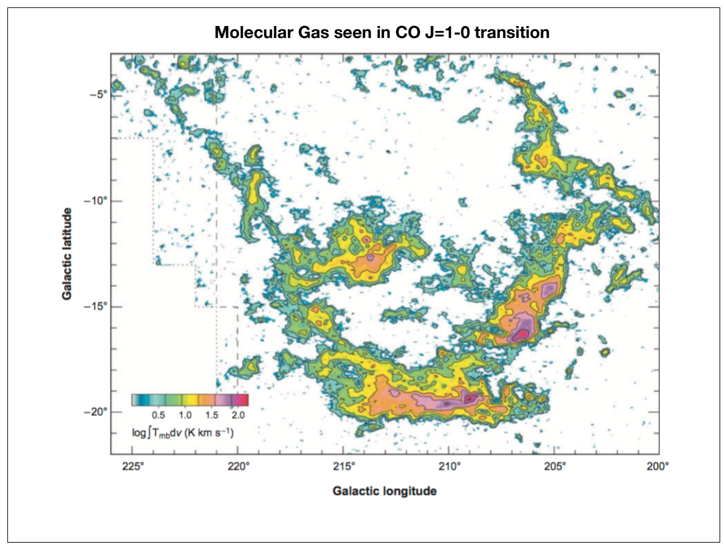
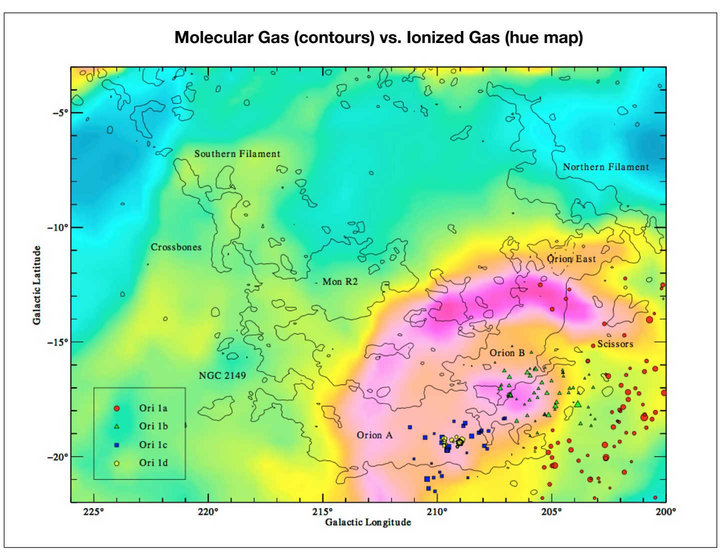
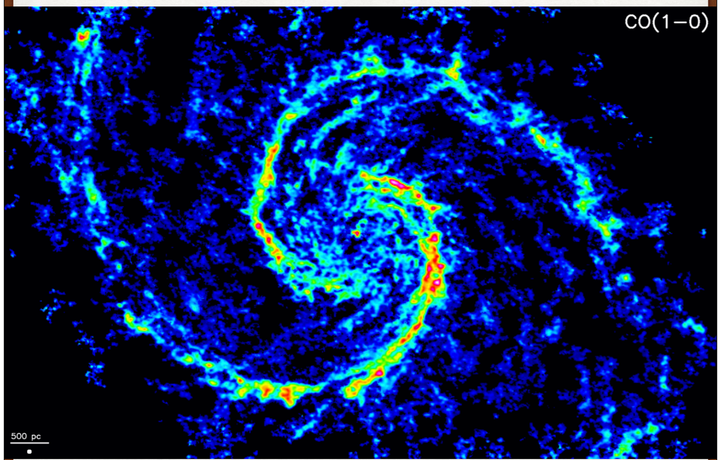
The Orion Star Forming Region



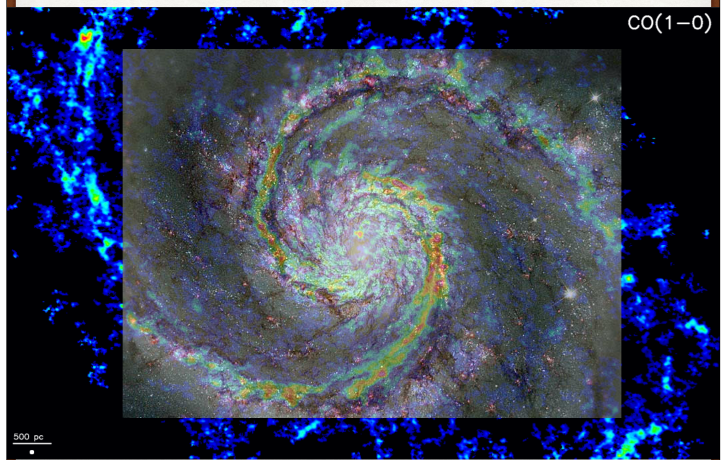


Star-Forming Regions in M51

M51 - WHIRLPOOL GALAXY

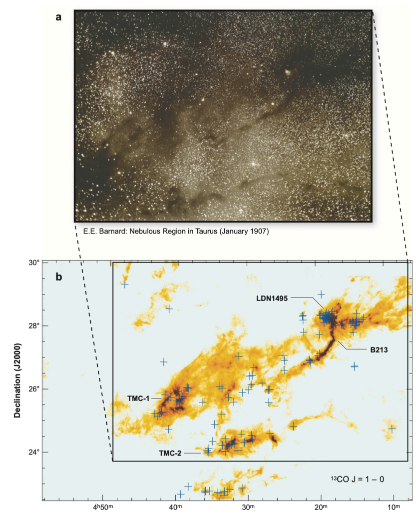


M51 - WHIRLPOOL GALAXY



Cold Molecular Cores

Bergin & Tafalla 2007

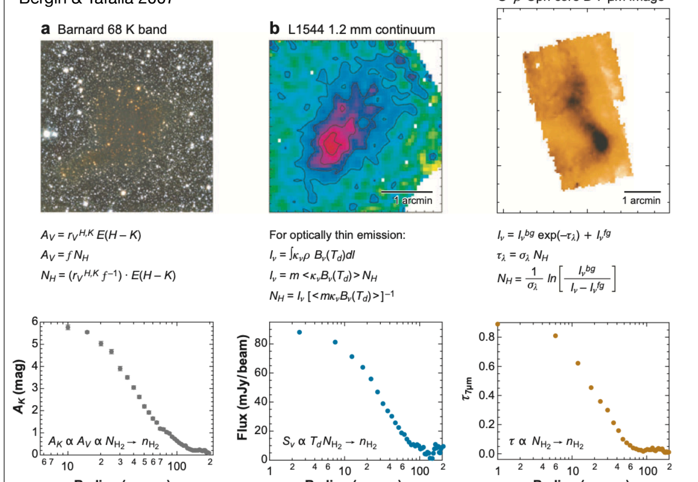


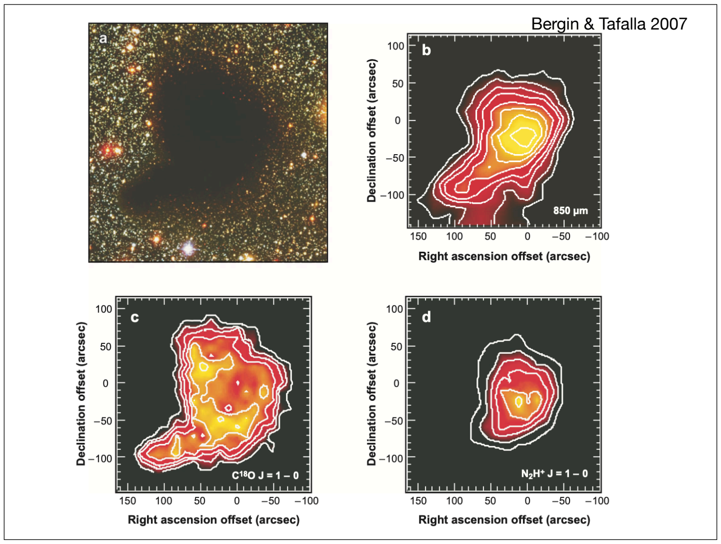
Optical photographic image of the Taurus molecular cloud, taken in 1907

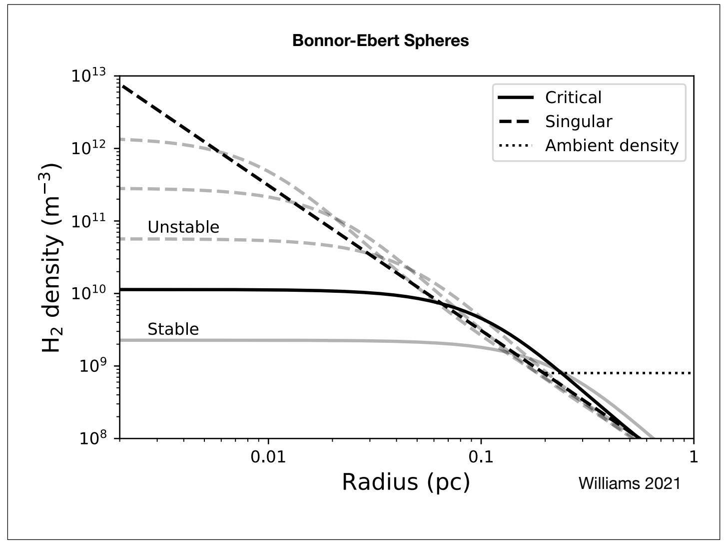
CO(1-0) emission crosses indicate known stellar and protostellar objects

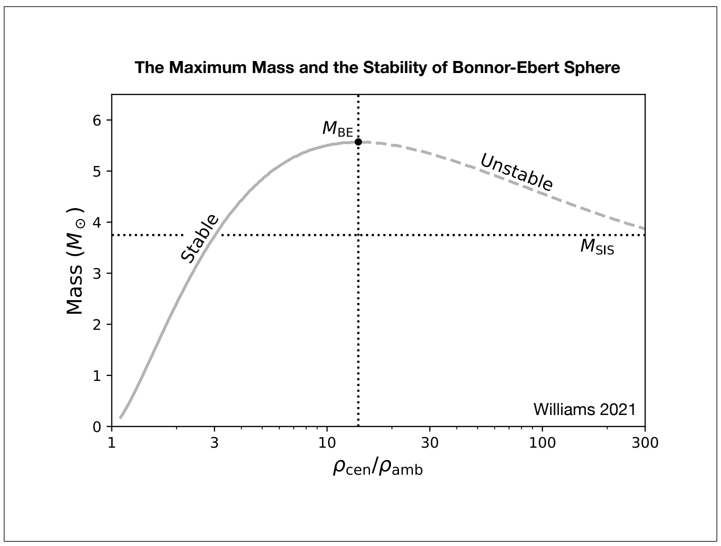
Barnard 68 dark core

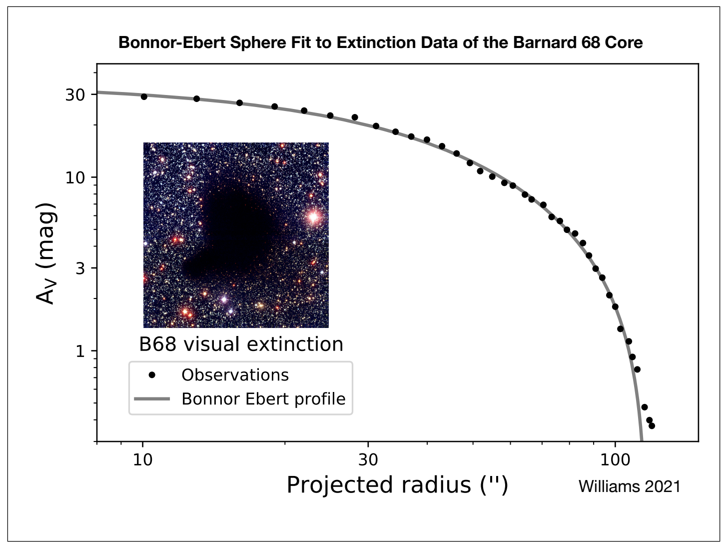
Bergin & Tafalla 2007




Bergin & Tafalla 2007
a
b
850 μm
Declination offset (arcsec)
Right ascension offset (arcsec)
c
C18O J = 1 – 0
d
N2H+ J = 1 – 0


Bonnor-Ebert Spheres
Critical
Singular
Ambient density
Unstable
Stable
H2 density (m−3)
Radius (pc)
Williams 2021


The Maximum Mass and the Stability of Bonnor-Ebert Sphere
MBE
Unstable
Stable
MSIS
Mass (M⊙)
ρcen/ρamb
Williams 2021


Bonnor-Ebert Sphere Fit to Extinction Data of the Barnard 68 Core
B68 visual extinction
Observations
Bonnor Ebert profile
AV (mag)
Projected radius ('')
Williams 2021

Star Formation Rate

Physical Properties of Gas

	$\langle n_H \rangle$	T
Pregalactic	$2\times10^{-4}\,cm^{-3}\left(\frac{1+z}{10}\right)^3$	$2\,K\cdot\left(\frac{1+z}{10}\right)^2$
Halo gas	$4\times10^{-2}\,cm^{-3}\left(\frac{1+z}{10}\right)^3$	$10^4 K\left(M_\Delta/10^8 M_\odot\right)^{2/3}\left(\frac{1+z}{10}\right)$
Solar neighborhood	$40\,cm^{-3}\left(\frac{M^*\text{density}}{1\,M_\odot/pc^3}\right)$	N/A
GMC (H_2)	$10^2\,cm^{-3}$	$15K \Rightarrow M_J = 80M_\odot$
Cores (H_2)	$10^4\,cm^{-3}$	$10K \Rightarrow M_J = 8M_\odot$
WNM (HI)	$0.3\,cm^{-3}$	$10^4 K$ } $P/k \sim 10^4\,cm^{-3}K$
CNM (HI)	$30\,cm^{-3}$	$100K$ }

Jeans Mass

$$M_J = \left(\frac{5kT}{\mu m_p G}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2} = 8M_\odot\left(\frac{T}{10K}\right)^{3/2}\left(\frac{\mu}{2}\right)^{-\frac{3}{2}}\cdot\left(\frac{n_{H_2}}{10^4 cm^{-3}}\right)^{-\frac{1}{2}}$$

$M_J \propto T^{3/2} n^{-\frac{1}{2}}$ → SF occurs in cold & dense gas clouds

molecular clouds are thus preferred sites of star formation

Free Fall Timescale

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho}} = 4\times10^5\,yr\left(\frac{n_{H_2}}{10^4 cm^{-3}}\right)^{-\frac{1}{2}}$$

SFR per core

$$M_J/t_{ff} \simeq 2\times10^{-5}\,M_\odot/yr\left(\frac{T}{10K}\right)^{3/2}\cdot\left(\frac{\mu}{2}\right)^{-3/2}$$

SFR of the Galaxy

$$SFR_{MW} = \frac{M_{H_2}}{M_J}\cdot\frac{M_J}{t_{ff}} = M_{H_2}/t_{ff} = 10^9 M_\odot/4\times10^5 yr \simeq 2\times10^3 M_\odot/yr$$

Key assumptions:

① All H_2 in molecular clouds are prone to gravitational collapse

② SF proceeds on the freefall timescale

Kelvin-Helmholtz timescale (cooling timescale)

$$t_{KH} = \frac{GM^2/R}{4\pi R^2 \sigma_{SB} T^4} \simeq 10^7\,\mathrm{yr} \left(\frac{M}{M_\odot}\right)^2 \cdot \left(\frac{R}{R_\odot}\right)^{-3} \left(\frac{T}{T_\odot}\right)^{-4} \propto M \cdot \rho \cdot T^{-4}$$

think about a proto star before fusion ignition

Turbulence Support against gravitational collapse

$$M_J = 8 M_\odot \left(\frac{T}{10K}\right)^{3/2} \left(\frac{10^4\,\mathrm{cm}^{-3}}{n_{H_2}}\right)^{1/2} \qquad C_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma k T}{\mu m_p}}$$

$$= 8 M_\odot \left(\frac{\sigma}{0.3\,\mathrm{km/s}}\right)^3 \left(\frac{10^4\,\mathrm{cm}^{-3}}{n_{H_2}}\right)^{1/2} \qquad = 0.54\,\mathrm{km/s} \left(\frac{\gamma}{7/5}\right)^{1/2} \left(\frac{2}{\mu}\right)^{1/2} \left(\frac{T}{50K}\right)^{1/2}$$

observed velocity dispersion in clouds $\gg$ thermal broadening, indicating supersonic turbulence in larger clouds dominates the kinetic energy.

$$\sigma = 3\,\mathrm{km/s} \left(\frac{R}{20\mathrm{pc}}\right)^{0.4} = \begin{cases} 0.36\,\mathrm{km/s} & \text{for } R \sim 0.1\,\mathrm{pc} \text{ (core size)} \\ 5.7\,\mathrm{km/s} & \text{for } R \sim 100\,\mathrm{pc} \text{ (GMC size)} \end{cases}$$

Larson (1981)

$\Rightarrow M_J \simeq 10^7 M_\odot$ for GMCs, which is greater than typical mass of GMCs ($10^5 M_\odot$)

$M_J \simeq 10 M_\odot$ for cores, comparable to the typical core mass ($10 M_\odot$)

Therefore, only the densest 1% molecular clouds can form stars.

Reevaluating MW SFR:

$$SFR_{MW} = N_{core} \cdot (SFR \text{ per core}) = \frac{\epsilon M_{H_2}}{M_{core}} \cdot \frac{M_{core}}{t_{SF}} = \epsilon \frac{M_{H_2}}{t_{SF}}$$

$\epsilon \simeq 1\%$ — core mass fraction $\qquad t_{SF} = \max(t_{\text{gas replenish}}, t_{cool}, t_{ff})$

$t_{KH} \simeq 10^7\,\mathrm{yr}$ — cooling timescale $\qquad$ how fast cores can be replenished.

$$\Rightarrow SFR_{MW} = 10^{-2} \cdot \frac{10^9 M_\odot}{10^7\,\mathrm{yr}} \simeq 1\,M_\odot/\mathrm{yr}$$

$$SFE_1 \equiv \frac{SFR}{M_{H_2}} = \epsilon / t_{SF} \simeq 1\,\mathrm{Gyr}^{-1}$$

$$SFE_2 \equiv \frac{SFR \cdot t_{ff}}{M_{H_2}} = \epsilon \cdot (t_{ff}/t_{SF}) \simeq 10^{-3}$$

Bonnor-Ebert Sphere

isothermal gas, self gravitating, hydrostatic equilibrium, spherical symmetry:

$$\frac{dP}{dr} = -\rho\frac{d\Phi}{dr}, \quad \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 4\pi G\rho, \quad P = \rho\cdot\frac{kT}{\mu m_p} = \rho c_s^2$$

isothermal sound speed

sigular isothermal sphere (SIS) is obtained when we try $\rho = \rho_0 \cdot r^\alpha$, which gives

$$P = \rho_0 c_s^2 r^\alpha$$

$$\frac{d\Phi}{dr} = -\frac{1}{\rho}\cdot\frac{dP}{dr} = -\alpha c_s^2/r$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -\alpha c_s^2\cdot r^{-2} = 4\pi G\rho_0 r^\alpha$$

$$\Rightarrow \alpha = -2 \quad \& \quad \rho_0 = \frac{c_s^2}{2\pi G} \quad \Rightarrow \rho = \frac{c_s^2}{2\pi G}r^{-2}$$

Other solutions exist for the same set of differential equations. In particular, we're interested in a solution where $\rho(r=0)$ is finite. To do that, we first combine the three equations into one equation. First, Eqs ① + ③ gives:

$$\frac{dP}{dr} = \boxed{c_s^2\frac{d\rho}{dr} = -\rho\frac{d\Phi}{dr}} \Rightarrow \rho = \rho_c\cdot\exp\left(-\frac{\Phi-\Phi_c}{c_s^2}\right)$$

define two dimensionless variables:

$$y = \frac{\Phi-\Phi_c}{c_s^2}, \quad x = \left(\frac{4\pi G\rho_c}{c_s^2}\right)^{1/2}\cdot r \qquad (y = \ln \rho_c/\rho)$$

logarithmic density

Poisson Eq. then can be written in a dimensionless form:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 4\pi G\rho_c\cdot\exp\left(-\frac{\Phi-\Phi_c}{c_s^2}\right)$$

$$\rightarrow \frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) = e^{-y} \quad \text{isothermal Lane-Emden Eq.}$$

Its numerical solution gives us the density profile:

$$r = \left(\frac{c_s^2}{4\pi G\rho_c}\right)^{1/2}x, \quad \rho = \rho_c\exp(-y)$$

Boundary condition: $y=0$ & $\frac{dy}{dx}=0$ at $x=0$. ρ_c is specified.

The profiles solved under the boundary condition is called Bonner-Ebert spheres (1955), representing non-singular isothermal spheres.

The enclosed mass within its boundary is:

$$M = 4\pi \int_0^{r_t} \rho(r)\, r^2 dr = 4\pi \left(\frac{c_s^2}{4\pi G \rho_c}\right)^{3/2} \int_0^{x_t} \rho(x)\, x^2 dx$$

$\rho(x) = \rho_c \cdot e^{-y}$

$$= \frac{1}{\sqrt{4\pi \rho_c}} \frac{c_s^3}{G^{3/2}} x_t^2 \left.\frac{dy}{dx}\right|_{x_t}$$

$$= \frac{c_s^3}{G^{3/2} \rho_t^{1/2}} \cdot m$$

where m is the dimensionless mass:

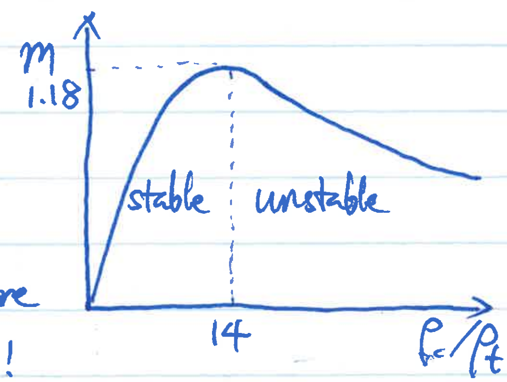
$$m = \frac{1}{\sqrt{4\pi}} \left(\frac{\rho_c}{\rho_t}\right)^{-\frac{1}{2}} x_t^2 \left.\frac{dy}{dx}\right|_{x_t}$$

and x_t is the radius at which the density equals the ambient ISM density

$$y(x_t) = \ln(\rho_c/\rho_t)$$

because x_t only depends on ρ_c/ρ_t, the density contrast, m is only a function of ρ_c/ρ_t. It first increases as the core becomes more centrally concentrated, reaches its peak at $\rho_c/\rho_t \simeq 14$, and begins to decrease at higher contrast!

This last regime is unstable because as the core gains mass, it has to reduce central density!



The Bonner-Ebert mass is the mass at the maximum, it is thus a lower limit for gravitational collapse, similar to Jeans Mass.

$$M_{BE} = \frac{c_s^3}{G^{3/2} \rho_t^{1/2}} m_{max} = 22\, M_\odot \left(\frac{\mu}{2.3}\right)^{-2} \left(\frac{n_t}{10^2\,\mathrm{cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{T}{15\,\mathrm{K}}\right)^{3/2}$$

$$M_J = 120\, M_\odot \left(\frac{\mu}{2.3}\right)^{-\frac{3}{2}} \left(\frac{n_{H_2}}{10^2\,\mathrm{cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{T}{15\,\mathrm{K}}\right)^{3/2}$$

Size of Protostellar Disk

Jeans length $R_J = \sqrt{\dfrac{15kT}{4\pi G \mu m_p \rho}} = 0.1\,pc\left(\dfrac{T}{10K}\right)^{1/2}\left(\dfrac{\mu}{2}\right)^{-\frac{1}{2}}\left(\dfrac{n_{H_2}}{10^4 cm^{-3}}\right)^{-\frac{1}{2}}$

conservation of angular momentum dictates the formation of a rotating disk

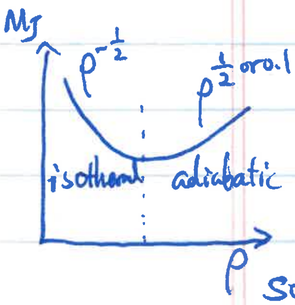
the edge of the disk has a radius of r_f & circular velocity v_f

the initial rotating velocity of the cloud is V_0. We have two equations:

$$\frac{GM_J}{r_f^2} = \frac{V_f^2}{r_f}, \qquad V_f \cdot r_f = V_0 \cdot R_J$$

$$\Rightarrow r_f = \frac{V_0^2 R_J^2}{GM_J} \simeq 600\,AU \left(\frac{V_0}{0.1 km/s}\right)^2 \left(\frac{R_J}{0.1pc}\right)^2 \cdot \left(\frac{M_J}{8M_\odot}\right)^{-1}$$

Cloud Fragmentation & the minimum Jeans Mass



$$M_J \propto T^{3/2} \rho^{-\frac{1}{2}} \propto \begin{cases} \rho^{-\frac{1}{2}} & \text{during isothermal collapse } (T = \text{const.}) \\ \rho^{(3\gamma-4)/2} & \text{in adiabatic collapse } (T \propto \rho^{\gamma-1}) \end{cases}$$

so the Jeans mass reaches its minimum when the collapse transitions to adiabatic.

This transition occurres when radiative cooling becomes inefficient:

$$t_{ff} < t_{cool} \text{ where } t_{cool} = t_{KH} = \frac{GM^2/R}{4\pi R^2 \sigma_{SB} T^4}, \quad t_{ff} = \sqrt{\frac{3\pi}{32 G\rho}}$$

We also know $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, and $R = \dfrac{GM}{5kT}\mu m_p$ from virial theorem

plug these in to replace ρ & R, we have

$$M > 0.1 M_\odot \left(T/1000K\right)^{1/4} \left(\mu/1\right)^{-9/4} \equiv M_{J,min}$$

Measuring the IMF

Ψ: Psi
Ξ/ξ: xi
$M_{V,\odot} = 4.8$

Observables : $\Psi(M_V)$ the present-day luminosity function
Models : $m - M_V$ relation & its derivative dm/dM_V
Equations :

$$\Psi(M_V)\, dM_V = dN = \Xi(m)\, dm \quad , \; \Xi(m): \text{present mass fun.}$$

$$\Xi(m) = \xi(m) \cdot \frac{1}{\tau_G} \int_{\tau_G - \tau(m)}^{\tau_G} b(t)\, dt \simeq \xi(m) \cdot \frac{\tau(m)}{\tau_G} \text{ or } \xi(m)$$

where τ_G is the age of the system (e.g. the MW), and $\tau(m)$ is the main-sequence life time of the star w/ mass of m.
$b(t)$ is the normalized star formation history :

$$\frac{1}{\tau_G} \cdot \int_0^{\tau_G} b(t)\, dt = 1.0$$

For MW disk stars, it's assumed to be $b(t) = 1.0$ (const. SF)
For star clusters, it's assumed to be $b(t) = \tau_G \cdot \delta(t - t_0)$ (burst)

Combining the two equations :

$$\xi(m) = \Psi(M_V) \cdot \left(\frac{dm}{dM_V}\right)^{-1} \times \begin{cases} 1.0 & , \tau(m) > \tau_G \text{ , low-mass stars} \\ \tau_G/\tau(m) & , \tau(m) < \tau_G \text{ , high-mass stars} \end{cases}$$

Kroupa (2002) IMF :

$$\xi(m) \propto \begin{cases} m^{-0.3} & m < 0.08\, M_\odot \\ m^{-1.3} & 0.08 < m < 0.5\, M_\odot \\ m^{-2.3} & m > 0.5\, M_\odot \end{cases}$$

Brown Dwarfs : $m < 0.072\, M_\odot$
Very low mass : $0.072 - 0.5\, M_\odot$
Low mass stars: $0.5 - 1\, M_\odot$
intermediate mass: $1 - 8\, M_\odot$
massive stars : $8 - 120\, M_\odot$

Initial Mass Function

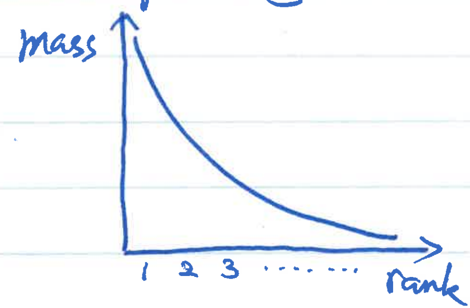
$$\Phi(m) \equiv \frac{dN}{dm} = \phi_0 \cdot m^{-2.35} \quad \text{Salpeter (1955)}$$

$$M^* \text{ mass density} = \int_{0.1M_\odot}^{100M_\odot} \phi(m)\cdot m\, dm \quad \text{unit: } M_\odot/pc^3$$

$$\text{mass per dex} = \frac{m\,dN}{d\log m} = \ln 10 \cdot m^2 \cdot \phi = \ln 10 \cdot \phi_0 \cdot m^{-0.35} \sim \text{const.}$$

A PDF with a powerlaw slope of -2 is the same as predicting the mass of any star using its rank:

$$m(i) = \frac{m_0}{\text{rank}(i)} \quad \text{or rank} \propto m^{-1}$$



because

$$\frac{dN}{dm} = \left|\frac{d\,\text{rank}}{dm}\right| = m_0\, m^{-2}$$

A similar PDF is seen in the size of cities:

city	pop	ratio	1/rank
NY	8.5	1.0	1.0
LA	3.9	0.46	0.5
CHI	2.7	0.32	0.33
Houston	2.2	0.26	0.25
Phil	1.6	0.18	0.20
Phoenix	1.5	0.18	0.17

A natural result of exponential growth ($\frac{dm}{dt} \propto m^2$) in environs w/ limited resources?

Bondi-Hoyle Accretion: $\dot{M} \simeq \pi R_B^2 \rho v$, $R_B \equiv 2GM/c_s^2$ ($v_{esc} = c_s$), $v \sim c_s$

$$\Rightarrow \dot{M} \simeq \frac{\pi \rho G^2 M^2}{c_s^3} \propto M^2$$