ASTR:4850 - Astrometry & Proper Motion

1 Introduction

In this Data Analysis lab, we will first calibrate the astrometry of a Landolt field image that we obtained, and then use the calibrated image to measure the proper motion of the standard star in the field. The astrometry calibration requires knowing *a priori* the sky coordinates of at least two stars in the image (assuming negligible proper motion), solving for the CCD-to-Sky transformation matrix, and inserting the matrix and tangent point into the FITS header. Once astrometry calibration is complete, we measure the position of the standard star relative to other stars in the field. This is so-called "differential astrometry". Finally, by comparing the J2000 position of the star measured in your image with that measured by Landolt in 1992, we can determine the proper motion in RA and Dec by assuming a straight-line motion and neglecting parallax.

2 Data Preparation

First, collect the following images:

- Lab Images: FITS files of the Landolt standard star field **G102 22** and calibration frames. For this exercise, all teams will use the images of the Landolt star **G102 22** because of its high proper motion. Fig. 1 shows the photographic plate image of the G102 22 field from Landolt (1992). The standard star is indicated by two short lines. When you compare this 3-decade-old image with your recent image, you would notice that this star has drifted a lot. The images may be flipped/mirrored in one of the two axes. If so, unflip your reduced image with `numpy.flipud` or `numpy.fliplr` and save the unflipped image in a new FITS file.

- POSS2 Images: Astrometry-calibrated archival image of the same field from the Second Palomar Observatory Sky Survey; download the FITS file from the STScI Digitized Sky Survey: http://archive.stsci.edu/cgi-bin/dss_form, choose "POSS2/UKSTU Red" and set Height and Width to roughly match the size of the Lab image. Recall your field of view calculations for the VAO and the IRO. The coordinates of the Landolt star are tabulated at this URL.

Next, follow the procedure described in the Photometry lab manual to reduce the Lab image of the Landolt G102 22 field. In short:

- calculate the master bias from the raw bias frames

- subtract master bias from raw flat field image

- average the bias-subtracted flat field images and then normalize the average by its median value

- subtract master bias from the raw Landolt field image, then divide bias-subtracted image by the normalized flat field.
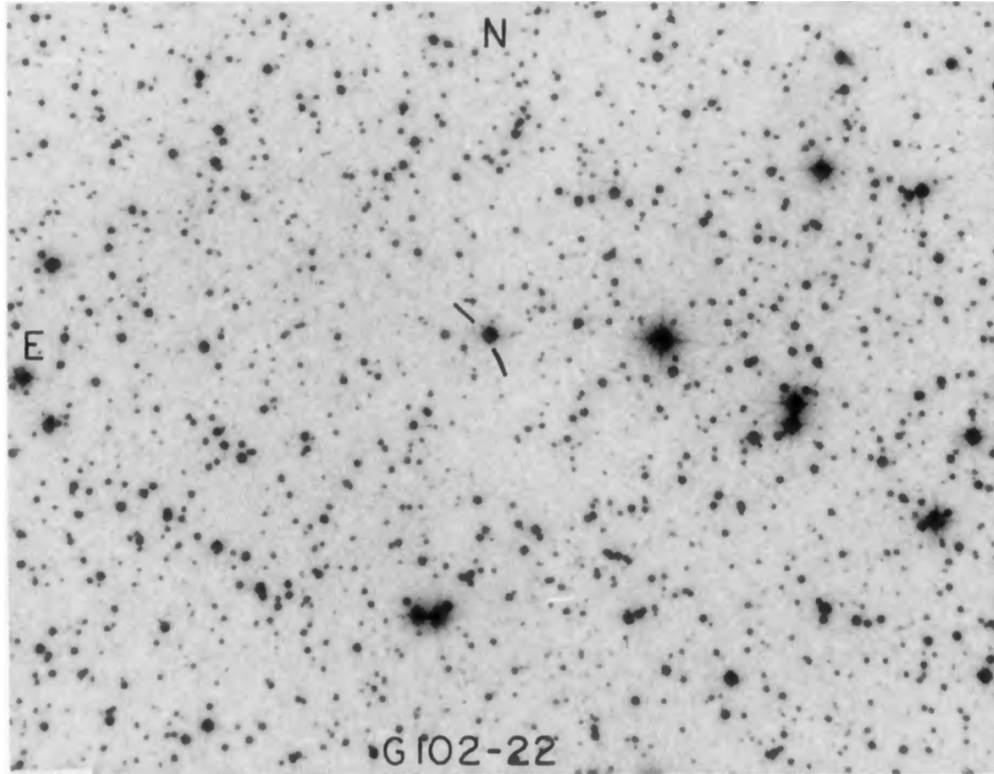
Figure 1: Photographic plate image of the Landolt (1992) G102 22 field. The standard star is indicated by two short lines near the center of the image.

3 World Coordinate System in FITS Header

Load the POSS2 image of G102 22 into DS9. As you move the pointer around on the image, you see the astronomical coordinates of the pointer position listed in the two boxes next

```
WCSAXES =                      2 /GetImage: Number WCS axes
WCSNAME = 'DSS            ' /GetImage: Local WCS approximation from full plat
RADESYS = 'ICRS           ' /GetImage: GSC-II calibration using ICRS system
CTYPE1  = 'RA---TAN       ' /GetImage: RA-Gnomic projection
CRPIX1  =            446.000000 /GetImage: X reference pixel
CRVAL1  =             85.538498 /GetImage: RA of reference pixel
CUNIT1  = 'deg            ' /GetImage: degrees
CTYPE2  = 'DEC--TAN       ' /GetImage: Dec-Gnomic projection
CRPIX2  =            447.000000 /GetImage: Y reference pixel
CRVAL2  =             12.489390 /GetImage: Dec of reference pixel
CUNIT2  = 'deg            ' /Getimage: degrees
CD1_1   =         -0.0002801756 /GetImage: rotation matrix coefficient
CD1_2   =          0.0000001550 /GetImage: rotation matrix coefficient
CD2_1   =          0.0000001359 /GetImage: rotation matrix coefficient
CD2_2   =          0.0002797326 /GetImage: rotation matrix coefficient
```

Figure 2: WCS keywords in the FITS header of the POSS2 image.

to the letters WCS or FK5. The astronomical coordinates are calculated from the pixel position using the so-called "World Coordinate System" (WCS) defined in the FITS header of the image. In DS9, do File/Display Header and then scroll down until you see the lines describing the FITS header that start with "WCSAXES". See Fig. 2 for an example of the WCS section in the FITS header. Let us go through these keywords.

WCSAXES gives the number of WCS axes, usually 2 when dealing with 2-dimensional images. WCSNAME gives the name of the WCS coordinate system. RADESYS gives the particular coordinate reference system used: e.g., the International Celestial Reference System (ICRS).

CTYPE1, CTYPE2 specify the projection used for the coordinate system. The projection is how pixels in the image are mapped onto the sky. Recall that all celestial objects are projected onto an imaginary sphere (the celestial sphere), but the surface of a CCD is usually a plane. For CCD images with relatively small fields of view, the CCD surface can be thought of as a small, flat square stuck onto the celestial sphere at one particular point, usually the center of the image. This is called the tangent projection because the flat square is tangent to the sphere at the point of attachment. In the POSS2 images, the tangent projection is indicated by "TAN" in the keywords CTYPE1="RA—TAN" and CTYPE2="DEC–TAN". These keywords also indicate that the first coordinate is Right Ascension and the second coordinate is Declination. The CUNIT1 and CUNIT2 keywords specify the units of the WCS coordinates, which are usually degrees.

The four keywords CRPIX1, CRVAL1, CRPIX2, CRVAL2 specify the tangent point where the flat CCD surface is stuck onto the celestial sphere. We need to know both the coordinates of the tangent point both on the CCD, given by the two CRPIXs, and the tangent on the celestial sphere, given by the CRVALs. The units for the CRPIX values are pixels. The units for the CRVAL values are given in the FITS keywords CUNIT1 and CUNIT2.

There are two things left to specify to complete the transformation from CCD coordinates to sky coordinates. One is how the CCD pixel size translates into angles on the sky, which is provided by the angular size of the image pixels, s (i.e., the plate scale). The other is how the CCD image is rotated relative to the axes of the celestial sphere. Both of these are captured in a linear transformation matrix encoded in the CD keywords in the FITS header, CD1_1, CD1_2, CD2_1, CD2_2.

To understand the meaning of these CD keywords, first recall the rotation matrix used to perform a rotation in Euclidean space (introduced in linear algebra):

$$\mathcal{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

For a *left-handed* coordinate system (y-axis is 90 deg clockwise from x-axis), pre-multiply $\mathcal{R}$ to a vector (x, y) rotates the point in the xy-plane *clockwise* by an angle θ with respect to the origin of the coordinate system to (x', y'):

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \tag{2}$$

Celestial coordinates are left-handed: Astronomical images oriented with North towards up (declination increasing towards up), usually have RA increasing to the left. Conversely,

CCD coordinates are right-handed: x increasing to the right and y increasing towards up. For simplicity, we do the substitution $x \to -x$ to convert CCD coordinates to a left-handed system.

Evidently, the rotation matrix's determinant equals to 1, which means that the magnitude of the vector is not changed when multiplied by the matrix. But if we pre-multiply the rotation matrix by a vector, then the length of the vector will scale accordingly. The first and second elements of the 2-element vector determine the scale factor in x and y directions, respectively. If the scale factors are the same in both directions, the vector is reduced into a scaler. For CCDs with square pixels, the following equation converts a distance in CCD coordinates (x, y) into a distance in sky coordinates (α, δ):

$$\begin{bmatrix} (\alpha - \alpha_0)\cos(\delta_0) \\ \delta - \delta_0 \end{bmatrix} = s \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} -(x - x_0) \\ y - y_0 \end{bmatrix} = \begin{bmatrix} -s\cos\theta & -s\sin\theta \\ -s\sin\theta & s\cos\theta \end{bmatrix} \cdot \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} \quad (3)$$

where α and δ are RA and Dec, x and y are the CCD coordinates, s is the pixel size in deg/pixel, θ is the rotation angle of the y-axis of the CCD (measured counterclockwise relative to the declination axis, i.e., the North), and $\alpha_0, \delta_0, x_0, y_0$ are CRVAL1, CRVAL2, CRPIX1, CRPIX2 that define the tangent point and the origin of the coordinate systems.

With the above equation, we are ready to define the linear transformation matrix:

$$\begin{bmatrix} -s\cos\theta & -s\sin\theta \\ -s\sin\theta & s\cos\theta \end{bmatrix} = \begin{bmatrix} \text{CD1\_1} & \text{CD1\_2} \\ \text{CD2\_1} & \text{CD2\_2} \end{bmatrix} \quad (4)$$

The meanings of the CD keywords are now evident.

Choosing the tangent point, thus evaluating CRVAL and CRPIX keywords, is more straightforward. The more difficult aspect of astrometry calibration is therefore to find the plate scale (s) and rotation angle (θ) so that we can evaluate the linear transformation matrix.

Before moving onto the next section, examine Eq. (3) and answer the following questions:

- Why is there a cosine term when we calculate distance in RA? *Understanding this is critical for the success of this lab.*

- Why is there a negative sign in the $(x - x_0)$ term when it is first multiplied by the rotation matrix $\mathcal{R}$?

- Is the definition of CD matrix consistent with the values in the FITS header? Did you notice anything strange? What are the plate scale and the rotation angle of the POSS2 image?

4 Astrometry Calibration

The goal of astrometry calibration is to add WCS information in the FITS header. Recall that when you move the cursor across the POSS2 image, DS9 shows the RA and Dec in the information box. When you move the cursor across your image, that field is blank. Once you have successfully saved a new FITS file of your image with WCS information in the header,

DS9 will automatically pick up that information and allow you to read off the RA and DEC of any position in the image. An example code block is provided in § 4.5 to show you how to write WCS keywords into the header.

When finding the WCS for an astronomical image, one usually knows the approximate pointing of the telescope as a starting point. One compares the CCD image to an existing image or catalog of stars in that field and then tries to find correspondences. Typically, one uses on the order of 8 to 30 matches to calculate the best WCS transformation. Here we first simplify that procedure by using only two stars and then use more stars to improve our results. We can break up this exercise into the following subsections.

4.1 Setting the Tangent Point

To set the tangent point, choose an unsaturated bright star near the center of the image. This is the tangent point reference star. Note that because our goal is to measure the proper motion of the standard star, we cannot use the standard star as a reference star in the astrometry calibration. The procedure is:

- Open the image in DS9 and choose a contrast scale that you are satisfied with.

- Choose the tangent point reference star and draw a circular region around it. Make sure the region is large enough to get a good centroid position (recall the centroid script you wrote in a previous lab).

- Click on the region to select it, then on the top menu, use Region-Centroid to center the region to its centroid. To see the effect of the centroid algorithm, shift the initial position of the circle so that it is clearly offset from the center of the star that you eyeballed.

- Double-click on the region to open the region's property window. Record the CCD coordinates of the center position in a table similar to Table 1.

Now you have the (x, y) position of the tangent point reference star, which sets the CRPIX1 and CRPIX2 keywords in the WCS header that you will create and add to the FITS file.

Next, we will use the archival POSS2 image to determine the sky coordinates of the reference star:

- Open the POSS2 image of the same field in DS9 in a separate frame.

- Find the tangent point reference star you had chosen in this new image and draw a circular region around it. Again, make sure the region is large enough to get a good centroid.

- Select the region and use DS9's top menu option Region-Centroid to shift the region to its centroid.

- Double-click on the region after it is shifted to the centroid

- Record the WCS/ICRS (RA,Dec) and the image (x,y) coordinates of the region's center position in your version of Table 1.

Table 1: Image and Sky Coordinates of Stars in the Field.
Filenames: —.fits (Lab image), —.fits (POSS2 image)

Reference Star	Lab x pixel	Lab y pixel	RA deg	Dec deg	POSS2 x pixel	POSS2 y pixel	Plate Scale arcsec/pix	θ deg
Tangent Point	...	...	...	...	...	...	N/A	N/A
Ref Star 2	...	...	...	...	...	...	...	...
Ref Star 3	...	...	...	...	...	...	...	...
Ref Star 4	...	...	...	...	...	...	...	...
Ref Star 5	...	...	...	...	...	...	...	...
Ref Star 6	...	...	...	...	...	...	...	...

You now have the (RA, Dec) of the tangent point from the POSS2 image, which sets the CRVAL1 and CRVAL2 keywords in the WCS header.

As an additional exercise, you're encouraged to use your centroid code from the previous lab to determine the CCD coordinates of the star in both images. Note that you need to be careful to make sure that your python code returns coordinates that are the same as the CCD coordinates reported by DS9. Note that DS9 and python may number pixels differently: e.g. is the center of the first pixel in the lower left corner at (0,0), (0.5, 0.5), or (1,1) in DS9? What about in python?

4.2 Calculating Plate Scale

Now pick a second star that is visible on both your image and the POSS2 image. For better results, pick a star far from the first star. Similar to your tangent point reference star, your second reference star should be well detected and relatively isolated. Find the centroid image and sky coordinates for the star using DS9, similar to what you did in the previous subsection. Record them in your version of Table 1.

With the POSS2 (RA, Dec) values you recorded in the table, find the angular distance between the tangent point and the second reference star in arcsecs. Note that you need to use spherical trigonometry. Write down the equation for the angular distance in terms the RA and Dec of the two stars and record your calculations. You may want to write a python function to calculate angular distances as you will be doing this for several stars below.

With the Lab (x, y) values, find the distance in pixels between the CCD coordinates of the two stars *in the Lab image* (not in the POSS2 image). Do you need to use spherical trigonometry here as well? Recall the fact that we are using a tangent projection for the coordinate system.

Note that you could use DS9's "ruler" region (on the top menu, select Region-Shape-Ruler) to carry out the same measurements.

Take the ratio of the two distances, this is your value for s, the angular size of pixels. Record the value in Table 1. Note the unit recorded here is arcsec/pixel and you will later convert this to degree/pixel.

4.3 Calculating Rotation Angle

With the POSS2 (RA, Dec) values you recorded in the table, find the angle between a line drawn from the tangent point star to your second star in sky coordinates and a line draw due north from the tangent point. With the Lab (x, y) values, find the angle between a line draw from the tangent point to your second star in CCD coordinates and a line draw towards positive y from the tangent point. The difference between these two angles is the rotation angle of the image (θ).

You can draw "ruler" regions in DS9 to measure these angles, but be careful about how DS9 defines angles. You may want to write a bit of python code to do this as you will be doing this for several stars below.

Note that the definitions of the angles are a bit tricky, you may want to draw a diagram to help you visualize the problem.

4.4 Use More Reference Stars

It is preferred to use more than two stars to obtain an accurate measurements of the CCD plate scale (s) and the rotation angle (θ). Keep using the same star as the Tangent Point, but measure the image and sky coordinates for at least four additional reference stars in the field. It is preferred to have at least one star in each quadrant of the Lab image.

Use Table 1 to record your data and copy the data into a text file. Write a python code to use the data in this table to calculate and report s and θ for each tangent-point−star pair. Of course, you may also use DS9 for this task if you don't want to use python. Calculate the mean and standard deviation of your measurements. In the next subsection, use the mean values to calculate the linear transformation matrix used for the WCS keywords.

4.5 Inserting WCS Keywords

Use python to read in the reduced image in FITS format (or calculate the reduced image from the object and bias images), add keywords to the header for the WCS based on the measurements that you made above, and then write out a new FITS file with the new header (as is traditional with FITS files, keep all the old keywords as long as they do not conflict with your new ones). Your code should specify the six input parameters for the transformation as listed above. Your code will need to use the plate scale and rotation angle you obtained in the previous subsection to calculate the transformation matrix.

You might find this page useful in learning about how to deal with FITS headers in python: http://pythonhosted.org/pyfits/index.html#tutorial. The following example code might be useful to get started:

```
1  # Python program to load two FITS images,
2  # take their difference,
3  # copy the header and add WCS keywords,
4  # then write out the difference image and header
5  # as a new FITS file
6
7  # import needed extensions
8  from numpy import *
```

```python
 9 #import matplotlib.pyplot as plt # plotting package
10 #import matplotlib.cm as cm # colormaps
11 import pyfits
12
13 # file names, change as appropriate
14 skyfile = 'm39-b-10s.fits'
15 biasfile = 'bias-medavg.fits'
16 outputfile = 'm39wcs.fits'
17
18 # read in the files
19 sky, header = pyfits.getdata(skyfile, 0, header=True)
20 bias = pyfits.getdata(biasfile)
21
22 # find the difference between the images
23 reduced = sky-bias
24
25 # add keywords to the header
26 # need to calculate values for CRPIX1/2, CRVAL1/2, CD's
27 header['WCSAXES'] = 2
28 header['WCSNAME'] = 'DSS'
29 header['RADESYS'] = 'ICRS'
30 header['CTYPE1'] = 'RA---TAN'
31 header['CUNIT1'] = 'deg'
32 header['CRPIX1'] = (1024.0, 'X reference pixel')
33 header['CRVAL1'] = ( 320.0, 'RA of reference pixel')
34 header['CTYPE2'] = 'DEC--TAN'
35 header['CUNIT2'] = 'deg'
36 header['CRPIX2'] = (1024.0, 'Y reference pixel')
37 header['CRVAL2'] = (  45.0, 'Dec of reference pixel')
38 header['CD1_1'] =  -0.0002802103
39 header['CD1_2'] =  -0.0000006392
40 header['CD2_1'] =  -0.0000006192
41 header['CD2_2'] =   0.0002797130
42
43 # write out the new FITS file
44 pyfits.writeto(outputfile, reduced, header, clobber=True)
```

4.6   Sanity Check

Do the following to check whether your inserted WCS information works and the WCS solution is accurate:

- Load your new image into DS9, does the information panel report WCS coordinates as you move your cursor around the image?

- Overlay an optical catalog on the image: In the top menu, select Analysis - Catalogs - Optical - GSC2.2. This will bring up a catalog window. It will automatically query the catalog database in the part of the sky that is covered by the image and overlay the catalog positions as green circular regions (you can change the style of these regions in DS9). Do you see good agreement between the catalog source positions and the objects in the image? Fig. 3 shows an example of catalog-image comparison.
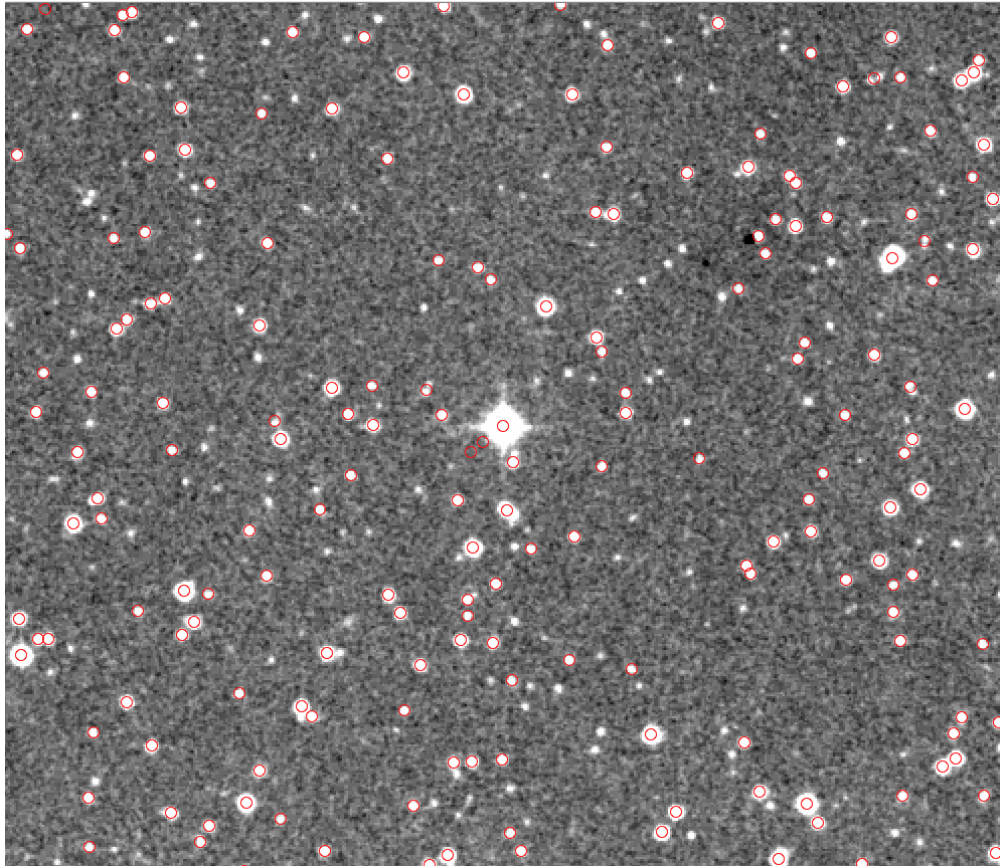
Figure 3: GSC 2.2 catalog overlaid on an archival POSS2 image. This figure shows excellent agreement between catalog positions and sources detected in the image. Overlay the GSC 2.2 catalog on your astrometry-calibrated image to check the accuracy of your WCS solution. Include a similar figure in your lab report.

- Estimate the accuracy of your WCS calibration by comparing the catalog (RA, Dec)'s and the centroid (RA, Dec)'s for 3+ randomly chosen stars. Note that you should not use any of the reference stars because they have been used to calibrate the WCS. Report the mean offset in arcsec and this is an estimate of the WCS uncertainty.

5 Calculate Proper Motion

Once you are satisfied with the astrometry calibration, you can measure the sky coordinates of the standard star G102 22:

- Identify the standard star in the image and use DS9's centroid tool to find the sky coordinates (RA, Dec) of the star in the Lab Image.

- Calculate and record the offsets in both RA and Dec directions between this new position and the position reported in Landolt (1992). Use arcsec as the unit and

record 2 decimal points.

- Calculate the time span between your observation and Landolt's photographic plate. Because the paper does not report the time of the observation, we will assume the position was measured in Feb 1991.

- Calculate the proper motion in both RA and Dec directions in either arcsec/yr or mas/yr.

- Compare your result with the GAIA proper motion result on Simard. This star's common name is Ross 47.

The proper motion measurement described above is limited by the uncertainty of the observation time of the photographic plate Landolt used for the initial position measurement. A better proper motion measurement can be obtained by comparing the star's position in your image and its position in the POSS2 image because the time of the POSS2 observation is kept in the FITS header. As the final exercise of this lab, measure the proper motion using the POSS2 image and your astrometry-calibrated image, quantify the uncertainties, and compare your results with GAIA again.