Shock heating & Virial Temperature

Recall that virial radius is the radius within which $\langle\rho\rangle\big|_{r_\Delta} = \Delta_c \Omega_m \rho_c$

where $\Delta_c(z) = 18\pi^2 + 82y - 39y^2$, $y = \Omega_m - 1$; $\Delta_c = 100 \sim 178$ at $z \approx 0 \sim 4$

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{is the critical density}$$

Based on this definition, we can calculate r_Δ for any M_Δ and z:

$$r_\Delta = \left(\frac{2GM_\Delta}{\Delta_c \Omega_m H^2}\right)^{1/3} = \left(\frac{2GM_\Delta}{\Delta_c \Omega_{m,0} H_0^2}\right)^{1/3} \cdot \frac{1}{1+z}$$

This redshift dependence is entirely due to our definition of r_Δ; This is not cosmic expansion

where we have used $\Omega_m H^2 = \Omega_{m,0} H_0^2 (1+z)^3 \Leftrightarrow \rho_m = \rho_{m,0}(1+z)^3$

The virial velocity is defined as the circular velocity at r_Δ:

$$V_\Delta = \sqrt{\frac{GM_\Delta}{r_\Delta}} = (GM_\Delta)^{1/3} \cdot \left(\frac{\Delta_c \Omega_m H^2}{2}\right)^{1/6}$$

$$= (GM_\Delta)^{1/3} \cdot \left(\frac{\Delta_c \Omega_{m,0} H_0^2}{2}\right)^{1/6} \cdot (1+z)^{1/2}$$

The virial temperature is defined as the temperature of isothermal gas in hydrostatic equilibrium: (self-consistent definition)

$$\nabla P = -\rho \nabla \Phi \quad \Rightarrow \quad \frac{k}{\mu m_p} \frac{d}{dr}(\rho_{gas} T_{gas}) = -\rho_{gas} \cdot \frac{GM(r)}{r^2}$$

$$\Rightarrow M(r) = -\frac{k T_{gas} r}{\mu m_p G}\left[\left(\frac{r}{T_{gas}\rho_{gas}}\right) \cdot \frac{d}{dr}(\rho_{gas} T_{gas})\right]$$

$$= -\frac{k T_{gas} r}{\mu m_p G} \cdot \left(\frac{d\ln\rho_{gas}}{d\ln r} + \frac{d\ln T_{gas}}{d\ln r}\right)$$

total mass profile inferred from $\rho_{gas}(r)$ & $T_{gas}(r)$

when there is no dark matter, $dM(r) = 4\pi \rho_{gas} r^2 dr$, then we can solve $\rho(r)$:

$$\frac{d}{dr}\left(r^2 \frac{d\ln\rho_{gas}}{dr}\right) = -\frac{4\pi G \mu m_p}{k T_{gas}} r^2 \rho_{gas} \Rightarrow \rho_{gas} = \frac{k T_{gas}}{2\pi G \mu m_p} \cdot r^{-2}$$

for isothermal gas, $\frac{dT}{dr} = 0 \Rightarrow$

$$-\frac{d\ln\rho_{gas}}{d\ln r} = \frac{\mu m_p}{k T_{gas}} \frac{GM(r)}{r} = \frac{\mu m_p}{k T_{gas}} \cdot V_c^2,$$

(for $\rho \propto r^{-2}$, we have:)

$$\boxed{T_\Delta \equiv \frac{\mu m_p}{2k} V_\Delta^2}$$

Virial Quantities Estimation Formulae

$$r_\Delta = \left(\frac{2GM_\Delta}{\Delta_c \Omega_{m,0} H_0^2}\right)^{1/3} \cdot \frac{1}{(1+z)}$$

$$= 1.4\,\mathrm{kpc} \cdot \left(\frac{h}{0.7}\right)^{-2/3} \left(\frac{\Delta_c}{200}\right)^{-\frac{1}{3}} \left(\frac{\Omega_{m,0}}{0.3}\right)^{-\frac{1}{3}} \left(\frac{1+z}{10}\right)^{-1} \left(\frac{M_\Delta}{10^8 M_\odot}\right)^{1/3}$$

$$V_\Delta = \sqrt{\frac{GM_\Delta}{r_\Delta}} = (GM_\Delta)^{1/3} \left(\frac{\Delta_c \Omega_{m,0} H_0^2}{2}\right)^{1/6} (1+z)^{1/2}$$

$$= 17.3\,\mathrm{km/s} \left(\frac{h}{0.7}\right)^{1/3} \left(\frac{\Delta_c}{200}\right)^{1/6} \cdot \left(\frac{\Omega_{m,0}}{0.3}\right)^{1/6} \cdot \left(\frac{M_\Delta}{10^8 M_\odot}\right)^{1/3} \cdot \left(\frac{1+z}{10}\right)^{1/2}$$

$$T_\Delta = \frac{\mu m_p}{2k} V_\Delta^2 = 3.6 \times 10^5\,\mathrm{K} \left(V_\Delta / 100\,\mathrm{km/s}\right)^2 \left(\mu / 0.6\right)$$

$$= 1.1 \times 10^4\,\mathrm{K} \left(\frac{\mu}{0.6}\right) \left(\frac{h}{0.7}\right)^{2/3} \left(\frac{\Delta_c}{200}\right)^{1/3} \left(\frac{\Omega_{m,0}}{0.3}\right)^{1/3} \cdot \left(\frac{M_\Delta}{10^8 M_\odot}\right)^{2/3} \left(\frac{1+z}{10}\right)$$

$$r_\Delta \propto M_\Delta^{1/3} (1+z)^{-1}, \quad V_\Delta \propto M_\Delta^{1/3} (1+z)^{1/2}, \quad T_\Delta \propto M_\Delta^{2/3} (1+z)$$

Mean molecular mass of primordial H & He composition:

$$N_H : N_{He} = 12 : 1 \quad \Rightarrow \quad \rho_H : \rho_{He} = 3 : 1$$

fully ionized gas:

$$\mu = \frac{(12+4)\,m_p}{24+3} \Big/ m_p = \frac{16}{27} \simeq 0.6$$

fully neutral gas:

$$\mu = \frac{(12+4)\,m_p}{12+1} \Big/ m_p = \frac{16}{13} \simeq 1.23$$

Note: the "evolution" of a non-evolving halo

this calculation assumes the SIS profile:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2} \equiv \rho_s (r/r_s)^{-2} \quad \text{where } \sigma^2 = 2\pi G \rho_s r_s^2$$

$$M(r) = \int_0^r 4\pi r^2 \rho(r)\, dr = \frac{2\sigma^2}{G} r = 4\pi \rho_s r_s^2 \cdot r$$

$$V_{circ} = \sqrt{\frac{G M(r)}{r}} = \sqrt{2}\,\sigma = 2\sqrt{\pi G \rho_s r_s^2} = \text{const.}$$

once such a halo is formed, its r_Δ & M_Δ continues to increase because the $\bar{\rho}_m$ of the universe continues to decrease as $(1+z)^3$, and more important, our static definition of r_Δ & M_Δ with Δ_c.

Mean density of the SIS within radius r is:

$$\bar{\rho}_h = \frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{3\sigma^2}{2\pi G r^2} = 3 \cdot \rho(r)$$

given the definition of r_Δ:

$$\bar{\rho}_h(r_\Delta) = \Delta_c \cdot \rho_{m,0} (1+z)^3$$

$$\Rightarrow \quad r_\Delta = \sqrt{\frac{3\sigma^2}{2\pi G \Delta_c \rho_{m,0} (1+z)^3}} \propto (1+z)^{-\frac{3}{2}}$$

$$\Rightarrow \quad M_\Delta = \frac{2\sigma^2}{G} r_\Delta \propto (1+z)^{-\frac{3}{2}}$$

between $z=10$ & $z=0$, the virial radius & mass of the halo increased by $36\times$

for a non-evolving SIS halo, $M_\Delta \cdot (1+z)^{3/2}$ is a preserved quantity.

This result is consistent with the formula for virial temperature:

$$T_\Delta = \frac{\mu m_p}{2k} V_\Delta^2 \propto M_\Delta^{2/3} (1+z) \Rightarrow T_\Delta^{3/2} \propto M_\Delta (1+z)^{3/2}$$

a non-evolving halo should have maintained its virial temperature.

collisional rate = # of coll. per unit vol. per unit time

$$= \frac{(\sigma V_e dt \cdot n_{HI}) \cdot n_e}{dt} = n_e n_{HI} \cdot V_e \cdot \sigma(V_e)$$

Cooling Function & CIE

Why Λ only depends on T ?

The ionization equation for pure Hydrogen :

$$\frac{dn_e}{dt} = n_{HI} \int_0^\infty \sigma_{CI}^{v} \, V_e \cdot n_e f(V_e) \, dV_e \quad \text{collisional ionization}$$

$$+ n_{HI} \int_{\nu_0}^\infty \sigma_{PI}^{\nu} \cdot c \cdot \frac{4\pi J_\nu}{c \cdot h\nu} \, d\nu \quad \text{photoionization}$$

$$- n_{HII} \int_0^\infty \sigma_{rec}^{v} \cdot V_e \cdot n_e f(V_e) \, dV_e \quad \text{recombination}$$

$$\equiv n_{HI} \cdot n_e \cdot \Gamma_{eHI} + n_{HI} \Gamma_{\gamma HI} - \alpha_H n_{HII} n_e$$

Γ_{eHI} & α_H are velocity-weighted cross-sections.

where $f(v)\,dv = \left(\frac{m}{2\pi k T_e}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT_e}} dv$ Maxwell-Boltzmann Dist.

Saha Eq. can be obtained when $T_e = T_{HI} = T_{HII} = T_\gamma$ & $\dot{n}_e = 0$

Collisional Ionization Equlibrium assumed instead $J_\nu = 0$ & $\dot{n}_e = 0$

Under CIE, we have

$$\Gamma_{eHI} \, n_e \, n_{HI} = \alpha_H \, n_e \, n_{HII}$$

$$\Rightarrow \quad \frac{n_{HII}}{n_{HI}} = \frac{\Gamma_{eHI}(T)}{\alpha_H(T)} \sim f(T) \text{ , no density dependency !}$$

$$\text{Radiative Cooling Rate} \sim n_e \, n_{ion} \int_0^\infty \sigma(v) \cdot v \cdot f(v) \, dv$$

$$\equiv \Lambda \cdot n_H^2$$

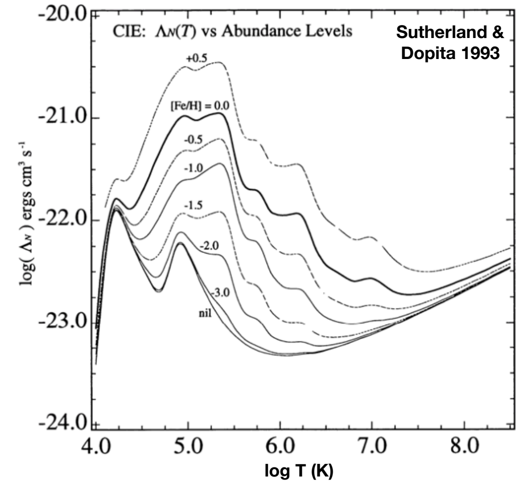
$$\Rightarrow \quad \Lambda = \frac{n_e n_{ion}}{n_H^2} \cdot \Gamma_{FF, FB, BB, BF}(T_e)$$

because the relative abundances of ionic species depends only on T in CIE, we conclude that Λ only depends on T : $\Lambda \equiv \frac{\text{Cooling Rate}}{n_H^2} = \Lambda(T)$

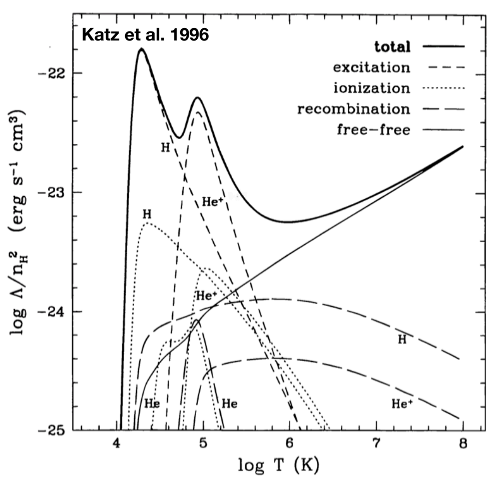
The Cooling Function: $\Lambda(T, Z)$

assume collisional ionization equilibrium (CIE)

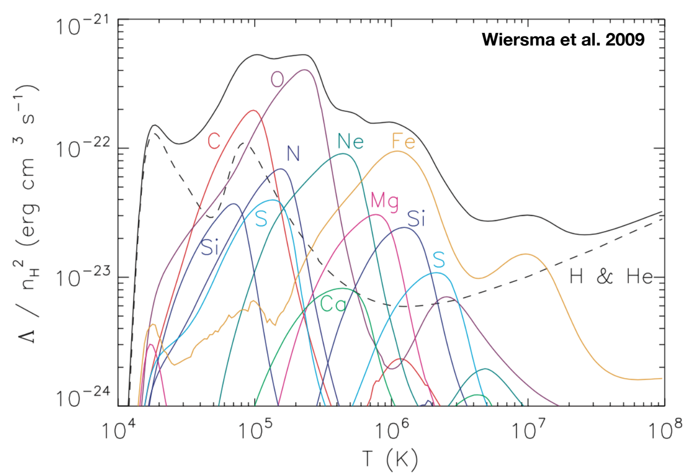
CIE Cooling Function: Metallicity Dependence



CIE Cooling Function: Contribution from different mechanisms



CIE Cooling Function: Contribution from Major Ions



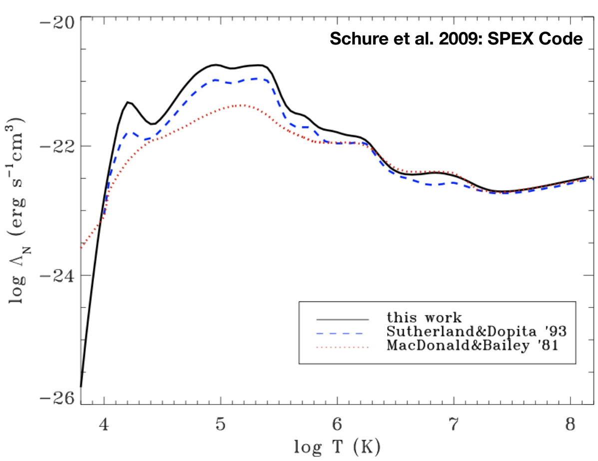


Fig. 1. Cooling curves compared: the higher cooling rates calculated with SPEX are mainly due to a more complete coverage of the line transitions, including Fe L and EUV lines.

Virial temperature (continued)

in general, $T_\Delta = \dfrac{\mu m_p}{-\gamma_\Delta k} V_\Delta^2$ where $\gamma_\Delta \equiv \left.\dfrac{d\ln\rho}{d\ln r}\right|_{r_\Delta} \simeq -2$ (SIS)

using the formula for V_Δ, we have:

$$T_\Delta = \frac{\mu m_p}{2^{4/3} k} (G M_\Delta)^{2/3} (\Delta_c H_0^2 \Omega_{m,0})^{1/3} \cdot (1+z)$$

$$\simeq 10^4 K \left(\frac{h}{0.7}\right)^{2/3} \left(\frac{\mu}{0.6}\right) \left(\frac{M_\Delta}{10^8 M_\odot}\right)^{2/3} \left(\frac{\Omega_{m,0}}{0.3} \cdot \frac{\Delta_c}{18\pi^2}\right)^{1/3} \cdot \left(\frac{1+z}{10}\right)$$

$T_\Delta \simeq 1.7 MK$ for $10^{12} M_\odot$ halo at $z \sim 2.5$.

Shock heating of infalling gas

consider a cloud of M_{gas} falling into a halo with M_Δ

as it crosses the virial radius, its infall velocity is the escape velocity

$$V_{sh} = \sqrt{-2\phi(r_{sh})} = \sqrt{2}\, V_\Delta \simeq 2\sqrt{kT_\Delta/\mu m_p}$$

which is much larger than its sound speed:

$$C_s \simeq \sqrt{\frac{5kT_{pre}}{3\mu m_p}}$$ because $T_{pre} \simeq 50K$ at $z \sim 50$ & $T_{pre} \sim (1+z)^2$

this develops an accretion shock near r_Δ, which thermalizes the kinetic E:

$$\frac{3}{2} k T_{sh} = \frac{1}{2} \mu m_p V_{sh}^2 = 2 k T_\Delta$$

$$\Rightarrow \quad \boxed{T_{sh} = \frac{4}{3} T_\Delta}$$ infalling gas shock-heated to virial temperature.

Radiative Cooling

(this def removes density dependency)

define cooling function Lambda: $\Lambda = \dfrac{C}{n_H^2} = \dfrac{\text{cooling rate}}{\text{H number density}^2} = \dfrac{\text{erg/s/cm}}{\text{cm}^{-6}}$

cooling timescale: $t_{cool} = \dfrac{\rho \varepsilon}{C} = \dfrac{3nkT}{2 n_H^2 \Lambda}$, where $n = \dfrac{\rho}{\mu m_p}$, $n_H = \dfrac{12}{27} n = \dfrac{4}{9}$

most of the mass is in H & He, we have (fully ionized gas) (H:He=12:1)

$$\begin{cases} \rho = n_H m_H + n_{He} \cdot m_{He} = n_H m_p \cdot (1 + 4 \times \frac{1}{12}) = \frac{4}{3} n_H m_p \\ \rho = n \cdot \mu m_p \Rightarrow n = \frac{4}{3\mu} n_H \end{cases}$$

therefore $t_{cool} = \dfrac{2kT}{\mu} \cdot \dfrac{1}{n_H \cdot \Lambda}$, $n_H \simeq 4 \times 10^{-5} \text{cm}^{-3} \left(\dfrac{1+\delta}{200}\right)(1+z)^3$ (after Helium correction)

Radiative Cooling (cont.)

$$t_{cool} = \frac{2kT}{\mu}\frac{1}{n_H \Lambda} \simeq 3\times 10^9 \, yr \left(\frac{T}{10^6 K}\right)\cdot\left(\frac{n}{10^{-3} cm^{-3}}\right)^{-1}\left(\frac{\Lambda}{10^{-23} erg \, s^{-1} \, cm^3}\right)^{-1}$$

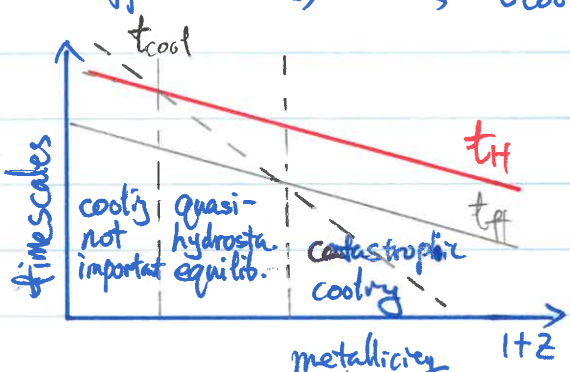
our goal is to compare t_{cool} with t_H & t_{ff}

$$t_H = \frac{1}{H(z)} = \sqrt{\frac{3}{8\pi G \bar{\rho}}} \propto \bar{\rho}^{-\frac{1}{2}} \quad \text{where } \bar{\rho} = \Omega_m \rho_c$$

$$t_{ff} = \sqrt{\frac{3\pi}{32 G \bar{\rho}_\Delta}} \propto \bar{\rho}_\Delta^{-\frac{1}{2}} \quad \text{where } \bar{\rho}_\Delta = \Delta_c \bar{\rho} \simeq 200 \bar{\rho}$$

$$t_{ff} \sim t_H/10 \propto \bar{\rho}_\Delta^{-\frac{1}{2}}, \quad t_{cool} \propto \bar{\rho}_\Delta^{-1}, \quad \bar{\rho}_\Delta \propto (1+z)^3$$

$$\Rightarrow t_{ff} \propto (1+z)^{-\frac{3}{2}}, \quad t_{cool} \propto (1+z)^{-3}$$



low z : $t_{cool} > t_H \rightarrow$ hydrostatic equilibrium

mid z : $t_{ff} < t_{cool} < t_H \rightarrow$ contract slowly

high z : $t_{cool} < t_{ff} \rightarrow$ gas cannot respond to loss of press collapses on ff timescale

If we know $\Lambda(T, Z)$, we can compute the density at any given T above which $t_{cool} < t_{ff}$ so that the accreted halo gas can form galaxies.

$$\frac{2kT}{\mu(T)\cdot n_H \Lambda(T,Z)} < \left(\frac{3\pi}{32 G}\cdot\frac{f_{gas}}{\frac{4}{3} n_H m_p}\right)^{1/2}, \quad \text{where } f_{gas} \equiv \frac{\Omega_{b,0}}{\Omega_{m,0}} \simeq 15\%$$

$$n_H > \frac{2^9 G m_p}{9\pi f_{gas}}\cdot\left(\frac{kT}{\mu\Lambda}\right)^2 \equiv n_H^{cc} \quad \text{density threshold for gal form}$$

Cooling Processes (how to decrease kinetic energy

free-free (bremsstrahlung): $e^- + X^+ \rightarrow e^- + X^+ + h\nu$

free-bound (recombination): $e^- + X^+ \rightarrow X + h\nu$

bound-free (colli. ionization): $e^- + X \rightarrow X^+ + 2e^- \rightarrow X + e^- + h\nu$

bound-bound (colli. excitation): $e^- + X \rightarrow X' + e^- \rightarrow X + e^- + h\nu$

spontaneous decay

free-bound

Cooling function & galaxy formation

$$n_H^{cc} = \frac{2^9 G m_p}{9\pi f_{gas}} \cdot \left(\frac{kT}{\mu\Lambda}\right)^2$$

Hydrogen density threshold for catastrophic cooling ($t_{cool} < t_{ff}$)

the temperature of gas in a halo is at virial temperature

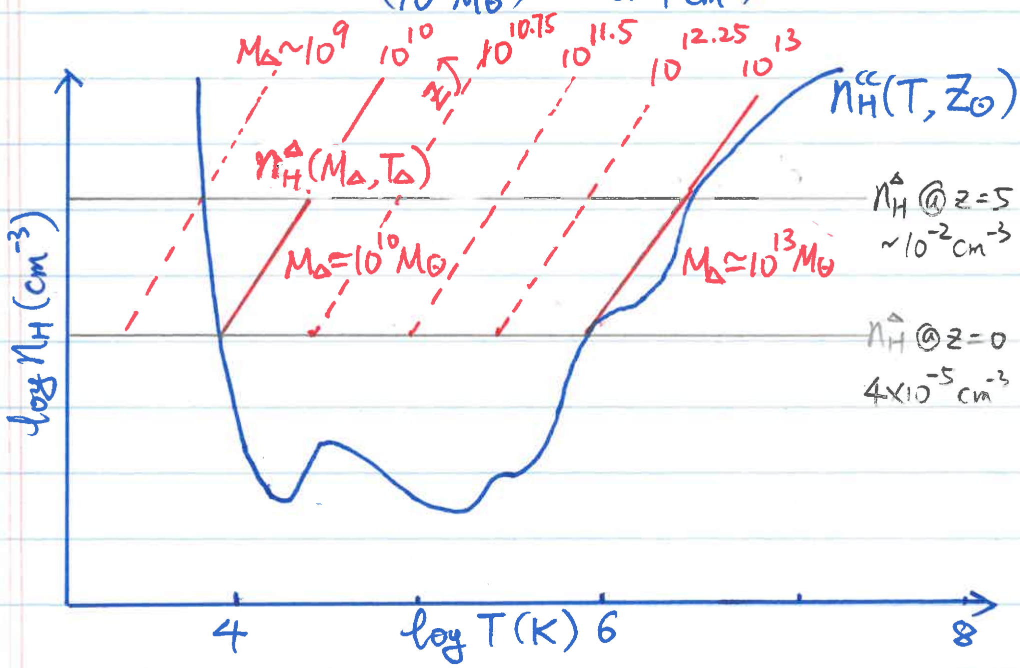
$$T_\Delta = \frac{\mu m_p}{2k} V_\Delta^2 \simeq 10^4 K \left(\frac{M_\Delta}{10^8 M_\odot}\right)^{2/3} \cdot \left(\frac{1+z}{10}\right)$$

$$\bar{\rho}_\Delta = \Delta_c \cdot \rho_c \cdot \Omega_m \cdot f_{gas} = \frac{4}{3} n_H^\Delta \cdot m_p$$

$$\Rightarrow n_H^\Delta = 4 \times 10^{-5} cm^{-3} \left(\frac{\Delta_c}{200}\right) \cdot (1+z)^3$$

therefore:

$$T_\Delta \simeq 10^4 K \left(\frac{M_\Delta}{10^8 M_\odot}\right)^{2/3} \cdot \left(\frac{n_H^\Delta}{0.04\, cm^{-3}}\right)^{1/3} \quad \text{or} \quad n_H^\Delta \propto M_\Delta^{-2} T_\Delta^3$$

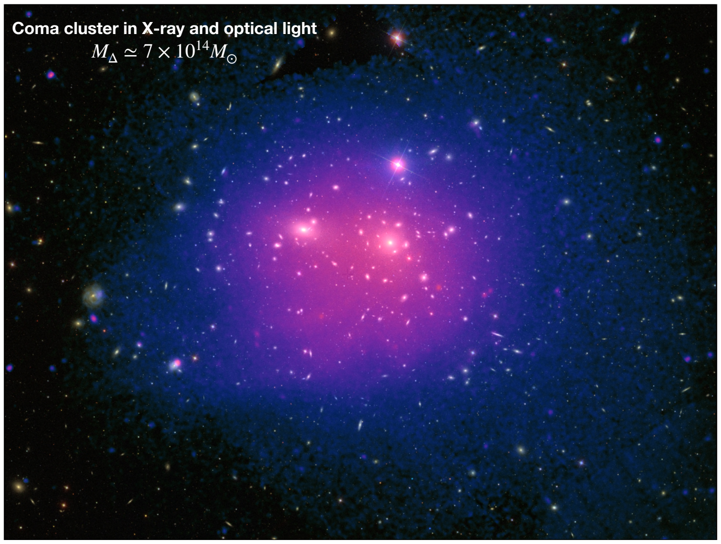


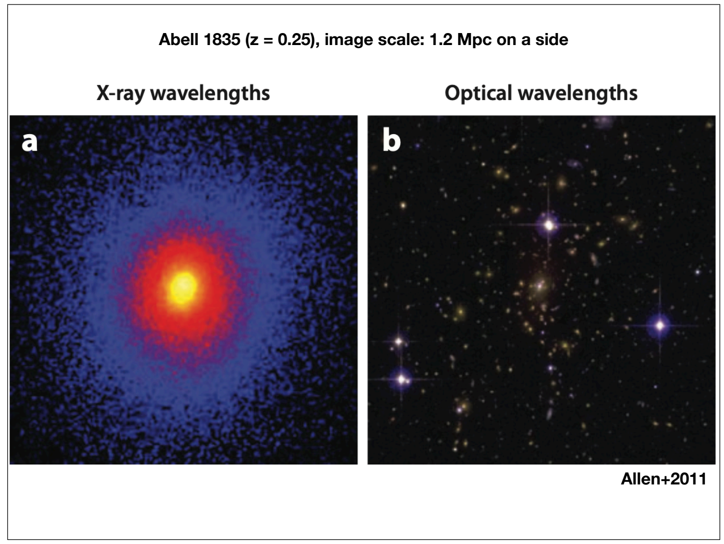
Between $0 < z < 5$, for solar metallicity gas, efficient cooling of accreted gas occurs in halos with $10^9 M_\odot < M_\Delta < 10^{13} M_\odot$

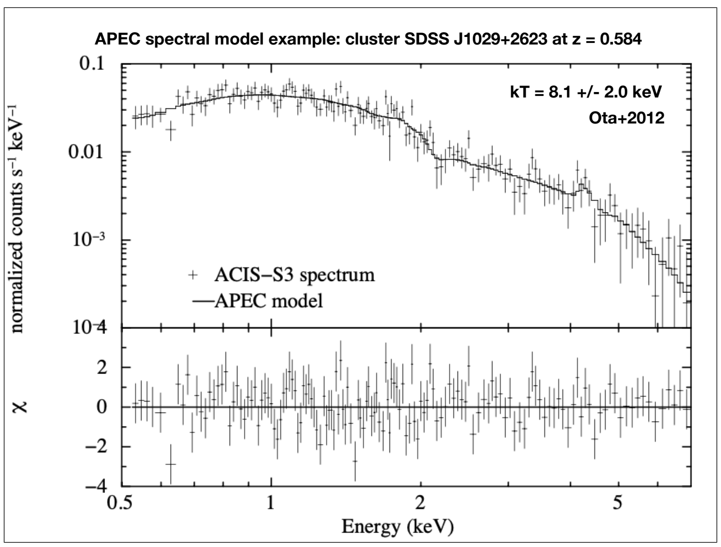
for primodial gas ($Z=0$), the range is $10^9 M_\odot < M_\Delta < 10^{12} M_\odot$

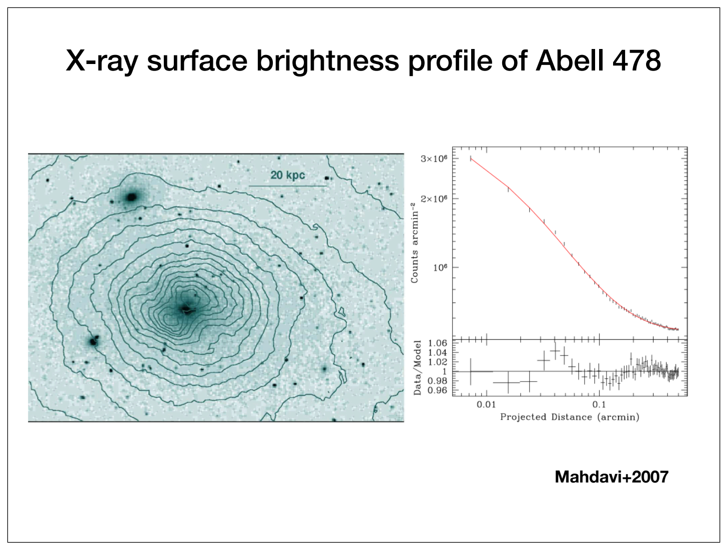
1970s belief: more massive galaxies cannot form because of cooling inefficiency

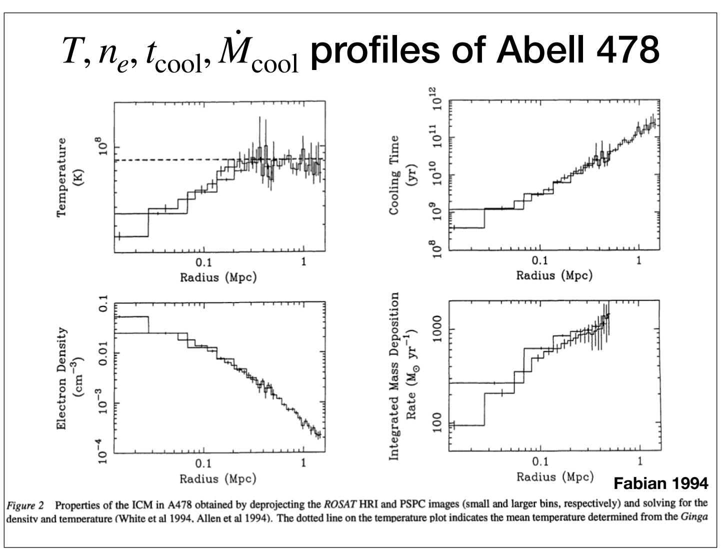
Problems: ① galaxies form hierarchically. ② denser gas near center can always cool.

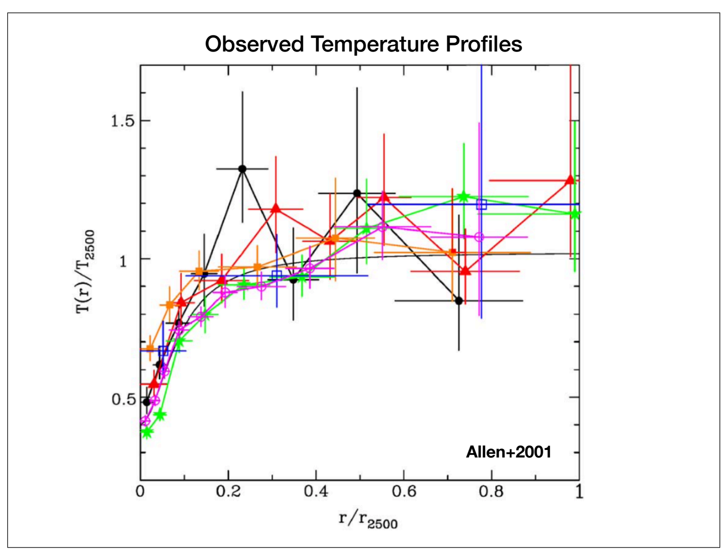

Coma cluster in X-ray and optical light
$M_\Delta \simeq 7 \times 10^{14} M_\odot$


Abell 1835 (z = 0.25), image scale: 1.2 Mpc on a side
X-ray wavelengths
Optical wavelengths
a
b
Allen+2011

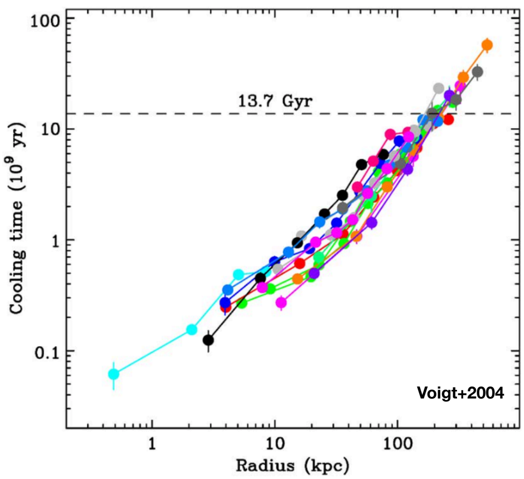

APEC spectral model example: cluster SDSS J1029+2623 at z = 0.584
kT = 8.1 +/- 2.0 keV
Ota+2012
ACIS−S3 spectrum
APEC model
normalized counts s^{-1} keV^{-1}
χ
Energy (keV)


X-ray surface brightness profile of Abell 478
20 kpc
Counts arcmin^{-2}
Data/Model
Projected Distance (arcmin)
Mahdavi+2007

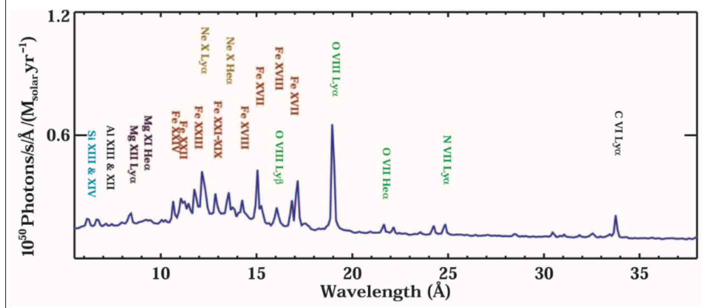

$T, n_e, t_{\rm cool}, \dot{M}_{\rm cool}$ profiles of Abell 478
Temperature (K)
Cooling Time (yr)
Electron Density (cm^{-3})
Integrated Mass Deposition Rate ($M_\odot$ yr^{-1})
Radius (Mpc)
Fabian 1994
Figure 2 Properties of the ICM in A478 obtained by deprojecting the ROSAT HRI and PSPC images (small and larger bins, respectively) and solving for the density and temperature (White et al 1994, Allen et al 1994). The dotted line on the temperature plot indicates the mean temperature determined from the Ginga


Observed Temperature Profiles
T(r)/T$_{2500}$
r/r$_{2500}$
Allen+2001

Cooling Time Profiles



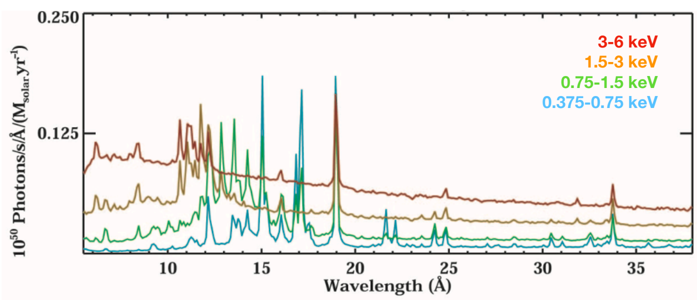
Predicted X-ray Spectra of Cooling Flow Models



Peterson+2003

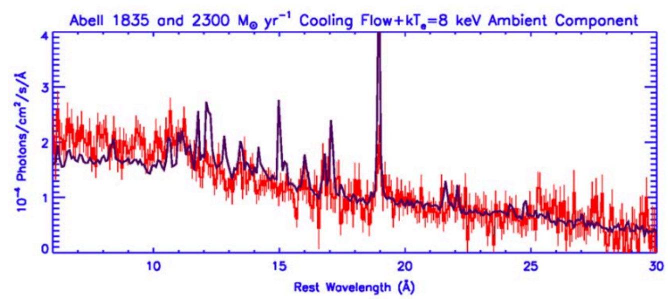
Predicted X-ray Spectra of Cooling Flow Models

Fe lines arise from cooler gas (kT < 1.5 keV or T < 1.5e7 K)



Peterson+2003

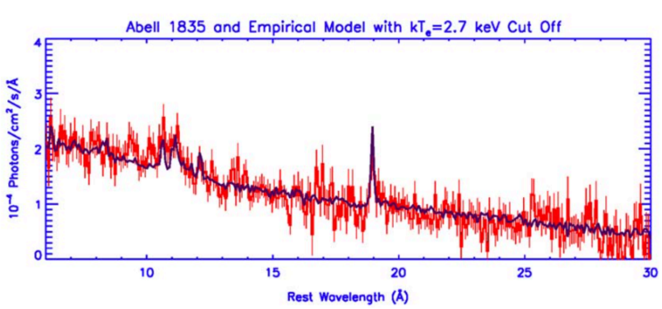
Cooling Flow Models vs. Data



Poor agreement w/ standard multi-temperature cooling flow model

Peterson+2003

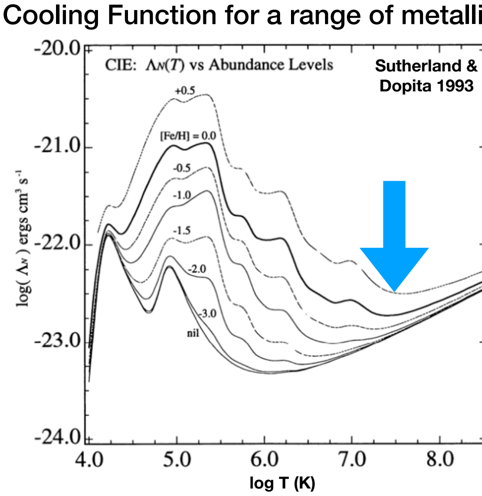
Cooling Flow Models vs. Data



Good agreement w/ a truncated cooling flow model: no gas with kT < 2.7 keV

Peterson+2003

CIE Cooling Function for a range of metallicities



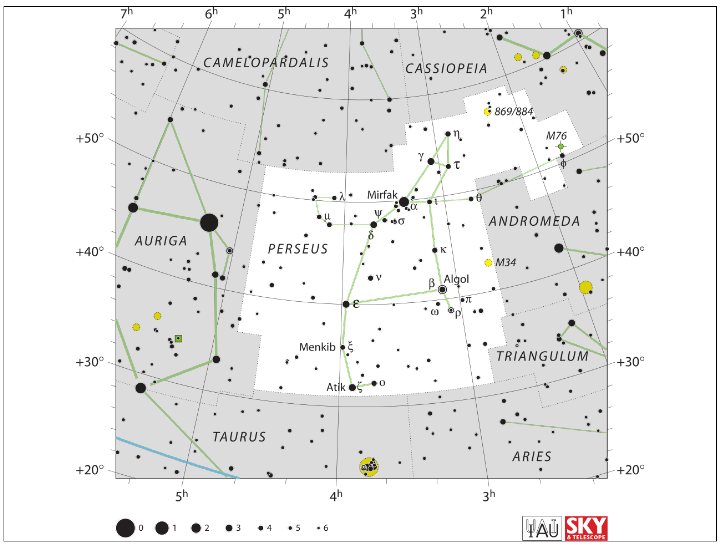
XMM-Newton high-res X-ray spectra of 14 clusters indicate that cooling flow stalls at

$$T < T_{\Delta}/3$$

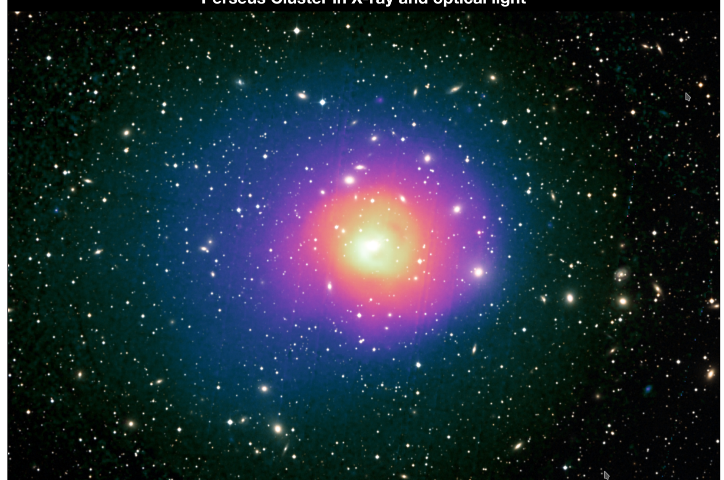
Why?

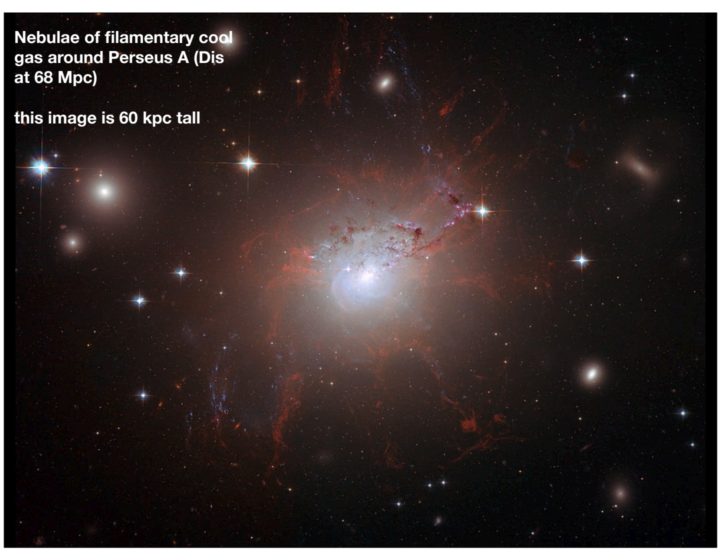
Possible solutions of the "Soft X-ray Deficit Problem"

- Rapid cooling:
  1. *Non-radiative cooling of T ~ 1 keV gas by mixing and conduction with preexisting colder gas at r < 100 kpc*
  2. *Chemically enriched clumps of gas cools much more rapidly and drop out from the rest of the ICM (a highly inhomogeneous ICM, i.e., lack of mixing between gas at the same temperature)*
- Keep it hot:
  Extra heating from active galactic nuclei (accreting supermassive black holes) in central cluster galaxies
- Observational evidence exist for both mechanisms (e.g., Perseus Cluster)

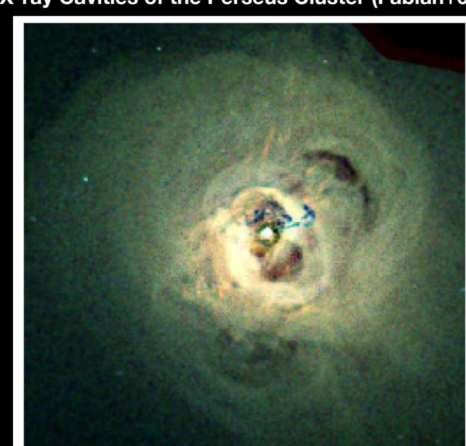


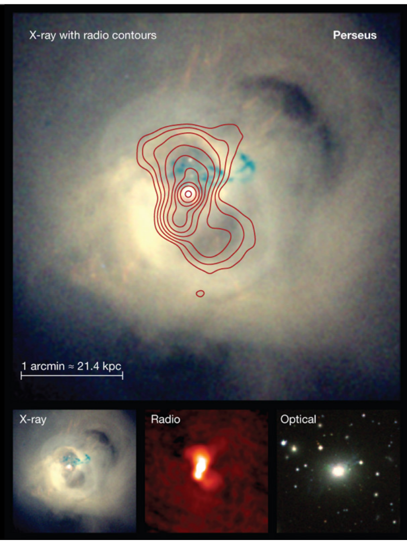
Perseus Cluster in X-ray and optical light





X-ray Cavities of the Perseus Cluster (Fabian+05)

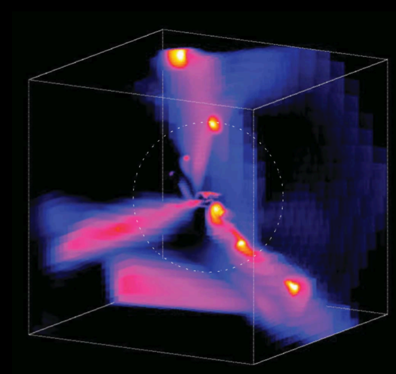




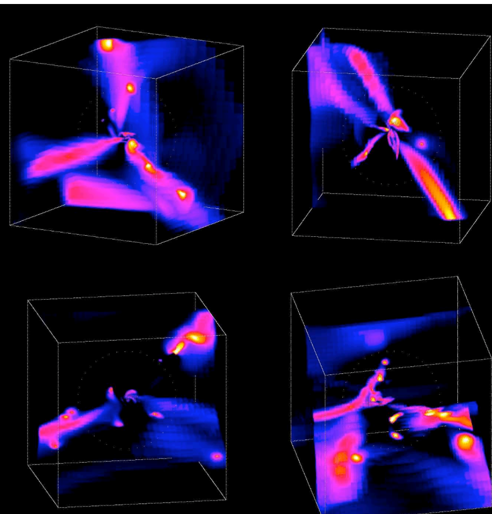
If cooling doesn’t provide enough cool gas to form galaxies in massive halos, what other mechanism(s) can?

Direct accretion from cold streams of the cosmic web

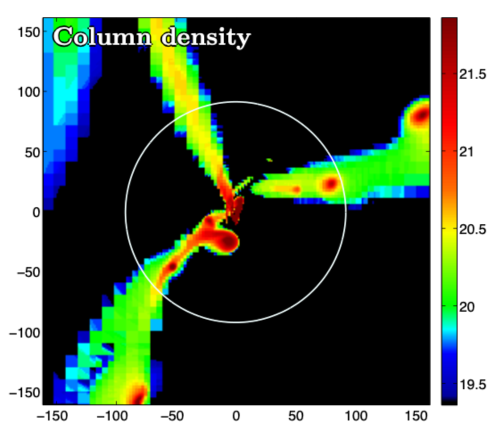
Radial streams of cool gas and galaxies can penetrate the shock-heated hot halo as seen in this 3D simulation of a halo with $M_\Delta = 10^{12} M_\odot$ *at* $z = 2.5$



Dekel+2009



Can we identify such cold streams in observations?



HI Lyman alpha filaments around a massive galaxy group at z=2.91 (Daddi+2021)

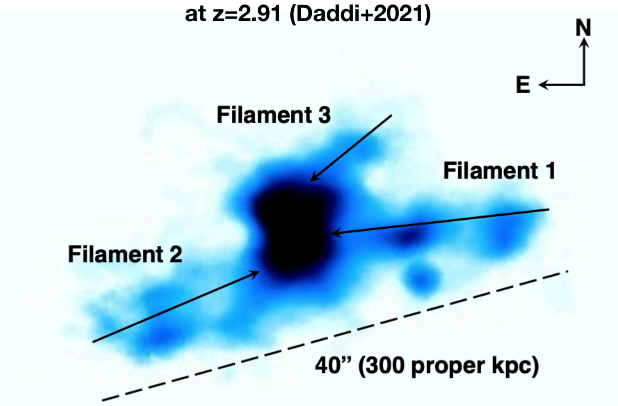
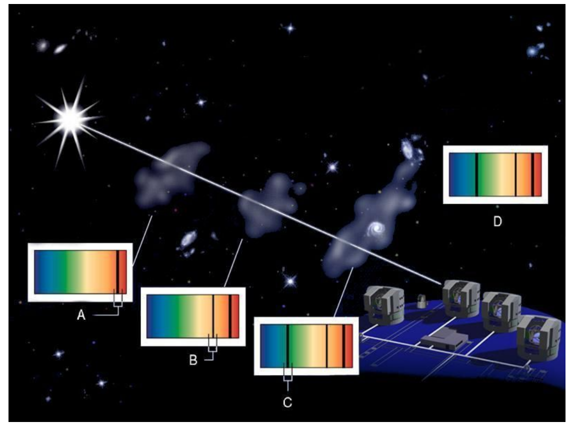
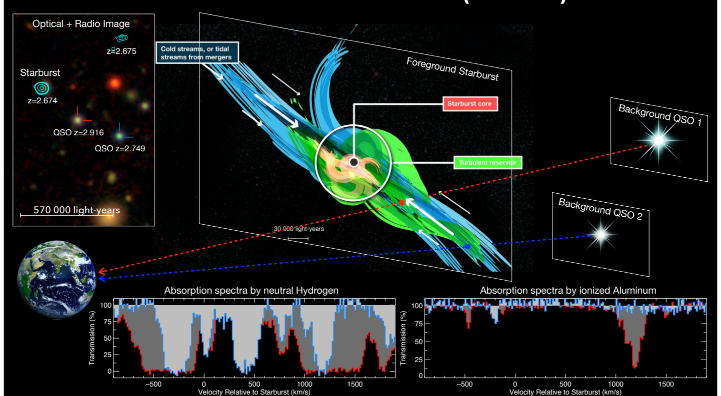


Fig. 1. Lyα image from KCWI observations of the RO-1001 group. The three filaments clearly traced by Lyα are labeled. Corresponding Lyα surface brightness levels can be gauged from Fig. 2.

Detection of Intervening Gas with Quasar absorption line spectroscopy

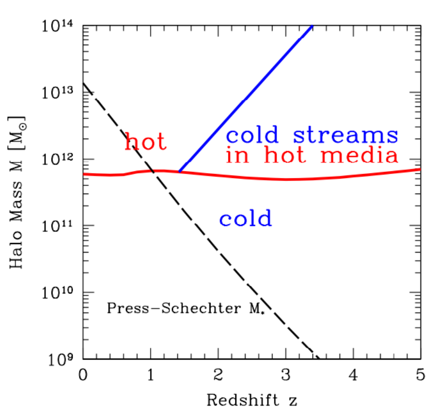


Detection of a 200-kpc-long, 0.01 $Z_{\odot}$, Cool Gas Stream in a Massive Halo at z = 2.7 (Fu+2021)



How do galaxies form in massive halos?

- Spherical accretion develops stable shocks near the virial radius, shock heat gas to virial temperature. The denser gas near the cores should cool efficiently and develop a cooling flow to form galaxies, but the cooling stalls at ~1 keV (10^7 K) likely due to extra heating.
- Non-spherical accretion filaments might penetrate the hot atmosphere of halo gas, the gas is never shock heated, and they deliver cool gas directly to the central tens of kpc to form galaxies.



Dekel+2009