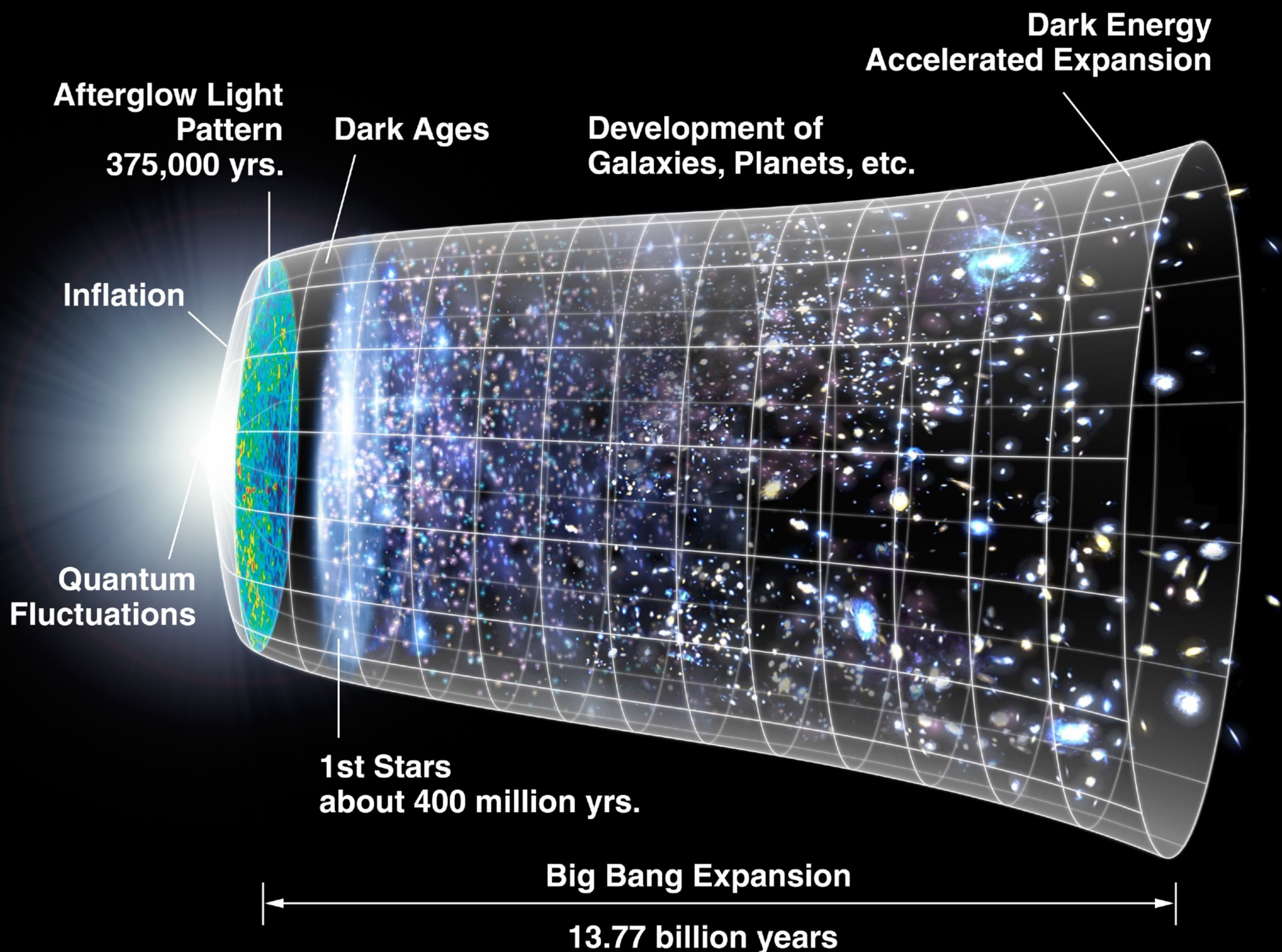
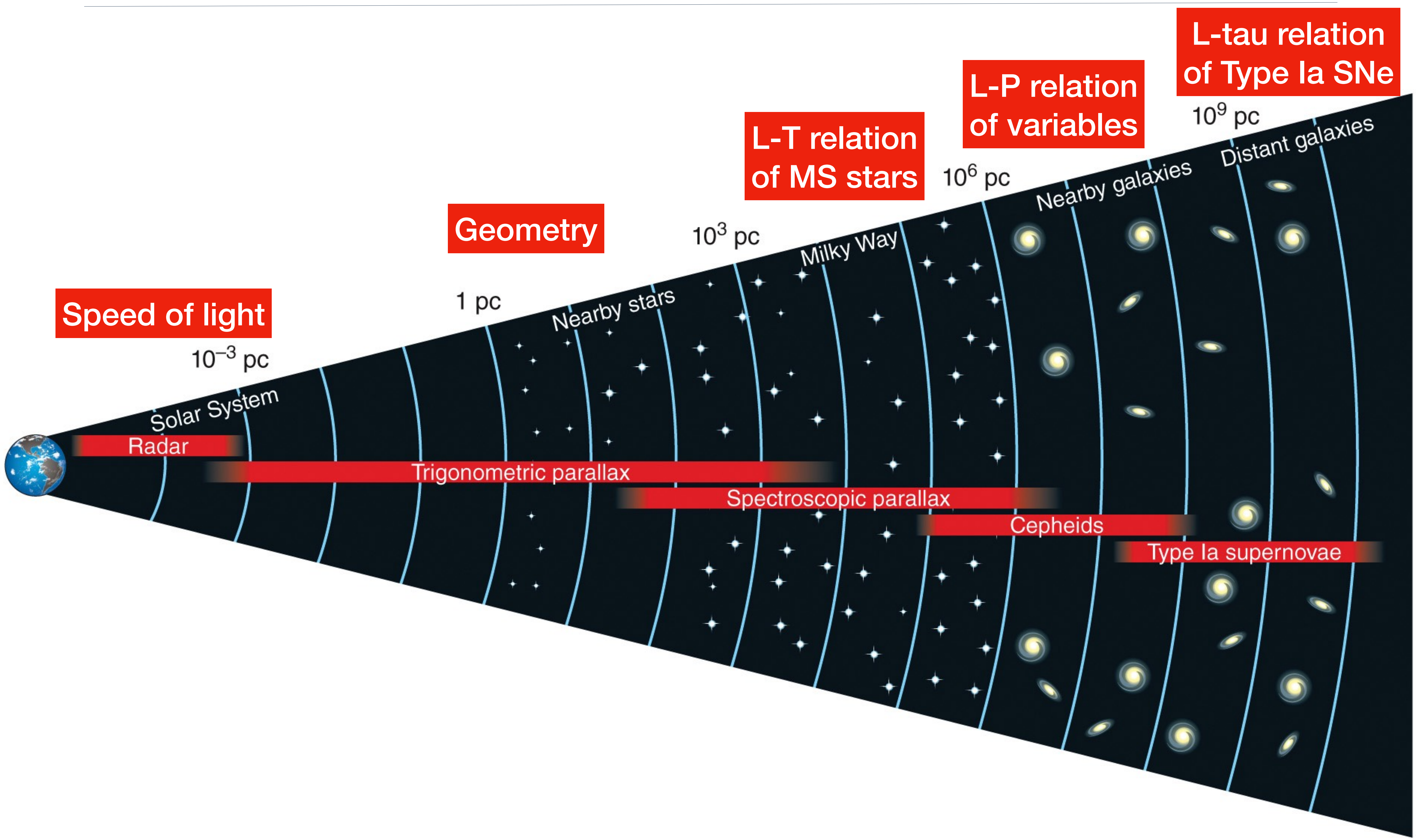
# Chap 21 & 22: The Expanding Universe

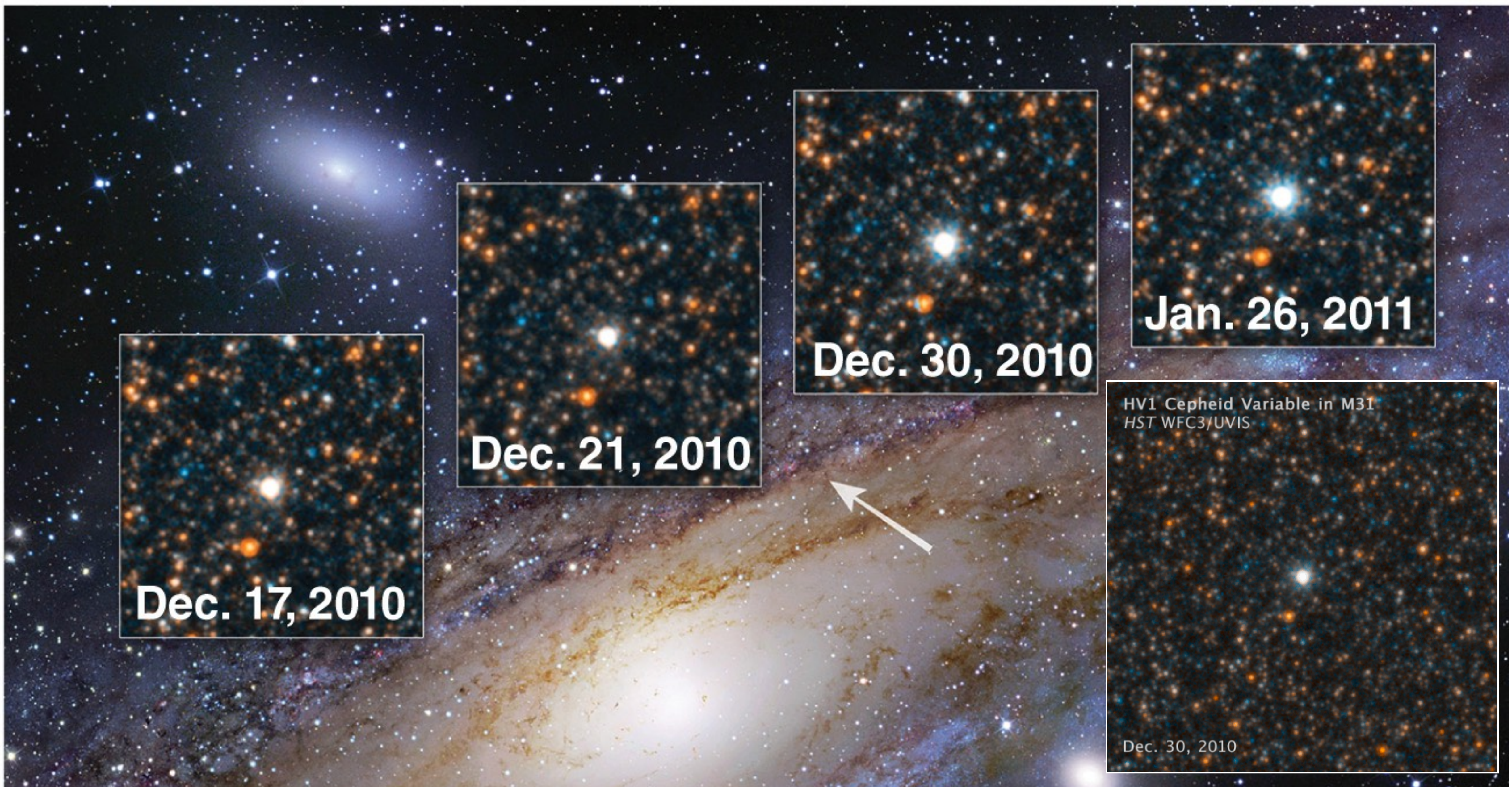


# Photometry: Distance measurements using standard candles



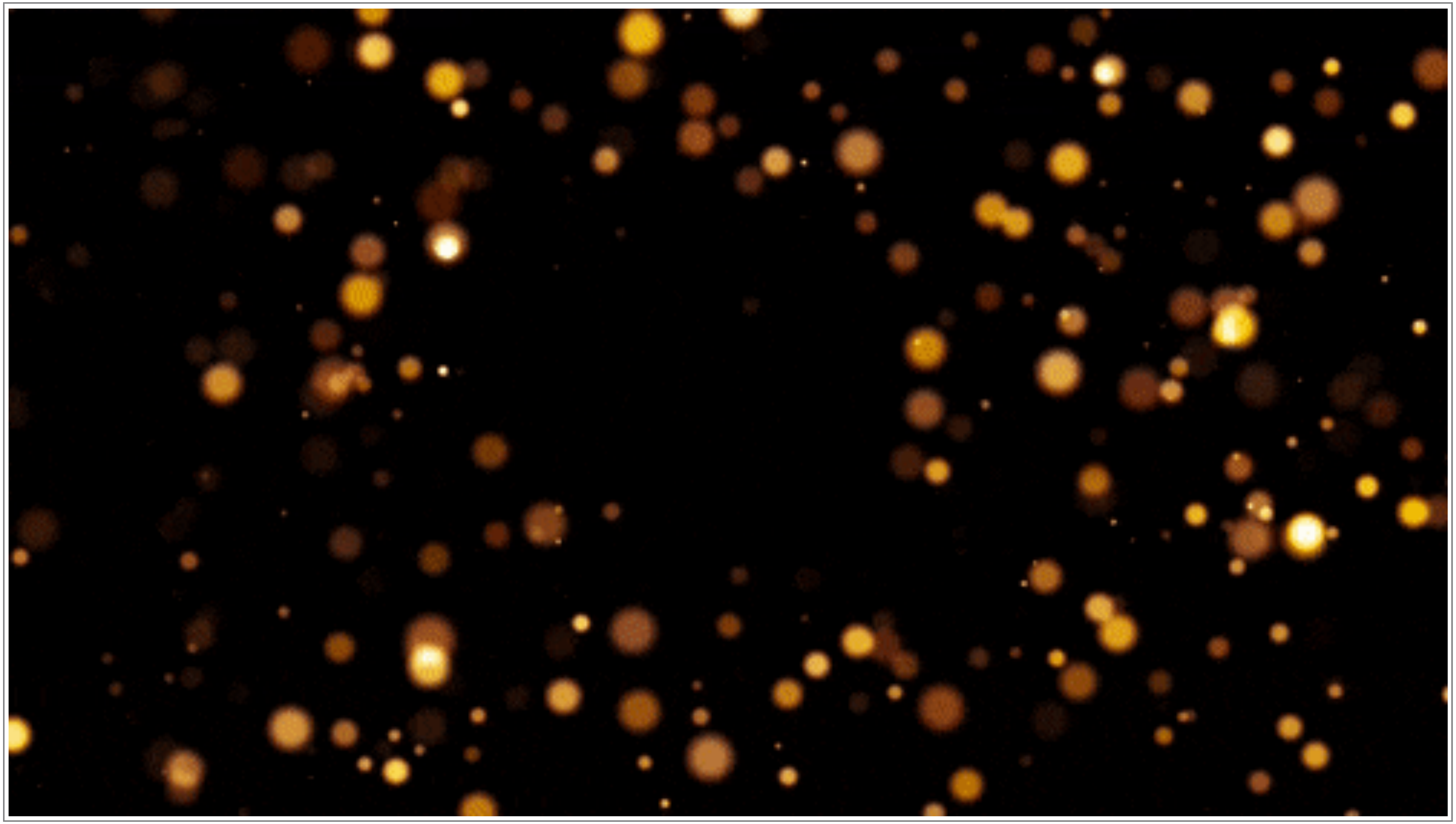
# The "1920 Great Debate" was resolved by distance measurements

- Hubble (1929): ***"A Spiral Nebula as a Stellar System, Messier 31"***
- He discovered a **Cepheid variable** inside of the Andromeda (**M31-V1**).
- He used Leavitt (1908)'s **Luminosity-Period relation** to calculate the distance to M31. This distance was much greater than the size of the Milky Way per Shapley.



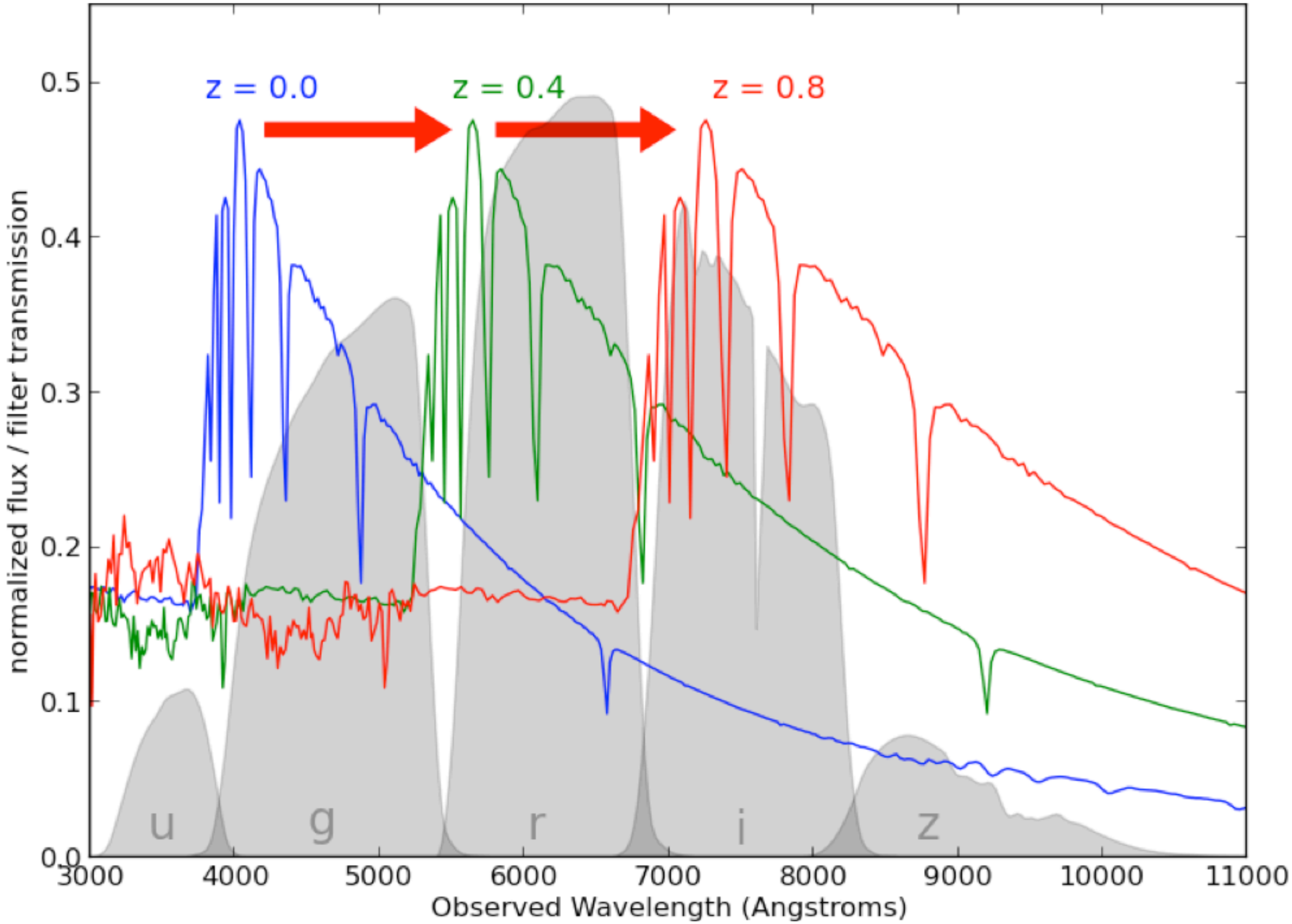
# Spectroscopy: radial velocity measurements to trace the flow

- Use galaxies as "massless" particles to trace the flow caused by gravity or other things; this is similar to measuring the peculiar velocities of solar neighborhood stars in the Milky Way.



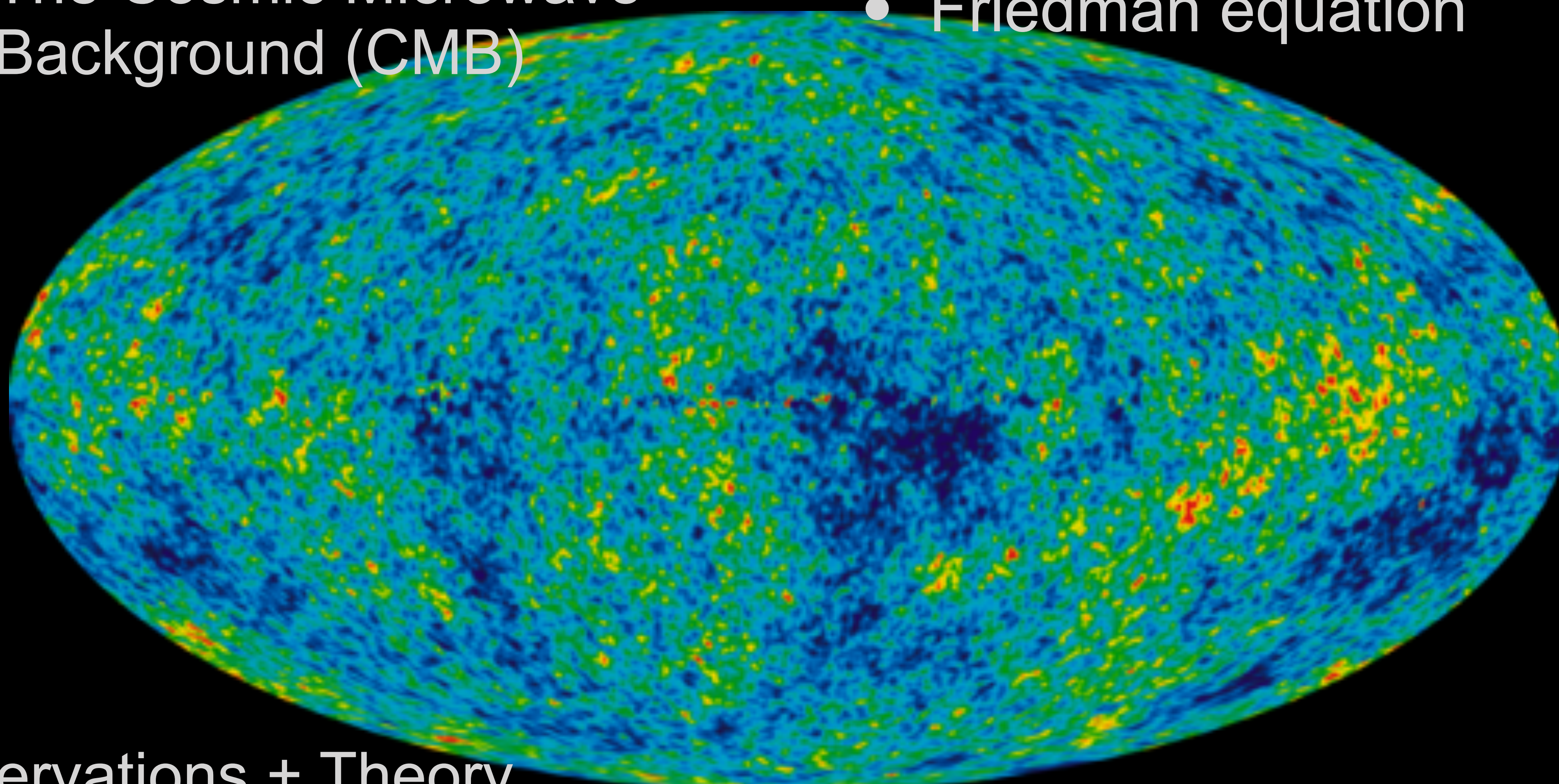
# Spectroscopy: radial velocity measurements to trace the flow

- Instead of finding similar numbers of *blueshifted* and *redshifted* galaxies, Hubble found that most of the galaxies are *redshifted* — i.e., they appear to be moving away from us.



## Chap 21 & 22: The Expanding Universe

- Observations (facts)
  - Hubble-Lemaître Law & the Hubble "constant"
  - The Cosmic Microwave Background (CMB)
- Interpretations (theories)
  - The cosmological principle
  - Robertson-Walker metric
  - Friedman equation



- Observations + Theory
  - Accelerating Expansion: Evidence of dark energy
  - The cosmic composition
- Predictions of the Big Bang theory: how everything began?

Evidence for an expanding Universe:

Discovery of Hubble's Law:
distance vs. redshift at $z \ll 1$

# The Prerequisites of the Discovery of the Hubble's Law

- In 1920s, two technological advancements enabled the discovery of Hubble's law
  - Distance *D* from standard candles like Cepheids (*L*-*P* relation)
  - redshift *z* from moderate resolution spectroscopy:

$$z = (\lambda_{observed} - \lambda_{rest})/\lambda_{rest}$$

- inspecting the definition of *z*, does it look similar to the Doppler shift equation?

$$v_r/c = (\lambda_{observed} - \lambda_{rest})/\lambda_{rest}$$

- so *small* redshifts are usually converted to velocities using the following formula:

$$v_r = z \times c \text{ (when } z << 1)$$

- **What about *z* > 1?**

# Classic vs. Relativistic Doppler Shift: Velocity-Redshift Relation

- If redshift was interpreted as a Doppler motion, then the velocity of a galaxy is given by

$$v_r = cz$$

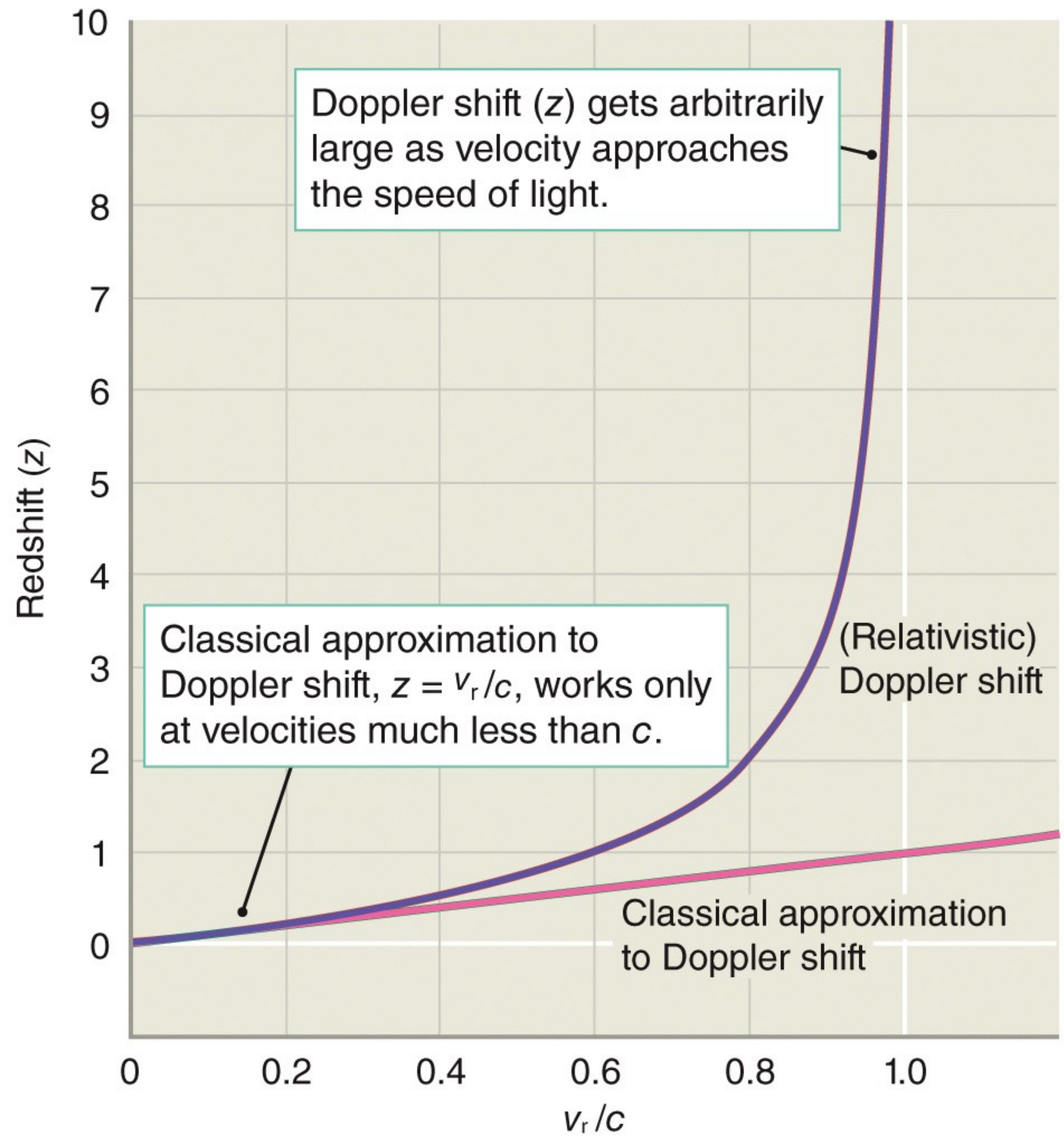
- At a redshift greater than 1, galaxies would be moving faster than speed of light!
- Using the relativistic Doppler shift formula,

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_0} = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}}$$

which gives:

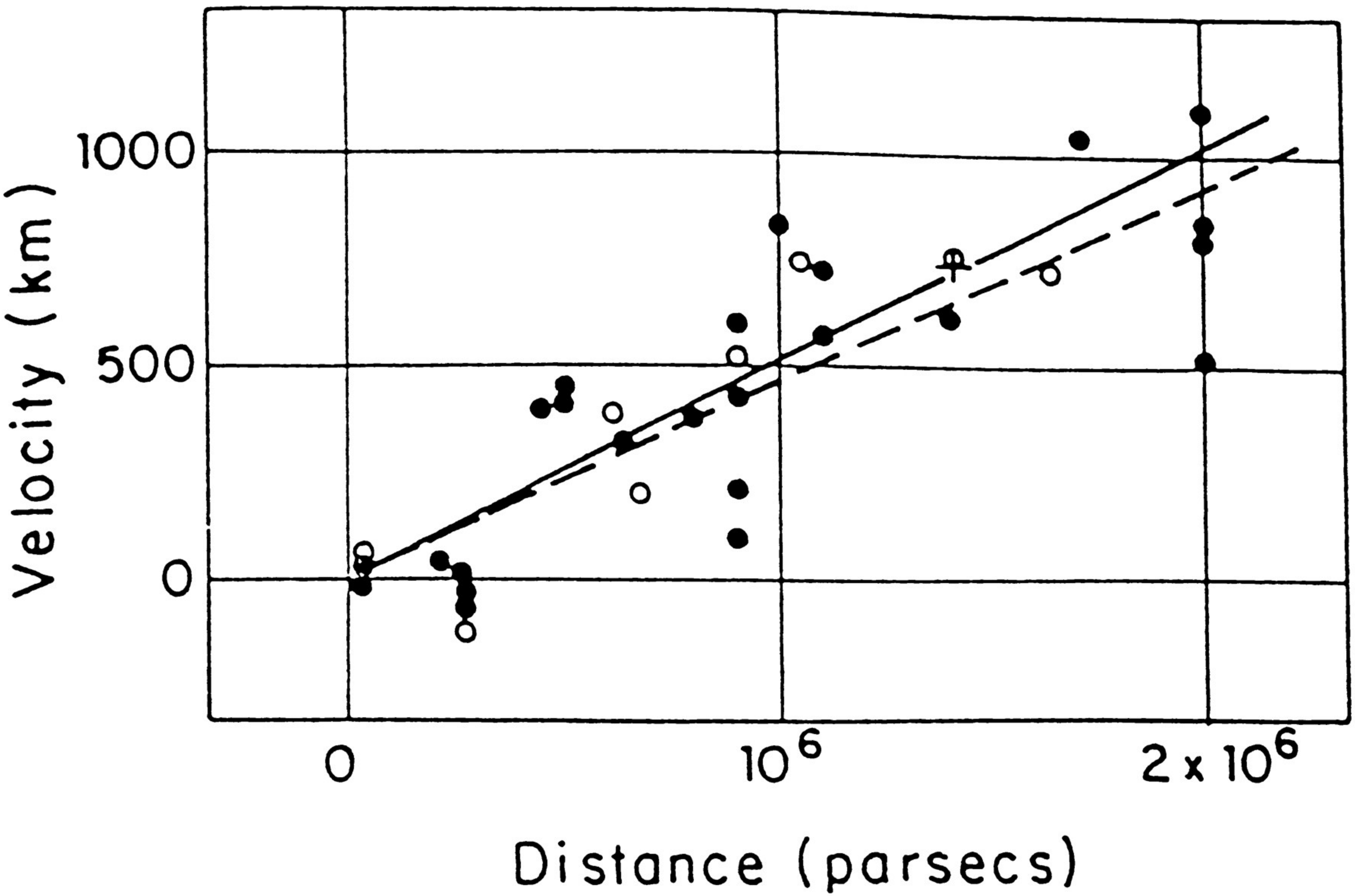
$$v_r = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$

- So for large redshifts, conversion to velocity is causing more complication than necessary.

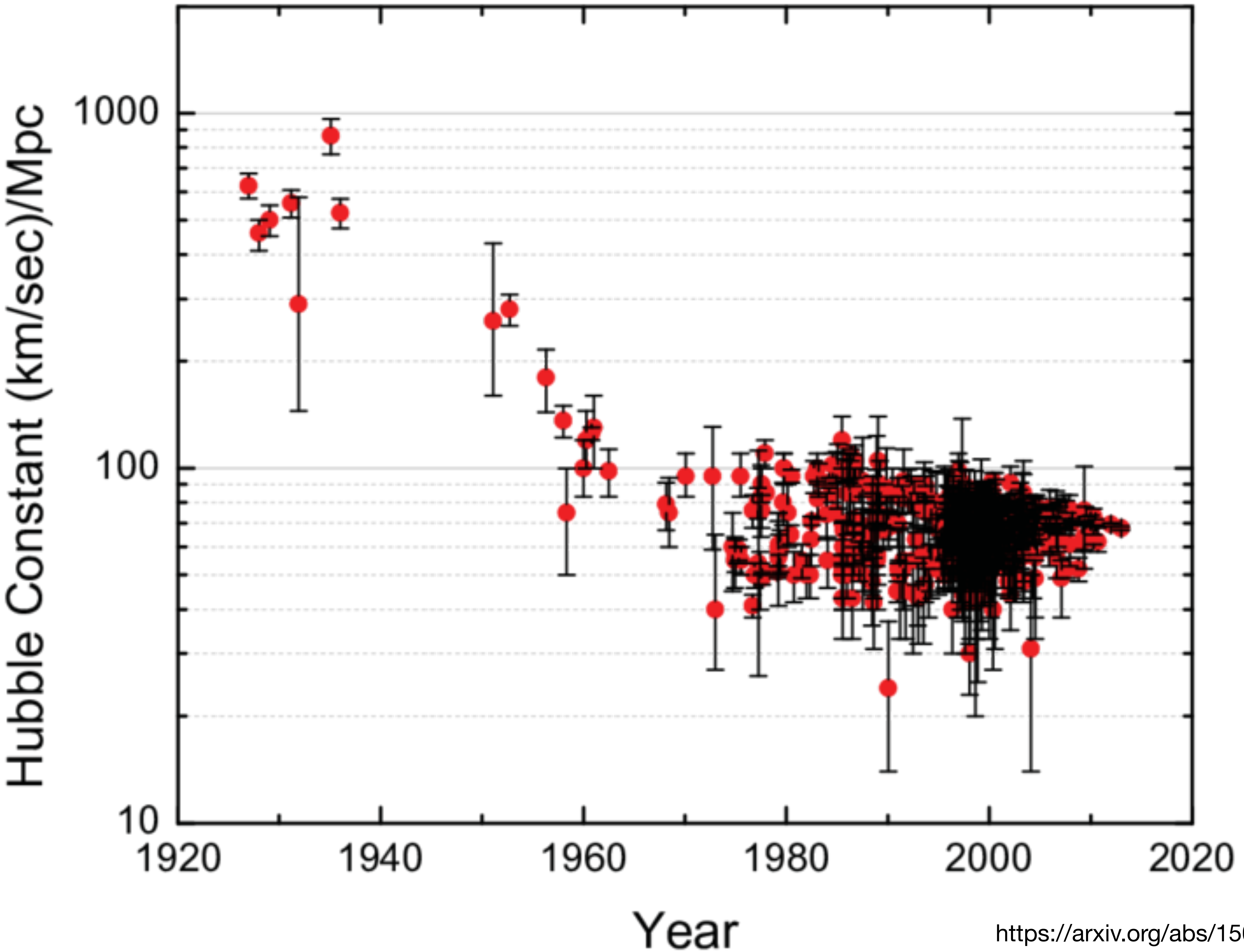


# Hubble-Lemaitre Law (1929; or Hubble's Law)

**Hubble & Lemaitre** discovered a **linear relation** between redshift and distance: $cz = H_0 D$, where $cz$ is redshift converted to velocity, $H_0$ is the slope of the linear relation with a unit of **km/s/Mpc** (the **Hubble constant**).

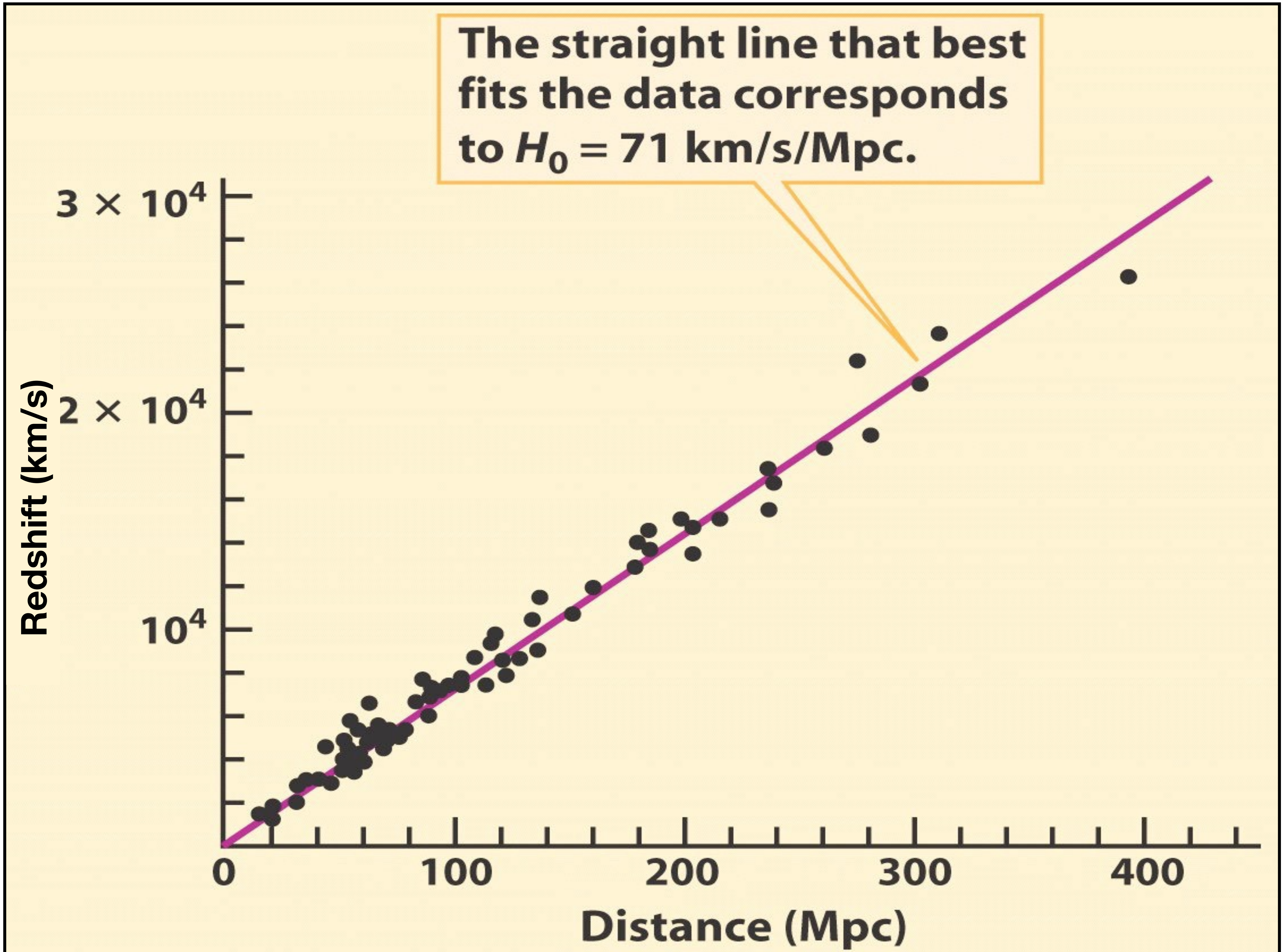


# Historical evolution of Hubble constant measurements




https://arxiv.org/abs/1506.02978

## What does Hubble's Law tell us about the Universe? Why is there a linear relation between redshift-converted velocity and distance?

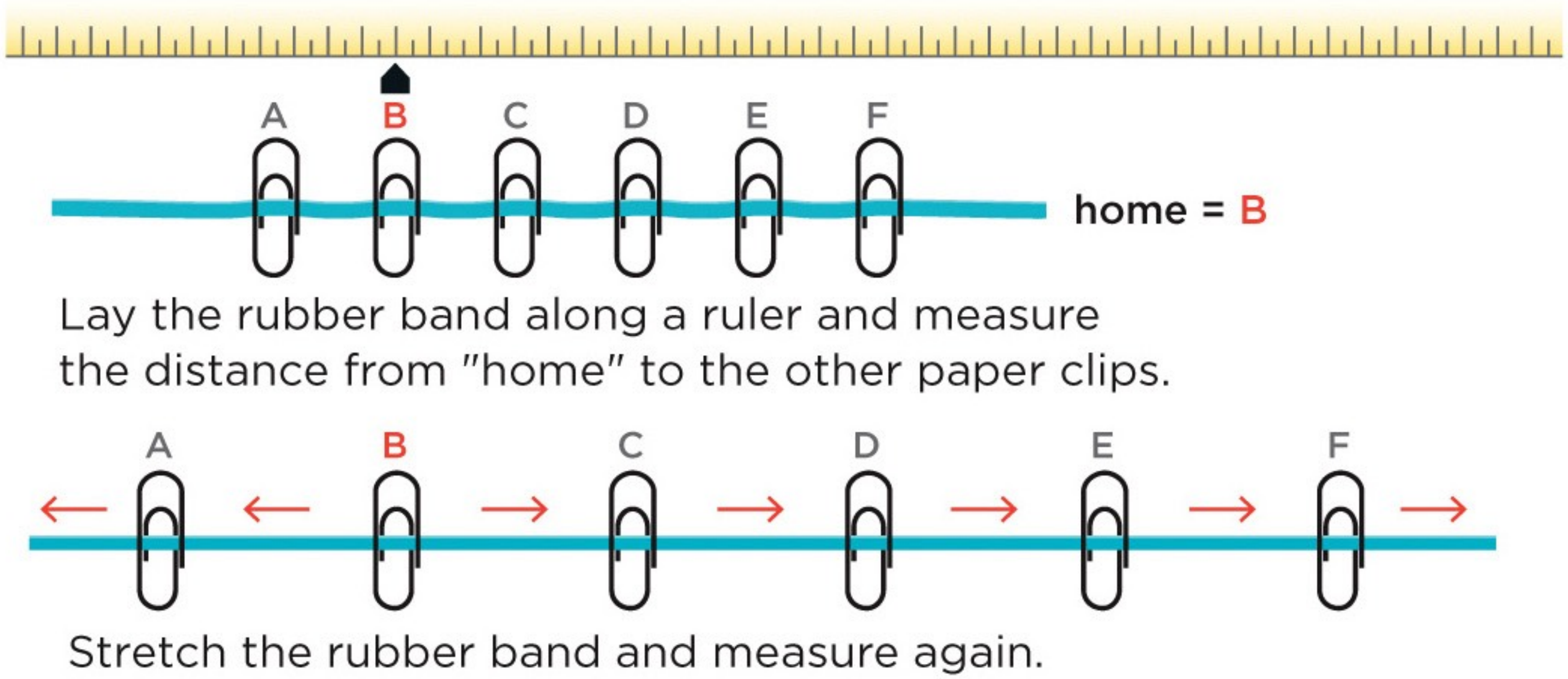


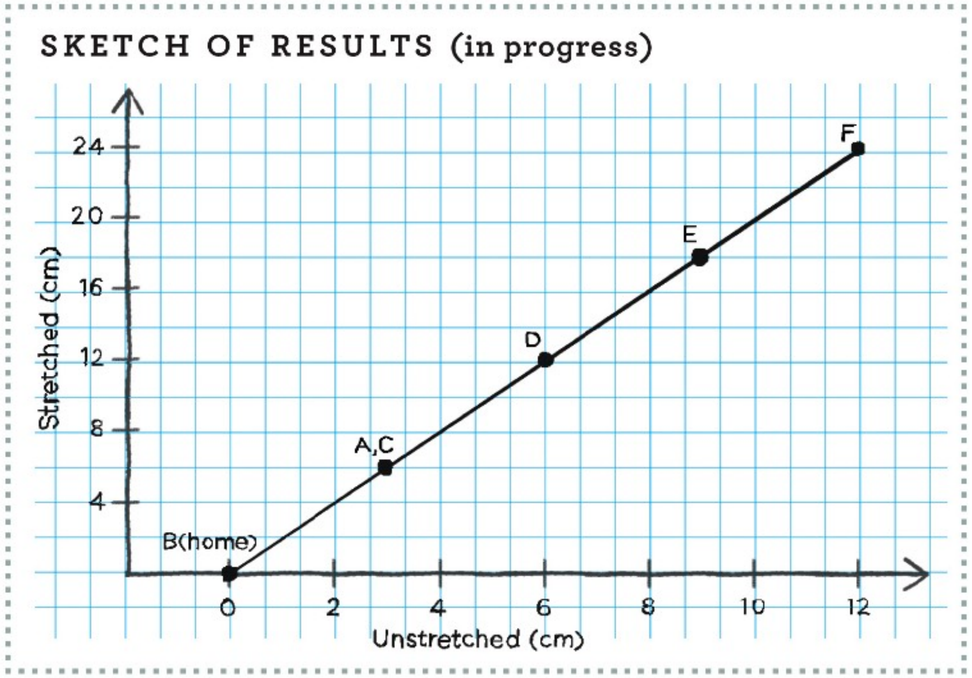
# So, what kind of flow does the galaxies trace?

- The idea is to use galaxies as "massless" particles to trace the flow caused by gravity and other things that we are currently unaware of;
- *Not only* all galaxies are redshifted, *but also* their redshifts increase with distance!
- The motion of galaxies due to the expansion of the universe is called the **Hubble flow**.

# Hubble Flow: Visualizing Expansion in 1D

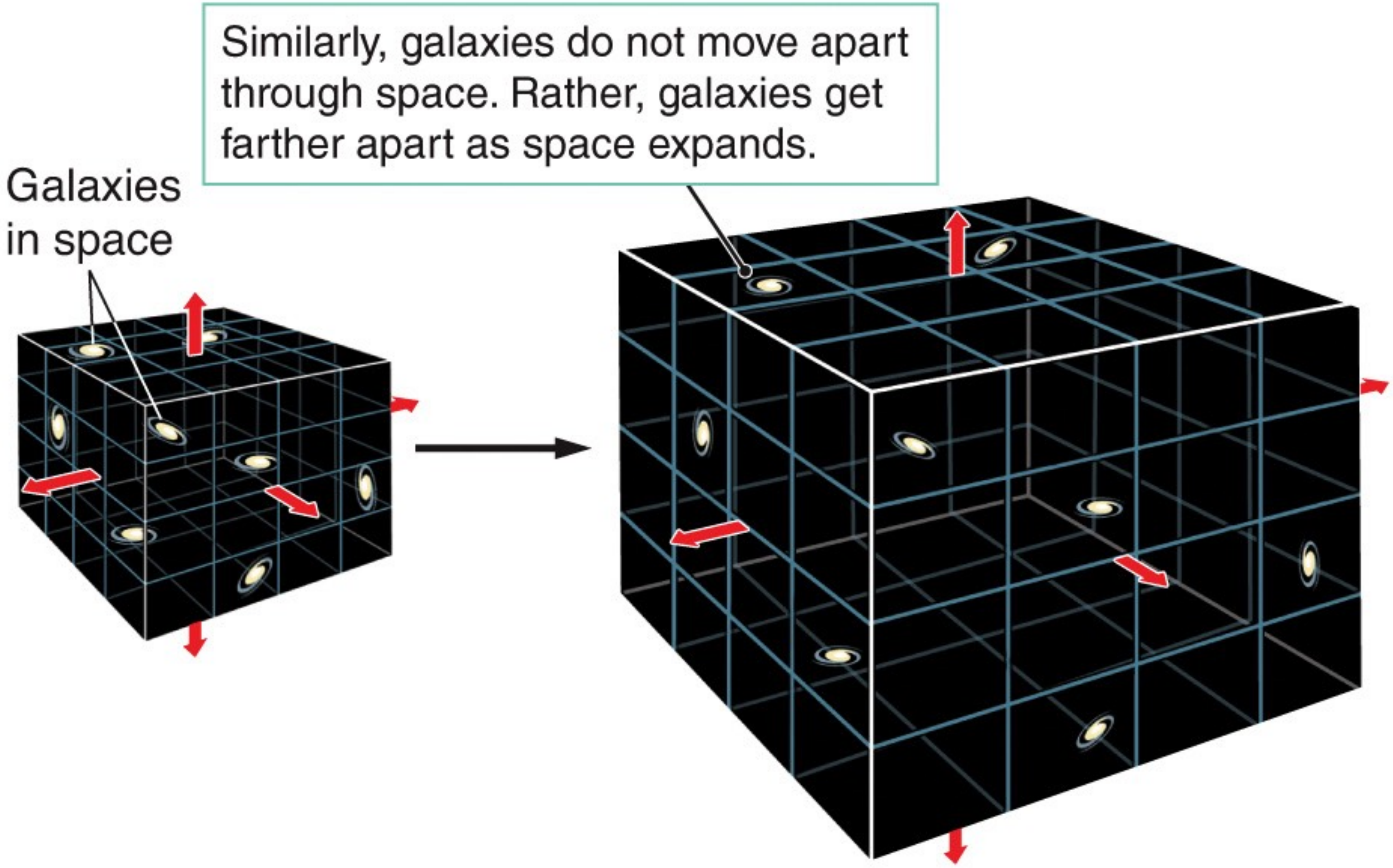
- Simple expansion model: paper clips on a rubber band
- As the rubber band stretches, an ant riding on clip B:
  - observes itself as stationary
  - observes clip F moving away twice as fast as clip D
  - observes clip A and C moving away at the same speed
- An ant on any paper clip would make similar observations.



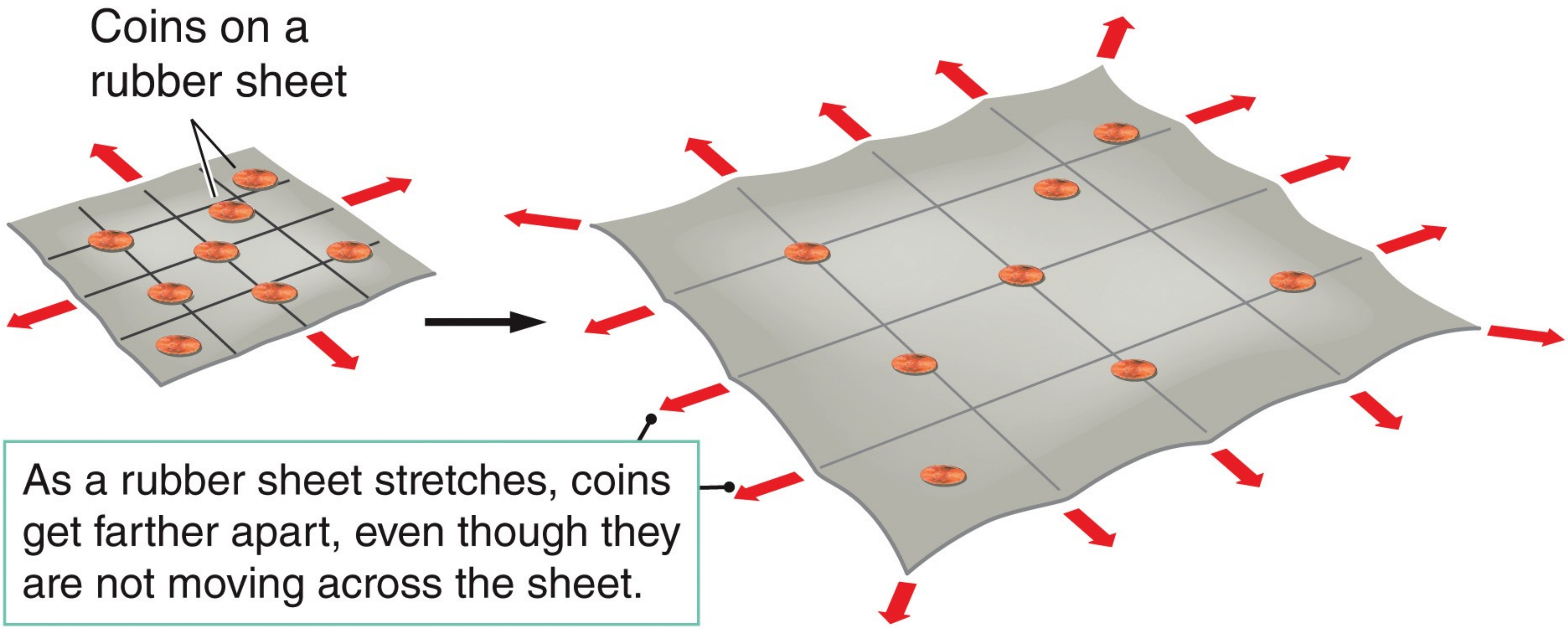
# Hubble Flow: Visualizing Expansion in 3D

- Galaxies as tracers of space shows that universe is expanding.
  - Galaxies are moving away from us because space is *created*
  - New space is created *uniformly* in the Universe, leading to the *linear* proportionality between redshift and distance.



## Hubble Flow: no "center" of the expansion, and non-Doppler redshifts

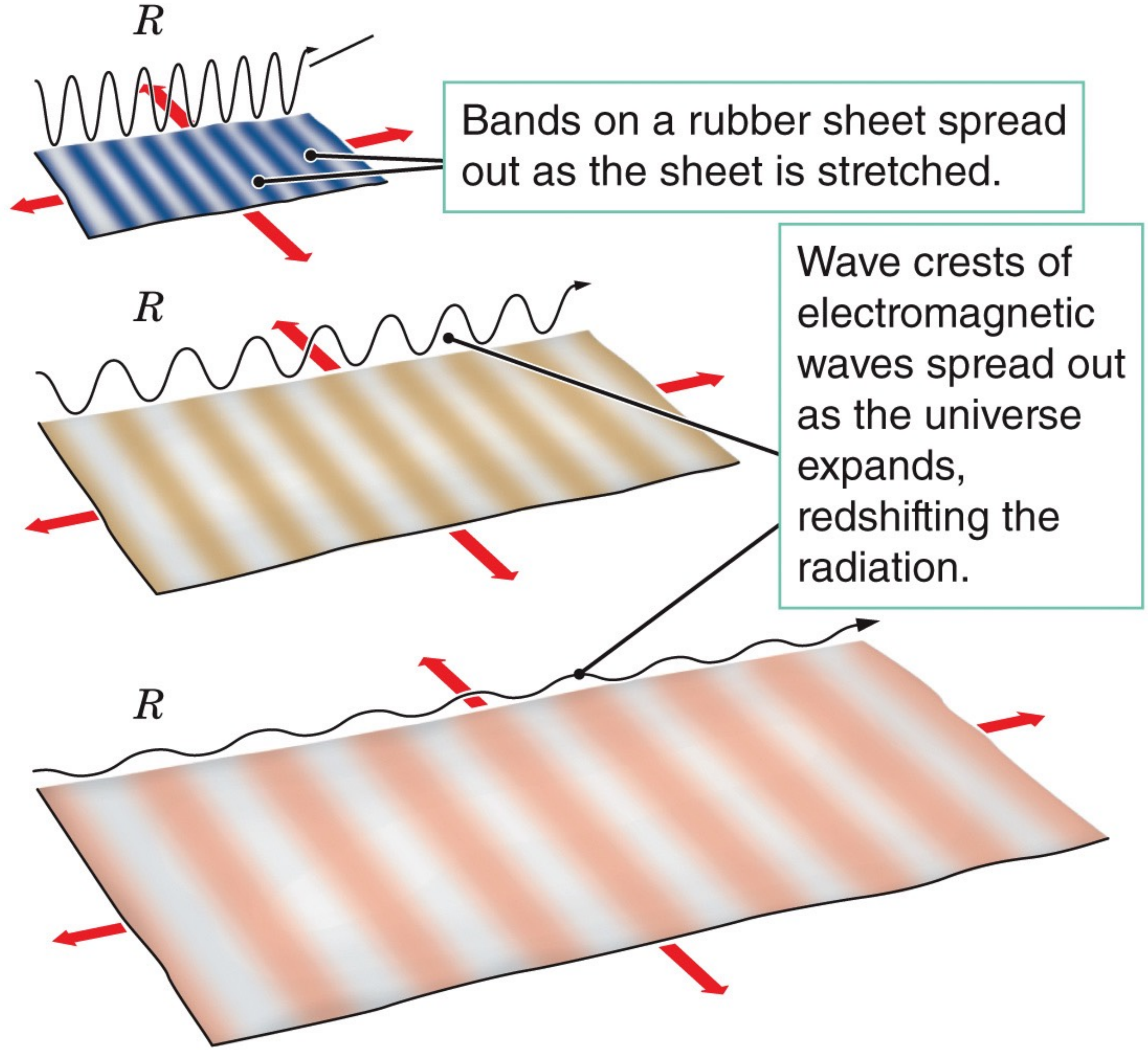
- It might appear that we are in the center of the universe, with all galaxies moving away. *But* there is no center: from any point in the universe, it would look the same.
- So the redshifts of galaxies are **not due to motion** (Doppler shift), but **due to space creation** (increasing in scale)



Cosmological redshift is not Doppler shift, it is caused by an increasing scale factor

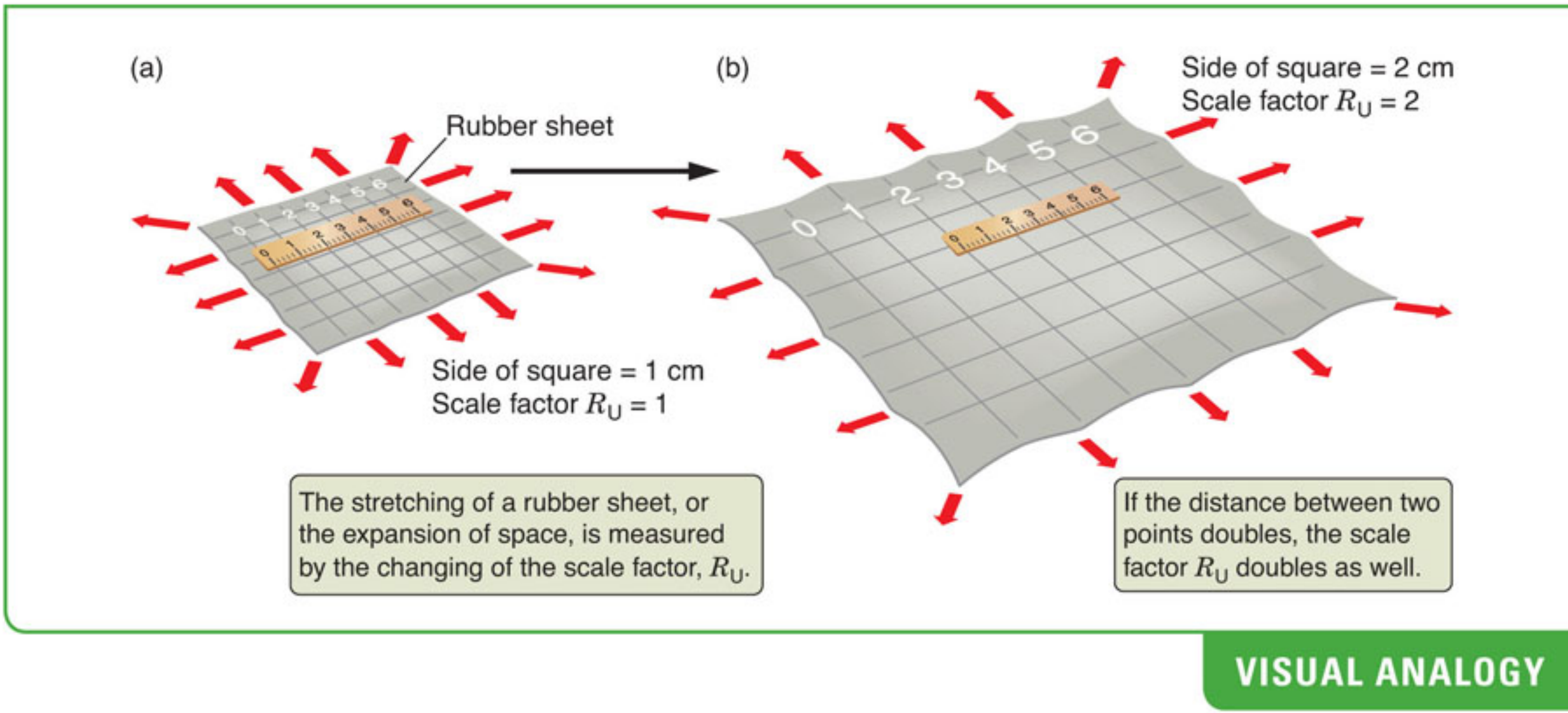
# Redshift and the Expansion

- Redshifts of galaxies are **not** due to Doppler shifts.
- Instead, the light is "stretched out" as it travels through the expanding universe: this is known as **cosmological redshift**.
- The wavelength of light is getting longer over time because the scale factor is increasing.
- A higher redshift indicates a smaller scale factor. Light emitted at high redshift will be very stretched out.



## An uniform expansion of the Universe must be scalable

- Imagine space like a rubber sheet, stretching the sheet increases the scale factor everywhere on the sheet, causing distances between grid points to increase

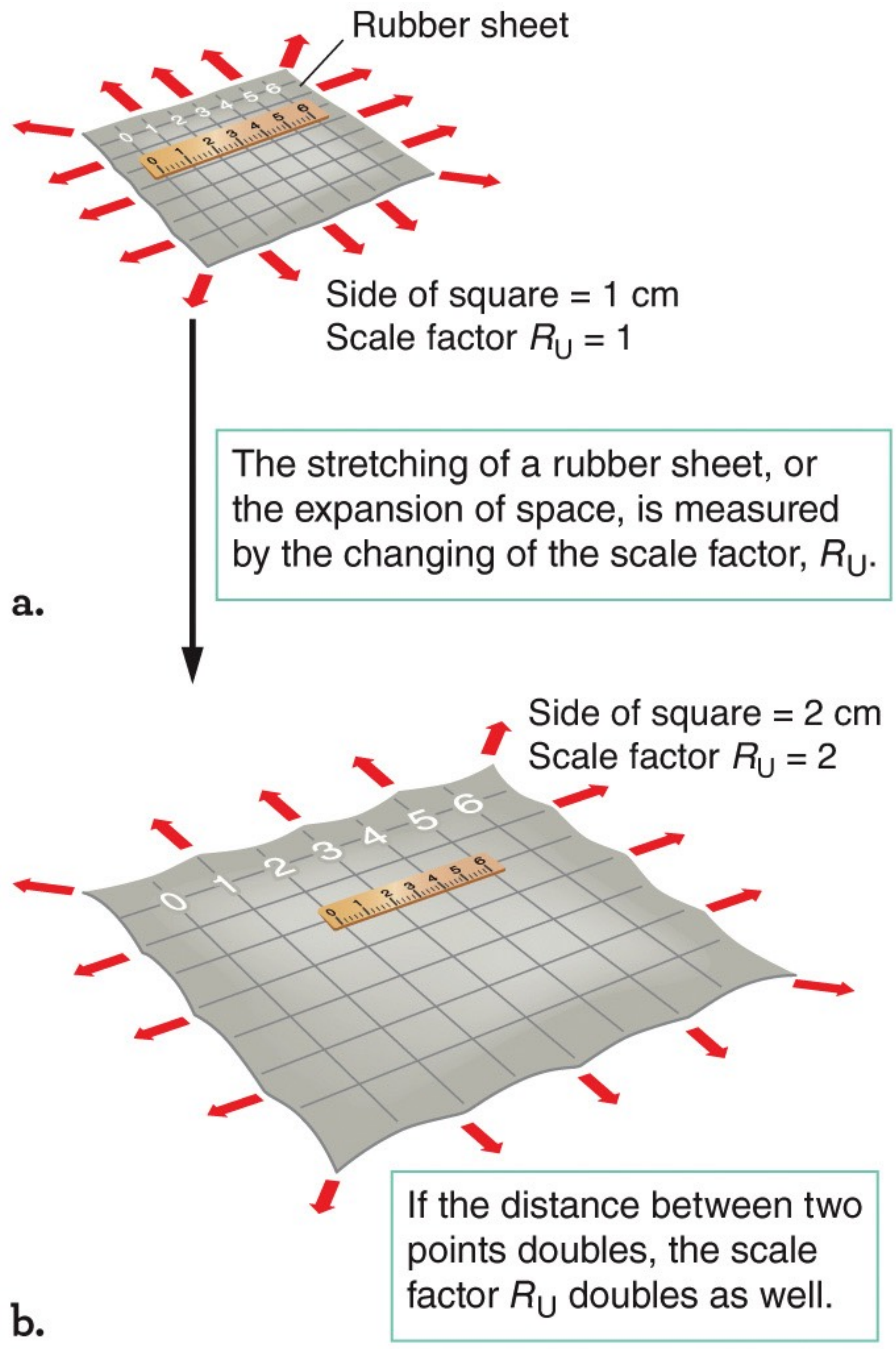


**Cosmological redshift** of photons emitted from a distant galaxy is caused by the increasing **scale factor** of the universe ($R_u$):

$$\frac{R_U(z)}{R_U(0)} = R_U(z) = \frac{1}{1+z} = \frac{\lambda_{\text{rest}}}{\lambda_{\text{observed}}} = \frac{\lambda(z)}{\lambda(0)}$$

# The Scale Factor

- The **scale factor** ($R_U$) is a measure of how much the universe has expanded.
- The scale factor gets smaller as we look back in time.
  - For example, when $R_U$ = 0.5, the universe was half of its current size.
- The expansion pulls galaxies apart but does not destroy galaxies (yet!):
  - At scales smaller than the Local Group, gravitational forces can overcome the space expansion.



## Redshift gives the scale factor of the Universe at the emitted time

- The redshift tells us how much the universe has expanded since a galaxy's light was emitted.
- Write $R_U$ as the *scale factor,* then

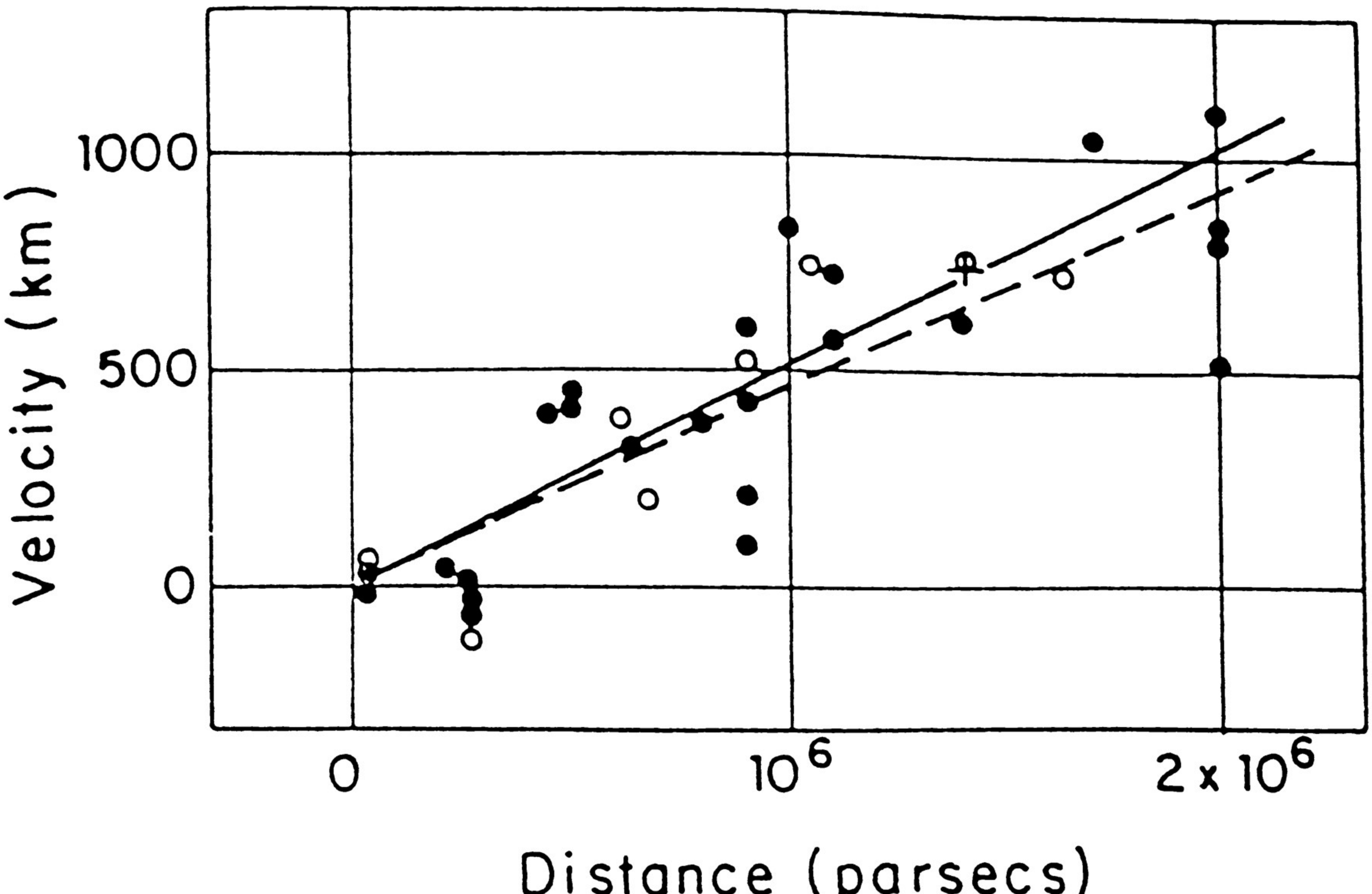
$$R_U = \frac{1}{1+z} = \frac{\lambda_{\text{rest}}}{\lambda_{\text{observed}}}$$

- Example: $z = 0$ means today, when $R_U = 1.0$. This is the maximum scale factor.
- Example: $z = 1$ means $R_U = 0.5$. The universe was half its current size when light was emitted from this galaxy.
- Example: $z = 10$, see *Working It Out 21.2*

# Recap: Lecture 1

# Hubble-Lemaitre Law (1929; or Hubble's Law)

**Hubble & Lemaitre** discovered a **linear relation** between redshift and distance: $c\,z = H_0\,D$, where ***cz*** is redshift converted to velocity, $H_0$ is the slope of the linear relation with a unit of **km/s/Mpc** (the **Hubble constant**).



# Hubble Flow: Homogeneous and Isotropic Expansion of Space

- Galaxies as tracers of space shows that universe is expanding.
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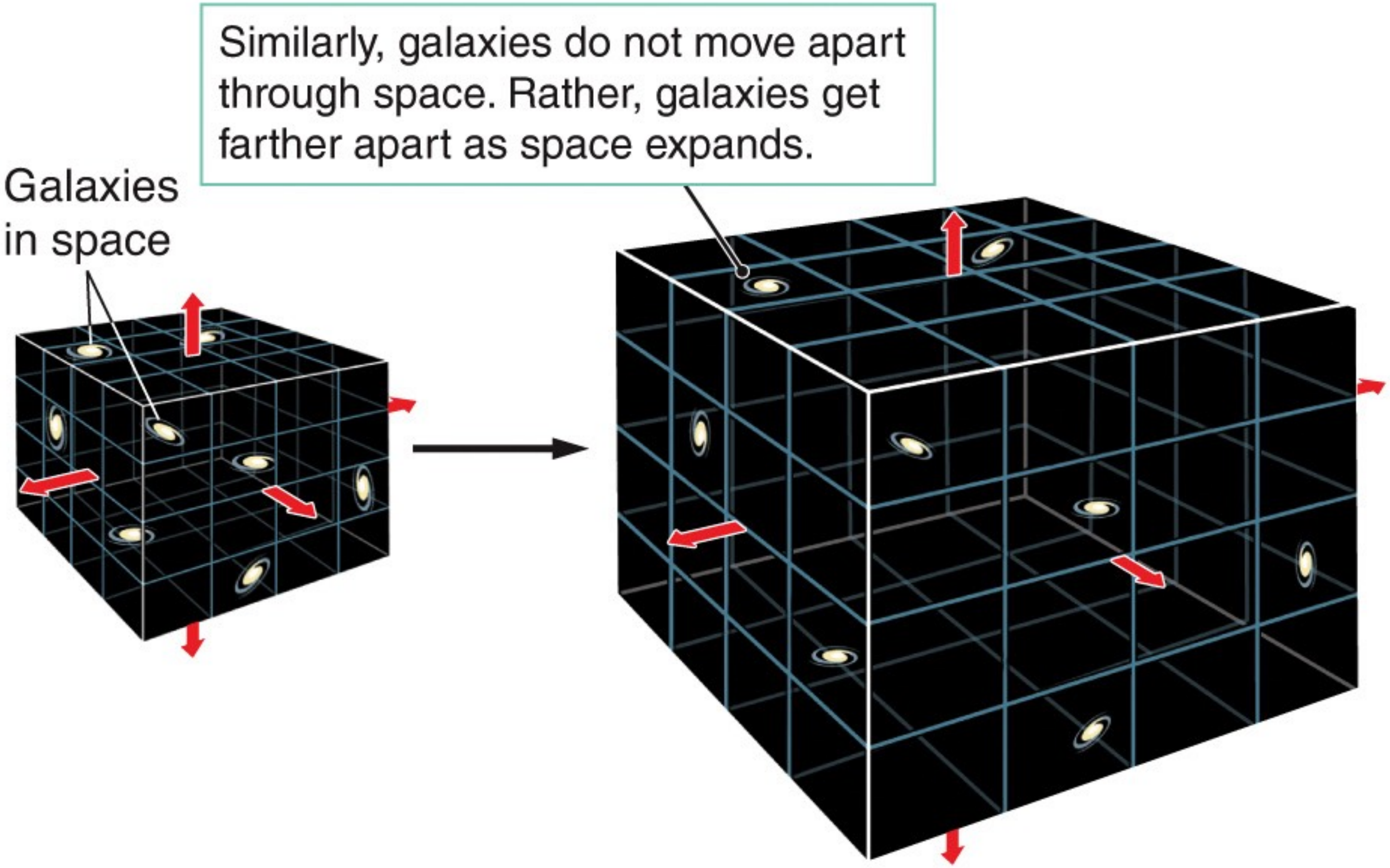


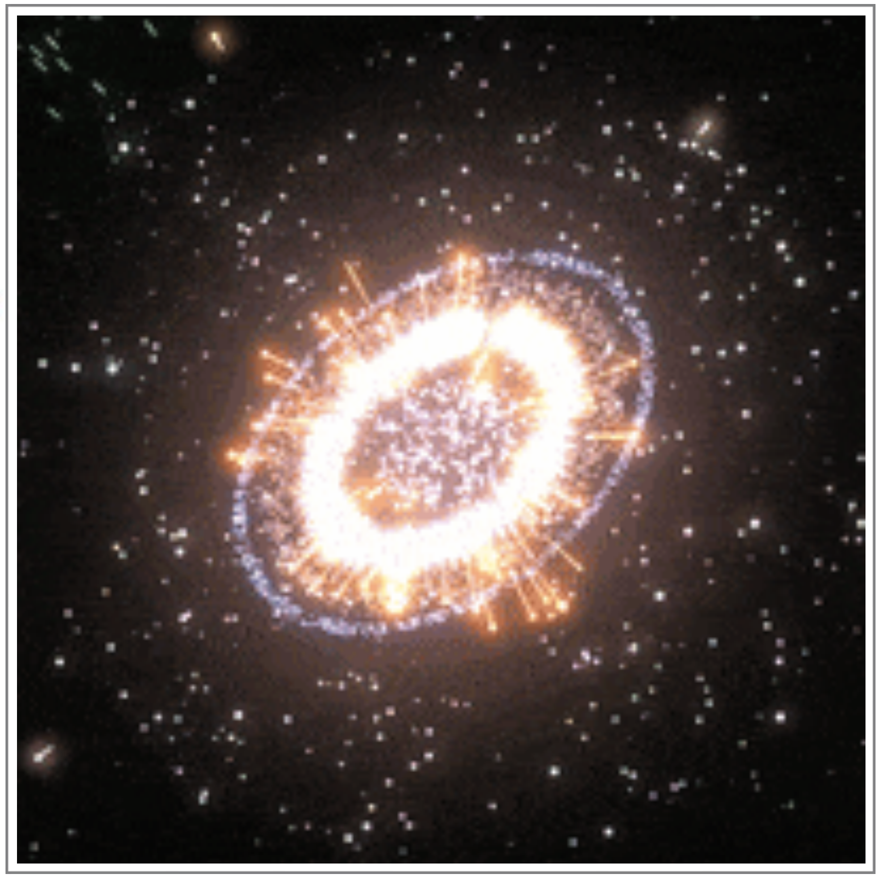

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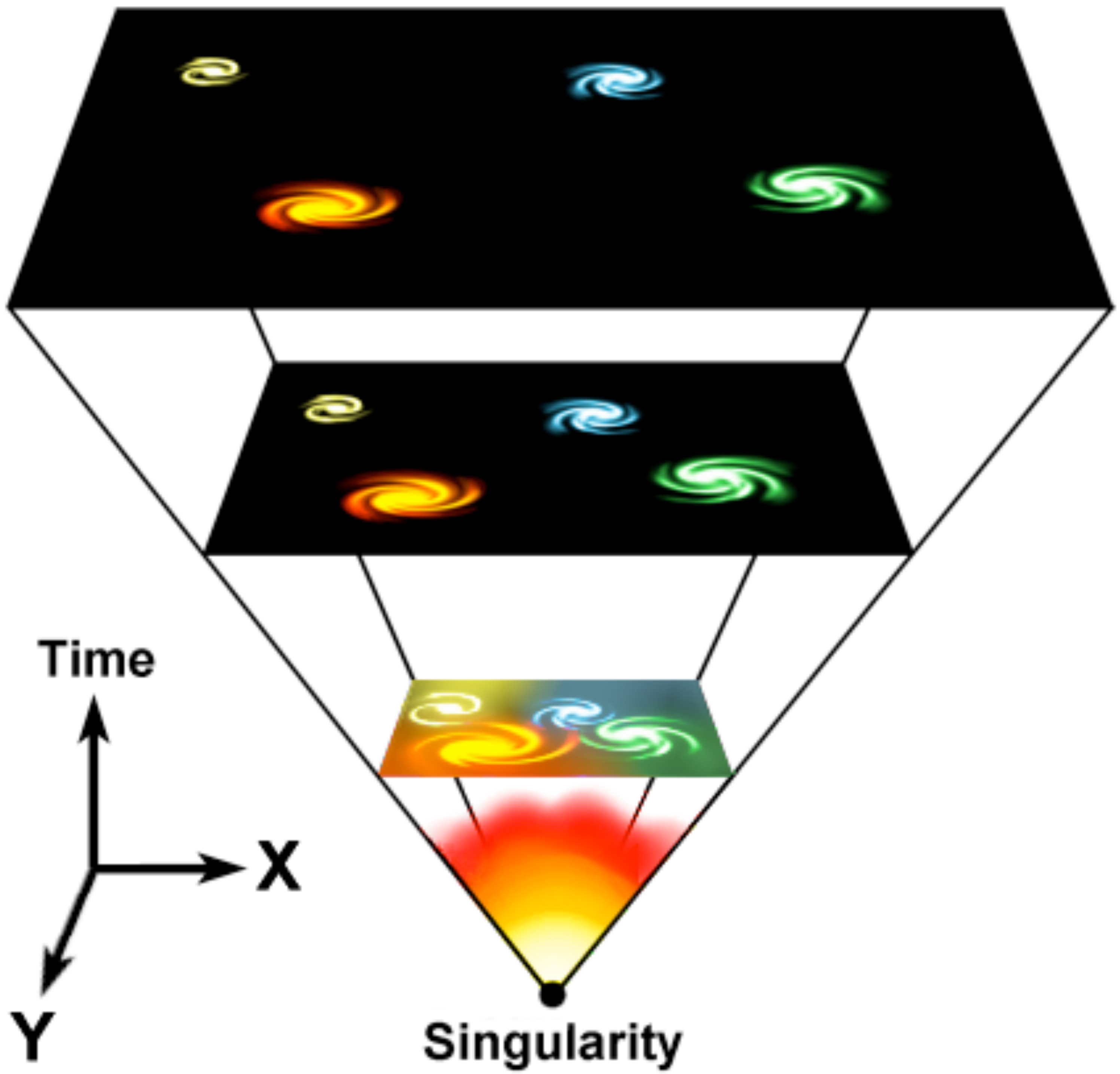
The Hubble Time

An estimate of the age of the Universe

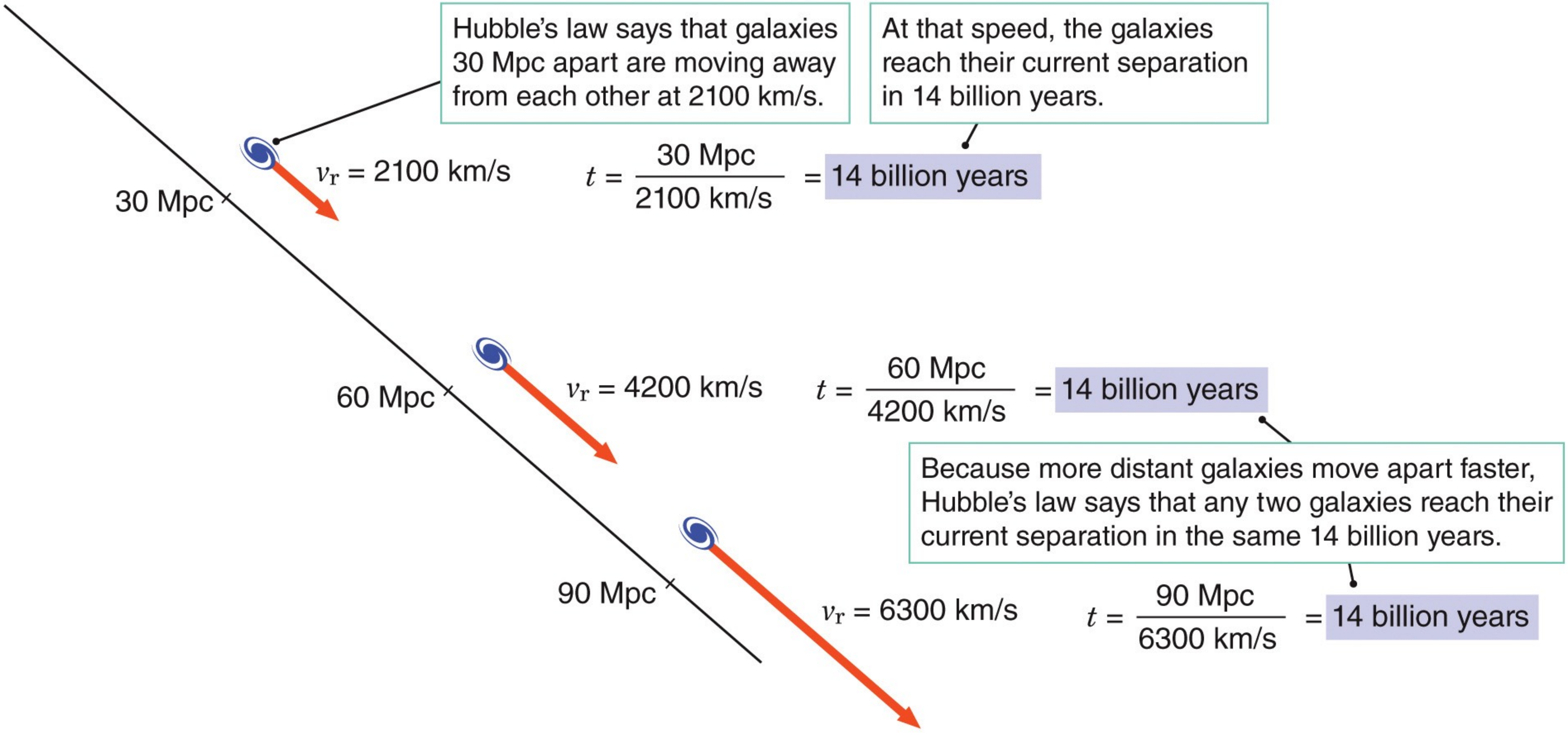
## Expansion of space means that Universe was very small in distant past

- If galaxies are getting farther apart now, they must be closer together in the past. Hubble's law implies that the entire Universe started from a single point. **Can we estimate when the Universe was a single point?**



# Working It Out 21.1: Visual Summary

- Because Hubble's law is a linear relation, it does not matter which two galaxies you use to make this measurement; the end result will be the same.



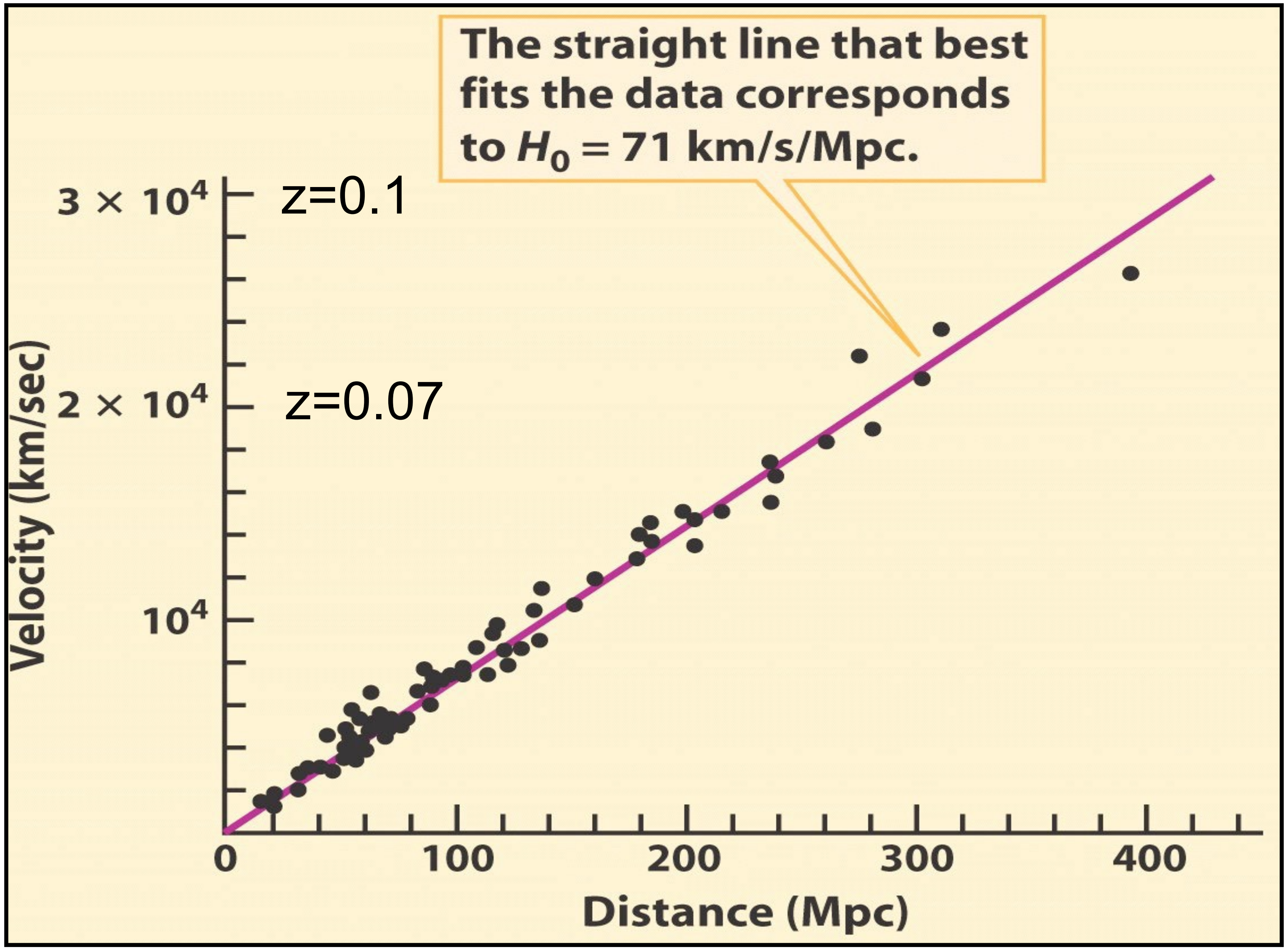
# The Hubble Time: How long ago did the Universe start expanding?

- ***Assume* space creation rate** is constant: $\frac{dD}{dt} = \text{constant} = H(t)D(t) = H_0 D(t_0)$
- Then, move ***dt*** to the right side and integrate both side from ***t=0*** to ***t=t₀***, we have:

$$D(t_0) - D(t=0) = H_0 D(t_0) \cdot (t_0 - 0)$$

- Given that ***D(t=0) = 0***, we can solve **the age of the Universe *t₀*** today:

$$t_0 = 1/H_0 = 1/(70\ \text{km/s/Mpc}) \approx 14\ \text{Gyr}$$

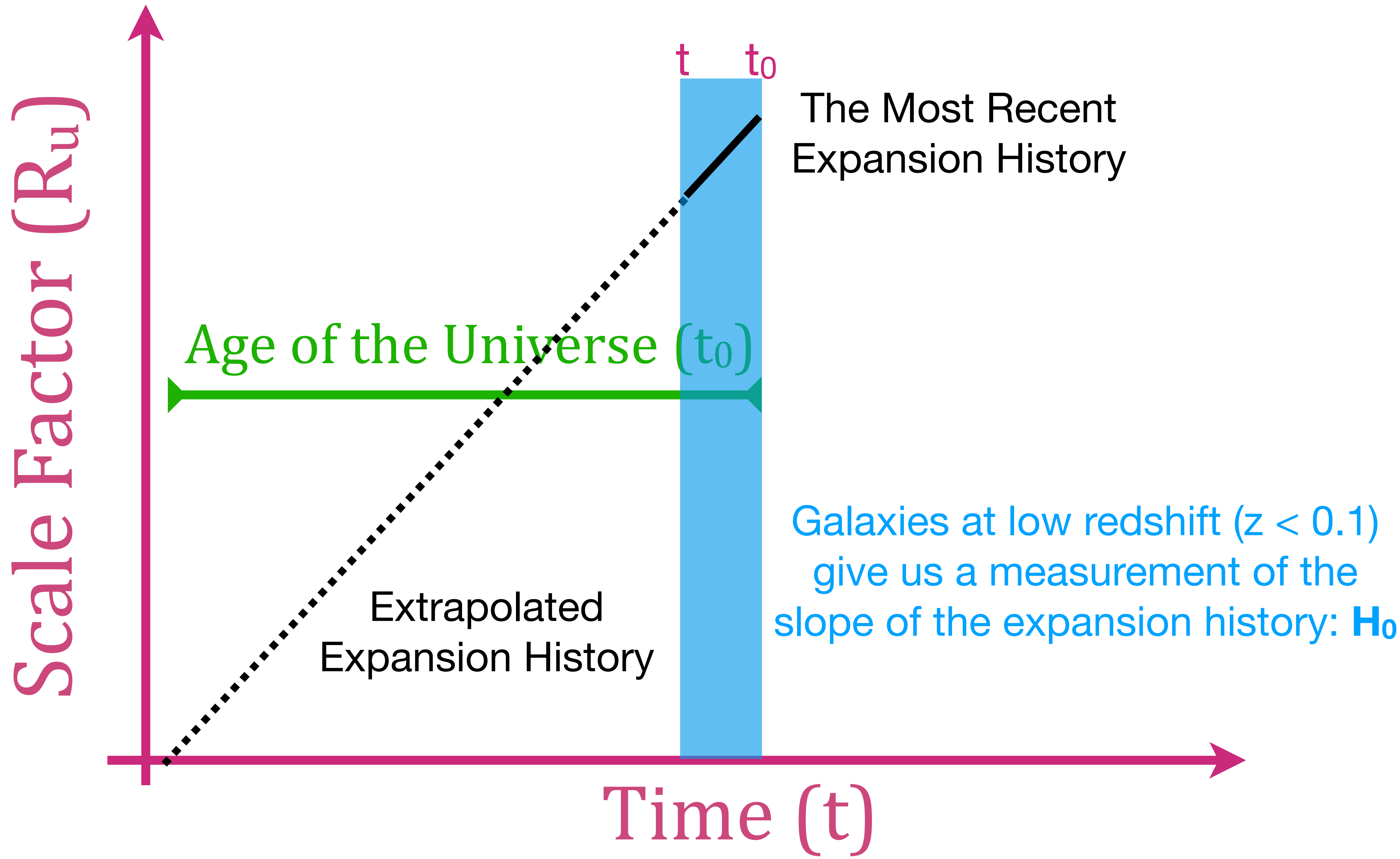


**Cosmological redshift** of photons emitted from a distant galaxy is caused by the increasing **scale factor** of the universe ($R_u$):

$$\frac{R_U(z)}{R_U(0)} = R_U(z) = \frac{1}{1+z} = \frac{\lambda_{\text{rest}}}{\lambda_{\text{observed}}} = \frac{\lambda(z)}{\lambda(0)}$$

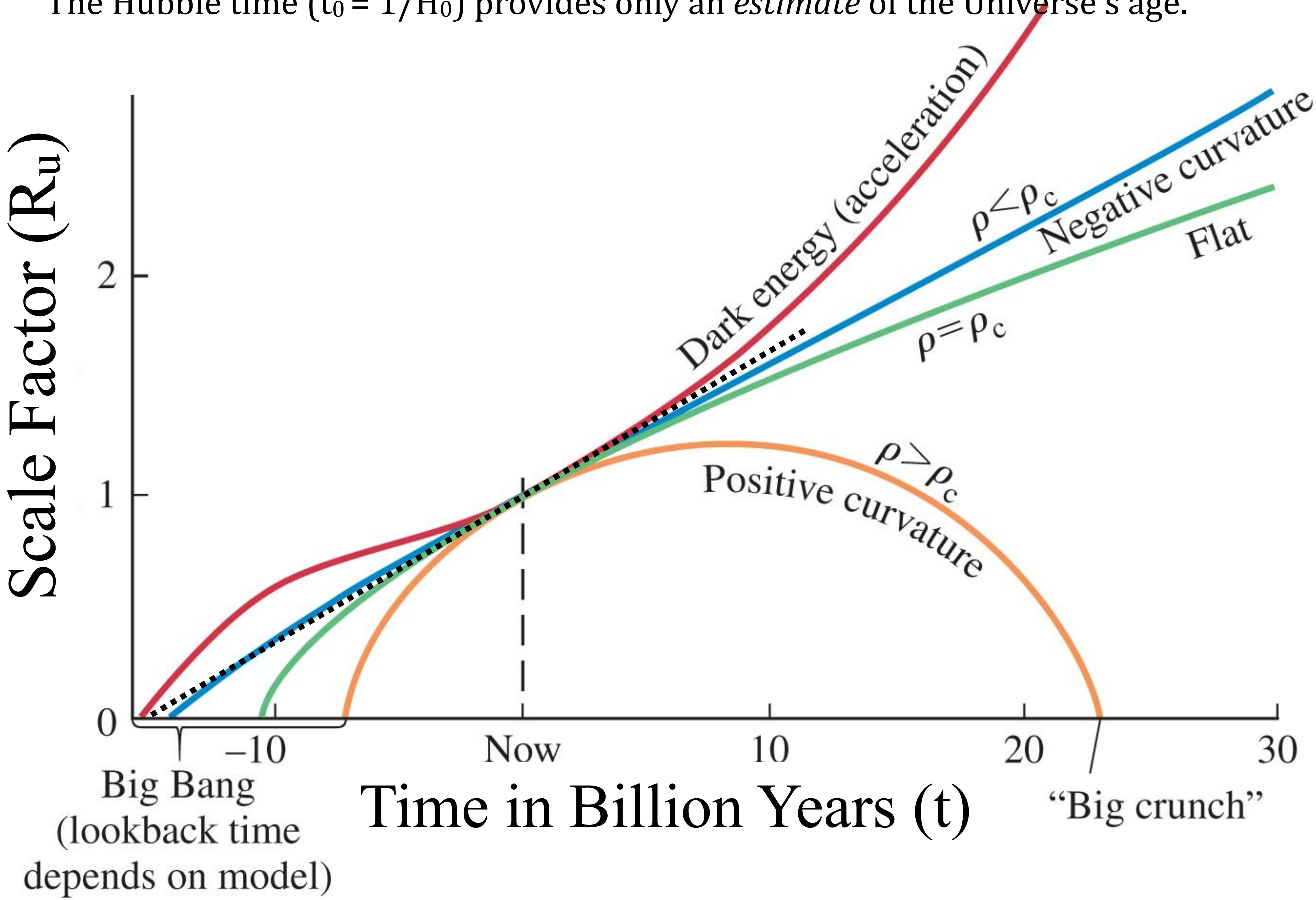
## Alternative derivation of the Hubble time using Scale Factor

- **Hubble's law**: $z\,c = H_0 D = H_0 [c\,(t_0 - t)]$, where we used the ***lookback time*** times *c* to replace distance *D*. Canceling *c* on both sizes, we have $\boldsymbol{z = H_0 (t_0 - t)}$
- **Scale factor**: $R_u = 1/(1+z) \approx 1 - z$ for $z \ll 1$
- which give the *most recent expansion history* of the Universe: $\boldsymbol{R_u = 1 - H_0 (t_0 - t)}$,
- Extrapolating the relation to $R_u(t=0) = 0$, we solve for *Hubble time*: $\boldsymbol{t_0 = 1/H_0}$



# The various possible expansion histories of the Universe

The Hubble time ($t_0 = 1/H_0$) provides only an *estimate* of the Universe's age.

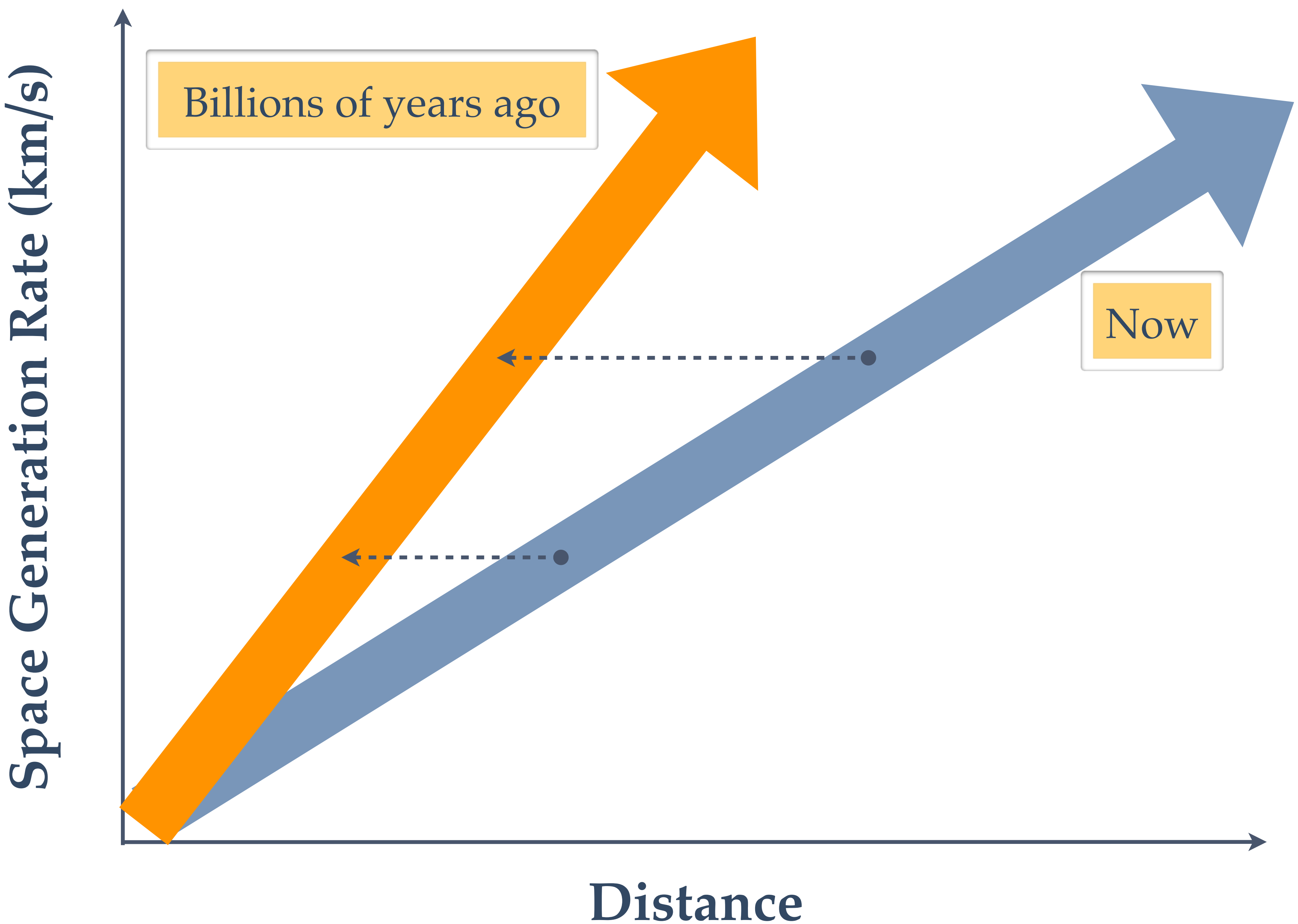


# Interesting implications of the Hubble constant, the size of the observable Universe, and the various distances

## Is Hubble Constant a constant? If not, how does it change over time?

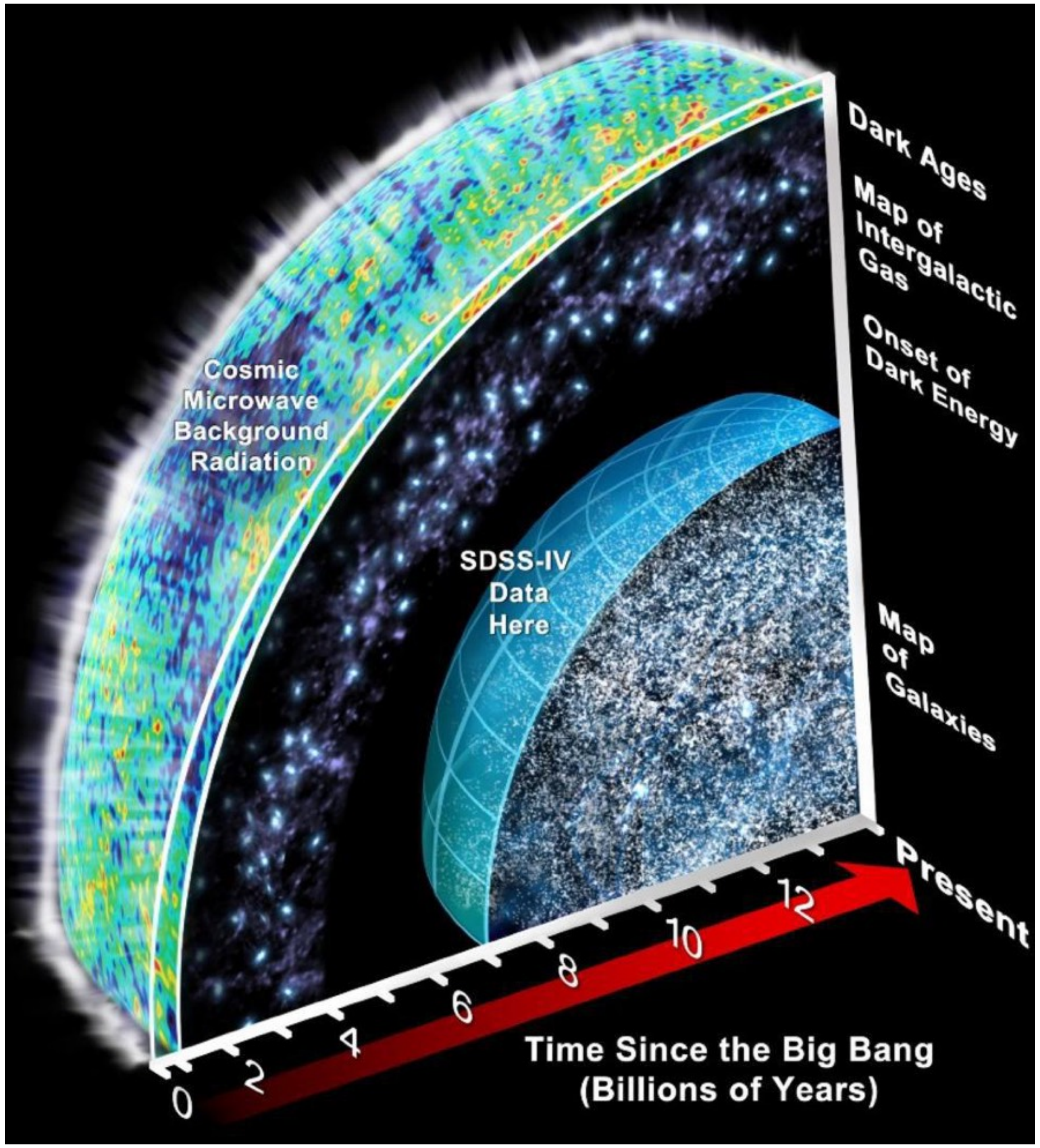
- The **Hubble "constant"** changes over time is expected because the universe obviously was younger in the past.
- We thus define $H(t) = \dot{R_U}(t)/R_U(t)$ as the **Hubble parameter**.
- At ***t = t₀*** (today), this definition gives **Hubble's Law**:

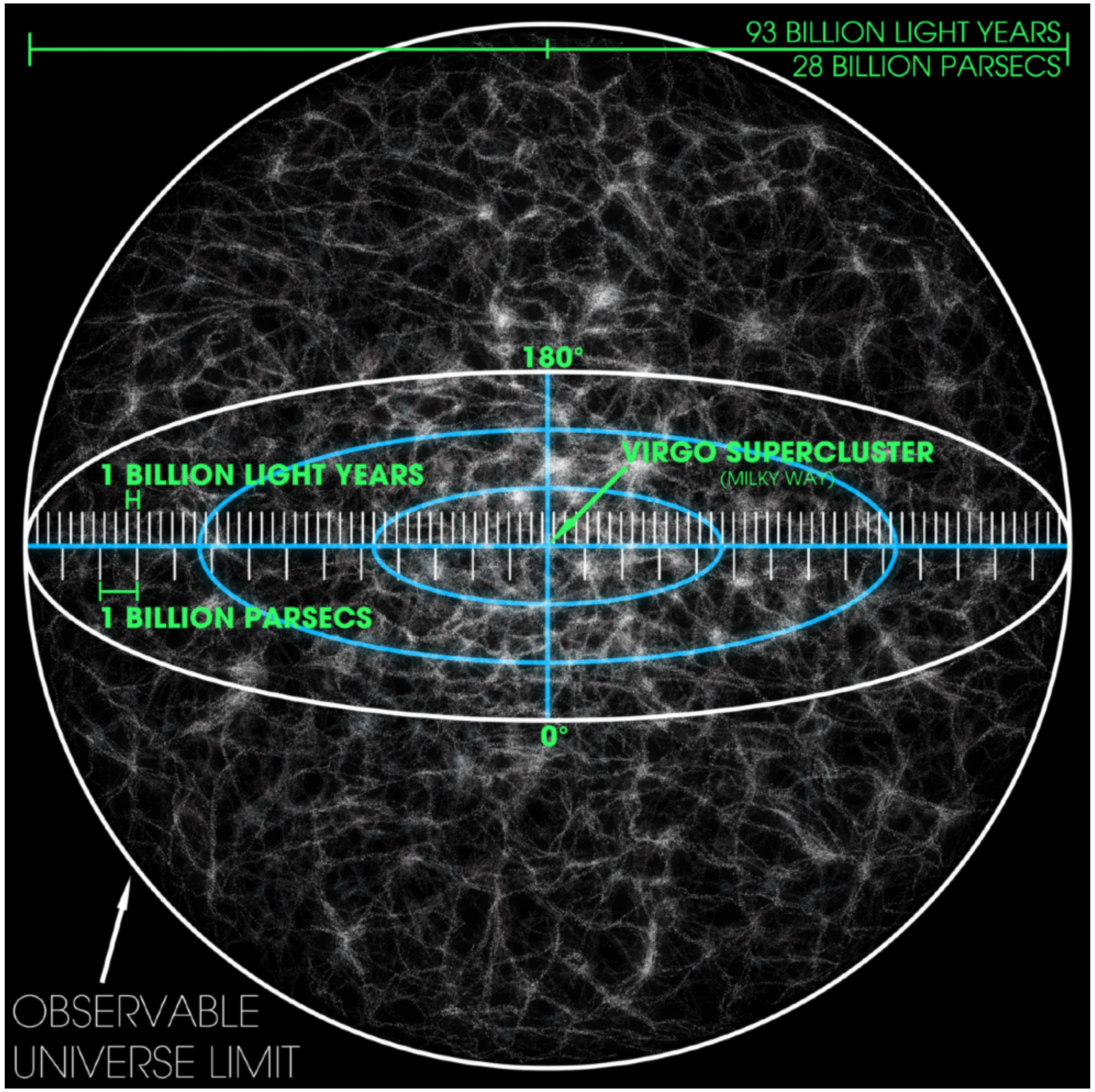
$$\dot{R_U}(t_0) = H(t_0)R_U(t_0) \Rightarrow \dot{D} = H_0 D \text{ where } D = R_U D_{\text{comoving}}$$



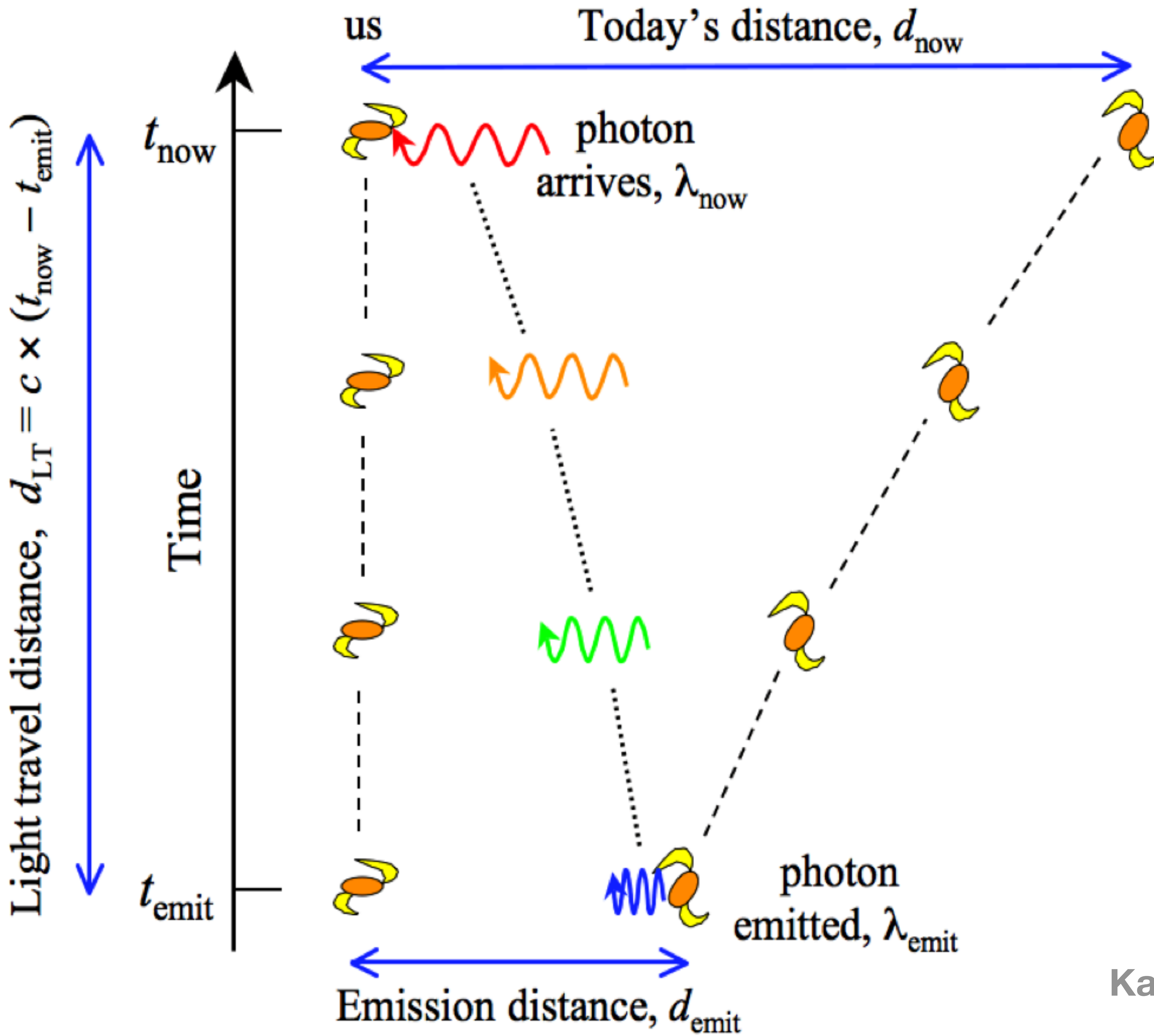
## The finite age of the Universe makes the observable universe finite in size

- The **observable universe** is the finite part of the Universe that we can see, since the earliest light was emitted 14 billion years ago.
  - The comoving radial distance is 14 Gpc (*not 14 Glyr*)



# Distances in Cosmology




Kauffmann

# Robertson-Walker Metric: Distance Measurements

- A **metric** is a function which measures ***differential space-time distance*** between two points. It is defined to be **Lorentzian invariant**.
- The **Robertson-Walker metric** is the metric that describes the geometry of a **homogeneous, isotropic, expanding** universe. In spherical coordinate system:

$$(ds)^2 = (c \cdot dt)^2 - R_U^2(t)\left[\left(\frac{dr_c}{\sqrt{1 - kr_c^2}}\right)^2 + (r_c d\theta)^2 + (r_c \sin\theta d\phi)^2\right]$$

where $R_U$ is the scale factor, $r_c$ is the comoving radial distance, $k$ is the time-independent curvature $k \equiv K(t)R_U(t)^2$. For **flat universe, $k$ = 0.**

  - Light travels along **null geodesics (ds = 0)**, so for photons traveling along the radial direction in a flat universe, we have:

$$dr_c = \frac{c\ dt}{R_U(t)}$$

    following the path of light by integrating this equation gives us the **scale-factor-redshift relation**.

  - **proper distance** (dt = 0; simultaneous measurements) along **radial direction** ($d\theta = d\phi = 0$):

$$D_P = \int \sqrt{-(ds)^2} = \int_0^{r_c} R_U(t) dr_c = R_U(t) r_c$$

    which is simply the **comoving** radial distance multiplied by the scale factor.

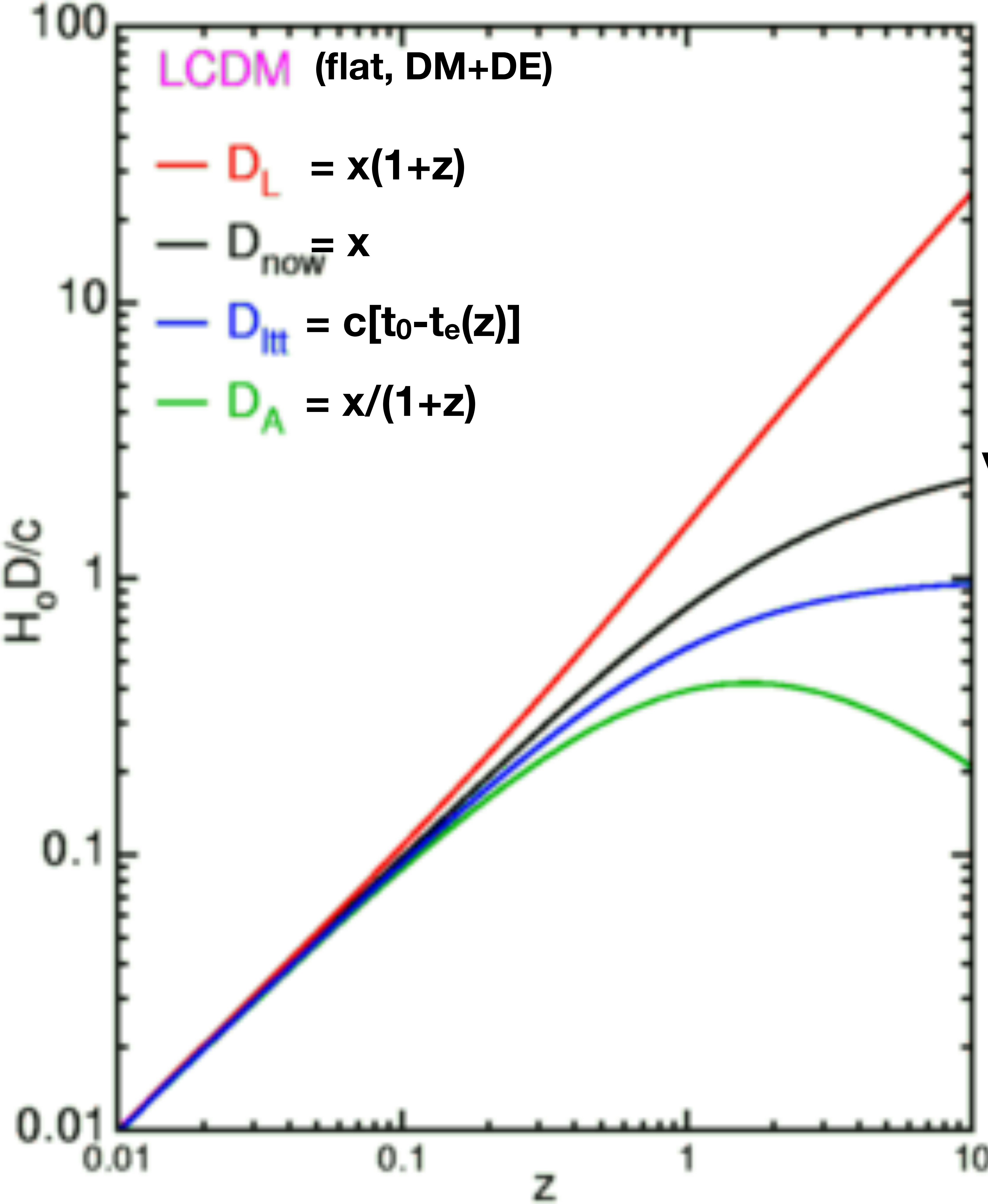
# Geometry is distorted in an expanding Universe

- **Any given object can have many different distances, and they do not equal!**
  - Comoving distance: $D_C = \int_{t_{\rm em}}^{t_{\rm obs}} \frac{c}{R_U(t)} dt = c \int_0^z \frac{dz}{H(z)}$
  - Luminosity distance: $D_L = \sqrt{L/(4\pi F)} = D_C \cdot (1+z)$
  - Angular size distance: $D_A = l/\theta = D_C/(1+z)$
  - Light travel distance: $D_{ltt} = c[t_0 - t(z)]$

https://www.astro.ucla.edu/~wright/CosmoCalc.html

For $H_o$ = 69.6, $\text{Omega}_M$ = 0.286, $\text{Omega}_{vac}$ = 0.714, z = 2.000

- It is now 13.721 Gyr since the Big Bang.
- The age at redshift z was 3.316 Gyr.
- The light travel time was 10.404 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 5273.0 Mpc or 17.198 Gly.
- The comoving volume within redshift z is 614.103 $\text{Gpc}^3$.
- The angular size distance $D_A$ is 1757.6 Mpc or 5.7326 Gly.
- This gives a scale of 8.521 kpc/".
- The luminosity distance $D_L$ is 15818.5 Mpc or 51.594 Gly.




# various distance vs. redshift

**Distance in unit of the Hubble scale: $D_H = c/H_0 = 3\ h^{-1}$ Gpc**

# Evidence for a Hot Beginning:

# Discovery of the Microwave Background

## If the universe started from a Big Bang, the hot early universe should have generated a blackbody radiation field that is detectable everywhere

- If all matter started in a small volume, conditions would be very hot. This hot, dense gas would have emitted blackbody radiation, which should be detectable even today (after the long expansion stretched these photons).
- In **1948**, **Alpher, Gamow**, and **Herman** made a prediction: The Planck spectrum of the thermal emission should be everywhere and it would be uniformly redshifted by the expansion of the universe to a temperature of about **5–50 K**, peaking at **microwave wavelengths.** *Their prediction was ignored or mostly forgotten.*

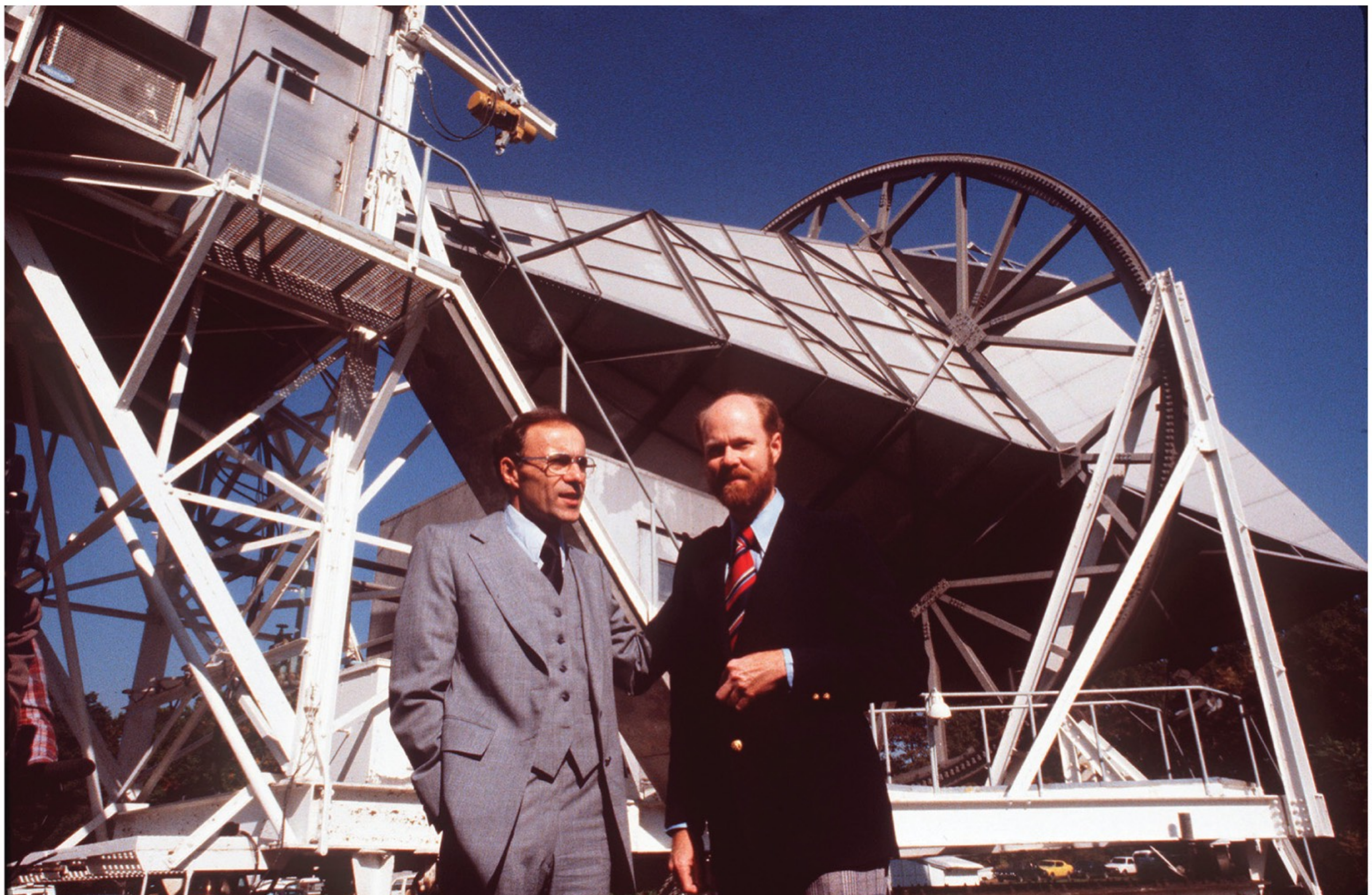
- Wien's displacement law: $\lambda_{\text{peak}} = (2.9 \text{ mm K})/T$
- Cosmological redshift: $\lambda_{\text{obs}} = \lambda_{\text{emit}}(1 + z)$
- Combining the above two, we have the relation the blackbody temperature when emitted and the observed temperature:

$$T_{\text{obs}} = T_{\text{emit}}/(1 + z)$$

  in other words, the observed temperature today is much lower than the original temperature due to cosmic expansion.

## Measuring the Cosmic Microwave Background

- This predicted spectrum was *accidentally* discovered in **1963** by **Arno Penzias** and **Robert Wilson**. They measured microwave emission in all directions and the observed temperature of the emission was about **3 K**.
- We call this predicted spectrum the **cosmic microwave background (CMB)**.
- This was the first clear evidence of **the Big Bang**.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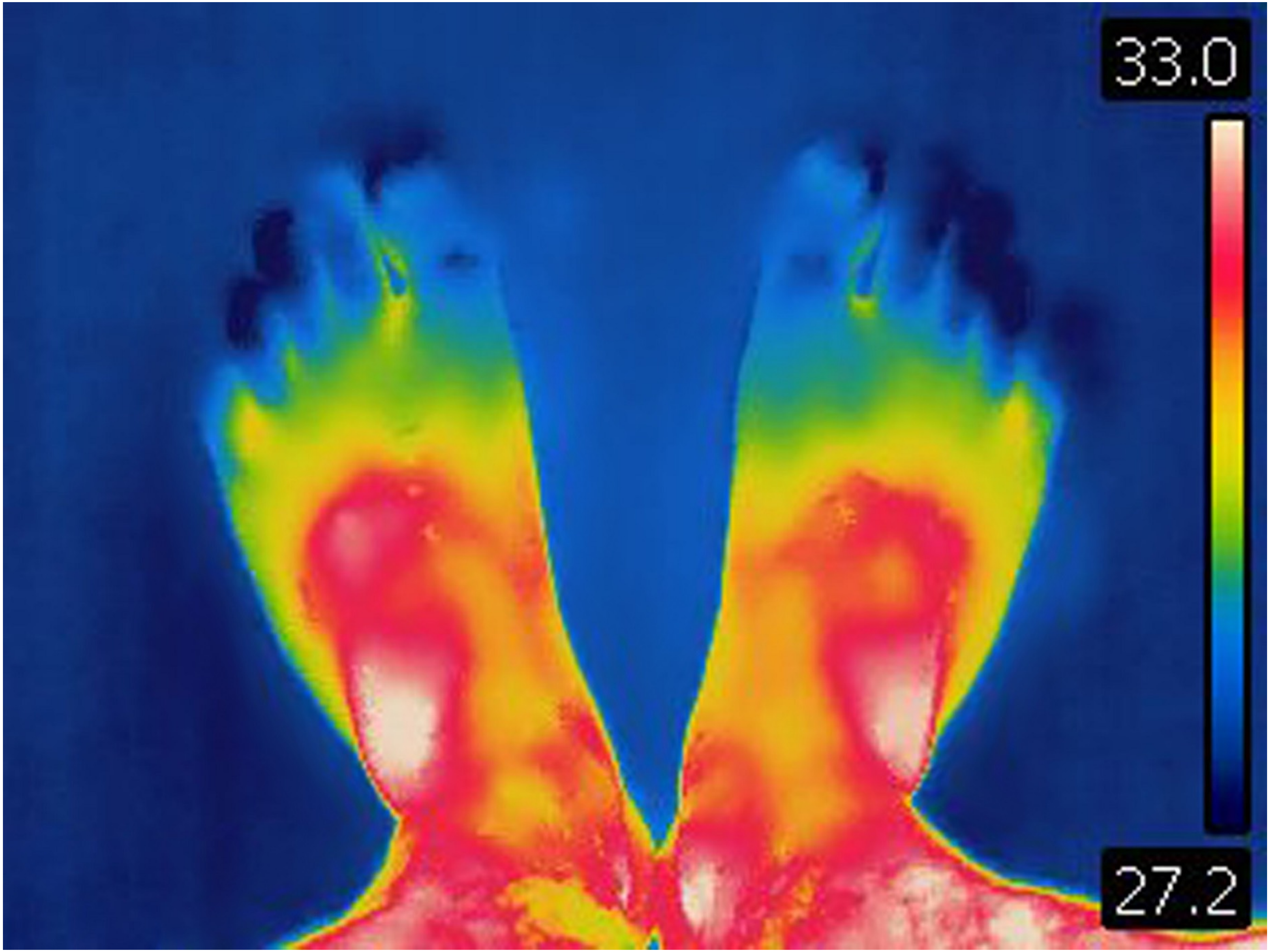
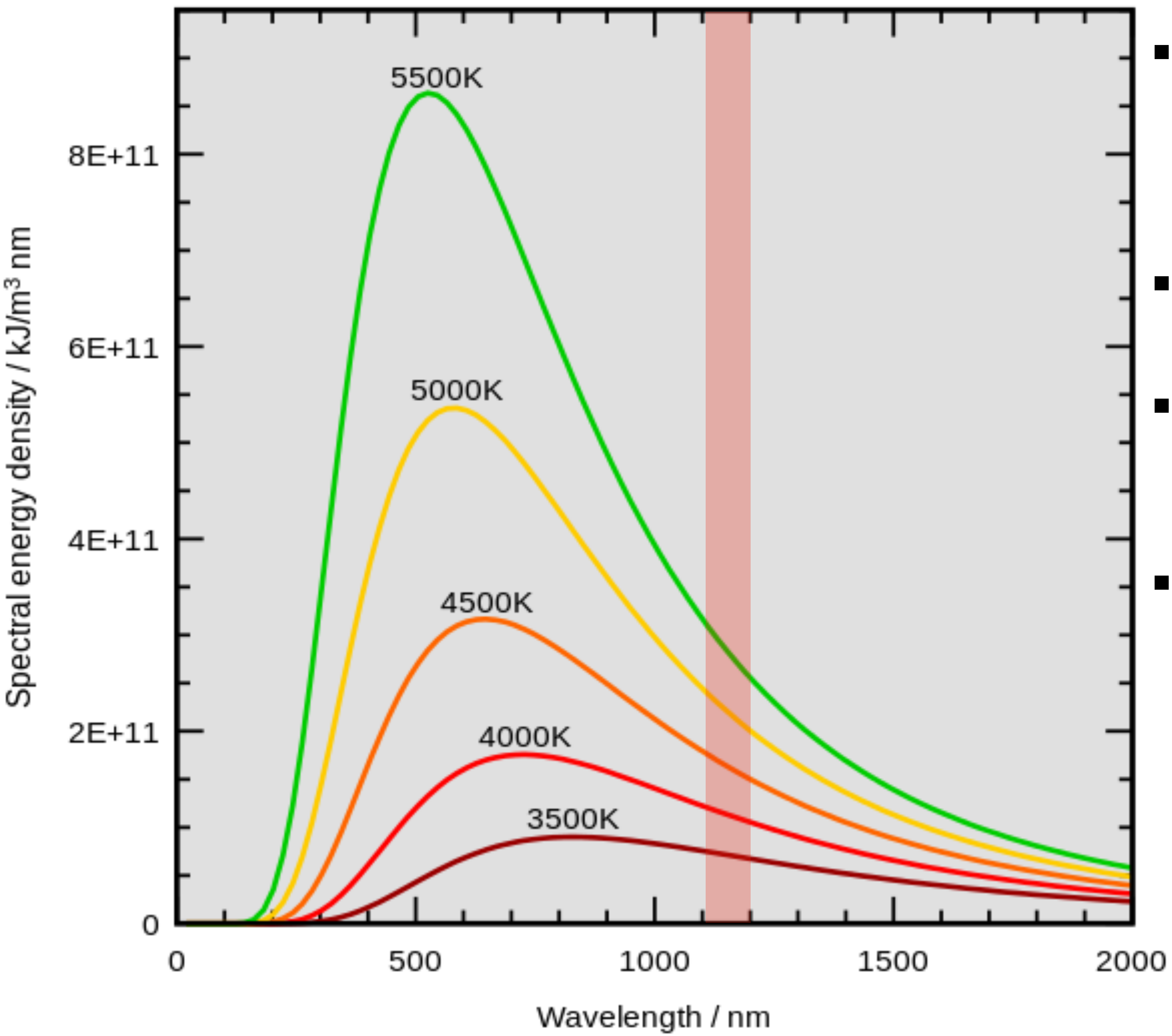
# Temperature Map from Single-wavelength Intensity Measurement



# The Planck Curve

$$I_\lambda = B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}} - 1}$$

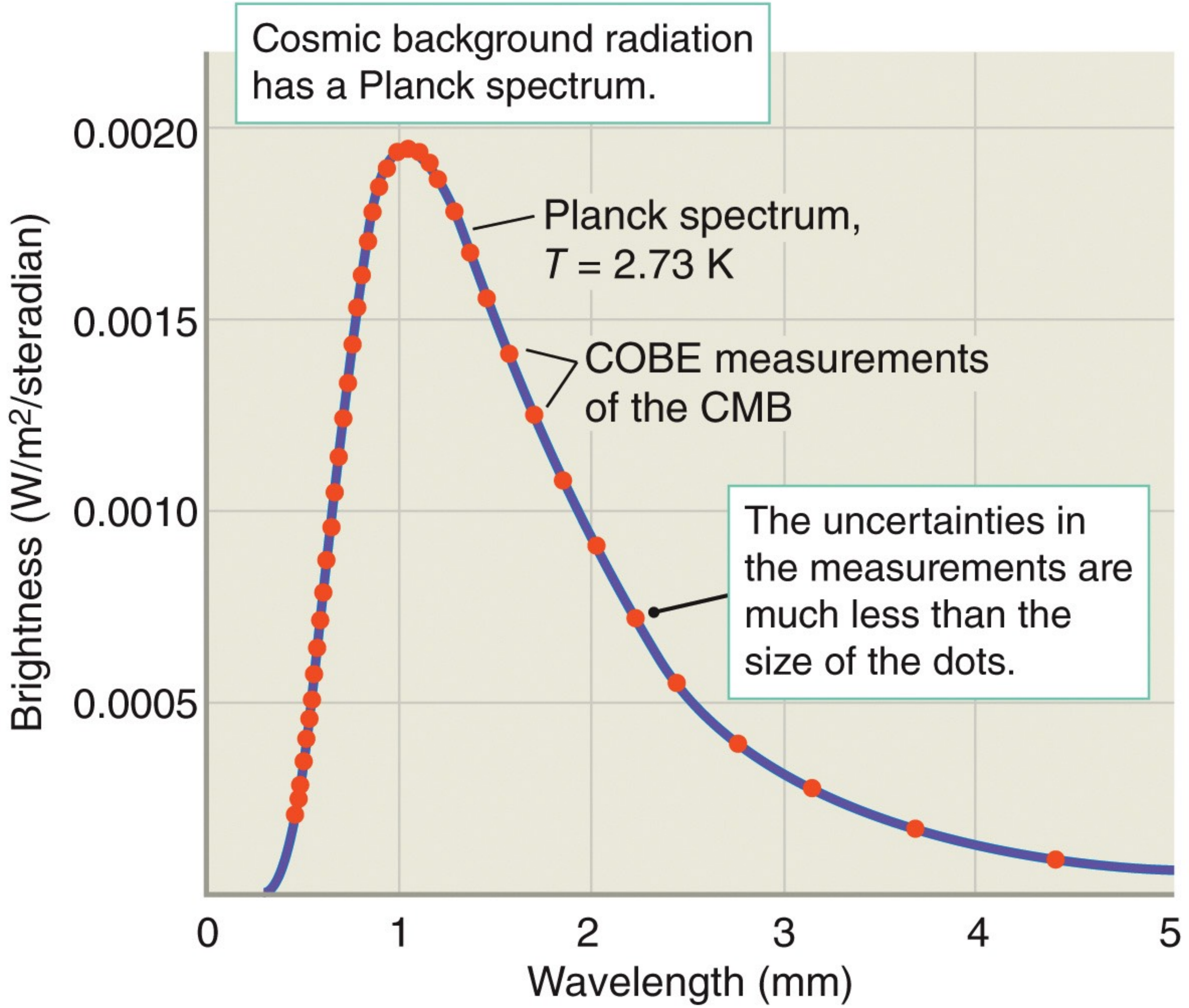



- **What happens when T increases?**

- **Peak** shifts to shorter wavelength
- **Total area** under the Planck curve increases
- Radiation **intensity** increases at all wavelengths

# The CMB Temperature from the Full Blackbody Spectrum

- **The COBE satellite (launched in 1989)** was the first instrument to provide very accurate measurements of **the CMB spectrum**. It determined the temperature of the CMB today is **2.73 K** by fitting a beautiful Planck curve to the data.



## Problem is that the observed CMB temperature can't tell the original temperature of the blackbody

- Wien's displacement law: $\lambda_{\rm peak} = (2.9 \text{ mm K})/T$
- Cosmological redshift: $\lambda_{\rm obs} = \lambda_{\rm emit}(1 + z)$
- Combining the above two, we have the relation the blackbody temperature when emitted and the observed temperature:

  $$T_{\rm obs} = T_{\rm emit}/(1 + z)$$

  in other words, the observed temperature today is much lower than the original temperature due to cosmic expansion.

- The above equation cannot constrain the temperature when the cosmic radiation background first emerged. In fact, infinite number of ($T_{emit}$, z) combinations could give us the same 3 K observed temperature, for examples:
  - $T_{emit}$ = 10 K, z = 2.3
  - $T_{emit}$ = 100 K, z = 32
  - $T_{emit}$ = 1000 K, z = 330
- We call that the two parameters ($T_{emit}$, z) are **degenerate.**

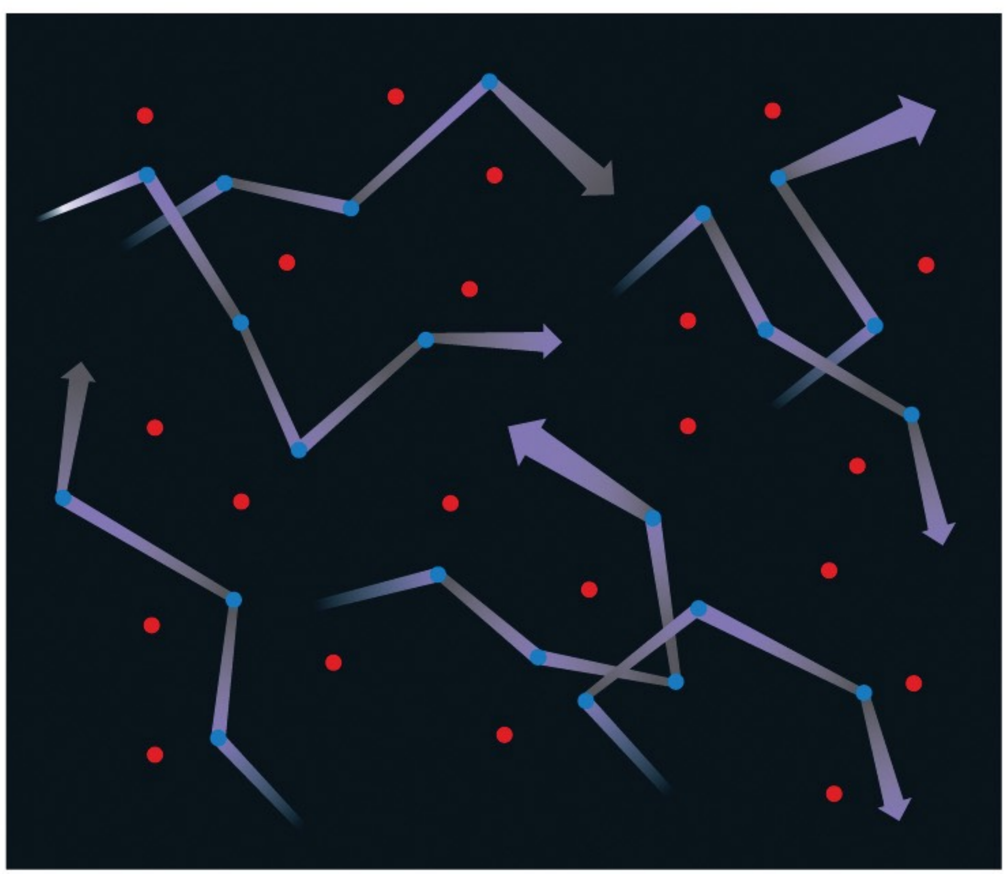
How do we know when the CMB first started to propagate in the universe?

The epoch of hydrogen recombination

## How did the CMB emerge?
## First, EM waves are trapped in an ionized universe

- When the universe was hot and the gas was ionized, photons were trapped with matter.
  - Free electrons interact strongly with photons (***Thomson scattering***).
  - We cannot observe anything during this era. It's as if the universe is filled with a dense fog.

In the ionized early universe, light was trapped by free electrons. Radiation had a blackbody spectrum.



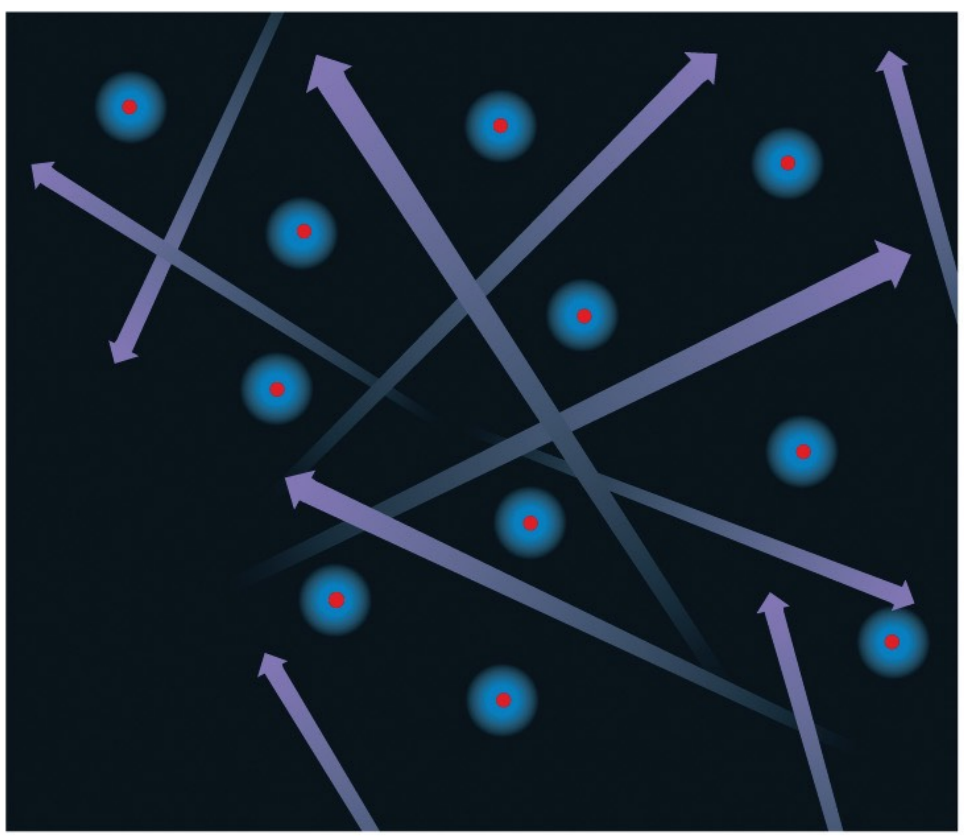
At that time, it was as though the universe was filled with a thick fog.


KEY • Proton • Electron Path of photon

## How did the CMB emerge?
## Then, protons and electrons recombined to form hydrogen

- Eventually, the expansion causes the temperature to cool enough that **protons** and **electrons** could form **neutral H atoms**: this phase-transition of the Universe is called **the epoch of recombination (EoR)**.
- At that time, light was no longer blocked from its travel by free electrons.
- **EoR** marks the earliest point in the universe that we can observe.

KEY • Proton • Electron Path of photon

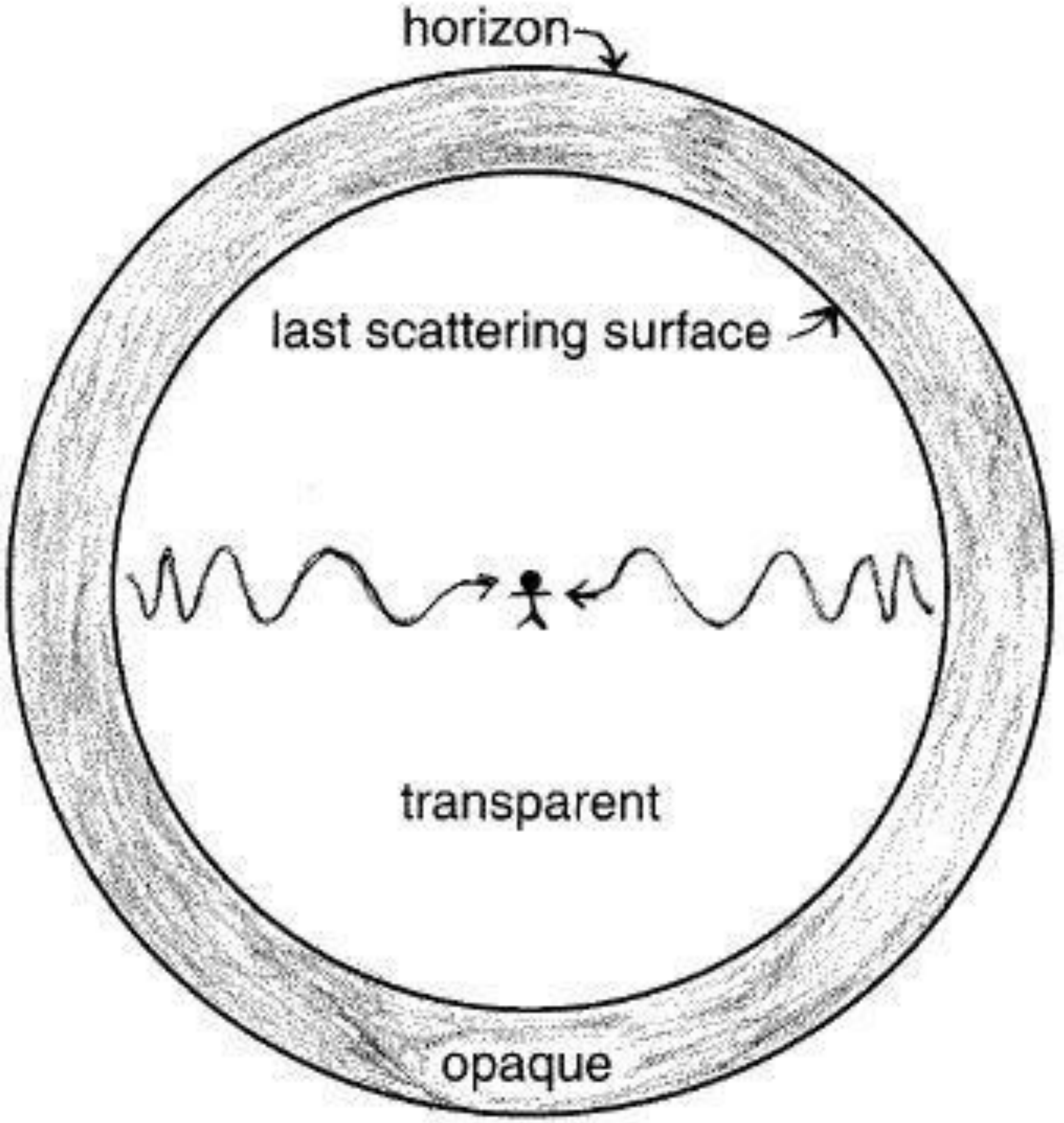


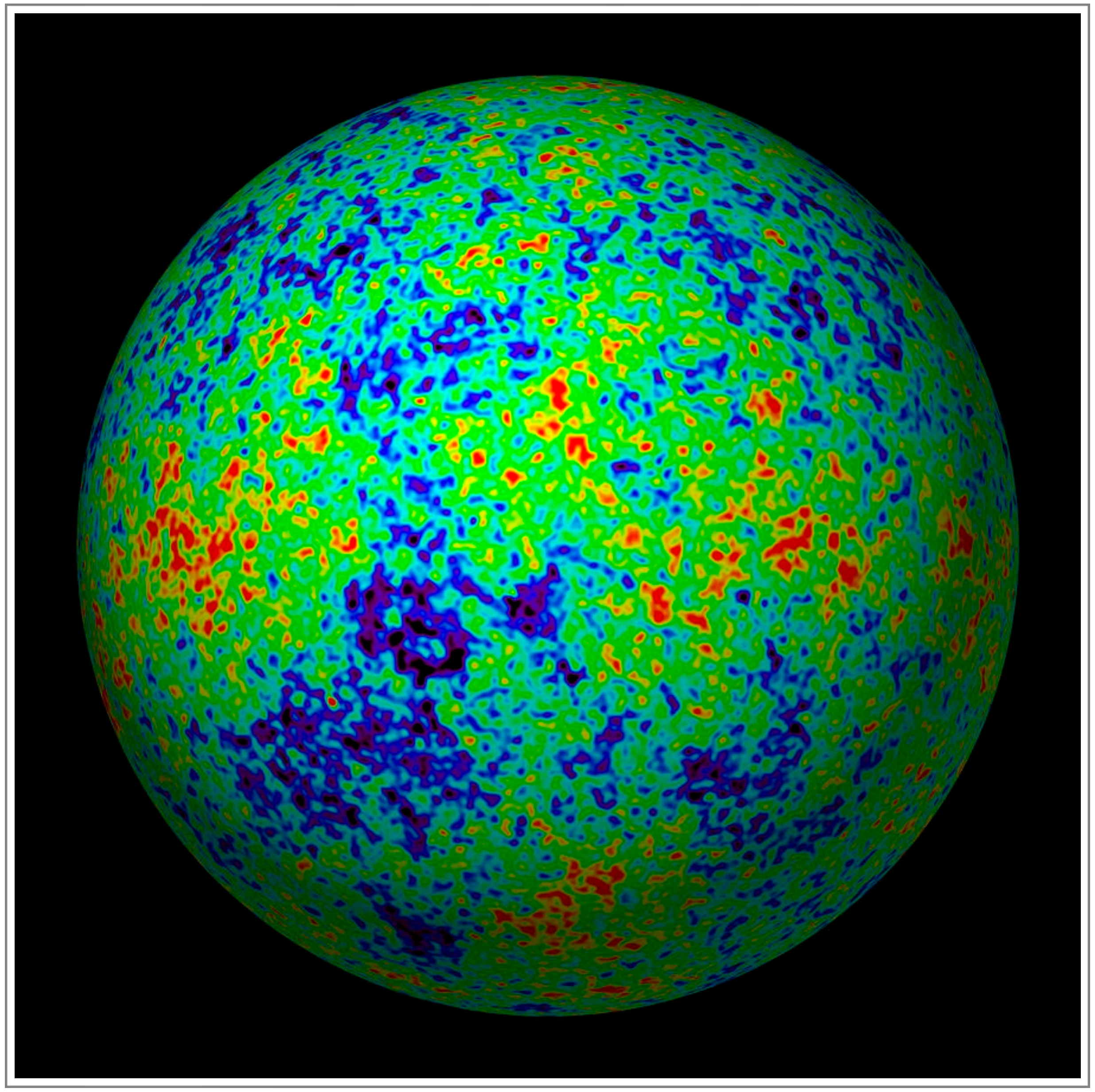
At recombination, the universe became transparent, and the blackbody radiation traveled freely through the universe.


Recombination was like the fog suddenly clearing.

## CMB Photons travel straight to us from the last scattering surface

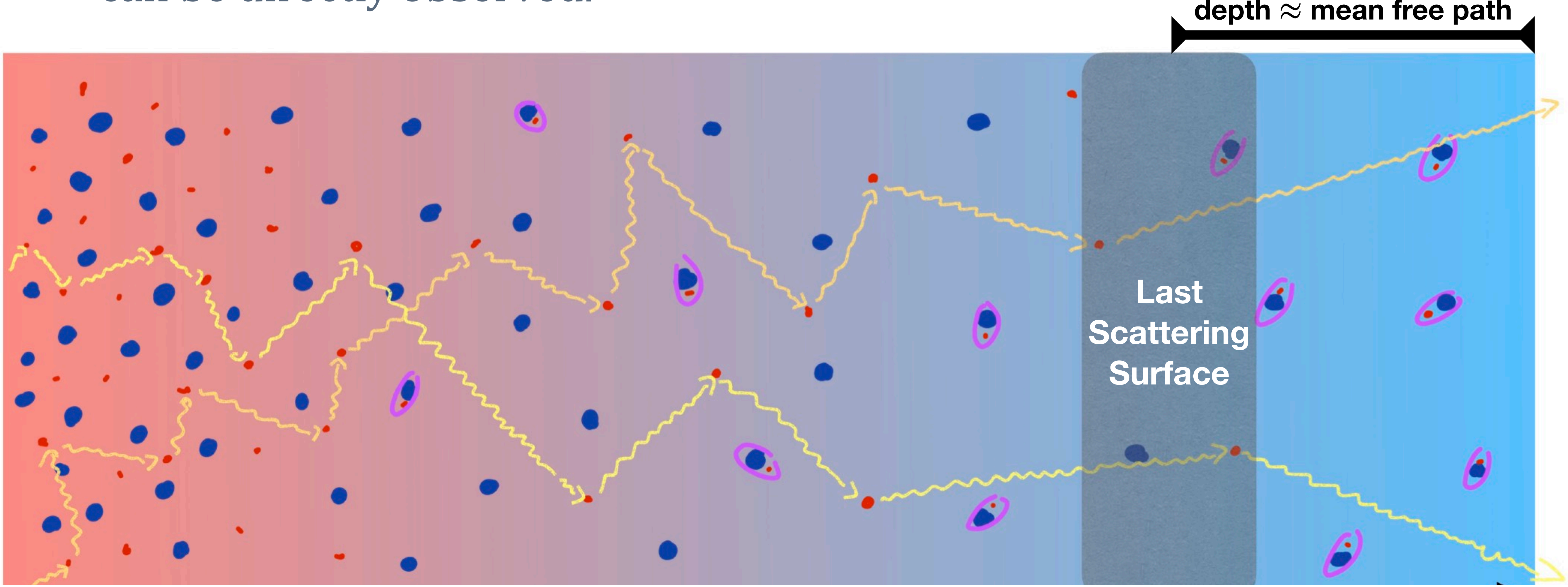
- Analogous to the *last scattering surface* that marks the surface of the Solar photosphere



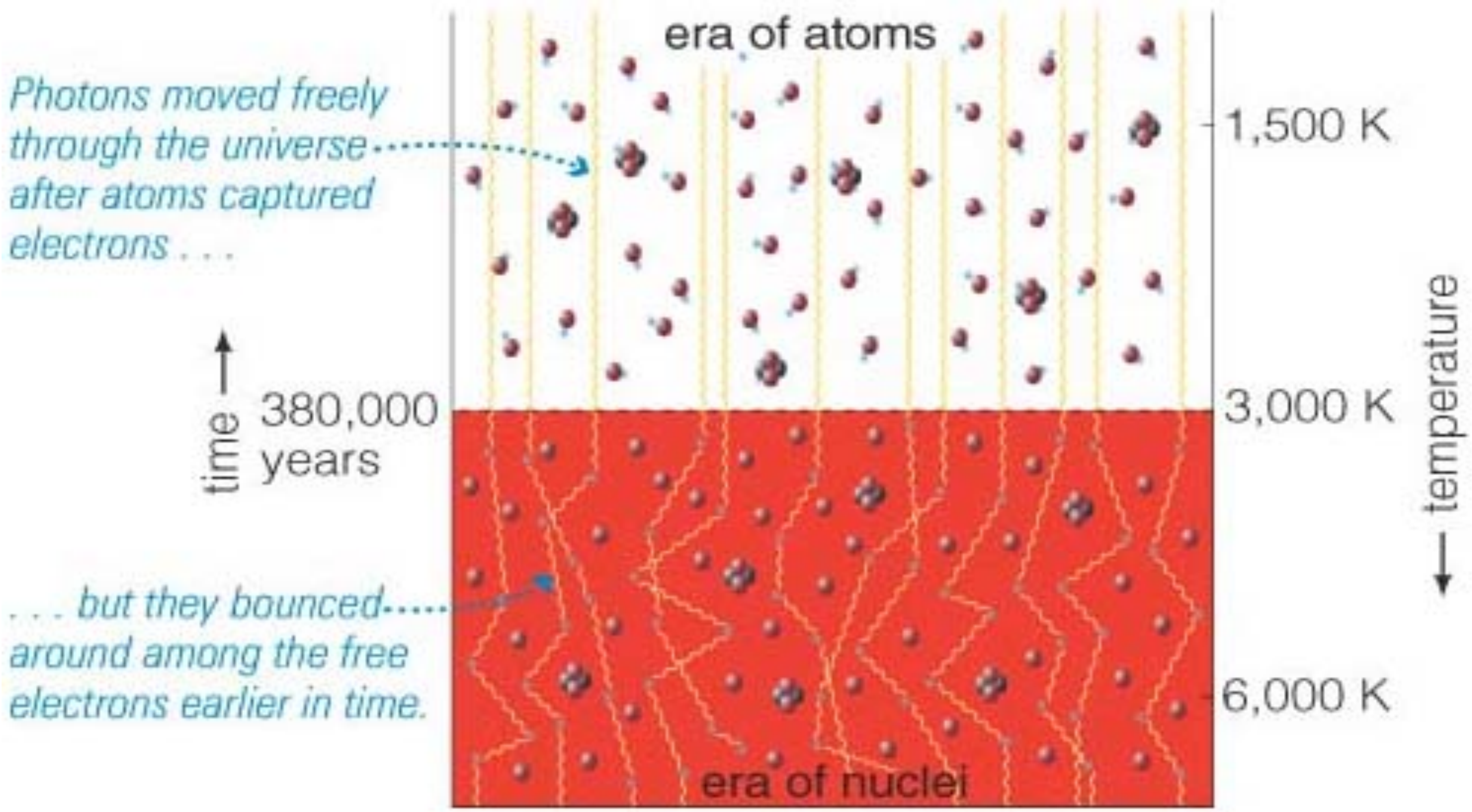
# Recall this slide in Chap 14? Last Scattering Surface

- The Sun has no solid surface, but the apparent surface of the Sun is the surface at which light can directly escape into space.
- Let's call this surface the **last scattering surface** (a concept also used in **cosmology**). Note that its depth depends on **(1) the angle we look into the Sun and (2) the wavelength of the photons**
- The layers above this point are known as the **atmosphere**, which can be directly observed.

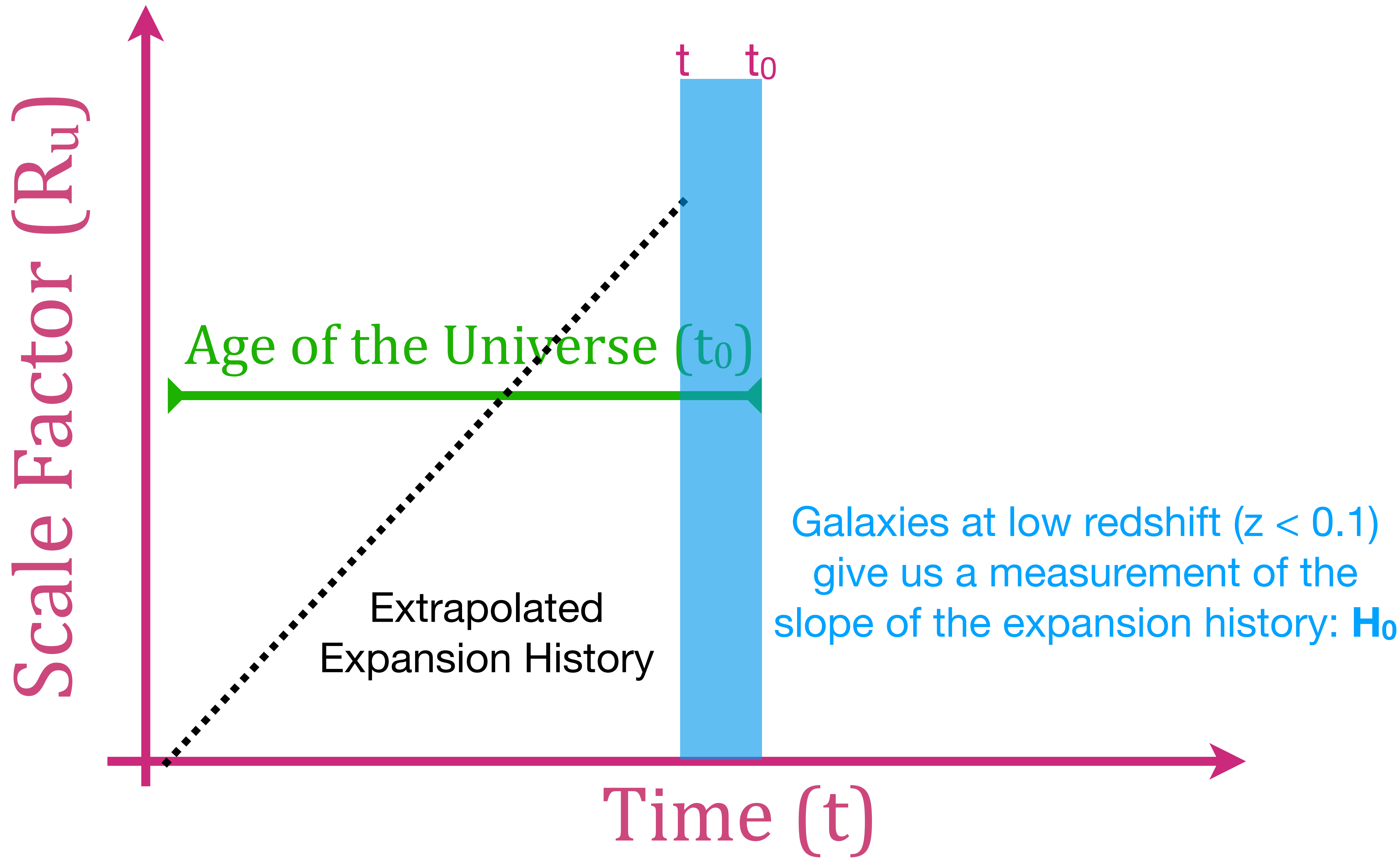


## The EM radiation background emerges when recombination completed

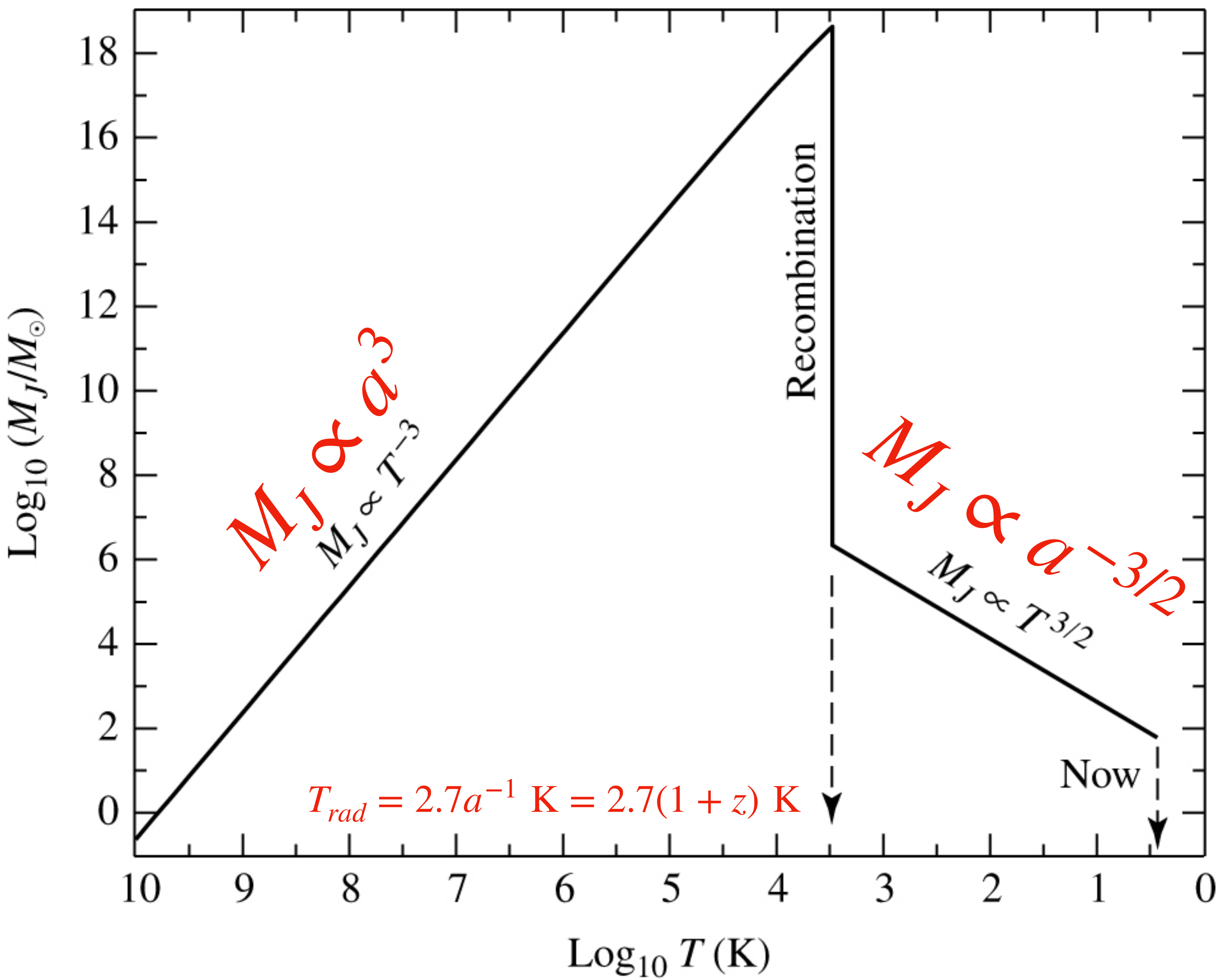
- Before **recombination**, photons cannot travel far before it is scattered by e-; after **recombination**, photons can freely travel and eventually reach us.
- Given the baryon density of the Universe, it can be shown that **Hydrogen recombination** completed when the universe was ~**3000 K.** This is **the original temperature of the cosmic EM radiation field**.
- Since we have proven $T_{observed} = T_{emitted}/(1+z)$, and we know the CMB has a temperature is 2.7 K today, the redshift at which the CMB emerged must be around 1100: $1+z = T_{emitted}/T_{observed} = 3000\text{ K}/ 2.7\text{ K} \sim 1100$



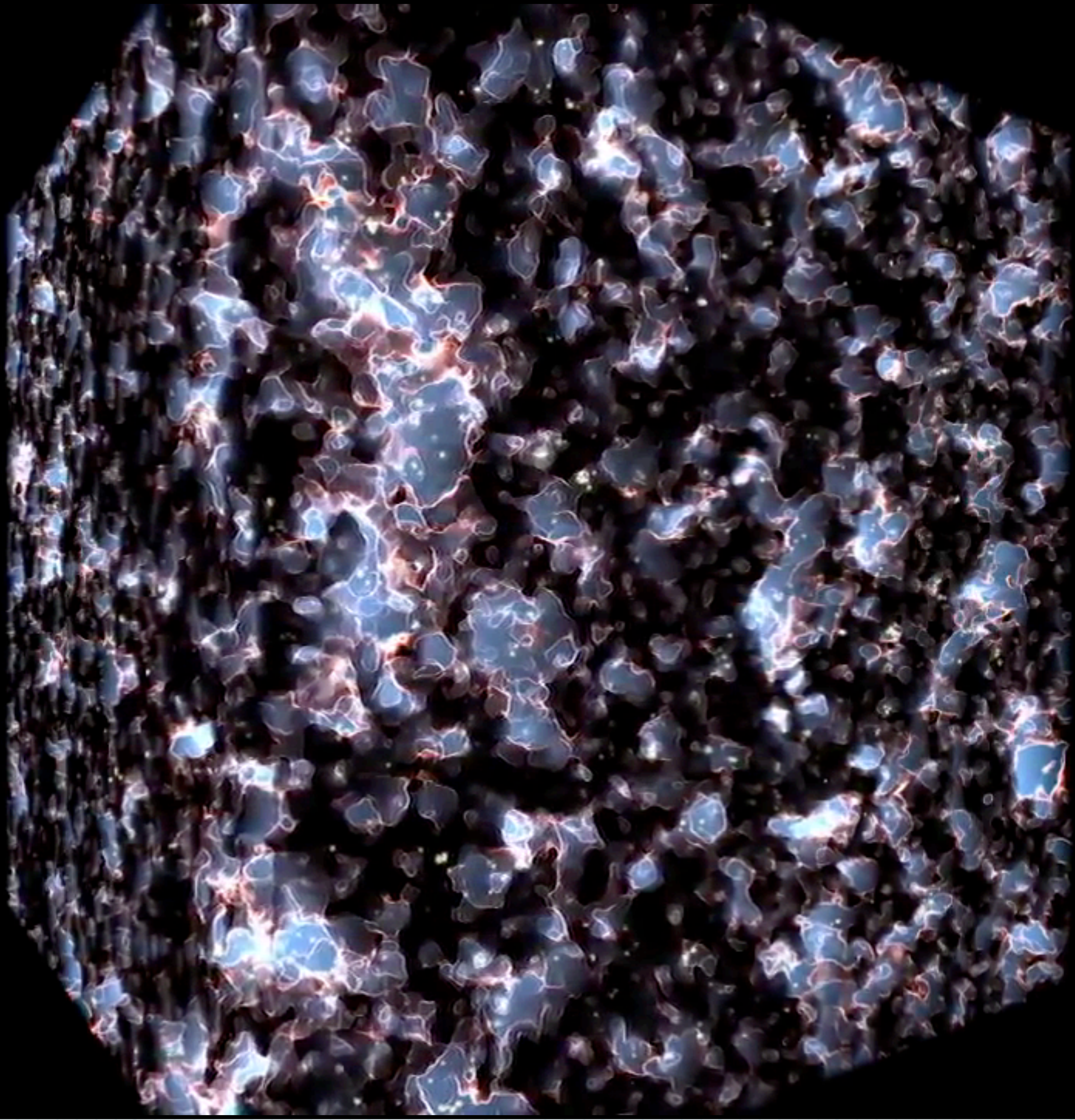
**Practice**: estimate the age of the Universe when the CMB first emerged. Think about the scale factor $R_u$ of the Universe at $z$ = 1100, and assume a **constant expansion rate** as we did when estimating the age of the Universe



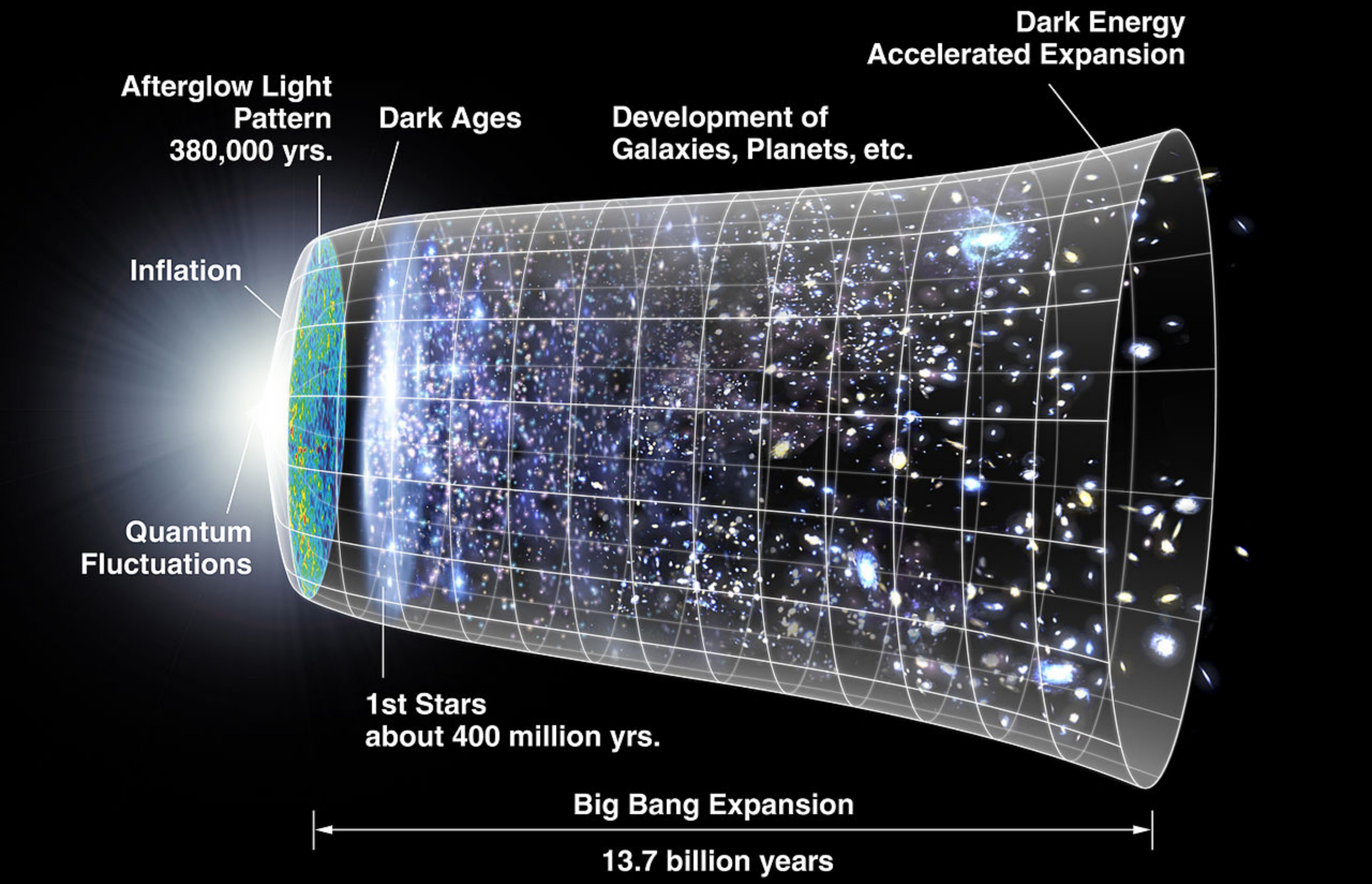
**Recombination drastically reduced the Jeans mass for gravitational instability, allowing galaxies to form.** ***What would happen next?***



# **Reionization** of the Universe by Galaxies



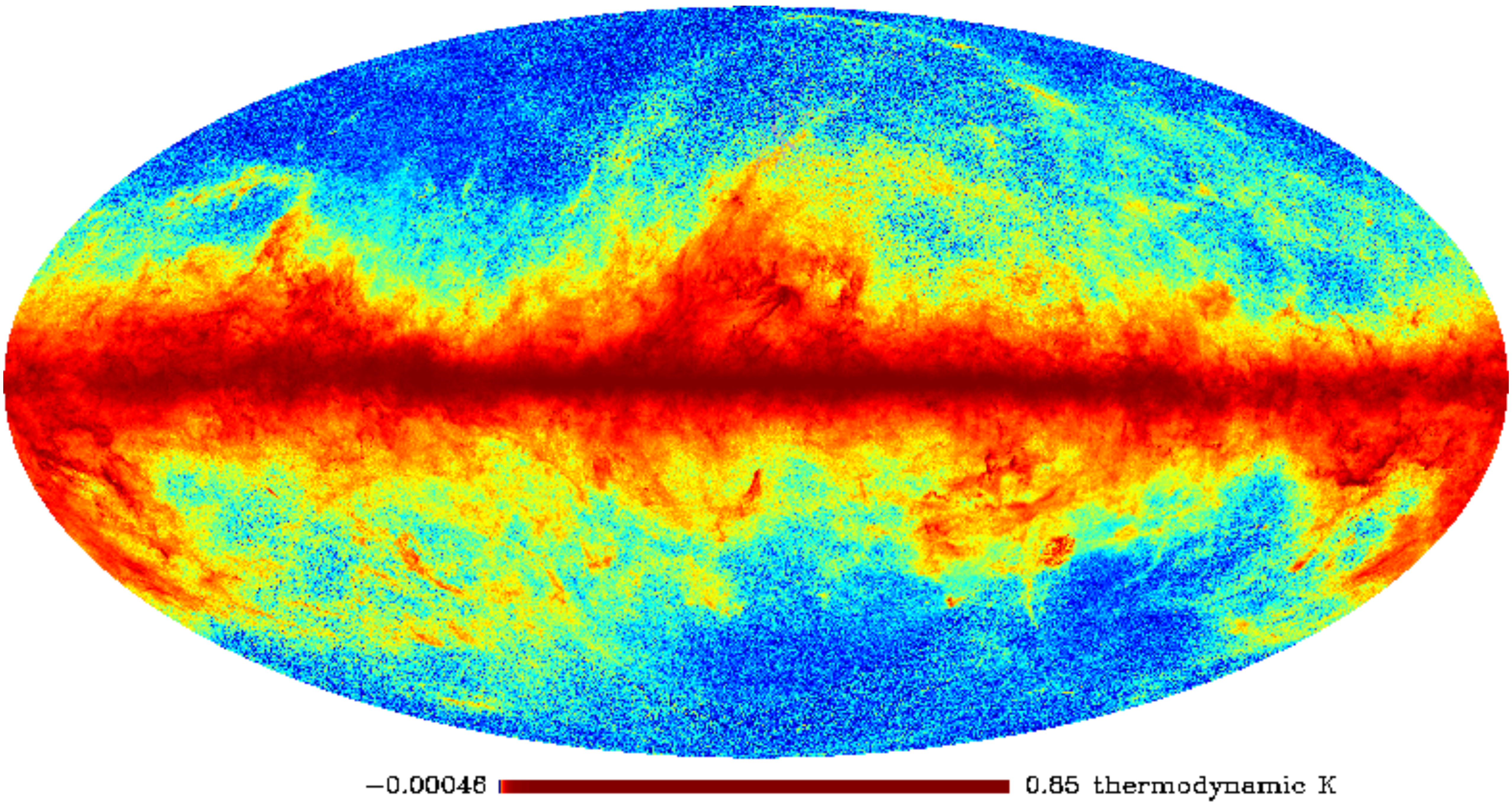
## *Big Bang - Particles created - Ionized universe (opaque) - Recombination (z ~ 1100) - Dark Ages - Reionization (z ~ 20-7)*



Since it were a background radiation, why don’t we get an all-sky map?

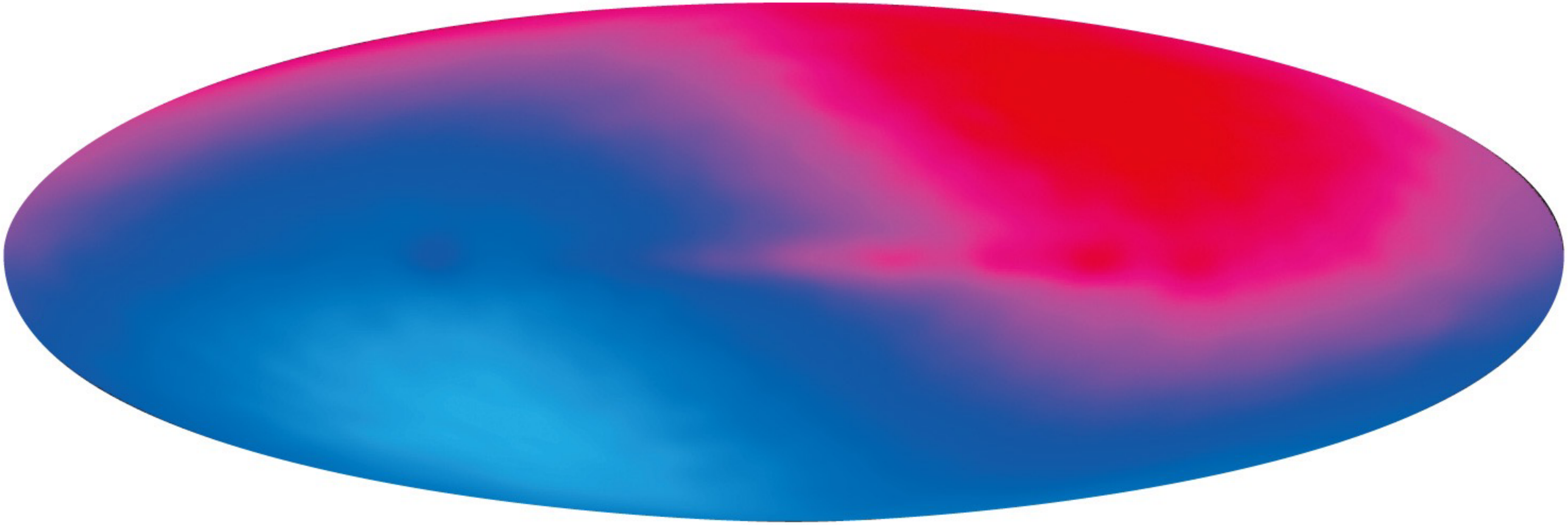
The discovery of CMB anisotropies

# An all-sky image in microwave (~1 mm)

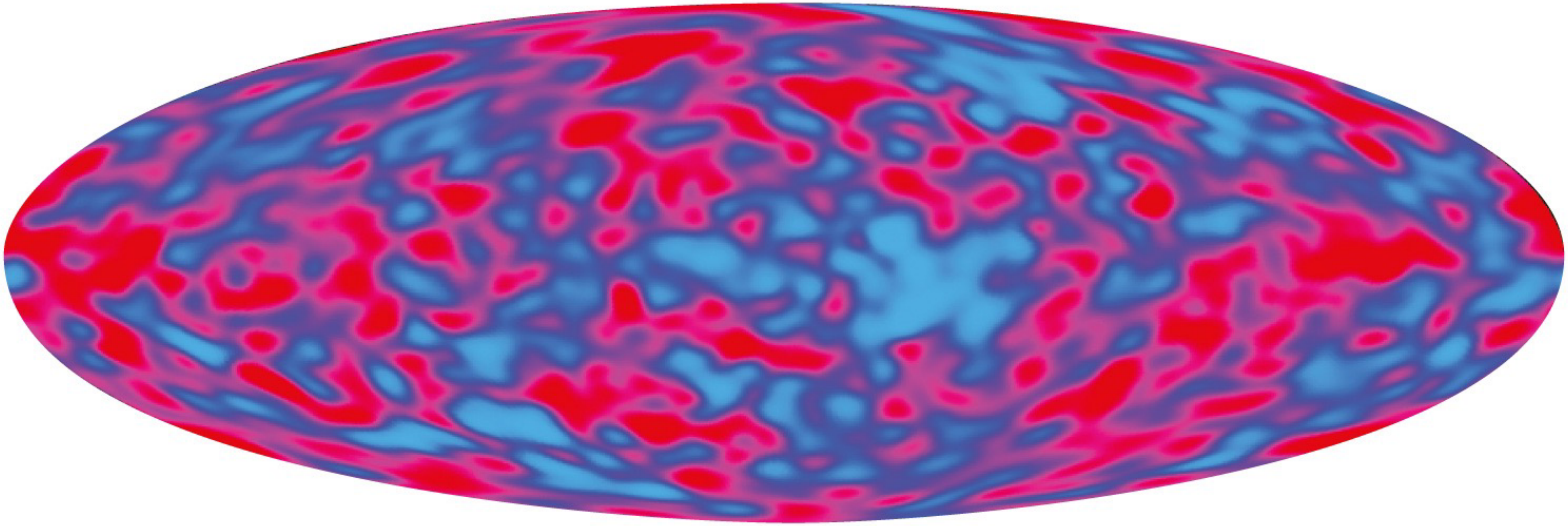


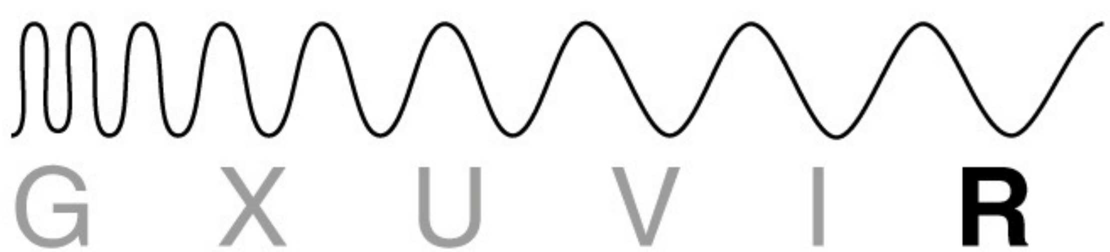
# A Strong Dipole in the CMB

suggests a relative velocity of 368 km/s between the Earth and the CMB.

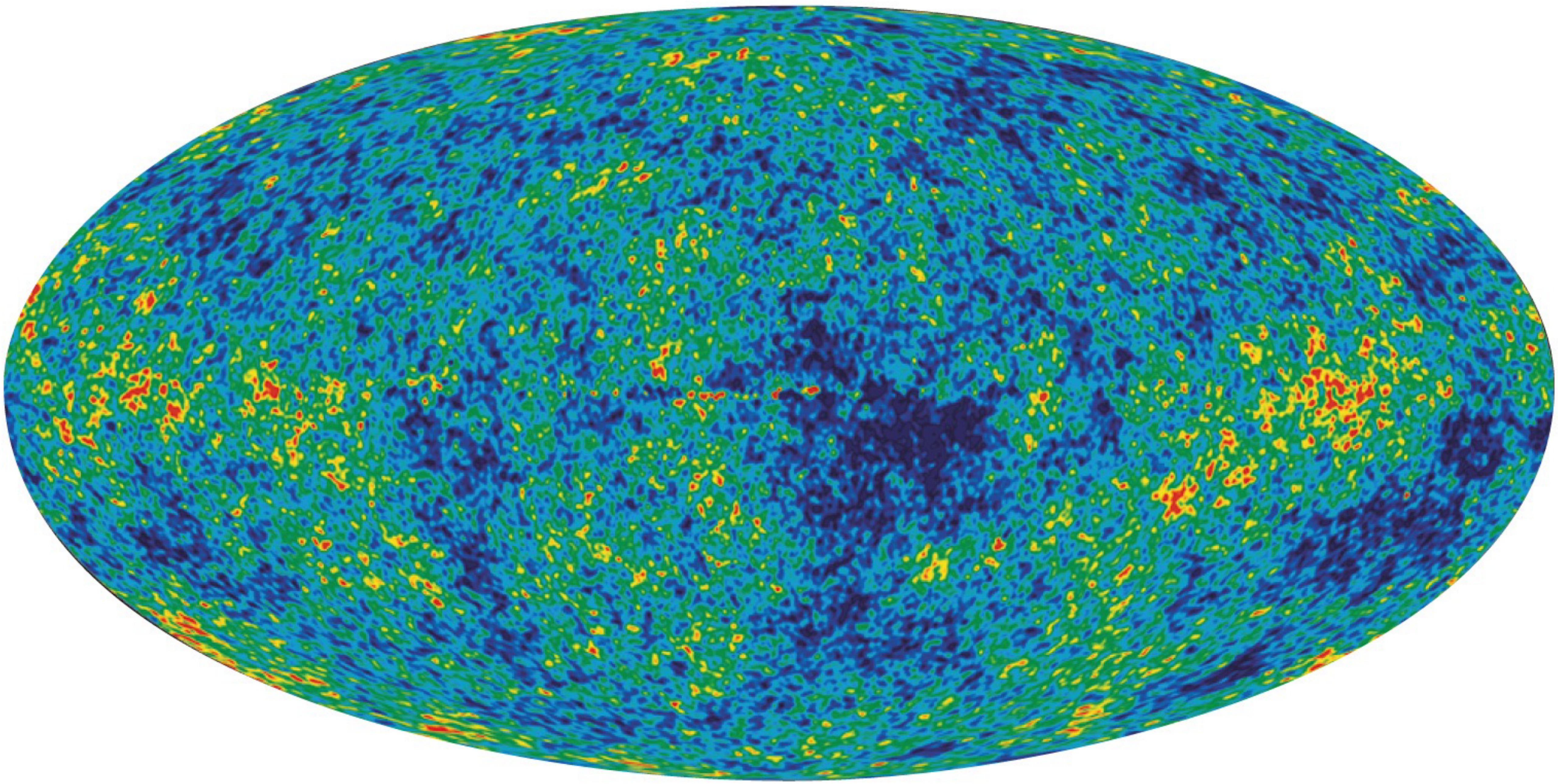


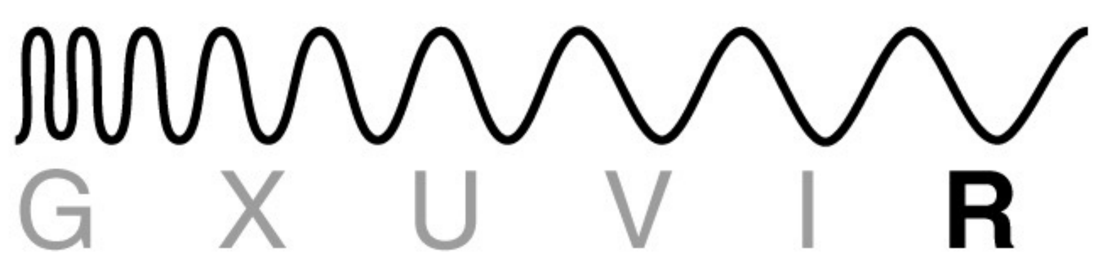
# Variations in the CMB





# A much sharper map: $\delta T/T \approx 0.00001$





# CMB anisotropy shows density fluctuations of $3 \times 10^{-5}$ at $z_{rec} \sim 1000$

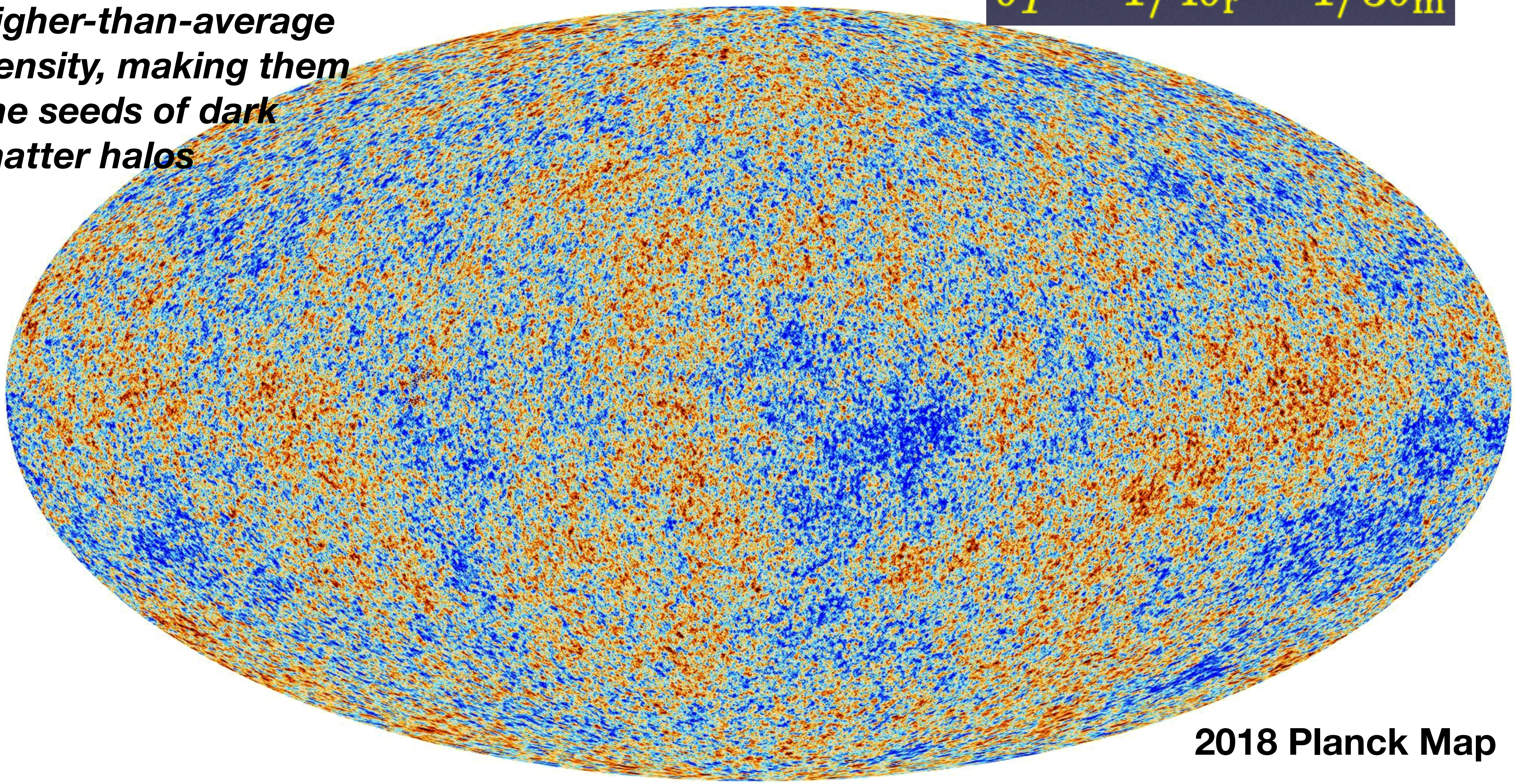
*isentropic/adiabatic perturbations: entropy per unit mass is preserved:*

$$S = \frac{s}{\rho_m} \propto \frac{\rho_r^{3/4}}{\rho_m}$$

$$\delta_S = \frac{\partial S}{S} = \frac{1}{S}\left[\frac{\partial S}{\partial \rho_r}\,\partial\rho_r + \frac{\partial S}{\partial \rho_m}\,\partial\rho_m\right] = \frac{3}{4}\delta_r - \delta_m$$

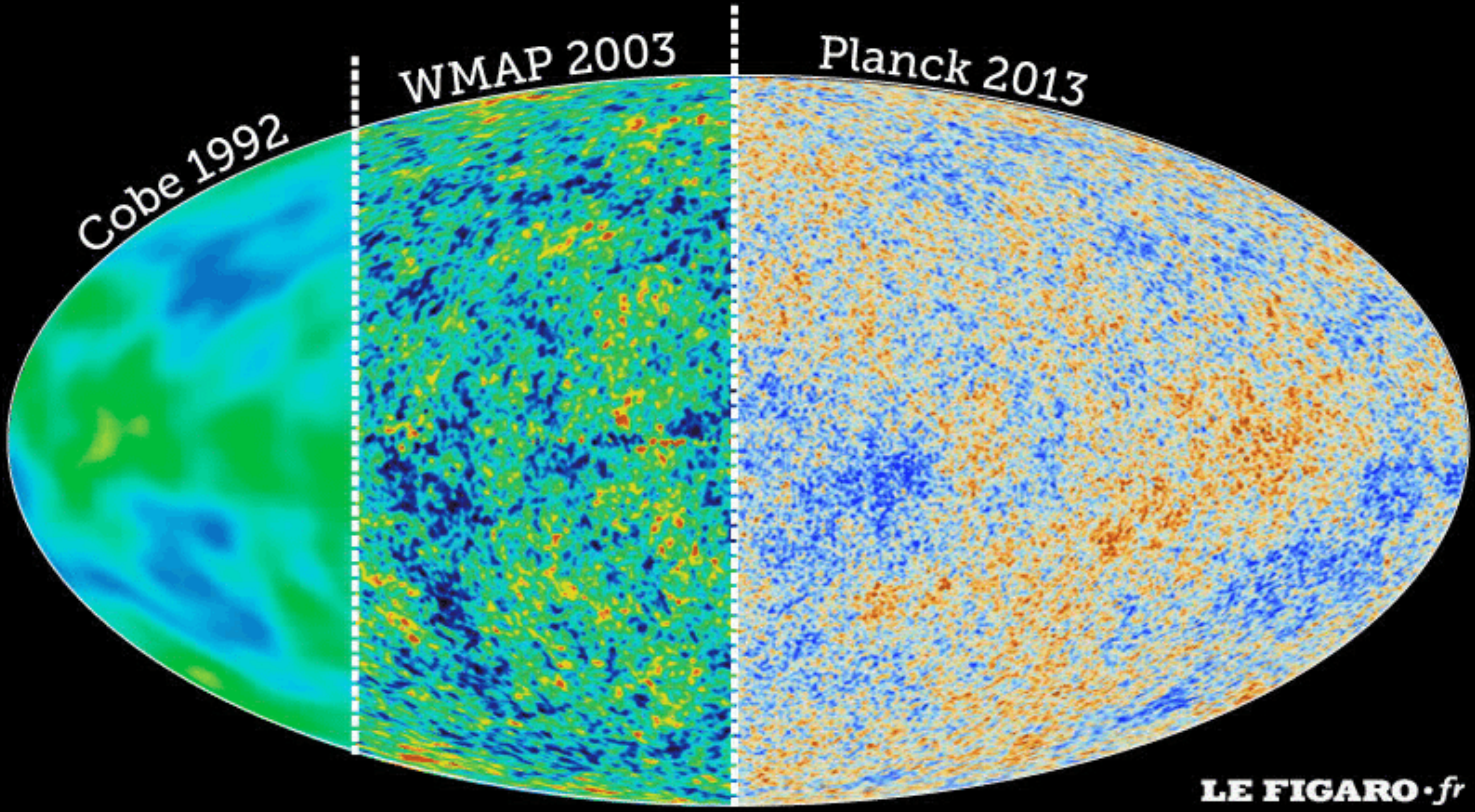
$$\delta_T = 1/4\delta_r = 1/3\delta_m$$

***Hotter T regions have higher-than-average density, making them the seeds of dark matter halos***



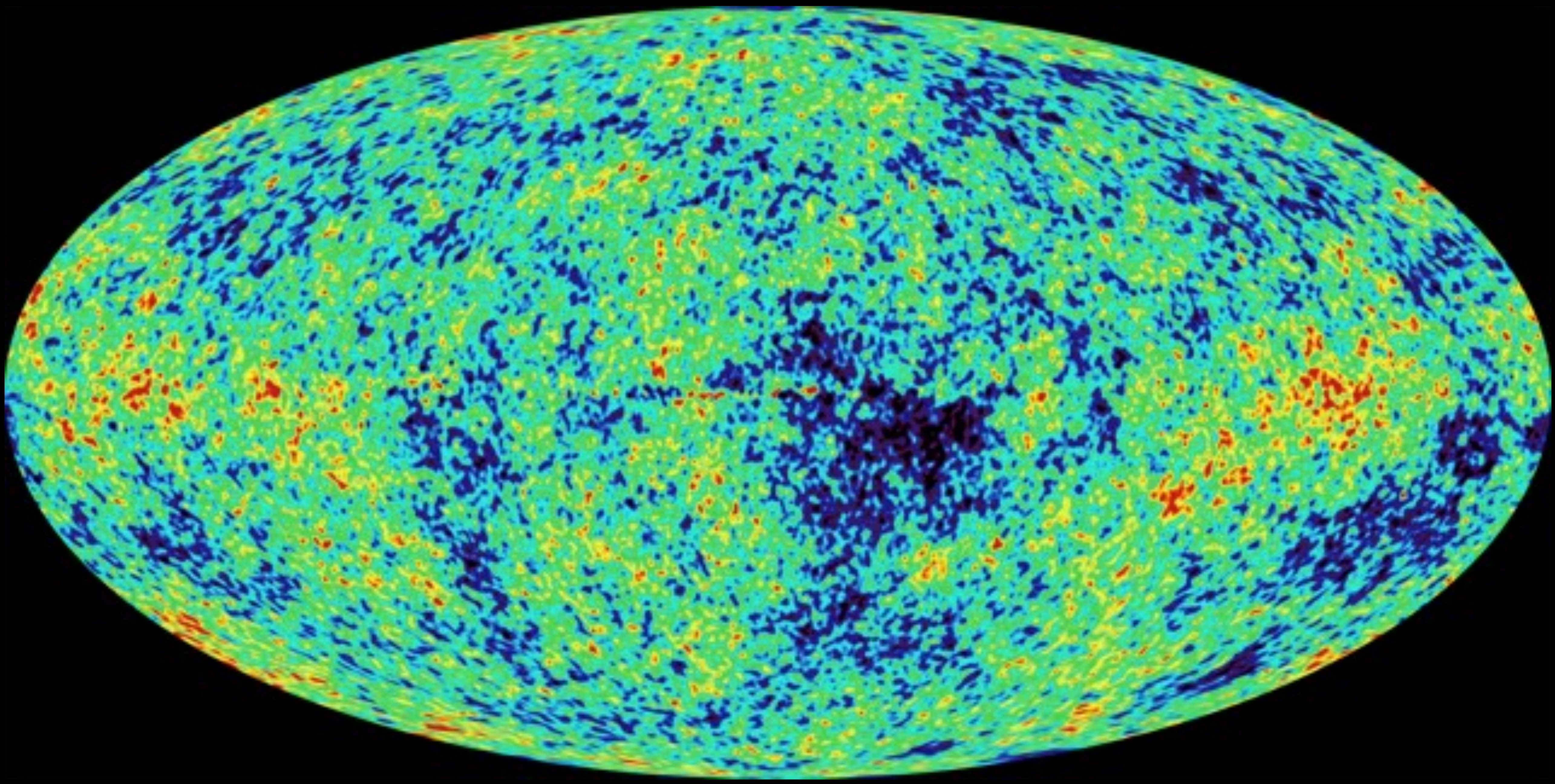
**2018 Planck Map**

# Improved angular resolutions over three generations of satellites



# Quantifying CMB anisotropies w/ its angular power spectrum

# CMB Anisotropy in Mollweide (equal-area) projection

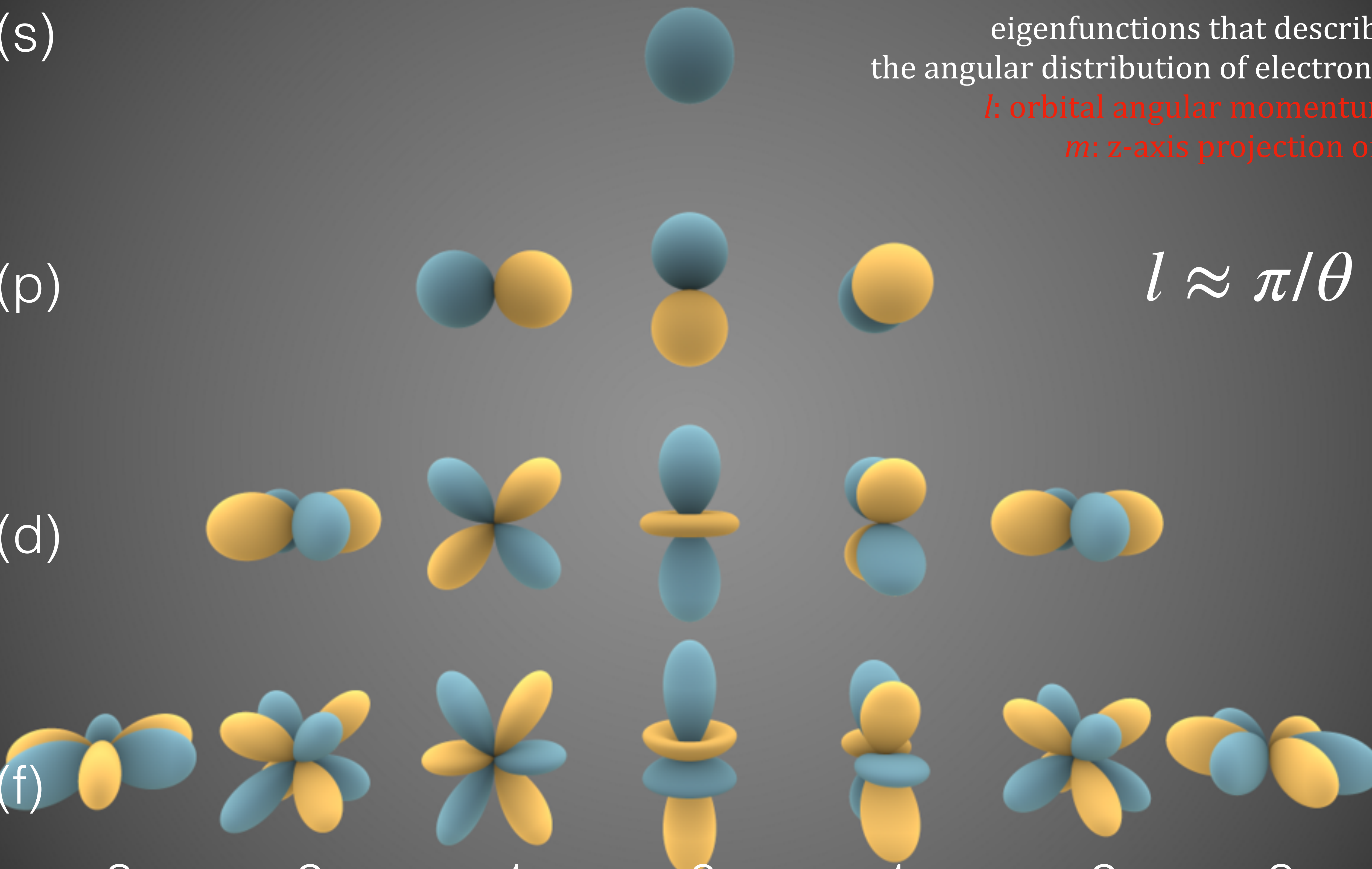


$$\delta T/T \approx 0.00001$$

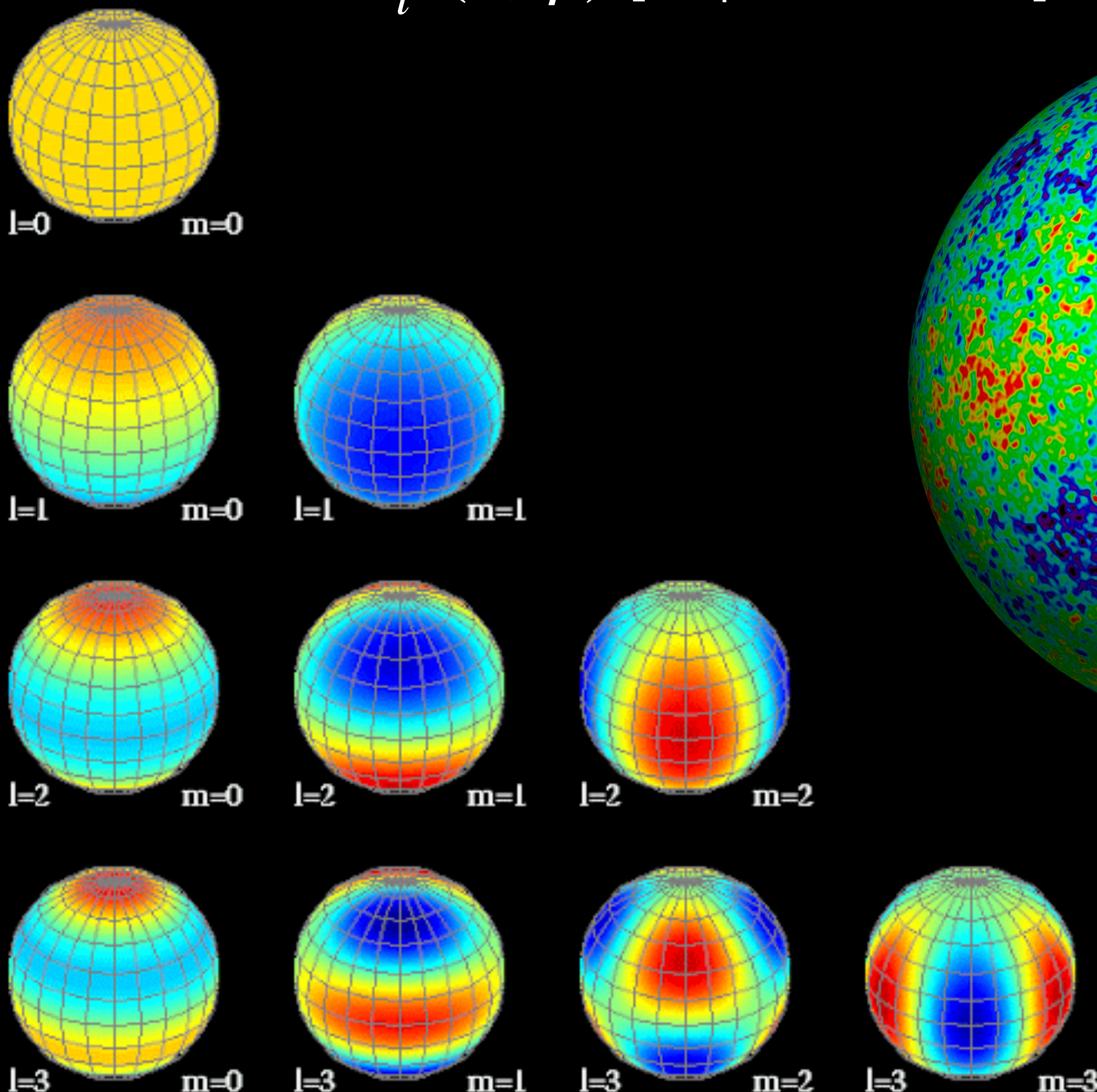
# Quantum Mechanics: spherical harmonics $Y_l^m(\theta, \phi)$

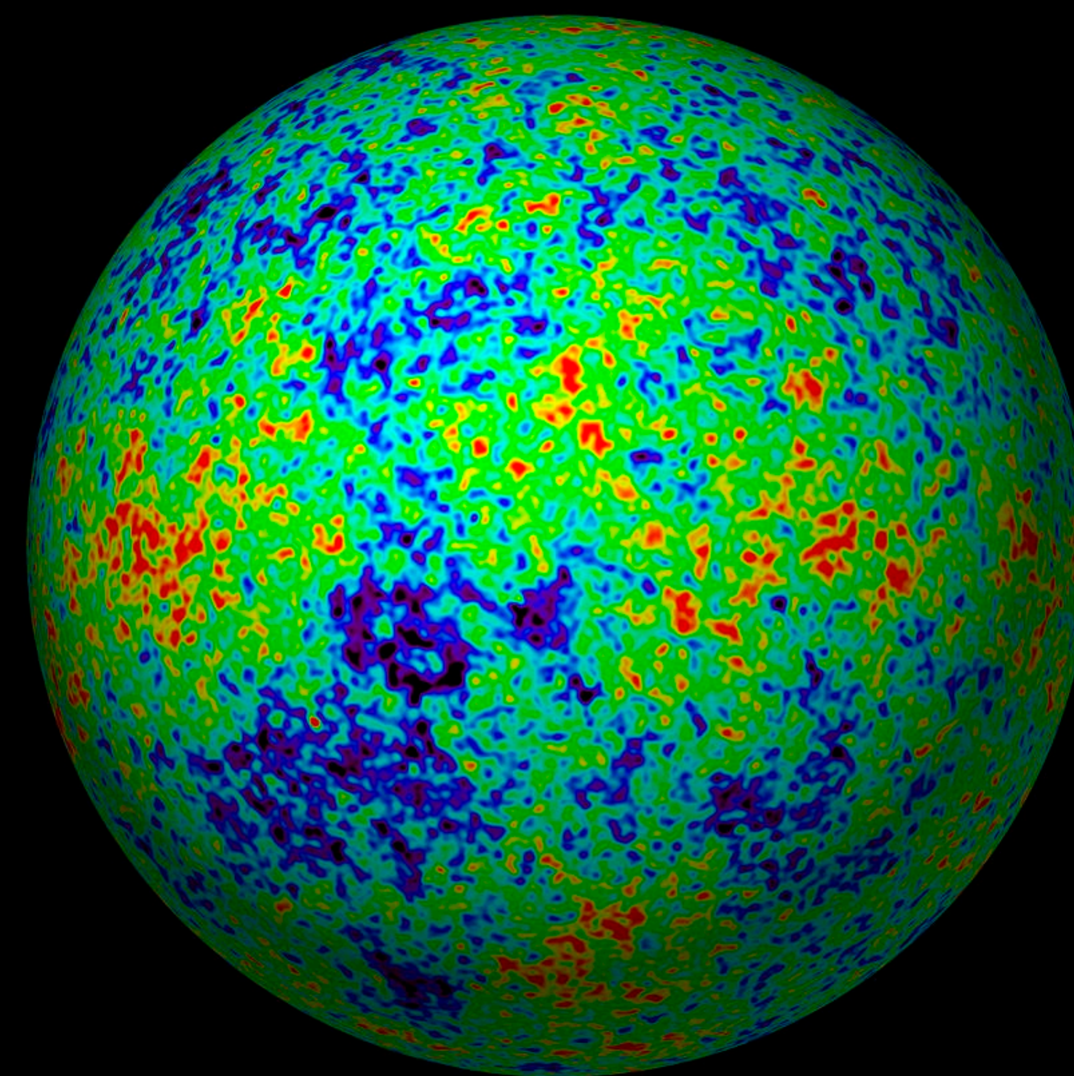
eigenfunctions that describe
the angular distribution of electrons:
*l*: orbital angular momentum
*m*: z-axis projection of *l*

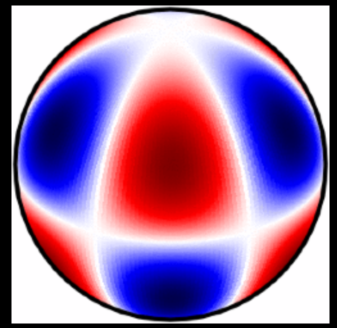
$$l \approx \pi/\theta$$



# Representing CMB anisotropies as a sum of spherical harmonics $Y_l^m(\theta, \phi)$ [Laplace 1782]

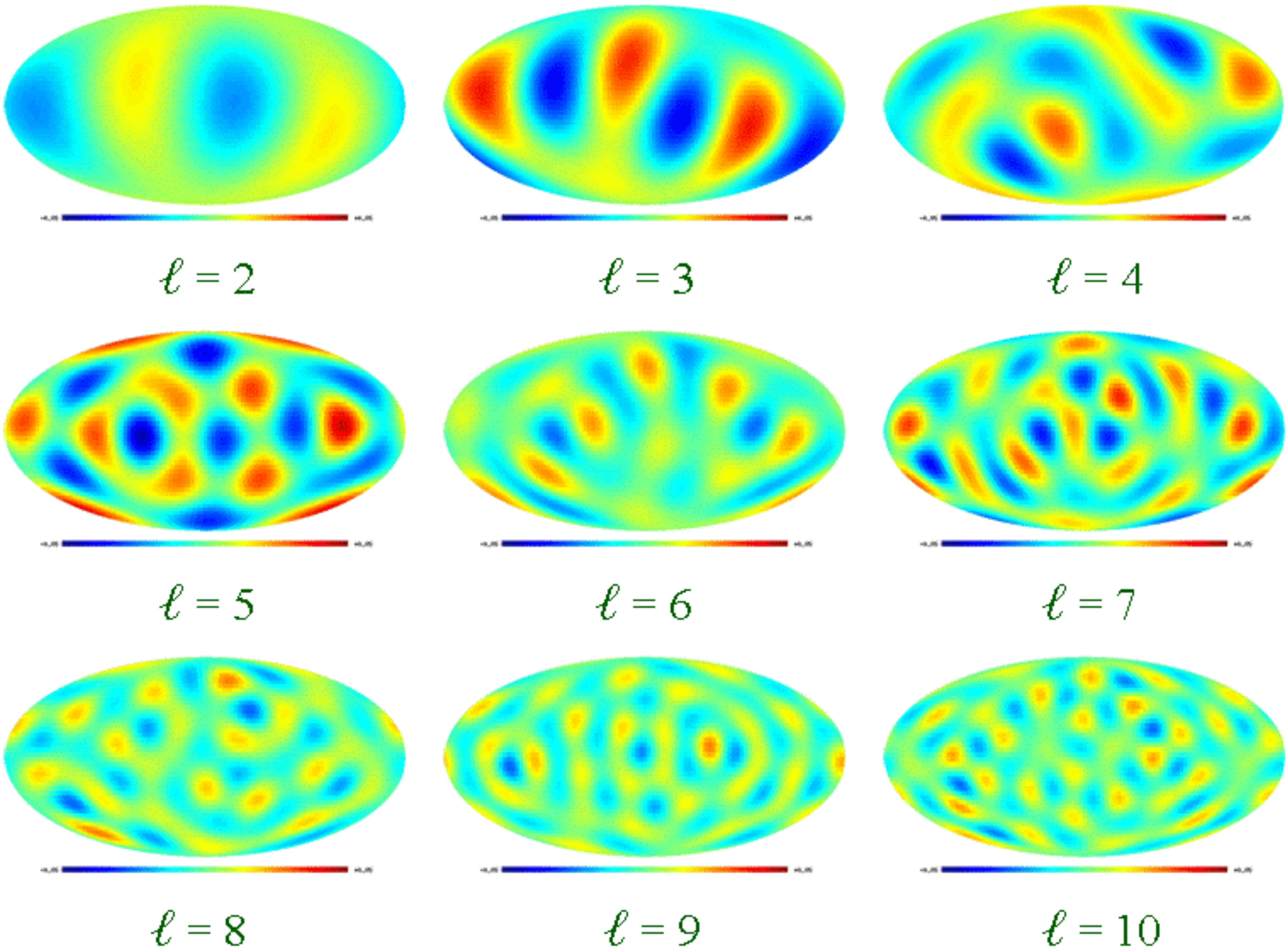


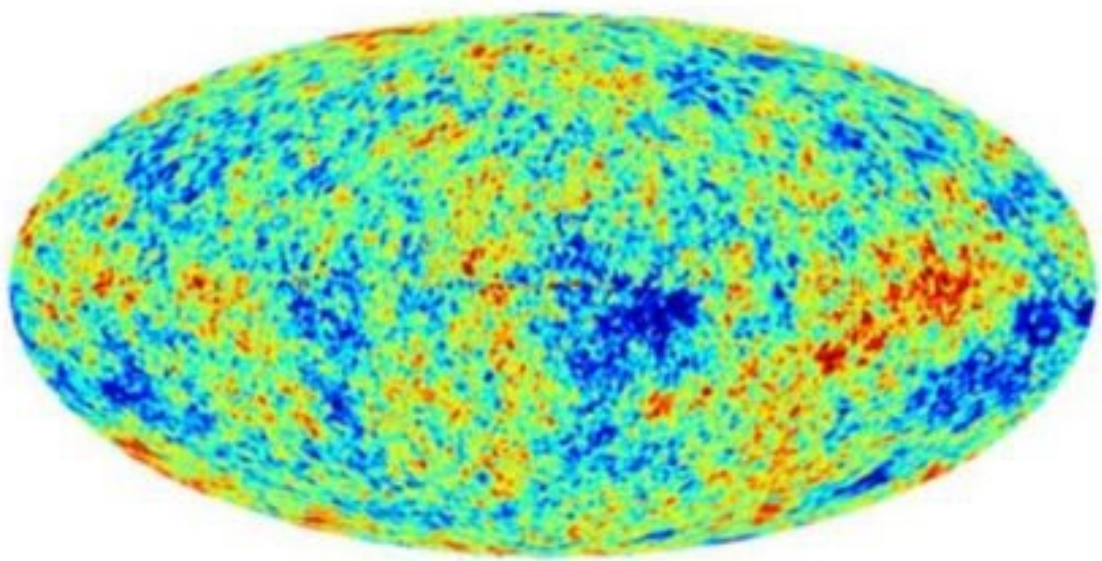


# Spherical harmonics in Mollweide projection

$m = l$



# Expressing anisotropies as sum of spherical harmonics



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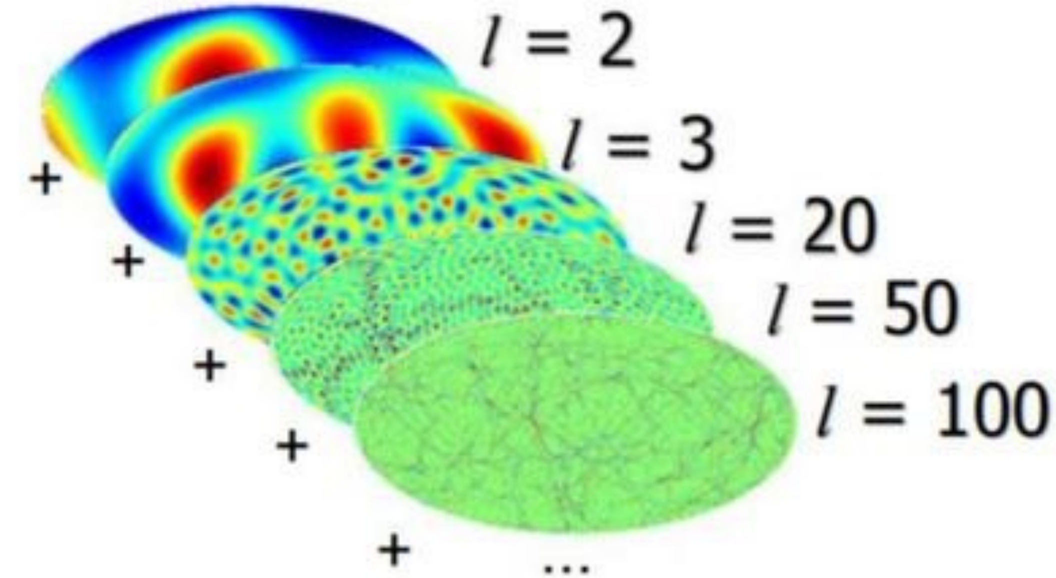


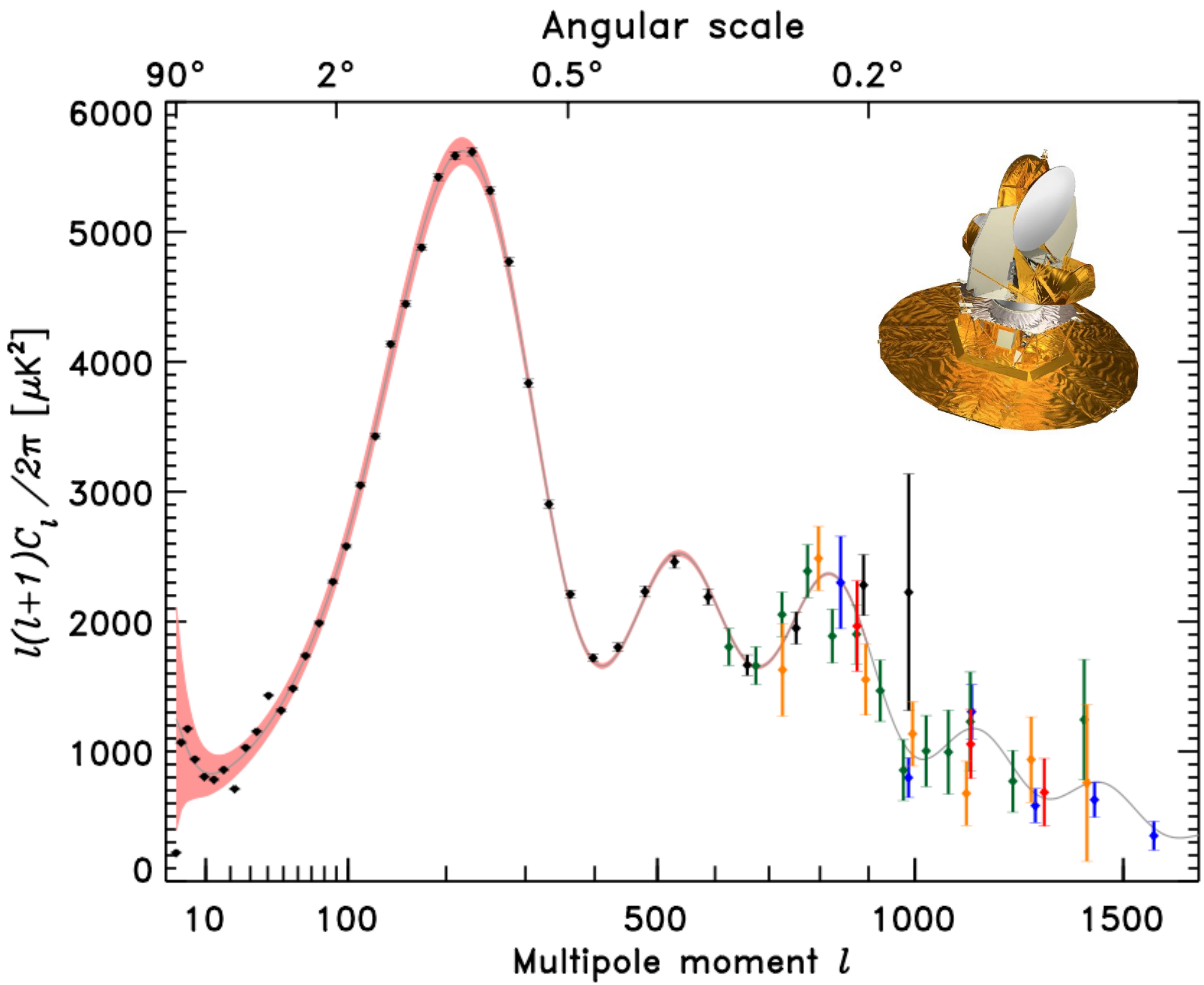

**Harmonic Decomposition:**

$$\delta_T(\theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{l,m} Y_l^m(\theta, \phi)$$

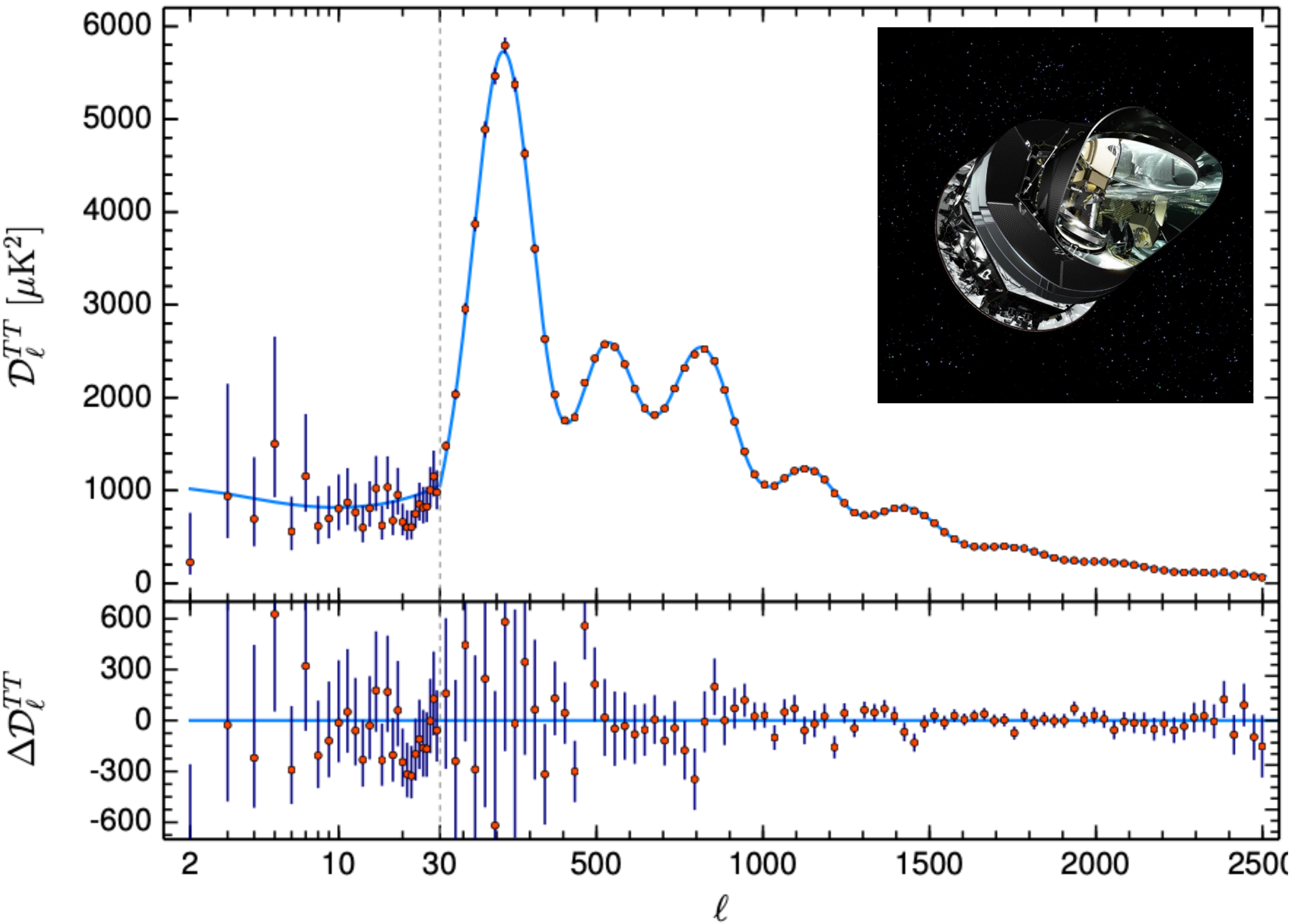
**Temp. Power Spectrum:**

$$P(l) = \frac{l(l+1)}{2\pi} C_l = \frac{l(l+1)}{2\pi} \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{l,m}|^2$$

# Power spectrum of CMB anisotropy (WMAP: launched in 2001)

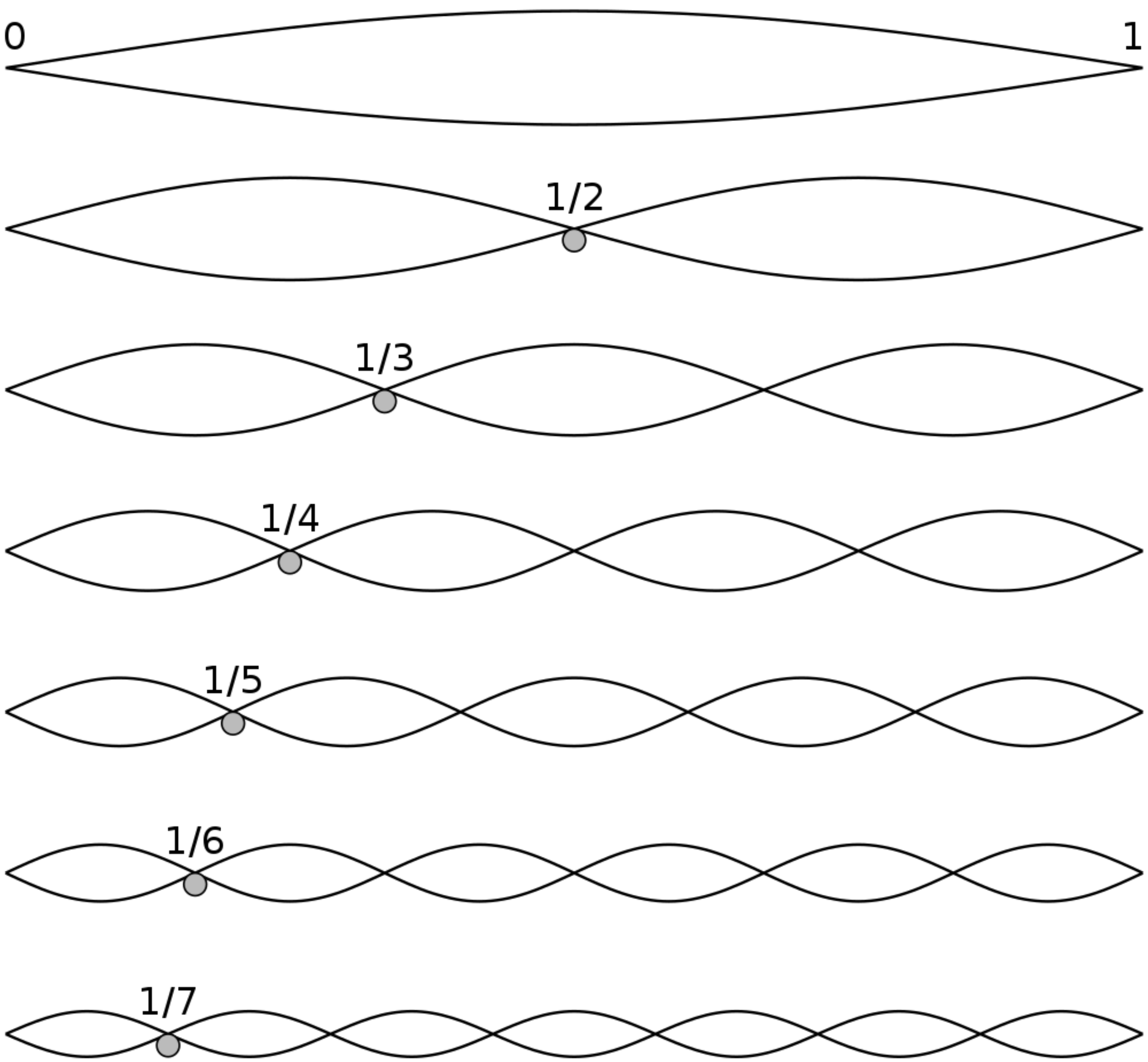


# Power spectrum of CMB anisotropy (Planck: launched in 2009)



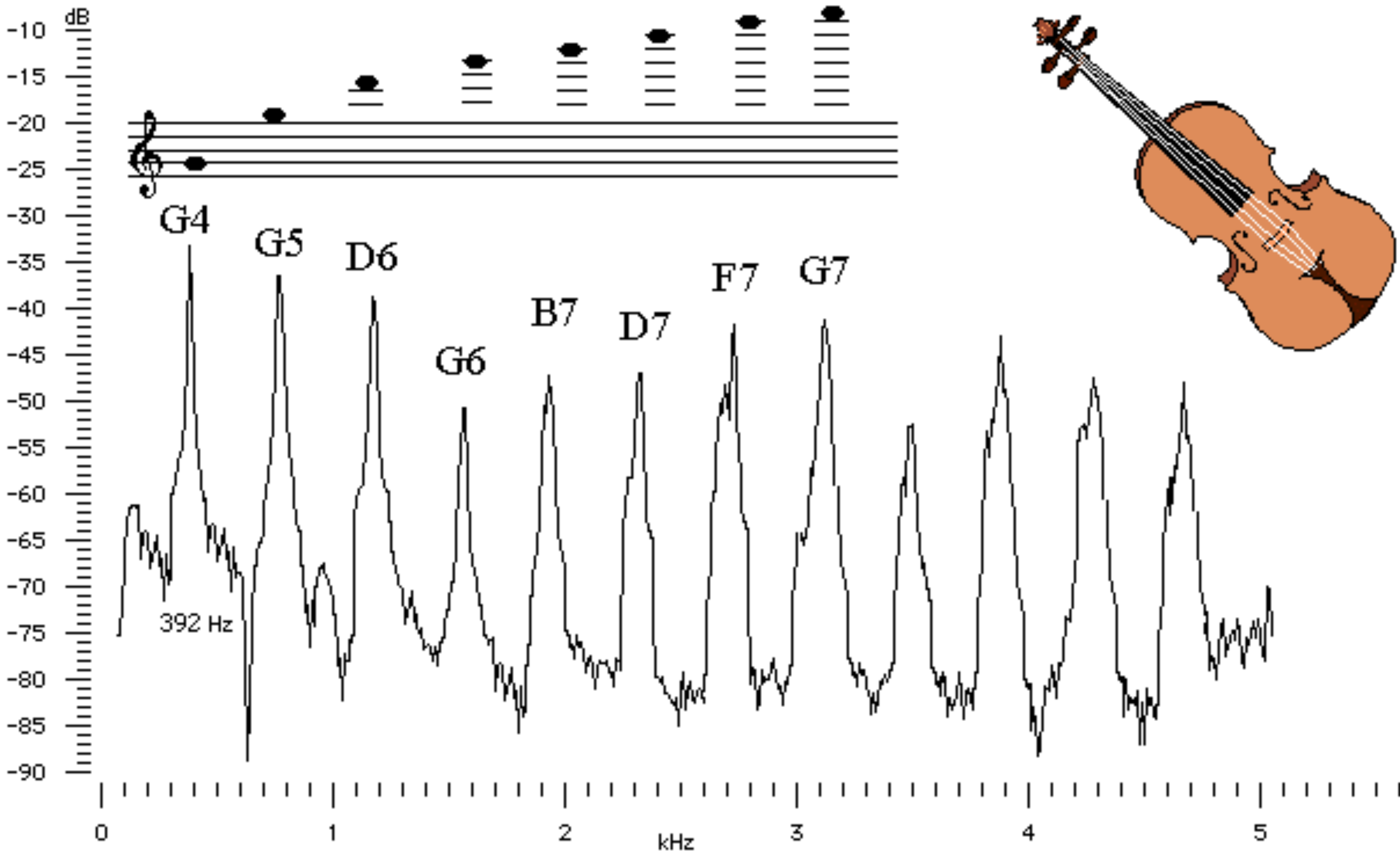
So, what are these peaks in the power spectrum?

# Harmonics of a string showing the periods of the pure-tone harmonics




acoustic resonators based on strings, which some call “music instruments”

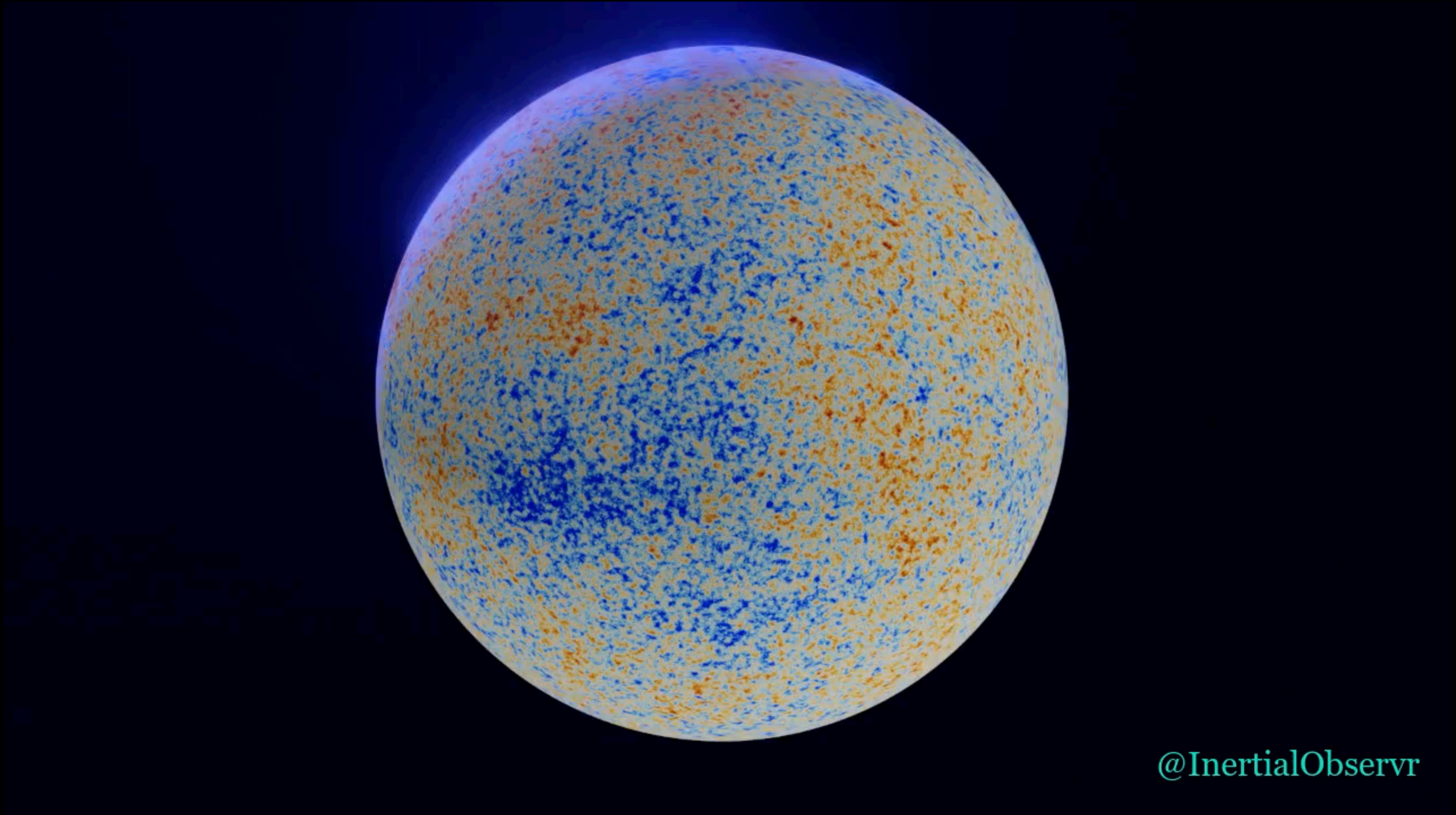
# The harmonic spectrum of a Violin



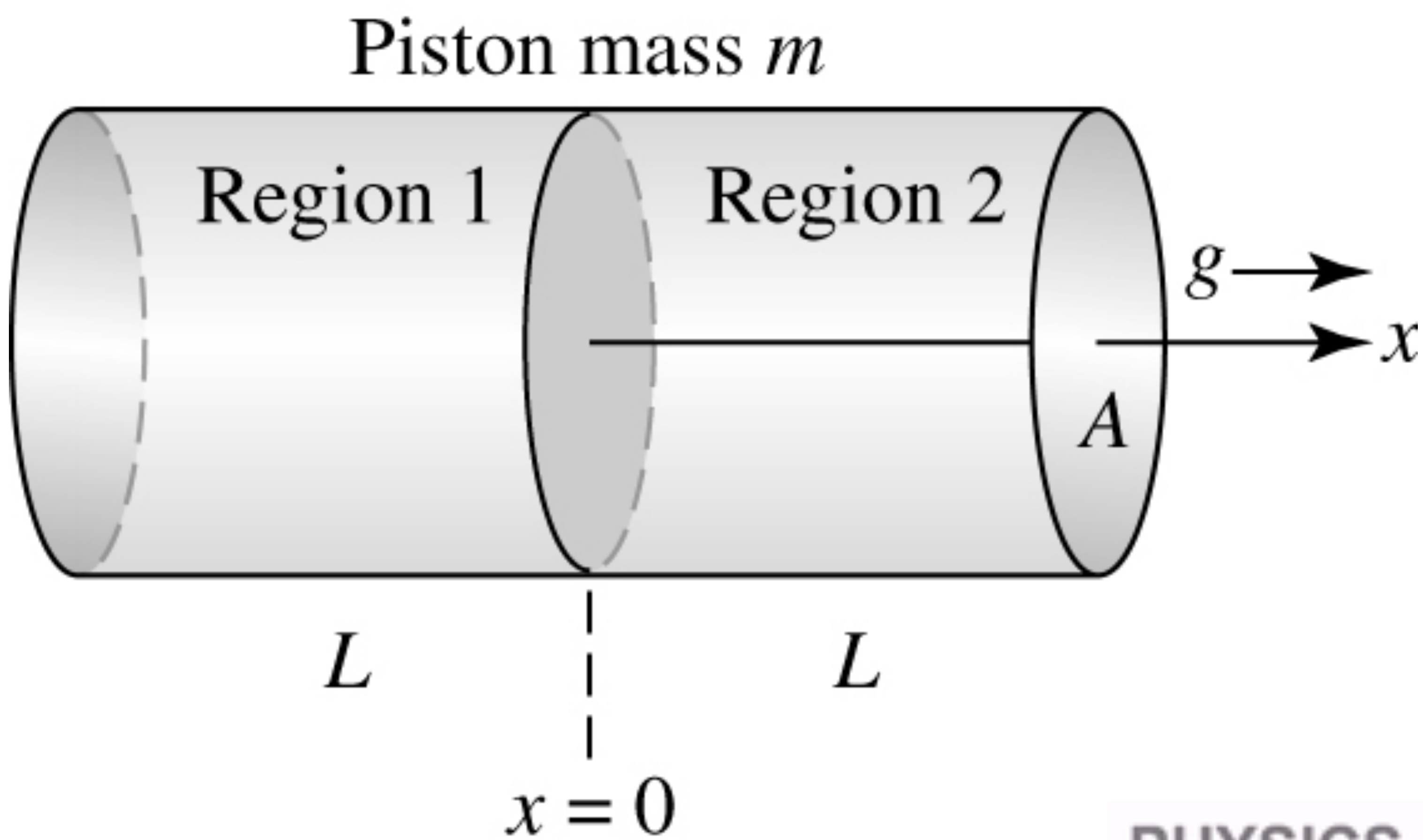

https://www.intmath.com/fourier-series/6-line-spectrum.php

# The cosmic harmonics frozen in time

*"What makes the music of heaven?" - Chuang Tzu (300 BC)*



**Because overdensities of the baryon+photon fluid cannot collapse (Jeans length > Horizon size), they undergo acoustic oscillations**




**Ideal Gas Solution**

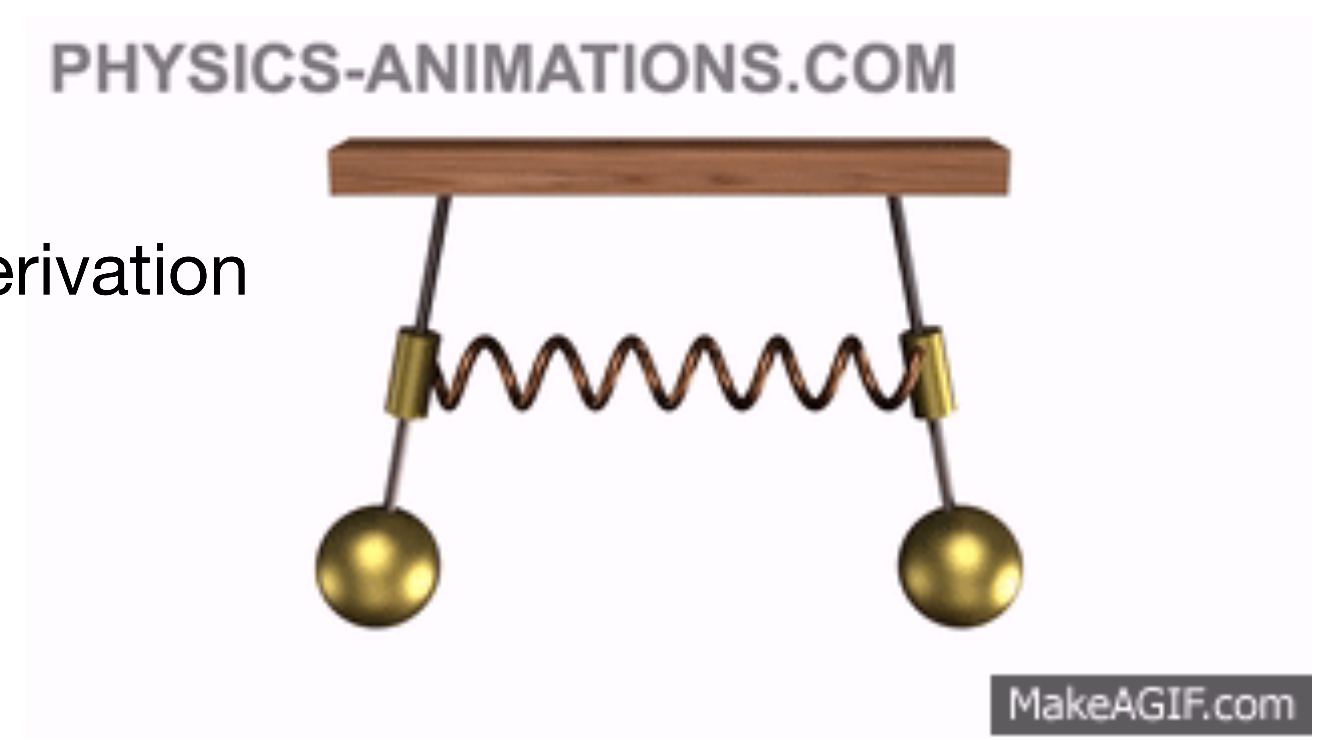
$$x(t) = x_0 \sin(c_s t/L)$$

$$\tau = 2\pi L/c_s$$

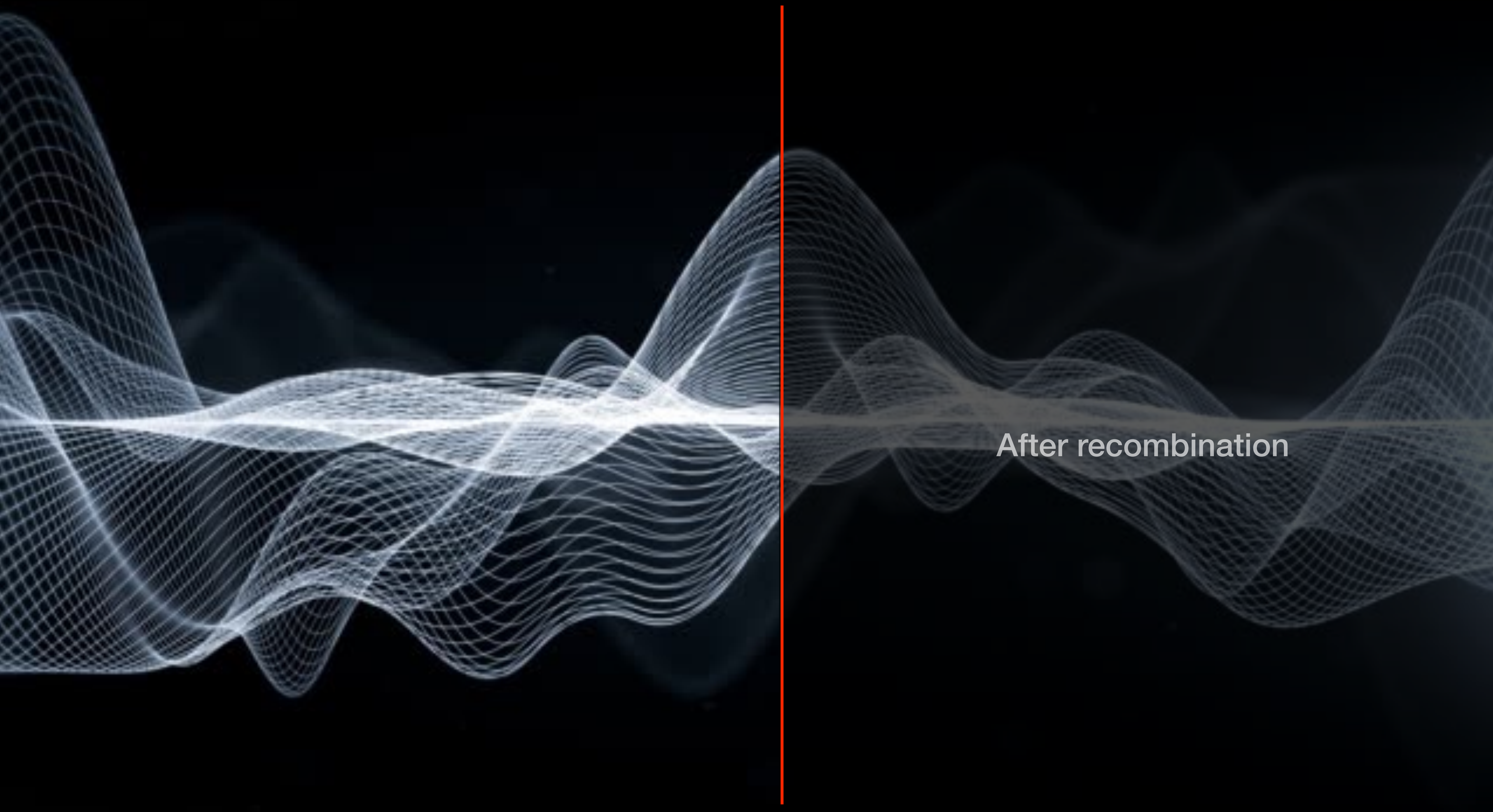
$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{\gamma k T}{\mu m_H}$$

$$\gamma = C_P/C_V$$

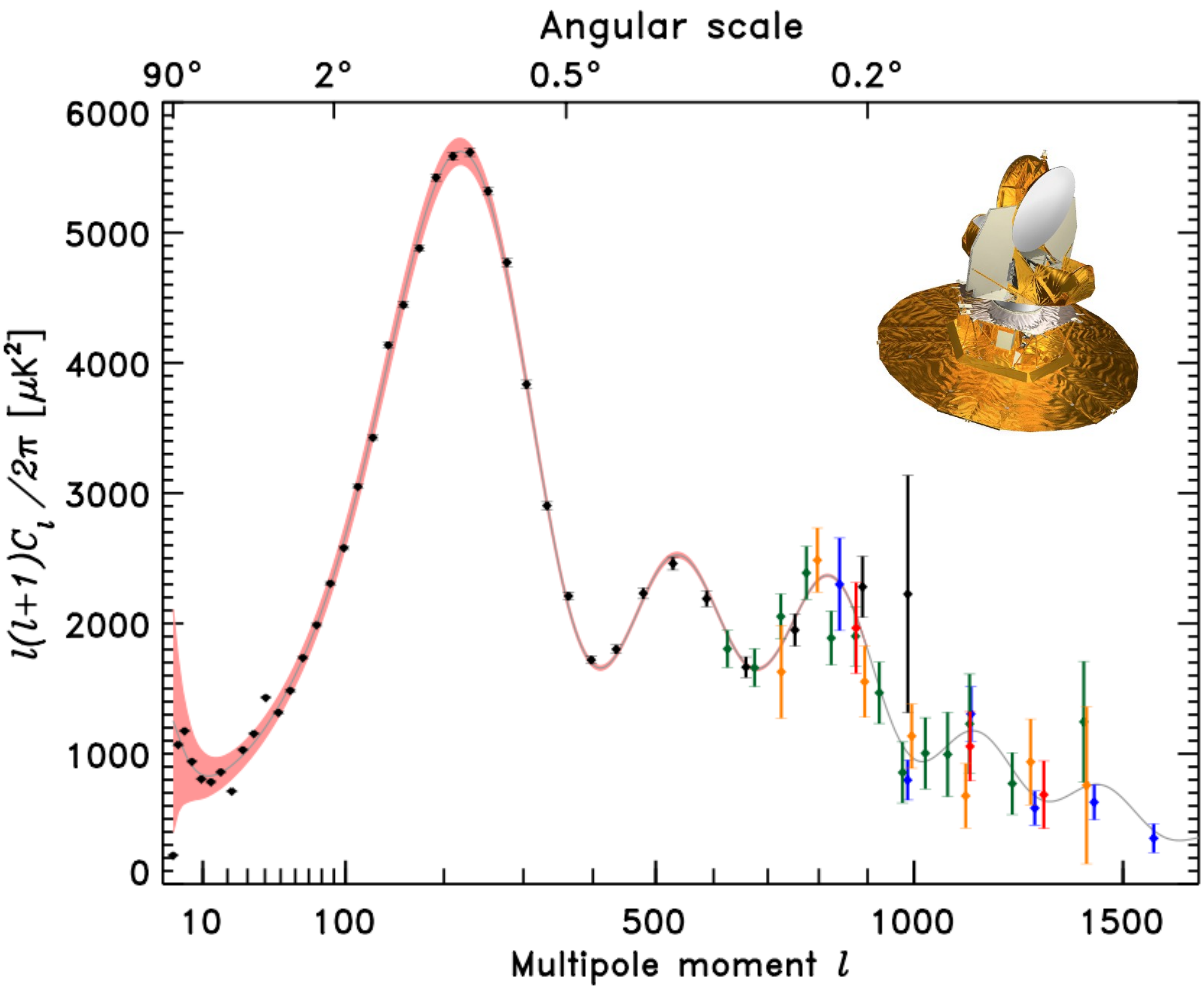
Simple gas cylinder + piston model derivation



# random acoustic oscillations frozen at recombination



# Power spectrum of CMB anisotropy (WMAP: launched in 2001)



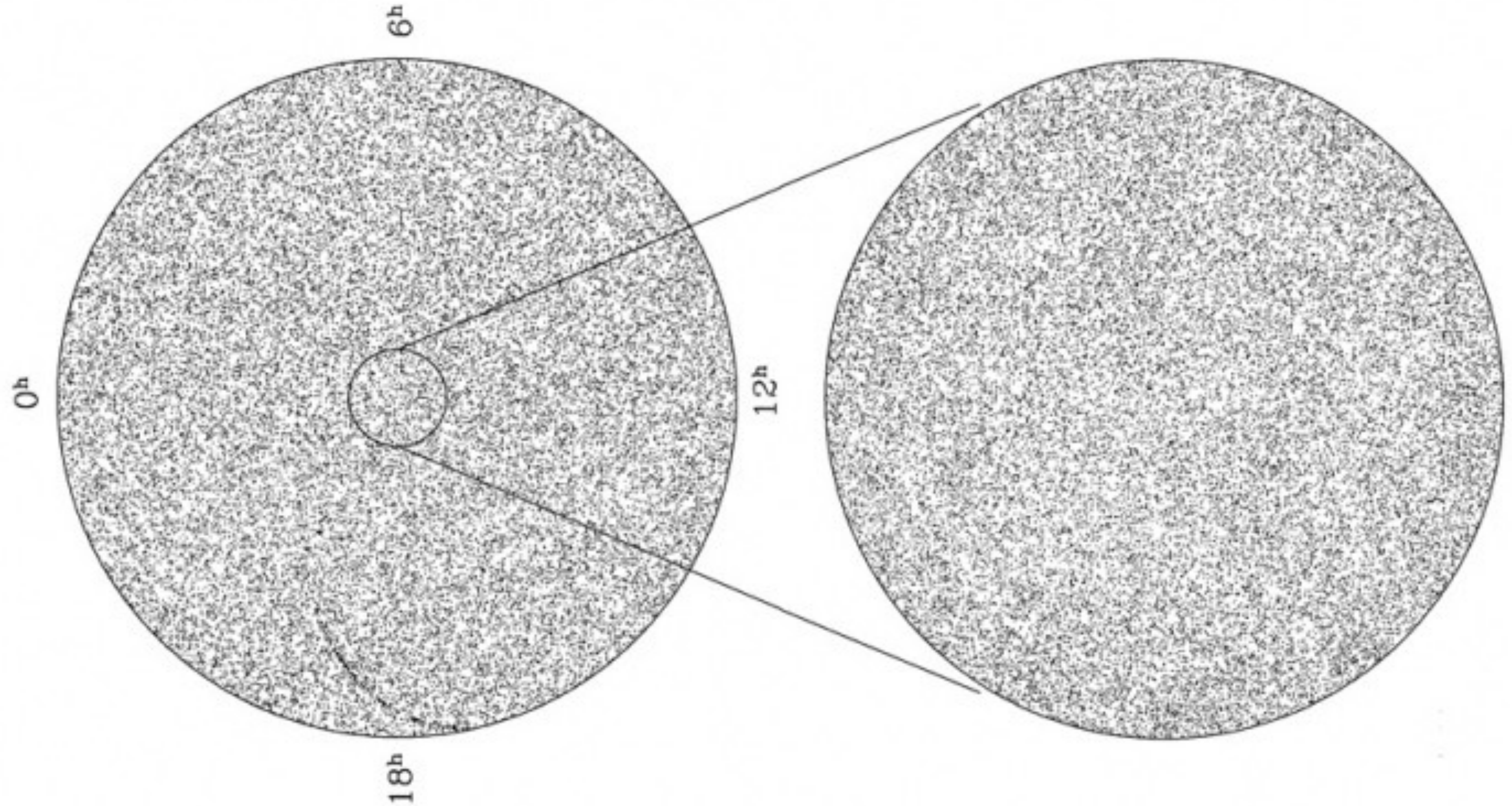
## Mathematical Description of the dynamics of the Universe:

## The Friedmann Equation (1922)

*note that this is before Hubble's law (1929)*

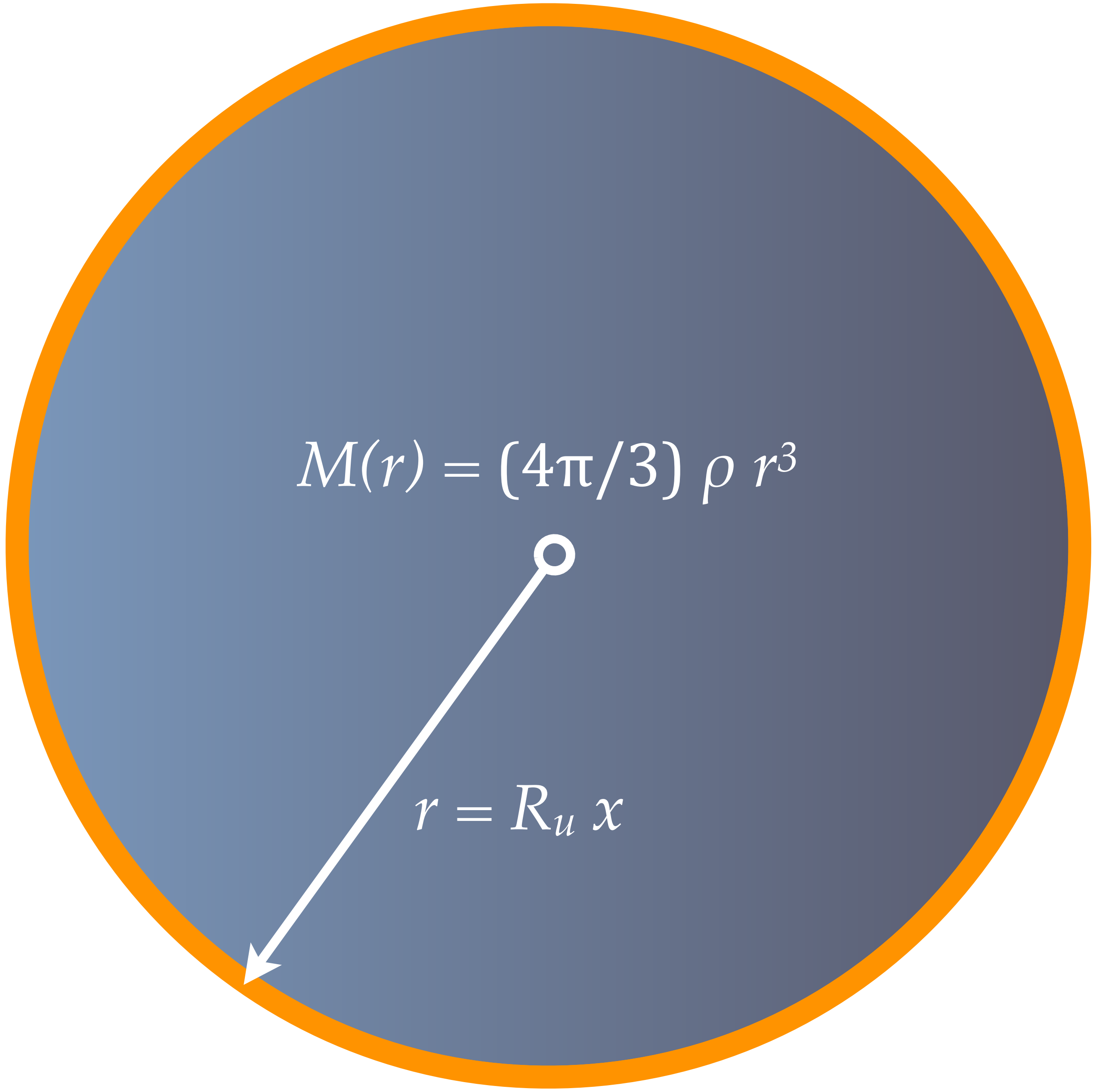
# Fundamental Assumption: The Cosmological Principle

- Although galaxies tend to clump, on the largest cosmic scales, the Universe is both **homogeneous** and **isotropic**
  - **Homogeneous:** there is no preferred **location** in the Universe
  - **Isotropic**: there is no preferred **direction** in the Universe



## Friedmann Equation: Classic Derivation based on Energy Conservation

- Imagine a **spherical shell** with **unit mass** in a **matter-only universe** with a **comoving radius** of **x**, as the universe expands:
  - its **physical radius** at time $t$ is $r(t) = R_U(t)\, x$,
  - its **expanding velocity** is $v(t) = \dot{R}_U(t) \cdot x$, and
  - the **mass enclosed** in the shell is
    $$M(r) = \frac{4\pi}{3}[R_U(t)x]^3 \cdot \rho(t)$$



## Friedmann Equation: Classic Derivation based on Energy Conservation

- Imagine a **spherical shell** with **unit mass** in a **matter-only universe** with a **comoving radius** of x, as the universe expands:
  - its **physical radius** at time *t* is $r(t) = R_U(t)\, x$,
  - its **expanding velocity** is $v(t) = \dot{R}_U(t) \cdot x$, and
  - the **mass enclosed** in the shell is $M(r) = \frac{4\pi}{3}[R_U(t)x]^3 \cdot \rho(t)$
- We can write down the **kinetic + gravitational potential energy** for the unit-mass spherical shell:

$$E = \frac{1}{2}v^2 - \frac{GM(r)}{r} = \frac{1}{2}\dot{R}_U(t)^2 x^2 - \frac{4\pi}{3}G\rho(t)R_U(t)^2 x^2$$

- This **energy per unit mass** must be the same for every shell with the same radius **x**, so we can define *E* with a *k* parameter:

$$E \equiv -\frac{1}{2}kc^2x^2$$

- Replacing E in the previous energy equation, we obtain:

$$\left(\frac{\dot{R}_U^{\,2}}{R_U^2} - \frac{8}{3}\pi G\rho\right) R_U^2 = -kc^2$$

## Friedmann Equation: Classic Derivation based on Energy Conservation

- Recall the definition of the **Hubble parameter**:

$$H(t) \equiv \dot{R}_U / R_U$$

- define a new parameter called **critical density**:

$$\rho_c = \frac{3H^2}{8\pi G}$$

note that because *H* varies, the critical density is not a constant.

- we can now rewrite the energy conservation

$$\left( \frac{\dot{R}_U^{\,2}}{R_U^2} - \frac{8}{3}\pi G \rho \right) R_U^2 = -\,kc^2$$

as:

$$H^2 \left( 1 - \frac{\rho}{\rho_c} \right) R_U^2 = -\,kc^2$$

- next, redefine the density ratio as a dimensionless **density parameter** called **Omega**:

$$\Omega_m \equiv \rho_m / \rho_c$$

- Finally, we have the **Friedmann equation** in a **matter-only universe**:

$$H^2 \; (1 - \Omega_m) \; R_U^2 = -\,kc^2$$

# Calculating the Critical Density Today (Working It Out 22.1)

- The value of the critical density, $\rho_c$, today is a key parameter in Alexander Friedmann's equation:

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

- If we rewrite $H_0 = 70$ km/s/Mpc as $H_0 = 2.3 \times 10^{-18}$/s by converting Mpc to km, then the critical density of Today's universe is:

$$\rho_c = \frac{3 \times \left(2.3 \times 10^{-18}/\text{s}\right)^2}{8 \times \pi \times \left[6.67 \times 10^{-20}\text{km}^3/\left(\text{kg s}^2\right)\right]}$$

$$\rho_c = 9.5 \times 10^{-27}\text{kg/m}^3$$

- This is equal to about **5.7 hydrogen atoms per cubic meter**.
- It seems small, but the observed mass density of **ordinary matter**, *averaged over large volumes*, is less than **one hydrogen atom per cubic meter**.

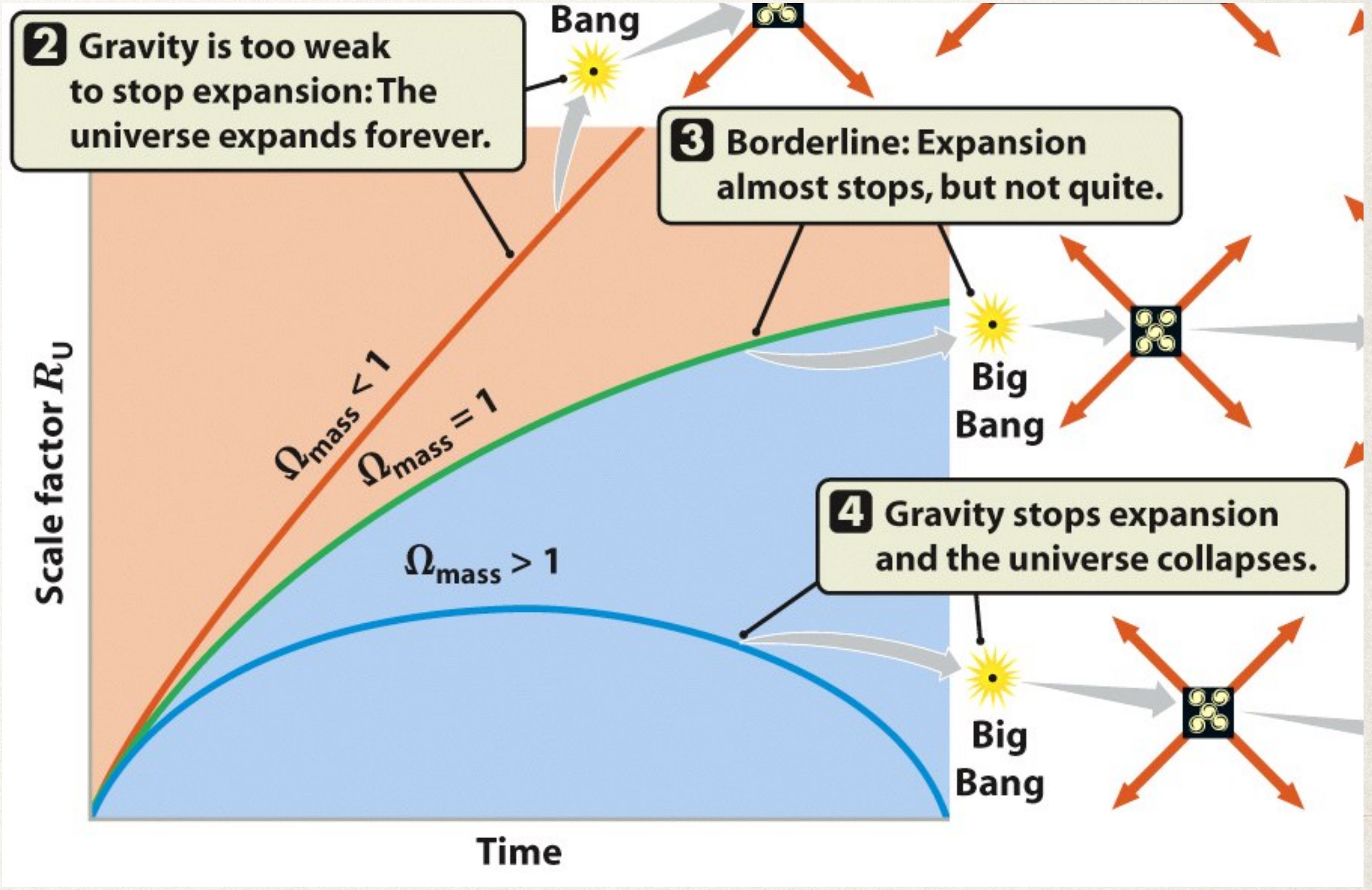
# Expansion histories predicted by Friedmann Equation: Part I

matter-only universe

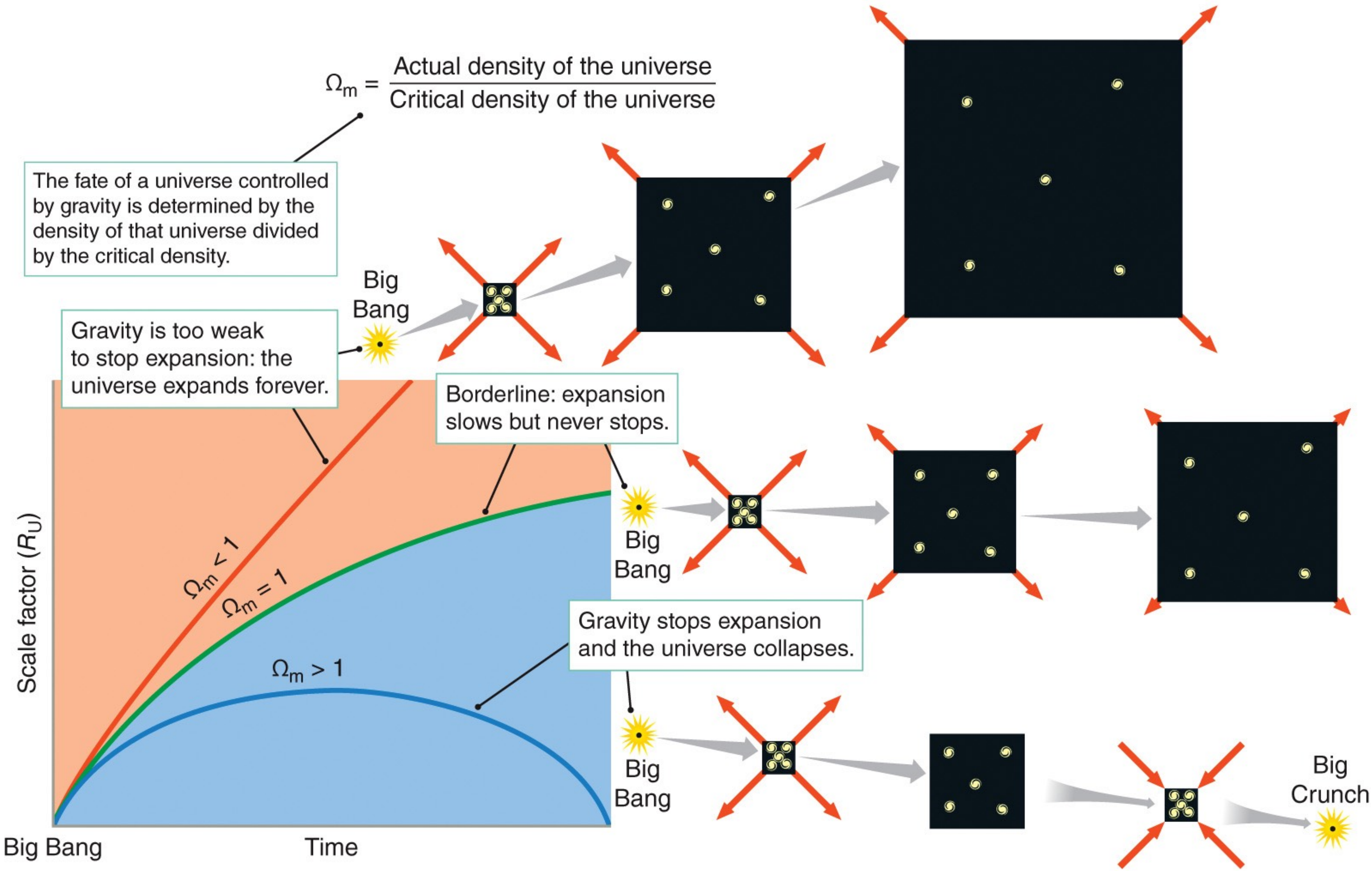
# The fate of a **matter-only** universe:

$$\rho_{c,0} = 3\,H_0^2/(8\pi G) \sim 10^{-26}\ kg/m^3$$

- The evolution depends on $\Omega_m \equiv \rho/\rho_{critical}$
- $\Omega_m < 1$: *expanding forever.*
- $\Omega_m = 1$: *expanding forever, but expansion speed approaches zero as time goes.*
- $\Omega_m > 1$: *expansion stops and the universe collapse.* ***(Big Crunch)***

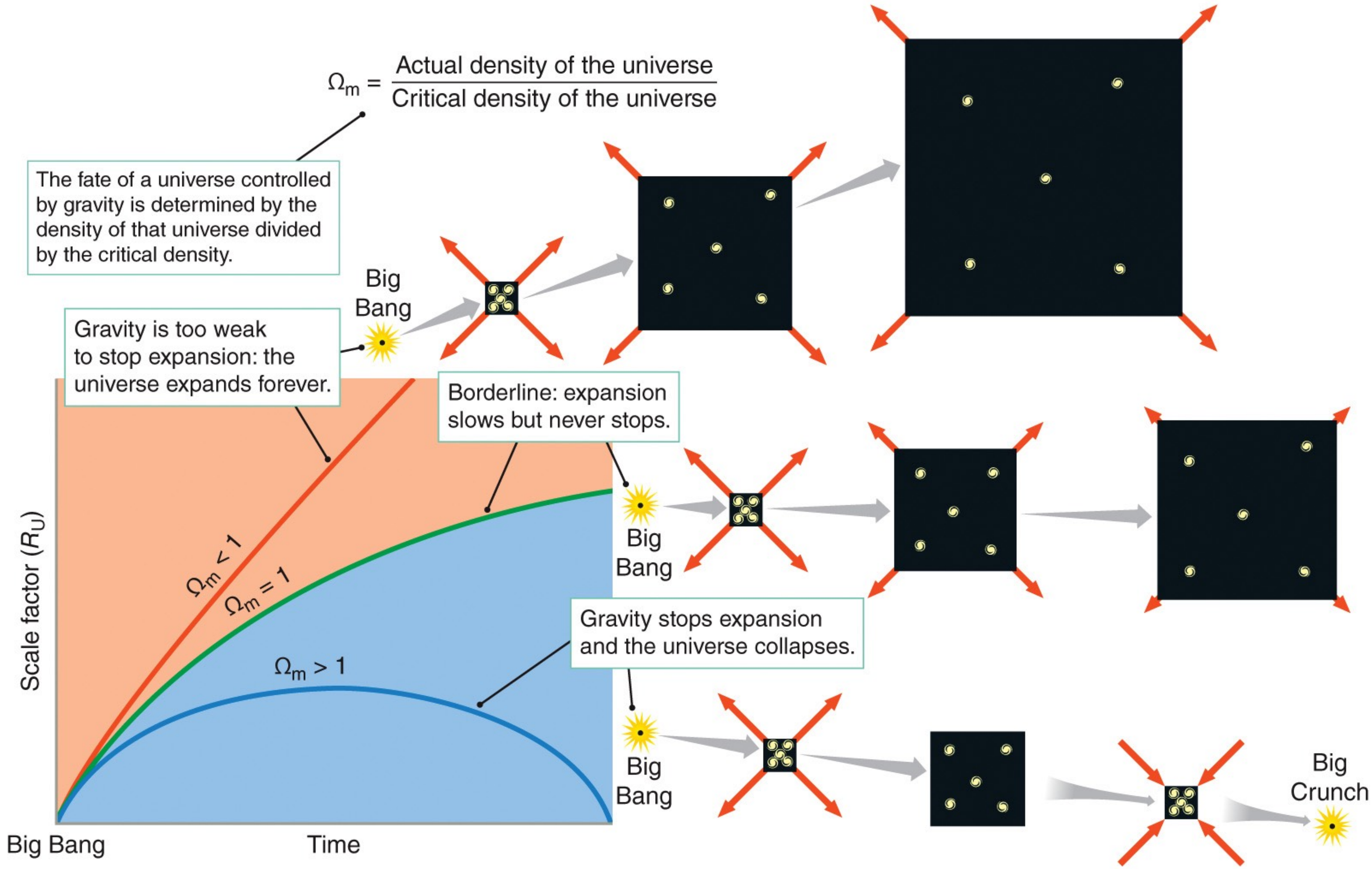


# Expansion History if Only Matter Is Involved



# The Total Mass in the Universe Today

- In ordinary stars and galaxies, $\Omega_m = 0.02$ (today).
- Dark matter in and between galaxies increases $\Omega_m$ to 0.3 (today).



# Expansion histories predicted by Friedmann Equation - Part II

## matter & dark energy universe

## The Complete Friedmann Equation with matter and dark energy

- By defining a new parameter called **critical density**: $\rho_c = \frac{3H^2}{8\pi G}$, we have derived the Friedmann Equation for matter-only universe:

$$H^2 \left(1 - \frac{\rho}{\rho_c}\right) R_U^2 = -kc^2$$

- The full GR version of the Friedmann Equation (1922) is:

$$H^2 \left[1 - \left(\frac{\rho_m}{\rho_c} + \frac{\rho_\gamma}{\rho_c} + \frac{\Lambda c^2}{8\pi G \rho_c}\right)\right] R_U^2 = -kc^2$$

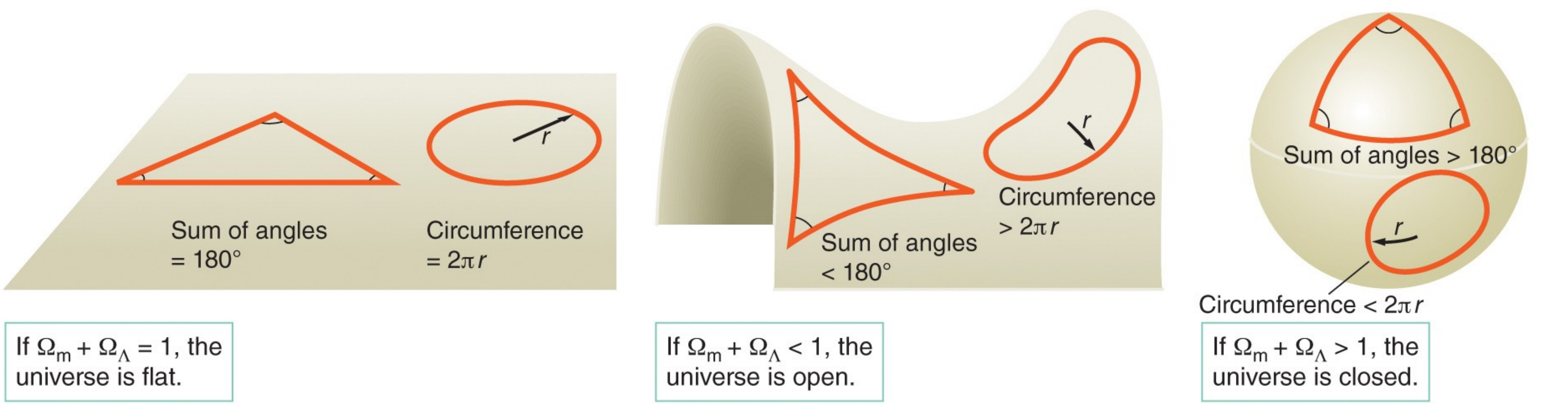
- There are now three density ratios, i.e., define three Omega's:
  - $\Omega_m \equiv \rho_m/\rho_c$ , **ordinary matter** (baryons and dark matter)
  - $\Omega_\gamma \equiv \rho_\gamma/\rho_c$, **relativistic matter** (light and neutrinos)
  - $\Omega_\Lambda \equiv \Lambda c^2/(8\pi G\rho_c)$, **cosmological constant** (dark energy)

- Replacing those, we have the final Friedmann Equation:

$$H^2 \left[1 - (\Omega_m + \Omega_\gamma + \Omega_\Lambda)\right] R_U^2 = -kc^2$$

# The curvature parameter *k* in Friedmann Equation

- The universe has three possible geometry types:
  - evaluating Friedmann Equation with the boundary condition today, we have: $k = -H_0^2(1-\Omega_0)/c^2$
  - **$k = 0$ ($\Omega_0 = 1$): Flat universe, infinite.**
  - $k < 0$ ($\Omega_0 < 1$): Open universe, infinite, like the surface of a saddle.
  - $k > 0$ ($\Omega_0 > 1$): Closed universe, finite, like the surface of a sphere.




**a.** Flat geometry **b.** Open (saddle) geometry **c.** Closed (spherical) geometry

# How to Solve the Friedmann Equation? H(z) solution

- **Boundary Condition at t = $t_0$:**

  H = $H_0$, $R_U$ = 1, thus $H_0^2(1-\Omega_0) = -kc^2$

- **Relations between density parameters and scale factor**:

$$\frac{\Omega_m}{\Omega_{m,0}} = \frac{\rho_m \rho_{c,0}}{\rho_{m,0}\rho_c} = \frac{\rho_m}{\rho_{m,0}}\frac{H_0^2}{H^2} = \frac{1}{R_U^3}\frac{H_0^2}{H^2}$$

$$\frac{\Omega_\gamma}{\Omega_{\gamma,0}} = \frac{1}{R_U^4}\frac{H_0^2}{H^2} \quad \text{and} \quad \frac{\Omega_\Lambda}{\Omega_{\Lambda,0}} = \frac{H_0^2}{H^2}$$

- Write down the Friedmann Equation with the boundary condition:

$$H^2\,[1-(\Omega_m+\Omega_\gamma+\Omega_\Lambda)]\,R_U^2 = -kc^2 = H_0^2(1-\Omega_0)$$

  then plug in the density parameter relations and rearrange:

$$H^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_0)+\Omega_{m,0}/R_U+\Omega_{\gamma,0}/R_U^2+\Omega_{\Lambda,0}R_U^2]$$

- Examples:
  - For **an empty universe**:

$$\Omega_0 = \Omega_{m,0} = \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0 \Rightarrow H = H_0/R_U = H_0(1+z)$$

  - For **a matter-only flat universe (Einstein-de Sitter universe)**:

$$\Omega_0 = \Omega_{m,0} = 1, \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0 \Rightarrow H = H_0/R_U^{3/2} = H_0(1+z)^{3/2}$$

# How to Solve the Friedmann Equation? t(z) or $R_u$(t) solution

- Write down the Friedmann Equation with the boundary condition and replace Hubble parameter with scale factor, $H(t) \equiv \dot{R}_U/R_U$, we have

$$\left(\frac{1}{R_U}\frac{dR_U}{dt}\right)^2 = \frac{H_0^2}{R_U^2}[(1-\Omega_0) + \Omega_{m,0}/R_U + \Omega_{\gamma,0}/R_U^2 + \Omega_{\Lambda,0}R_U^2]$$

- For simplicity, assume a **flat universe:** *k = 0 and* $\Omega_0 = 1$ *at all times.*
- Separate time and scale factor into two sides of the equation:

$$dt = \frac{1}{H_0}\frac{R_U dR_U}{\sqrt{\Omega_{m,0}R_U + \Omega_{\gamma,0} + \Omega_{\Lambda,0}R_U^4}}$$

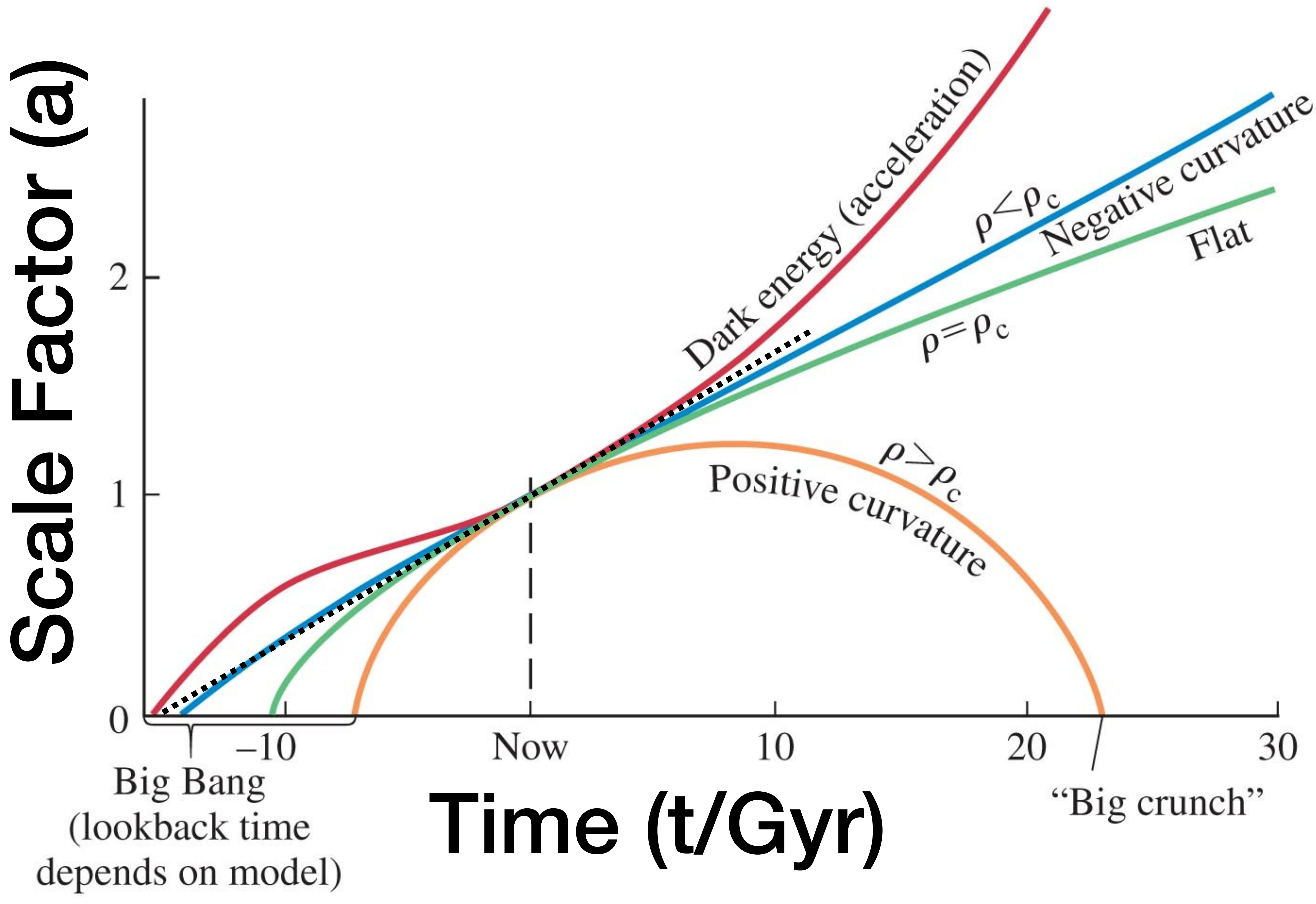
- Integrating it from $R_U$=0 (i.e., t=0) to $R_U$ = 1/(1+z) [i.e., t(z)], we can solve for the t(z) relation for *any given values of the density parameters*.
- For example, for **a matter-only flat universe (Einstein-de Sitter universe),** we have solved for both H(z) and t(z):

$$\Omega_0 = \Omega_{m,0} = 1, \Omega_{\gamma,0} = \Omega_{\Lambda,0} = 0$$

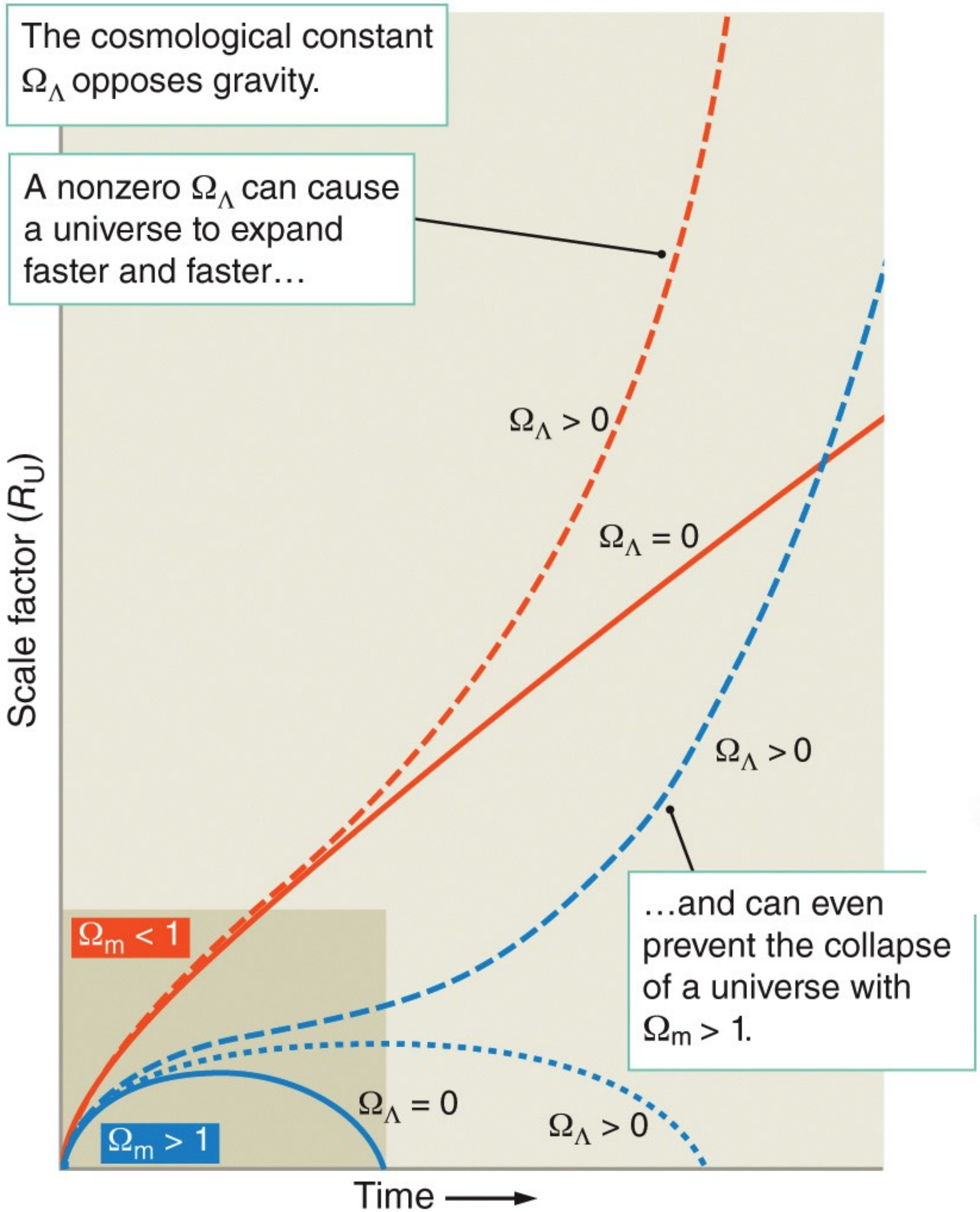
$$\Rightarrow H = H_0/R_U^{3/2} = H_0(1+z)^{3/2}$$

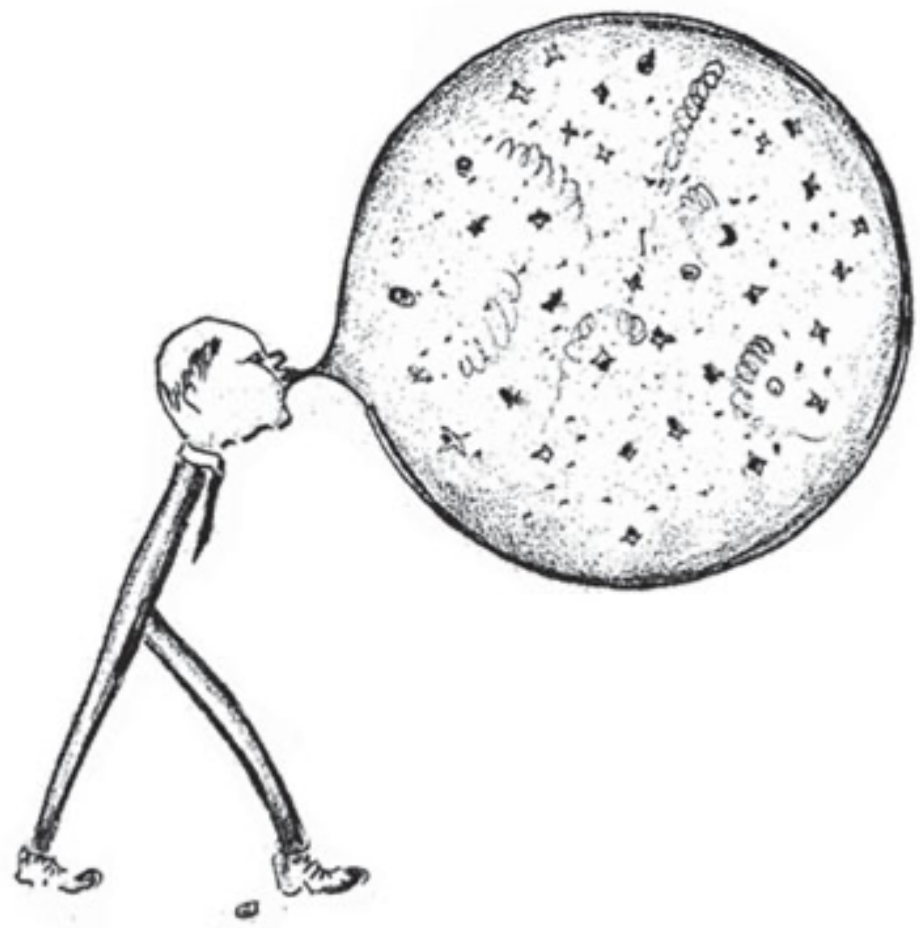
$$\Rightarrow t(z) = \frac{2}{3}t_H(1+z)^{-3/2}$$

# $R_u(t)$: The Expansion History



# $R_u(t)$ solutions: Expansion history with the cosmological constant



$\Lambda$ as the source of cosmic expansion: De Sitter in 1930

# Constraints on Cosmological Parameters:

# distance-redshift relation up to $z \sim 1$

# Robertson-Walker Metric: Distance Measurements

- A **metric** is a function which measures ***differential space-time distance*** between two points. It is defined to be **Lorentzian invariant**.
- The **Robertson-Walker metric** is the metric that describes the geometry of a **homogeneous, isotropic, expanding** universe. In spherical coordinate system:

$$(ds)^2 = (c \cdot dt)^2 - R_U^2(t)\left[\left(\frac{dr_c}{\sqrt{1 - kr_c^2}}\right)^2 + (r_c d\theta)^2 + (r_c \sin\theta d\phi)^2\right]$$

where $R_U$ is the scale factor, $r_c$ is the comoving radial distance, $k$ is the time-independent curvature $k \equiv K(t)R_U(t)^2$. For **flat universe, $k$ = 0.**

  - Light travels along **null geodesics (ds = 0)**, so for photons traveling along the radial direction in a flat universe, we have:

$$dr_c = \frac{c\ dt}{R_U(t)}$$

  following the path of light by integrating this equation gives us the **scale-factor-redshift relation**.

  - **proper distance** (dt = 0; simultaneous measurements) along **radial direction** ($d\theta = d\phi = 0$):

$$D_P = \int \sqrt{-(ds)^2} = \int_0^{r_c} R_U(t) dr_c = R_U(t) r_c$$

  which is simply the **comoving** radial distance multiplied by the scale factor.

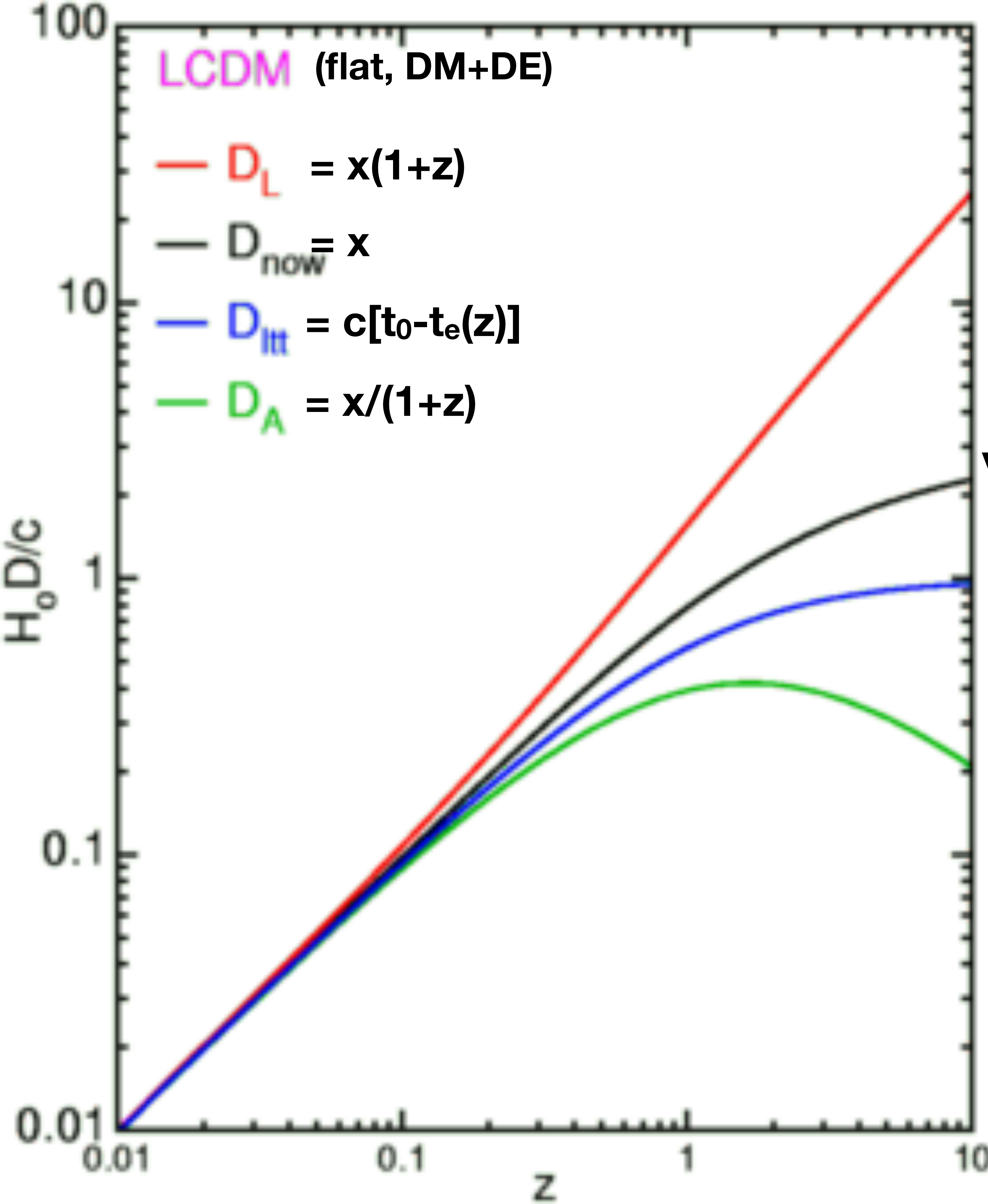
# Geometry is distorted in an expanding Universe

- **Any given object can have many different distances, and they do not equal!**
  - Comoving distance: $D_C = \int_{t_{\rm em}}^{t_{\rm obs}} \frac{c}{R_U(t)} dt = c \int_0^z \frac{dz}{H(z)}$
  - Luminosity distance: $D_L = \sqrt{L/(4\pi F)} = D_C \cdot (1+z)$
  - Angular size distance: $D_A = l/\theta = D_C/(1+z)$
  - Light travel distance: $D_{ltt} = c[t_0 - t(z)]$

https://www.astro.ucla.edu/~wright/CosmoCalc.html

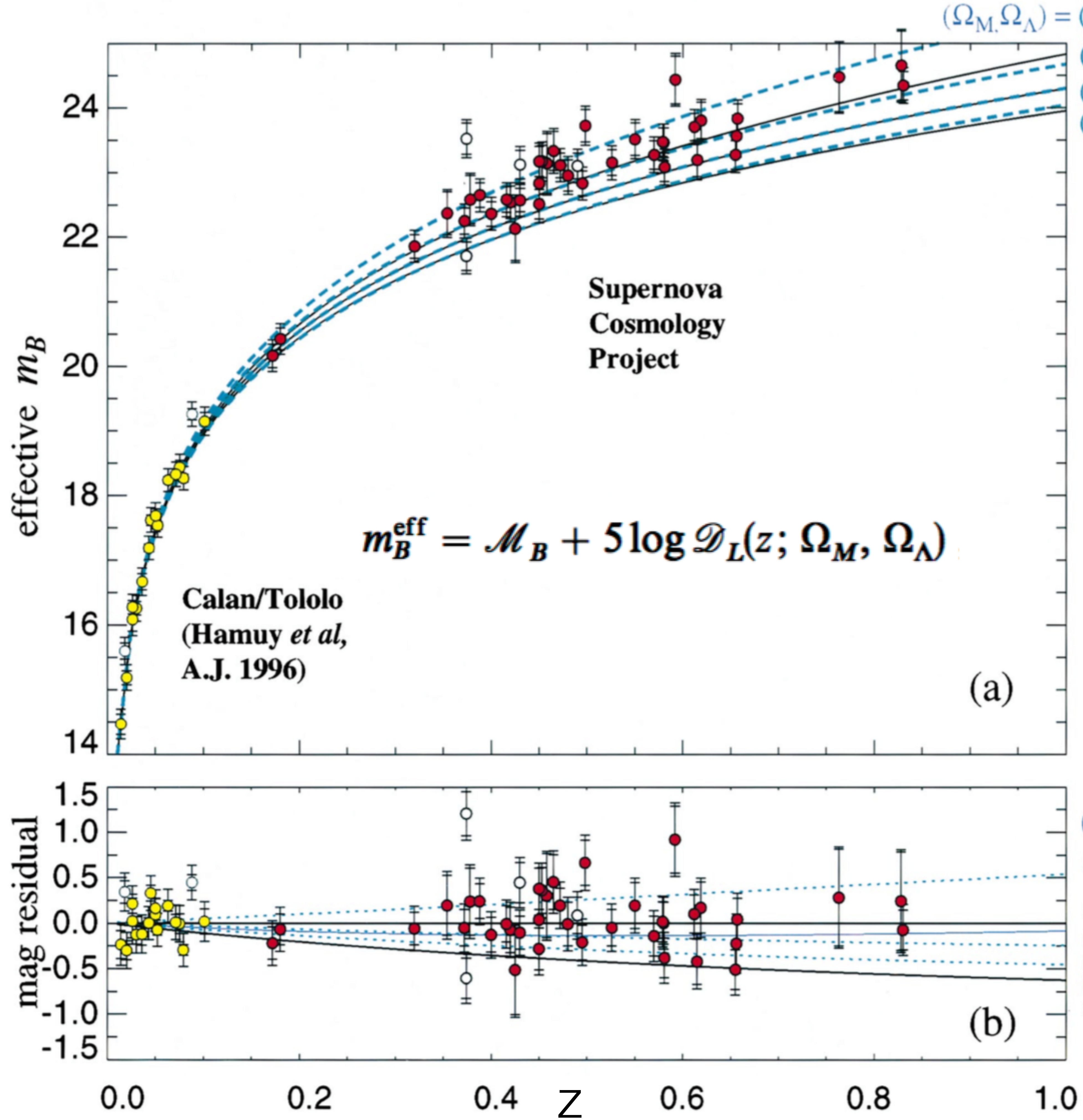
For $H_o$ = 69.6, $\text{Omega}_M$ = 0.286, $\text{Omega}_{vac}$ = 0.714, z = 2.000

- It is now 13.721 Gyr since the Big Bang.
- The age at redshift z was 3.316 Gyr.
- The light travel time was 10.404 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 5273.0 Mpc or 17.198 Gly.
- The comoving volume within redshift z is 614.103 $\text{Gpc}^3$.
- The angular size distance $D_A$ is 1757.6 Mpc or 5.7326 Gly.
- This gives a scale of 8.521 kpc/".
- The luminosity distance $D_L$ is 15818.5 Mpc or 51.594 Gly.




# various distance vs. redshift

**Distance in unit of the Hubble scale: $D_H = c/H_0 = 3\ h^{-1}$ Gpc**





**Nobel Prize in Physics 2011**

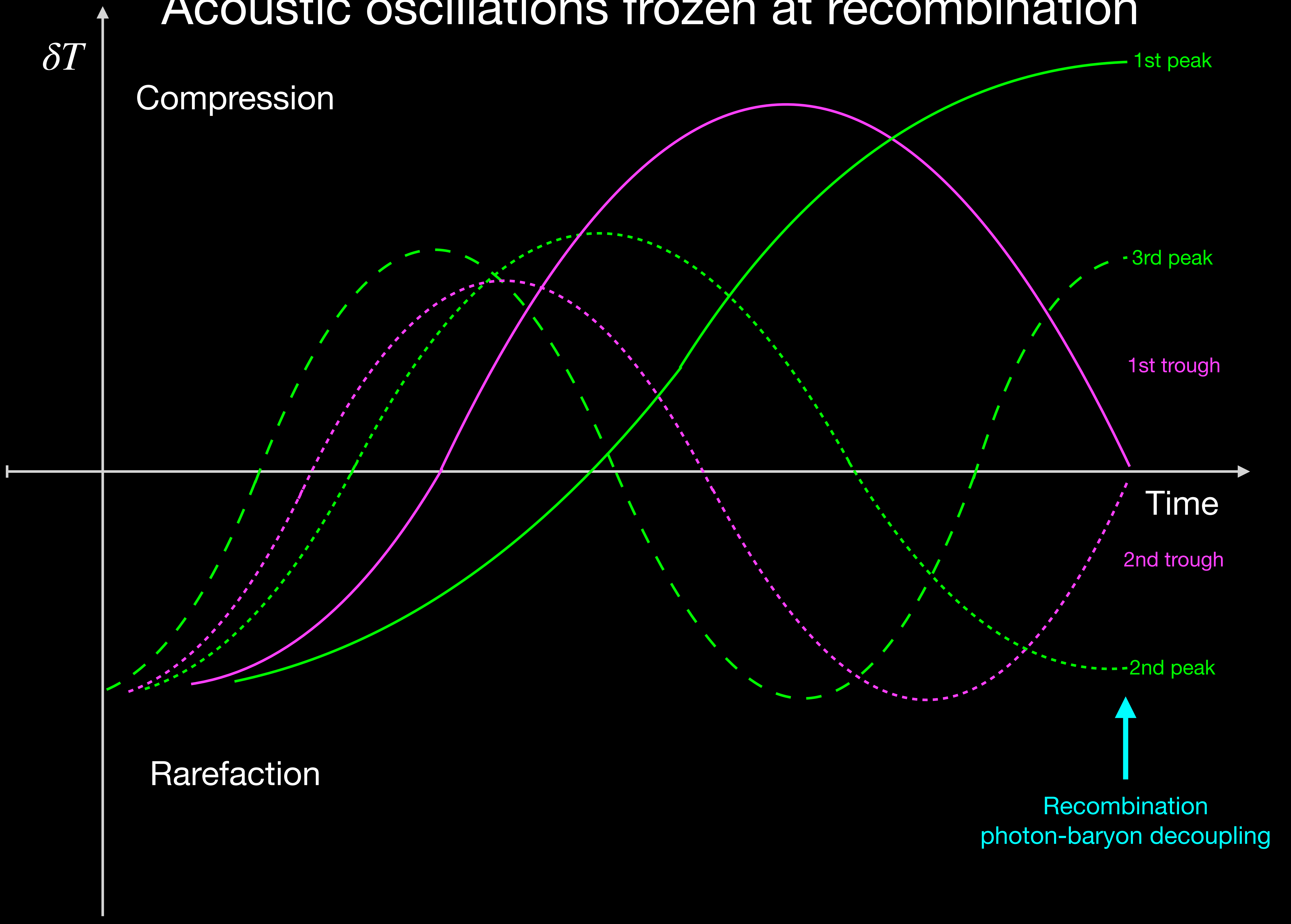
Riess+1998 AJ

Perlmutter+1999 ApJ

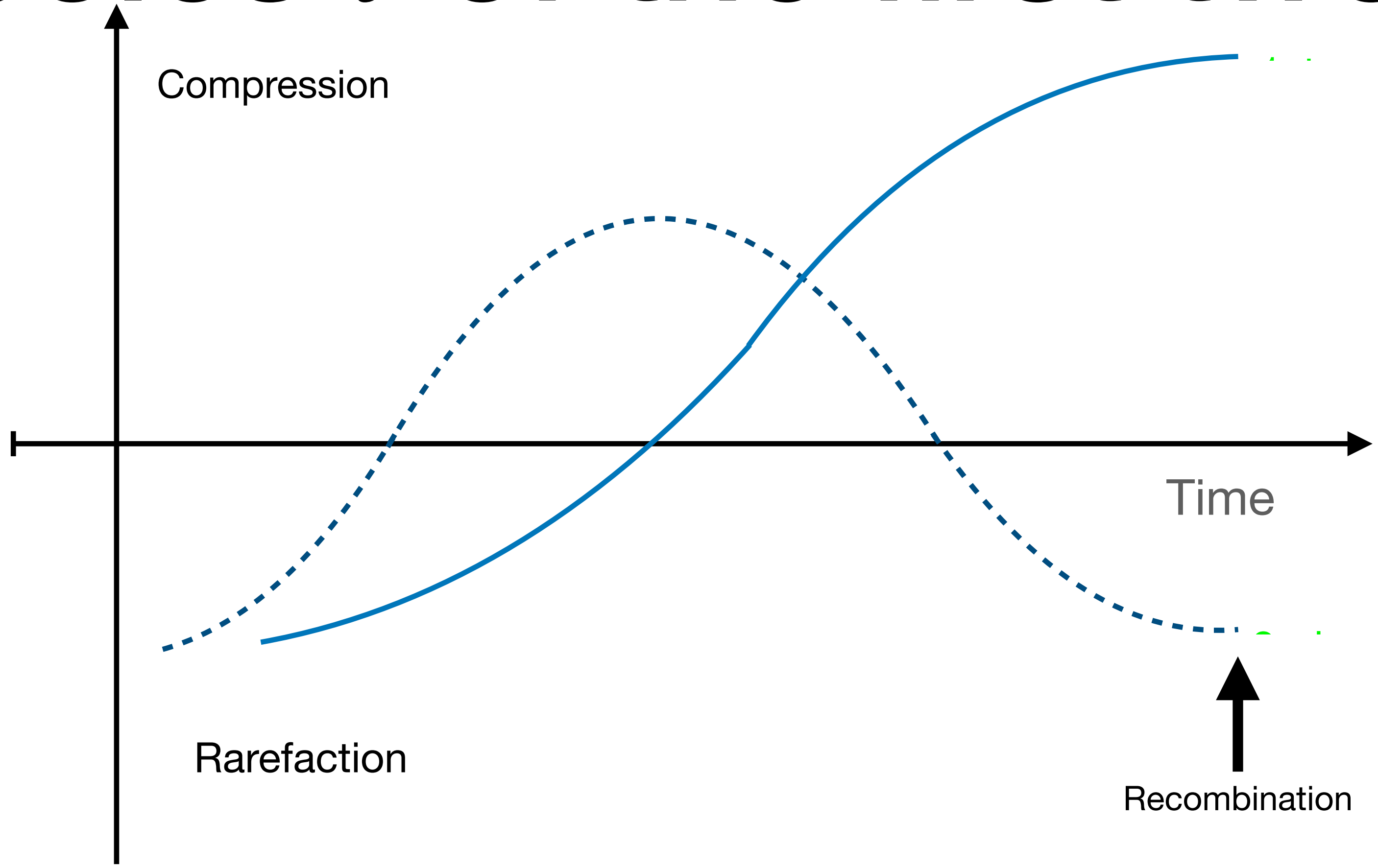
# Constraints on Cosmological Parameters:

# CMB Anisotropies

# Acoustic oscillations frozen at recombination



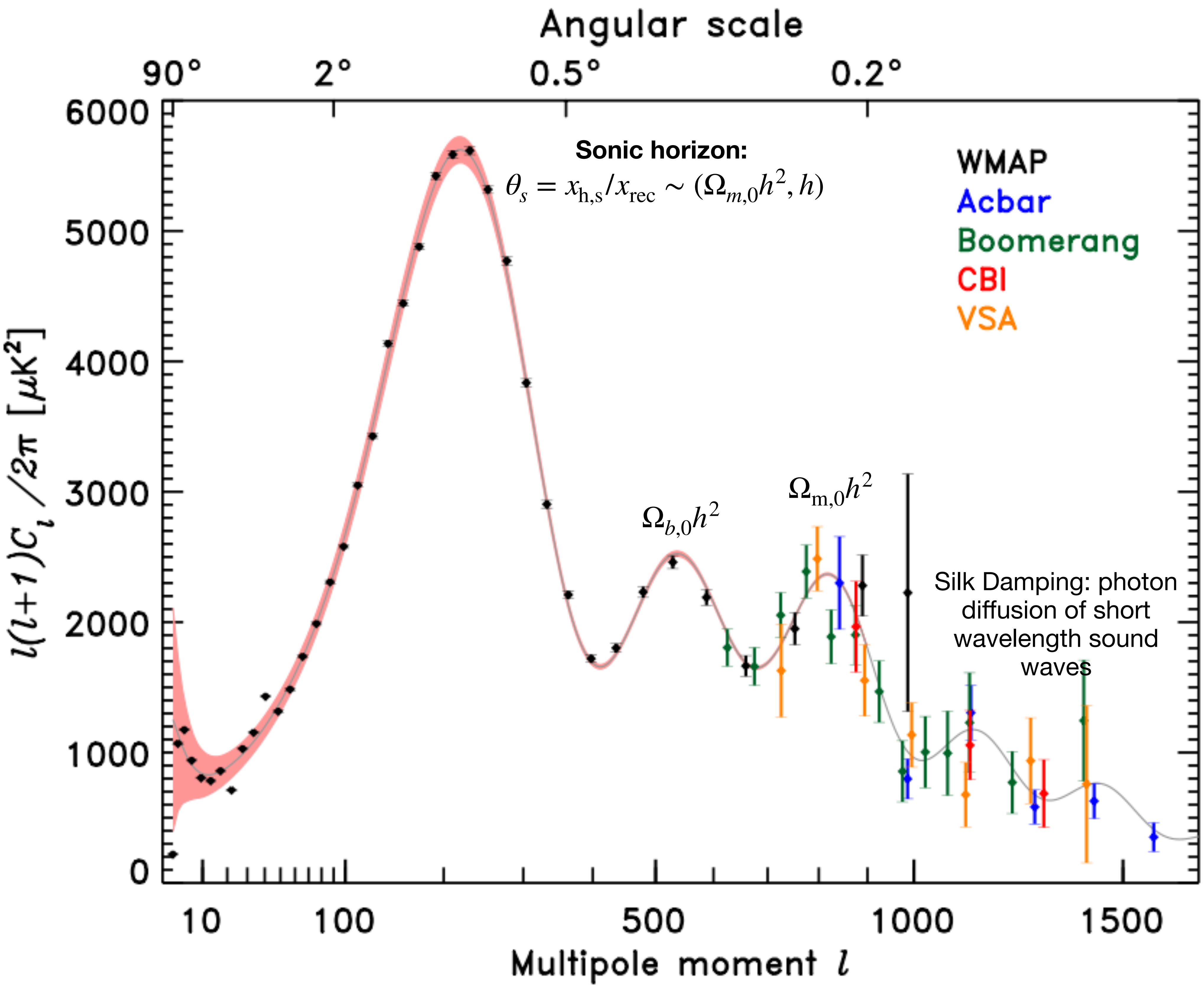
# Multipoles $l$ of the first two peaks



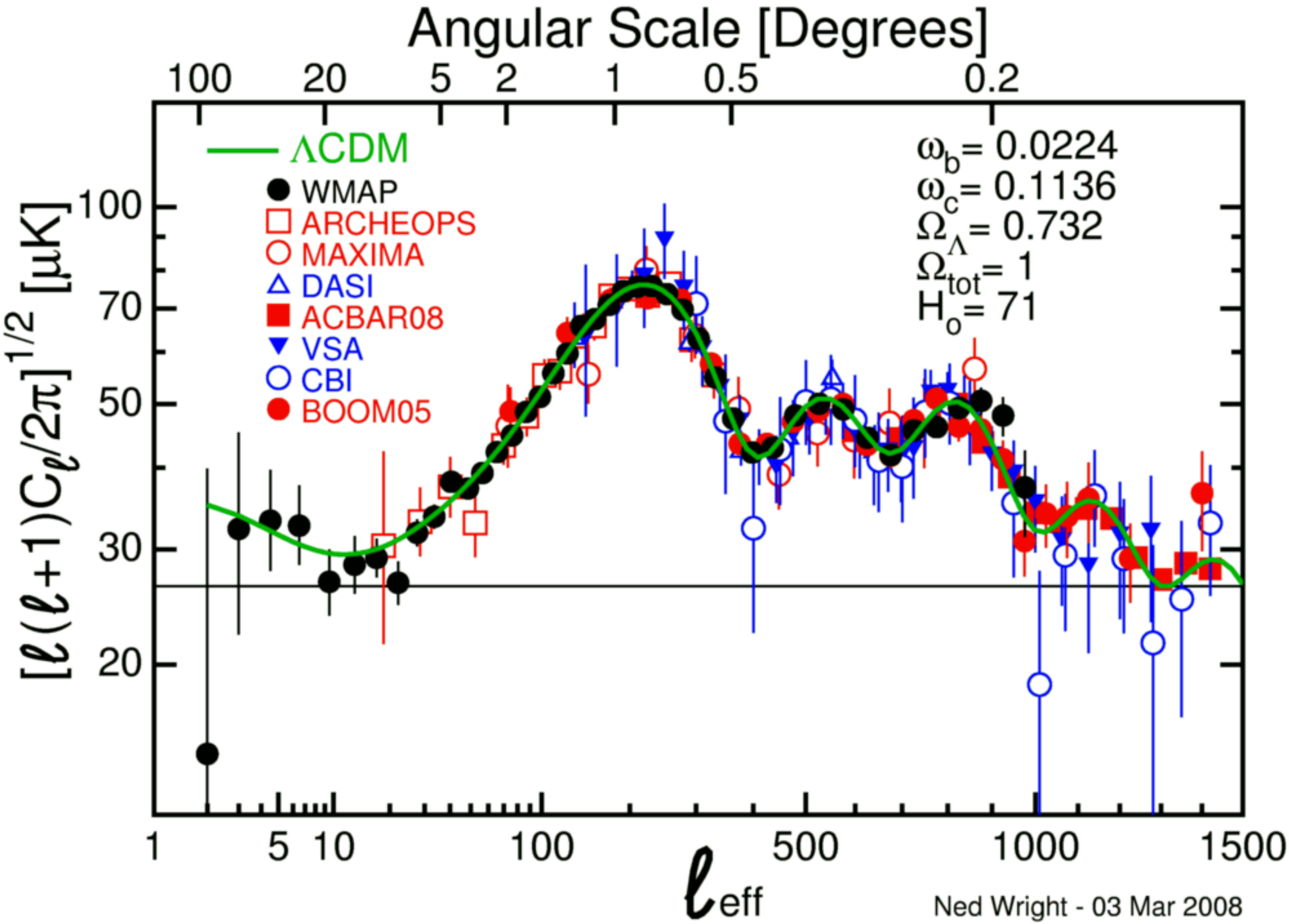

- First peak ($l \sim 200$): the largest structures that could have reached maximum compression at recombination: $\tau = 2\pi L/c_s = 2t_{\rm rec} \rightarrow L \sim c_s t_{\rm rec} \sim$ sonic horizon

- Second peak ($l \sim 500$): the largest structures that could have reached maximum rarefaction $\tau = 2\pi L/c_s = t_{\rm rec} \rightarrow L \sim c_s t_{\rm rec}/2 \sim$ sonic horizon/2

# CMB power spectrum & cosmological parameters



# The Power Spectrum of the CMB and the best-fit Parameters



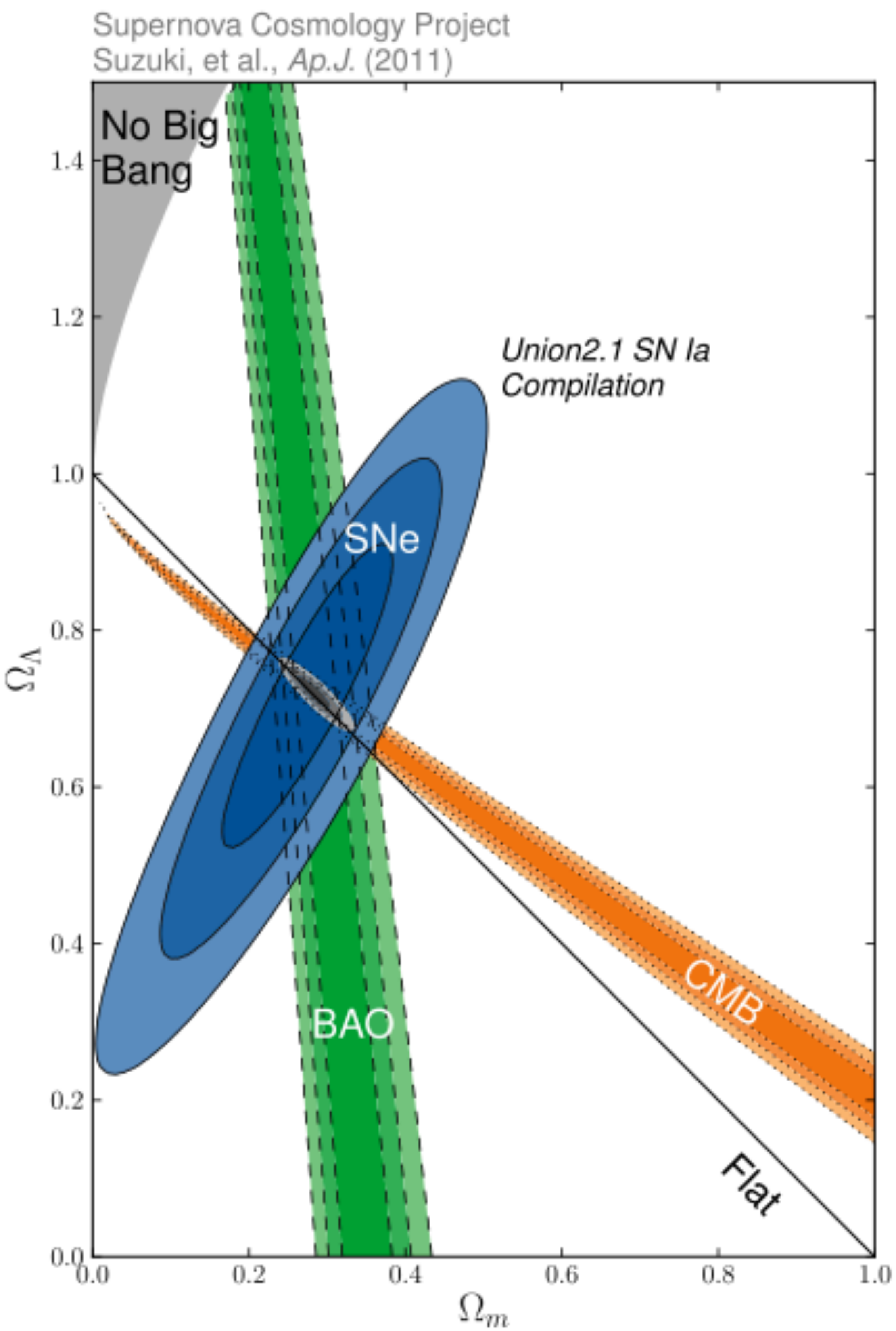



Figure 10.6: Joint likelihood contours (68%, 95%, and 99.7% confidence limits) in the $\Omega_{m,0} - \Omega_{\Lambda,0}$ plane for a recent compilation of SN Ia data, together with the WMAP measure of the temperature anisotropies of the CMB, and the large-scale distribution of galaxies in the nearby Universe (BAO).

## The Density Parameters of the Universe Today

*How do they evolve over time?*
*What's the predicted future?*

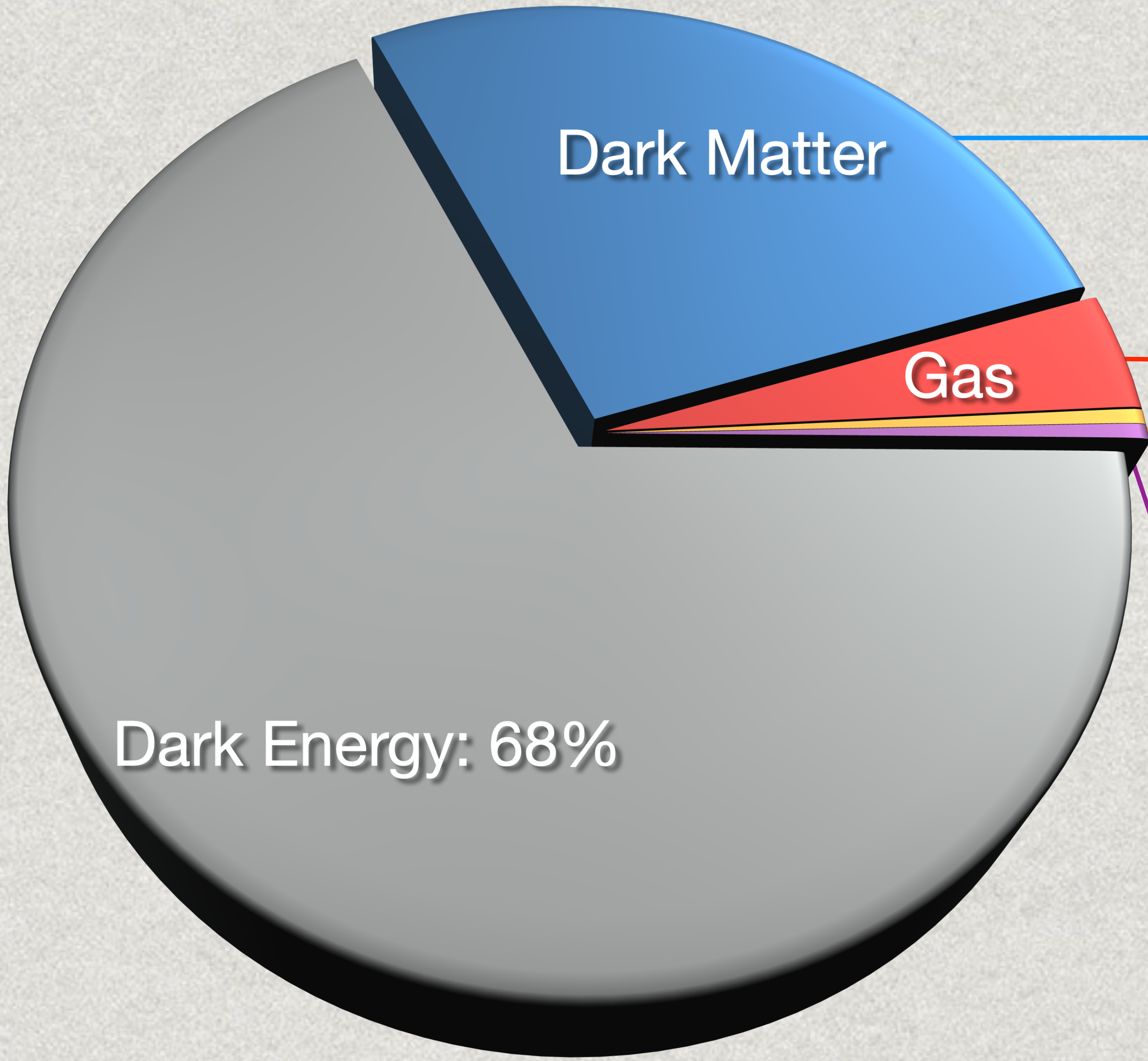
# Density Parameters Today

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

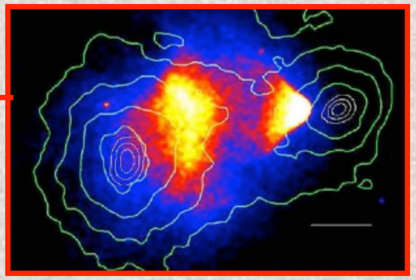
Critical density as a function of time. Value below is present value, based on present value of the Hubble parameter H

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 9.47x10^{-27}\, kg\, /\, m^3$$

Planck Collaboration (2013)

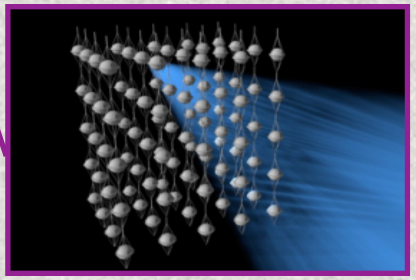



**Dark matter: 27%**



**Gas: 4%**

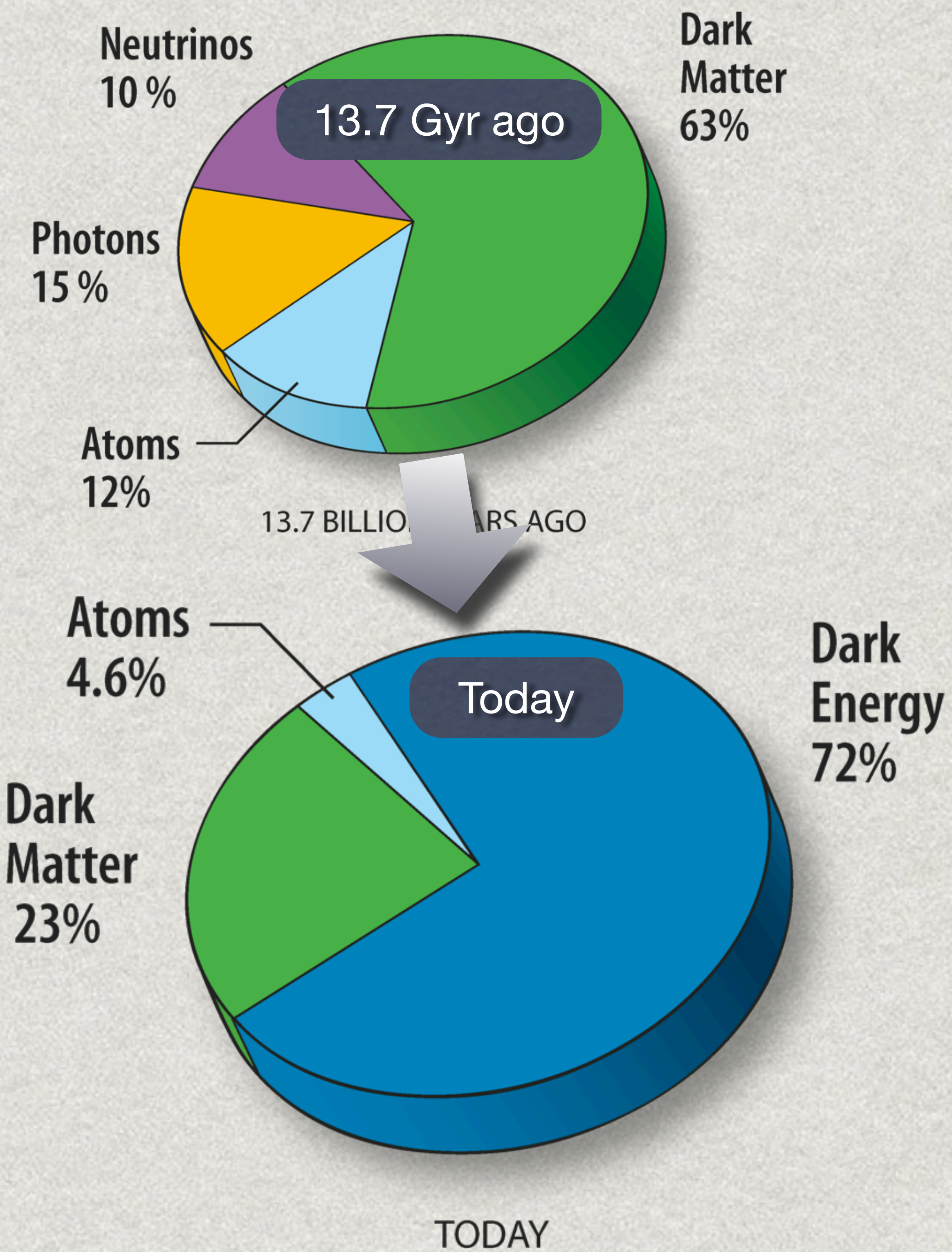
**Galaxies: 0.5%**

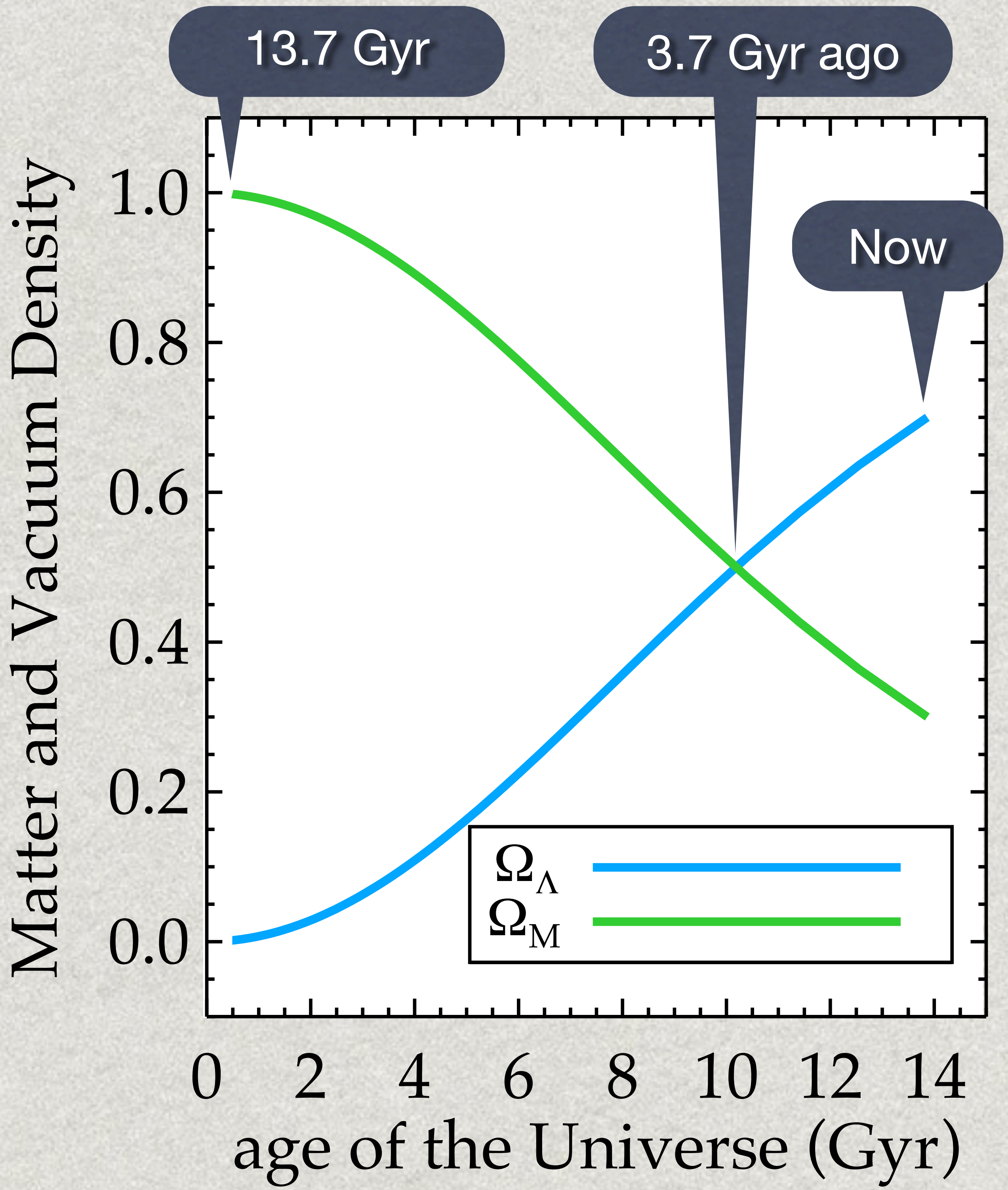


**Neutrinos: <0.5%**

# Density Parameters vs. Time

- Dark Matter-dominated in the first 10 Gyrs, then Dark Energy dominated



# Predicted Evolution of Density Parameters

- Dark Matter-dominated in the first 10 Gyrs, then Dark Energy dominated

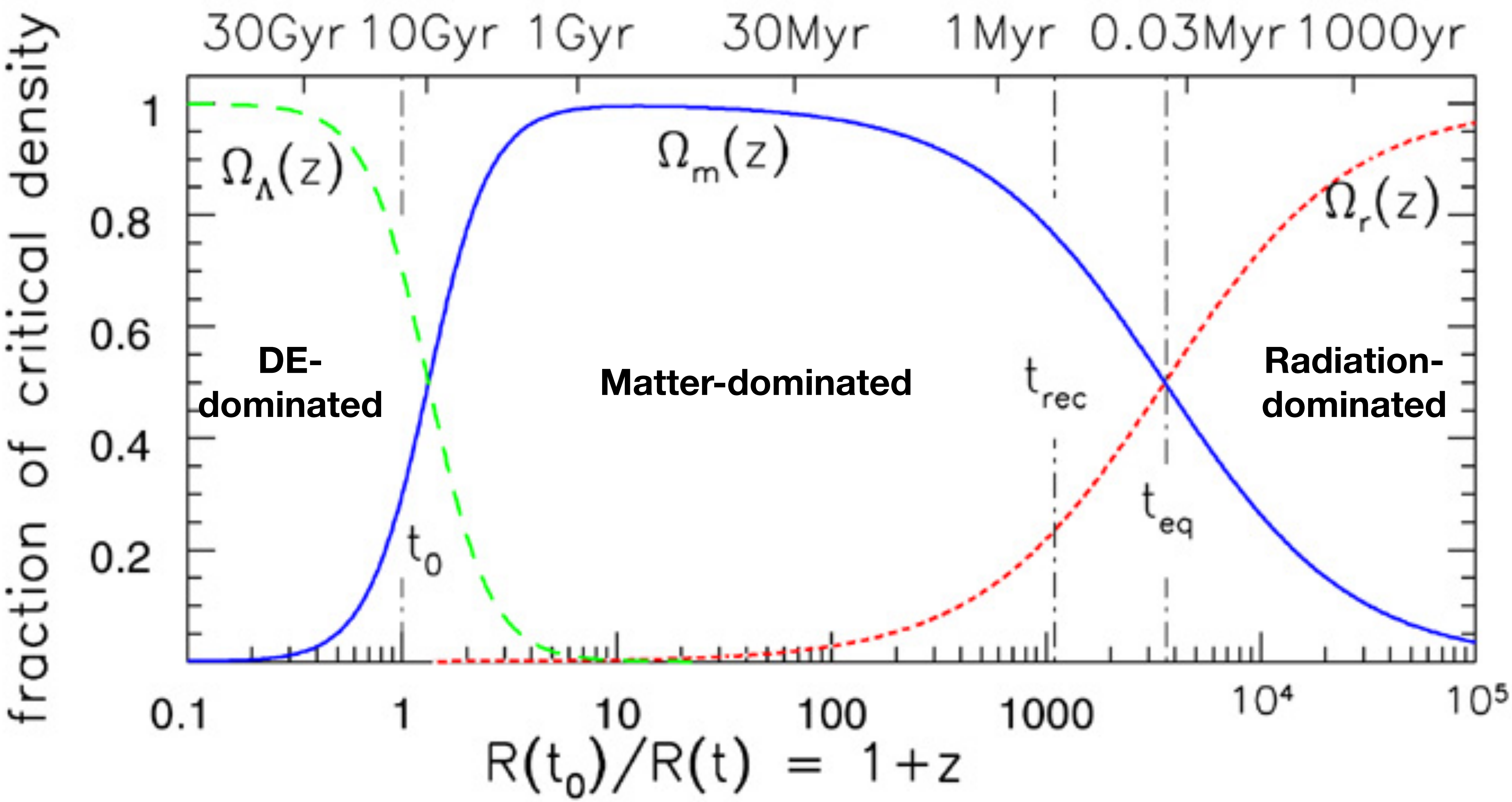
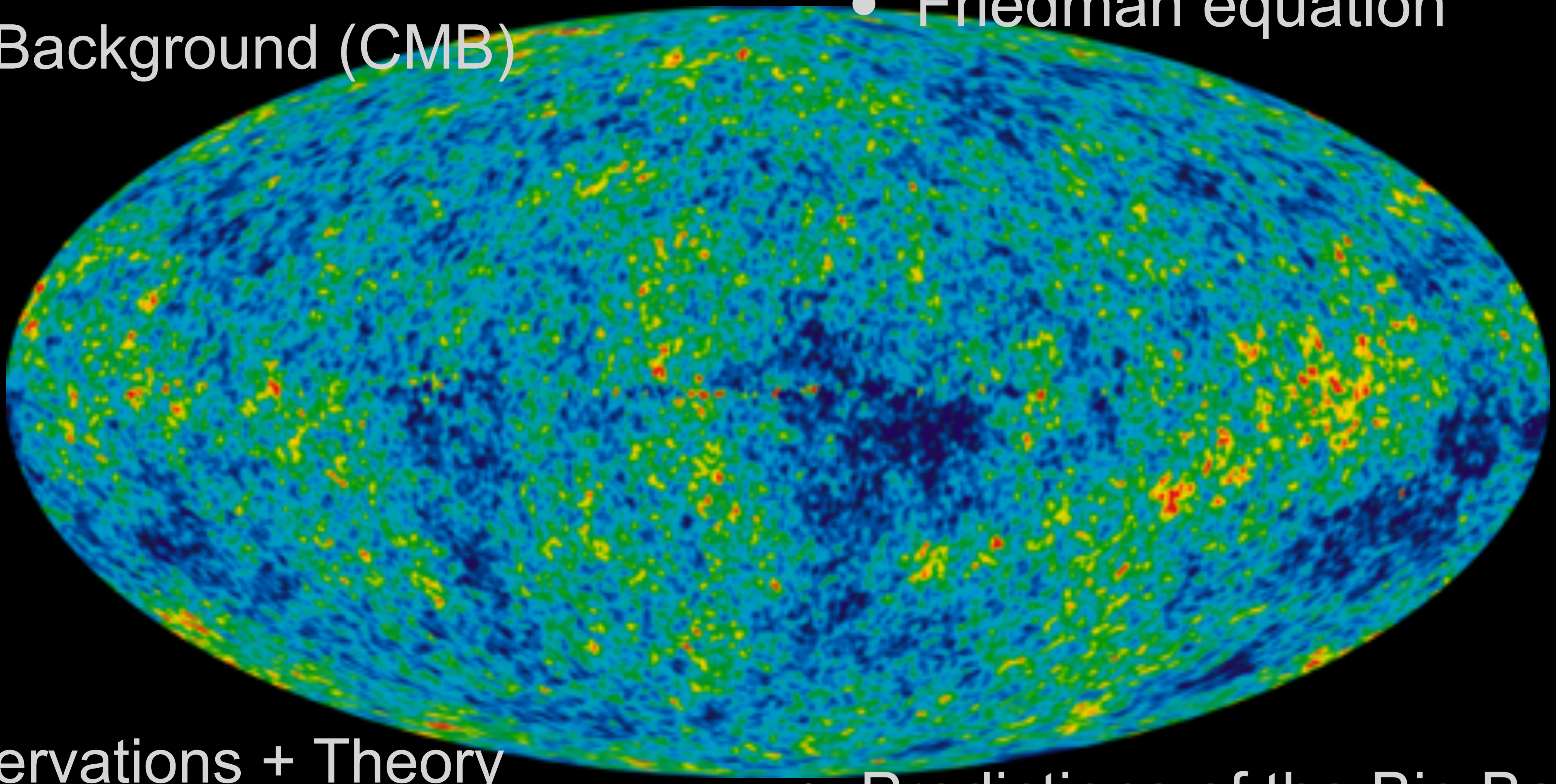



Fig 8.7 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

# Chap 21 & 22: The Expanding Universe

- Observations (facts)
  - Hubble-Lemaître Law & the Hubble "constant"
  - The Cosmic Microwave Background (CMB)
- Interpretations (theories)
  - The cosmological principle
  - Robertson-Walker metric
  - Friedman equation



- Observations + Theory
  - Accelerating Expansion: Evidence of dark energy
  - The cosmic composition
- Predictions of the Big Bang theory: how everything began?

The scientist does not study nature because it is useful to do so.

He studies it because he takes pleasure in it,

and he takes pleasure in it because it is beautiful.

-Henri Poincare, 1908

Henri Poincare (1854-1912)
French mathematician and physicist
Known for three-body problem, topology, and Lorentz transformation