Halo Mass Function - Press-Schechter 1974

Results from SC model:

$$\begin{cases} a = A(1-\cos\theta) \\ t = B(\theta - \sin\theta) \end{cases} \quad A = a_i/2\delta_i \quad B = 1/2H_i\delta_i^{3/2} \quad ; \quad 1+\delta = \frac{9}{2}\frac{(\theta-\sin\theta)^2}{(1-\cos\theta)^3}$$

	θ	a	t	$1+\delta$	$(1+\delta)_{lin}$
turn-around / max radius	π	a_i/δ_i	$\pi/2H_i\delta_i^{3/2}$	$\frac{9}{16}\pi^2$	2.062
collapse / virialization	2π	$a_i/2\delta_i$	$\pi/H_i\delta_i^{3/2}$	$18\pi^2$	2.686

Mass of the virialized halo:

$$M_i = \frac{4\pi}{3}\bar{\rho}_i (a_i r_o)^3 (1+\delta_i) \simeq \frac{4\pi}{3}(\bar{\rho}_i a_i^3)\cdot r_o^3 \propto r_o^3$$

$$M_V = \frac{4\pi}{3}(1+\delta)_{vir}\cdot\bar{\rho}_{vir}\cdot(a_{vir}\cdot r_o)^3 = \frac{4\pi}{3}\left[\bar{\rho}_{vir}(1+\delta)_{vir}a_{vir}^3\right]\cdot r_o^3 \propto r_o^3$$

it's clear that $M_i = M_V$ because $1+\delta = \frac{\rho}{\bar{\rho}} = \frac{\bar{a}^3}{a^3}$

final mass of the halo depends only on the comoving radius of the overdensity.

Press-Schechter Formalism (1974)

Given an initial Gaussian random density field that's described by $P(k)$, linearly extrapolate it to present day $\delta_0 = \delta_i/a_i$, smooth δ_0 on a size scale $R \equiv (M/\gamma\bar{\rho}_m)^{1/3}$ [spatial/mass smoothing] any peaks with $\delta_M \equiv \delta_0 \otimes W(R) > \delta_c(z) = 1.686(1+z)$ should have collapsed by redshift z & formed halos w/ mass $> M$.

$$F(>M, z) = 2\cdot \mathrm{Prob}\left[\delta_M > \delta_c(z)\right]$$

- $F(>M,z)$ ↑ fraction of mass locked in halos w/ $>M$
- 2 ↑ fudge factor to include masses in underdense regions that get accreted.
- Prob ↑ probability of overdensity above threshold, after mass smoothing

Procedure:

After mass/spatial smoothing, the initial Gaussian random field still maintains the Gaussian PDF, although w/ lower σ_M that depends on M/R. So the probability of the occurrance of $\delta_M > \delta_c$ is:

$$\mathrm{Prob}(\delta_M > \delta_c) = \frac{1}{\sqrt{2\pi}\,\sigma_M}\int_{\delta_c}^{\infty} \exp\left(-\frac{\delta_M^2}{2\sigma_M^2}\right) d\delta_M = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_M}\right)$$

$$\Rightarrow F(>M, z) = \mathrm{erfc}\left[\frac{\delta_c(z)}{\sqrt{2}\sigma_M}\right] \text{ ; fraction of mass locked}$$

therefore, the cumulative halo mass function is:

$$n(>M, z) = \frac{\bar{\rho}}{M}\cdot F(>M, z) \text{ ; comoving volume density}$$

and the differential halo mass function is:

$$\phi(M, z) = \frac{\bar{\rho}}{M}\frac{\partial F}{\partial M} = \sqrt{\frac{2}{\pi}}\cdot\frac{\bar{\rho}}{M^2}\frac{\delta_c(z)}{\sigma_M}\exp\left[-\frac{\delta_c^2(z)}{2\sigma_M^2}\right]\left|\frac{d\ln\sigma_M}{d\ln M}\right|$$

$\phi(M,z)\,dM$ = comoving volume density of halos w/ mass $\in [M, M+dM]$ at z

the exp fun makes it natural to define M^*, characteristic mass, as:

$\sigma_M(M^*) = \delta_c(z) = 1.686(1+z) \Rightarrow M^* \downarrow$ as $z \uparrow$

at low-mass end:

$M \ll M^*$, $\phi(M) \propto M^{\alpha-2}$ where $\alpha = \left|\frac{d\ln\sigma_M}{d\ln M}\right|$ or $\sigma_M \propto M^{-\alpha}$

because $\sigma_M \gg \delta_c(z)$ at low mass or large $k\ (\equiv 2\pi/R)$

for power-law power spectrum $P(k) \propto k^n$, $\sigma_M \propto M^{-(3+n)/6}$

at high-mass end:

$$M \gg M^*,\quad \phi(M) \propto \exp\left[-\frac{1}{2}\left(\frac{M}{M^*}\right)^{2\alpha}\right]\cdot M^{\alpha-2},\quad \alpha = \frac{3+n}{6}$$

powerlaw at low mass end, exponential cutoff at high mass end

How to calculate σ_M? The mass variance

$$\sigma_M^2 = \langle \delta_M^2 \rangle = \frac{1}{V}\int_{-\infty}^{\infty} \delta_M^2(\vec{x})\, d^3\vec{x} = \langle \delta_M(\vec{x}) \cdot \delta_M(\vec{x}+0) \rangle = \xi_M(0)$$

the variance of a density field equals the 2-point correlation function at zero dist.

$$\delta_M(\vec{x}) = \delta(\vec{x}) \otimes W_M(\vec{x}) = \int \delta(\vec{x}')\, W(\vec{x}-\vec{x}'; M)\, d^3\vec{x}'$$

mass smoothed density field is a convolution w/ a window function at all locations.

because convolution is computationally expensive, and $\delta(\vec{x})$ is a random Gaussian field that is completely described by the correlation function or the power spectrum, we can utilize the convolution theorem & the total energy theorem to calculate σ_M in its Fourier space:

$$\delta_M(\vec{x}) = \delta(\vec{x}) \otimes W_M(\vec{x}) \longleftrightarrow \delta_M(\vec{k}) = \delta(\vec{k}) \cdot \tilde{W}_M(\vec{k})$$

$$\sigma_M^2 = \frac{1}{V}\int_{-\infty}^{\infty} \left[\delta(\vec{x}) \otimes W_M(\vec{x})\right]^2 d^3\vec{x}$$

$$= \frac{V}{(2\pi)^3}\int_{-\infty}^{\infty} \delta^2(\vec{k}) \cdot \tilde{W}_M^2(\vec{k})\, d^3\vec{k}$$

$$= \frac{1}{2\pi^2}\int_0^{\infty} P(k)\, \tilde{W}_M^2(\vec{k})\, k^2\, dk$$

$$\mathcal{F}\{g \otimes h\} = \mathcal{F}(g) \cdot \mathcal{F}(h) \qquad \begin{cases} g = \delta(\vec{x}) \\ h = W_M(\vec{x}) \end{cases}$$

total energy is the same:

$$\frac{1}{V}\int |g \otimes h|^2 d^3x = \frac{V}{(2\pi)^3}\int |\mathcal{F}(g) \cdot \mathcal{F}(h)|^2 d^3k$$

For power spectrum that follows a power-law:

$$P(k) \propto k^n$$

$$\sigma_M^2 \propto \int_0^K k^{n+2}\, dk \qquad \text{because } \tilde{W}_K(k) \to 0 \text{ when } k > K = \frac{2\pi}{R}$$

$$\propto K^{n+3} \propto M^{-(n+3)/3}$$

monotonically decreasing function with M when $n > -3$

Summary: Press-Schechter HMF.

① fraction of mass locked in halos of masses greater than M at z is

$$F(>M, z) = \mathrm{erfc}\left[\frac{\delta_c(z)}{2\sigma_M}\right]$$

② differential halo mass function:

$$\phi(M, z) = \frac{\bar{\rho}_m}{M} \cdot \frac{\partial F}{\partial M} = \frac{\bar{\rho}_m}{M} \frac{\partial F}{\partial \sigma_M} \cdot \frac{\partial \sigma_M}{\partial M}$$

$$= \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}_m}{M^2} \cdot \frac{\delta_c(z)}{\sigma_M} \cdot \exp\left[-\frac{\delta_c^2(z)}{2\sigma_M^2}\right] \left|\frac{d\ln\sigma_M}{d\ln M}\right|$$

③ calculate σ_M^2, mass variance (dimensionless)

$$\sigma_M^2 \equiv \frac{1}{V}\int \delta_M^2(\vec{x})\, d^3\vec{x} = \frac{1}{V}\int |\delta(\vec{x}) * W_M(\vec{x})|^2 d^3\vec{x}$$

$$= \xi_M(0) \quad \leftarrow \text{correlation function at zero separation}$$

$$= \frac{1}{(2\pi)^3}\int P_M(k)\, e^{+i\vec{k}\cdot\vec{r}}\, d^3\vec{k} \quad \text{for } \vec{r} = 0$$

$$= \frac{1}{2\pi^2}\int_0^\infty P_M(k)\, k^2 dk$$

$$= \frac{1}{2\pi^2}\int_0^\infty P(k) \cdot \tilde{W}_M^2(k) \cdot k^2 dk$$

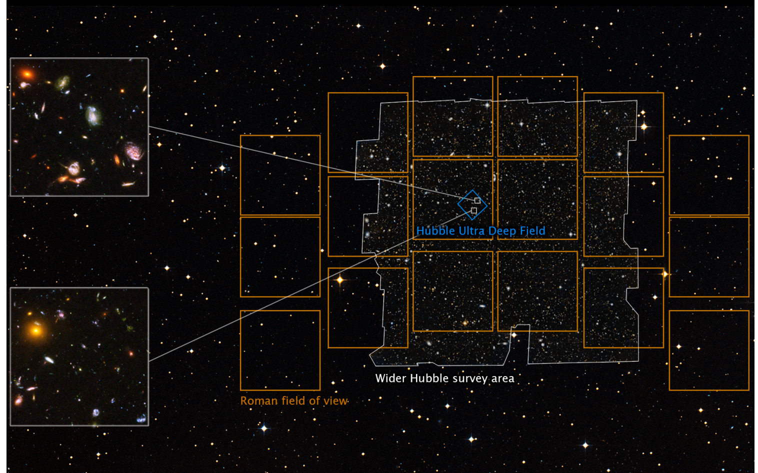
$$\simeq \frac{1}{2\pi^2}\int_0^{2\pi/R} P(k)\, k^2 dk, \quad R = (M/\gamma\bar{\rho}_m)^{1/3}, \gamma \simeq \frac{4}{3}\pi$$

④ higher overdensities collapse earlier

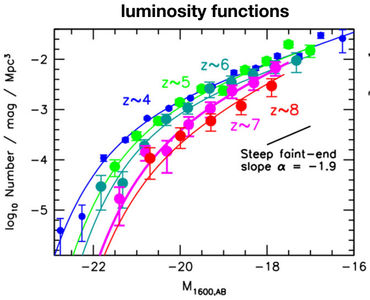
$$\delta_c(z) = 1.686\,(1+z)$$

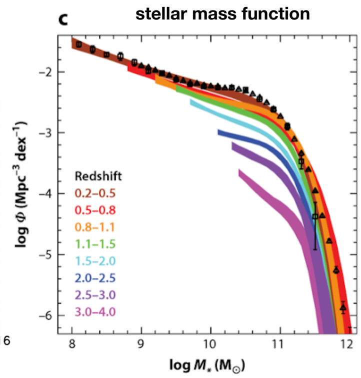
⑤ evaluate $\phi\left(M, z; \sigma_M, \frac{d\ln\sigma_M}{d\ln M}\right)$

Motivation: explaining the observed galaxy luminosity / stellar mass functions



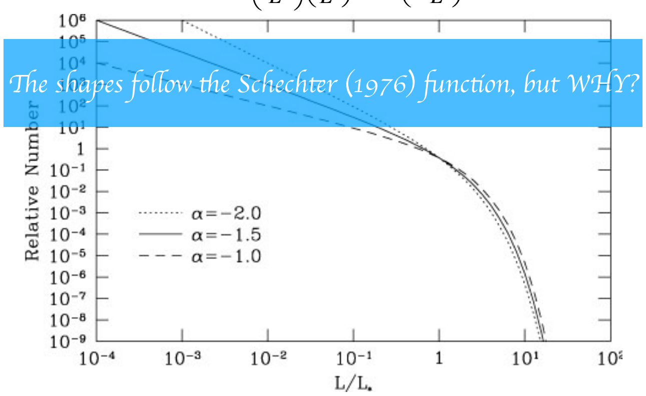
Observed evolution of galaxy luminosity functions and stellar mass functions





comoving density of galaxies with luminosity between L and L+dL is

$$\Phi(L)\mathrm{d}L = \left(\frac{\Phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-\frac{L}{L^*}\right)\mathrm{d}L,$$



The shapes follow the Schechter (1976) function, but WHY?

The Basic Idea of the Press-Schechter Formalism

The Goal

Based on the results from the spherical collapse model:

- Estimate the comoving volume density of collapsed halos more massive than M, at any z:

$$n_{halo}(>M, z)$$

cumulative mass function

- Estimate the comoving volume density of collapsed halos within a mass range of [M, M+dM], at any z:

$$\phi_{halo}(M, z)dM = \frac{dn_{halo}(>M, z)}{dM}dM$$

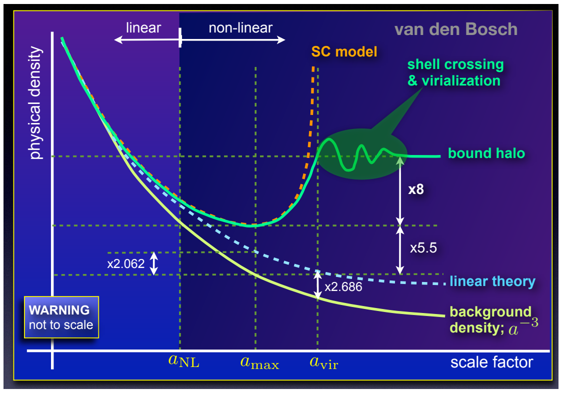
differential mass function

Spherical Collapse Model

- any region in the density field (*linearly extrapolated to today*) denser than certain threshold should have collapsed by redshift z:

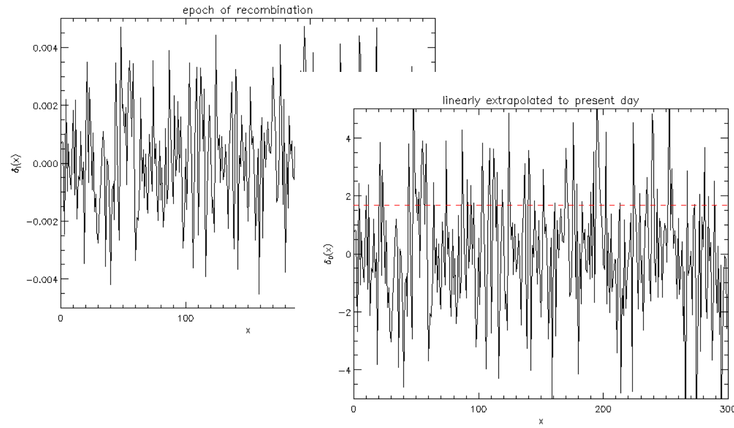
$$\delta(t_0) > \delta_c(z) = 1.686(1+z)$$

- collapsed halo mass: $M = \gamma \bar{\rho} R^3$



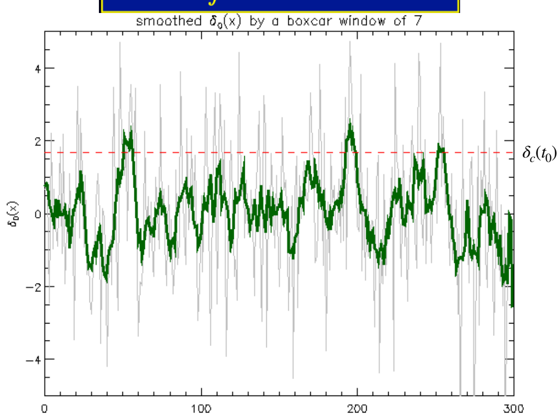
Linearly Extrapolated Density Perturbation Field

$$\delta(x;t_0) = \delta(x;t_i)\ a(t_0)/a(t_i) = \delta(x;t_i)\ (1+z_i)$$



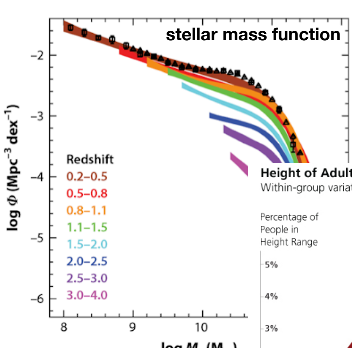
Density field smoothed on a scale of R ~ (M/rho)$^{1/3}$

$$\delta(\vec{x};R) \equiv \int \delta(\vec{x}')\, W(\vec{x}-\vec{x}';R)\, \mathrm{d}^3\vec{x}'$$

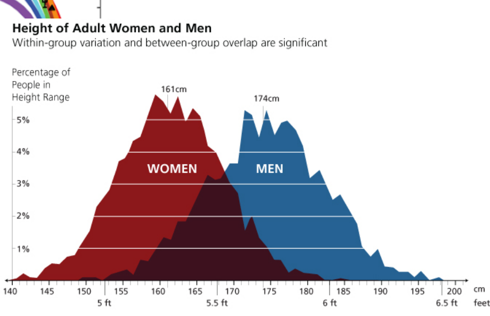


The Basic Idea of the Press-Schechter Formalism (continued)

Observed galaxy stellar mass functions



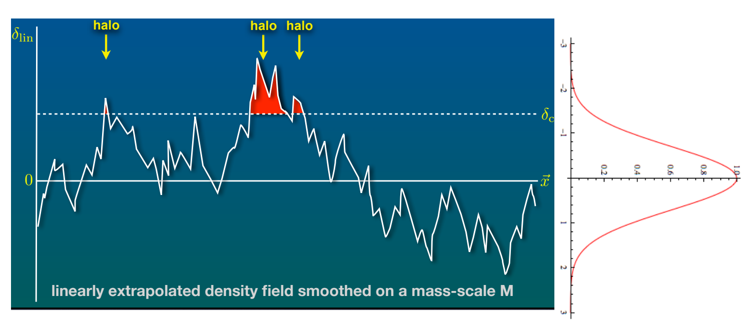
$$\Phi(L)\mathrm{d}L = \left(\frac{\Phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-\frac{L}{L^*}\right)\mathrm{d}L,$$



The Basic Idea of Press-Schechter Formalism (1974)

Let δ_M be the linear density field smoothed on a mass scale M, i.e., $\delta_M = \delta(\vec{x};R)$ where $M = \gamma_f\,\bar{\rho}\,R^3$, then those locations where $\delta_M = \delta_c(t)$ are the locations where, at time t, a halo of mass M condenses out of the evolving density field....

$$F(>M) = \mathscr{P}[\delta_M > \delta_c(z)];\ \text{where}\ \delta_c(z) = 1.686\ (1+z)$$

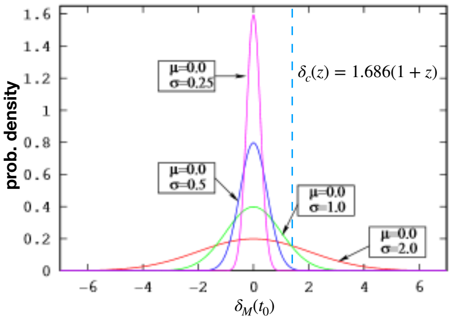


Calculate the probability above the collapse barrier

For Gaussian random fields, the prob. of finding an overdensity greater than a threshold is:

$$\mathcal{P}(\delta_M > \delta_c) = \frac{1}{\sqrt{2\pi}\,\sigma_M}\int_{\delta_c}^{\infty} \exp\left[-\frac{\delta_M^2}{2\sigma_M^2}\right] \mathrm{d}\delta_M = \frac{1}{2}\mathrm{erfc}\left[\frac{\delta_c}{2\sigma_M}\right]$$

which only depends on (1) the threshold $\delta_c(z)$ and (2) the variance of the smoothed field σ_M; note that $\delta_M(x)$ has been integrated out.



Convolution in spatial dimension decreases the width of the Gaussian PDF (prob. density function); i.e., higher mass halos are more unlikely than lower mass halos

From Probability to *Differential* Halo Mass Function

The PS postulate: the fraction of mass locked up in halos w/ mass > *M* is (*fudge factor 2 is used to account for mass in underdense regions*):

$$F(>M, z) = 2\mathscr{P}[\delta_M > \delta_c(z)]$$

the fraction of mass locked up in halos in the mass range [*M*,*M*+*dM*] is:

$$\frac{dF(>M)}{dM}dM = 2\frac{d\mathscr{P}}{dM}dM = 2\frac{d\mathscr{P}}{d\sigma_M}\frac{d\sigma_M}{dM}dM$$

multiplying the above by the $\bar{\rho}$ gives the total locked mass per unit volume, which is then divided by *M* to give the comoving volume density of halos with masses between [*M, M+dM*], i.e., $\phi(M,z)dM$:

$$\phi dM = \frac{\bar{\rho}}{M}\frac{dF(>M,z)}{dM}dM = 2\frac{\bar{\rho}}{M}\frac{d\mathscr{P}}{d\sigma_M}\frac{d\sigma_M}{dM}dM;$$

$$\frac{d\mathscr{P}}{d\sigma_M} = \frac{1}{\sqrt{2\pi}}\frac{\delta_c}{\sigma_M^2}\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right);\ \text{also}\ \frac{d\sigma_M}{dM} = \frac{\sigma_M}{M}\frac{d\ln\sigma_M}{d\ln M}$$

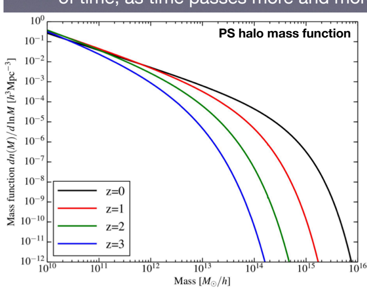
we have the **final result**:

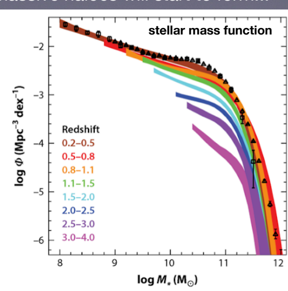
$$\phi(M,z) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma_M}\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)\left|\frac{d\ln\sigma_M}{d\ln M}\right|$$

Differential Halo Mass Function: $\phi(M,z) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma_M^2}\exp\left(-\frac{\delta_c^2}{2\sigma_M}\right)\left|\frac{d\ln\sigma_M}{d\ln M}\right|$

If we define a characteristic mass, M^*, by $\sigma(M^*) = \delta_c(t)$

- For $M \ll M^*$ we have that $n(M,t) \propto M^{\alpha-2}$, where $\alpha = \mathrm{d}\ln\sigma/\mathrm{d}\ln M$
  For a CDM cosmology $\alpha \to 0$ at low mass end so that $n(M) \propto M^{-2}$
- For $M \gg M^*$ the abundance of haloes is exponentially suppressed.
- Since $\delta_c(t)$ decreases with time, the characteristic halo mass grows as function of time; as time passes more and more massive haloes will start to form...





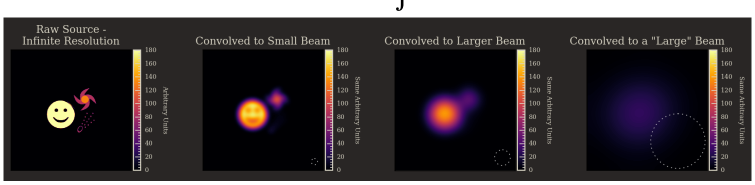
$$\phi(M,z) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma_M}\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)\left|\frac{d\ln\sigma_M}{d\ln M}\right|$$

To make quantitative comparisons one needs to calculate σ_M

Variance of a smoothed density field

smoothing is a **convolution** of the density field w/ a window function of width R ($M = \gamma\bar{\rho}R^3$):

$$\delta_M(x) = \delta(x) * W_M(x) = \int \delta(x')W_M(x'-x)d^3x'$$



the **variance** of the **smoothed** density field is then:

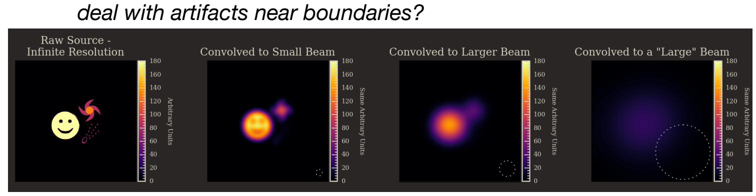
$$\sigma_M^2 = \langle\delta_M^2(x)\rangle = \frac{1}{V}\int \delta_M^2(x)d^3x = \frac{1}{V}\int |\delta(x) * W_M(x)|^2 d^3x$$

Variance of a smoothed density field

$$\sigma_M^2 = \frac{1}{V}\int_{-\infty}^{\infty}\delta_M^2(x)d^3x = \frac{1}{V}\int_{-\infty}^{\infty}|\delta(x) * W_M(x)|^2 d^3x$$

There are some major problems going forward:

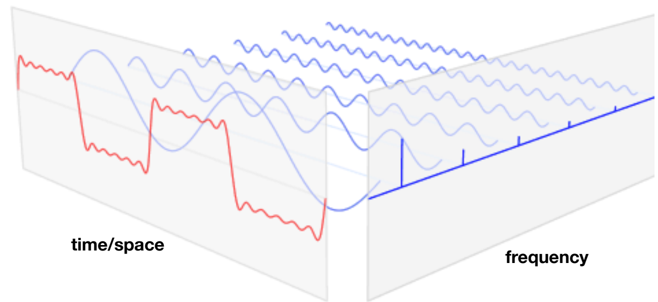
- *the density field is a random field, so would require many realizations, and how to realize a random field that matches the initial conditions of the universe?*
- *what would be the appropriate volume and what scale to use to sample this random field?*
- *numerical convolution is computationally expensive and how to deal with artifacts near boundaries?*



the calculation of σ_M

Simplify the calculation by converting to Fourier space

Consider transforming to Fourier space



Decomposing a *periodic* time/space signal into Fourier series

The Fourier transform

we'll be interested in signals defined for all t

the **Fourier transform** of a signal f is the function

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

- F is a function of a *real* variable ω; the function value $F(\omega)$ is (in general) a complex number

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cos\omega t \, dt - j \int_{-\infty}^{\infty} f(t) \sin\omega t \, dt$$

- $|F(\omega)|$ is called the *amplitude spectrum* of f; $\angle F(\omega)$ is the *phase spectrum* of f
- notation: $F = \mathcal{F}(f)$ means F is the Fourier transform of f; as for

FT and inverse FT in 1D

time domain:

$$F(\omega) = \int f(t)\ e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int F(\omega) e^{i\omega t} d\omega$$

angular freq.: $\omega = \dfrac{2\pi}{\Delta t}$

1D space domain:

$$F(k) = \int f(x)\ e^{-ikx} dx$$

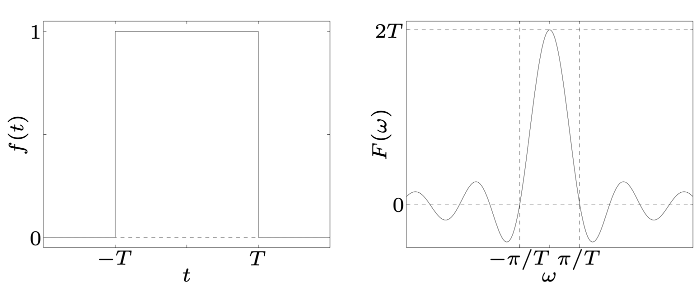
$$f(x) = \frac{1}{2\pi} \int F(k) e^{ikx} dk$$

wave number: $k = \dfrac{2\pi}{\Delta x}$

Fourier transform of a top-hat filter

rectangular pulse: $f(t) = \begin{cases} 1 & -T \le t \le T \\ 0 & |t| > T \end{cases}$

$$F(\omega) = \int_{-T}^{T} e^{-j\omega t} \, dt = \frac{-1}{j\omega}\left(e^{-j\omega T} - e^{j\omega T}\right) = \frac{2 \sin \omega T}{\omega}$$



Fourier Transform Pairs

random density field in real and frequency space:

$$\delta(\vec{x}) = \sum_k \delta_{\vec{k}}\, e^{+i\vec{k}\cdot\vec{x}} \qquad \delta_{\vec{k}} = \frac{1}{V} \int \delta(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x}$$

window function and its F.T.:

Top Hat Filter: $\gamma_f = 4\pi/3$

$$W(\vec{x}; R) = \begin{cases} \frac{3}{4\pi R^3} & r \le R \\ 0 & r > R \end{cases} \qquad \widetilde{W}(kR) = \frac{3}{(kR)^3}\left[\sin(kR) - (kR)\cos(kR)\right]$$

power spectrum (P) and correlation function (xi):

$$P(k) = V\langle\delta(k')\delta(k'+k)\rangle = \int \xi(x) e^{-i\vec{k}\vec{x}} d^3\vec{x}$$

$$\xi(x) = \langle\delta(x')\delta(x'+x)\rangle = \frac{1}{(2\pi)^3} \int P(k) e^{i\vec{k}\vec{x}} d^3\vec{k}$$

Calculation of σ_M in Fourier space

Convolution theorem: convolution in real space = multiplication in Fourier space

Frequency components of periodic density fluctuations:
Fourier transform of smoothed density field

$$\delta_M(k) = \mathscr{F}\{\delta_M(x)\} = \mathscr{F}\{\delta(x) * W_M(x)\} = \delta(k)\tilde{W}_M(k)$$

Power spectrum of the smoothed density field:
Fourier transform of the correlation function of the smoothed density field

$$P_M(k) = \mathscr{F}\{\xi_M(x)\} = P(k)\tilde{W}_M^{\,2}(k)$$

σ_M expressed in correlation function & power spectrum

definition of the **variance** of the density field:

$$\sigma^2 = \langle \delta^2 \rangle = \frac{1}{V}\int \delta^2(\vec{x})\, \mathrm{d}^3\vec{x}$$

definition of the **two-point correlation function**:

$$\xi(x) = \langle \delta(x')\delta(x'+x) \rangle = \frac{1}{V}\int \delta(x')\delta(x'+x) d^3x$$

$$\text{therefore, } \sigma^2 = \xi(0)$$

Correlation Function is the Fourier transform of the **Power Spectrum** *(vice versa)*

$$\xi(r) = \langle \delta(\vec{x})\delta(\vec{x}+\vec{r}) \rangle = \frac{1}{(2\pi)^3}\int P(k) e^{+i\vec{k}\cdot\vec{r}} \mathrm{d}^3\vec{k}$$

evaluating the **correlation function** at 0 using the **power spectrum**

$$\sigma^2 = \xi(0) = \frac{1}{(2\pi)^3}\int P(k)\mathrm{d}^3\vec{k} = \frac{1}{2\pi^2}\int P(k)\, k^2\, \mathrm{d}k$$

therefore, the variance of the smoothed density field is:

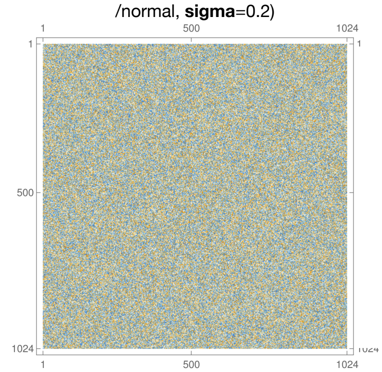
$$\sigma_M^2 = \frac{1}{2\pi^2}\int_0^\infty P_M(k)k^2 dk = \frac{1}{2\pi^2}\int_0^\infty P(k)\tilde{W}_M^{\,2}(k)k^2 dk$$

the calculation of σ_M

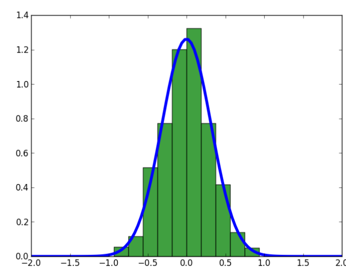
the power spectrum of the Gaussian random field

Gaussian random field: white noise

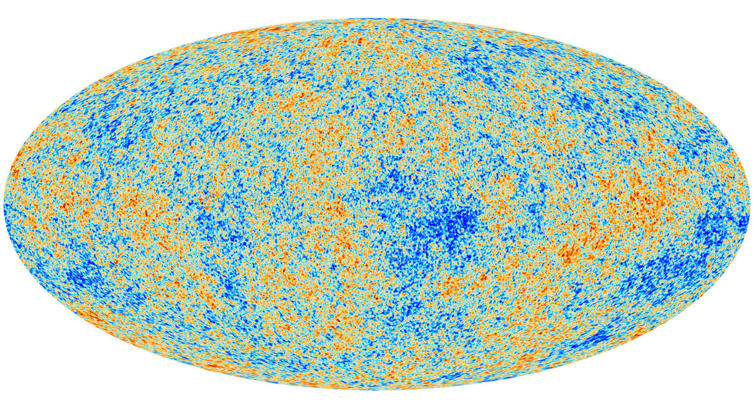
Img = randomn(seed, nx=1024, ny=1024, /normal, **sigma**=0.2)



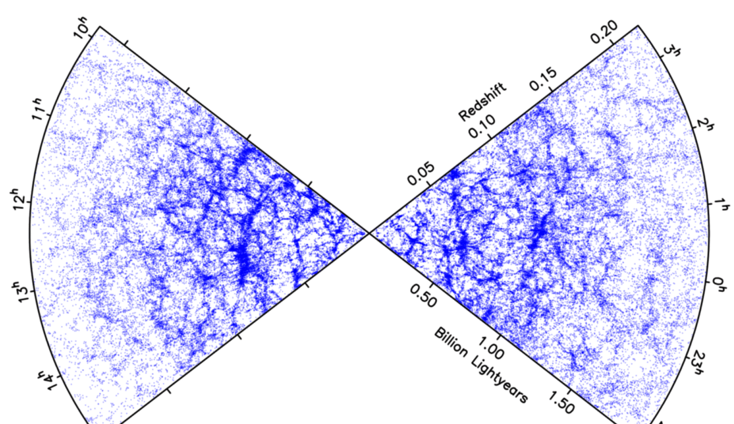
histogram(img, bin=0.2)



Planck CMB delta T Map is not white noise



Galaxy distribution also doesn't look like white noise



2dF Galaxy Redshift Survey (1997-2002)

A Gaussian random field is fully described by the correlation function or its F.T. the power spectrum

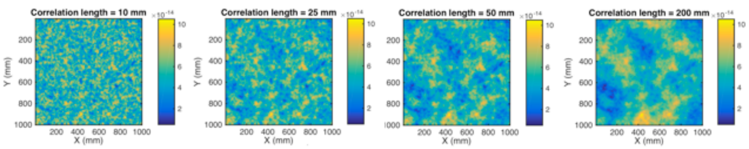
Define two-point **correlation function**:

$$\xi(x) = \langle \delta(x')\delta(x'+x) \rangle = \frac{1}{V}\int \delta(\vec{x'})\delta(\vec{x'}+\vec{x})d^3\vec{x'}$$

and its Fourier transform is the **power spectrum:**

$$P(k) = V\langle \delta(k')\delta(k'+k) \rangle = \int \xi(x) e^{-i\vec{k}\vec{x}} d^3\vec{x}$$

Gaussian random fields w/ increasing correlation lengths



A Gaussian random field is fully described by the correlation function or its F.T. the power spectrum

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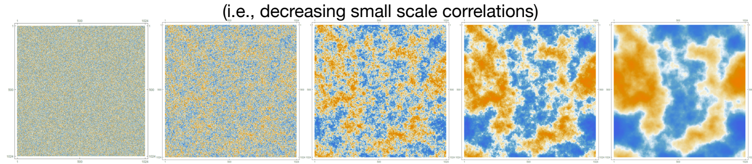
$$\xi(x) = \langle \delta(x')\delta(x'+x) \rangle = \frac{1}{V}\int \delta(\vec{x'})\delta(\vec{x'}+\vec{x})d^3\vec{x'}$$

and its Fourier transform is the **power spectrum:**

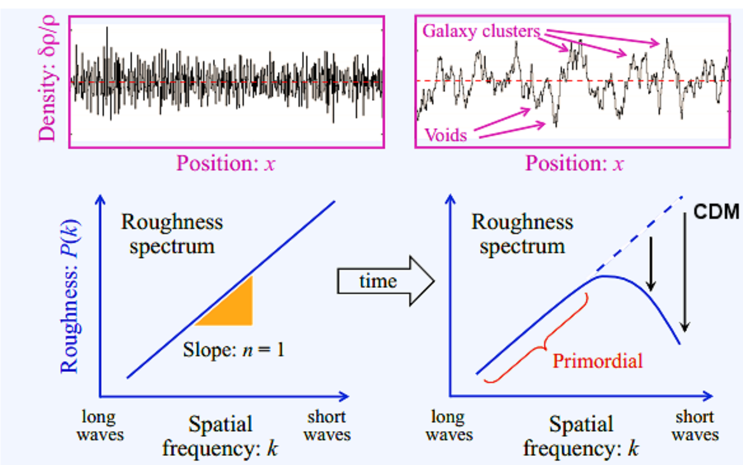
$$P(k) = V\langle \delta(k')\delta(k'+k) \rangle = \int \xi(x) e^{-i\vec{k}\vec{x}} d^3\vec{x}$$

Gaussian random fields w/ ever steeper power spectrum
(i.e., decreasing small scale correlations)

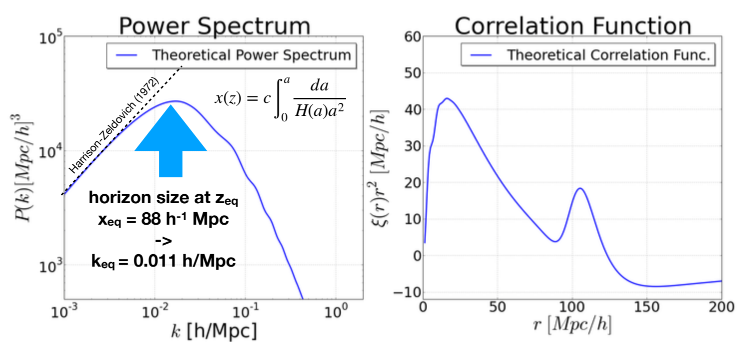


$P(k) = k^0$ $P(k) = k^{-1}$ $P(k) = k^{-2}$ $P(k) = k^{-3}$ $P(k) = k^{-4}$

Evolution of the power spectrum from the primordial density perturbations (due to inflation) to the present day



Linearly evolved correlation function and power spectrum in the Lambda CDM universe



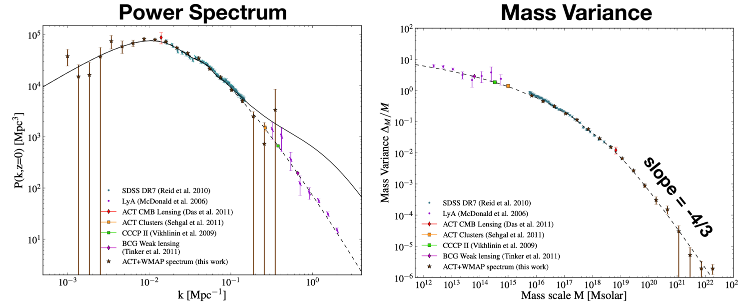
https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

the calculation of σ_M

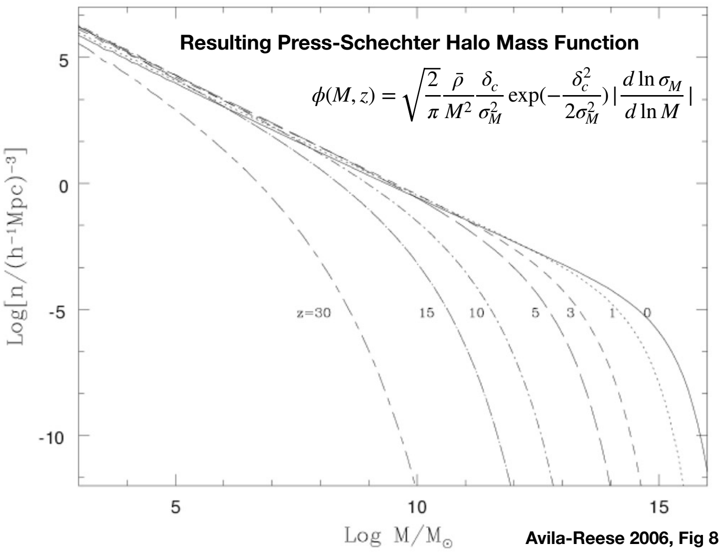
resulting sigma_M function and halo mass function

Resulting mass variance by integrating power spectrum: a monotonically decreasing function of mass scale

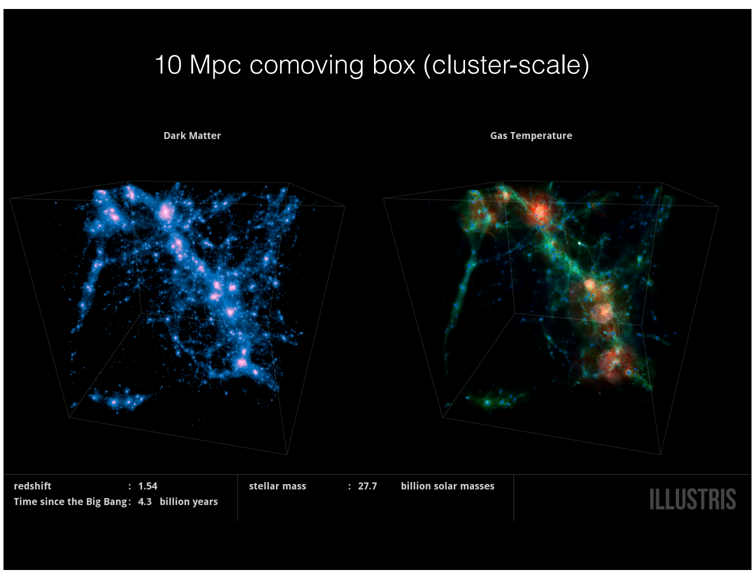
$$\sigma_M^2 = \frac{1}{2\pi^2}\int_0^\infty P(k)\tilde{W}_M^2(k)k^2dk \approx \frac{1}{2\pi^2}\int_0^{2\pi/R} P(k)k^2dk \propto M^{-(3+n_s)/3} \text{ for } P(k) \propto k^{n_s}$$

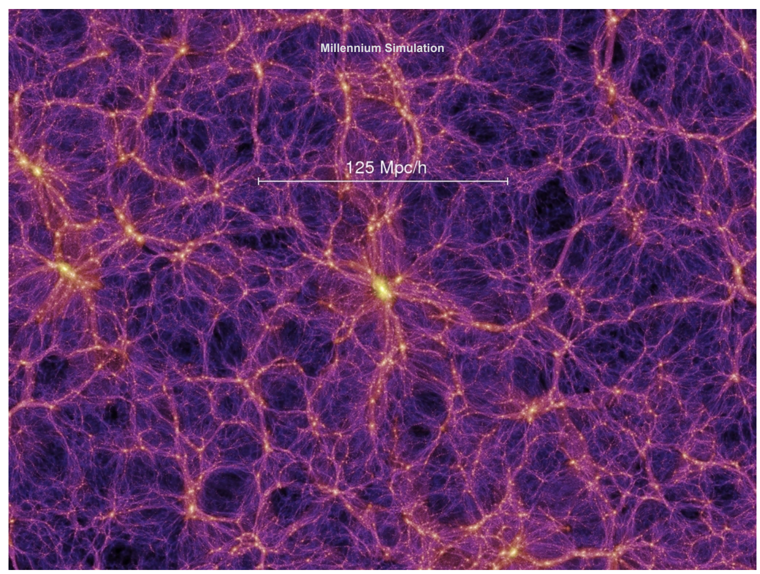


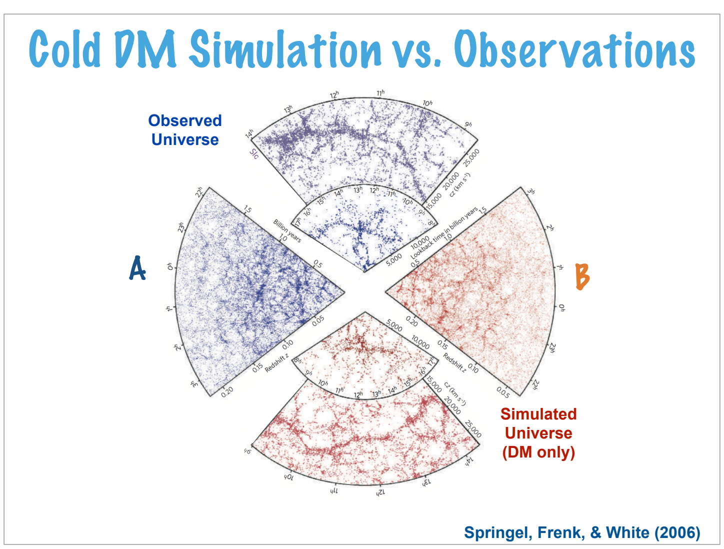
Hlozek et al. 2012: The Atacama Cosmology Telescope: A Measurement of the Primordial Power Spectrum

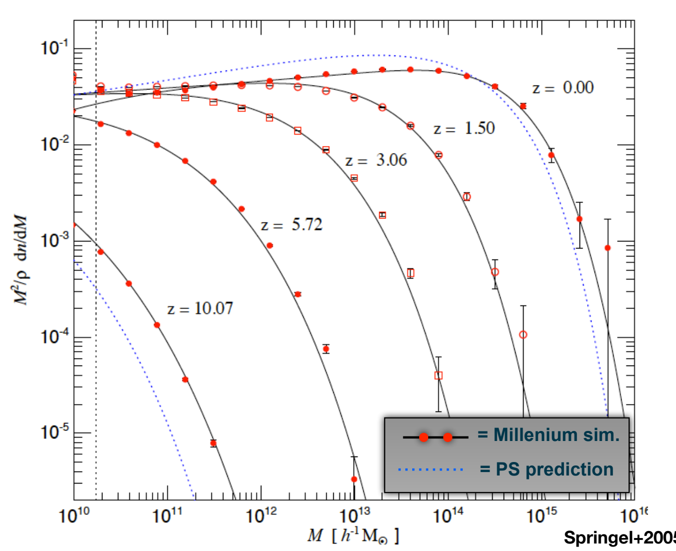


Comparison w/ N-body Simulations

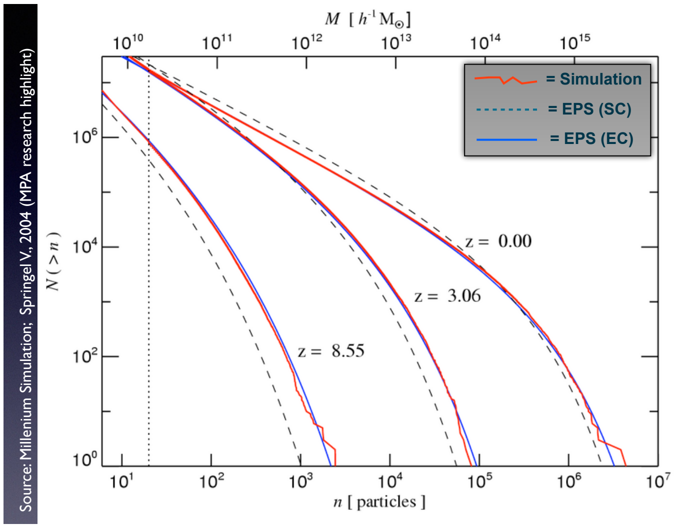








Extended Press-Schechter formalism improves the agreement w/ N-body simulations



Comparison w/ Galaxy Stellar Mass Function

Predicted halo mass function vs. *Observed* galaxy mass function

galaxies turned out to be ***biased*** tracers of DM halos