The Epoch of Recombination

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Success: N-body Simulation agrees with PS Prediction



Problem: both the P-S and N-body halo mass function strongly disagrees w/ Observed galaxy mass function



The PS halo mass function does not have bounds. But it must truncate at some point.

What is the minimum mass of halos?

The Jeans Criteria of Gravitational Instabilities

Stars form in the coldest and densest gaseous regions in the Galaxy



Jeans Criteria: Systems with 2K + U < 0 are unstable and will collapse



i.e., for typical conditions in a dense molecular core, the **lower mass limit** (the **Jeans mass**) is about **8** M_{Sun}.

In the above, we used the density relation: $\rho_0 = n_0 \cdot \mu m_H$

Wikipedia: Barnard 68's interior is extremely cold, its temperature being about 16 K. Its mass is about twice that of the Sun and it measures about half a light-year across (0.15 parsec).

Barnard 68 *a Bok globule*

Jeans Length: the initial radius of the parent cloud

We have derived the **Jeans mass**:

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi\rho_0}\right)^{\frac{1}{2}} = 8 M_{\odot} \left(\frac{T}{10 \text{ K}}\right)^{3/2} \left(\frac{n_0}{10^4 \text{ cm}^{-3}}\right)^{-1/2} \left(\frac{\mu}{2}\right)^{-2}$$

We also know the cloud radius *R* is simply a ratio between *M* and the mean mass density ρ_0 :

$$R = \left(\frac{3M}{4\pi\rho_0}\right)^{\frac{1}{3}}$$

Thus, the Jeans Mass and the initial mean density gives us the **initial radius** of the unstable cloud (**Jeans length**): $R_J = \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{\frac{1}{2}} = 0.1 \text{ pc}\left(\frac{T}{10 \text{ K}}\right)^{1/2} \left(\frac{n_0}{10^4 \text{ cm}^{-3}}\right)^{-1/2} \left(\frac{\mu}{2}\right)^{-1}$

i.e., for typical conditions in dense molecular cores, the **Jeans length** is about **0.1 parsec** or **20,000 AU**.

In the above, we used density relation: $\rho_0 = n_0 \cdot \mu m_H$

Alternative way to think about instabilities: overdensities do not have to collapse, they can also undergo acoustic oscillations



Simple gas cylinder + piston model derivation



MakeAGIF.com

Jeans Criteria: acoustic oscillation vs. gravitational collapse

• Based on continuity equation (mass conservation), Euler's equation (momentum conservation), and Poisson Equation (density-potential pair), the evolution of a density perturbation ($\delta \equiv \rho - \rho_0$) is described by this DE:

$$\frac{\partial^2 \delta}{\partial t^2} = c_s^2 \nabla^2 \delta + 4\pi G \rho_0 \delta$$

• Plug in the general solution $\delta = \overline{\delta}e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, so that $\partial\delta/\partial t = -i\omega\delta$, $\overrightarrow{\nabla}\delta = i\delta\vec{k}$ one can obtain the **dispersion relation**:

$$\omega^2(k) = c_s^2 k^2 - 4\pi G \rho_0$$

The solution has the following scenarios:

- when $\omega^2 > 0$, the perturbation oscillates in time (**acoustic oscillations**)
- when $\omega^2 < 0$, the perturbation grows exponentially (**Jeans criteria**)
- Jeans (minimum) length:

$$\lambda_{J} = \frac{2\pi}{k_{J}} = \sqrt{\frac{\pi c_{s}^{2}}{G\rho_{0}}} = \sqrt{\frac{32}{3}} c_{s} t_{ff} = \sqrt{\frac{32}{3}} \sqrt{\frac{3\pi}{32G\rho_{0}}} \sqrt{\frac{\gamma kT}{\mu m_{H}}} \propto \left(\frac{15kT}{4\pi G\mu m_{H}\rho_{0}}\right)^{\frac{1}{2}}$$

• Jeans (minimum) mass:

$$M_J = \frac{4\pi}{3} \rho_0 \lambda_J^3$$

Chronology of the Universe Diagram





The decoupling epoch marks both the recombination of Hydrogen and the last scatter of CMB photons

 $p^+ + e^- \rightarrow H + \gamma$



Strongly coupled photon-baryon fluid

Evolution of the radial mass profile (comoving) of an initially overdensity

Near the initial time, the photons



Jeans Length vs. Time



Jeans Mass vs. Temperature



The Temperature at Recombination

The Temperature Corresponding to the Ionization Energy of Hydrogen



- To calculate the ionization energy, we can plug in $n_{low} = 1$ and $n_{high} = infinity$.
- What temperature does 13.6 eV correspond to?

$$T = 11604 \text{ K} \left(\frac{E}{1 \text{ eV}}\right)$$

mm-wavelength spectrum of the cosmic microwave background



The CMB spectrum should not be confused with the CMB power spectrum, which is a Fourier transform of the temperature anisotropy



Ionization Balance: Saha Eq. & Boltzmann Dist.

Solar Atmosphere: Understanding the Absorption Line Spectrum

- The Sun displays a complex absorption spectrum from the presence of over 70 elements.
- The strongest lines are from singly-ionized Calcium ions at 3968.5 and 3933.7 Angstroms. This seems quite strange given that Calcium is much more rare than Hydrogen.



Solar Atmosphere: Understanding the Absorption Line Spectrum

• Why the strongest lines in the Solar spectrum are from **singly-ionized Calcium** instead of **neutral hydrogen**? Given that for every 1 Ca atom there are 500,000 H atoms.



The Excitation of Neutral Hydrogen to Higher Energy Levels

 As temperature increases, more and more remaining neutral hydrogen are excited (**Boltzmann Distribution**):

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

where g represent the degeneracy of each excitation state, and N2/N1 is called the excitation ratio.



Ionization Equilibrium: Balance between ionization and recombination

• Saha (1920) Ionization Equation:

$$\frac{N_{II}}{N_I} = \frac{2Z_{II}}{n_e Z_I} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{kT}\right)$$

 N_{II}/N_{I} : number density ratio of two adjacent ionization states, Z_{II}/Z_{I} : Partition functions or degeneracies of the ionization states, χ_{i} : ionization energy to go from I to II state, $n_{e}m_{e}$ are electron density and electron mass



The Combined Effect of Ionization and Excitation

• Causes a peak in the number of desired species that creates the H Balmer lines: neutral hydrogen (H I) at the first excited state (n=2)



Line Strength vs. Temperature / Spectral Type



First Ionization Energies of Common Elements

Element	Symbol	Atomic Number (Z)	First Ionization Energy (eV)
Hydrogen	Н	1	13.5984
Helium	He	2	24.5874
Lithium	Li	3	5.3917
Carbon	С	6	11.2603
Nitrogen	Ν	7	14.5341
Oxygen	0	8	13.6181
Sodium	Na	11	5.1391
Calcium	Ca	20	6.1132
Iron	Fe	26	7.9024

Solar Elemental Abundance (Relative to Hydrogen)

Element	Symbol	Atomic Number (Z)	Logarithmic Abundance ($\log_{10}(N_X/N_H)+12$)	Number Abundance (N_X/N_H)
Hydrogen	Н	1	12.00	1.000
Helium	He	2	10.93	0.0851
Lithium	Li	3	1.05	$1.12 imes 10^{-11}$
Carbon	С	6	8.43	$2.69 imes10^{-4}$
Nitrogen	Ν	7	7.83	$6.76 imes10^{-5}$
Oxygen	0	8	8.69	$4.90 imes10^{-4}$
Sodium	Na	11	6.24	$1.74 imes10^{-6}$
Calcium	Ca	20	6.34	$2.19 imes10^{-6}$
Titanium	Ti	22	4.95	$8.91 imes 10^{-8}$
Iron	Fe	26	7.50	$3.16 imes 10^{-5}$

Solar Elemental Abundance (Relative to Hydrogen)



Dependence of spectral line strengths on temperature



Figure 8.11 Carroll & Ostlie

Why H Balmer lines are strongest in A-type stars?



normalized flux

Apply Saha (1920) Ionization Eq. to the Universe to calculate the ionization fraction vs. redshift

Saha equation result: ionization fraction vs. redshift



When did the CMB emerge?

the last scattering surface

How did the CMB emerge? First, EM waves are trapped in an ionized universe

- When the universe was hot and the gas was ionized, photons were trapped with matter.
 - Free electrons interact strongly with photons (*Thomson scattering*).
 - We cannot observe anything during this era. It's as if the universe is filled with a dense fog.

In the ionized early universe, light was trapped by free electrons. Radiation had a blackbody spectrum.



At that time, it was as though the universe was filled with a thick fog.



Emil Oprisa/Alamy Stock Photo

KEY • Proton • Electron


How did the CMB emerge?

KEY

Then, protons and electrons recombined to form hydrogen

- Eventually, the expansion causes the temperature to cool enough that protons and electrons could form neutral H atoms: this phase-transition of the Universe is called the epoch of recombination (EoR).
- At that time, light was no longer blocked from its travel by free electrons.
- **EoR** marks the earliest point in the universe that we can observe.

Proton
 Electron



Path of photon



At recombination, the universe became transparent, and the blackbody radiation traveled freely through the universe. Recombination was like the fog suddenly clearing.

CMB Photons travel straight to us from the last scattering surface

• Analogous to the *last scattering surface* that marks the surface of the Solar photosphere



Solar Atmosphere: Last Scattering Surface

- The Sun has no solid surface, but the apparent surface of the Sun is the surface at which light can directly escape into space.
- Let's call this surface the **last scattering surface** (a concept also used in cosmology). Note that its depth depends on **(1) the angle we look into the Sun and (2) the wavelength of the photons**
- The layers above this point are known as the atmosphere, which can be directly observed.
 depth ≈ mean free path







(It takes a photon 100,000 years to travel from the core to the surface of the sun)

The CMB emerges when the mean free path of photons reaches the size of the cosmic horizon (~*ct*)



How gas become fully ionized at a temperature 10x lower than ionization energy divided by *k*?

 $T_{ionized} \ll \chi_i / k$

Ionization of Hydrogen in Stellar Photospheres



Ionization of Hydrogen in the Early Universe, Even lower T!



Collisional Ionization vs. Recombination



Maxwell-Boltzmann Velocity Distribution Function



How to determine $\Omega_{b,0}$? Big Bang Nucleosynthesis & CMB Power spectrum

Baryonic Matter Density from Big Bang Nucleosynthesis



CMB power spectrum & cosmological parameters



The Era of Precision Cosmology (1-2% errors)

Parameter	TT+lowP+lensing 68% limits	TT,TE,EE+lowP+lensing+ext 68% limits
$n_{\rm s}$	0.9677 ± 0.0060	0.9667 ± 0.0040
H_0	67.81 ± 0.92	67.74 ± 0.46
Ω_{Λ}	0.692 ± 0.012	0.6911 ± 0.0062
$\Omega_{\rm m}$	0.308 ± 0.012	0.3089 ± 0.0062
$\Omega_{ m b} h^2$	0.02226 ± 0.00023	0.02230 ± 0.00014
$\Omega_{\rm c} h^2$	0.1186 ± 0.0020	0.1188 ± 0.0010
σ_8	0.8149 ± 0.0093	0.8159 ± 0.0086
$z_{\rm re}$	$8.8^{+1.7}_{-1.4}$	$8.8^{+1.2}_{-1.1}$
Age/Gyr	13.799 ± 0.038	13.799 ± 0.021

https://arxiv.org/abs/1502.01589 Planck 2015 Results. Table 4

Quantifying CMB anisotropies w/ temperature **power spectrum**

Analogy: Expressing periodic density fluctuations as the sum of Fourier bases



Fourier Transform:
$$\delta(\mathbf{x}) = \sum_{k=2\pi/l}^{\infty} \delta(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

Power Spectrum:
$$P(k) = V \langle |\delta(k)|^2 \rangle$$



Representing CMB anisotropies as a sum of spherical harmonics $Y_l^m(\theta, \phi)$

m=2

m=2









CMB - Scaler - Dipole = MW + Anisotropies



Source: NASA/WMAP Science Team

Mollweide (equal-area) projection



Spherical harmonics in Mollweide projection m = l



 $\ell = 2$

 $\ell = 3$

 $\ell = 4$







 $\ell = 5$

 $\ell = 6$

 $\ell = 7$







 $\ell = 9$

 $\ell = 10$

Expressing anisotropies as sum of spherical harmonics



Harmonic
Decomposition:
$$\delta_T(\theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{l,m} Y_l^m(\theta, \phi)$$

Temp. Power
Spectrum:
$$P(l) = \frac{l(l+1)}{2\pi}C_l = \frac{l(l+1)}{2\pi}\frac{1}{2l+1}\sum_{m=-l}^l |a_{l,m}|^2$$

WMAP temperature power spectrum



Planck temperature power spectrum



Harmonic spectrum of a flute



https://www.intmath.com/fourier-series/6-line-spectrum.php

Explaining the peaks in the temperature power spectrum

Jeans Length vs. Horizon



random acoustic oscillations frozen at recombination





Multipoles l of the first two peaks



- First peak ($l \sim 200$): the largest structures that could have reached maximum compression at recombination: $\tau = 2\pi L/c_s = 2t_{rec} \rightarrow L \sim c_s t_{rec} \sim \text{sonic horizon}$
- Second peak ($l \sim 500$): the largest structures that could have reached maximum rarefaction $\tau = 2\pi L/c_s = t_{rec} \rightarrow L \sim c_s t_{rec}/2 \sim \text{sonic horizon}/2$

CMB power spectrum & cosmological parameters



CMB power spectrum & cosmological parameters



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 $\Omega_b/\Omega_m = 0.157 \approx 1/6$

https://arxiv.org/abs/1502.01589 Planck 2015 Results. Table 4 Phase Coherence Problem: who is the conductor?

random acoustic oscillations frozen at recombination



WMAP temperature power spectrum


Implications of the strong harmonic peaks

- Oscillations of <u>all density fluctuations of a given size</u> (thus having the same frequency) must reach their maximum compressions / rarefactions <u>at the same time</u>.
- This requires that they begin their oscillations simultaneously and with <u>coherent phases</u>
- In other words, to play the cosmic symphony the universe needs a conductor



The inflation theory vs. standard model



Inflation also solves the phase coherence problem by expanding density fluctuations to super-horizon sizes



Inflation made sure that oscillations of <u>all density fluctuations of a</u> <u>given size</u> reach their maximum compressions / rarefactions <u>at the</u> <u>same time</u>, leading to the strong peaks in the power spectrum



The cosmic harmonics frozen in time "What makes the music of heaven?" - Chuang Tzu (300 BC)



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