Measuring SMBH mass

requires resolving the sphere of influnece

observational signatures : (1) a spike in σ_* (2) Keplerian $1/r$ velocity

r_{inf} defined as the radius at which $-U_{BH} = 2K_*$:

$$r_{inf} = \frac{GM_{BH}}{\sigma_*^2} \iff \frac{GM_{BH}}{r_{inf}} = \sigma_*^2$$

$$= 11\,\mathrm{pc}\ \frac{M_{BH}}{10^8 M_\odot}\left(\frac{\sigma_*}{200\,\mathrm{km/s}}\right)^{-2}$$

which is 10^6 times greater than the Schwarzschild radius :

$$r_{Sch} = \frac{2GM_{BH}}{c^2} = 2\,\mathrm{AU}\ \frac{M_{BH}}{10^8 M_\odot}$$

BH feedback energy

total energy released throughout the history of the universe per unit volume :

$$E_{BH} = \epsilon \cdot \rho_{BH} \cdot c^2 \quad , \epsilon : \text{radiative efficiency} \simeq 0.1$$

$$E_{SN} = \frac{10^{51}\,\mathrm{erg}}{100\,M_\odot} \cdot \rho_* \quad , 1\ \text{SN per } 100\,M_\odot \text{ of stars formed}$$

$$\Rightarrow \frac{E_{BH}}{E_{SN}} = \frac{\epsilon \cdot 100\,M_\odot \cdot c^2}{10^{51}\,\mathrm{erg}} \cdot \frac{\rho_{BH}}{\rho_*} = 1.8\times10^4 \cdot \frac{\rho_{BH}}{\rho_*}$$

Radiative efficiency

$$\epsilon = \frac{L}{\dot{m}c^2} = \frac{\Delta E}{mc^2}$$ [where $\dot{m}$ is accretion rate, m the mass of a gas cloud]

consider a patch of cloud that is accreted from a distance of ∞, its original energy is : $E_0 = U_0 + K_0 = 0$

the accretion disk ends at R, where the energy of the cloud becomes :

$$E_R = U_R + K_R = \frac{1}{2}U_R = -\frac{GMm}{2R}$$

if all the energy loss is radiated away, we have (with Virial theorem) :

$$\epsilon = \frac{\Delta E}{mc^2} = \frac{GM}{2Rc^2} = 0.074\,\frac{M/1M_\odot}{R/10\mathrm{km}} = \frac{1}{12} \text{ when } R = 3r_{Sch} = \frac{6GM}{c^2}$$

Accretion disk temperature profile:

assume an optically thick, steady-state disk

↓ black body   ↓ $\dot{M}$ is independant of r

consider the energy loss of inflowing gas from r+dr to r

$$\frac{dE}{dt} = \frac{d}{dr}\left[-\frac{GM\dot{M}}{2r}\right]\cdot dr = \frac{GM\dot{M}}{2r^2}dr$$

this energy has to be radiated away locally:

$$\frac{dE}{dt} = \sigma_{SB} T^4 \cdot 4\pi r\, dr$$

$$\Rightarrow \quad T(r) = \left(\frac{GM\dot{M}}{8\pi\sigma_{SB}\cdot r^3}\right)^{1/4} = \left(\frac{GM\dot{M}}{8\pi\sigma_{SB} R^3}\right)^{1/4}\cdot\left(\frac{r}{R}\right)^{-\frac{3}{4}} \equiv T_c\cdot\left(\frac{r}{R}\right)^{-\frac{3}{4}}$$

where R is the innermost radius of the disk

For neutron stars:

$$T_c = \left(\frac{GM\dot{M}}{8\pi\sigma_{SB}R^3}\right)^{1/4} = 10^7 K\left(\frac{M}{1.4M_\odot}\right)^{1/4}\left(\frac{\dot{M}}{1.6\times10^{-9}M_\odot/yr}\right)^{1/4}\cdot\left(\frac{R}{10km}\right)^{-\frac{3}{4}}$$

For SMBHs:

$$R \simeq R_{ISCO} = 3\,r_{Sch} = \frac{6GM}{c^2} \simeq 6\,AU\left(\frac{M}{10^8 M_\odot}\right)$$

$$M \simeq 10^8 M_\odot, \quad \dot{M} = 1\, M_\odot/yr$$

$T_c = 4\times10^5\ K$ (much cooler than that of neutron stars)

Eddington Luminosity Limit

① single particle derivation:

grav. force on proton : $F_{grav} = \frac{GM}{r^2} \cdot m_p$

radi. force on e^- : $F_{rad} = \frac{dP}{dt} = \frac{d(E/c)}{dt} = \frac{1}{c} \cdot \overset{L/4\pi r^2}{\overset{\downarrow}{\text{flux}}} \cdot \sigma_T$

outflow happens when $F_{rad} > F_{grav}$, which gives

$$L > \frac{4\pi G M c}{\sigma_T / m_p} \equiv L_{Edd}$$

② hydrostatic equilibrium derivation:

imagine atmosphere of a massive star, hydro equil. requires:

$$\frac{dP}{dr} = -\rho g = -G\frac{M\rho}{r^2}$$

where the pressure gradient comes from P_{rad} & P_{gas}.
if the star is extremly luminous, dP/dr is dominated by $\frac{dP_{rad}}{dr}$, which equals:

$$\frac{dP_{rad}}{dr} = -\frac{\sigma n}{c} \cdot \text{Flux}$$ (radiative pressure gradient drives flux)

outflow happens when

$$-\frac{dP}{dr} \simeq -\frac{dP_{rad}}{dr} > \rho g$$

$$\Rightarrow \quad \frac{\sigma n}{c} \cdot \frac{L}{4\pi r^2} > G\frac{M\rho}{r^2}$$

$$\Rightarrow \quad L > \frac{4\pi G M c}{\sigma/(\rho/n)} = \frac{4\pi G M c}{\sigma_T/m_p} \equiv L_{Edd}.$$

M_{BH} - σ_* relation:

$$\frac{M_{BH}}{10^9 M_\odot} = (0.310 \pm 0.035)\left(\frac{\sigma_*}{200\,\mathrm{km/s}}\right)^{4.4\pm 0.3}$$

theory based on radiative feedback (King 2003):

consider both the gas and the stars in the bulge follow SIS profile:

$$\rho_{gas}(r) = f_{gas}\cdot\frac{\sigma_*^2}{2\pi G r^2}$$

which gives:

$$M_{gas}(r) = \frac{2 f_{gas}\,\sigma_*^2\, r}{G} \qquad \& \qquad V_{cir}(r) = \sqrt{2}\,\sigma_*$$

consider, if the surrounding gas absorbs a fraction of the AGN luminosity, and if the SMBH is at the Eddington limit (i.e., max power), we have the momentum deposition of swepted up gas in a shell at r:

$$\underset{\substack{\uparrow \\ \text{total momentum of gas}}}{M_{gas}(r)\cdot\frac{dr}{dt}} = \delta\int_0^t \frac{L_{Edd}}{c}\,dt \simeq \frac{\delta L_{Edd}}{c}\cdot t$$

$$\Rightarrow \quad \frac{r}{t}\cdot\frac{dr}{dt} = \frac{\delta\, G\, L_{Edd}}{2 f_{gas}\cdot\sigma_*^2\cdot c} \qquad (1)$$

also we know the Eddington luminosity:

$$L_{Edd} = \frac{4\pi G M_{BH}\cdot c}{\sigma_T/m_p}$$

we have the terminal velocity as [by integrating Eq (1)]:

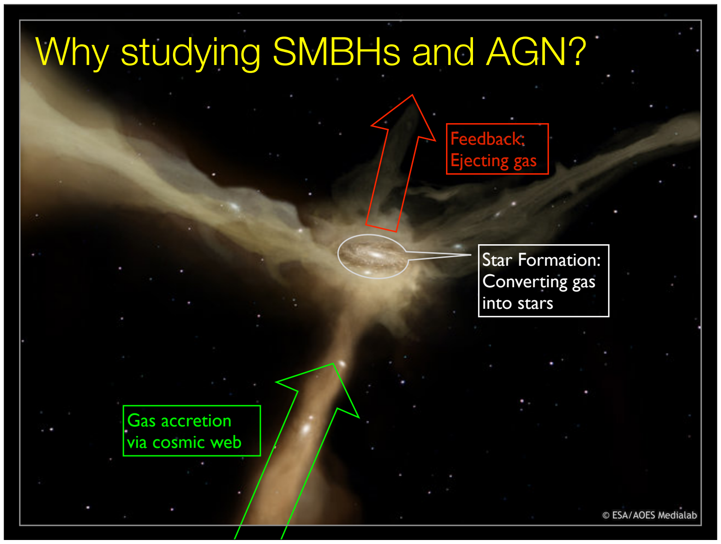
$$\left(\frac{r}{t}\right)^2 = \frac{\delta G}{2 f_{gas}\sigma_*^2 \cancel{c}}\cdot\frac{4\pi G M_{BH}\cdot\cancel{c}}{\sigma_T/m_p}$$

mean velocity is half of current velocity

for the gas to be ejected, we need $V_m = \frac{r}{t} \simeq \sigma_* \Rightarrow \frac{dr}{dt} = 2\sigma_*$

this criterion leads to $M_{BH} \geq \frac{f_{gas}}{2\pi\delta}\cdot\frac{\sigma_T/m_p}{G^2}\cdot\sigma_*^4$

Why studying SMBHs and AGN?



Feedback: Ejecting gas

Star Formation: Converting gas into stars

Gas accretion via cosmic web

© ESA/AOES Medialab

The importance of SMBHs

compare the total amount of feedback energies from SMBHs and SNe:

$$E_{\rm BH}/V \simeq 0.1\,\rho_{\rm BH}\,c^2$$

$$E_{\rm SN}/V \simeq \frac{10^{51}\,{\rm erg}}{100\,{\rm M}_\odot}\,\rho_\star$$

$$\Rightarrow \frac{E_{\rm BH}}{E_{\rm SN}} = \frac{10 M_\odot c^2}{10^{51}{\rm erg}}\frac{\rho_{\rm BH}}{\rho_\star} = 1.8\times10^4\frac{\rho_{\rm BH}}{\rho_\star}$$

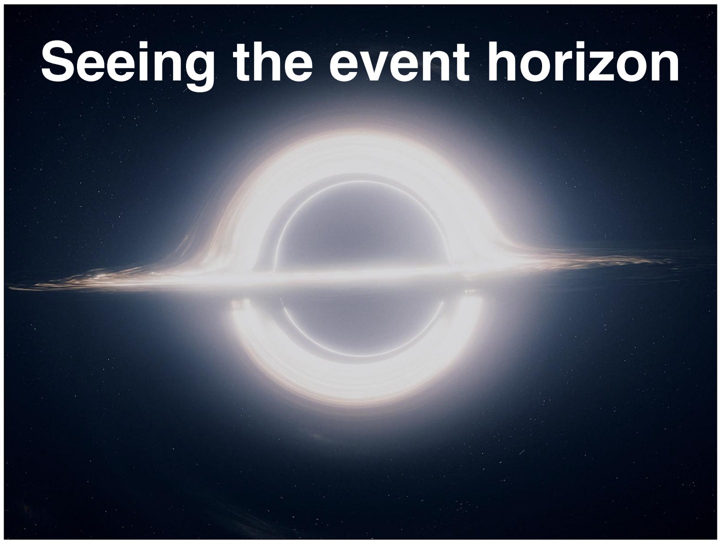
If $\rho_{\rm BH} \simeq 0.001\,\rho_\star$

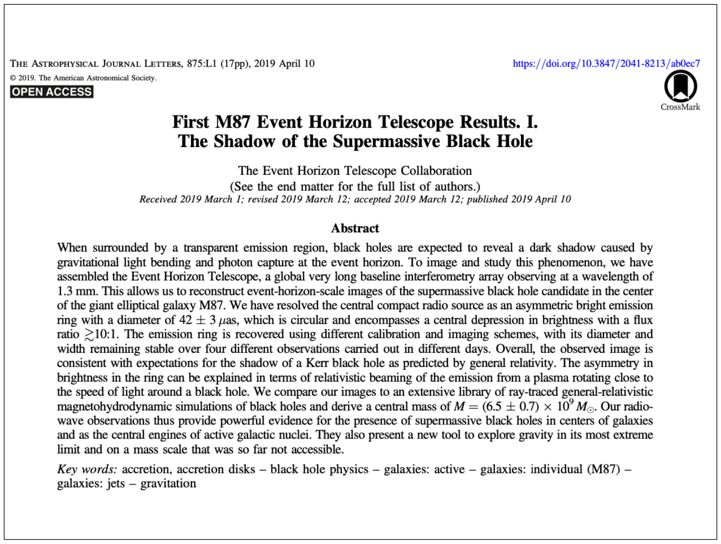
then $\frac{E_{\rm BH}}{E_{\rm SN}} \simeq 1.8$

SMBHs and AGN

- ***Measuring the mass of SMBHs in normal galaxies***
  - resolving the gravitational sphere of influence
  - enabled by AO and HST observations (1989)
  - $M_{\rm BH}$-sigma* relation (2000)
- **Understanding the $M_{\rm BH}$-sigma* relation**
  - How SMBHs moderate the growth of galaxies
- ***Active Galactic Nuclei (AGN): Quasars (1963)/QSOs 1965)/Radio Galaxies (1949)/Seyferts (1943)***
  - Accretion disk physics
  - Deducing the innermost structures of AGN
  - Measuring BH masses in AGN: reverberation mapping and the BLR size-luminosity relation

Seeing the event horizon





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First M87 Event Horizon Telescope Results. I.
The Shadow of the Supermassive Black Hole

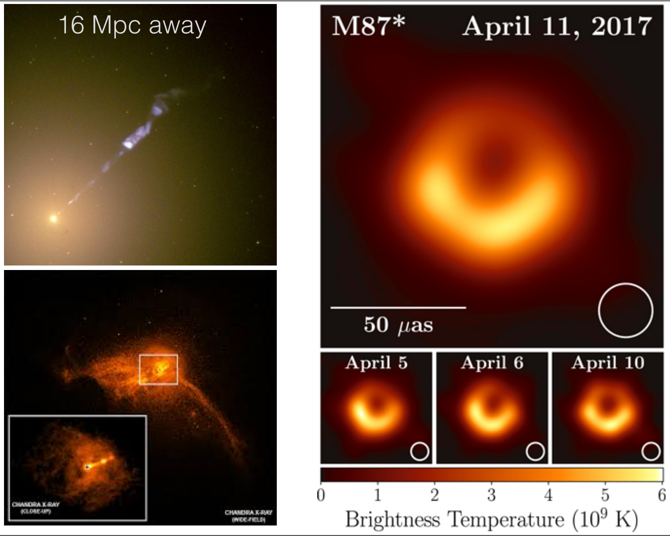
The Event Horizon Telescope Collaboration
(See the end matter for the full list of authors.)

Received 2019 March 1; revised 2019 March 12; accepted 2019 March 12; published 2019 April 10

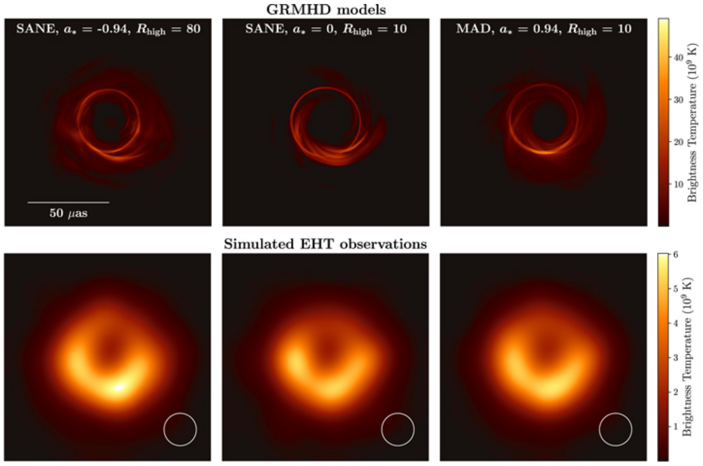
Abstract

When surrounded by a transparent emission region, black holes are expected to reveal a dark shadow caused by gravitational light bending and photon capture at the event horizon. To image and study this phenomenon, we have assembled the Event Horizon Telescope, a global very long baseline interferometry array observing at a wavelength of 1.3 mm. This allows us to reconstruct event-horizon-scale images of the supermassive black hole candidate in the center of the giant elliptical galaxy M87. We have resolved the central compact radio source as an asymmetric bright emission ring with a diameter of $42 \pm 3\,\mu$as, which is circular and encompasses a central depression in brightness with a flux ratio $\gtrsim$10:1. The emission ring is recovered using different calibration and imaging schemes, with its diameter and width remaining stable over four different observations carried out in different days. Overall, the observed image is consistent with expectations for the shadow of a Kerr black hole as predicted by general relativity. The asymmetry in brightness in the ring can be explained in terms of relativistic beaming of the emission from a plasma rotating close to the speed of light around a black hole. We compare our images to an extensive library of ray-traced general-relativistic magnetohydrodynamic simulations of black holes and derive a central mass of $M = (6.5 \pm 0.7) \times 10^9\,M_\odot$. Our radio-wave observations thus provide powerful evidence for the presence of supermassive black holes in centers of galaxies and as the central engines of active galactic nuclei. They also present a new tool to explore gravity in its most extreme limit and on a mass scale that was so far not accessible.

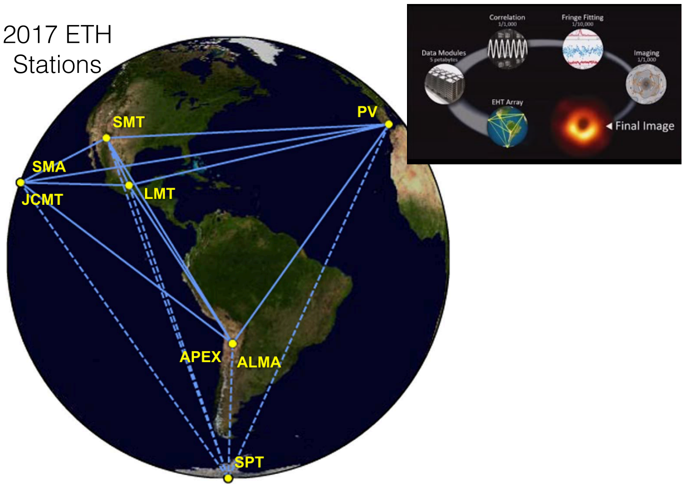
Key words: accretion, accretion disks – black hole physics – galaxies: active – galaxies: individual (M87) – galaxies: jets – gravitation



GRMHD simulations of different spin parameters and accretion flows



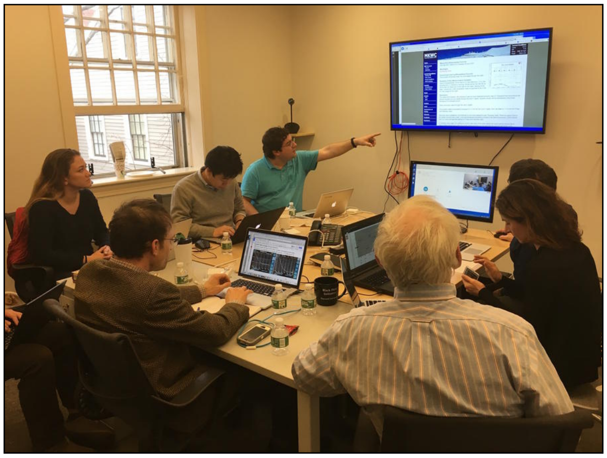
2017 ETH Stations



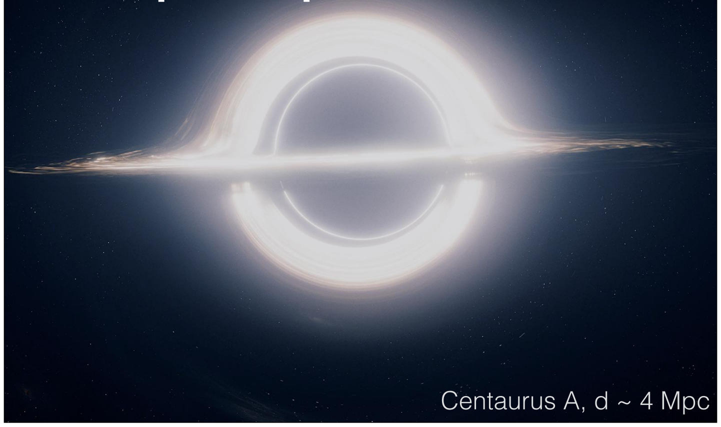
https://iopscience.iop.org/journal/2041-8205/page/Focus_on_EHT

5 petabytes (PB) of data

"More than **200 members from 59 institutes in 20 countries and regions** have devoted years to the effort, all unified by a common scientific vision."

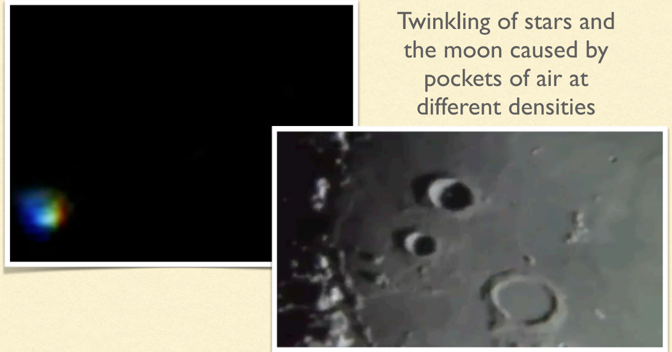


Resolving the sphere of influence: Adaptive Optics and Hubble



Centaurus A, d ~ 4 Mpc

ATMOSPHERE TURBULENCE

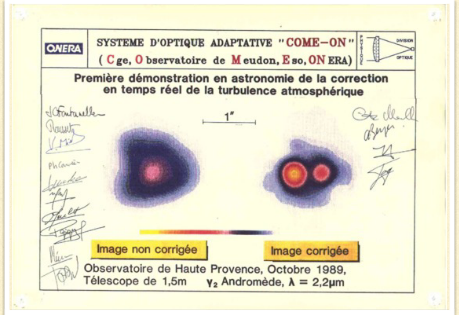


Twinkling of stars and the moon caused by pockets of air at different densities

THE IDEA OF ADAPTIVE OPTICS

- “If we had the means of continually **measuring** the deviation of rays from all parts of the mirror, and of amplifying and **feeding back** this information so as to correct locally the figure of the mirror in response to the schlieren pattern, we could expect to **compensate both for the seeing and for any inherent imperfection of the optical figure**” (**Horace Babcock 1953 PASP**)

1989/10, THE BREAK-THROUGH



ONERA SYSTEME D'OPTIQUE ADAPTATIVE "COME–ON"
(C ge, O bservatoire de M eudon, E so, ON ERA)

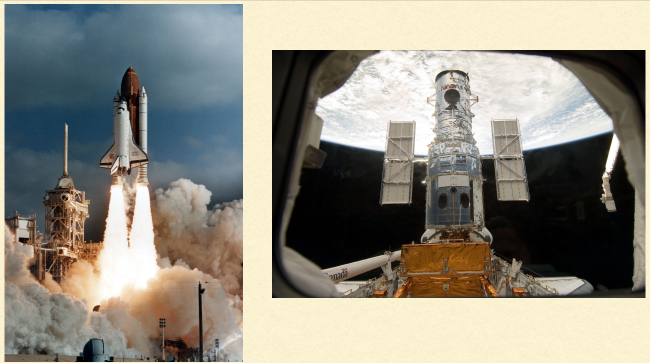
Première démonstration en astronomie de la correction en temps réel de la turbulence atmosphérique

Observatoire de Haute Provence, Octobre 1989,
Télescope de 1,5m γ_2 Andromède, $\lambda = 2{,}2\mu m$

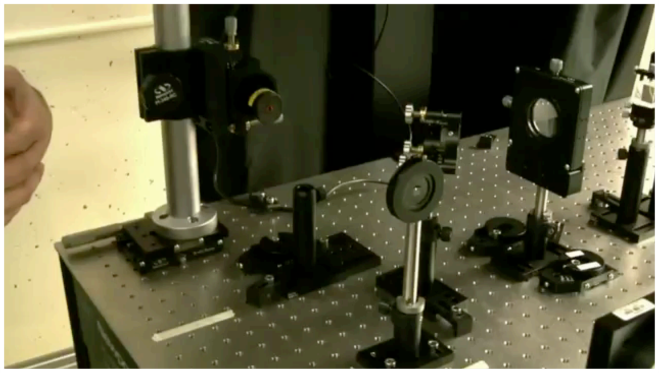
- 1989, the first AO for astronomy worked: COME-ON on the 1.5-m telescope of Haute-Provence Observatory in southern France.
- In 1991, the US military declassified most of the development work on AO to astronomers.

“An old dream of ground-based astronomers has finally come true”, Merkle+1989

1990/4/24: HST LAUNCHED



A SIMPLIFIED AO OPTICAL BENCH



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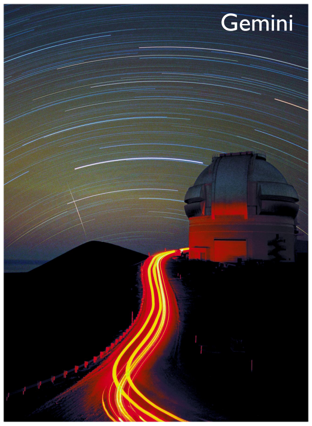
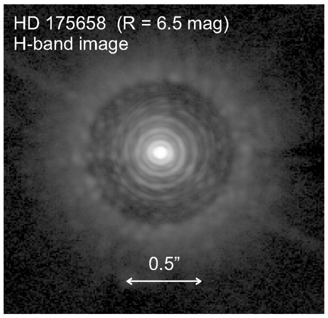
20 YEARS LATER, AO HAS BECOME STANDARD



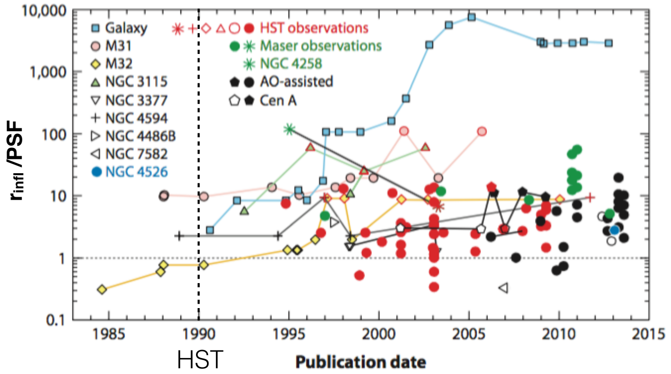
STATE-OF-THE-ART



Deformable secondary mirror of LBT, 1-m across, costs $1m!

Esposito+2010

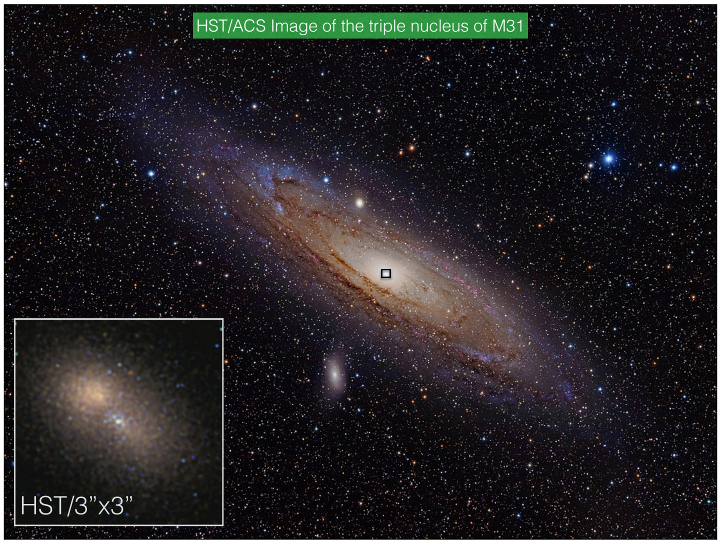
Key: Resolving the Sphere of Influence

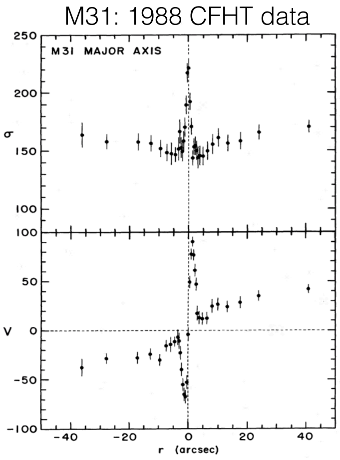


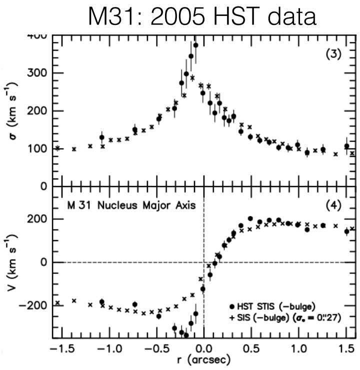
HST

AO

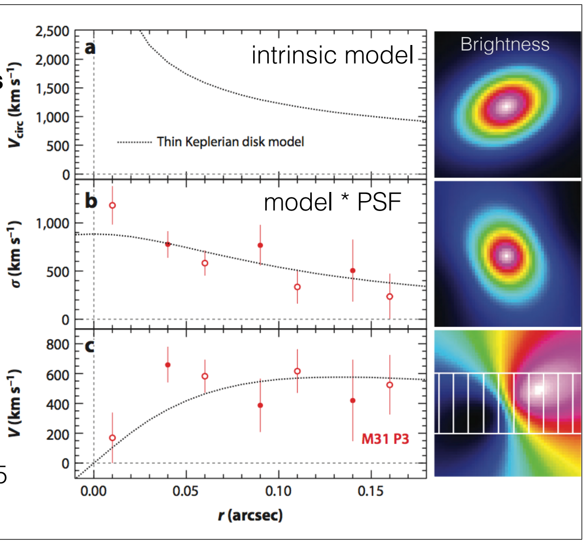
Kormendy & Ho 2013





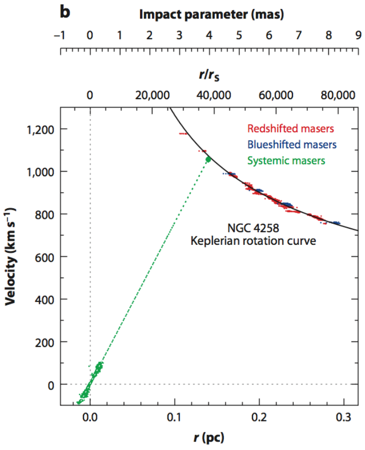


Stellar Dynamics



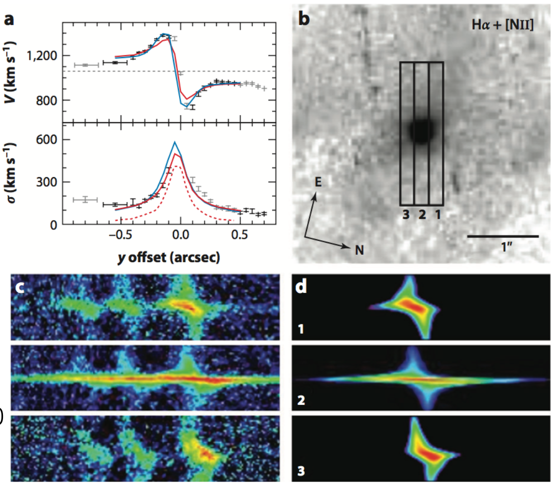
Bender+05

H_2O Maser Dynamics in NGC 4258



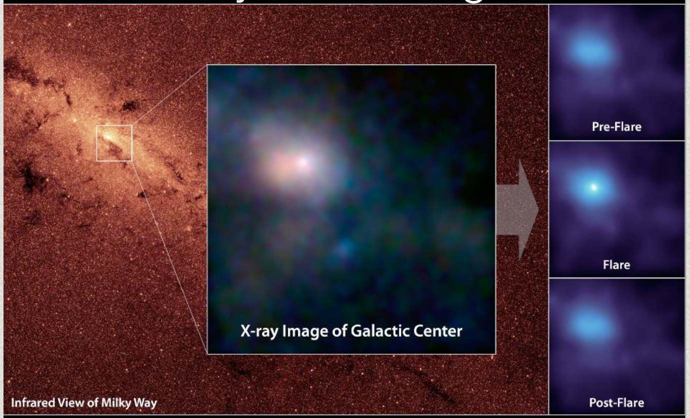
Argon+07, Moran 08

Ionized Gas Dynamics

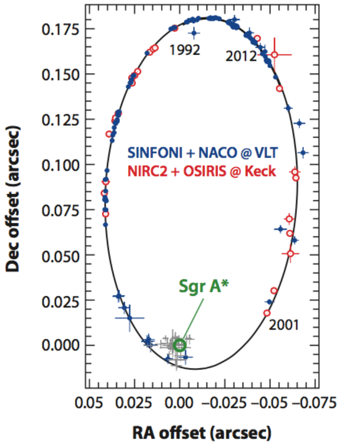


NGC 4374
D = 17 Mpc
HST/STIS
Walsh+2010

X-ray Flare of Sgr A*



NuSTAR, October 2012

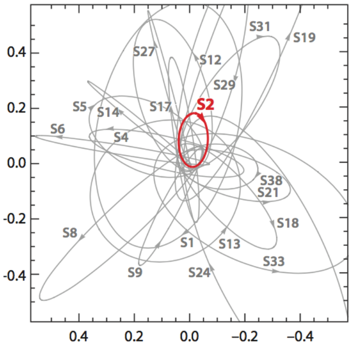


$$M_\bullet = 4.30 \pm 0.20(\mathrm{stat}) \pm 0.30(\mathrm{sys}) \times 10^6\ \mathrm{M}_\odot$$

$$R_0 = 8.28(\pm 0.15)_{\mathrm{stat}}(\pm 0.29)_{\mathrm{sys}}\ \mathrm{kpc}$$

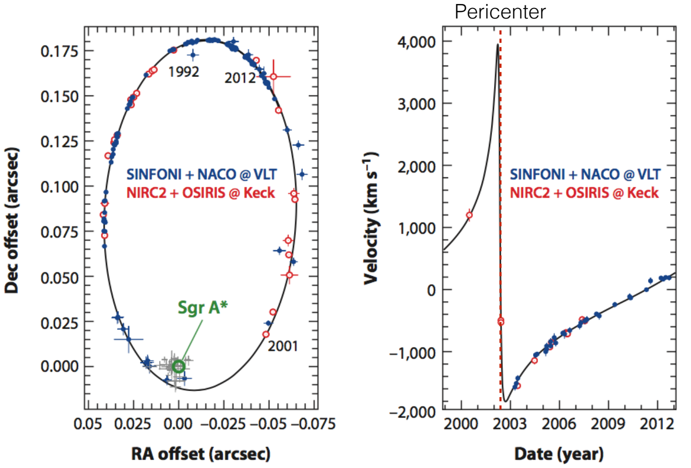
$$\rho_\bullet > 10^{19.5}\ \mathrm{M}_\odot\ \mathrm{pc}^{-3}$$

$$M_{\mathrm{extended}}/M_\bullet < \text{a few} \times 10^{-2}$$



Genzel & Ghez Groups

Pericenter



Genzel & Ghez Groups

M$_{\mathrm{BH}}$-sigma* relation

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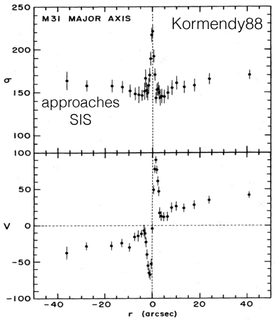
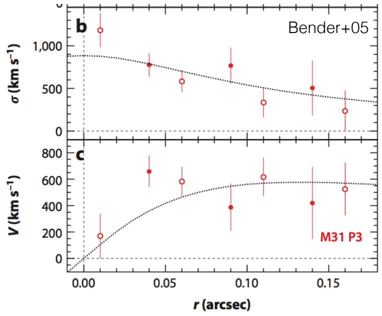
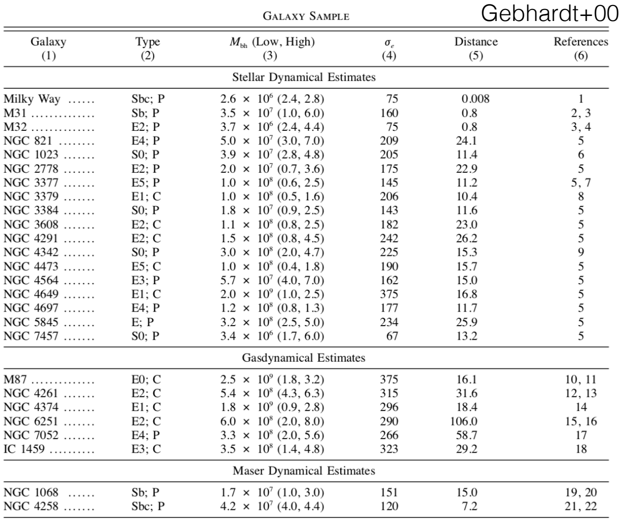
on larger scale



on SMBH scale

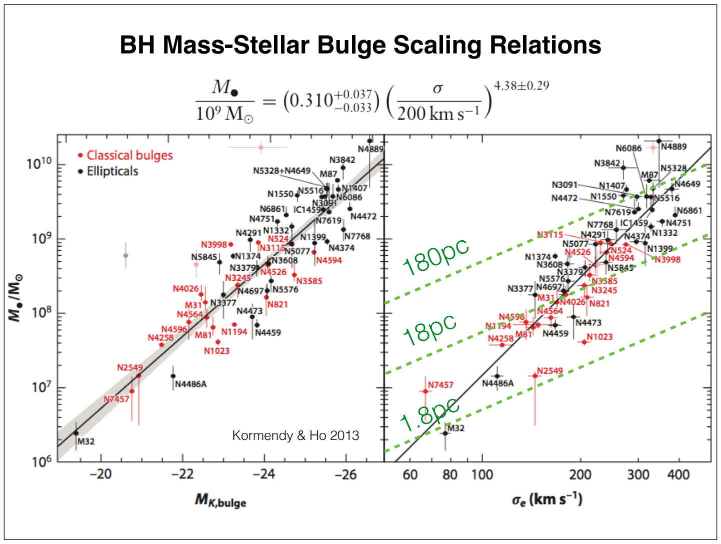


Gebhardt+00



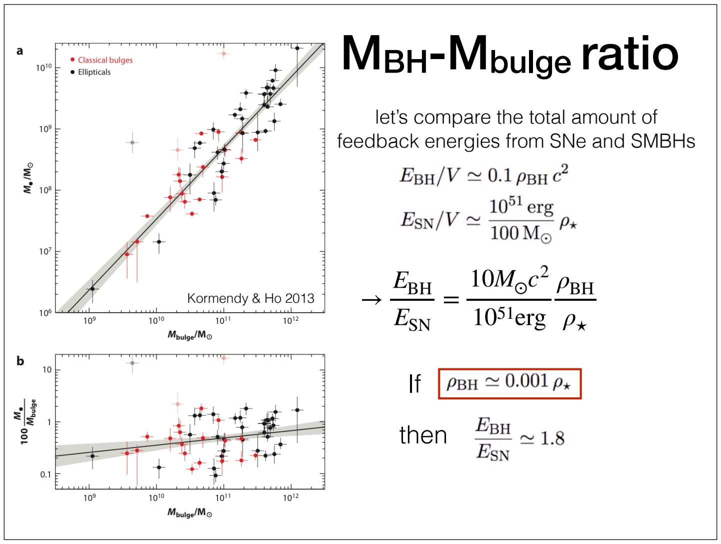
Galaxy Sample

Galaxy (1)	Type (2)	M_{bh} (Low, High) (3)	σ_e (4)	Distance (5)	References (6)
Stellar Dynamical Estimates					
Milky Way ……	Sbc; P	2.6×10^6 (2.4, 2.8)	75	0.008	1
M31 …………..	Sb; P	3.5×10^7 (1.0, 6.0)	160	0.8	2, 3
M32 …………..	E2; P	3.7×10^6 (2.4, 4.4)	75	0.8	3, 4
NGC 821 ……..	E4; P	5.0×10^7 (3.0, 7.0)	209	24.1	5
NGC 1023 …….	S0; P	3.9×10^7 (2.8, 4.8)	205	11.4	6
NGC 2778 …….	E2; P	2.0×10^7 (0.7, 3.6)	175	22.9	5
NGC 3377 …….	E5; P	1.0×10^8 (0.6, 2.5)	145	11.2	5, 7
NGC 3379 …….	E1; C	1.0×10^8 (0.5, 1.6)	206	10.4	8
NGC 3384 …….	S0; P	1.8×10^7 (0.9, 2.5)	143	11.6	5
NGC 3608 …….	E2; C	1.1×10^8 (0.8, 2.5)	182	23.0	5
NGC 4291 …….	E2; C	1.5×10^8 (0.8, 4.5)	242	26.2	5
NGC 4342 …….	S0; P	3.0×10^8 (2.0, 4.7)	225	15.3	9
NGC 4473 …….	E5; C	1.0×10^8 (0.4, 1.8)	190	15.7	5
NGC 4564 …….	E3; P	5.7×10^7 (4.0, 7.0)	162	15.0	5
NGC 4649 …….	E1; C	2.0×10^9 (1.0, 2.5)	375	16.8	5
NGC 4697 …….	E4; P	1.2×10^8 (0.8, 1.3)	177	11.7	5
NGC 5845 …….	E; P	3.2×10^8 (2.5, 5.0)	234	25.9	5
NGC 7457 …….	S0; P	3.4×10^6 (1.7, 6.0)	67	13.2	5
Gasdynamical Estimates					
M87 …………..	E0; C	2.5×10^9 (1.8, 3.2)	375	16.1	10, 11
NGC 4261 …….	E2; C	5.4×10^8 (4.3, 6.3)	315	31.6	12, 13
NGC 4374 …….	E1; C	1.8×10^9 (0.9, 2.8)	296	18.4	14
NGC 6251 …….	E2; C	6.0×10^8 (2.0, 8.0)	290	106.0	15, 16
NGC 7052 …….	E4; P	3.3×10^8 (2.0, 5.6)	266	58.7	17
IC 1459 ……….	E3; C	3.5×10^8 (1.4, 4.8)	323	29.2	18
Maser Dynamical Estimates					
NGC 1068 ……	Sb; P	1.7×10^7 (1.0, 3.0)	151	15.0	19, 20
NGC 4258 …….	Sbc; P	4.2×10^7 (4.0, 4.4)	120	7.2	21, 22



BH Mass-Stellar Bulge Scaling Relations

$$\frac{M_\bullet}{10^9\,\mathrm{M}_\odot} = \left(0.310^{+0.037}_{-0.033}\right)\left(\frac{\sigma}{200\,\mathrm{km\,s^{-1}}}\right)^{4.38\pm0.29}$$



MBH-Mbulge ratio

let's compare the total amount of feedback energies from SNe and SMBHs

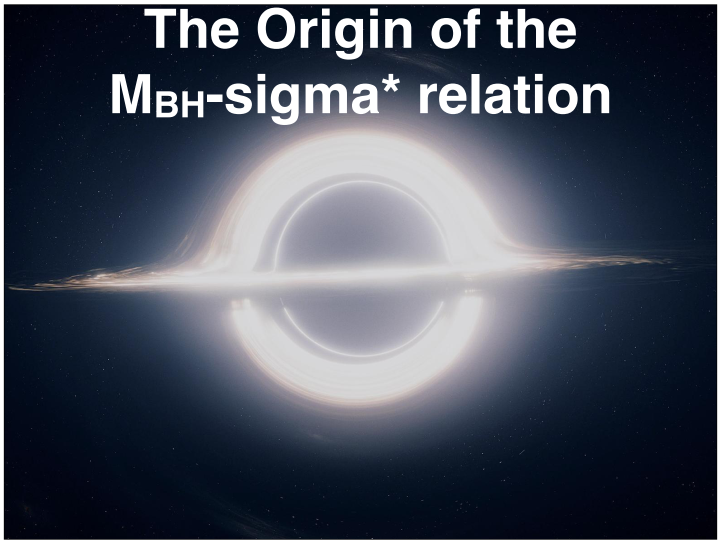
$$E_{\mathrm{BH}}/V \simeq 0.1\,\rho_{\mathrm{BH}}\,c^2$$

$$E_{\mathrm{SN}}/V \simeq \frac{10^{51}\,\mathrm{erg}}{100\,\mathrm{M}_\odot}\,\rho_\star$$

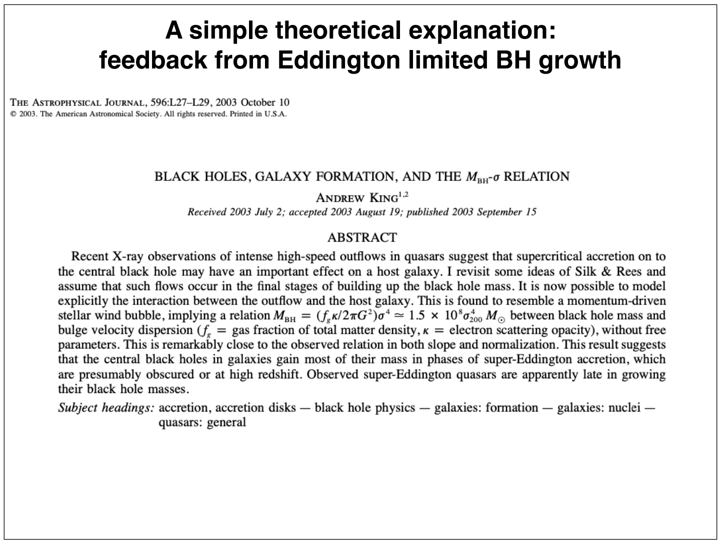
$$\rightarrow \frac{E_{\mathrm{BH}}}{E_{\mathrm{SN}}} = \frac{10 M_\odot c^2}{10^{51}\mathrm{erg}}\frac{\rho_{\mathrm{BH}}}{\rho_\star}$$

If $\boxed{\rho_{\mathrm{BH}} \simeq 0.001\,\rho_\star}$

then $\frac{E_{\mathrm{BH}}}{E_{\mathrm{SN}}} \simeq 1.8$



The Origin of the MBH-sigma* relation



A simple theoretical explanation: feedback from Eddington limited BH growth

The Astrophysical Journal, 596:L27–L29, 2003 October 10

BLACK HOLES, GALAXY FORMATION, AND THE M_{BH}-σ RELATION

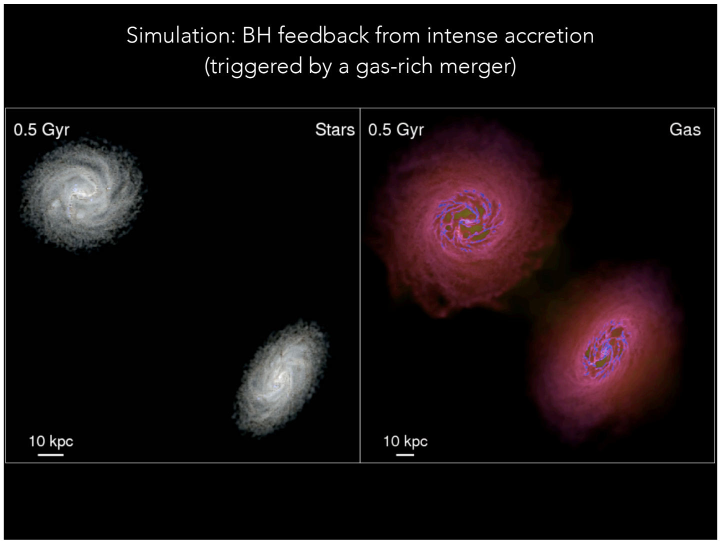
Andrew King[1,2]

Received 2003 July 2; accepted 2003 August 19; published 2003 September 15

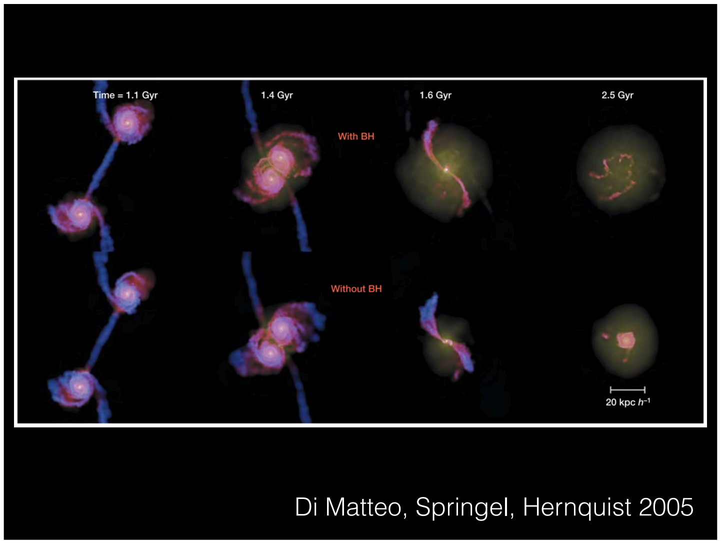
ABSTRACT

Recent X-ray observations of intense high-speed outflows in quasars suggest that supercritical accretion on to the central black hole may have an important effect on a host galaxy. I revisit some ideas of Silk & Rees and assume that such flows occur in the final stages of building up the black hole mass. It is now possible to model explicitly the interaction between the outflow and the host galaxy. This is found to resemble a momentum-driven stellar wind bubble, implying a relation $M_{\mathrm{BH}} = (f_g\kappa/2\pi G^2)\sigma^4 \simeq 1.5 \times 10^8\sigma^4_{200}\,M_\odot$ between black hole mass and bulge velocity dispersion (f_g = gas fraction of total matter density, κ = electron scattering opacity), without free parameters. This is remarkably close to the observed relation in both slope and normalization. This result suggests that the central black holes in galaxies gain most of their mass in phases of super-Eddington accretion, which are presumably obscured or at high redshift. Observed super-Eddington quasars are apparently late in growing their black hole masses.

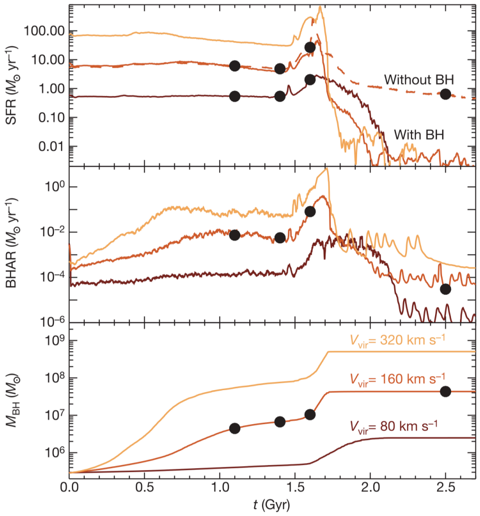
Subject headings: accretion, accretion disks — black hole physics — galaxies: formation — galaxies: nuclei — quasars: general



Simulation: BH feedback from intense accretion
(triggered by a gas-rich merger)

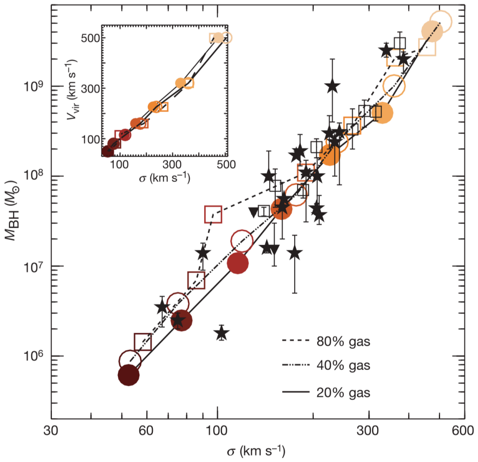


Di Matteo, Springel, Hernquist 2005



Di Matteo, Springel, Hernquist 2005

Merger models w/ feedback reproduces the M-sigma relation



Di Matteo, Springel, Hernquist 2005

SMBHs and AGN

- ***Measuring the mass of SMBHs in normal galaxies***
  - resolving the gravitational sphere of influence
  - enabled by AO and HST observations (1989)
  - M_{BH}-sigma* relation (2000)
- **Understanding the M_{BH}-sigma* relation**
  - How SMBHs moderate the growth of galaxies
- ***Active Galactic Nuclei (AGN): Quasars (1963)/QSOs 1965)/Radio Galaxies (1949)/Seyferts (1943)***
  - Accretion disk physics
  - Deducing the innermost structures of AGN
  - Measuring BH masses in AGN: reverberation mapping and the BLR size-luminosity relation

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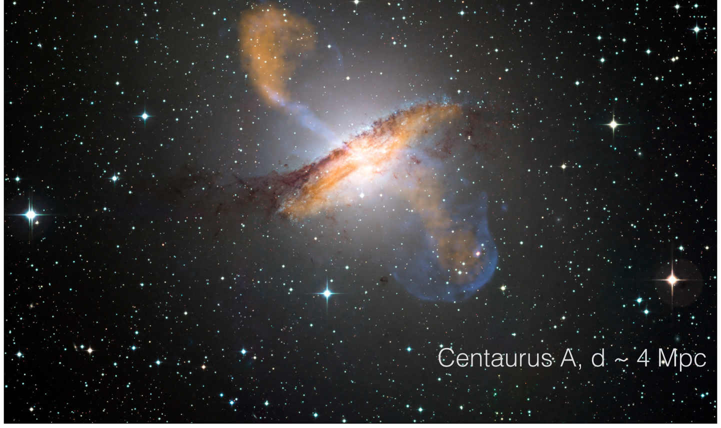
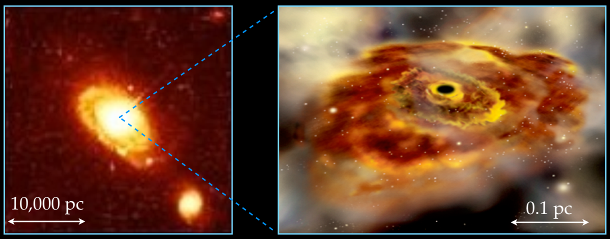
Structure of the AGN Central Engine



Centaurus A, d ~ 4 Mpc

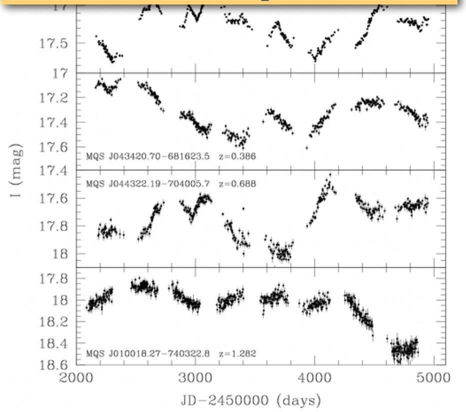
Why do we believe that quasars are powered by accreting SMBHs?

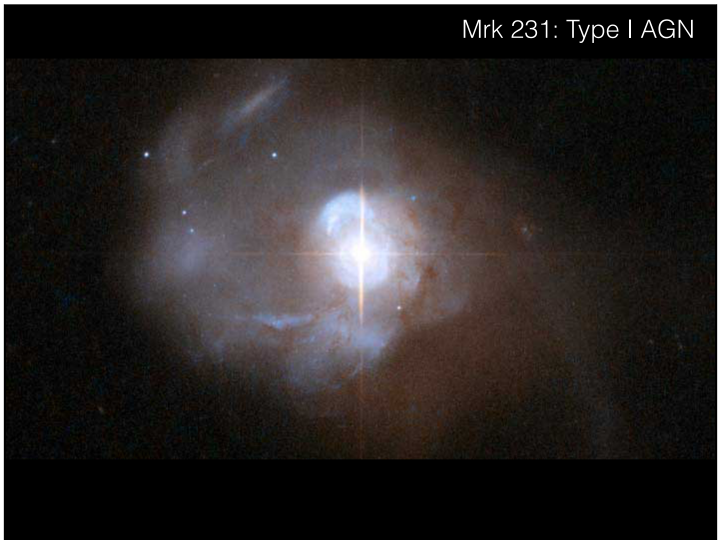


Hoyle et al (1964), Lynden-Bell (1969)

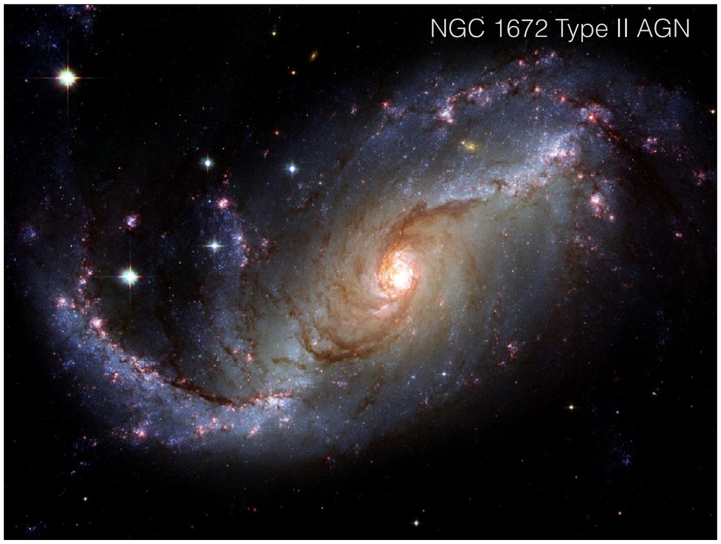
Quasar Light Curves: Rapid Variations

$c \times 1$ week = 0.006 pc = 1212 AU

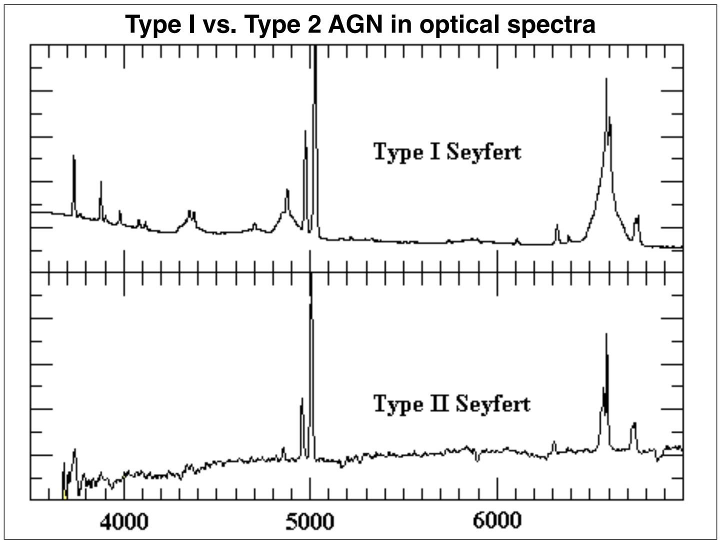




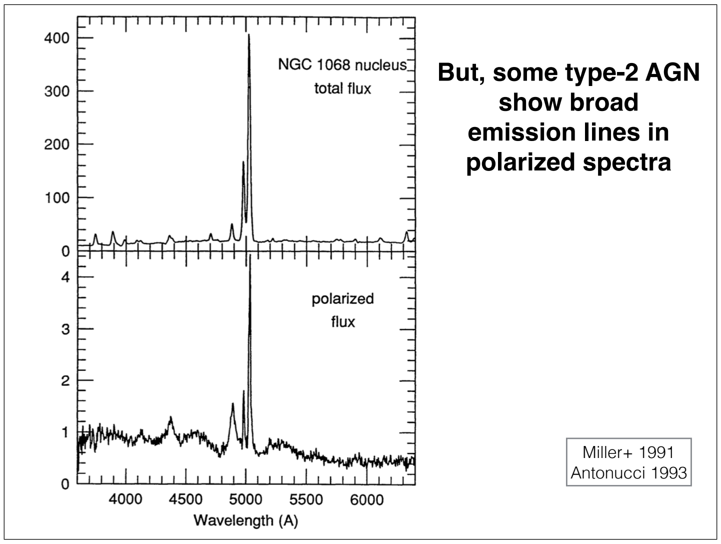
Mrk 231: Type I AGN



NGC 1672 Type II AGN

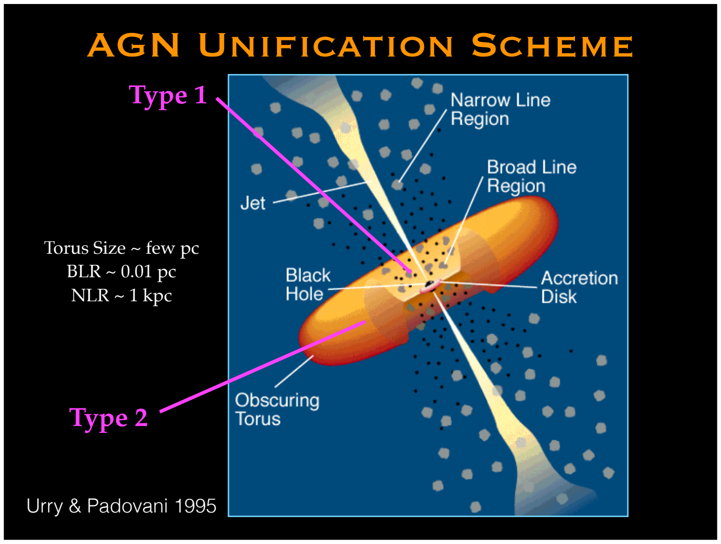


Type I vs. Type 2 AGN in optical spectra



But, some type-2 AGN show broad emission lines in polarized spectra

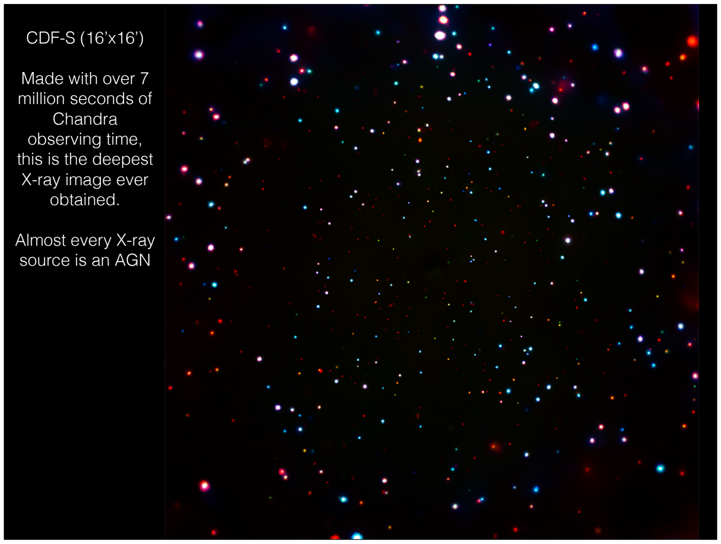
Miller+ 1991
Antonucci 1993



AGN Unification Scheme

Torus Size ~ few pc
BLR ~ 0.01 pc
NLR ~ 1 kpc

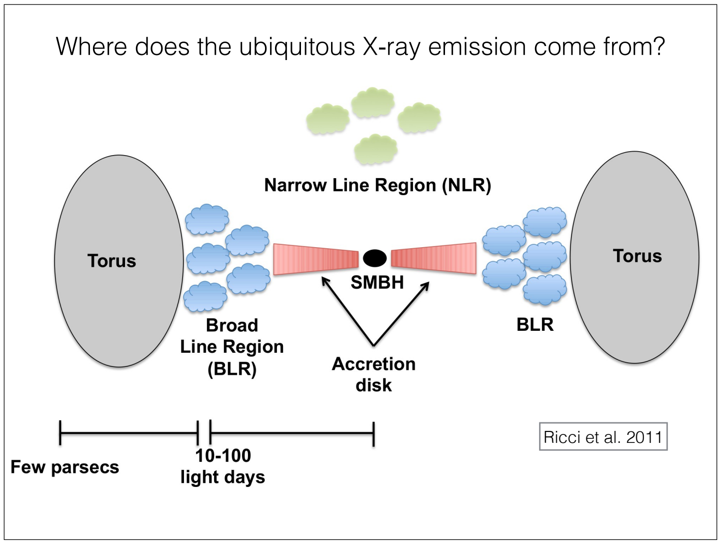
Urry & Padovani 1995



CDF-S (16'x16')

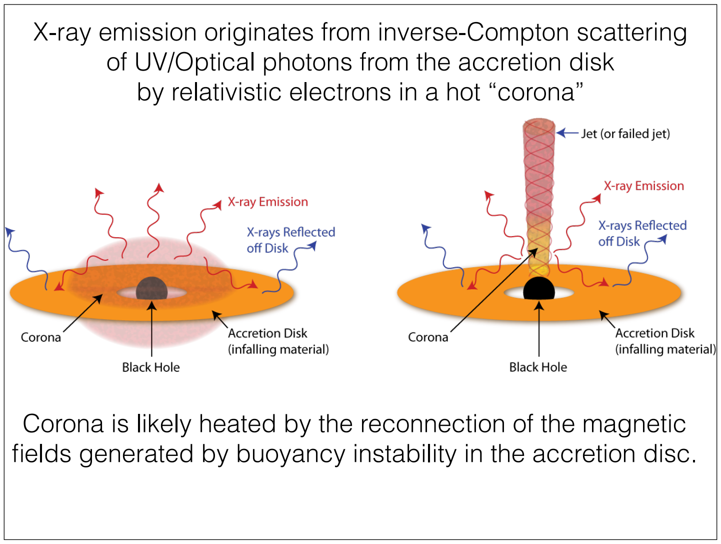
Made with over 7 million seconds of Chandra observing time, this is the deepest X-ray image ever obtained.

Almost every X-ray source is an AGN



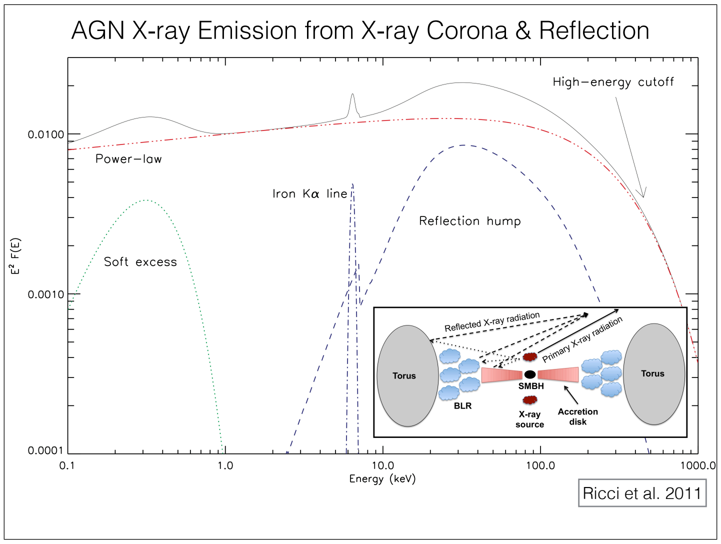
Where does the ubiquitous X-ray emission come from?

Ricci et al. 2011



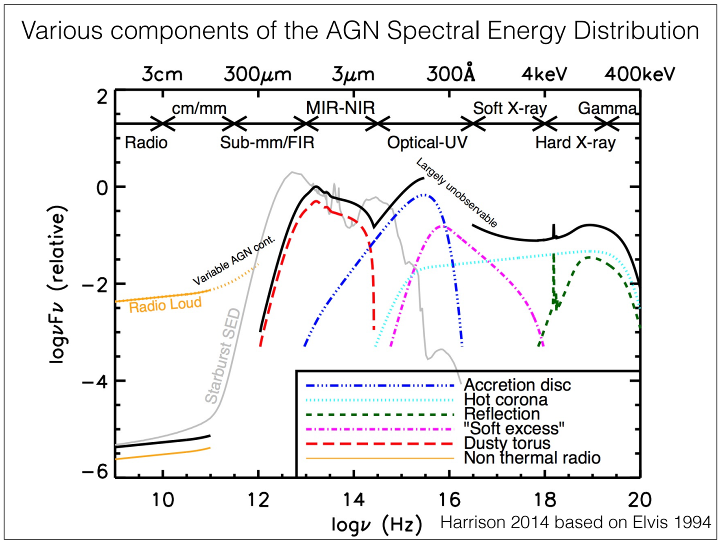
X-ray emission originates from inverse-Compton scattering of UV/Optical photons from the accretion disk by relativistic electrons in a hot “corona”

Corona is likely heated by the reconnection of the magnetic fields generated by buoyancy instability in the accretion disc.



AGN X-ray Emission from X-ray Corona & Reflection

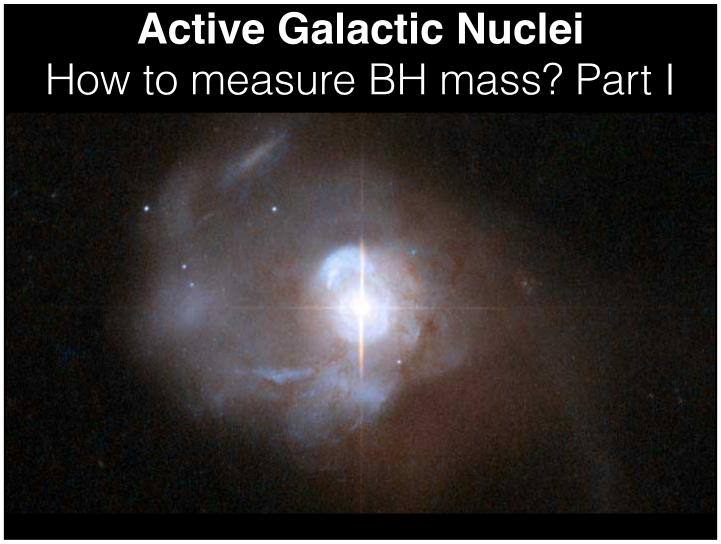
Ricci et al. 2011



Various components of the AGN Spectral Energy Distribution

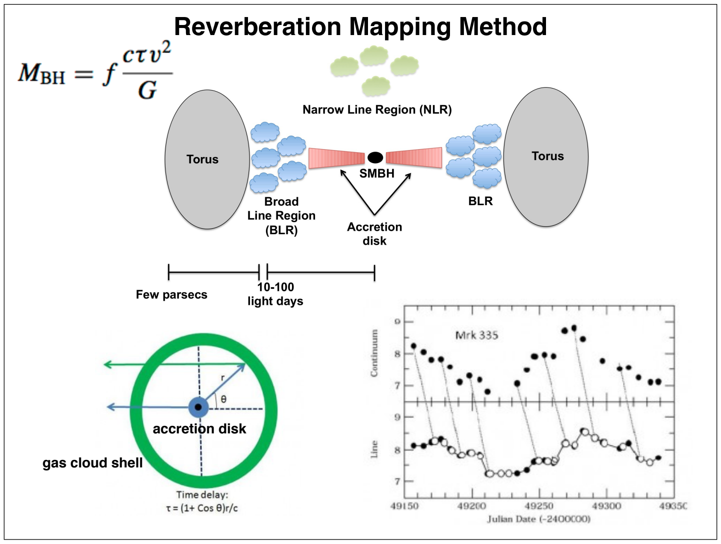
- · · - · ·	Accretion disc
· · · · · ·	Hot corona
- - - - - -	Reflection
- · - · - ·	"Soft excess"
— — — —	Dusty torus
———	Non thermal radio

Harrison 2014 based on Elvis 1994



Active Galactic Nuclei

How to measure BH mass? Part I

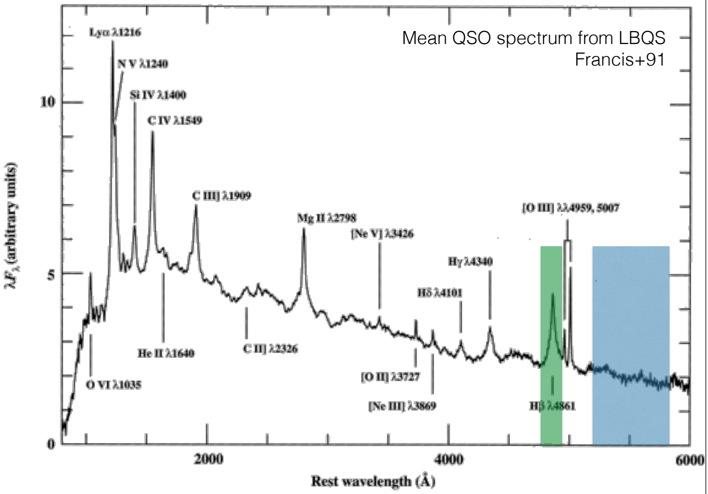


Reverberation Mapping Method

$$M_{\rm BH} = f\,\frac{c\tau v^2}{G}$$

Time delay:
τ = (1+ Cos θ)r/c

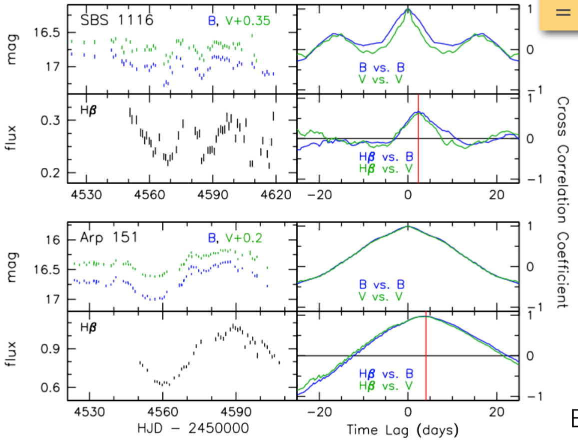
Continuum from accretion disk, emission lines from gas around



Mean QSO spectrum from LBQS
Francis+91

Continuum vs. Line Light Curves
luminosity-weighted time lags

1 light week
= 0.006 pc
= 1212 AU



Bentz+09

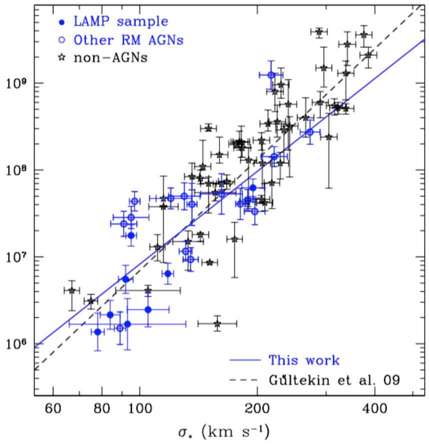
Constrain the virial coefficient

$$M_{\rm BH} = f\,\frac{c\tau v^2}{G}$$

f: virial coefficient

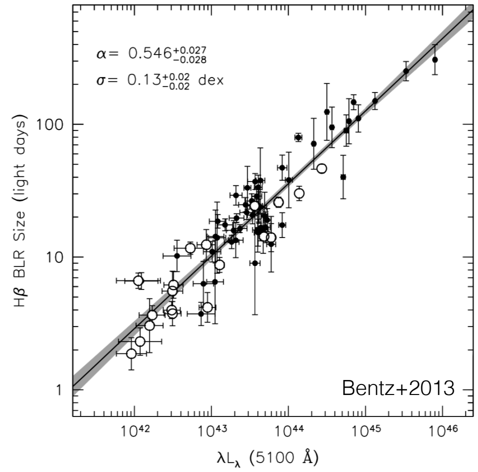
Sample: 24 nearby
broad-line AGNs

By matching the
M-sigma relation
of inactive galaxies
f = 5.2
Woo+2010



Active Galactic Nuclei
How to measure BH mass? Part II

With the size-luminosity relation, one can estimate BH mass without



Bentz+2013

$$M_{\rm BH} = f\,\frac{c\tau v^2}{G}$$

BH mass from **single-epoch** spectroscopy
(Vestergaard & Peterson 2006)

$$\log M_{\rm BH}({\rm H}\beta) = \log\left\{\left[\frac{{\rm FWHM}({\rm H}\beta)}{1000\ {\rm km\ s^{-1}}}\right]^2 \left[\frac{\lambda L_\lambda(5100\ {\rm \AA})}{10^{44}\ {\rm ergs\ s^{-1}}}\right]^{0.50}\right\} + (6.91 \pm 0.02). \qquad (5)$$

$$\log\left(\frac{M_{\rm BH}}{M_\odot}\right) = 6.66 + 0.53\log\left(\frac{L_{1350}}{10^{44}\ {\rm erg\ s^{-1}}}\right) + 2\log\left(\frac{{\rm FWHM}_{\rm C\,IV}}{10^3\ {\rm km\ s^{-1}}}\right).$$

Summary

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