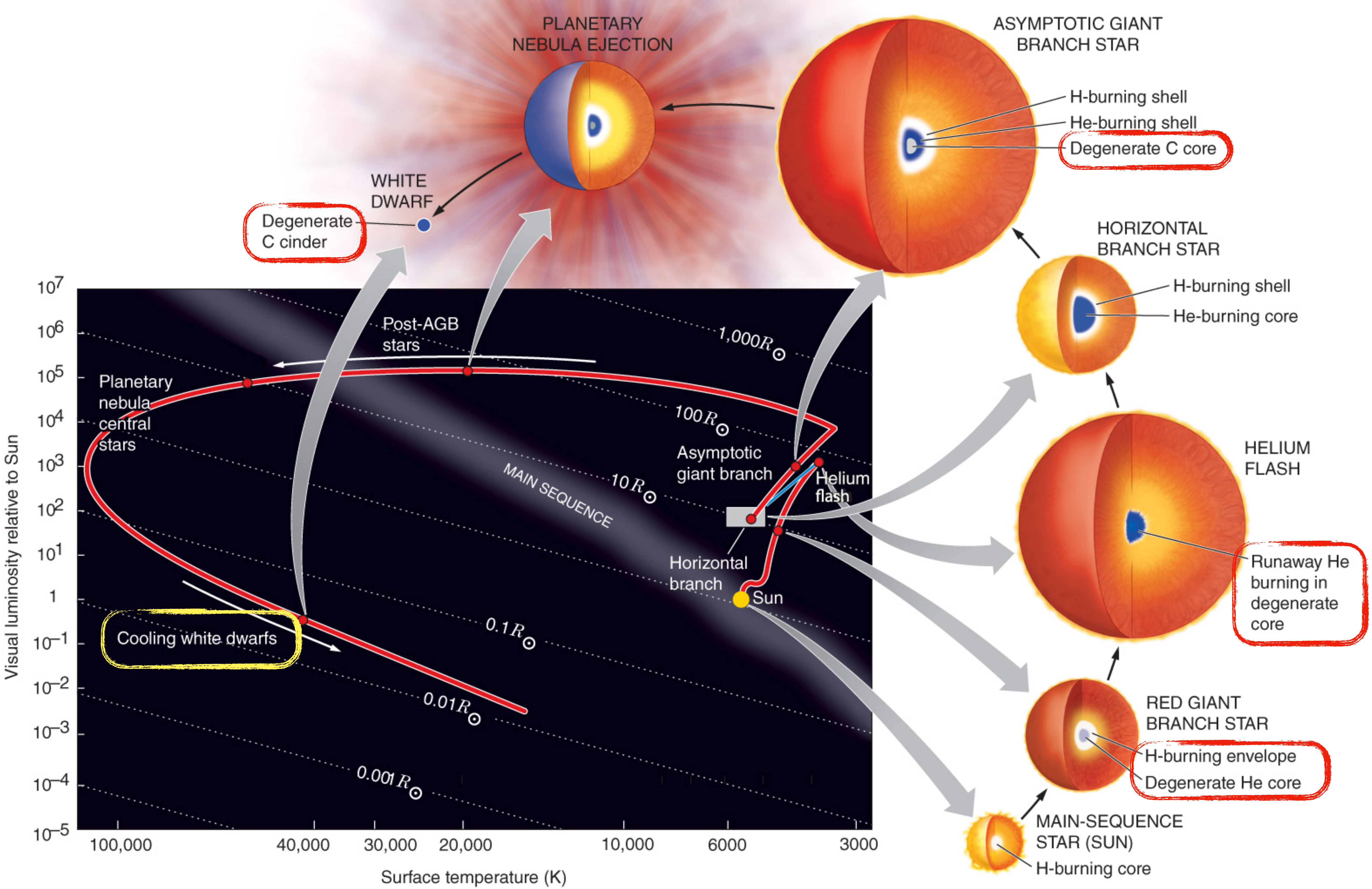
- This is in contrast to main sequence stars, where **R ~ M$^{0.8}$**

Mass-Size Relation of White Dwarfs & the Chandrasekhar Limit

- When degenerate electrons provide the pressure, the size of the core decreases as the mass increases, $\mathbf{R \sim M^{-1/3}}$. This is in contrast to main sequence stars, where $\mathbf{R \sim M^{0.8}}$
- The relation eventually breaks when degenerate electrons become **relativistic**, and that places a limit on the maximum mass of the white dwarf, the **1.4 Solar Mass Chandrasekhar limit**

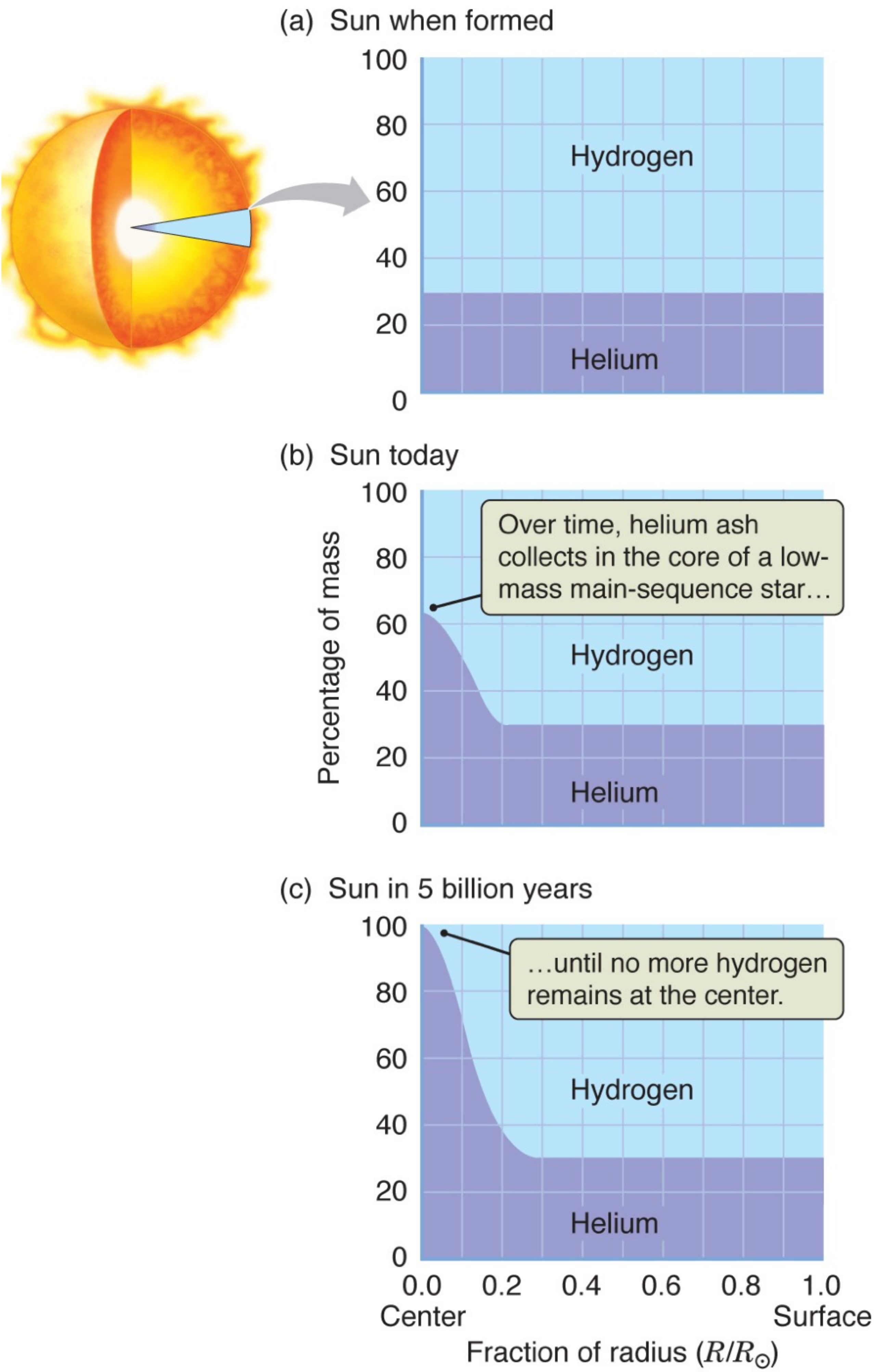


But white dwarfs are the end state of low-mass star evolution, how did the stellar core first become degenerate?



Inference from the best-fit numerical models

What happens inside the stars as they evolve into each of the stages?



- Main-sequence stars fuse hydrogen to helium in their cores.
- At the end of MS, a non-fusing core of Helium ash starts to built up over time
- The He core is surrounded by a H-burning shell (originally thick and later thin) that keep dumping Helium ash onto the core.
- **Can the non-fusing core keep growing or is there a limit?**

Schonberg-Chandrasekhar Limit

- In 1942, Schonberg and Chandrasekhar found that when the non-fusing Helium core reaches ~8% of the total mass of the star, the core will contract because under isothermal condition, its pressure can no longer support the envelope:

$$\left(\frac{M_{\rm core}}{M}\right)_{SC} = 0.37\left(\frac{\mu_{\rm env}}{\mu_{\rm core}}\right)^2$$

- The mass-ratio limit above is derived in the following way:
  - Virial theorem (self-gravitating) with external pressure at the core-envelop boundary to calculate the maximum pressure that the core can support:

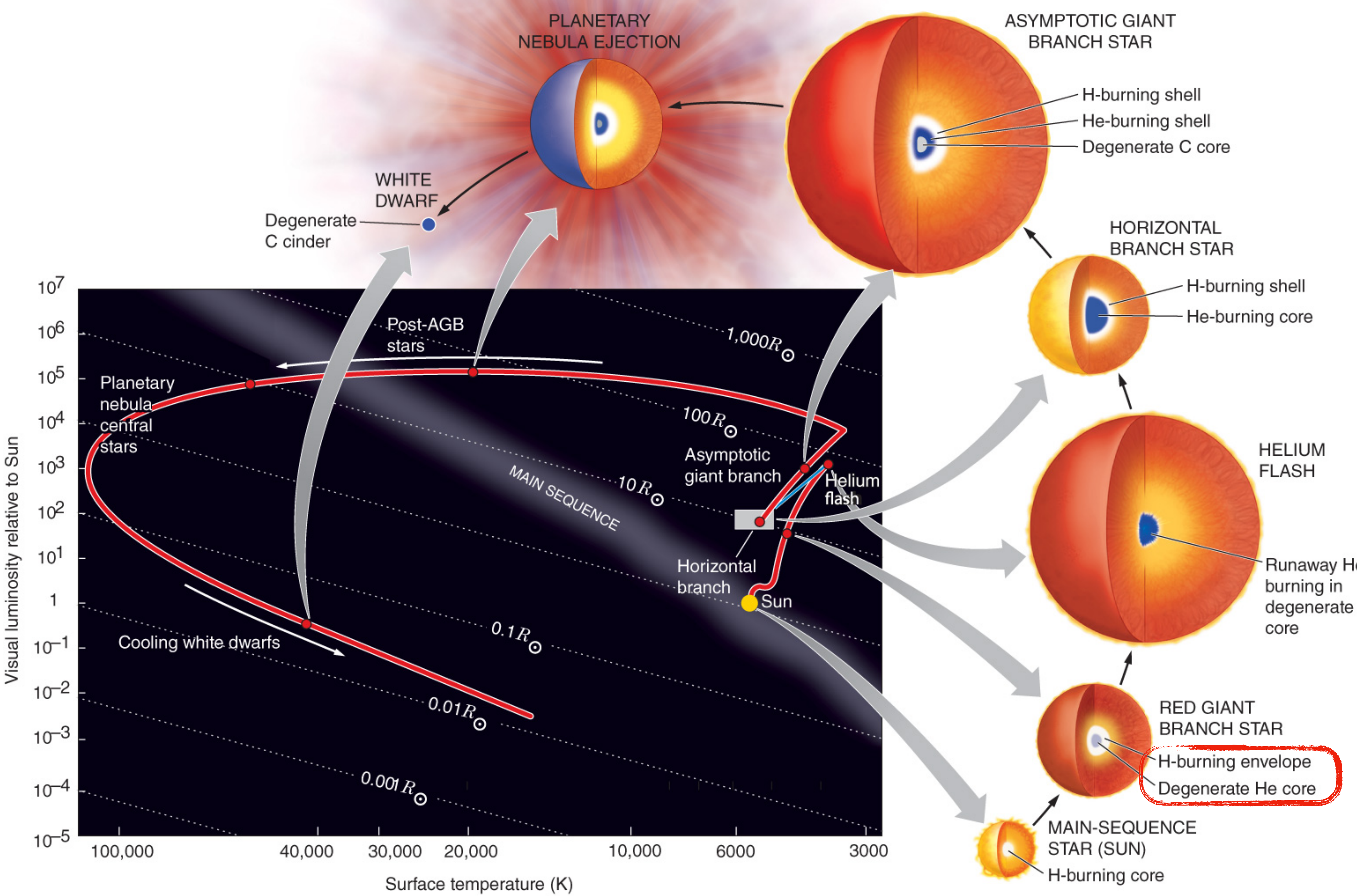
$$P_{\rm core,max} = \frac{A}{G^3 M_{\rm core}^2}\left(\frac{kT_{\rm core}}{\mu_{\rm core} m_H}\right)^4$$

  - Hydrostatic equilibrium to calculate the actual pressure from the envelope plus the ideal gas law:

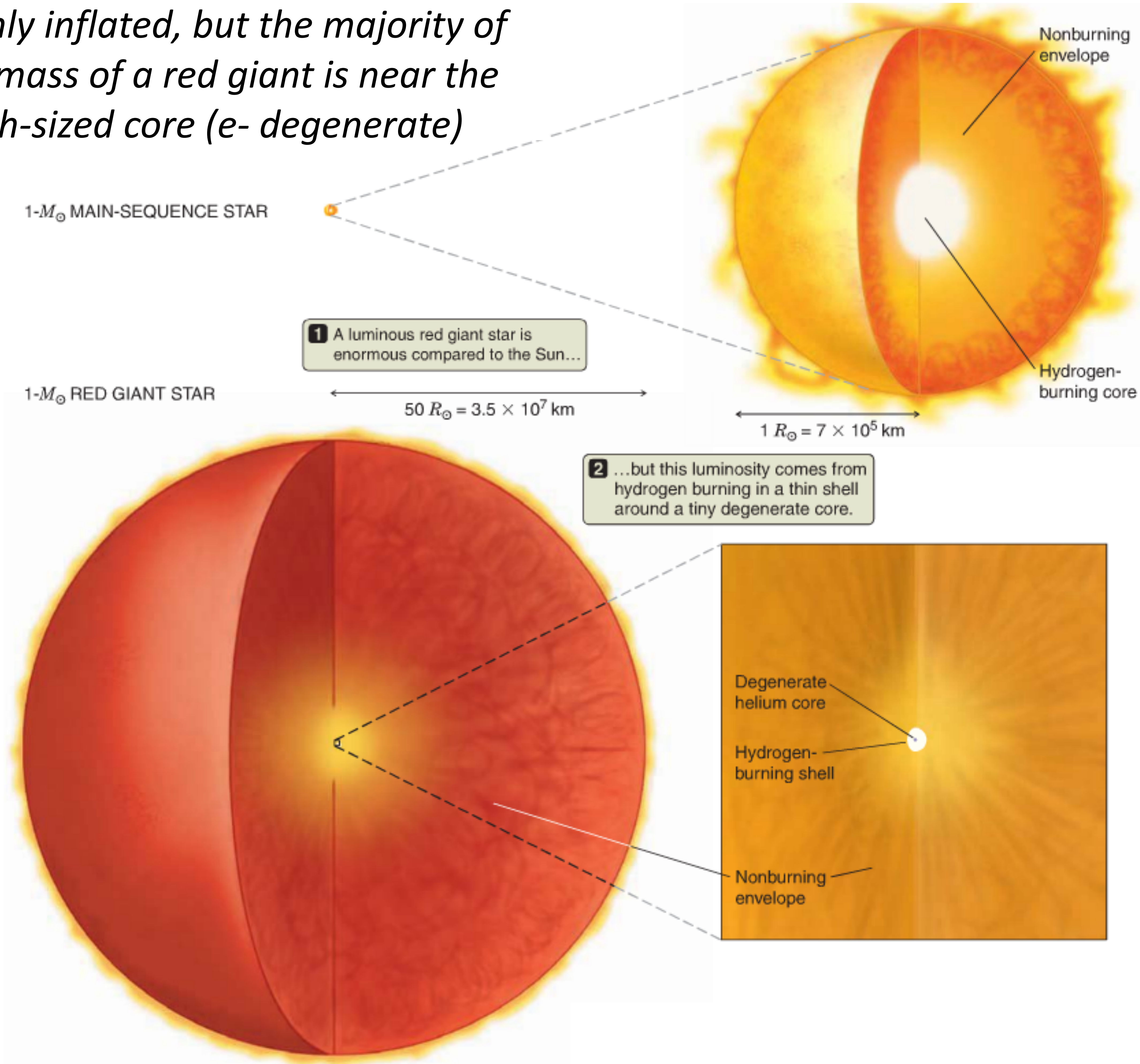
$$P_{\rm env} \approx \frac{G}{4\pi R^4}(M^2 - M_{\rm core}^2) \approx \frac{B}{G^3 M^2}\left(\frac{kT_{\rm boundary}}{\mu_{\rm env} m_H}\right)^4$$

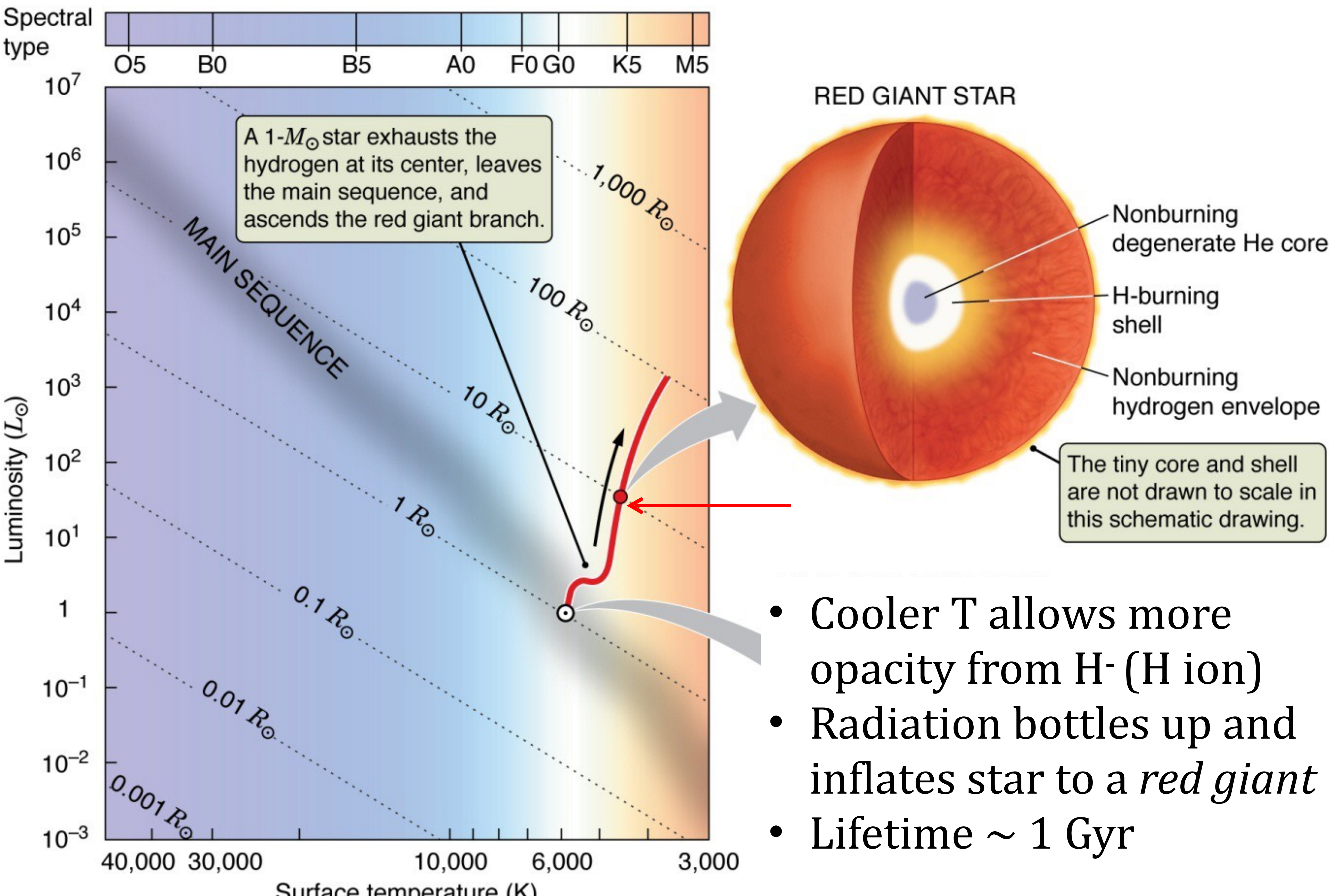
  - Given that at the boundary, the core and the envelope have the same temperature, the condition for collapse ($P_{\rm env} > P_{\rm core,max}$) gives us a maximum core-mass-total-mass ratio that is *inversely* proportional to the square of the ratio of the mean molecular mass.

Violation of the SC limit causes the core to contract and the packed electrons become degenerate before ions because of their smaller masses



Highly inflated, but the majority of the mass of a red giant is near the Earth-sized core (e- degenerate)





- Cooler T allows more opacity from H^- (H ion)
- Radiation bottles up and inflates star to a *red giant*
- Lifetime ~ 1 Gyr

The H^- thermostat in the atmosphere of a red giant

Surface temperature decreases

H- increases

Atmosphere becomes more opaque

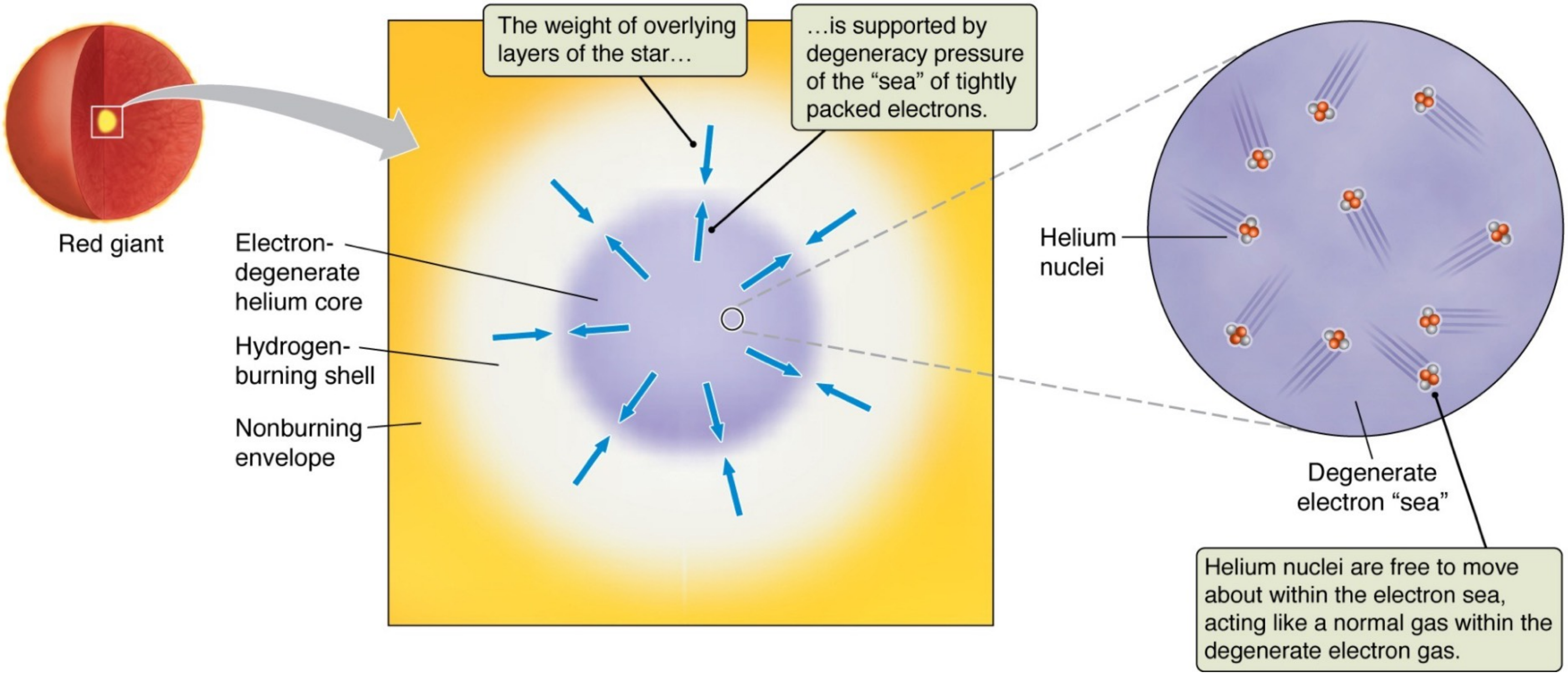
Surface temperature increases

H- decreases

Atmosphere becomes more transparent

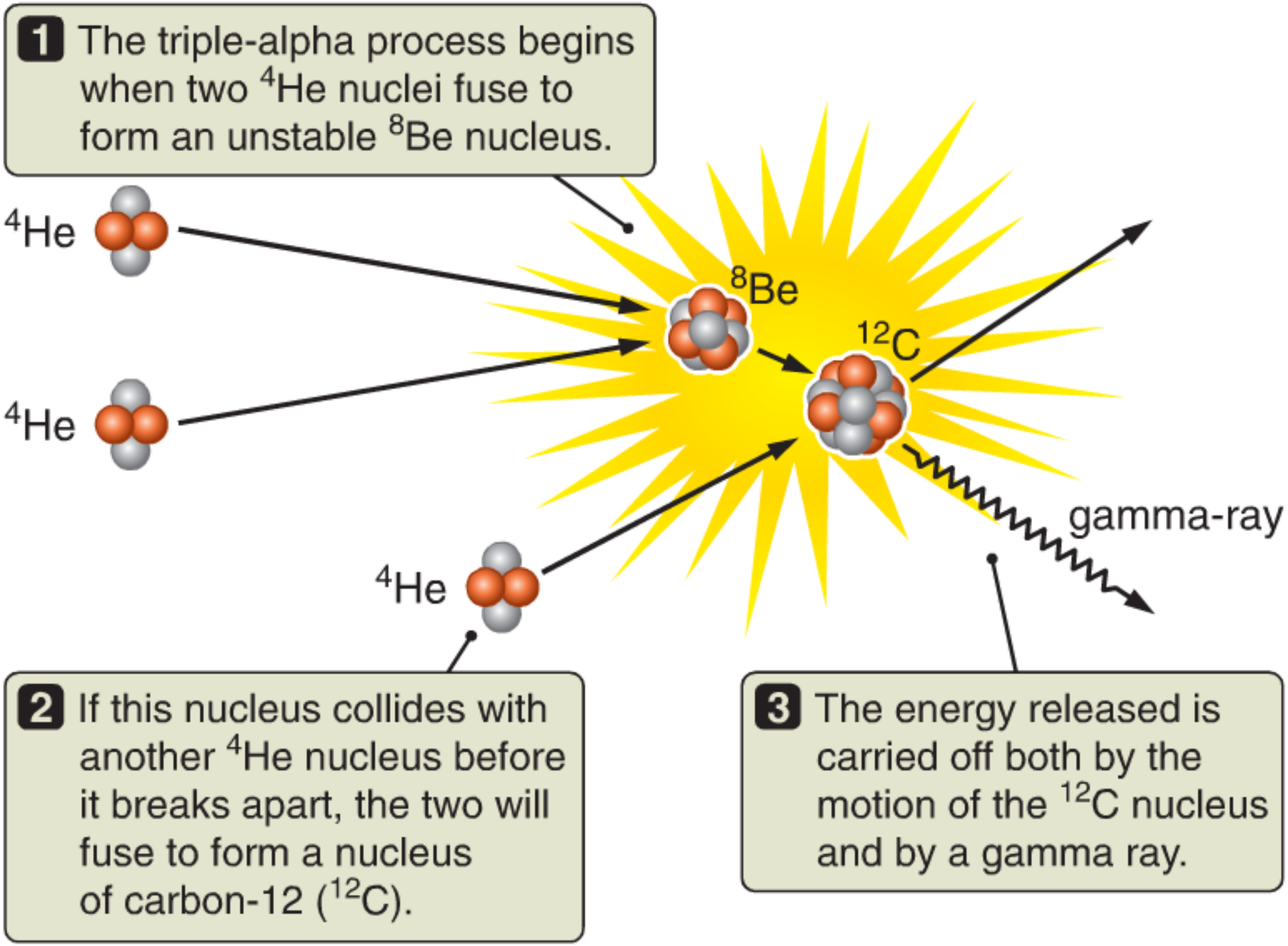
Surface temperature decreases

Helium Flash: The End of the Red Giant Phase



- Fusion of H in shell creates more He, making the core heavier.
- Pressure doesn’t increase past the electron-degenerate limit, but core temperature is allowed to increase without bound.
- …**until T reaches ~ 100 million K, at which time Helium can fuse.**

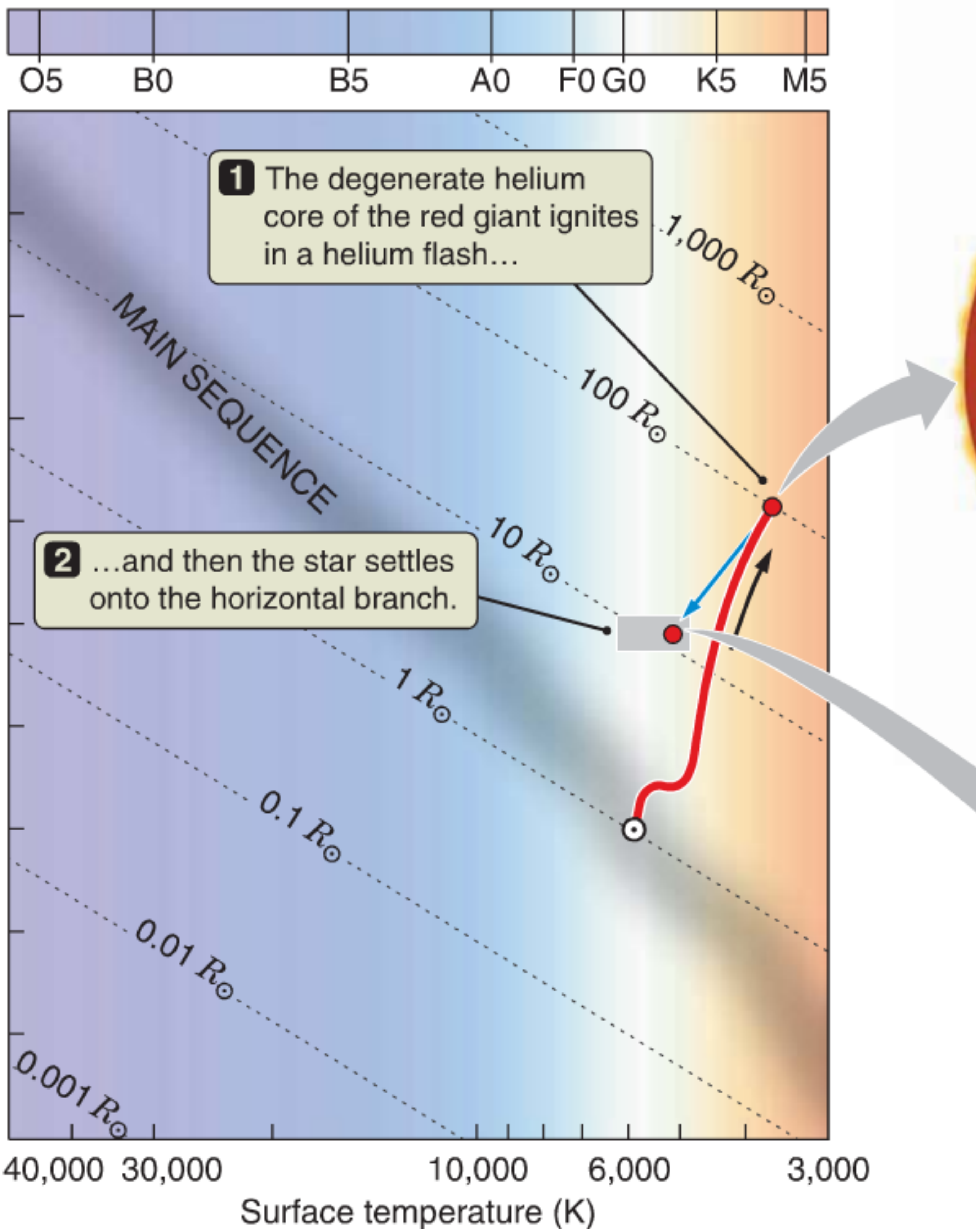
He burning – the triple-alpha process

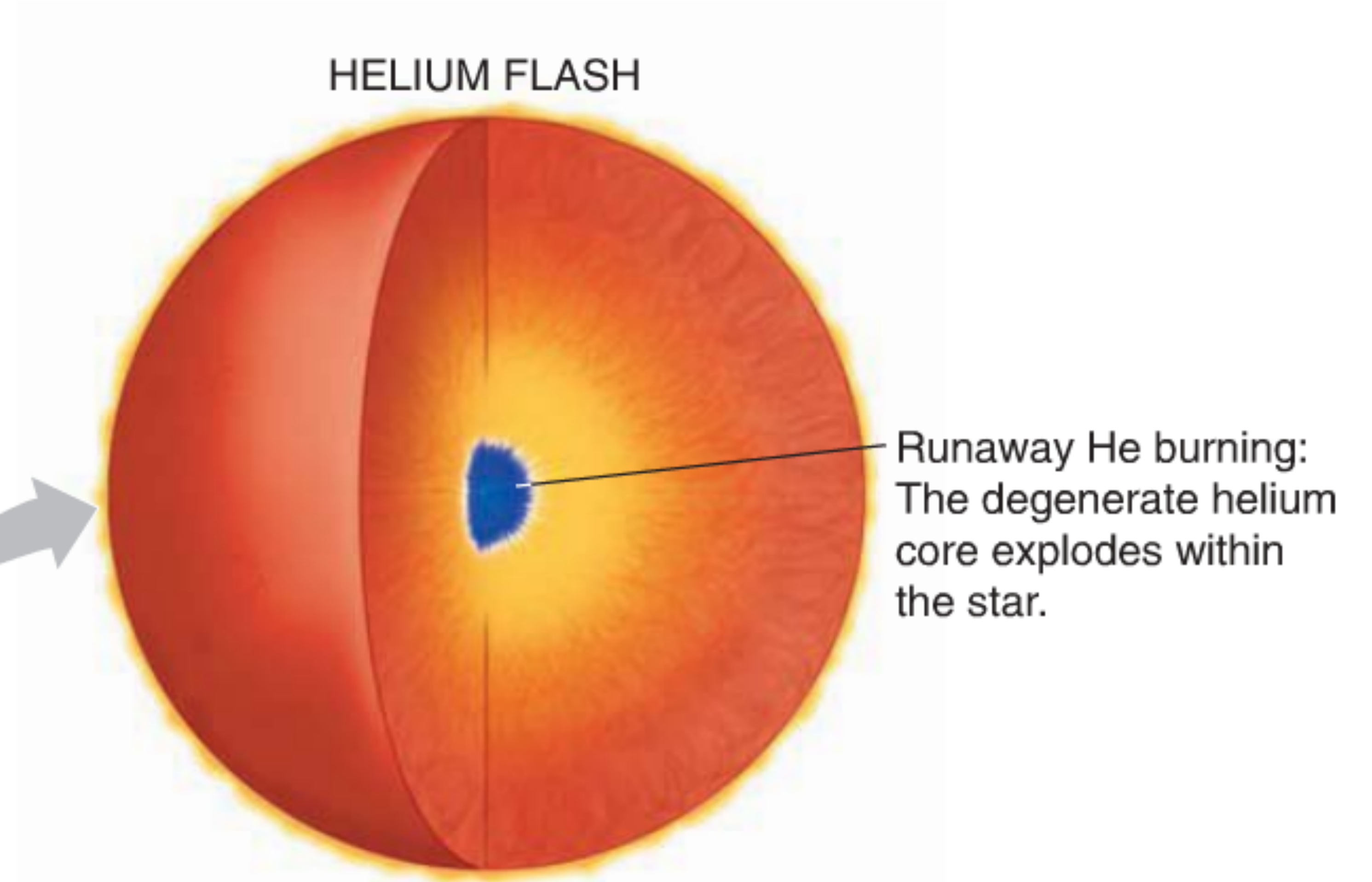


When the gravitational thermostat is out of order

non-degenerate core	electron-degenerate core
H fusion rate increases	He fusion rate increases
T & P increases	$T(He^{2+})$ increases, $P(e-)$ do not change
Core expands, work against Gravity	Core doesn't expand
T decreases	$T(He^{2+})$ still high
H fusion rate decreases	He fusion rate increases
Steady H Burning	He Flash

He flash - a thermonuclear runaway



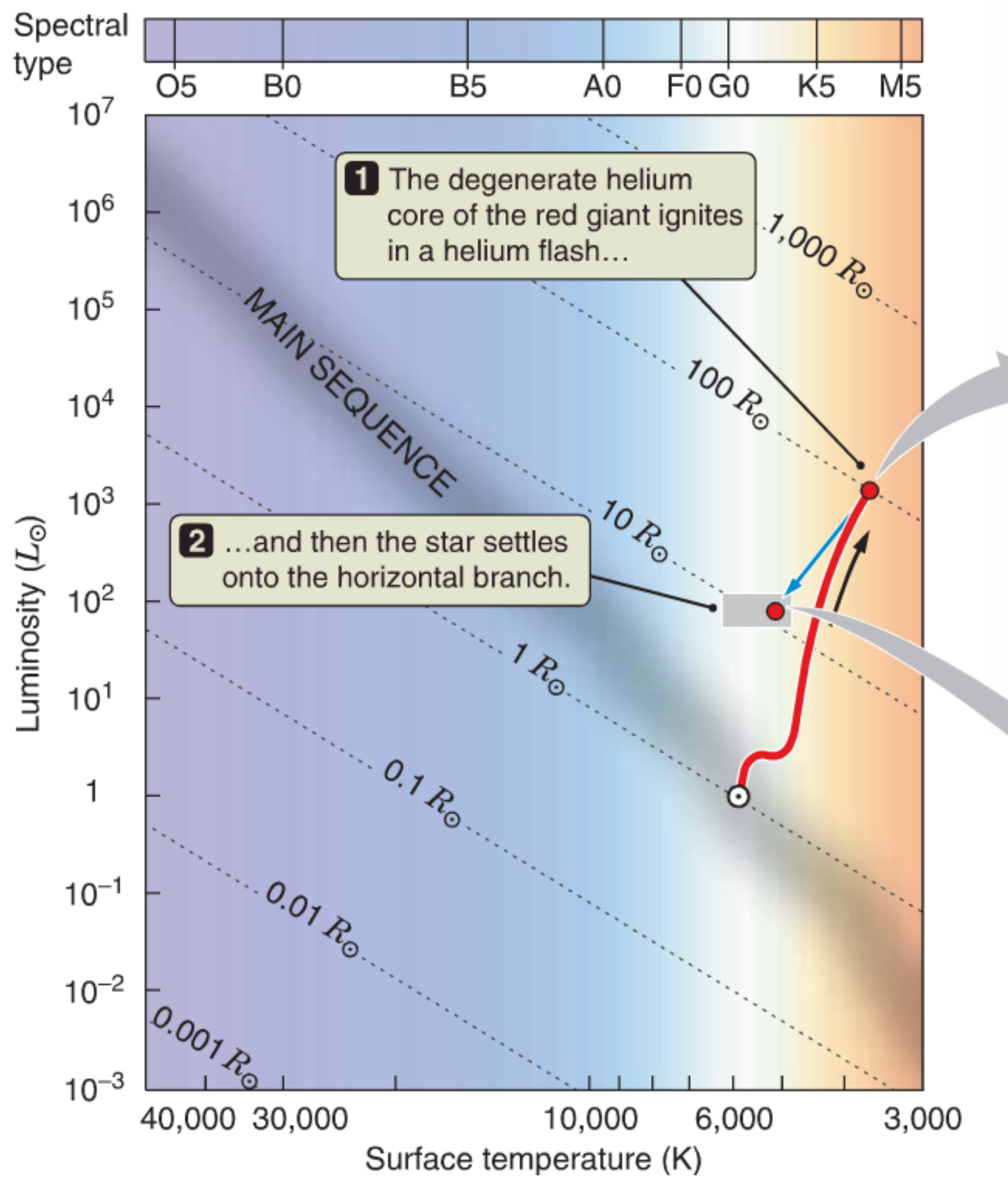


- Nuclear fusion out of control (a runaway process)
- The degenerate helium core explodes within the star
- The whole process lasts only **a few hours**

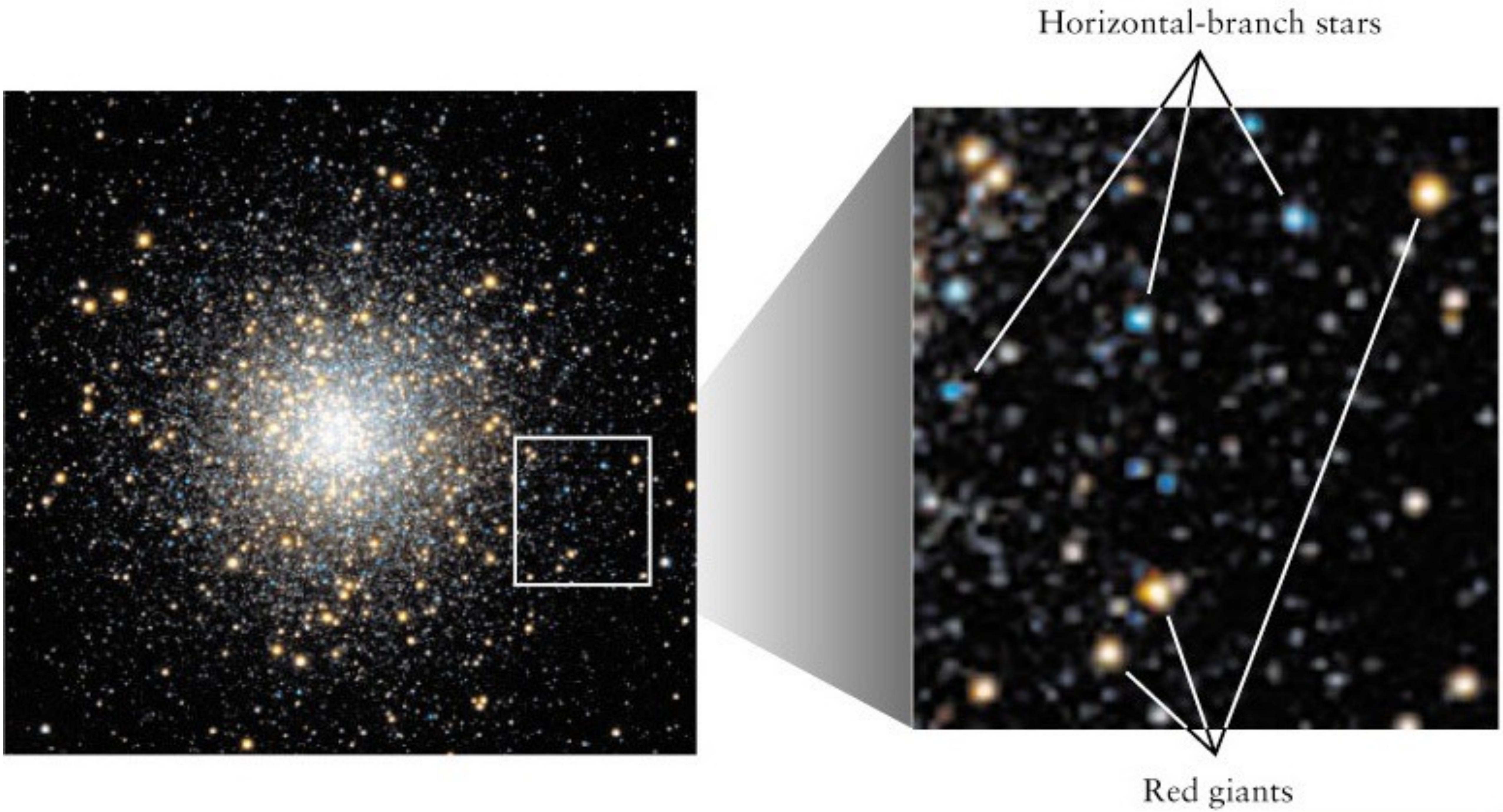
After Helium flash

- Settles onto ***horizontal branch***
- Stable Helium core burning, similar to a main sequence star
- H burning continues in shell surrounding the He burning core

- Main Sequence – 10 Gyr
- Red Giant – 1 Gyr
- He flash - a few hours
- He flash to HB – 100 Kyrs
- HB - 100 Myrs, a new “MS”

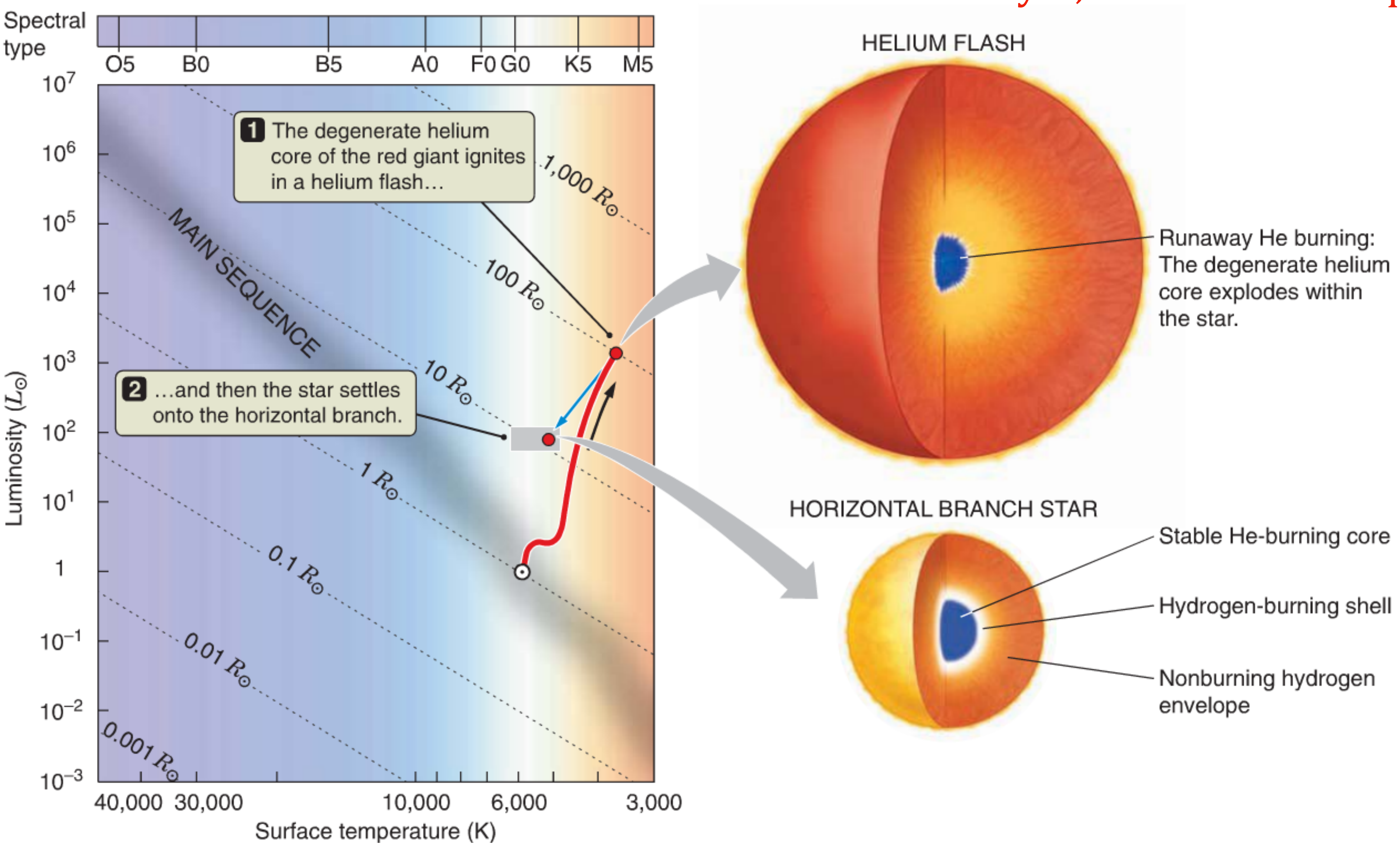


Horizontal branch stars are hotter (bluer) than RGs, and are evolving horizontally to left on HR diagram

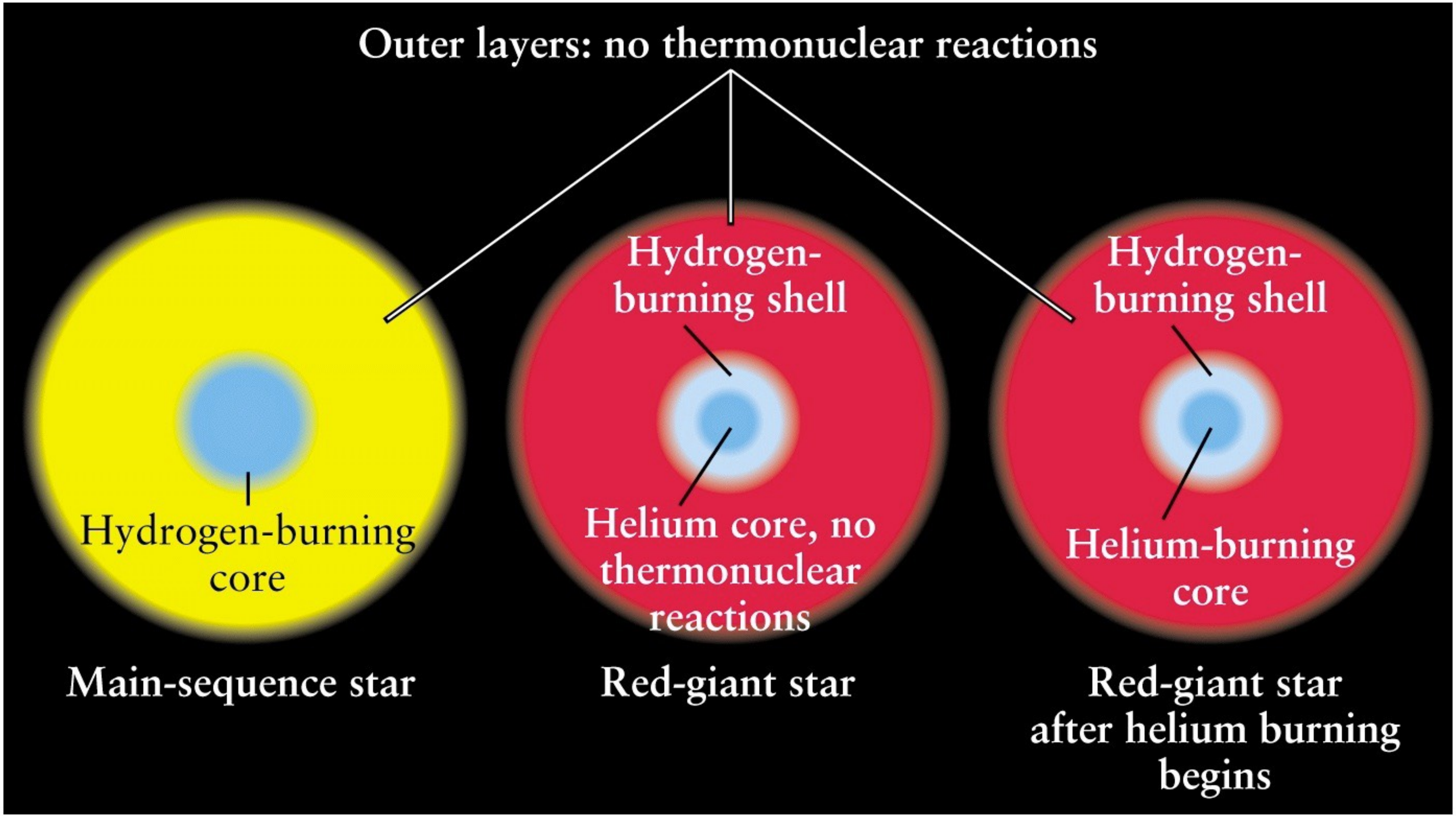


Post-**MS** evolution

- Red Giant – 1 Gyr
- Helium Flash - a few hours
- Readjust to HB – 100 Kyrs
- HB - 100 Myrs, a new “main-seq”



Main Sequence → Before He flash: Red Giant → After He flash: Horizontal Branch



Eventually, Helium will be exhausted at the center of a Horizontal Branch star, and a non-fusing Carbon core will form under a Helium-burning shell?

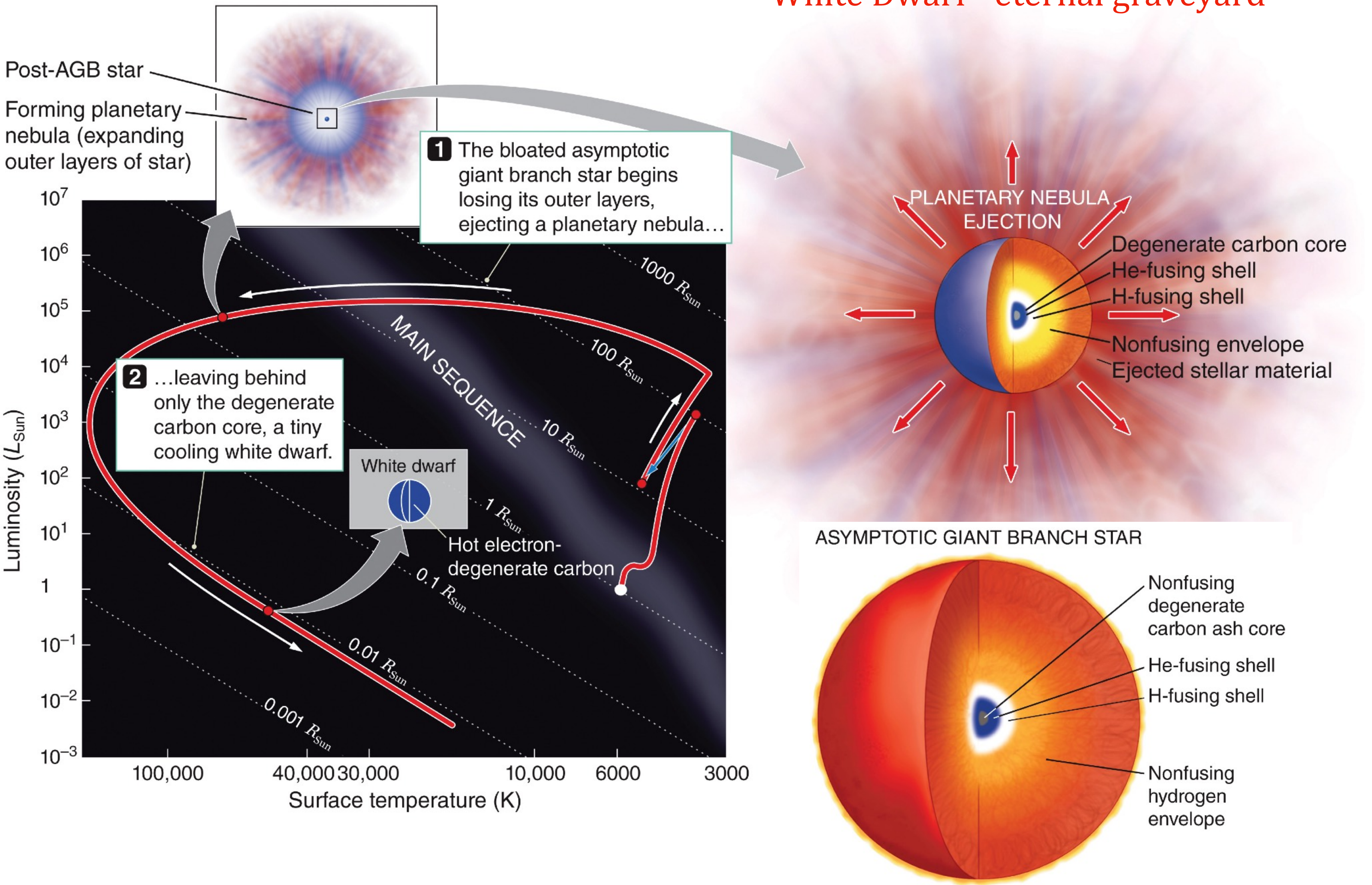
This marks the end of the horizontal branch phase. Based on what you learned about the post-MS evolution, deduce the post-HB evolution of the star.

Post-**MS** evolution:

- non-fusing Helium core, H-burning shell
- Helium-core contract and become **e- degenerate**
- **Red giant** phase (H^- controls surface T as L increases)
- uncontrolled Helium-burning in the e- degenerate core (thermonuclear runaway, **Helium flash**)
- core expands and become non-degenerate, allowing steady Helium burning (**Horizontal branch**)

Post-**HB** evolution

- Asymptotic Giant Branch – 100 Kyr
- Post-AGB / Planetary Nebulae - 10 Kyr
- White Dwarf - eternal graveyard



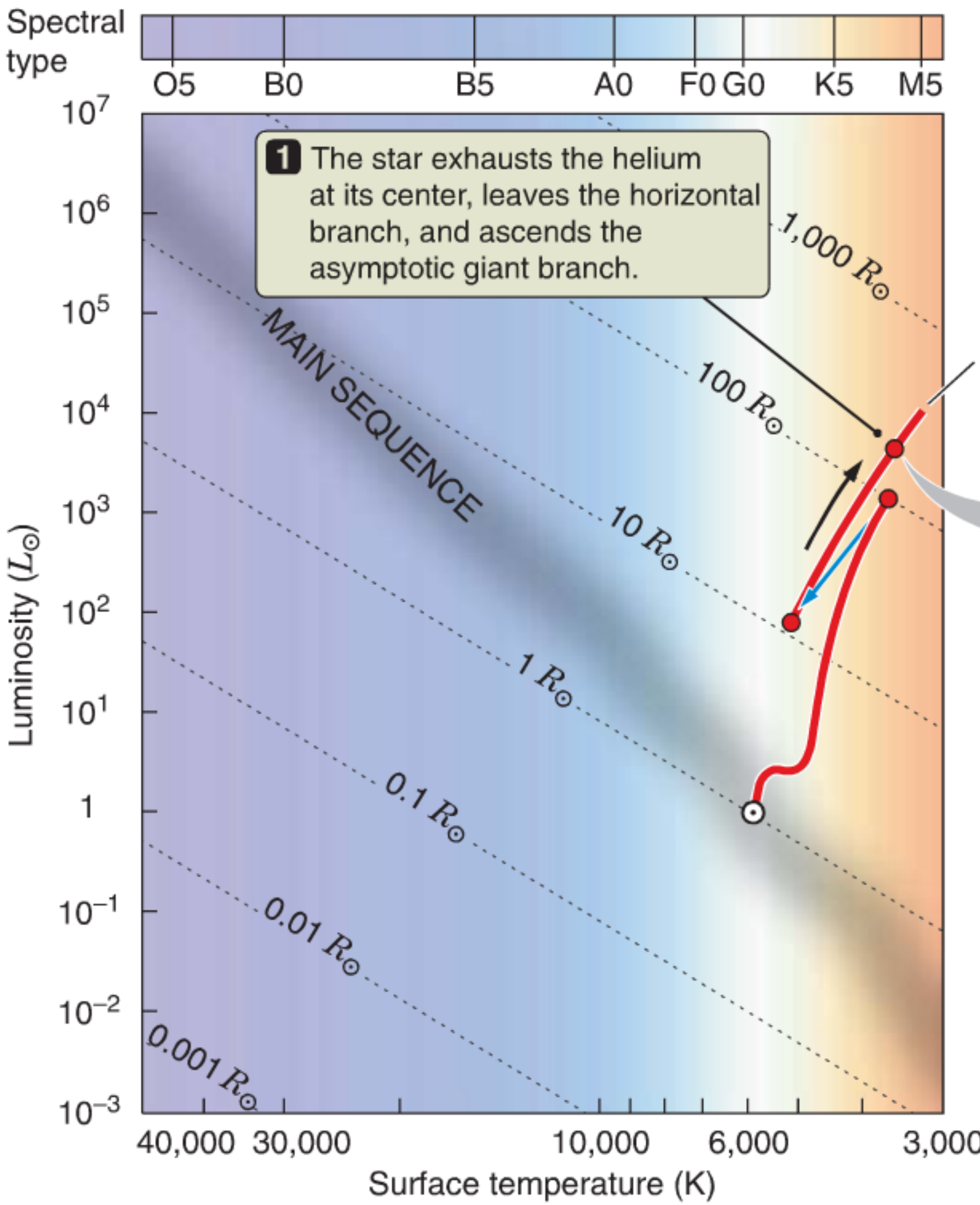
Post-**MS** evolution:

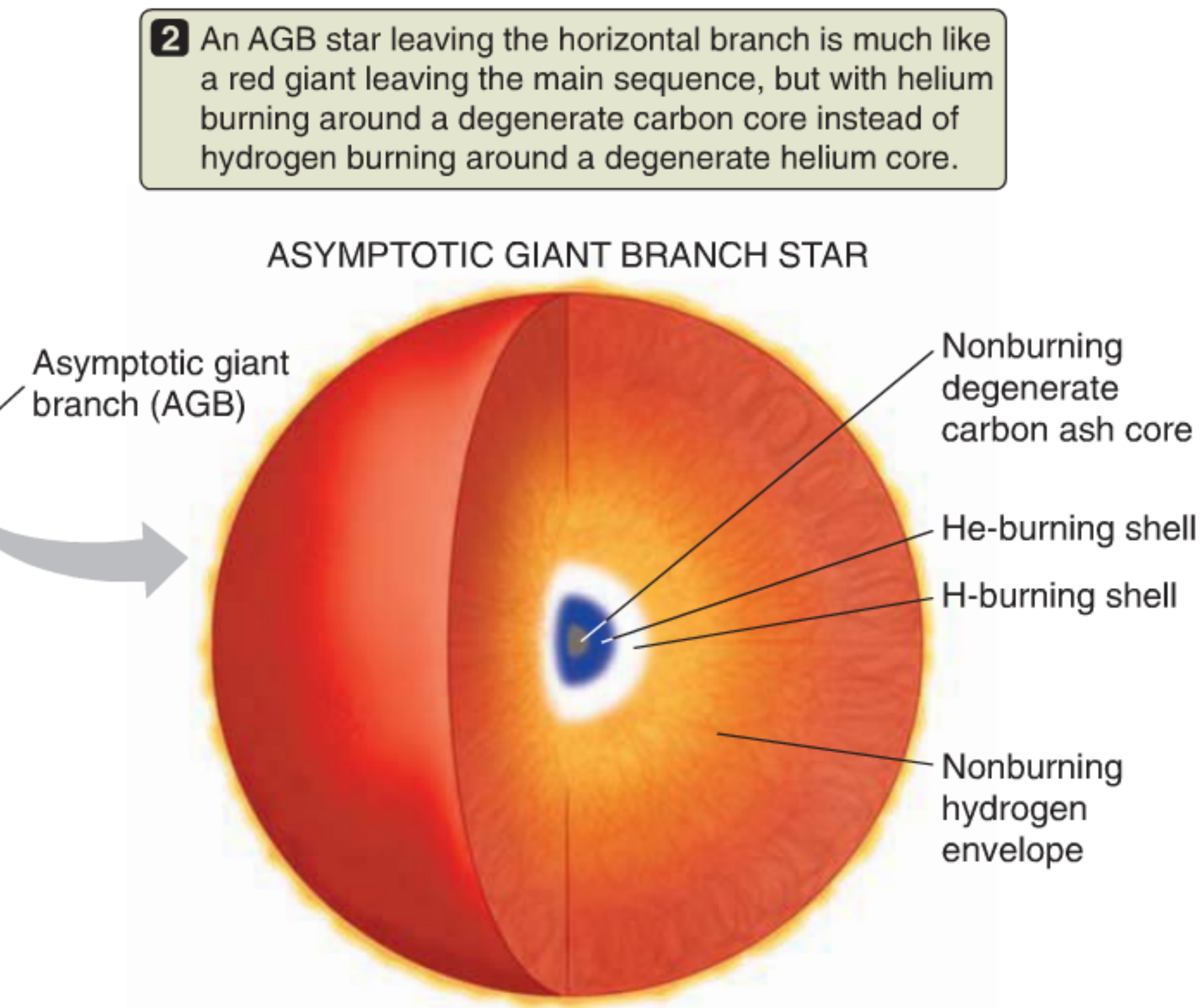
- non-fusing Helium core, H-burning shell
- He-core contracts and become **e- degenerate**
- **Red giant** phase (H^- controls surface T as L increases)
- uncontrolled Helium-burning in the e- degenerate core (thermonuclear runaway, **Helium flash**)
- core expands and become non-degenerate, allowing steady Helium burning (**Horizontal branch**)

Post-**HB** evolution:

- non-fusing C core, He-burning shell, H-burning shell
- C-core contracts and become **e- degenerate**
- **Asymptotic Giant Branch** Phase (H^- controls surface T as L increases, the radius of the star increases even more)
- core temperature never reaches 500 million K needed for Carbon-burning (**no Carbon flash**)
- But eventually the AGB star becomes so bloated that it starts to lose its mass (post-AGB phase)

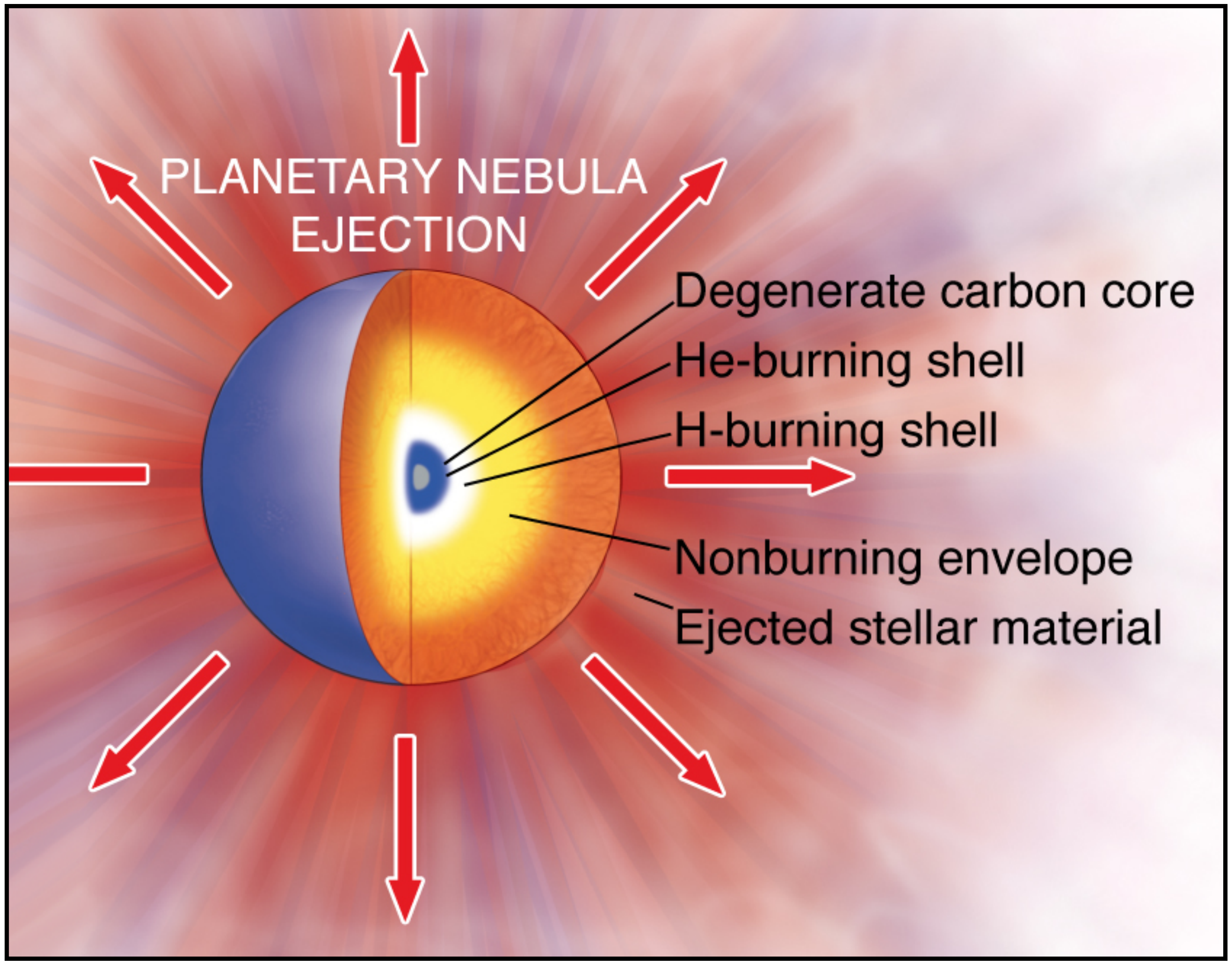
Asymptotic Giant Branch (AGB)





- Star gets more luminous and cool, and enters the asymptotic giant branch (AGB).
- Electron-degenerate Carbon core with no nuclear fusion (T not hot enough)
- H^- keeps the surface temperature almost constant, like in the Red Giant Branch (RGB)

Post-AGB - No "Carbon Flash" but a Planetary Nebula



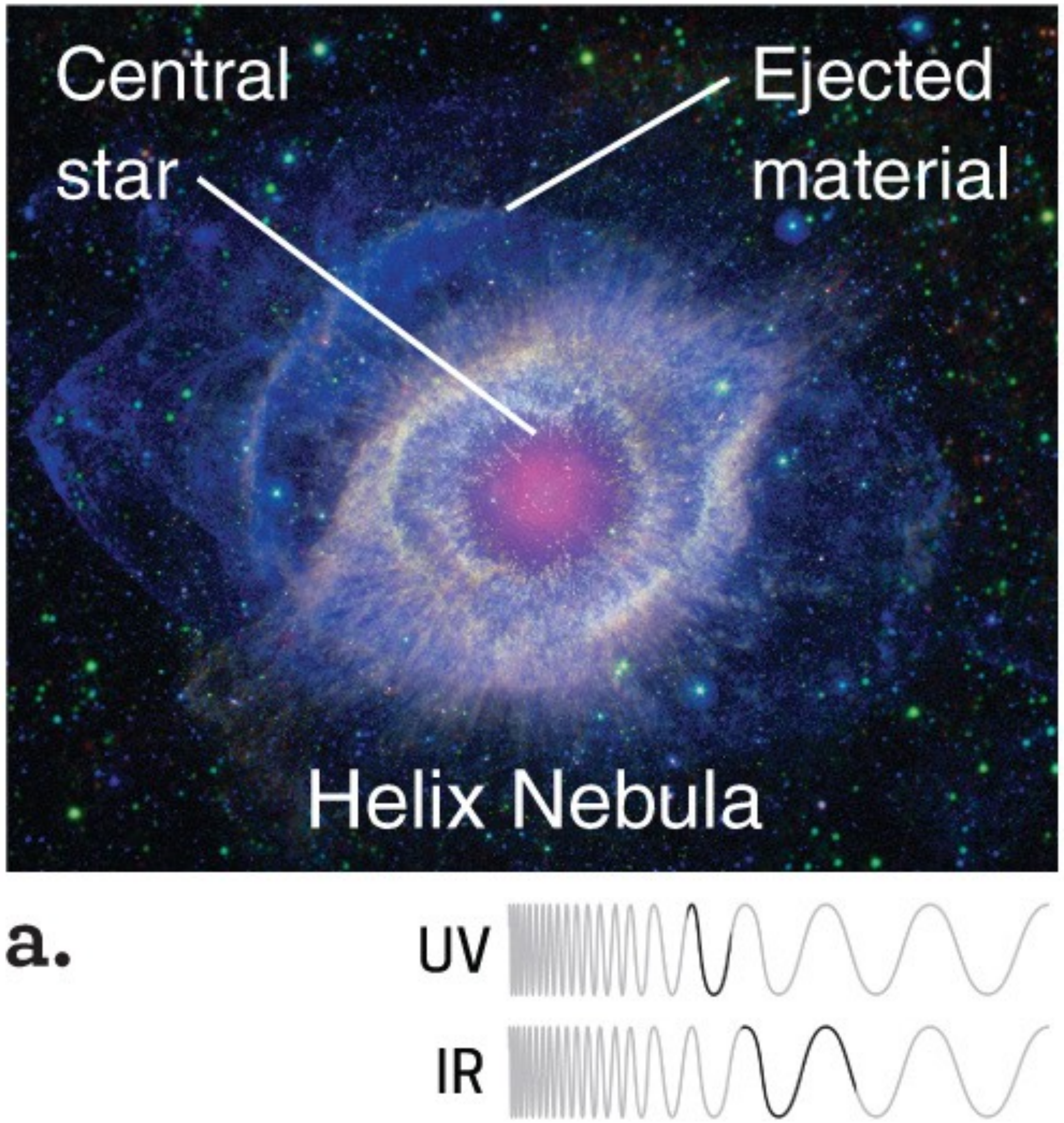
- Temperature never gets high enough to initiate C burning (500 million K at core density)

$${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{24}_{12}Mg + \gamma$$

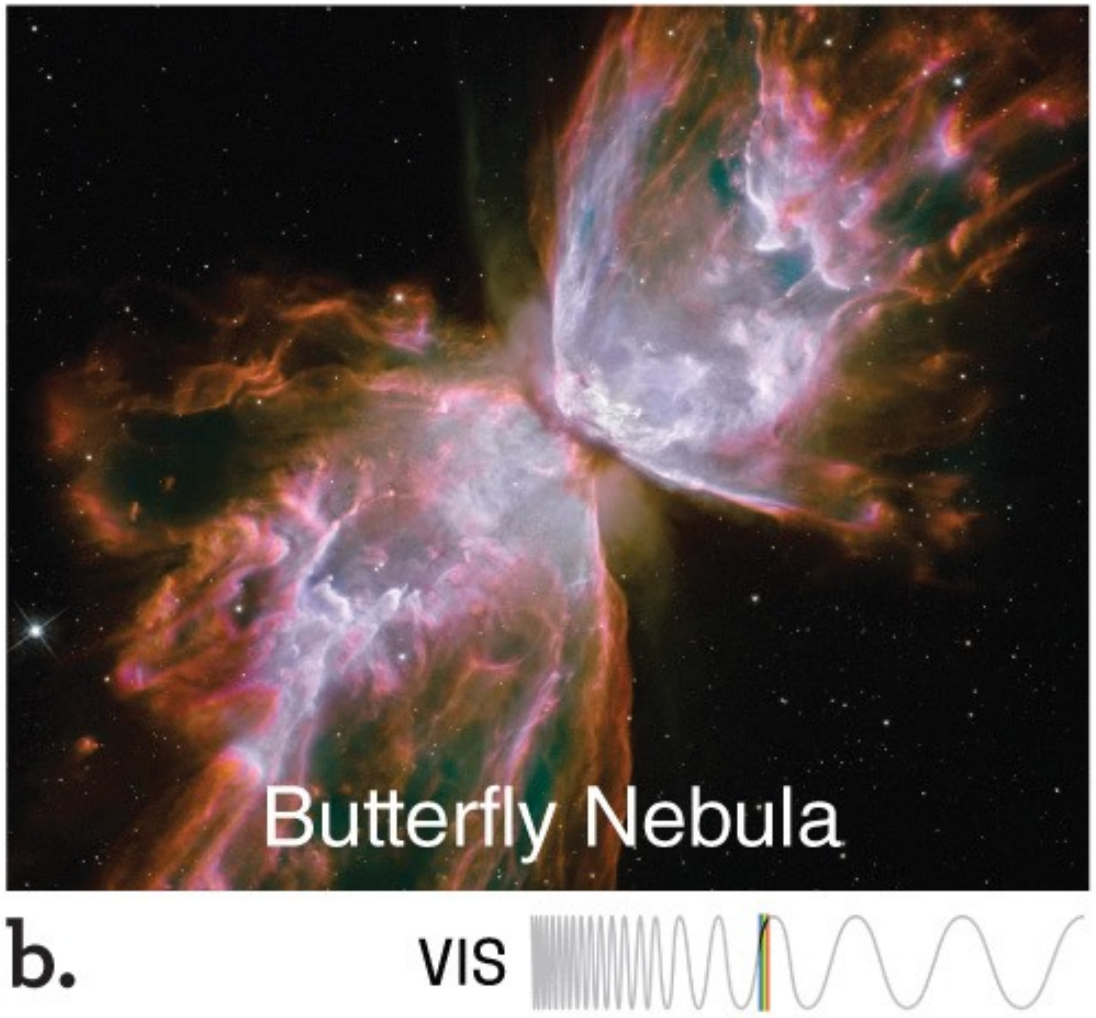
- **Mass loss becomes a runaway process** – forming **planetary nebulae**
- mass loss ->
  star puffs up ->
  less gravity ->
  more mass loss

Planetary Nebulae

- Material farther from the star was ejected earlier.
- Radiation from the white dwarf ionizes the gas. The colors are due to specific atoms and bright spots indicate areas of denser gas.

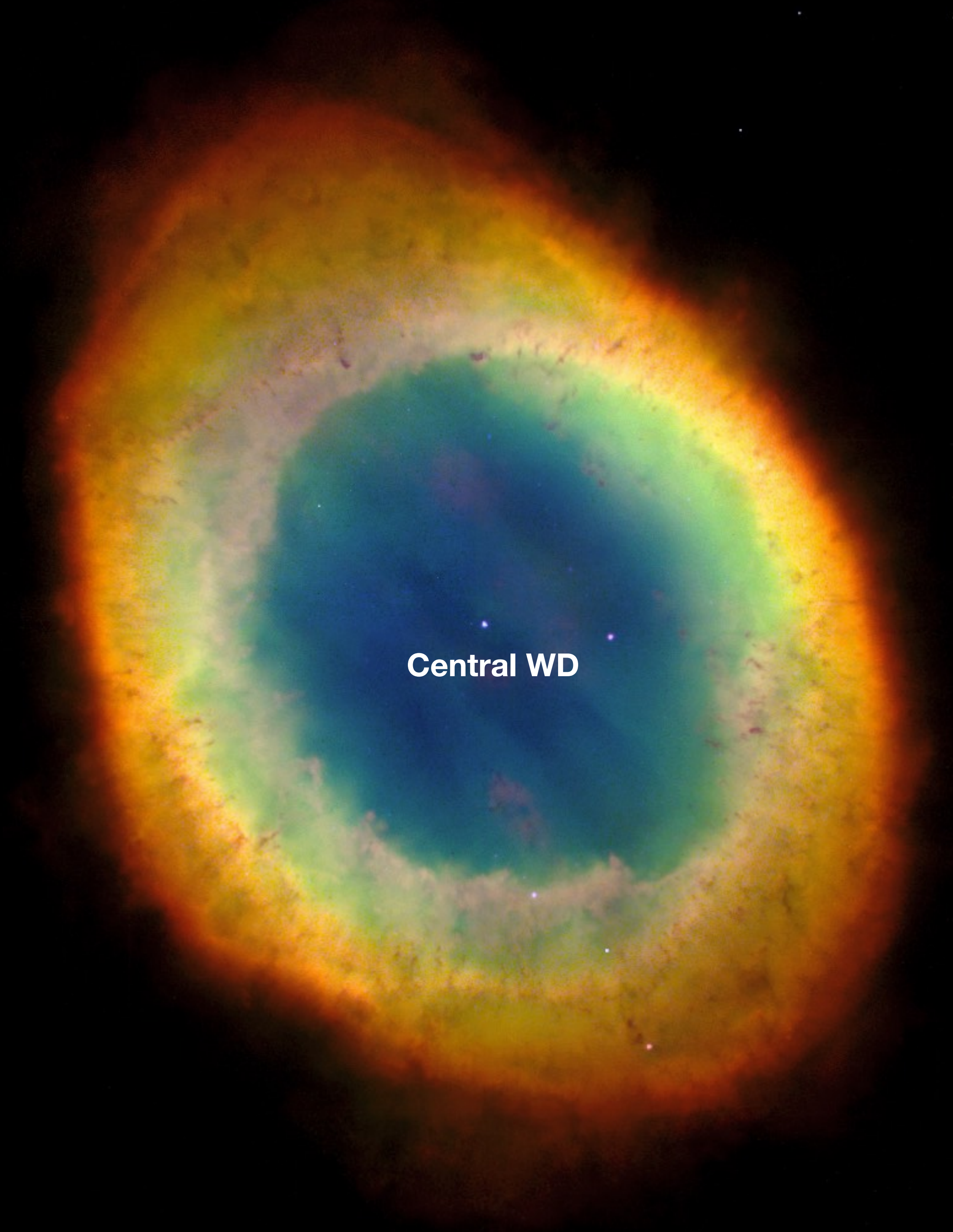


a.



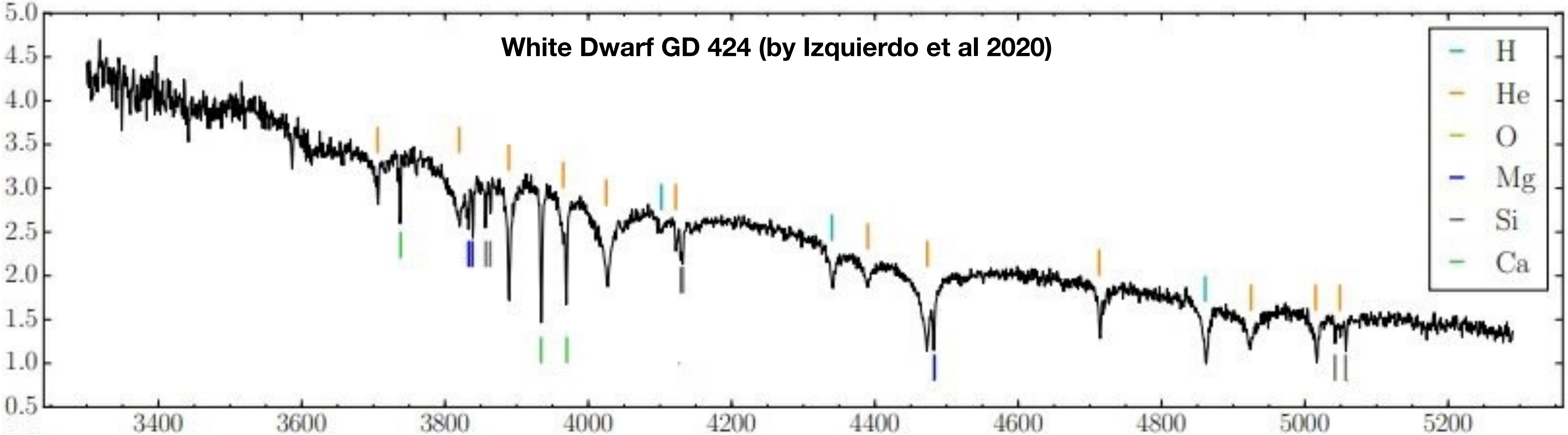
b.

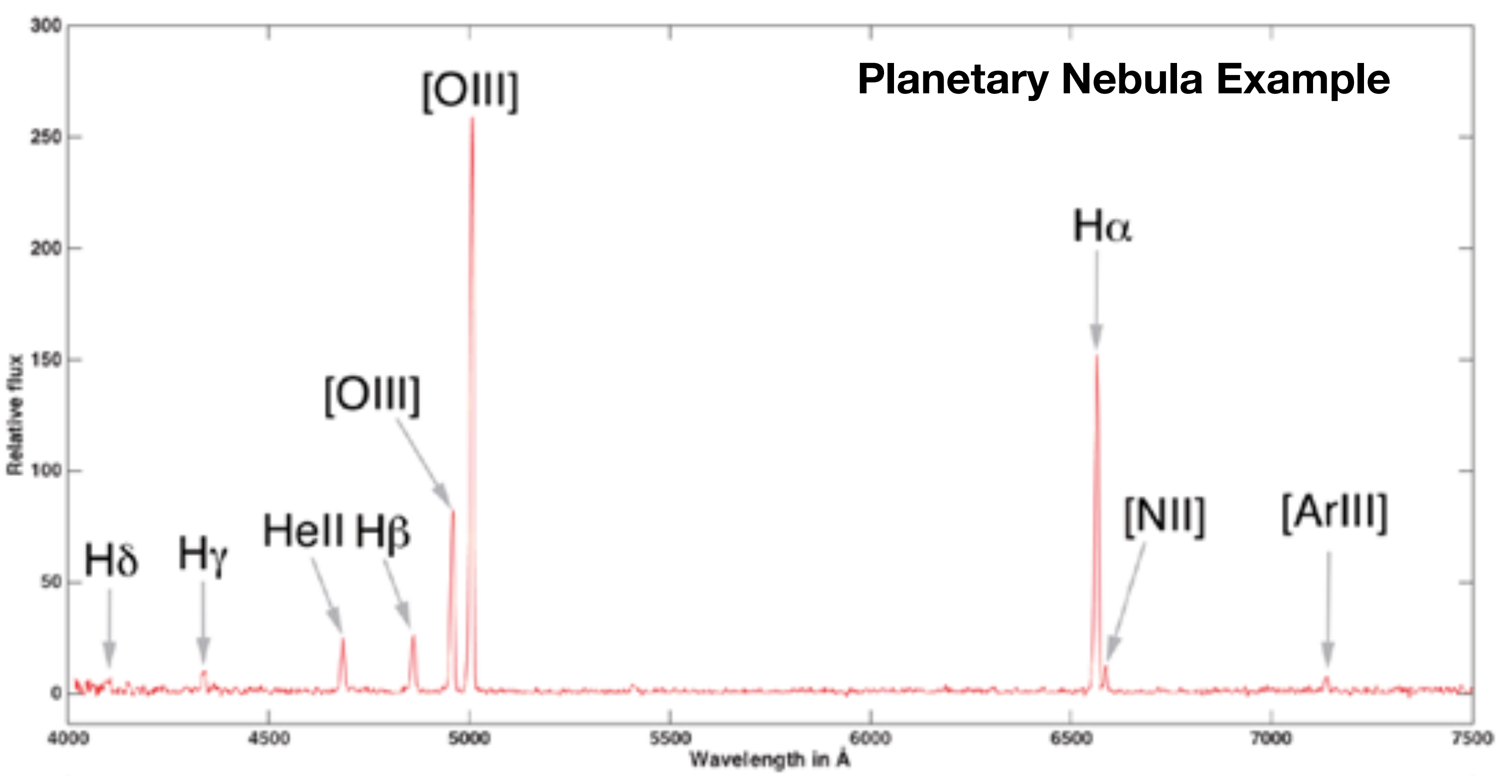
M57 - the Ring Nebula in Lyra near Vega

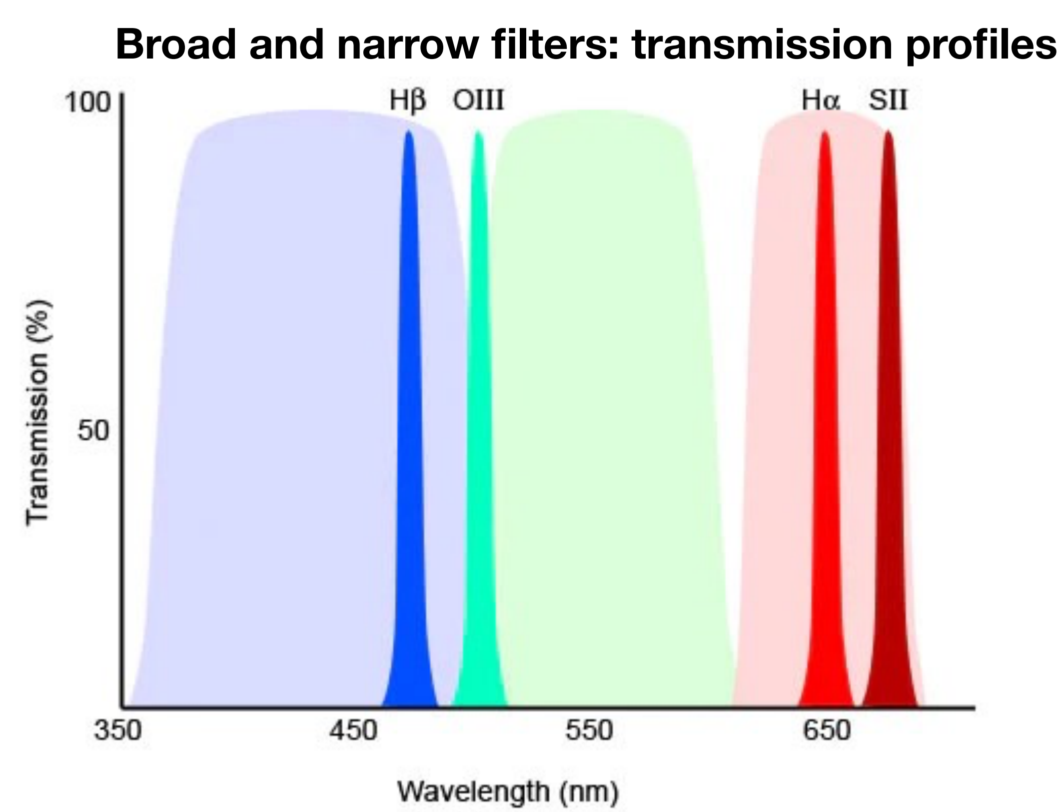


Why the central star appears so dim here?

How narrow-band filters decrease the contrast between the hot central star and the surrounding planetary nebula?

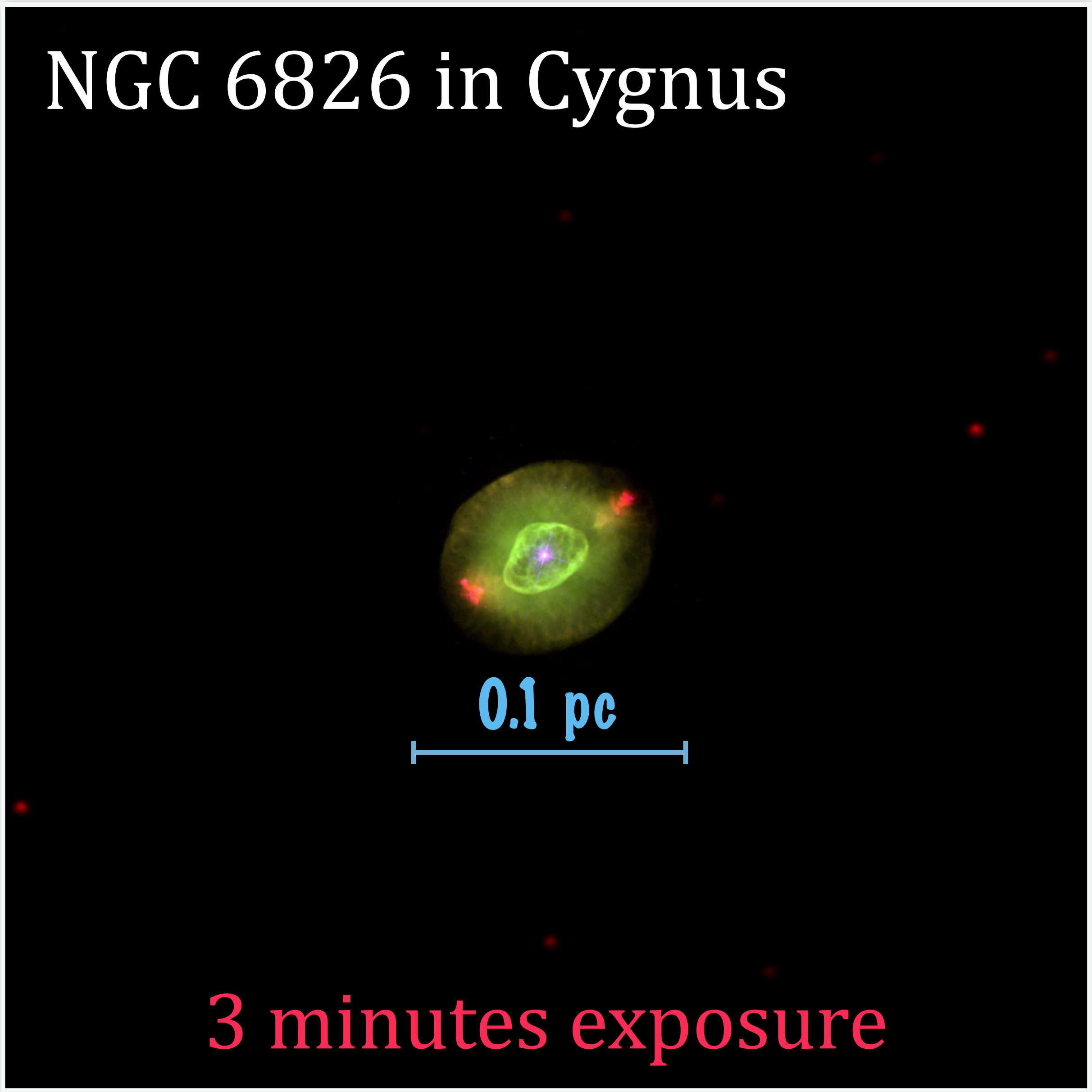


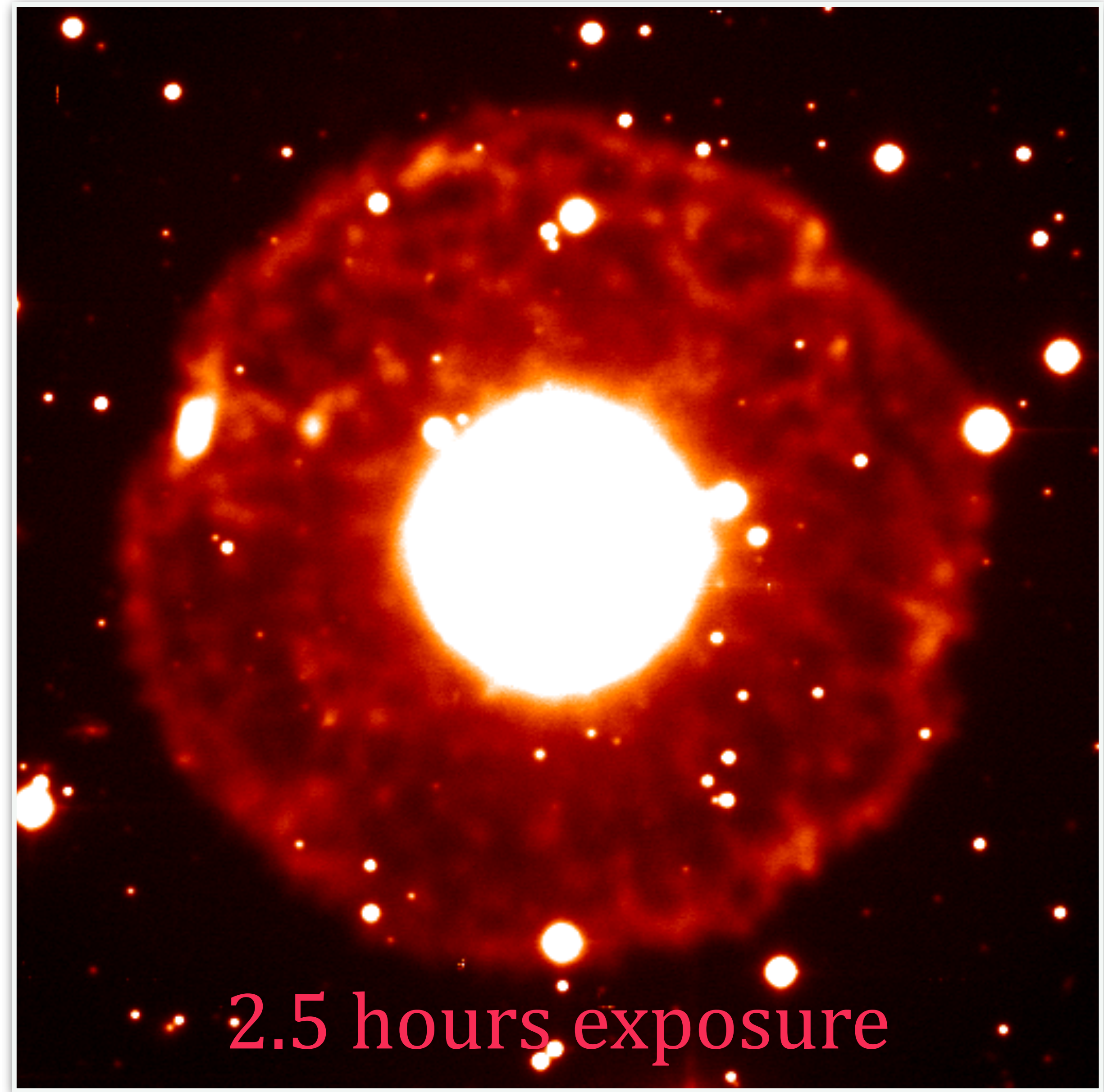




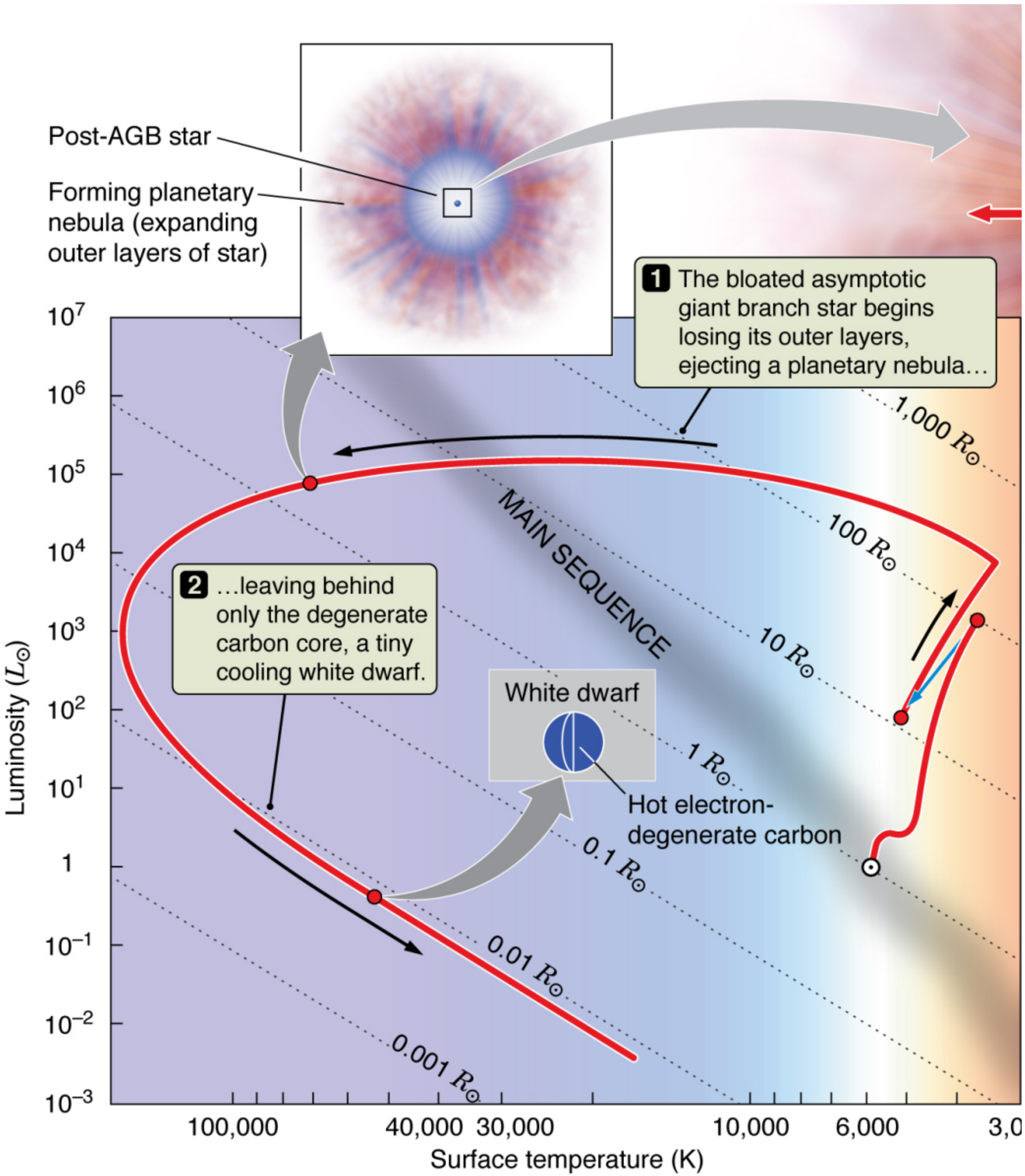
Severe Stellar Mass Loss illustrated in a Planetary Nebula

The planetary nebula extends much farther than the central bright area





Post-AGB and White Dwarfs



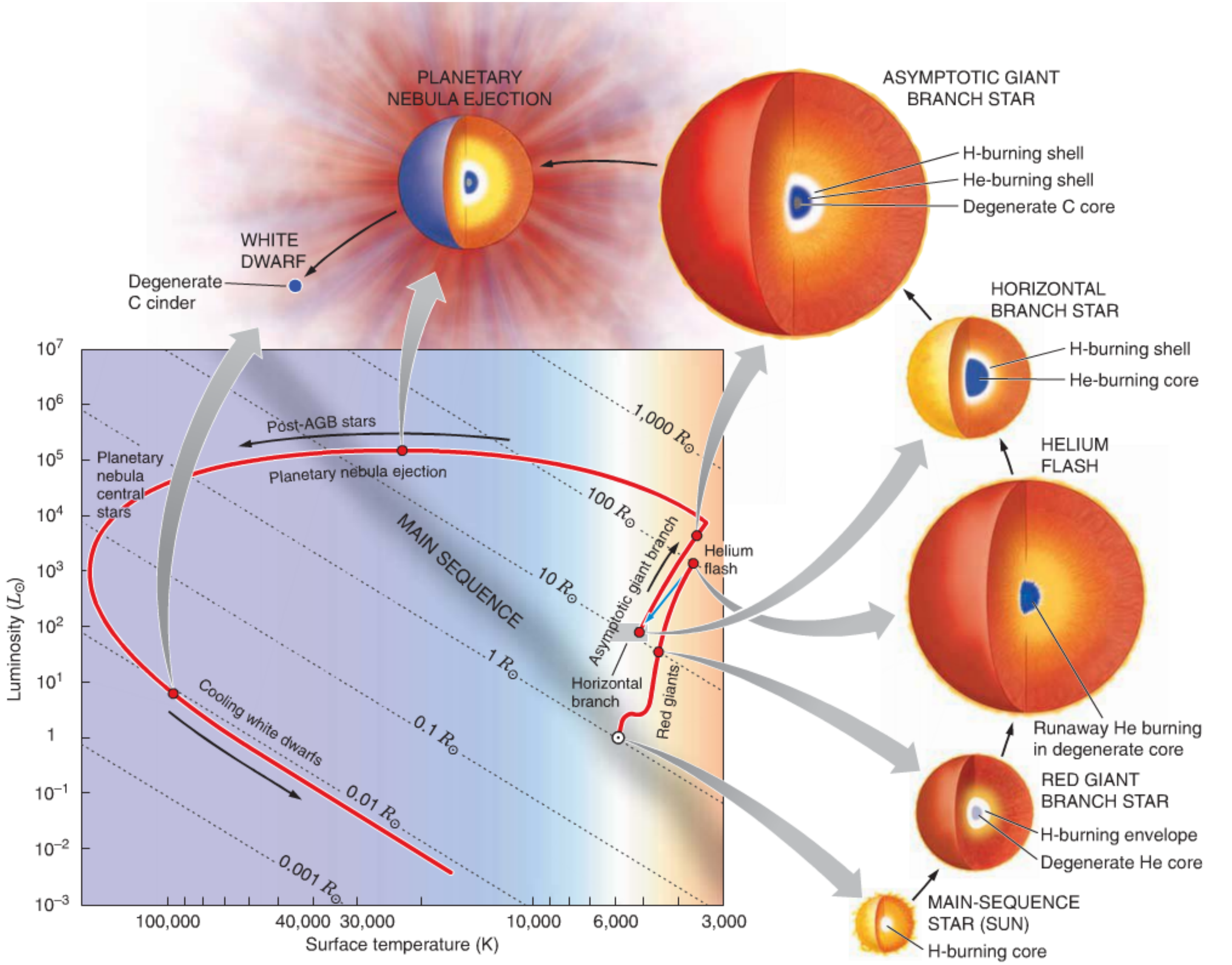
- Lasts **~10,000 yrs** before the gas expands too far and disperses into the ISM
- The hot **electron-degenerate Carbon core** gradually reveals itself as the star's outer envelope disperses.
- Observer finds a **white dwarf** that will cool for eternity
- Can you calculate the time it takes for the temperature to half?

End Product: White Dwarfs (e.g., Sirius B)

- The leftover electron-degenerate carbon core of a star remains as a white dwarf.
  - It is initially very hot but because there is no energy production, it must cool off eventually.
  - It is small: inverse mass-radius relation, about Earth size
  - It is roughly half the mass of its progenitor MS star: 0.6–1.4 M_{Sun}.
  - Its density is a ton per teaspoonful.
  - Its size remains constant as it cools (degenerate pressure).
  - It has much lower luminosity compared to MS stars (why?)

- Models predict that all low-mass stars (Mass < 3 M_{Sun}) evolves into white dwarfs. So they carry lots of mass in a galaxy.

Model-Computed Evolutionary Track of a Low-Mass Star

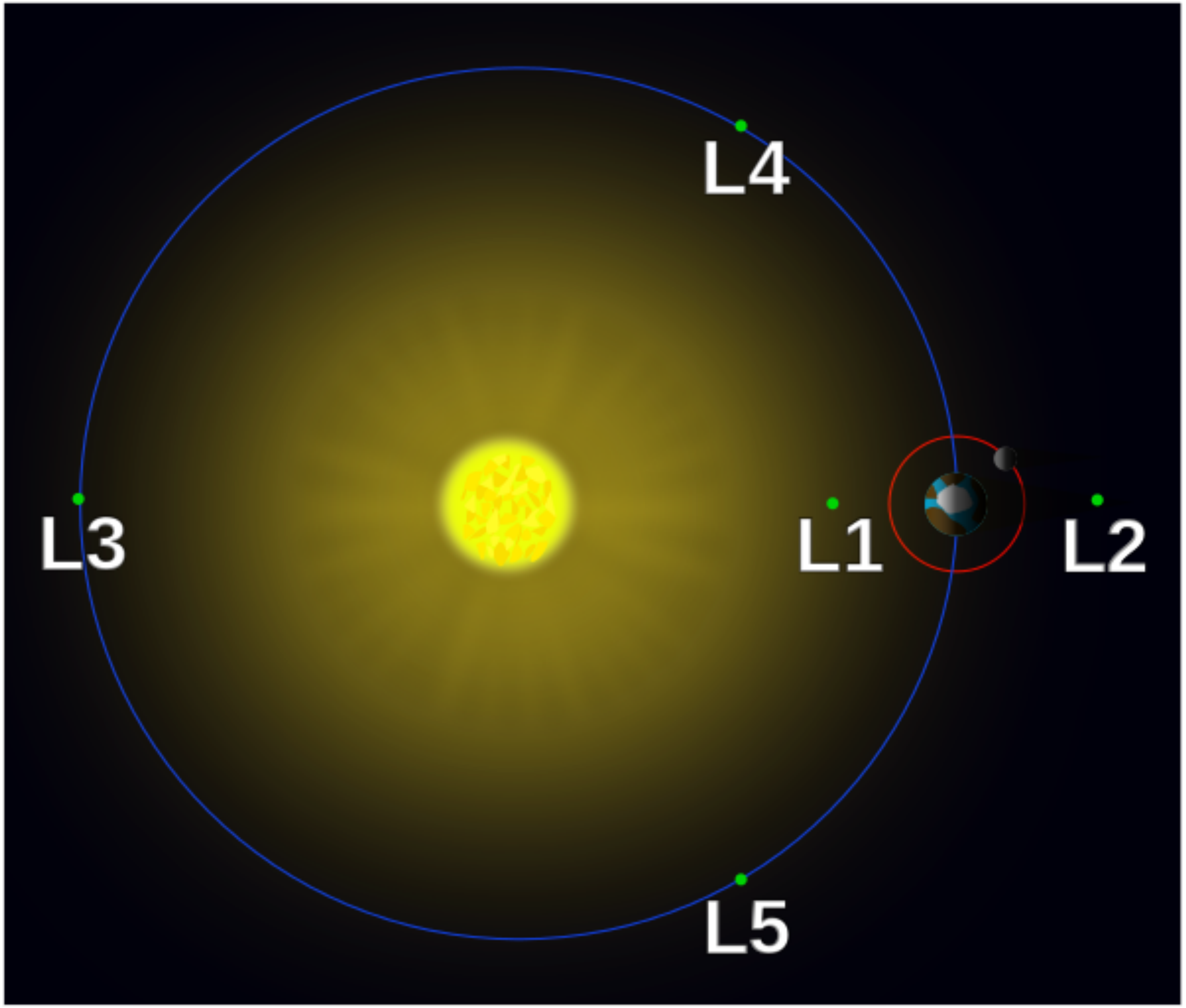


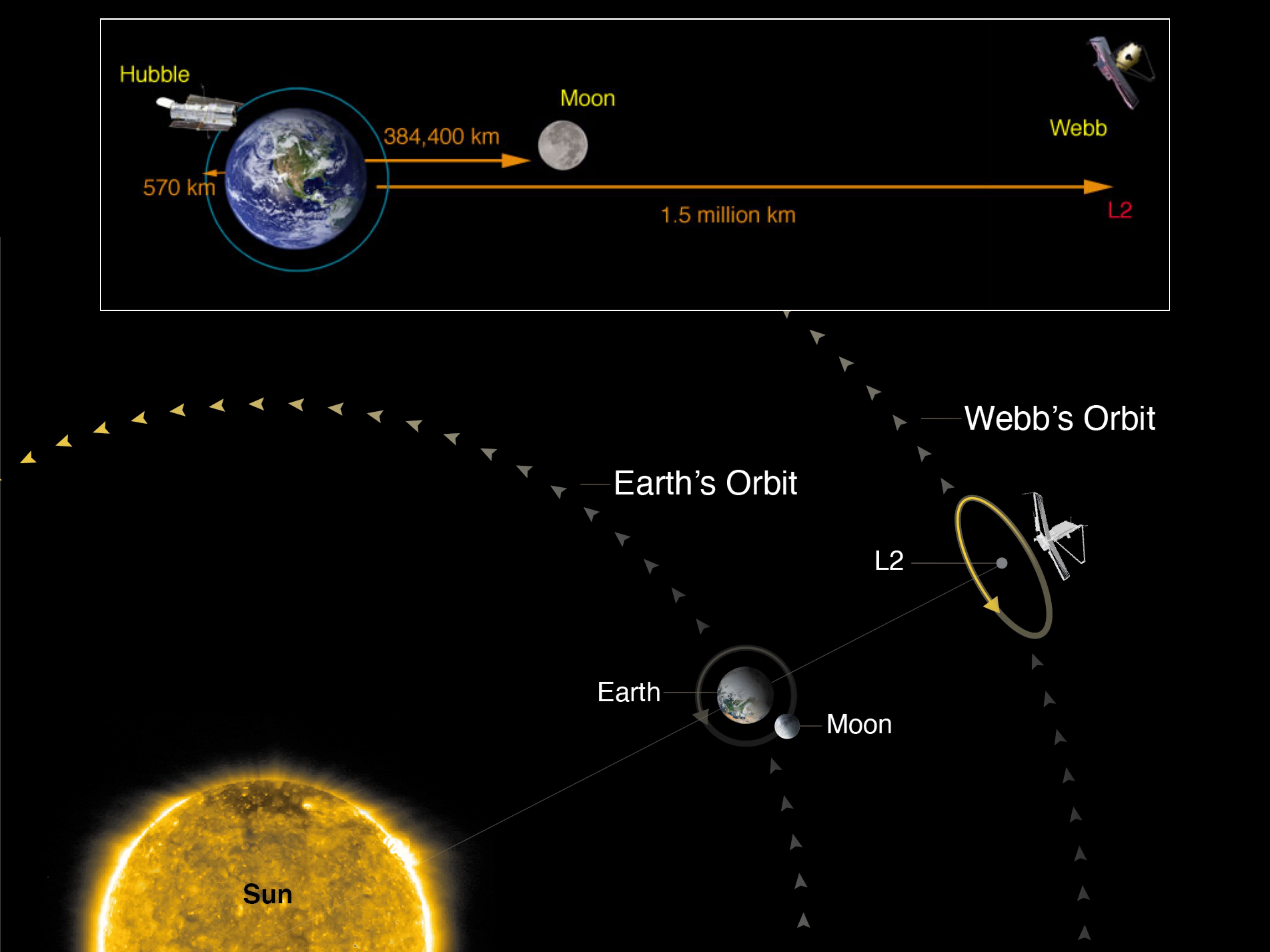
Mass-Transfer Binary Stars

Blue stragglers and Type-Ia SNe

Lagrange Points of a Binary System

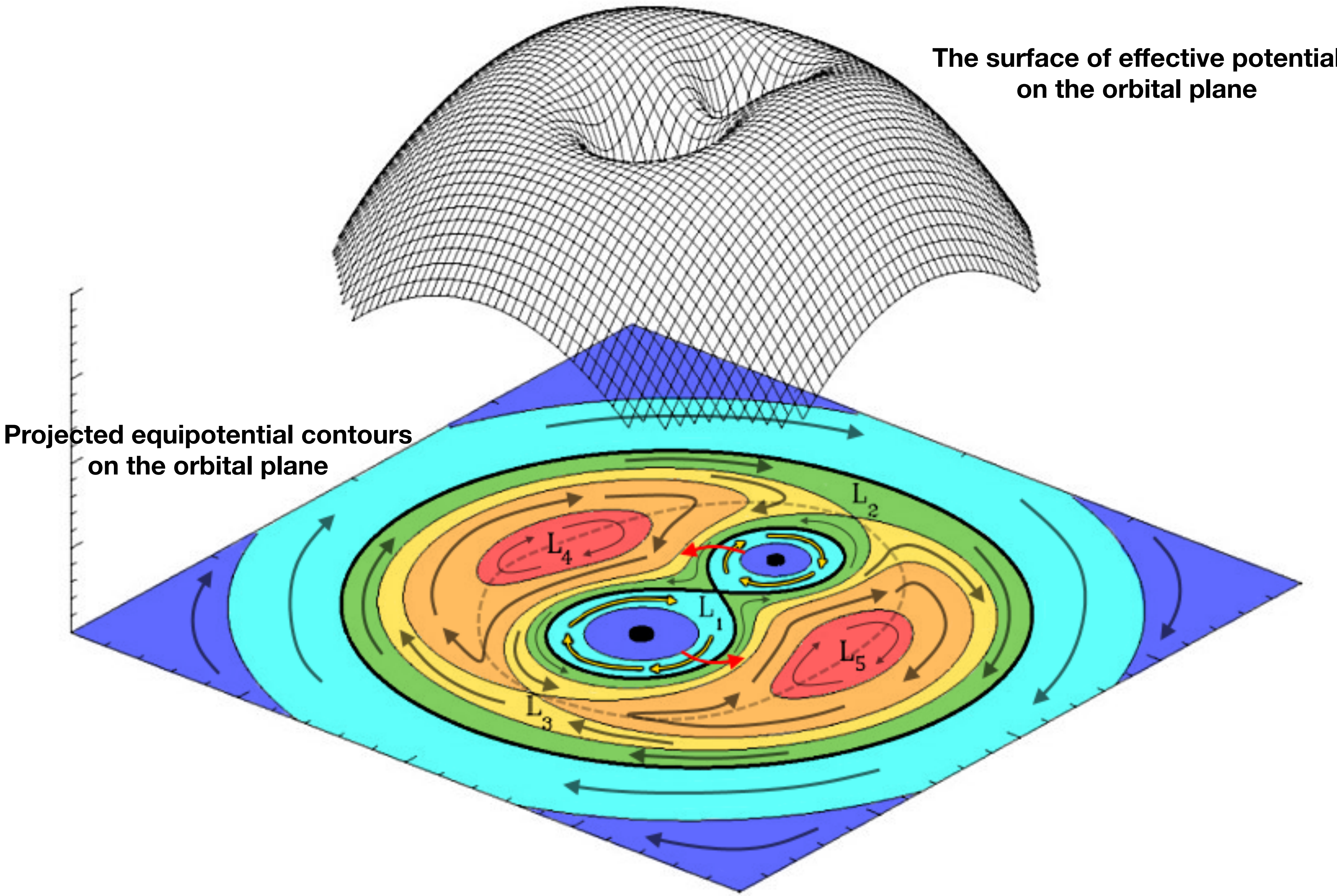
• Equilibrium points for small mass objects under the influence of two massive orbiting bodies




Hubble
Moon
Webb
384,400 km
570 km
1.5 million km
L2
Webb's Orbit
Earth's Orbit
L2
Earth
Moon
Sun

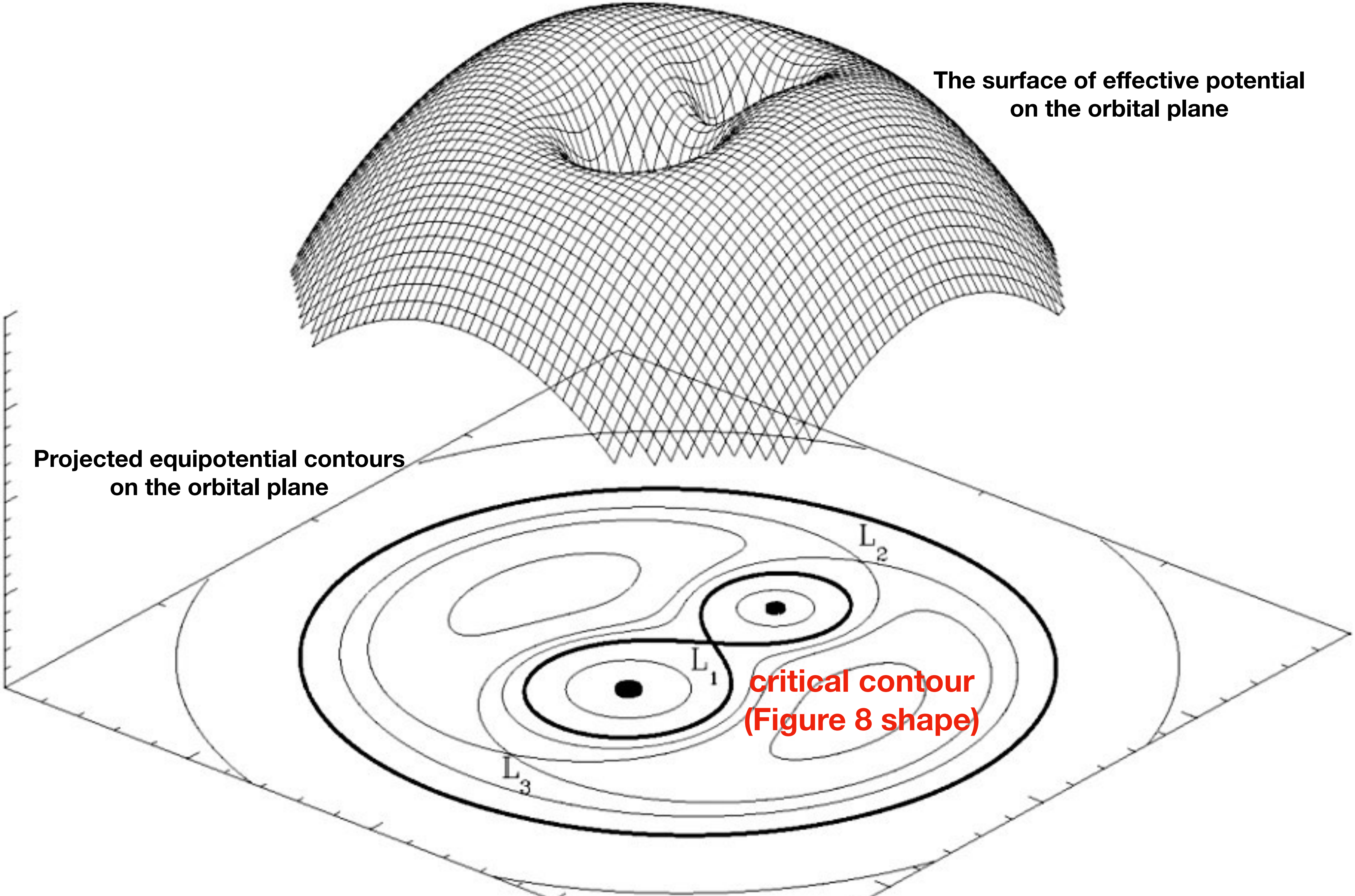
Lagrange Points of a Binary System

The contours of **equipotential** show the ***effective*** **gravitational potential** on the orbital plane, the **Lagrange points** are the **local maxima** where the gradient of effective potential is zero (**no acceleration** in the **co-rotating non-inertial reference frame**).



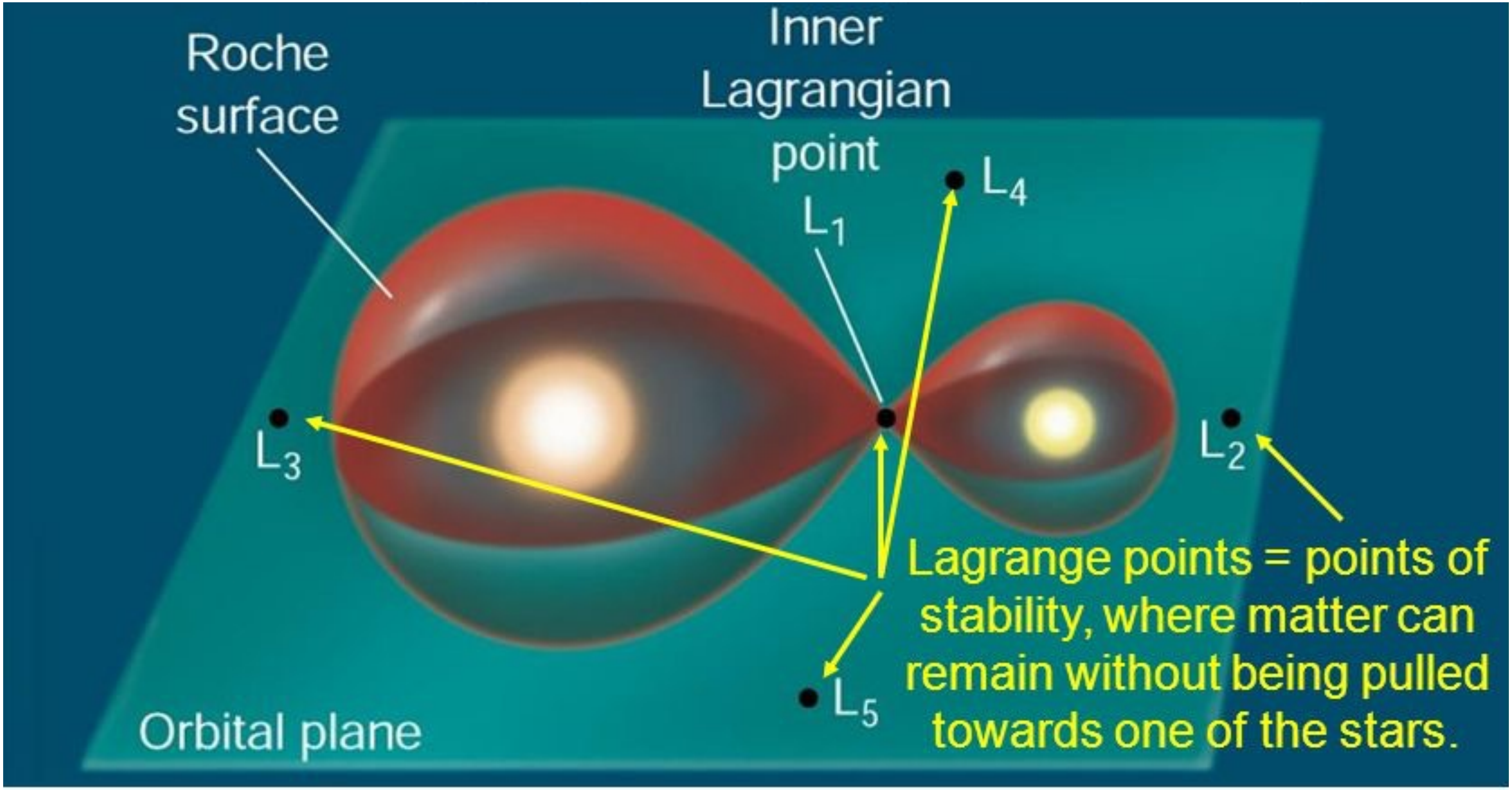
Roche Lobe (or Roche Surface)

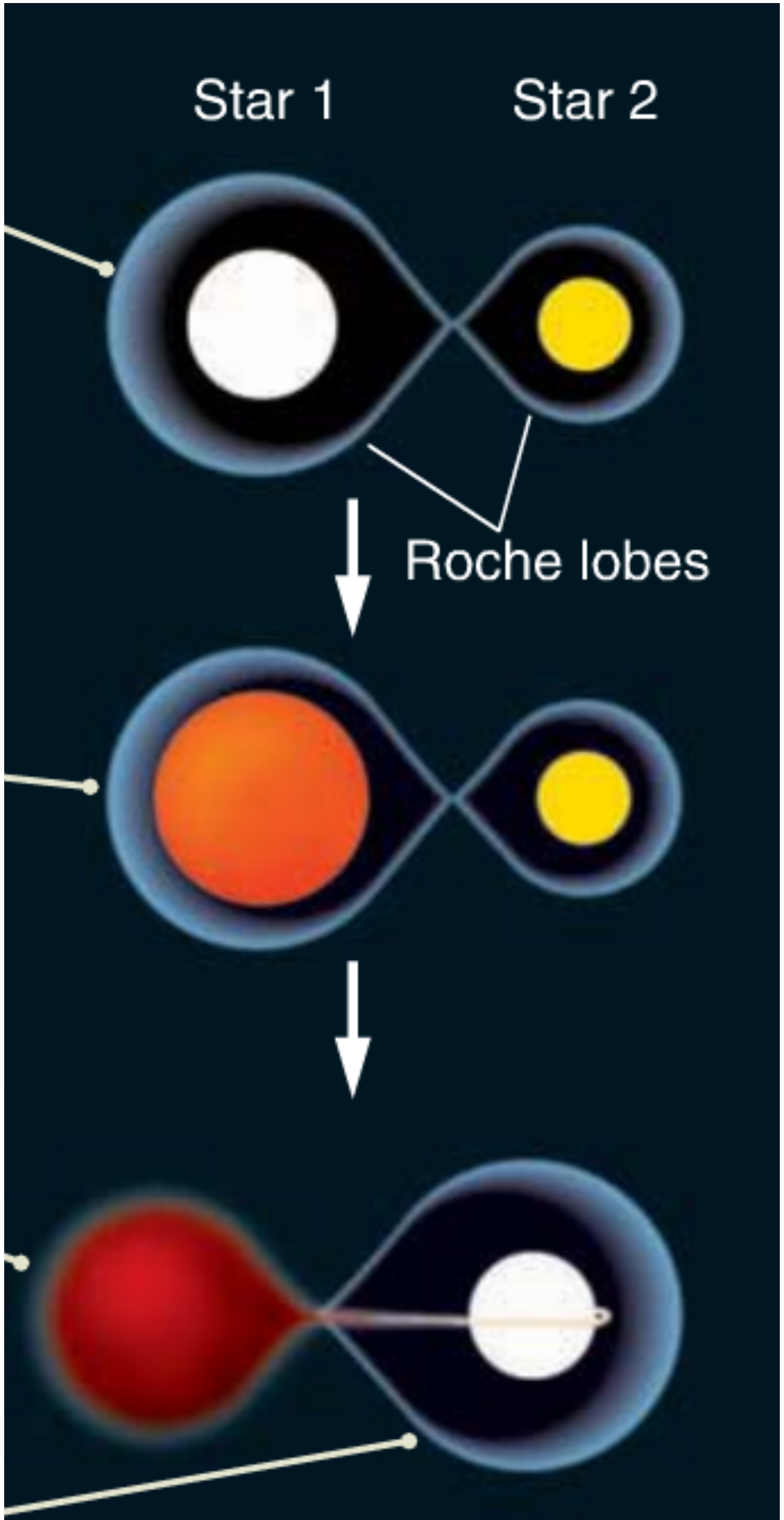
- On the orbital plane, there is a critical equipotential contour that intersects itself at the L1 point, forming a figure-of-eight.



Roche Lobe (or Roche Surface)

- In 3D, the critical equipotential surface delineates two lobes in a binary system, in each lobe, small-mass objects are gravitationally bounded to the massive object at the center.

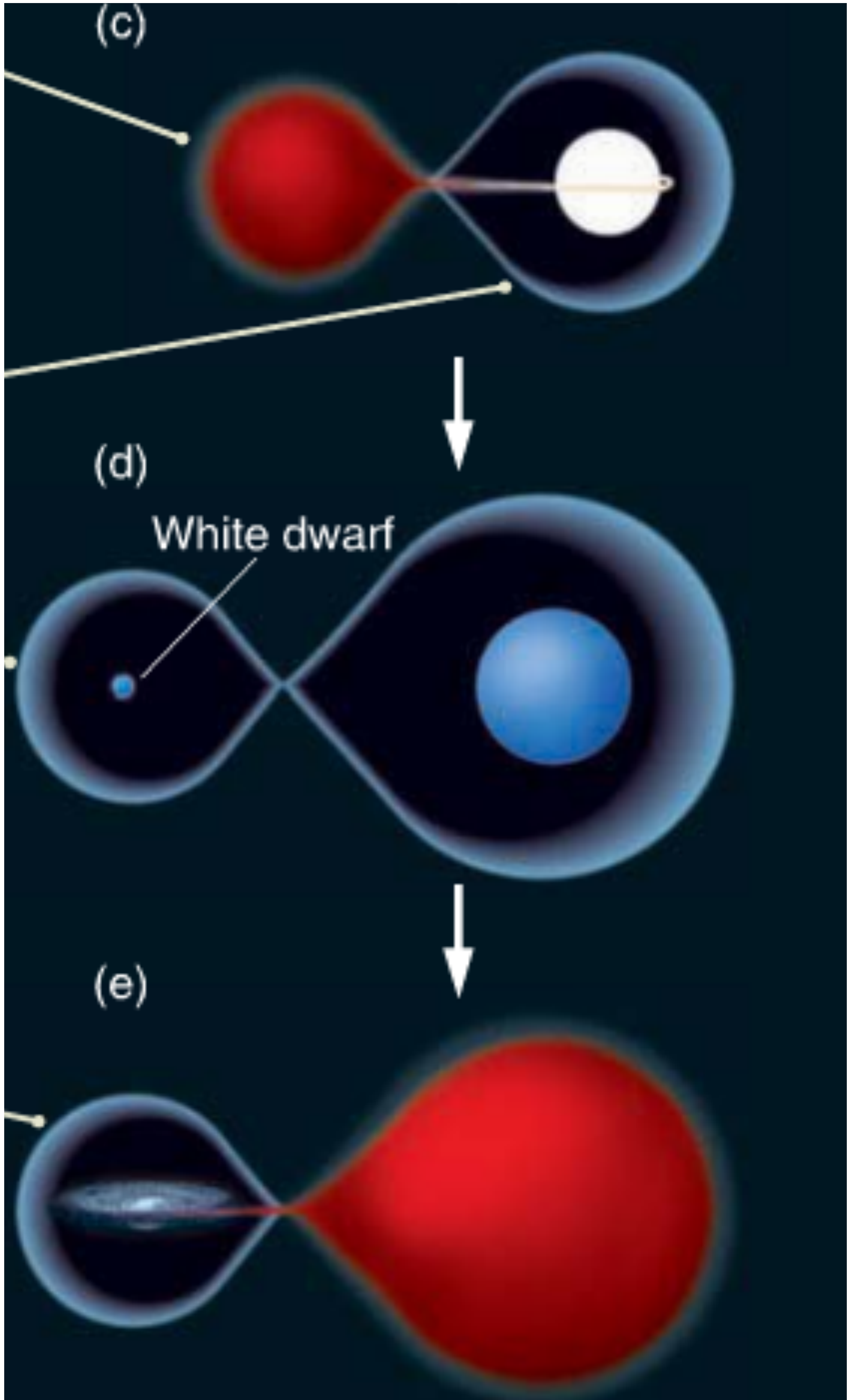




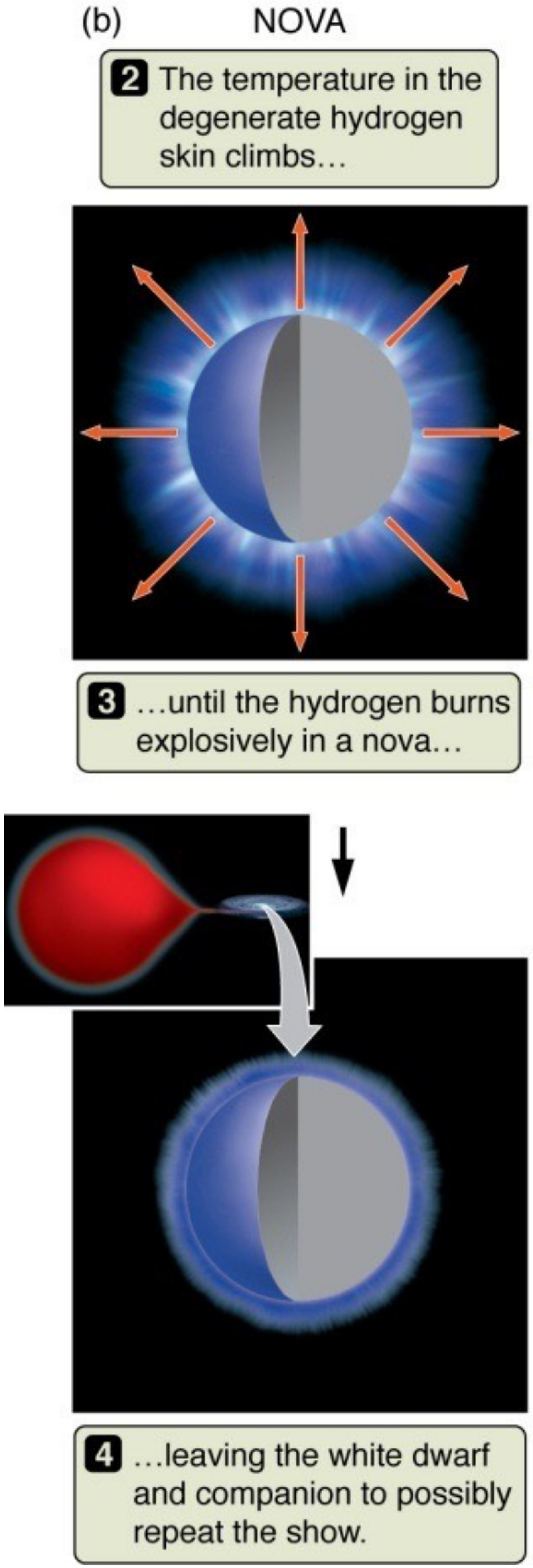
Mass-Transfer Binary Stars

- ~60% of stars are in binaries, a small fraction of which are very close binaries.
- The two stars in a binary have different MS lifetimes because of their different *initial* masses
- The more massive **primary** evolves into a **RGB** while the less massive **secondary** remains on the **MS** (middle figure)
- If the **Roche lobe** is smaller than the possible size of the RGB, the **red giant primary** can only expand so much before material is lost to the **MS secondary's** gravity (bottom fig)

Mass-Transfer Binary Stars



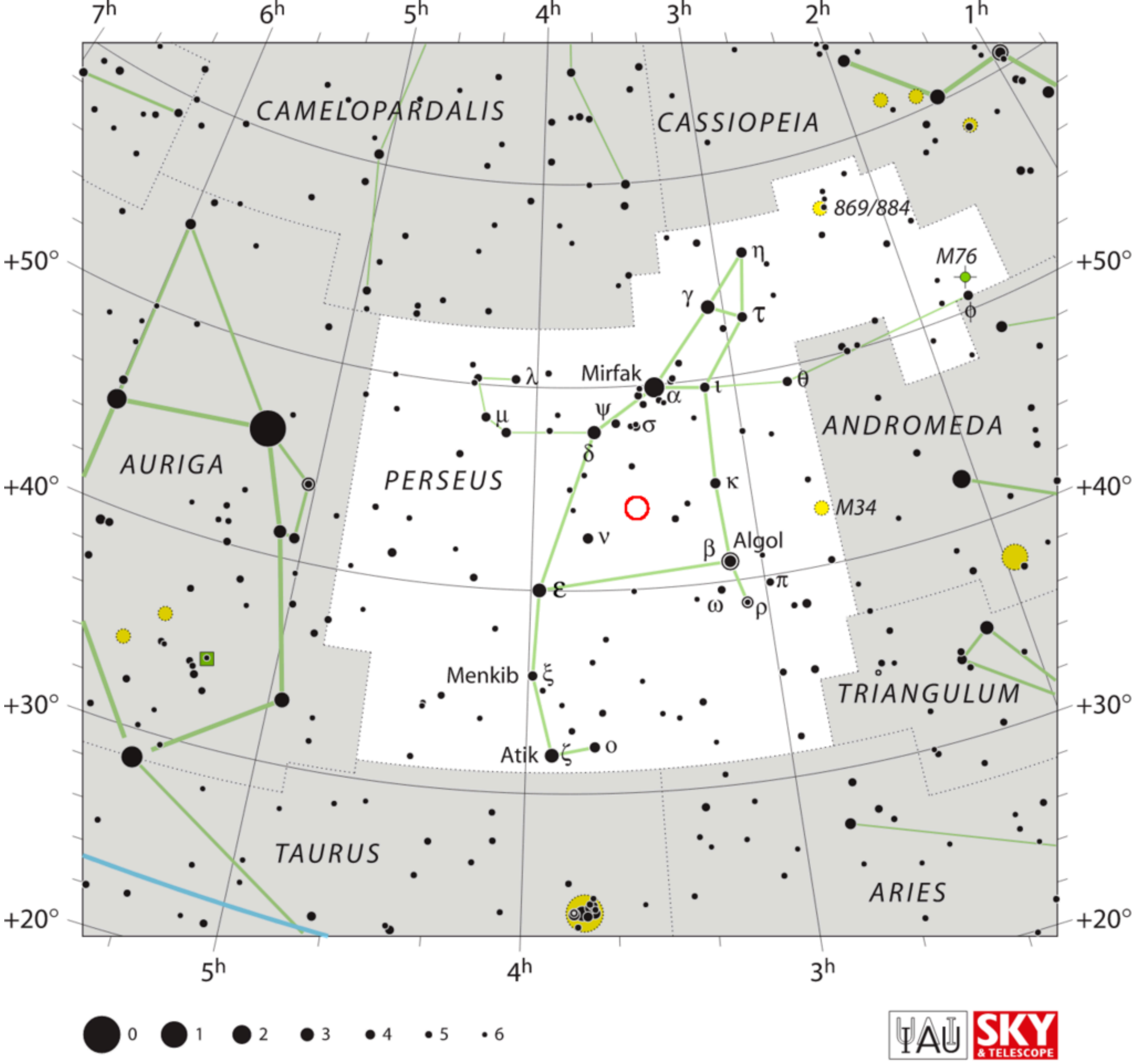
- It's likely that by the time the less massive star evolves to a red giant, the originally more massive star already evolved into a WD (Fig d)
- So mass transfer reverses: the secondary star begins to lose its envelope to the WD's gravity, forming an accretion disk (Fig e)
- As the WD grows in mass because of accretion, there are two possible consequences.



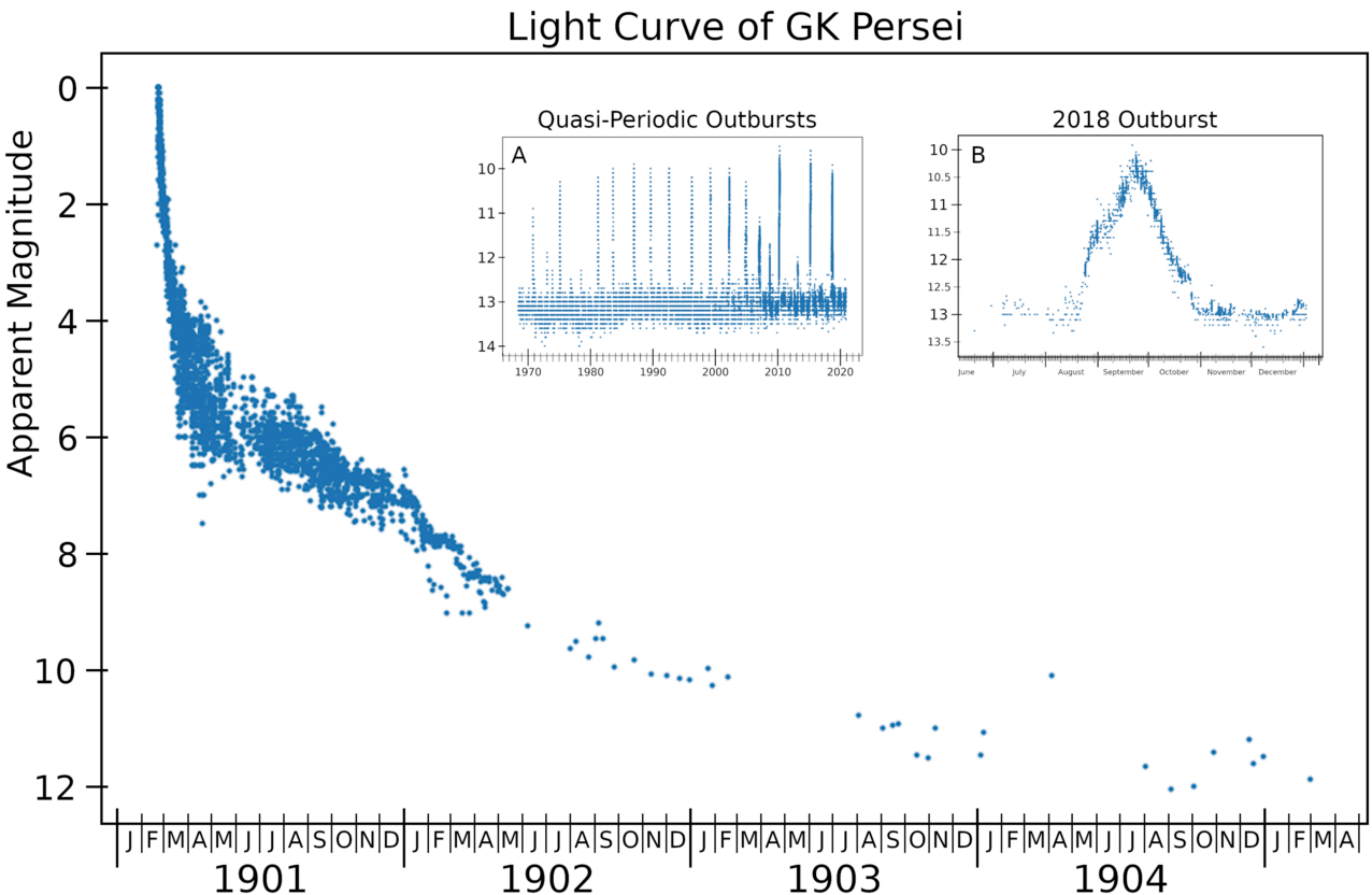
I: Classical Novae

- H deposition on the surface of the white dwarf from the red giant star
- Condenses onto degenerate core and explosively burns episodically: **Nova**
- For a few hours, a Nova can be 10^5 times more luminous than the Sun.

GK Persei: Nova Persei 1901 red circle on the chart

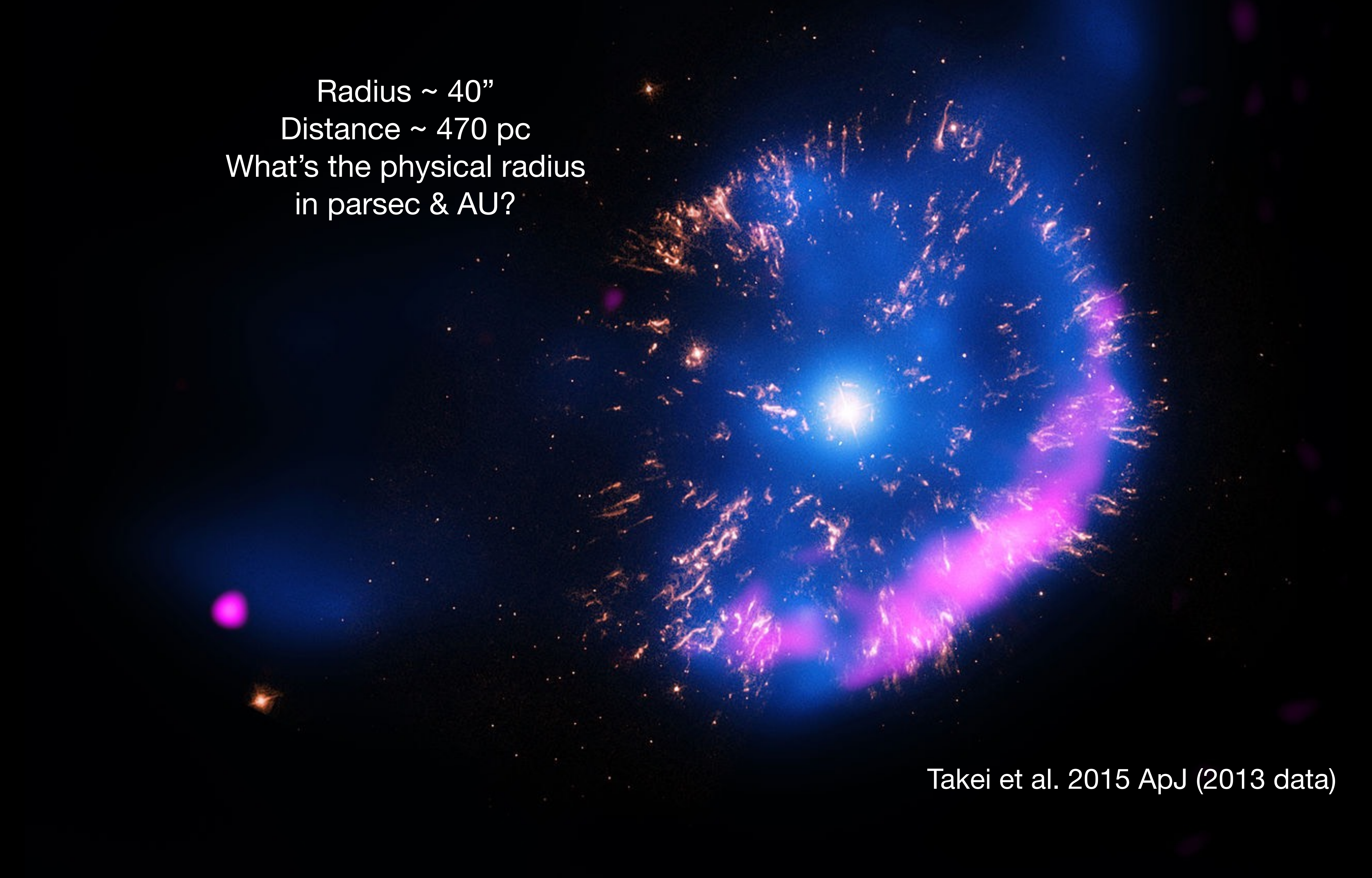


In 1901, GK Persei was one of the brightest star on the sky (for a few days)

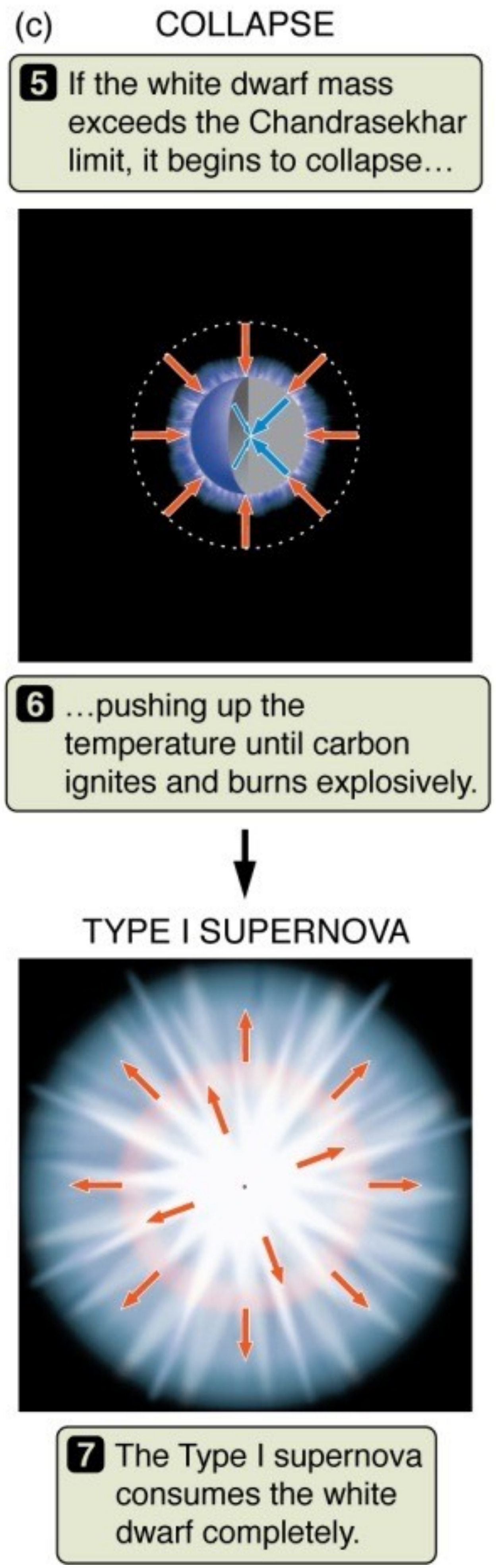


GK Persei: Nova of 1901
X-ray (blue), optical (yellow), radio (pink)

Radius ~ 40"
Distance ~ 470 pc
What's the physical radius
in parsec & AU?



Takei et al. 2015 ApJ (2013 data)

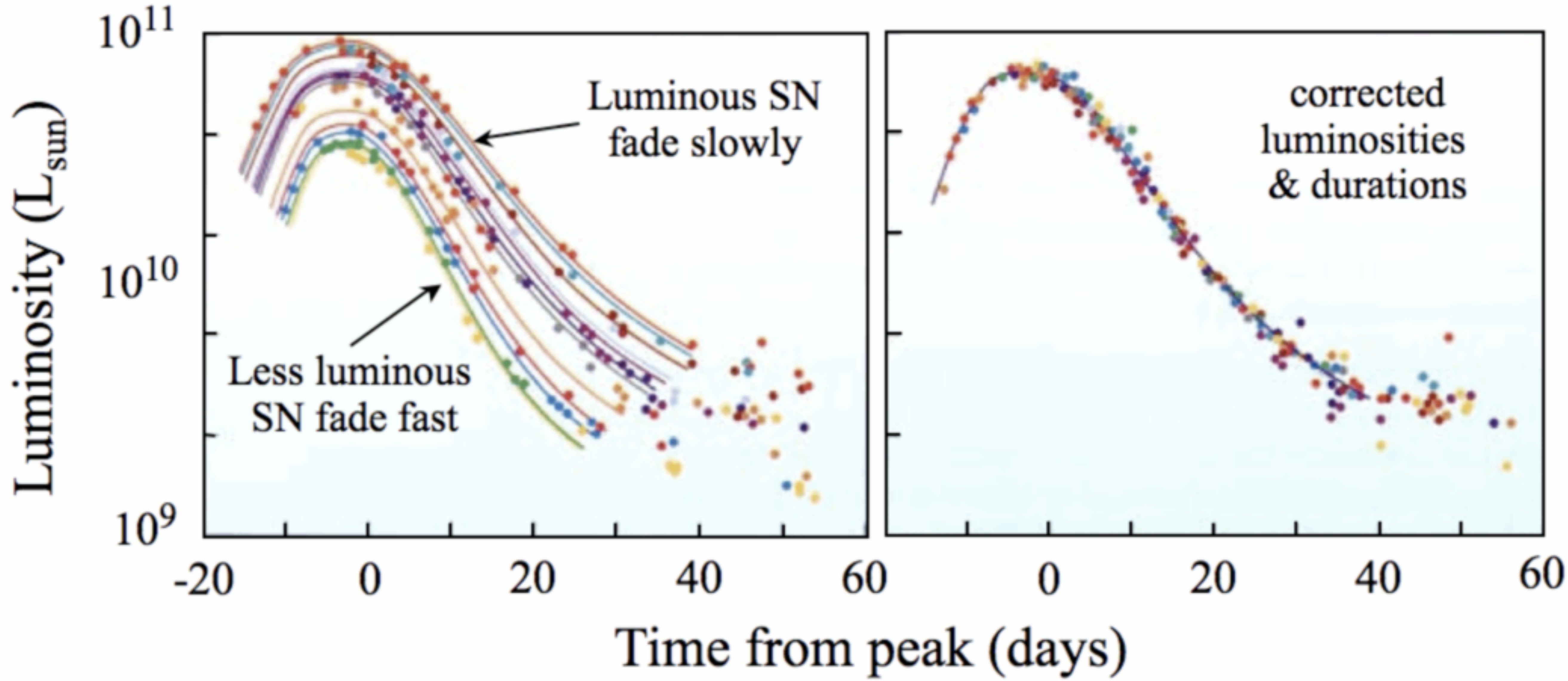


II: Type Ia Supernovae

- WD *mass increases over time* because of accretion from the RGB
- When its mass reaches 1.4 M_{sun}, the **Chandrasekhar limit**, gravity overcomes the ***relativistic*** **electron degeneracy pressure**
- The WD collapses, heats up and triggers a **thermonuclear runaway**:
  - **"C Flash"**: C core burns out in <1 sec! Forms Mg, Ne, Na, Ni, Fe.
  - This is a Type Ia supernova.
- 10^{10} times brighter than the Sun – comparable to the luminosity of a galaxy!
- Note: Type Ia may also be WD mergers

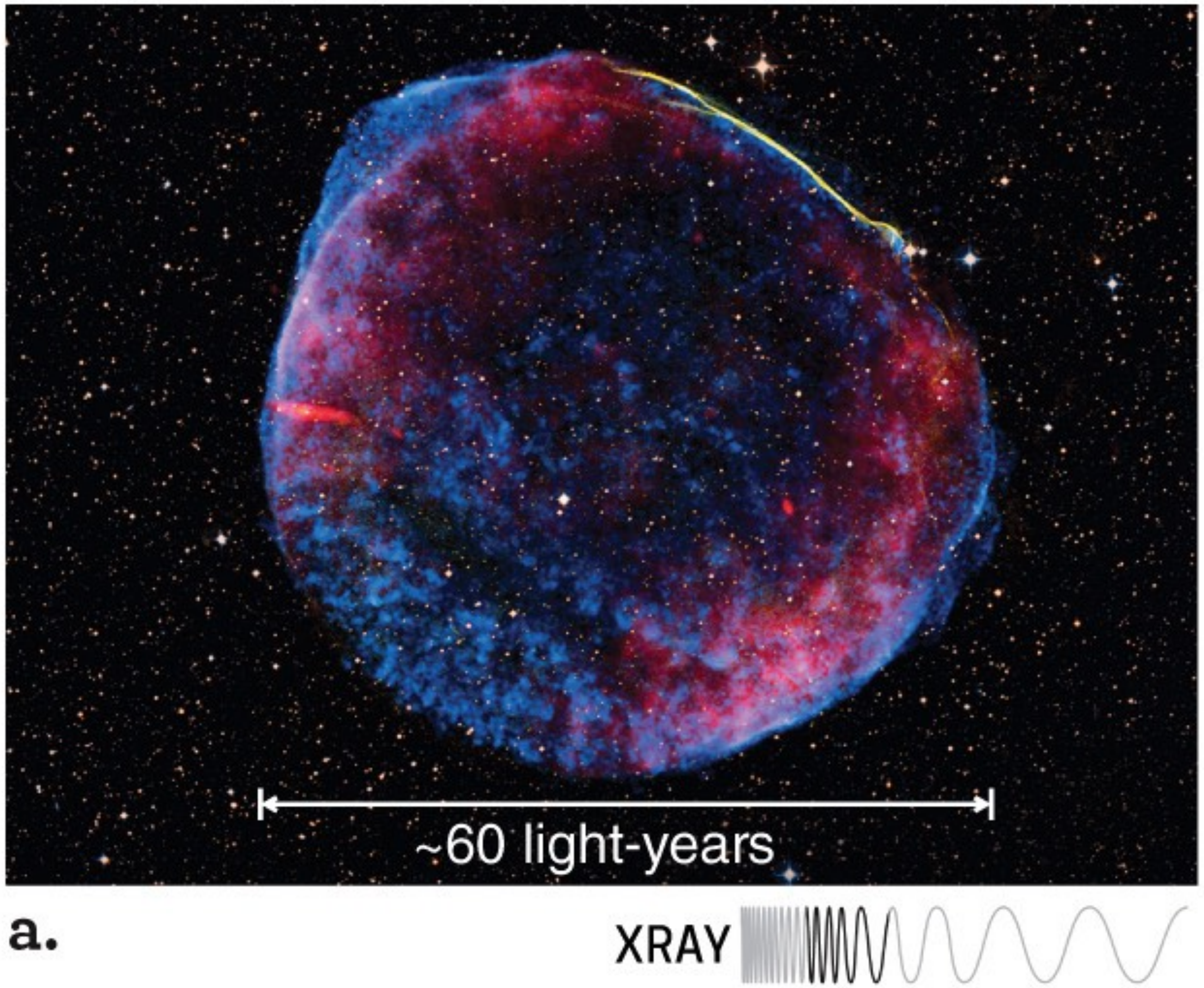
Type Ia Supernovae

- Over just a few days, the explosion releases about the same amount of energy as the Sun does over its entire main-sequence lifetime (10^{44} Joules).
- Type Ia supernovae are excellent **distance indicators** because they are **standard candles** (luminosity can be inferred from the shape of its light curve)

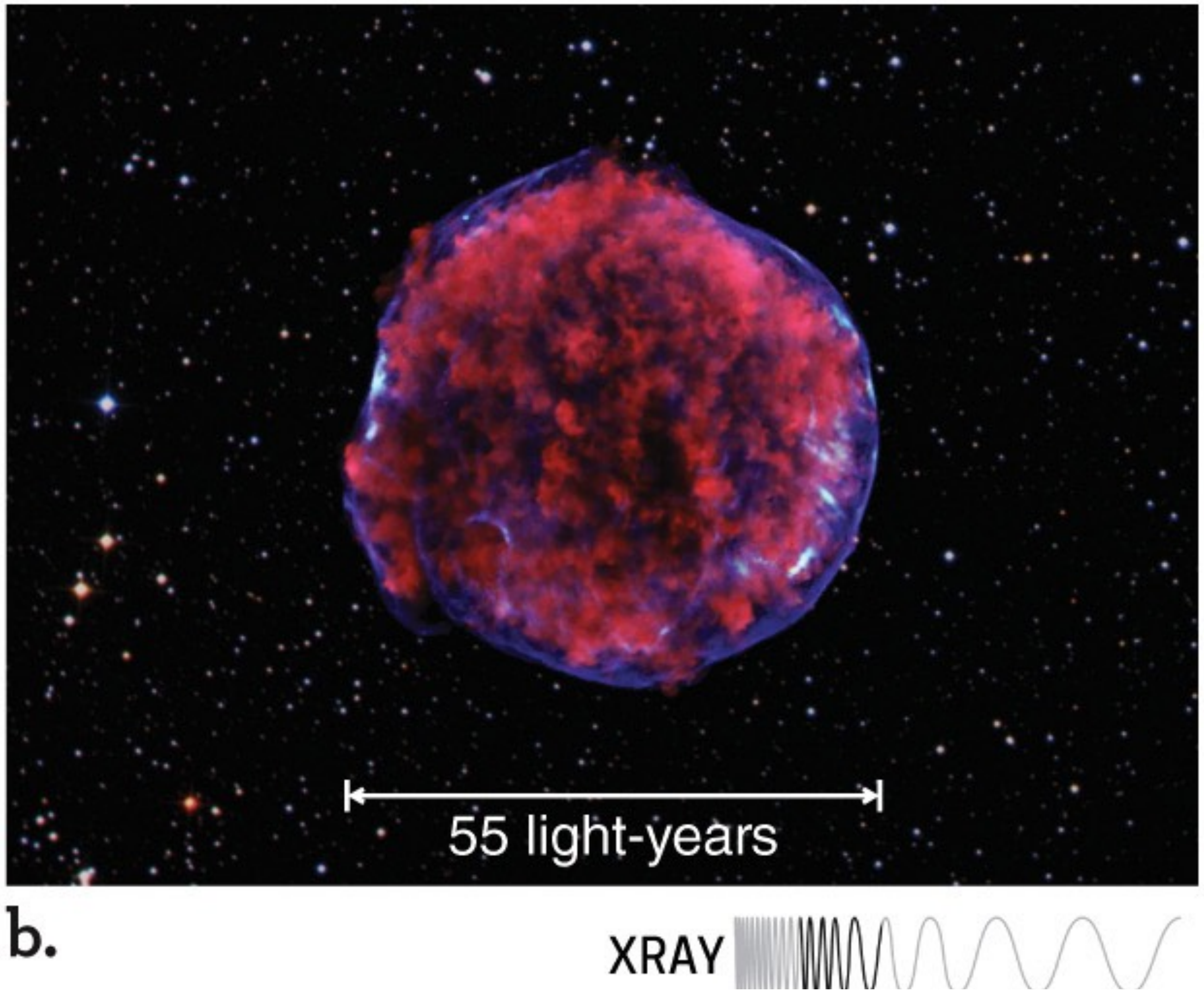


Supernova Remnants

- The entire star explodes in the thermonuclear runaway — there is no central star left (unlike planetary nebulae)
- **Supernova remnants** are leftover shells of dust and gas from the explosion

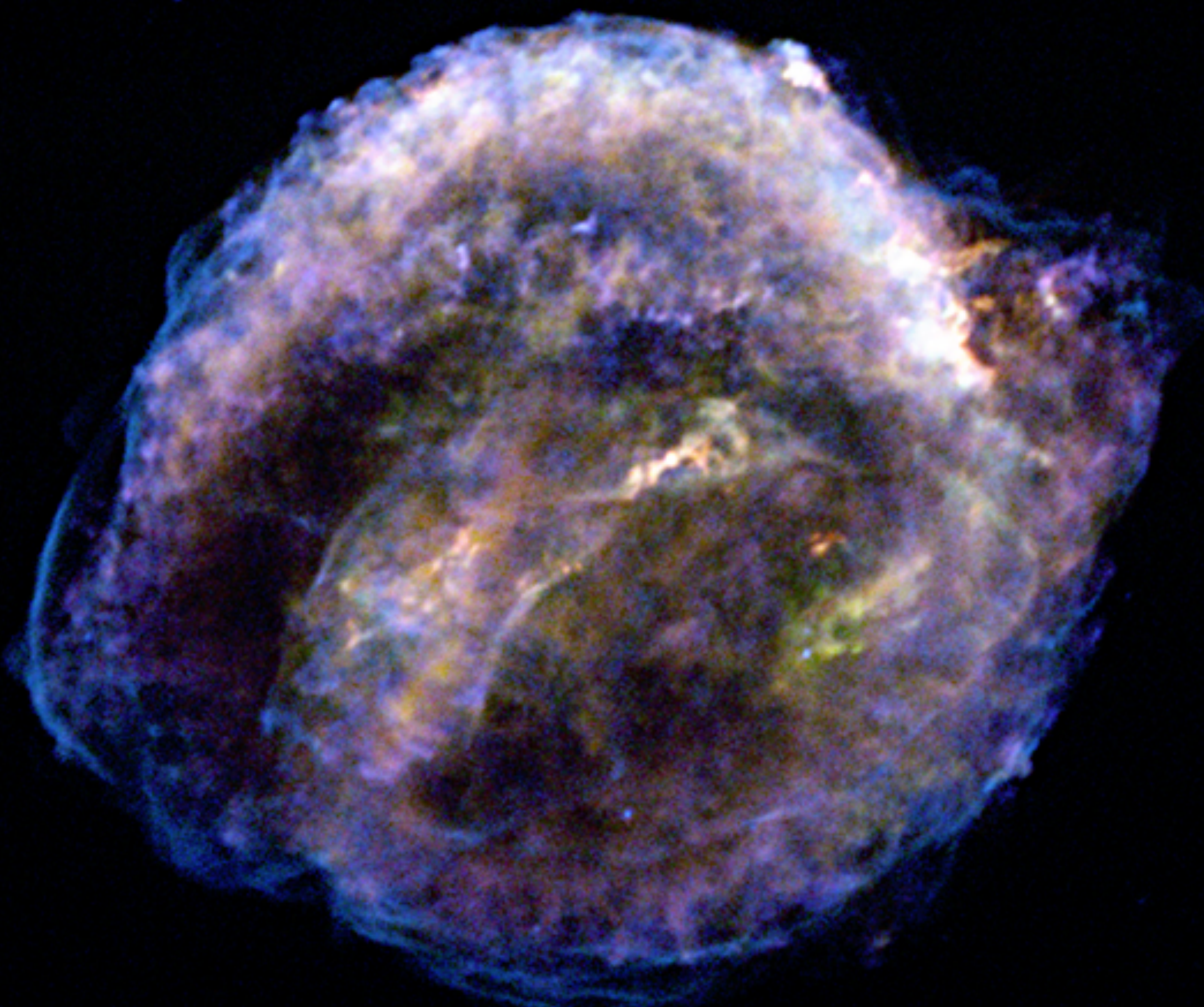


a.

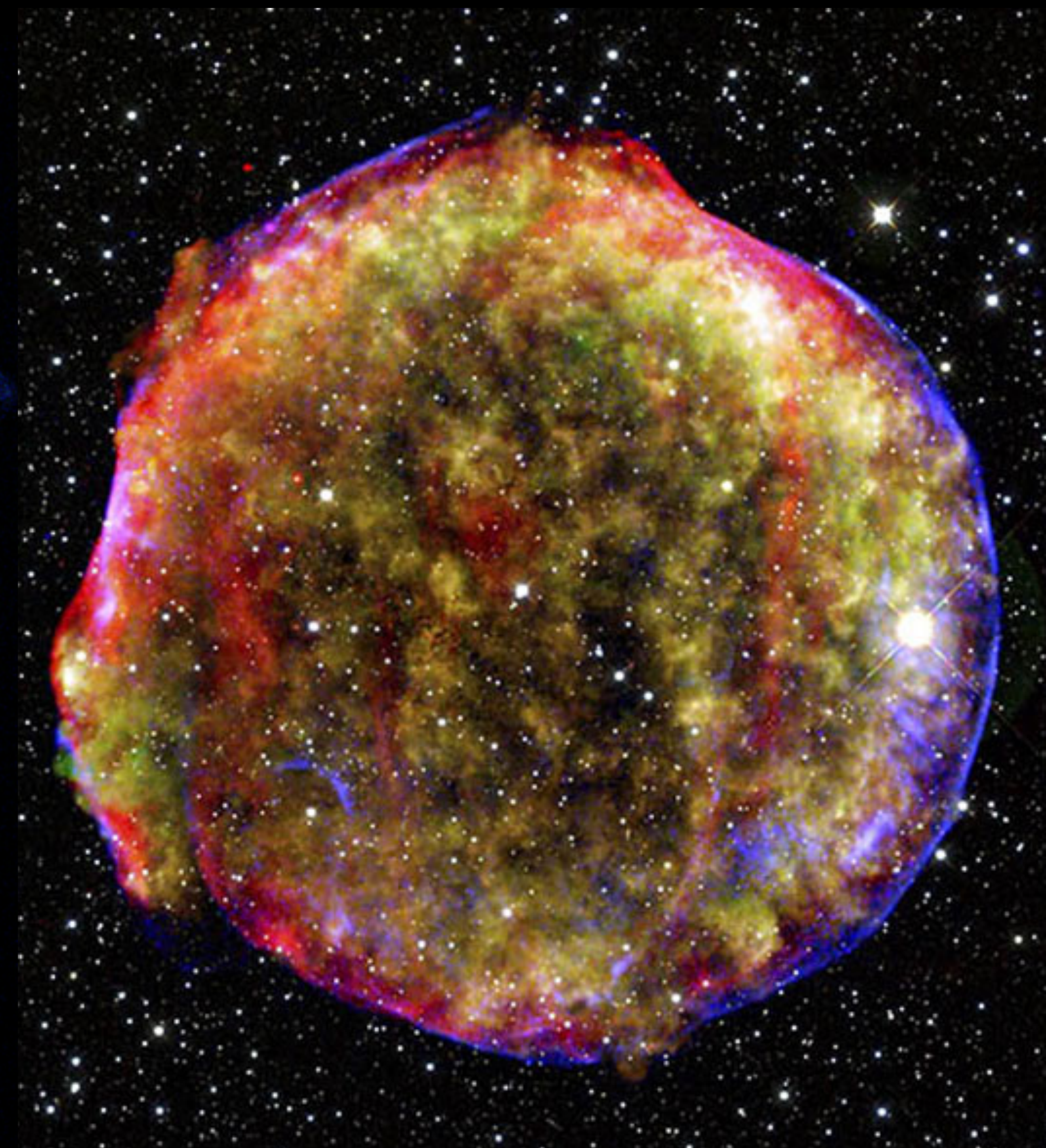


b.

More X-ray Images of Type Ia Supernova Remnants

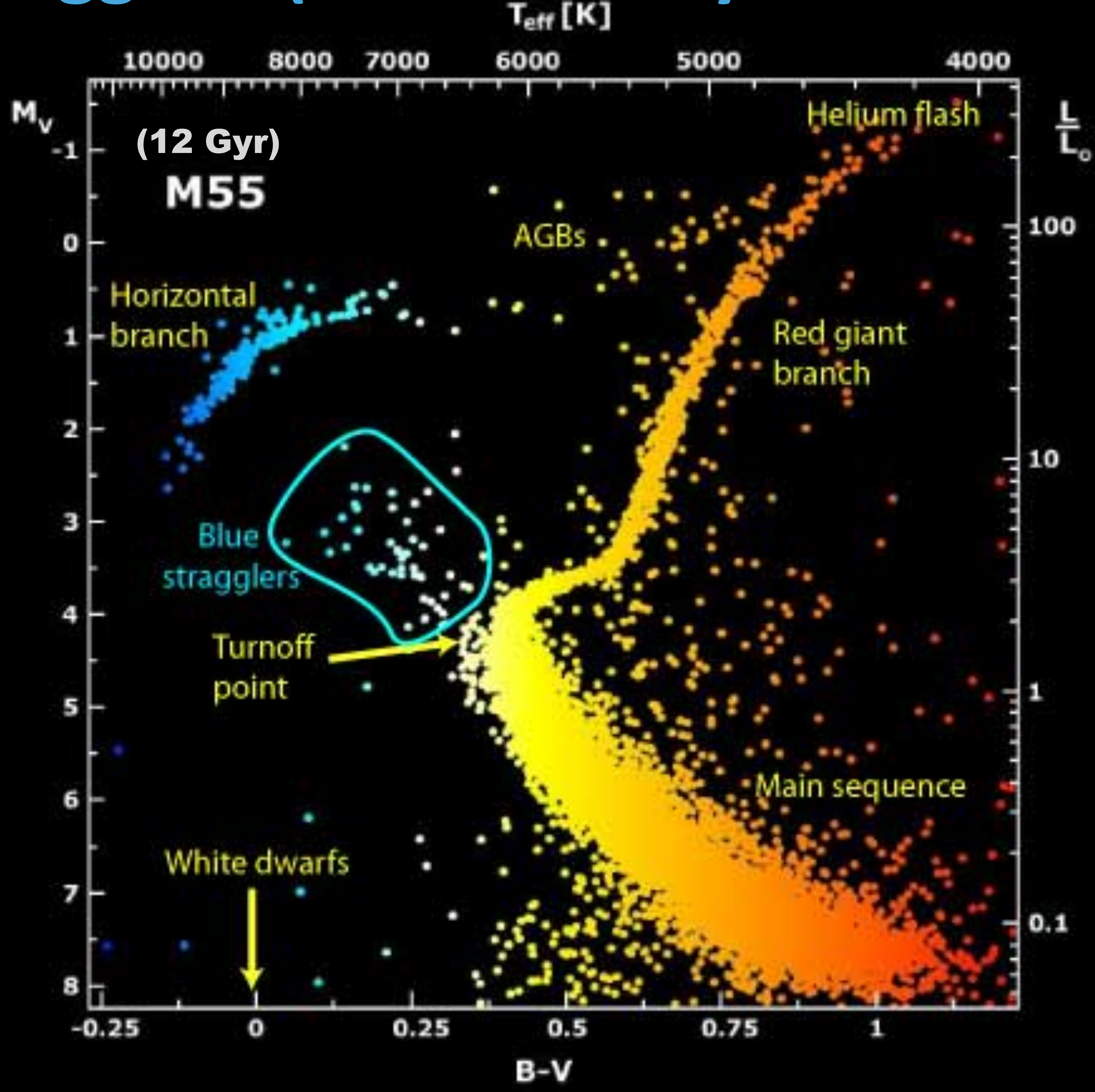


Kepler supernova (1604)



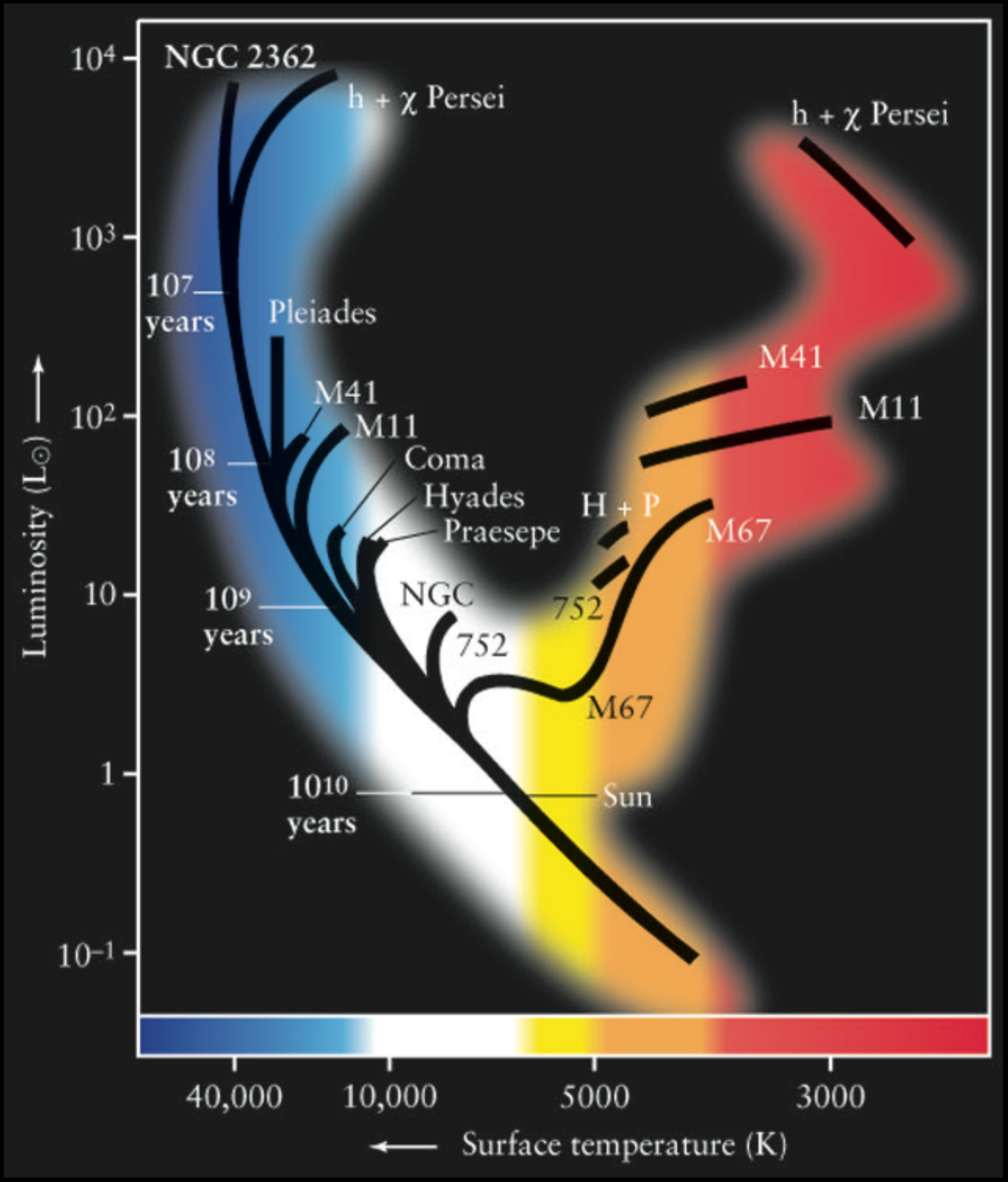
Tycho supernova (1572)

Can you use mass-transfer to explain blue stragglers (MS stars beyond the turnoff)



Chap 16: Key Concepts

- Observations
  - Nothing last forever, even stars
  - H-R diagram of star clusters
- Numerical Models
  - Equations of stellar structure and evolution
  - Stellar evolutionary tracks
- Fine-Tune Models
  - Isochrones (equal-age lines)
  - Fitting cluster H-R diagrams
  - Cluster age estimates
- Model Inferences
  - Main stages and rough lifetimes
  - Changes in the interiors of the stars: e- degenerate core + fusion shells
- Mass-Transfer Binaries
  - Roche Lobe, Lagrange Points
  - Novae, Type Ia SNe, Blue Stragglers



Chap 16: Key Equations

- Hydrostatic Equilibrium:

$$\frac{dP}{dr} = -\rho g(r) = -\rho \frac{GM_r}{r^2}$$

- The pressure from non-relativistic degenerate gas is:

$$P_{\mathrm{degen}} = \frac{2}{3} n \frac{p^2}{2m} \approx \frac{h^2}{4\pi^2} \frac{n^{5/3}}{m}$$

- The pressure from ideal gas is:

$$P_{\mathrm{ideal}} = \frac{2}{3} n \left(\frac{3}{2} kT \right) = nkT$$

- The condition for degeneracy is

$$\frac{h^2}{4\pi^2} \frac{n^{2/3}}{m} > kT$$

- Mass-Radius relation of White Dwarfs:

$$R \propto M^{-1/3} \text{ when } M < 1.4 M_{\odot}$$