ASTR:4850 - Signal to Noise Ratio

1 Required Data

In this lab, you will learn about various noise sources in a CCD image, calculate the signal to noise ratio (S/N) of stellar sources, and explore the dependency of S/N on source aperture size and exposure time. The data files are provided as a downloadable zip file on Canvas. All of the images were obtained using the same instrument and at the same CCD temperature.

2 Introduction

An ideal CCD detector would record the number of photons striking each pixel and nothing else - no counts due to read noise or dark current. In that case, and if the sky were perfectly dark, we could measure the brightness of a star by drawing a circle around the star on a CCD image and then summing up all the counts inside the circle.

However, as you found in the previous lab, the measurements produced by a CCD will also have contributions due to noise associated with the electronics that amplifies and digitizes the charge signal in the CCD readout (read noise) and due to electrons released by thermal fluctuations in the silicon (dark current). In order to measure the brightness of a star these contributions must be measured and subtracted off. Since the read noise and dark current arise due to random process, they will fluctuate and it will never be possible to subtract them off exactly. However, if we can determine the magnitude of the read noise and dark current, we can estimate how much they affect the measurement of the stellar brightness, i.e. how much uncertainty or noise they add to the measurement.

In addition, the sky isn't perfectly dark, so we need to subtract off the background from the sky. To do this, we estimate the brightness of the sky near the star, being careful to exclude the star and any other stars, and then subtract that off. The sky background also contributes to the noise in any measurement of stellar brightness.

Usually, one does dark and bias subtractions from the whole image. Below, we will handle each image individually, so that you can see all of the various contributions to the noise.

2.1 CCD Signal to Noise Equation

The signal from the star is the total number of electrons (Note: not counts or ADUs), N_T, recorded by the CCD from the star (or other astronomical target) after subtraction of the bias, the dark electrons, and the sky background. Photon counting is a random process and if we take repeated measurements of the same star the value of N_T will fluctuate according to a Poisson distribution. In the limit of large N_T, the Poisson distribution is well approximated by a Gaussian distribution with a standard deviation equal to $\sqrt{N_T}$. This photon-counting noise exists even if we had an ideal CCD and telescope and with zero sky background. These intrinsic fluctuations contribute a limiting noise term $\sigma_T = \sqrt{N_T}$. A 'bright' source is defined as one where the intrinsic noise dominates and, thus, the signal to noise ratio, S/N, for a measurement of the source is:

$$S/N = N_T/\sqrt{N_T} = \sqrt{N_T}. \tag{1}$$

For dimmer sources, we need to worry about noise from the sky background, the dark current, and the readout noise. The number of electrons recorded by the CCD from the dark current and the sky background are also Poisson distributed and fluctuate in the same way as the signal. Looking at each pixel individually, we write the number of dark current electrons per pixel as N_D. Even though we subtract off an estimate of this number, the fluctuations in the number of dark current electrons remain and add noise per pixel, $\sigma_D = \sqrt{N_D}$. The same is true for electrons produced by the sky background. The number of electrons per pixel from sky background is N_S and it adds noise per pixel, $\sigma_S = \sqrt{N_S}$.

The readout noise, σ_R, is the noise associated with the electronics that amplifies and digitizes the charge signal in the CCD readout. Readout noise is present even for zero signal and is usually assumed to be independent of the magnitude of the signal. Recall that in a previous lab we estimated the readout noise of a different camera using the histogram of bias frames. You can use the same technique to estimate σ_R of any CCD camera.

Noises from dark current, sky background, and readout arise from every pixel, so one needs to sum up the read noise for all the pixels in the source aperture. How do we add noises from multiple pixels and add noises from different contributing sources? In probability theory, the probability distribution function (PDF) of the sum of two independent random variables U and V, each of which has its own probability density function, is the convolution of their separate density functions: $f_{U+V}(x) = (f_U * f_V)(x)$. As mentioned above, all of the terms contributing to our total noise follow approximately Gaussian PDFs. The convolution of two Gaussians is another Gaussian (`https://en.wikipedia.org/wiki/Gaussian_function`). If the width (standard deviation) of the initial Gaussians are σ_1 and σ_2, then their convolution has a width $\sigma_{1+2} = \sqrt{\sigma_1^2 + \sigma_2^2}$. This sort of addition is called "addition in quadrature".

Noting that we need to sum up the noise contributions from the source itself (σ_T), the sky background (σ_S), the dark current (σ_D), and readout noise (σ_R). The last three terms are measured per pixel, so we need to sum their contribution from the n_{pix} pixels in the aperture containing the star. Therefore, the total noise is:

$$\sigma = \sqrt{\sigma_T^2 + n_{\text{pix}} \times (\sigma_S^2 + \sigma_D^2 + \sigma_R^2)} = \sqrt{N_T + n_{\text{pix}} \times (N_S + N_D + \sigma_R^2)} \tag{2}$$

The reason that the readout noise term is unlike the other terms in the expression on the right, is simply because it is a directly measured noise, while the other terms are estimates of noise derived from the measurements assuming Poisson distribution.

Because the total signal (S) is N_T and the total noise (N) is σ, the signal to noise equation for CCDs or the "CCD equation" is then:

$$S/N = \frac{N_T}{\sigma} = \frac{N_T}{\sqrt{N_T + n_{\text{pix}} \times (N_S + N_D + \sigma_R^2)}} \tag{3}$$

3 S/N Measurements with DS9

The goal in this section is to use DS9 to estimate the S/N's of two stars in your sky image using Eq. 3. You need a sky image in any filter, an average dark frame with the same exposure time, and an average bias frame. S/N measurements do not require flat field corrections. The sky images were taken at a range of exposure times; for this DS9 exercise, you need to choose the sky frame that has the same exposure time as the dark frames.

3.1 Set Apertures

The first step in aperture photometry is to decide on the source aperture and the background aperture in the image. Load your sky image into DS9. Pick a bright star on your image that is not saturated, i.e., no bleeding features like those seen in Fig 2.6 on the CCD textbook. You may want to set the image scale to zscale (Scale - zscale) and/or play with Scale - Scale Parameters.

- Draw the source aperture. Zoom in on your star so that you can see the individual pixels. Draw a circle around the star that contains most of the counts from the star. Depending on the version of DS9 that you have, you may need to select Edit - Region to change the cursor into region mode. In DS9, do Region - Shape - Circle, then left click at the center of the circle and drag it out to your desired radius. You should center the aperture on the star and make the radius of the circle large enough to contain most of the counts from the star. If you want to adjust your circle after drawing it, left click inside and you can then move and resize. Doubling left clicking inside brings up a dialog box where you can adjust the parameters by hand.
- Draw the background aperture. Select Region - Shape - Annulus, then draw an annulus centered on the star. Double-click the annulus to bring up the dialog box to set parameters. Use an inner radius large enough not to include any counts from the star, i.e. bigger than the radius of the circle drawn around the star (make the inner radius at least 20 pixels). Set a large enough outer radius so that the area of the annulus is at least several times the area of your circle. Make sure no visible stars are in the annulus.
- If you drew a bunch of extra regions, delete them now.
- Save your regions: Region - Save Regions. Pick a file name of your choice and use DS9 format and physical coordinate system.

3.2 Measure Counts inside Apertures

In this subsection, we use the analysis tool in DS9 to measure the counts in the source circular aperture and the background annulus aperture. The counts measured in the sky image include counts from the bias level, the dark current, and the sky background. To find out their contributions and to calculate their noises, we measure these counts in the sky

Table 1: Counts Measurements from Frames.
Filenames: a.fits (Sky), b.fits (Master Dark), c.fits (Master Bias)

	Src Aperture (Circle)	Bkg Aperture (Annulus)	Unit
Aperture Position	(x, y)	(x, y)	pixels
Aperture Size	r	$r_{\text{in}}, r_{\text{out}}$	pixels
# of Pixels	...	...	pixels
Counts in Sky Frame	...	...	ADUs
Counts in Dark Frame	...	...	ADUs
Counts in Bias Frame	...	...	ADUs

image, the average dark frame, and the average bias frame. In your notebook, record your measurements in a table like Table 1.

First, we measure counts in the Sky image. Hopefully the image is still displayed in your DS9 window.

- Left-click on your circle, then do Region - Get Information - Analysis - Statistics. This should bring up a new window with information about the region and the counts inside. Record the 'sum', which is the sum of the image counts in the region, and the 'area', which is the number of pixels in the region.

- Do the same for the annulus.

Then, load the average dark frame into DS9 taken with the same exposure time and same CCD temperature as your Sky image (File - Open). If you want to look at both frames simultaneously, do Frame/New Frame before loading the dark frame and then afterwards do Frame - Tile Frames and then Frame - Match - Frame - Physical. You might want to re-size your DS9 window to see both images.

- Load the region file that you saved into the DS9 frame with the dark image: do Region - Load Regions, use DS9 format and 'Load into Current Frame'.

- Find and record the statistics (sum and area) for the circle and annulus on the dark frame. Note that the area for the circles should be the same in both images (also for the annuli).

Finally, do the same for the average bias frame. Find and record the statistics (sum and area) for the circle and annulus on the bias frame.

3.3 Calculating the Signal to Noise Ratio

To calculate the S/N for the chosen source and the aperture size, we need to obtain the values for all the parameters in Table 2. Note that you need to convert the counts recorded in Table 1 to electrons by multiplying the counts with the CCD gain (which is given in the FITS header).

Table 2: CCD Photometry Noise Terms

Parameter	Measured Value	Unit
Source Aperture Radius, r	...	pixels
# of Pixels in Aperture, $n_{\rm pix}$	...	pixels
CCD Gain	...	e^-/ADU
Readout Noise per Pixel, $\sigma_{\rm R}$	...	e^-/pixel
Signal in Aperture, $N_{\rm T}$	...	e^-
Sky Background per Pixel, $N_{\rm S}$	...	e^-/pixel
Dark Current per Pixel, $N_{\rm D}$	...	e^-/pixel
Noise from Signal, $\sqrt{N_T}$	...	e^-
Noise from Sky, $\sqrt{n_{\rm pix} N_S}$	...	e^-
Noise from Dark, $\sqrt{n_{\rm pix} N_D}$	...	e^-
Noise from Readout, $\sqrt{n_{\rm pix} \sigma_R^2}$	...	e^-
Total noise, σ	...	e^-
Signal to Noise, S/N	...	...

- CCD Gain - this is the conversion factor from electrons to ADU in terms of electrons per ADU.

- Readout Noise - follow the steps outlined in Lab 2 to determine the CCD readout noise. Note that you should use the bias frames provided in this lab. Does it agree with the value in the camera's manual?

- Sky Background per Pixel = (Sky Counts - Dark Counts)/(# of Pixels in Annulus)×Gain, where both counts are measured in the background annulus.

- Dark Current per Pixel = (Dark Counts - Bias Counts)/(# of Pixels in Annulus)×Gain, where both counts are measured in the background annulus.

- Signal in Aperture = [(Sky Counts in Circle) - (# of Pixels in Circle)×(Sky Counts in Annulus)/(# of Pixels in Annulus)]×Gain. This is the number of electrons generated by the photons emitted by the star.

Now calculate the S/N of your selected star with your selected aperture using the numbers in Table 2 and the CCD Equation (Eq. 3). Record your calculations in Table 2. Which component of the noise dominates?

Once you are done with the bright star. Repeat the measurements for another star. This time choose one of the dimmest star you can see in your sky image. To make the results comparable, use the same sizes for the source aperture and the background annulus.

4 S/N measurements with Python

4.1 S/N vs. aperture size

By now, you should have realized that it is not easy to do a lot of S/N measurements with DS9. The biggest advantage of the process you just went through is that it is transparent, meaning that you should be fairly confident of the results, and therefore, you could use this method to do sanity checks of your python code.

In this section, we will take advantage of the automation provided by python programming to explore (1) how the signal varies as a function of aperture size (i.e., How to recover all of the signal through aperture correction?), (2) how S/N varies as a function of aperture size (i.e., Is there the optimal source aperture to use to maximize S/N?), and (3) how S/N varies with source brightness? Our goal here is simply to reproduce the plots in Fig. 5.6 and Fig. 5.7 in the e-book *Handbook of CCD Astronomy*.

Now write your own python program to do the aperture photometry repeatedly. The Astropy-affiliated package `photutils` has useful functions for this purpose: photutils.aperture.

First make sure your python program can reproduce the results you obtained in the previous section with DS9, then use it to calculate the S/Ns of three different stars with a range of aperture radii. Use the same Sky image as you used before. Choose three stars that about 1 magnitude apart in magnitude (i.e., about 2.5× difference in counts). The typical seeing is about ∼2 arcsec in FWHM (*How many pixels is that?*). Note that this is a guess of the seeing FWHM and you need to use your data to determine the actual seeing FWHM next. So set about ten aperture radii that cover the range between ∼0.5 and ∼4 times the seeing FWHM. As in the previous section, for each source, use the same background annulus region at all radii.

Now plot your results in three figures:

1. the signal from the star (i.e., N_T) vs. aperture radius (r).
2. fractional signal for the star vs. aperture radius (r), like in Fig. 5.6 but for three stars.
3. S/N vs. aperture radius (r).

Use different symbols for the stars and plot vertical error bars for your data points. Include your plots in your lab report and write some discussion comparing them with Fig. 5.6 and the upper panel of Fig. 5.7 in the CCD textbook. What aperture radius gives the best S/N (i.e., the optimal aperture)? Does this change depending on the brightness of the star? Using the optimal aperture, how much fraction of the total stellar flux is missing?

4.2 Atmosphere Seeing Measurements

The stars are at such great distances that it is safe to assume that their angular sizes are far below the diffraction limit of the telescope; hence they are often referred to as *unresolved sources* or *point sources*. As part of the exercise, use the formula for diffraction limit, $\theta = 1.22\lambda/D$, to calculate the diffraction limit of the telescope used to obtain the CCD images, and compare it with the angular size of a star of 1 solar radius at a distance of 1 parsec. Report your results in unit of milliarcsec (mas).

Without the atmosphere, a point source should be recorded as an Airy disk with a size of the diffraction limit you just calculated. But because of turbulence in the atmosphere, it is recorded as a Gaussian disk with a size much greater than the diffraction limit. The full-width-at-half-maximum (FWHM) of the Gaussian is called the atmospheric seeing, which is usually given in unit of arcsec. Ground-based telescopes without adaptive optics (AO) are thus *seeing limited*.

Assuming the camera is correctly focused, you can use the data in your version of textbook Fig. 5.6 (fraction signal vs. aperture radius) to determine the atmosphere seeing at the time of your observation. Analytically, we can show that a circular aperture (centered on the peak) with a radius of FWHM/2 encloses 50% of the total volume under a 2D Gaussian surface. We thus have:

$$\text{seeing} = 2 \times r_{50}, \tag{4}$$

where r_{50} is the aperture radius that encloses 50% of the total signal. The native units of both the FWHM and r_{50} are CCD pixels, but you can convert them to arcsec using the plate scale. With some interpolation method, you can measure r_{50} using the data in your Fig. 5.6. You will obtain three measurements of r_{50} with the three different stars. Record them and use a vertical dashed line to indicate the mean FWHM in all three plots you produced in the previous section. Finally, convert the r_{50} measurements in pixels to three seeing FWHM measurements in arcsec. Provide both the mean and the uncertainty of your seeing FWHM measurements.

4.3 S/N vs. exposure time

In the previous subsections, you explored the S/N's dependency on aperture radius using a sky image with an exposure time that matches those of your dark images. Now let's explore how S/N varies with exposure time. This is an important relation because it allows an observer to estimate the required telescope time to reach the S/N desired for their science goals, before they start the observations and sometimes during their observations.

Before starting the measurements, think about this: Given the S/N equation (Eq. 3) and how N_T, N_S, N_D depend on exposure time, what sort of dependency do you expect to find between S/N and time?

In the data folder, you should find sky images of the same field at five different exposure times. Choose two stars of drastically different brightness, fix your aperture size to that gives the highest S/N, and calculate their S/N's in the images with different exposure times. Note that because the exposure time of the science image no longer matches that of the dark image, you have to scale the number of dark electrons to the exposure time of the science frame (i.e., making the reasonable assumption that the dark current in e^-/pixel/second is constant). Plot your results of two stars at five exposure times in a figure of S/N vs. exposure time. Use logarithmic scale for the axes if necessary.