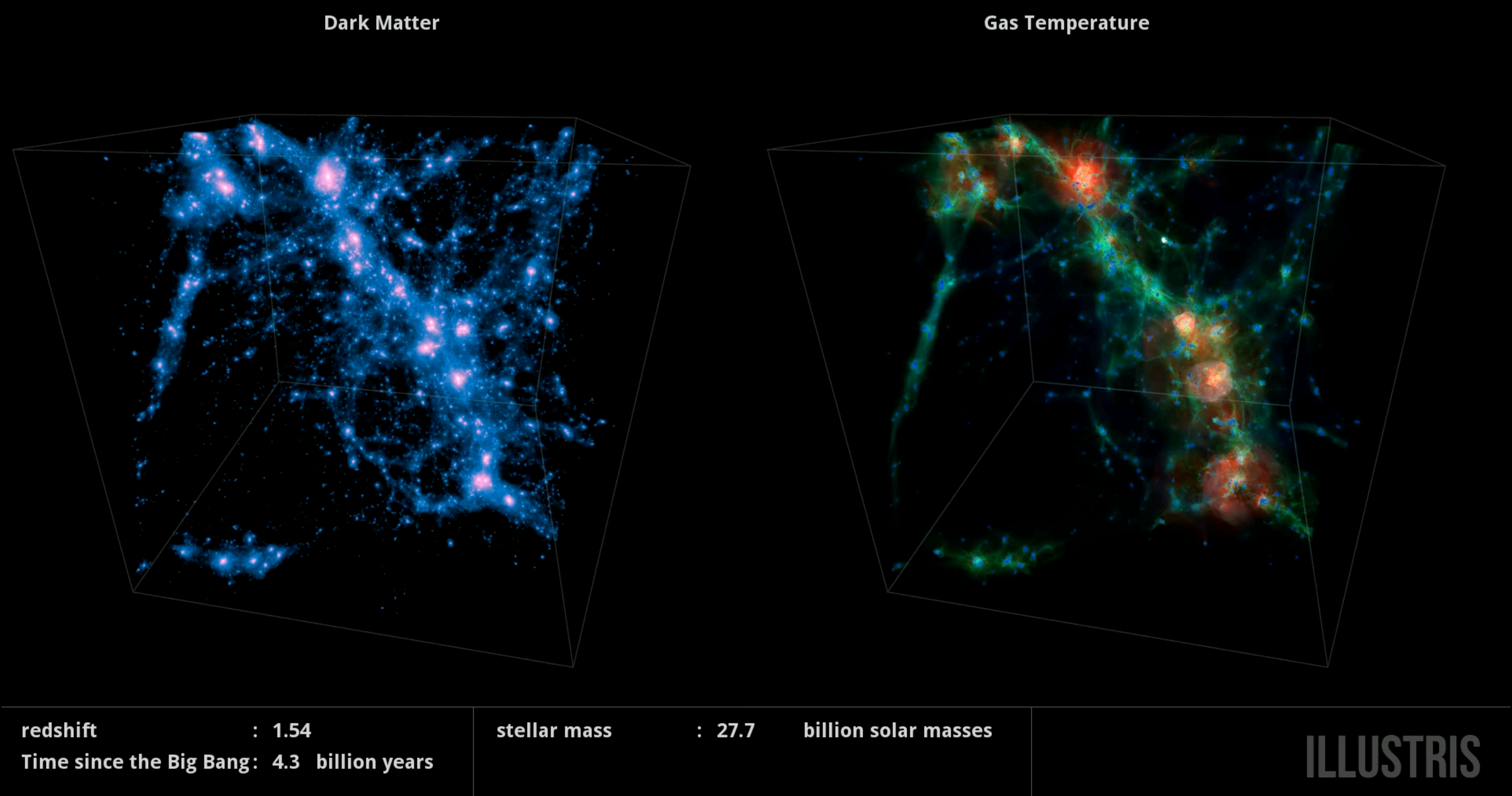
A Brief Review of the Course



Robertson-Walker Metric: Differential Space-Time Distance

- In **General Relativity**, a **metric** is a function which measures ***differential space-time distance*** between two events**:**

$$(ds)^2 = (c \cdot dt)^2 - (dl)^2$$

- The **Robertson-Walker metric** is the metric that describes the geometry of a **homogeneous, isotropic, expanding** universe. The metric in *spherical coordinate system* is:

$$(ds)^2 = (c \cdot dt)^2 - R_U^2(t)\left[\left(\frac{dx}{\sqrt{1-kx^2}}\right)^2 + (xd\theta)^2 + (x\sin\theta d\phi)^2\right]$$

where

R_U is the **scale factor,** defined to be 1 at present day, and <1 in the past
x is the **comoving** radial distance, $x \equiv r(t)/R_U(t)$,
k is the **comoving** curvature, $k \equiv \frac{1}{R^2} = \frac{R_U^2}{R^2 R_U^2} = K(t)R_U^2(t)$,
R is the **comoving** radius of the curvature.

The same terms are in **Friedmann Equation**.

Solution of FE1: Hubble Parameter vs. redshift - E(z)

- The **FE1 in density parameters and Hubble parameter**:

$$R_U^2 H^2 \left[1 - (\Omega_m + \Omega_\gamma + \Omega_\Lambda)\right] = -kc^2$$

- **Boundary condition** at $t = t_0$ gives the value of the curvature**:**

$$H_0^2 (1 - \Omega_0) = -kc^2$$

- **Equations of state** gives density-scale-factor relations:

$$\frac{\Omega_m}{\Omega_{m,0}} = \frac{\rho_m \rho_{c,0}}{\rho_{m,0} \rho_c} = \frac{\rho_m}{\rho_{m,0}} \frac{H_0^2}{H^2} = \frac{1}{R_U^3} \frac{H_0^2}{H^2}$$

$$\frac{\Omega_\gamma}{\Omega_{\gamma,0}} = \frac{1}{R_U^4} \frac{H_0^2}{H^2} \quad \text{and} \quad \frac{\Omega_\Lambda}{\Omega_{\Lambda,0}} = \frac{H_0^2}{H^2}$$

- Plug in and rearrange:

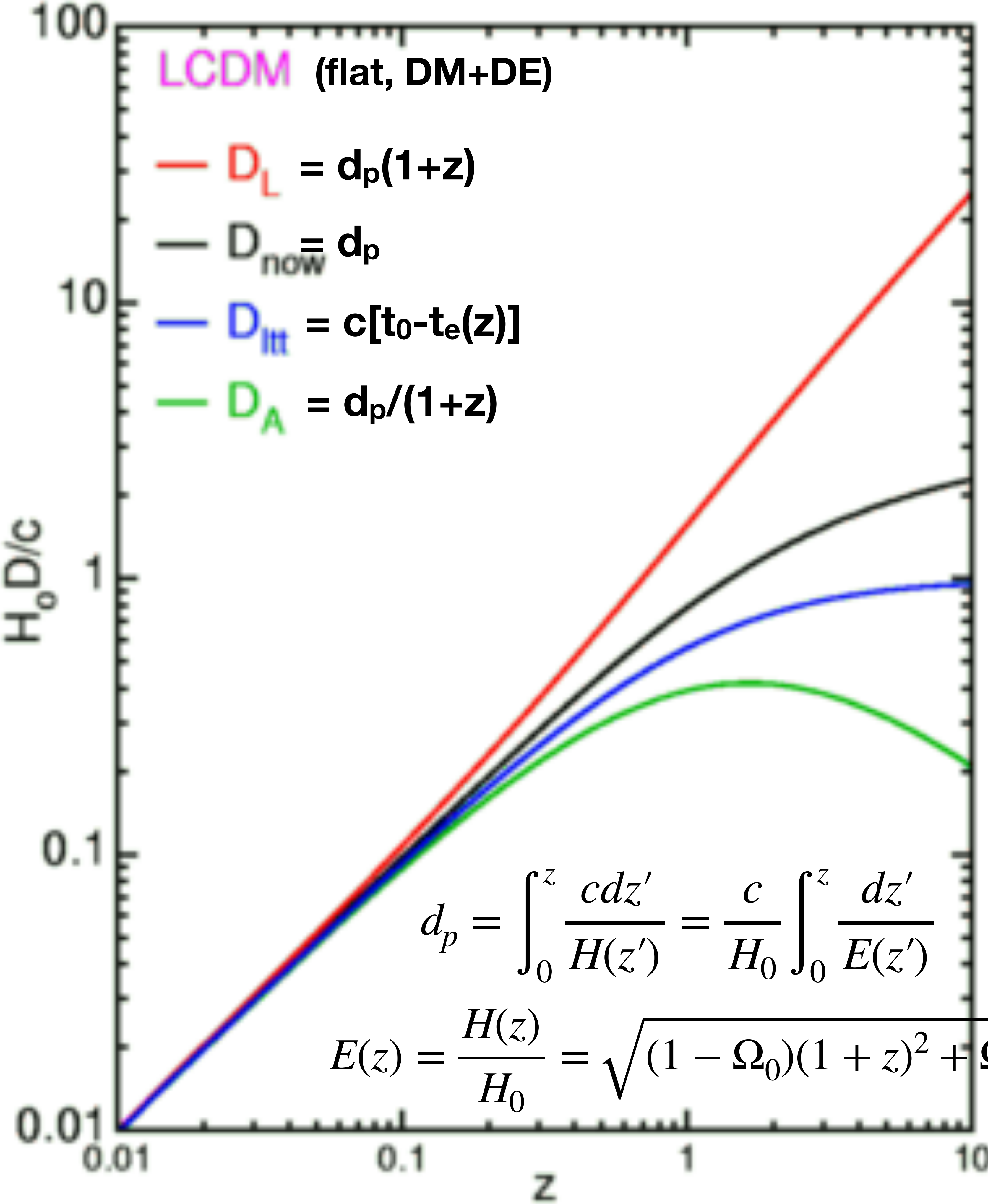
$$H^2 = H_0^2 \left[(1 - \Omega_0)/R_U^2 + \Omega_{m,0}/R_U^3 + \Omega_{\gamma,0}/R_U^4 + \Omega_{\Lambda,0}\right]$$

- Expressed scale factor as redshift and define the **dimensionless Hubble parameter** E(z):

$$E(z) = \frac{H(z)}{H_0} = \sqrt{(1 - \Omega_0)(1+z)^2 + \Omega_{m,0}(1+z)^3 + \Omega_{\gamma,0}(1+z)^4 + \Omega_{\Lambda,0}}$$

D vs. z relation

Distance in unit of the Hubble scale: $D_H = c/H_0 = 3\ h^{-1}$ Gpc

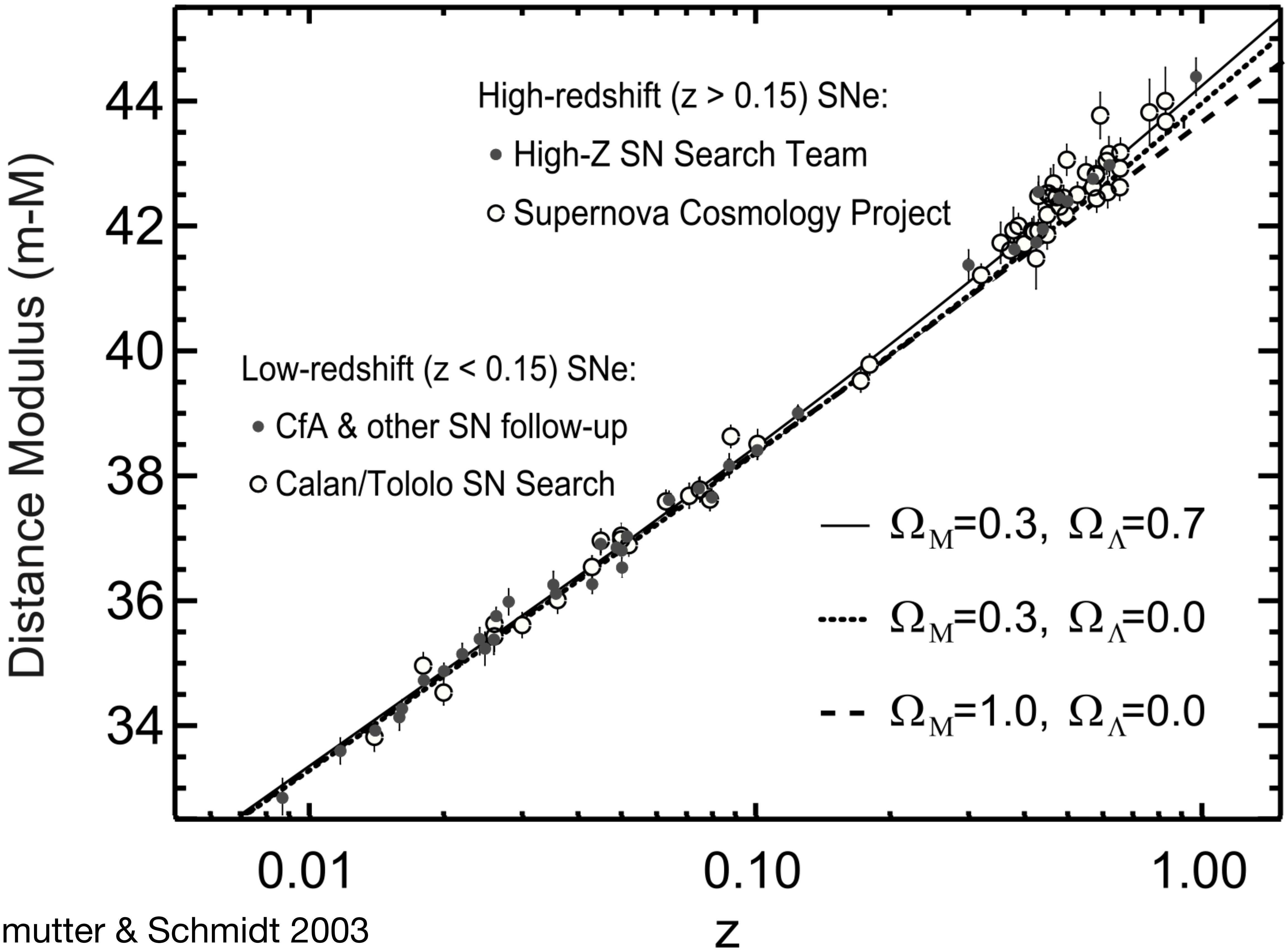


$$d_p = \int_0^z \frac{c\,dz'}{H(z')} = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$$

$$\frac{t}{t_H} = \int_z^\infty \frac{dz'}{(1+z')E(z')}$$

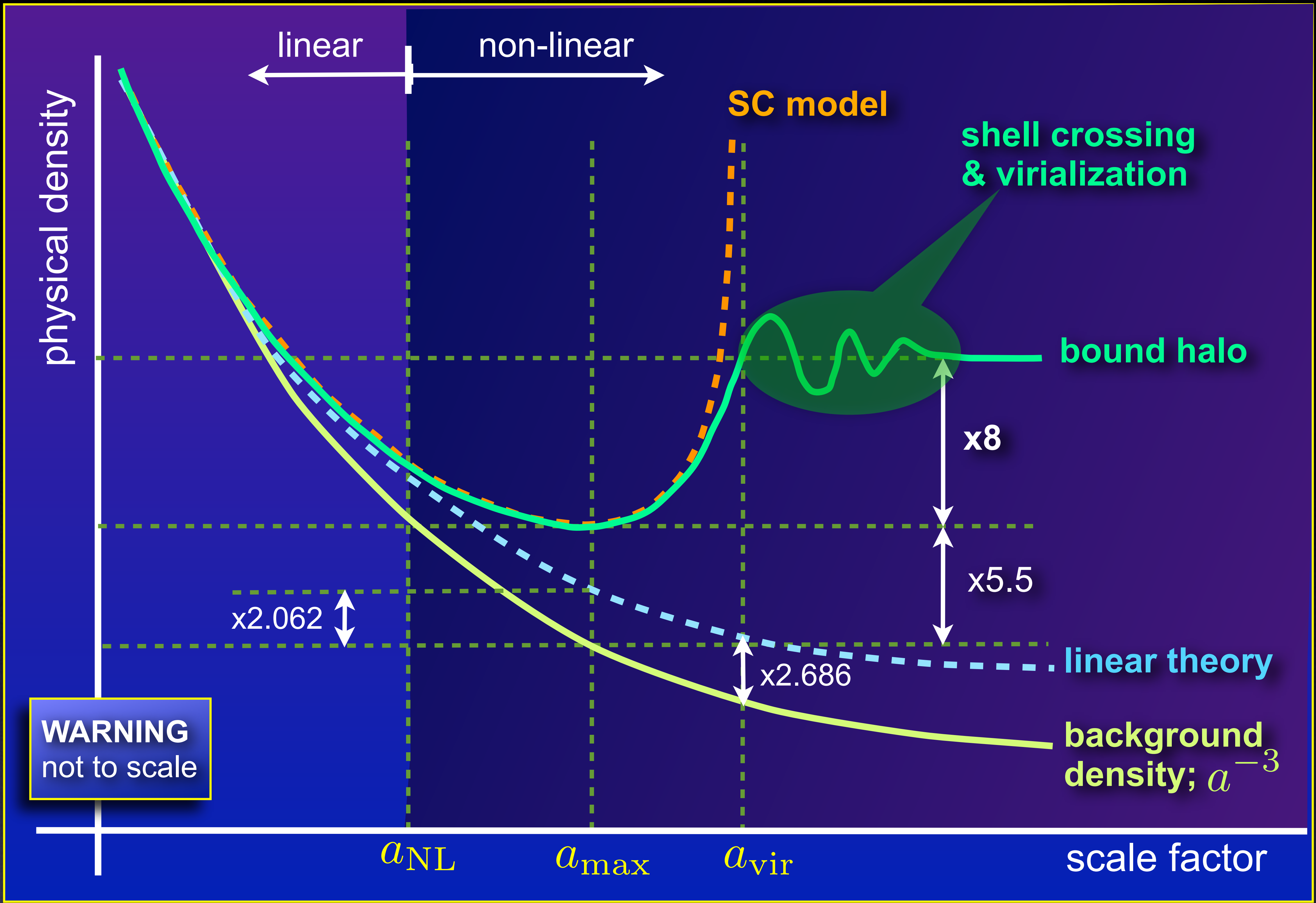
$$E(z) = \frac{H(z)}{H_0} = \sqrt{(1-\Omega_0)(1+z)^2 + \Omega_{m,0}(1+z)^3 + \Omega_{\gamma,0}(1+z)^4 + \Omega_{\Lambda,0}}$$

When we have distance measurement from galaxies at **z > 0.1**, **cosmological density parameters** can be constrained by **the same Hubble diagram**



Perlmutter & Schmidt 2003

Top-Hat Spherical Collapse Model:
First expands then collapses: $18\pi^2$



van den Bosch

NFW Profile vs. SIS Profile, using consistent definitions

Density distribution:

$$\rho(r) = \frac{4\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

Density distribution:

$$\rho(r) = \frac{\rho_s}{(r/r_s)^2}$$

Enclosed Mass:

$$M(r) = 16\pi\rho_s r_s^3 \left(\ln(1 + \frac{r}{r_s}) - \frac{r/r_s}{1 + r/r_s} \right)$$

Enclosed Mass:

$$M(r) = 4\pi\rho_s r_s^2 r$$

Potential - const.:

$$\phi(r) = -16\pi G\rho_s r_s^2 \frac{\ln(1 + r/r_s)}{r/r_s}$$

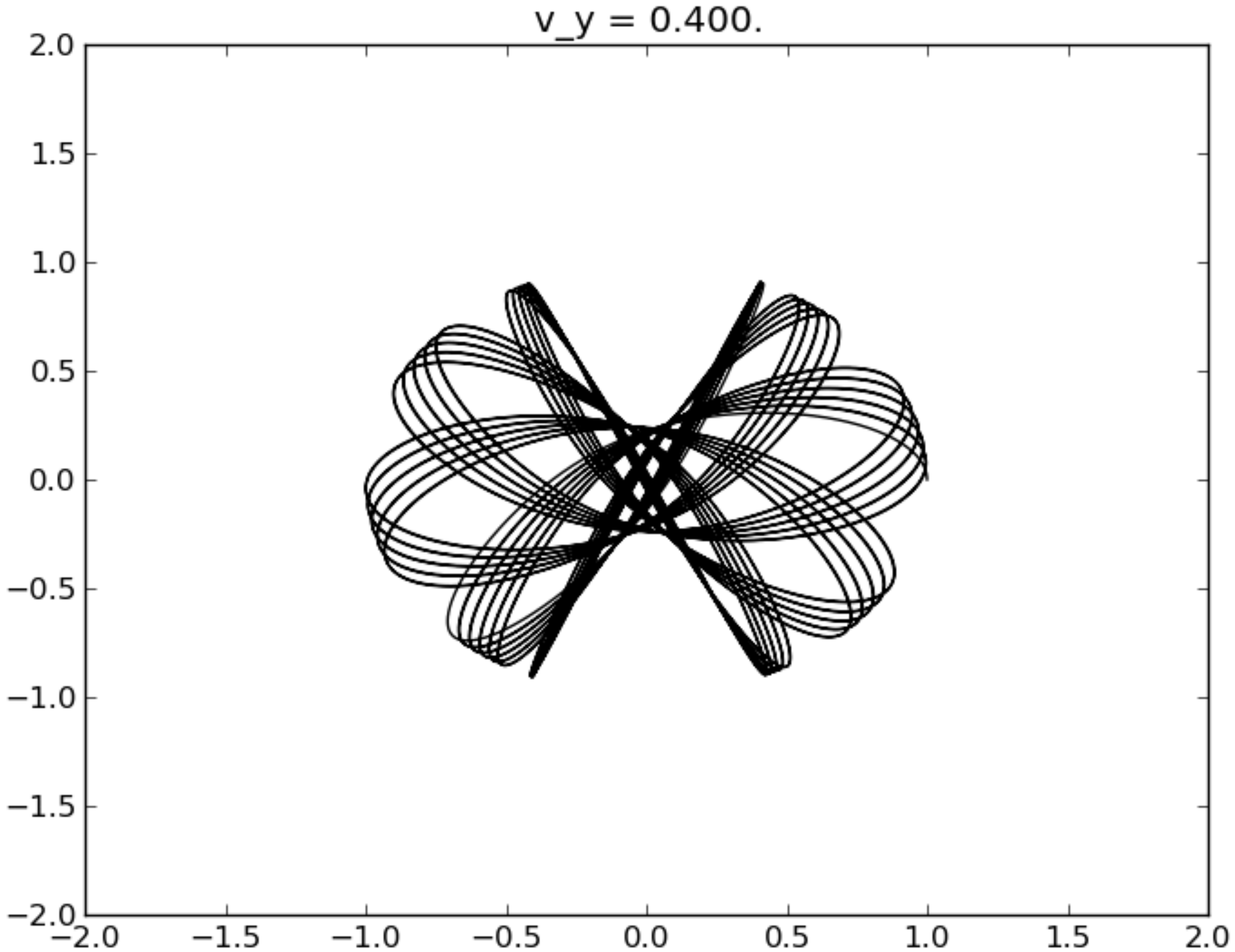
Potential - const.:

$$\phi(r) = 4\pi G\rho_s r_s^2 \ln(r/r_s)$$

What about non-circular orbits in a logarithmic potential field?

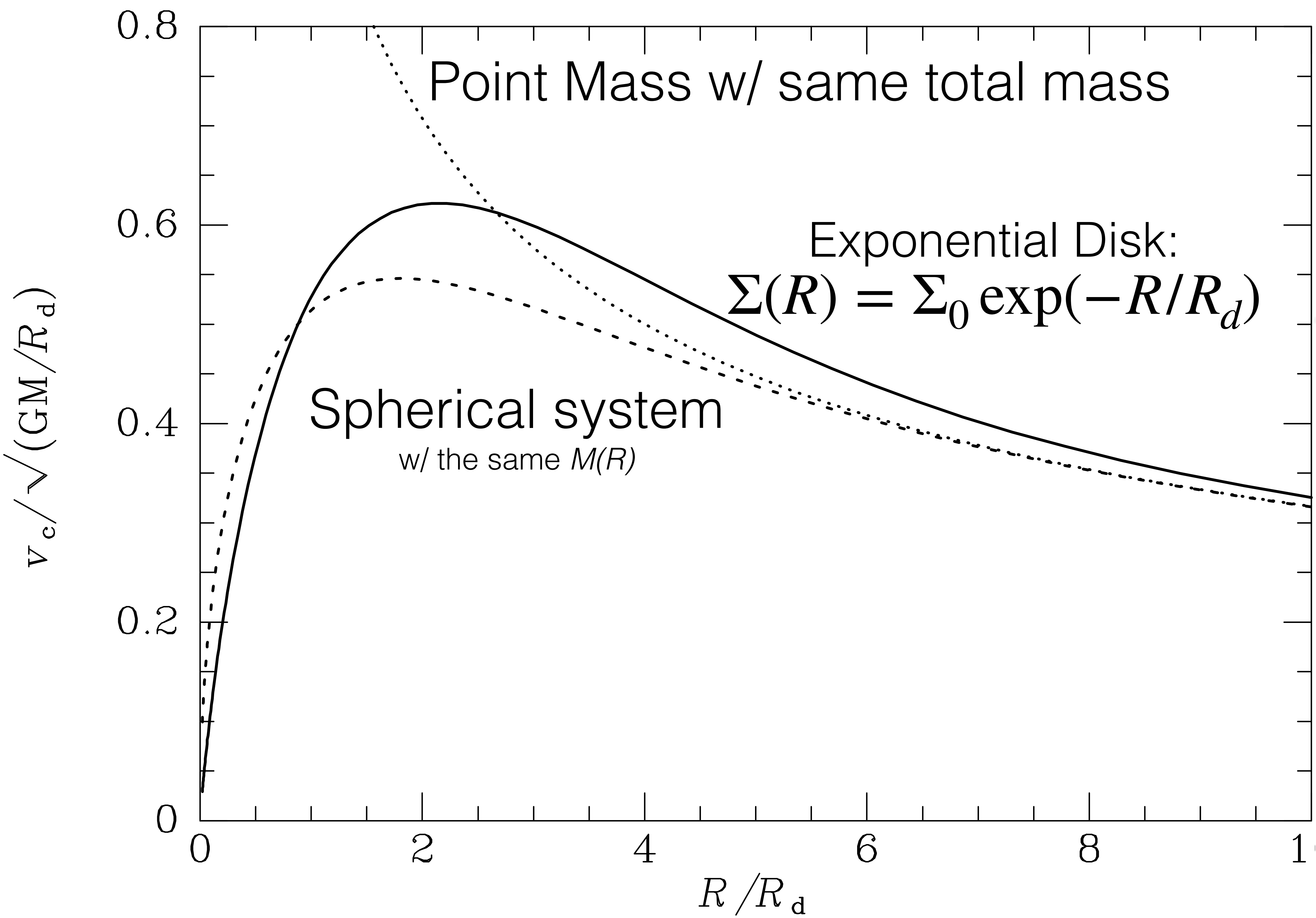
General form: $\phi(x, y, z) = 0.5 \ln(R_c^2 + x^2 + (y/b)^2 + (z/c)^2)$

When b=c=1: $\phi(r) = 0.5 \ln(R_c^2 + r^2)$



Circular Velocity Profiles: Axial Symmetry vs. Spherical Symmetry

The spherical system has the same enclosed mass profile as the disk

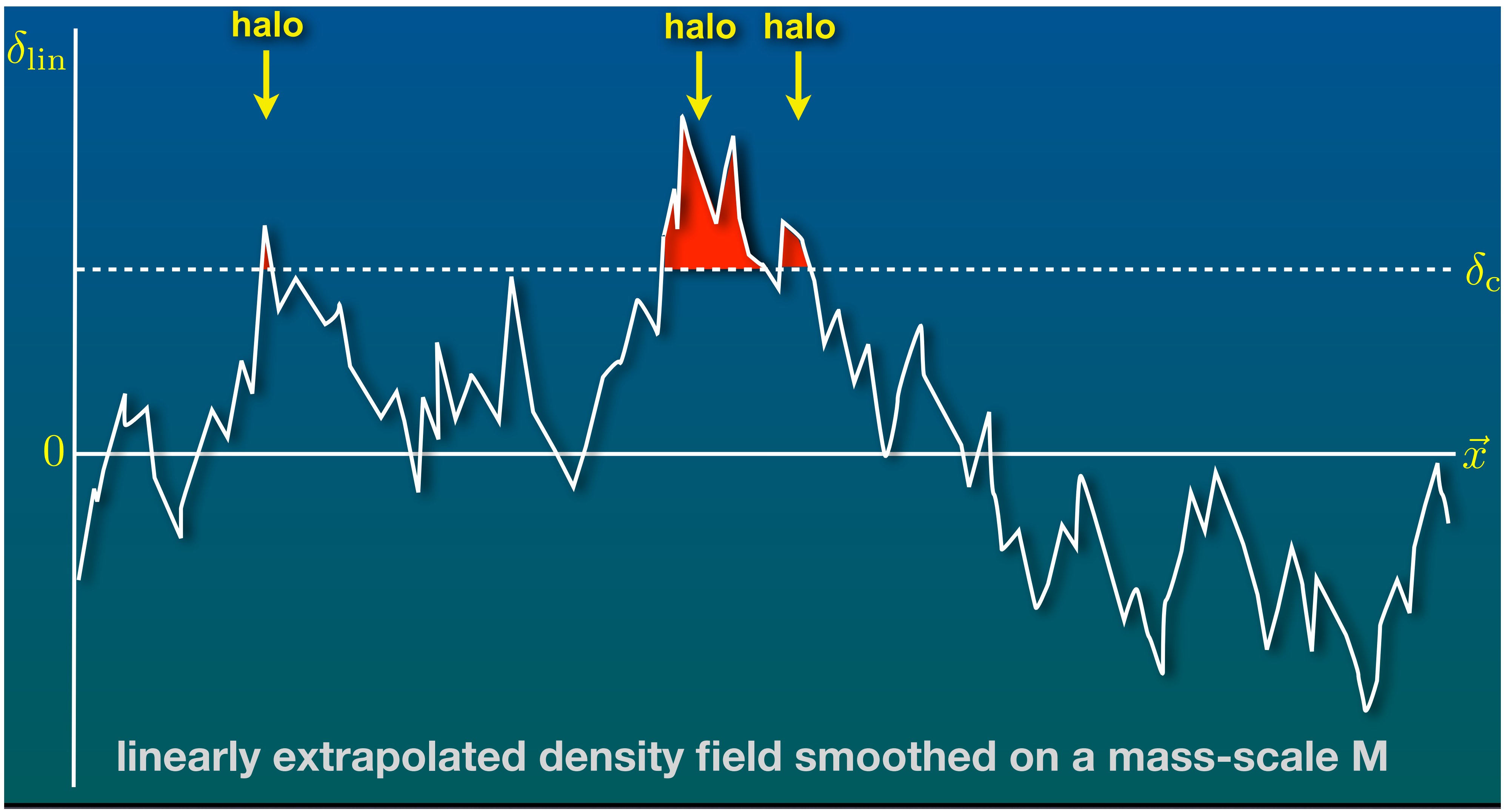


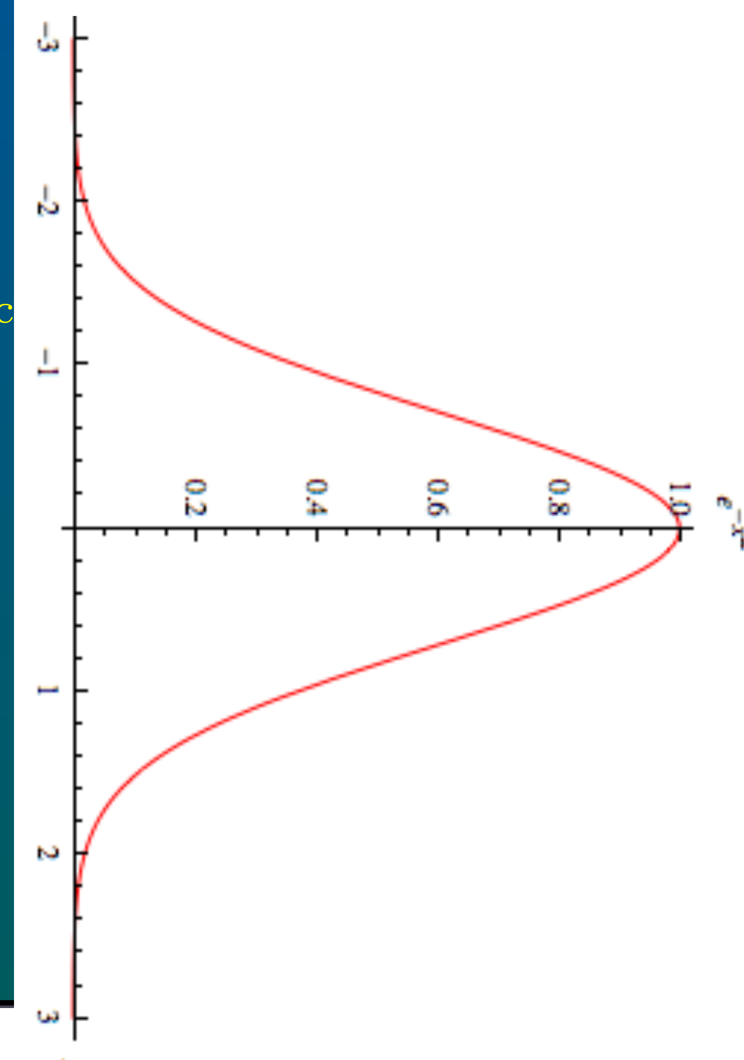
Counting Peaks in the Mass-Smoothed Density Field

The physical scale *R* used to smooth the linearly extrapolated density field $\delta_0(x)$ today determines the minimum mass of all collapsable regions with $\delta_M > \delta_c(z)$, so the fraction of mass locked in halos with masses greater than $M \equiv \gamma \rho_c R^3$ is

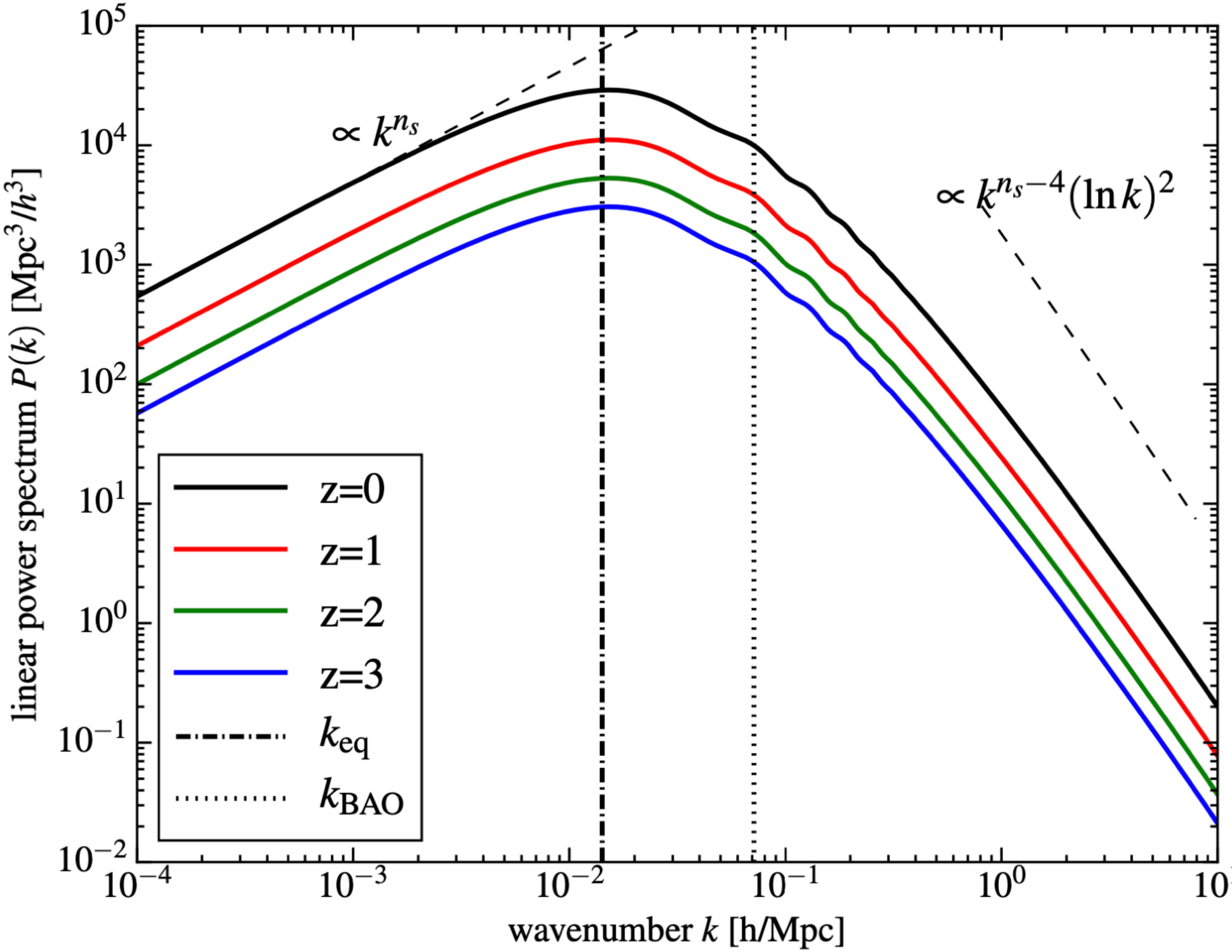
$$F(>M) = P[\delta_M > \delta_c(z)]; \text{ where } \delta_M(x) = \int \delta_0(x') w[x - x'; (M/\gamma\rho_c)^{1/3}] d^3x'$$

where w(x; R) is the window function used for smoothing



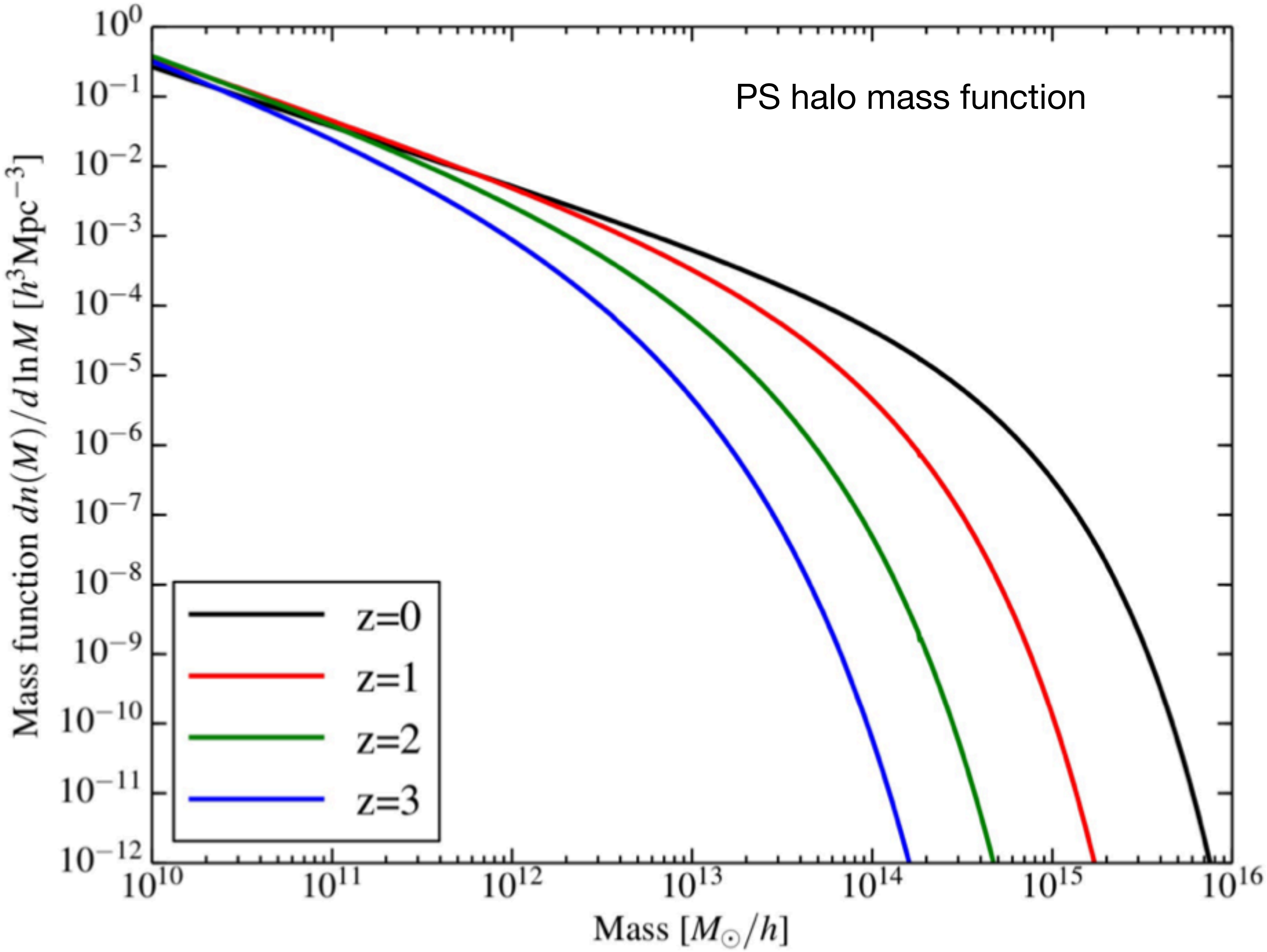


mass variance from integrating the power spectrum:
a. power spectra at different redshifts

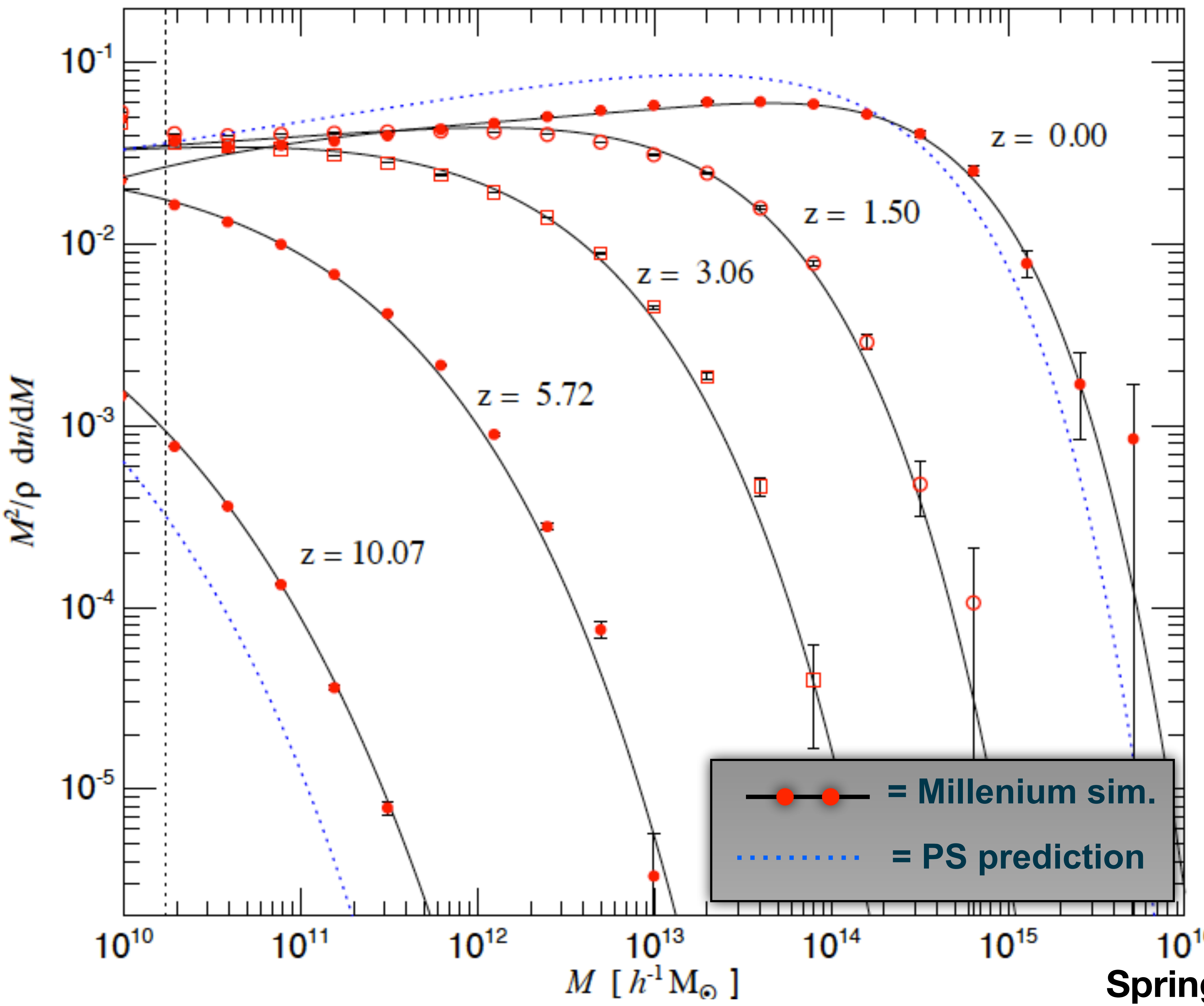


Resulting PS Halo Mass Function at Various Redshifts

$$\phi(M,z) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma_M}\left|\frac{d\ln\sigma_M}{d\ln M}\right|\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

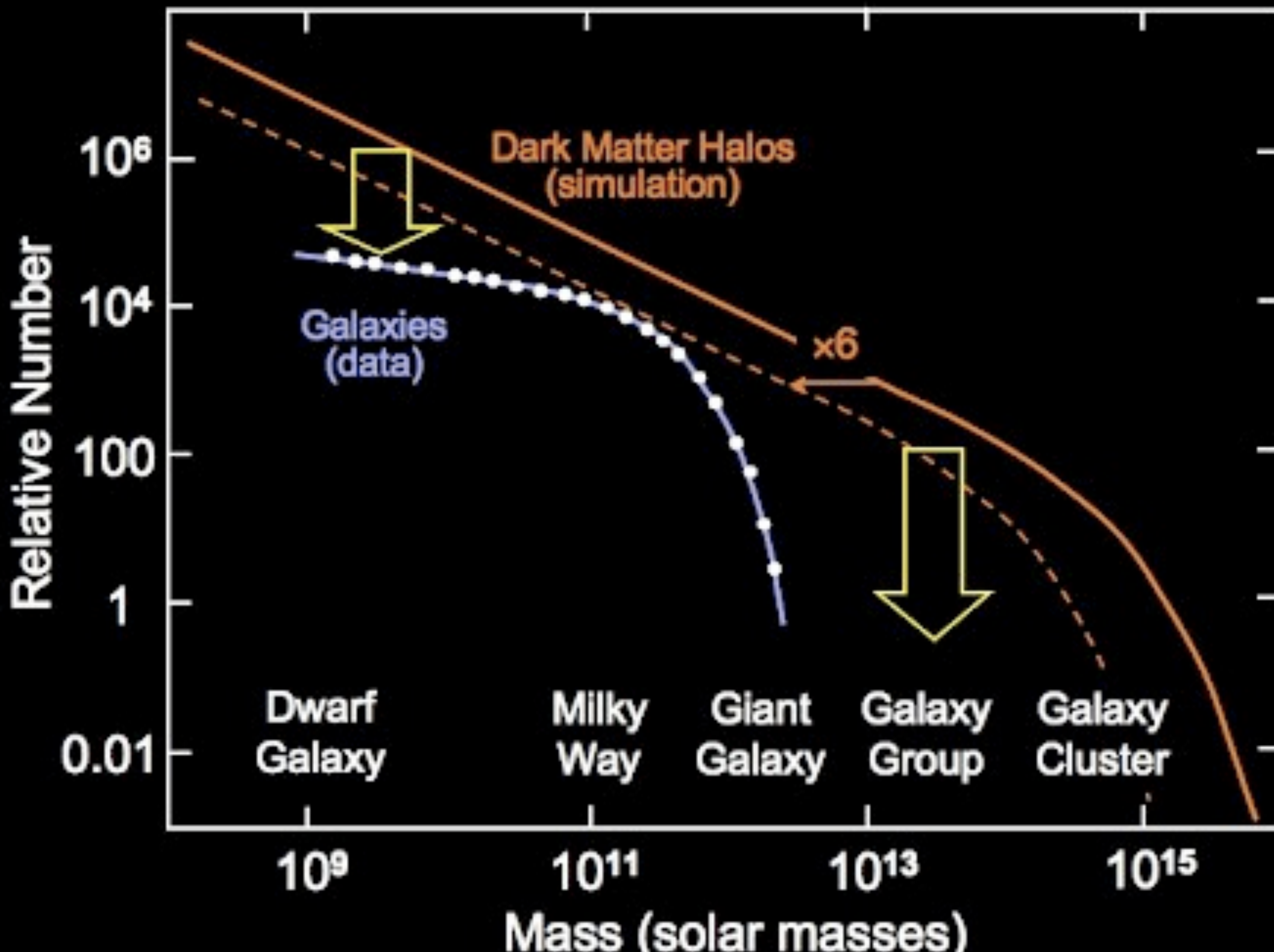


Halo Mass Functions: N-body Simulation vs. Analytical Predictions



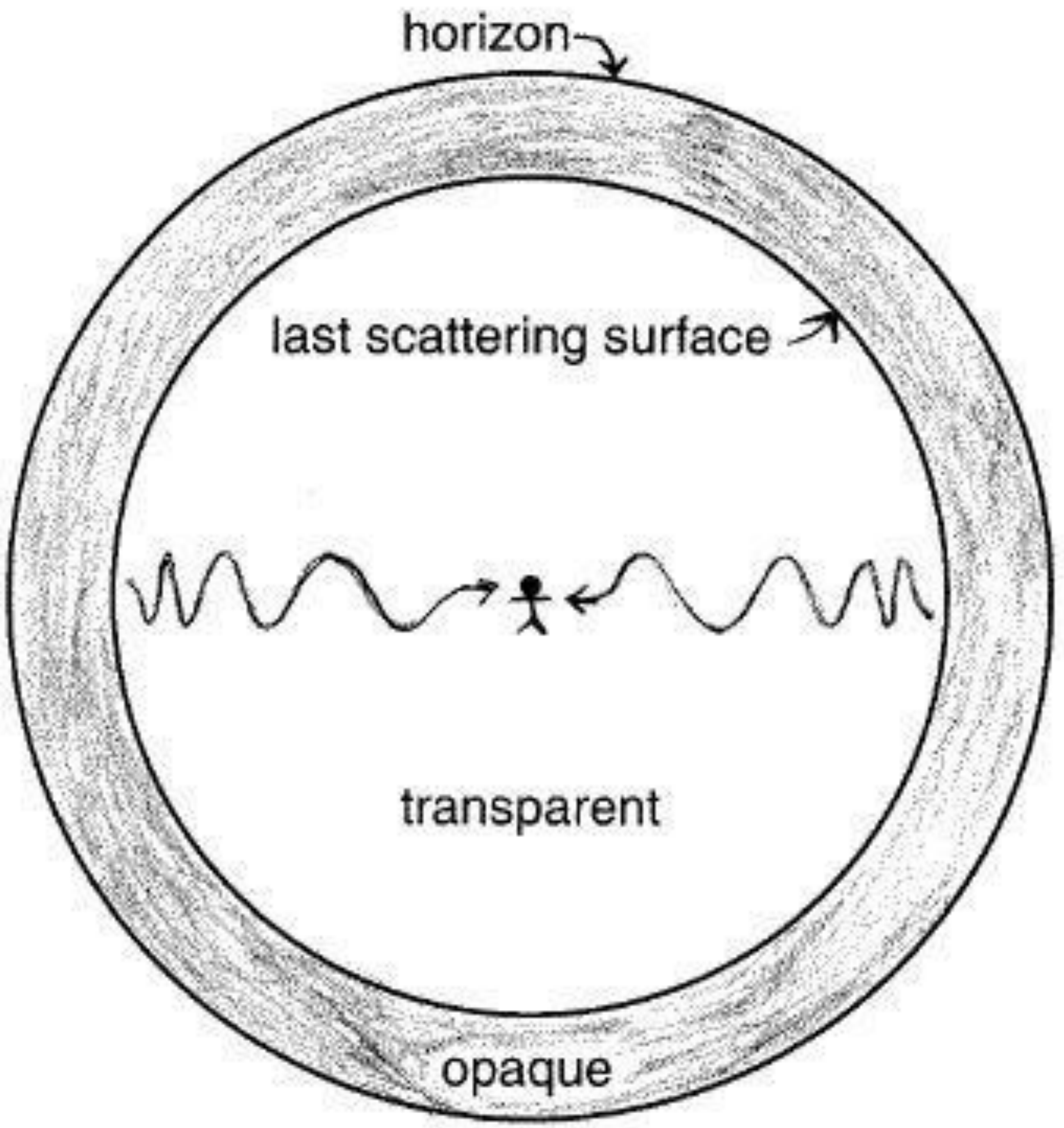
Springel+2005

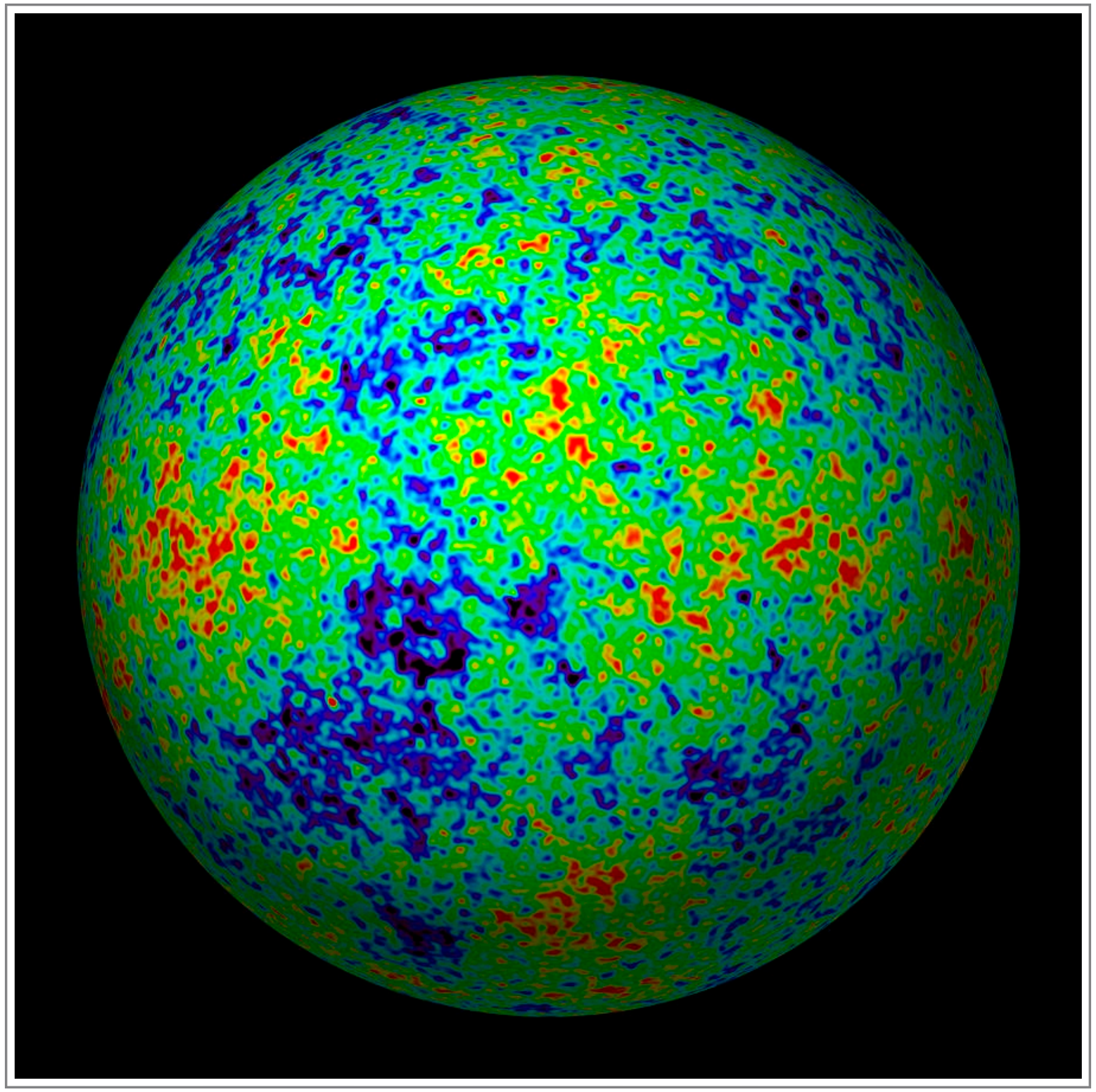
Problem: both the P-S and N-body halo mass function strongly disagrees w/ *Observed* galaxy mass function



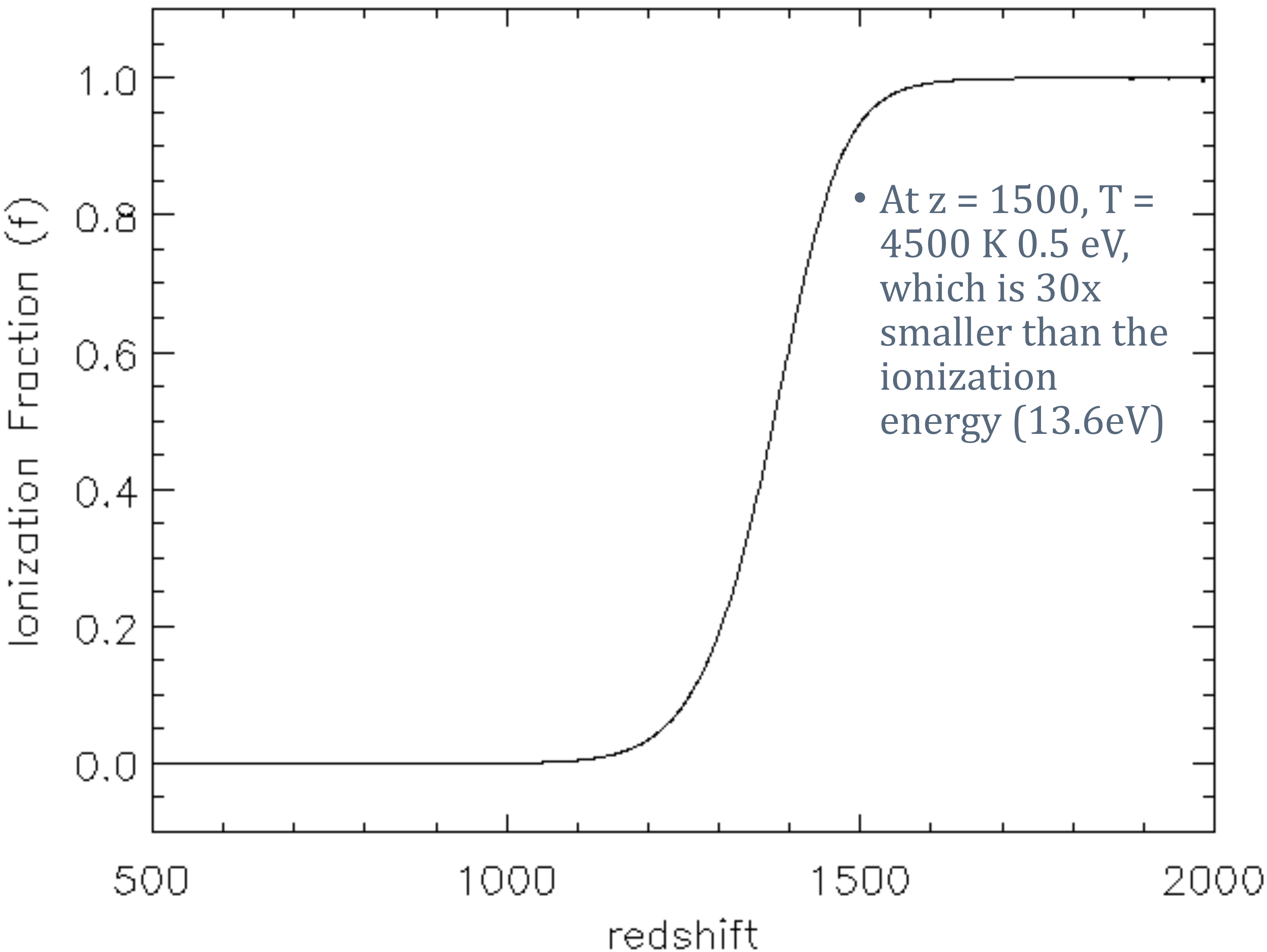
CMB Photons travel straight to us from the last scattering surface

- Analogous to the *last scattering surface* that marks the surface of the Solar photosphere



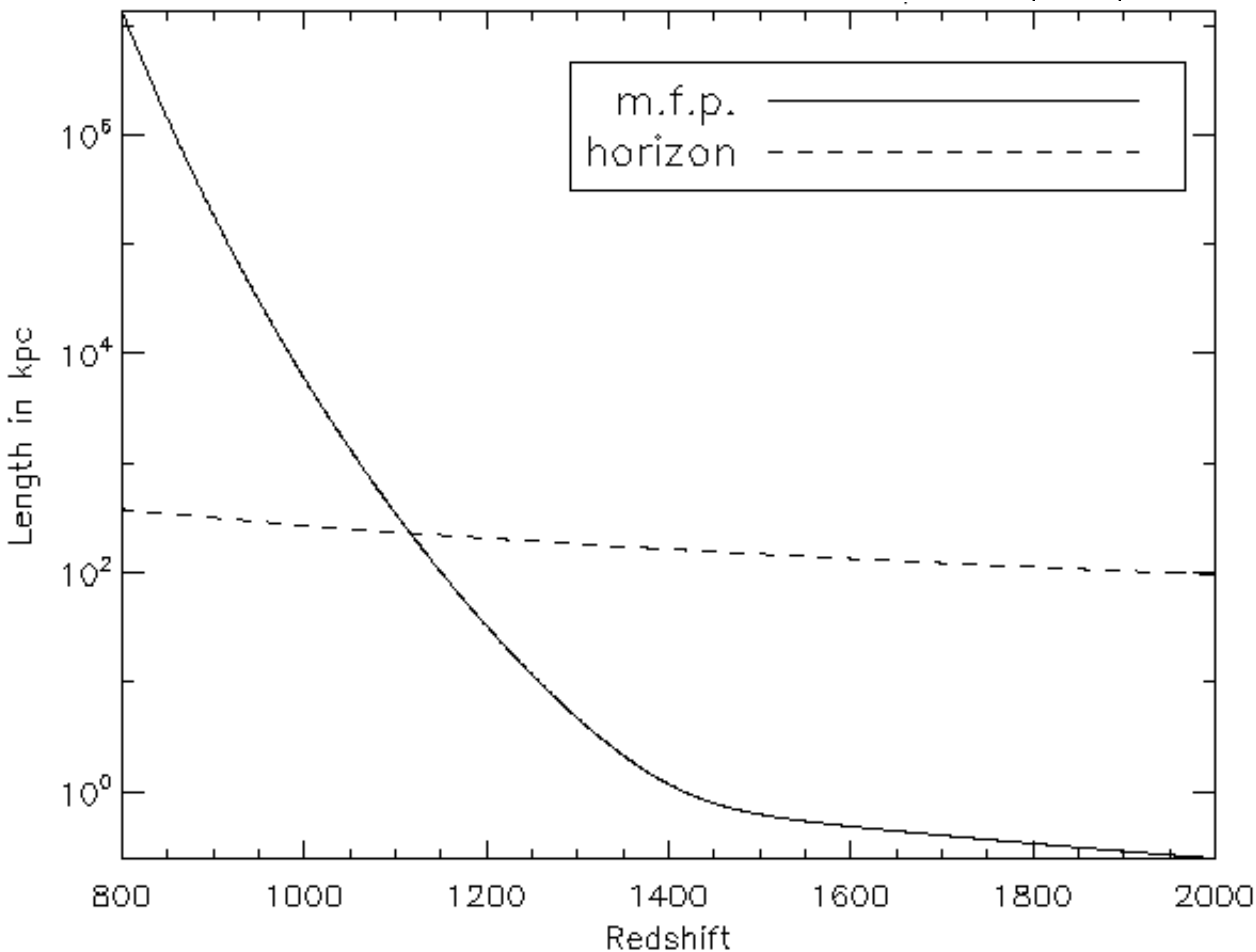


Ionization Fraction of Hydrogen vs. redshift in the Early Universe

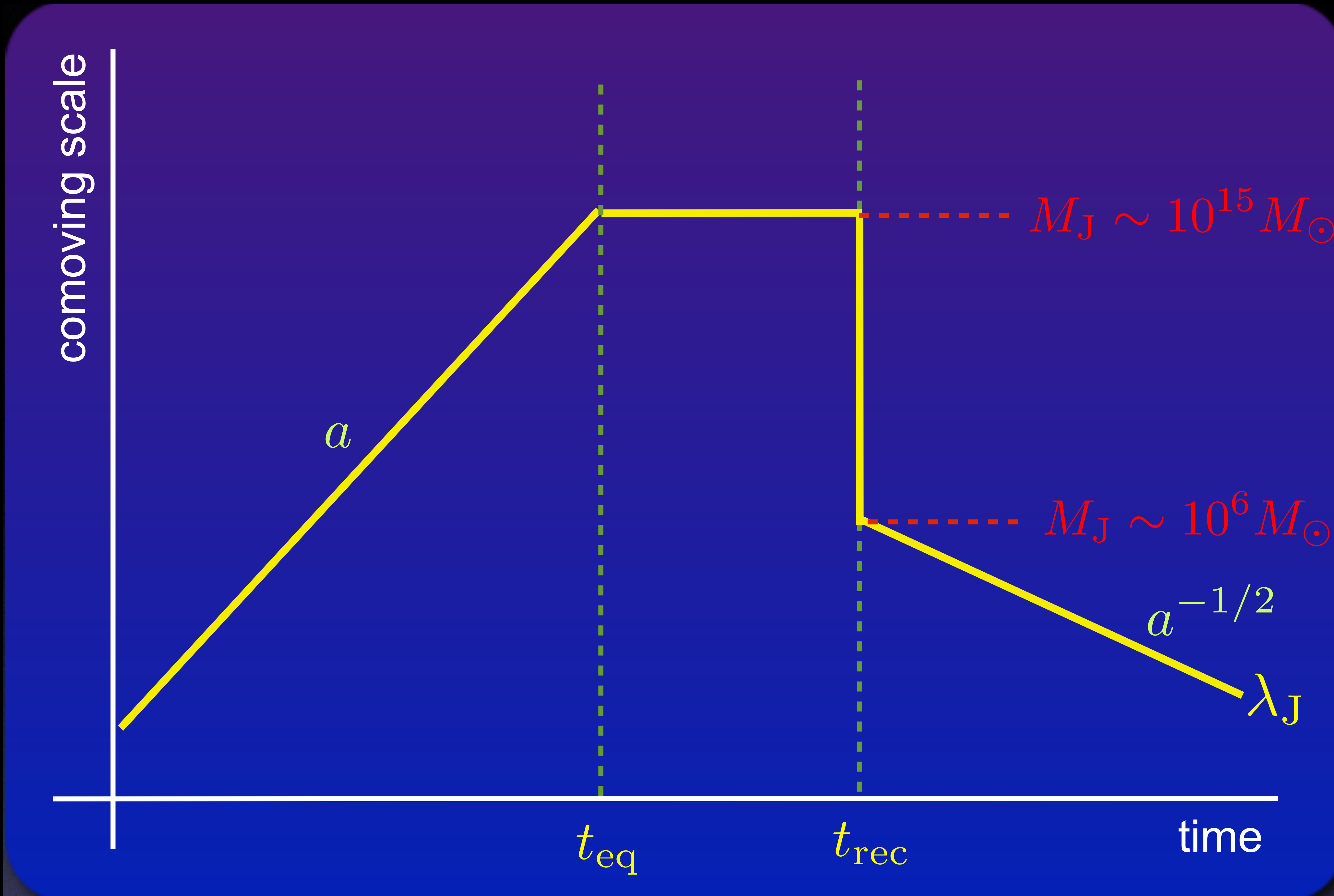


- At z = 1500, T = 4500 K 0.5 eV, which is 30x smaller than the ionization energy (13.6eV)

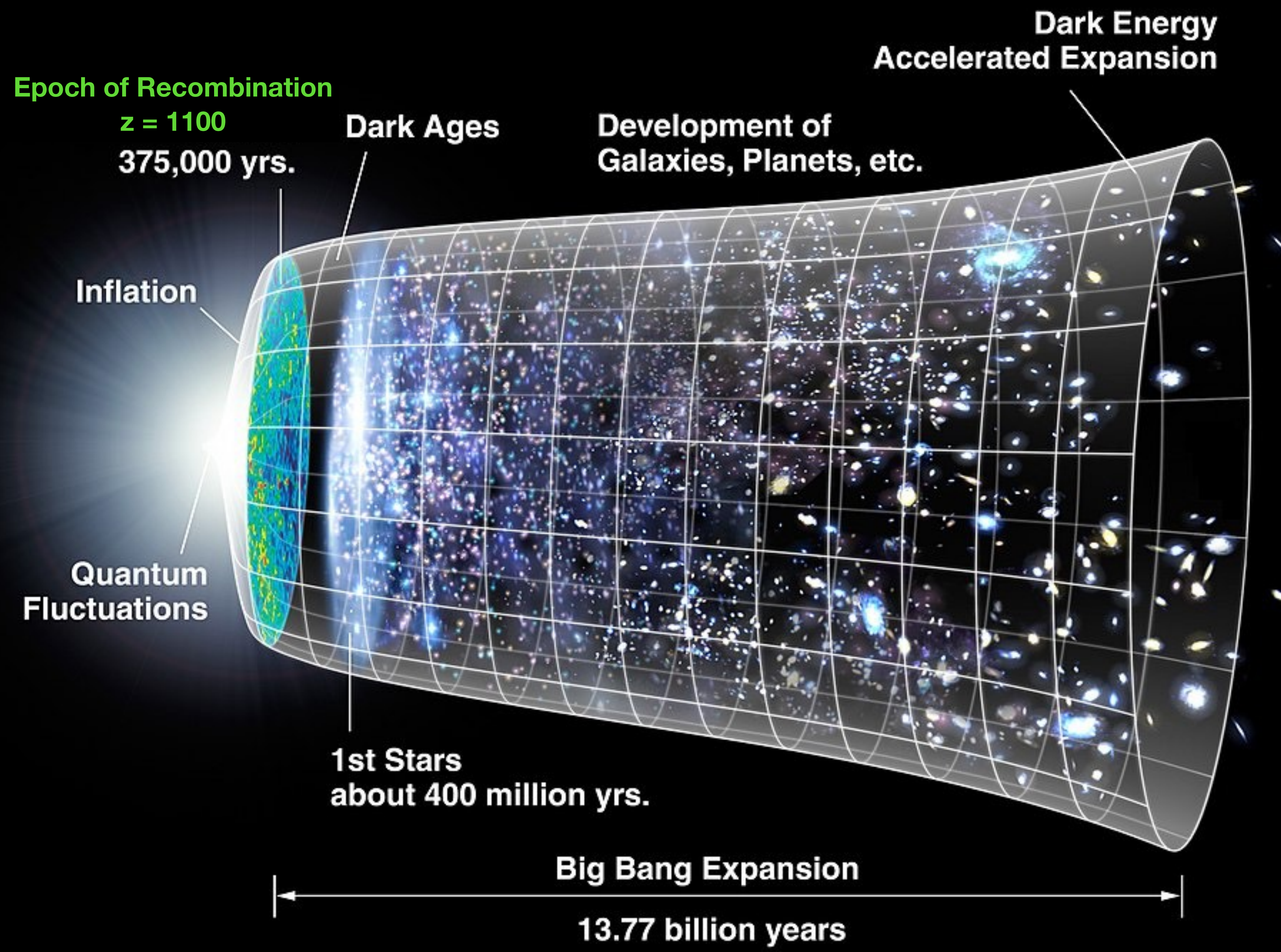
The CMB emerges when the mean free path of photons reaches the size of the cosmic horizon (~*ct*)



Jeans Length vs. Time



Chronology of the Universe Diagram



Virial Velocity & Virial Temperature Expressed Directly with Virial Mass

- **Virial radius:**

$$r_\Delta = \left(\frac{2GM_\Delta}{\Delta_c \Omega_{m,0} H_0^2} \right)^{1/3} (1+z)^{-1} \propto M_\Delta^{1/3} (1+z)^{-1}$$

- **Virial (circular) velocity:**

$$V_\Delta = \sqrt{\frac{GM_\Delta}{r_\Delta}} = (\Delta_c \Omega_{m,0} H_0^2/2)^{1/6} (GM_\Delta)^{1/3} (1+z)^{1/2}$$

- **Virial Temperature:**

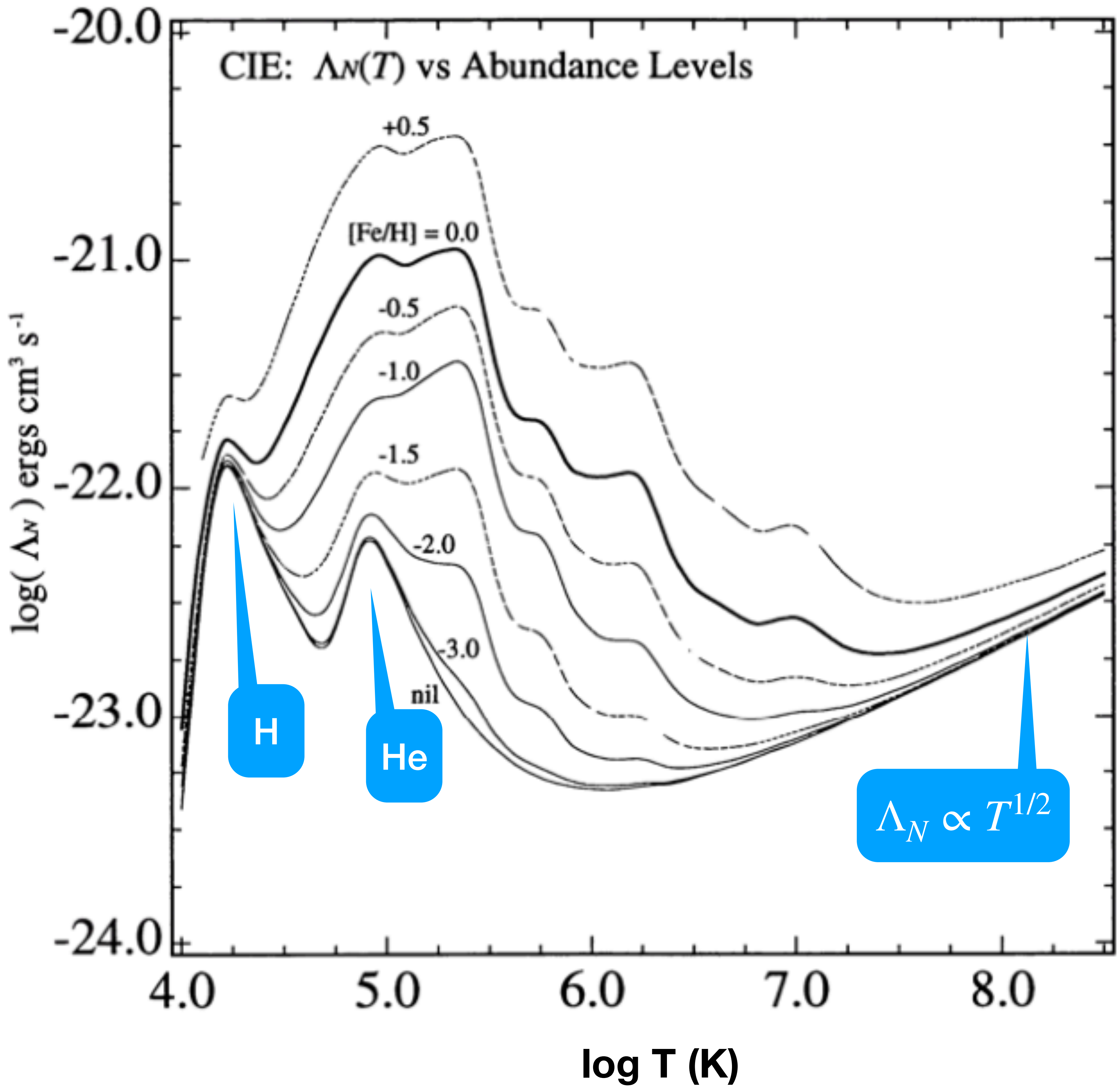
$$T_\Delta = \frac{\mu m_p}{2k} V_\Delta^2 \propto M_\Delta^{2/3} (1+z)$$

note: $\frac{1}{2}\mu m_p V_\Delta^2 = kT_\Delta \neq \frac{3}{2} kT_\Delta$ because it's **circular** not **rms** vel.

- **Virial temperature** is defined as the temperature of self-gravitating isothermal gas in hydrostatic equilibrium**.** It is **preserved** for a **non-evolving halo** because $M_\Delta \propto r_\Delta \propto (1+z)^{-3/2}$. It is also the **expected temperature of baryons** in the halo once shock-heated.

Cooling Function = Cooling Rate / Hydrogen Density Squared

- **Cooling Rate Λ**
  unit: erg/s/cm³
- Hydrogen Density n_H
  unit cm⁻³
- **Cooling Function:**
  $\Lambda_N \equiv \Lambda / n_H^2$
  unit: erg/s cm³
- **Normalization** makes cooling function depend only on T and Z



CC Density Threshold vs. Halo Gas Density and Temperature

- Catastrophic cooling occurs when $t_{\rm cool} < t_{ff}$, which leads to a hydrogen density threshold above which baryons cool rapidly in a halo:

$$n_H^{cc} = \frac{2^9 G m_p}{9\pi f_{\rm gas}} \left(\frac{kT_\Delta}{\mu \Lambda(T_\Delta, Z)} \right)^2$$

- This threshold is then compared to the mean hydrogen density of the halo to decide the mass range of the halos over which galaxies form:

$$\frac{4}{3} n_H^\Delta m_p = \rho_b^\Delta = \Delta_c f_{\rm gas} \rho_{c,0} \Omega_{m,0} (1+z)^3$$

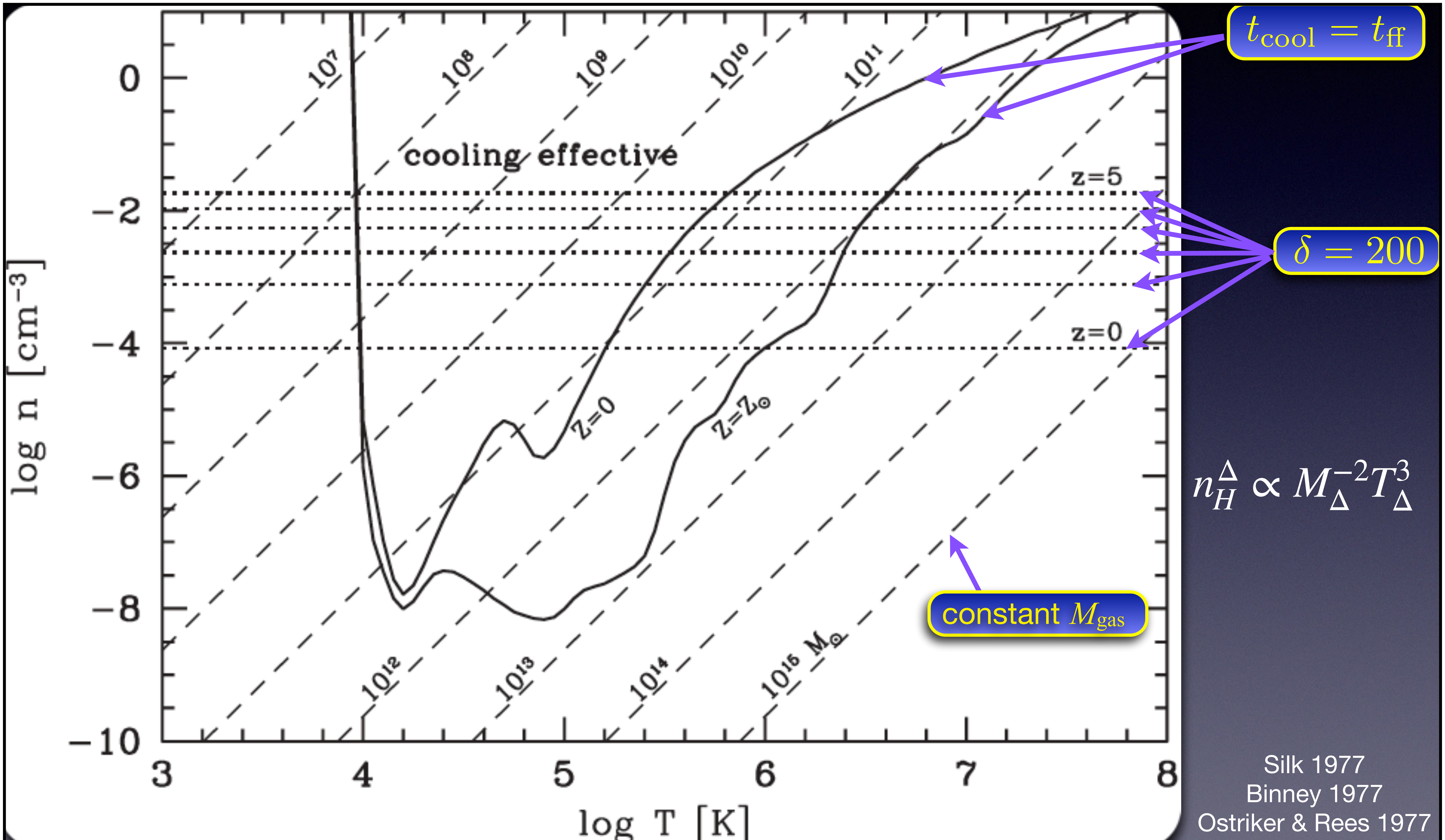
given the definition of virial radius, halos of all masses at a given redshift should have the same mean density!

- The mean density can be expressed with virial mass and virial temperature given that $T_\Delta \propto M_\Delta^{2/3}(1+z)$ and $n_H^\Delta \propto (1+z)^3$:

$$\left(\frac{n_H^\Delta}{0.04 cm^{-3}} \right) = \left(\frac{M_\Delta}{10^8 M_\odot} \right)^{-2} \left(\frac{T_\Delta}{10^4 K} \right)^3$$

CC Density Threshold vs. Halo Gas Density and Temperature

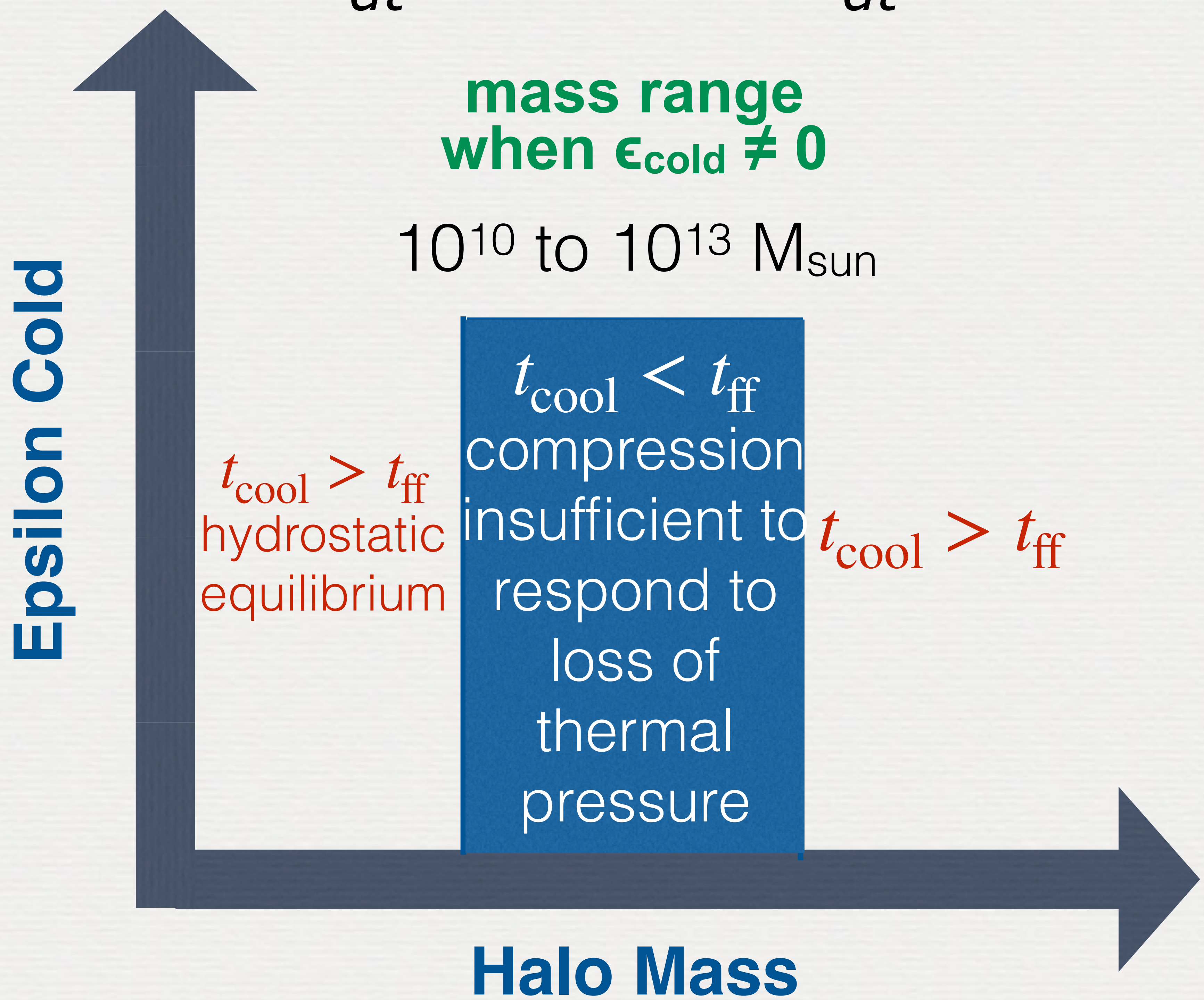
For solar-metallicty gas, efficient cooling at 0 < z < 5 occurs in halos between $10^9 M_\odot < M_{gas} < 10^{12} M_\odot$



Mo, van den Bosch, White (2010), Cambridge Press

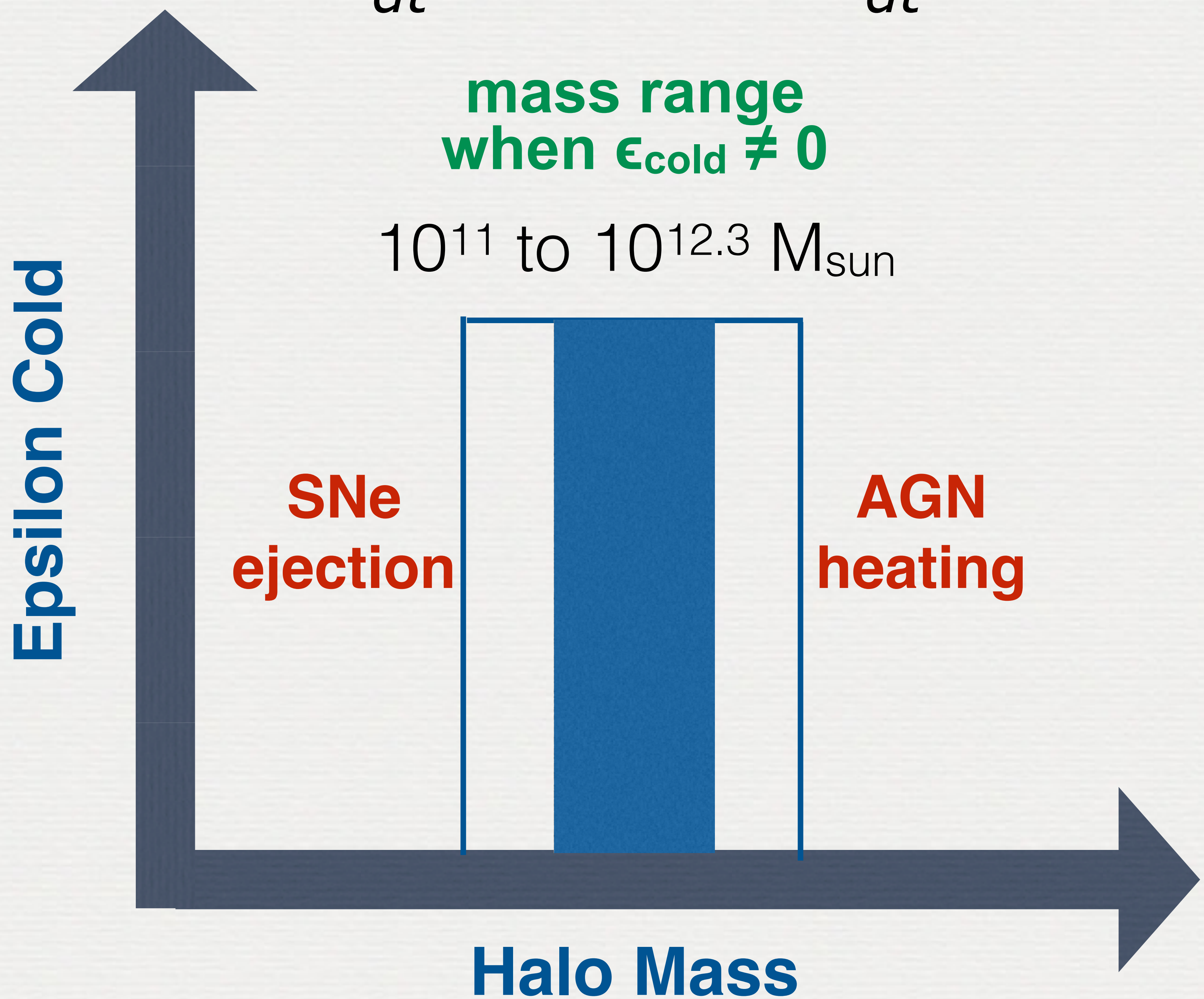
The Cold Gas Accretion Rate

$$\frac{dM_{\text{gas}}}{dt} = \epsilon_{\text{cold}} f_{\text{baryon}} \frac{dM_{\text{halo}}}{dt}$$

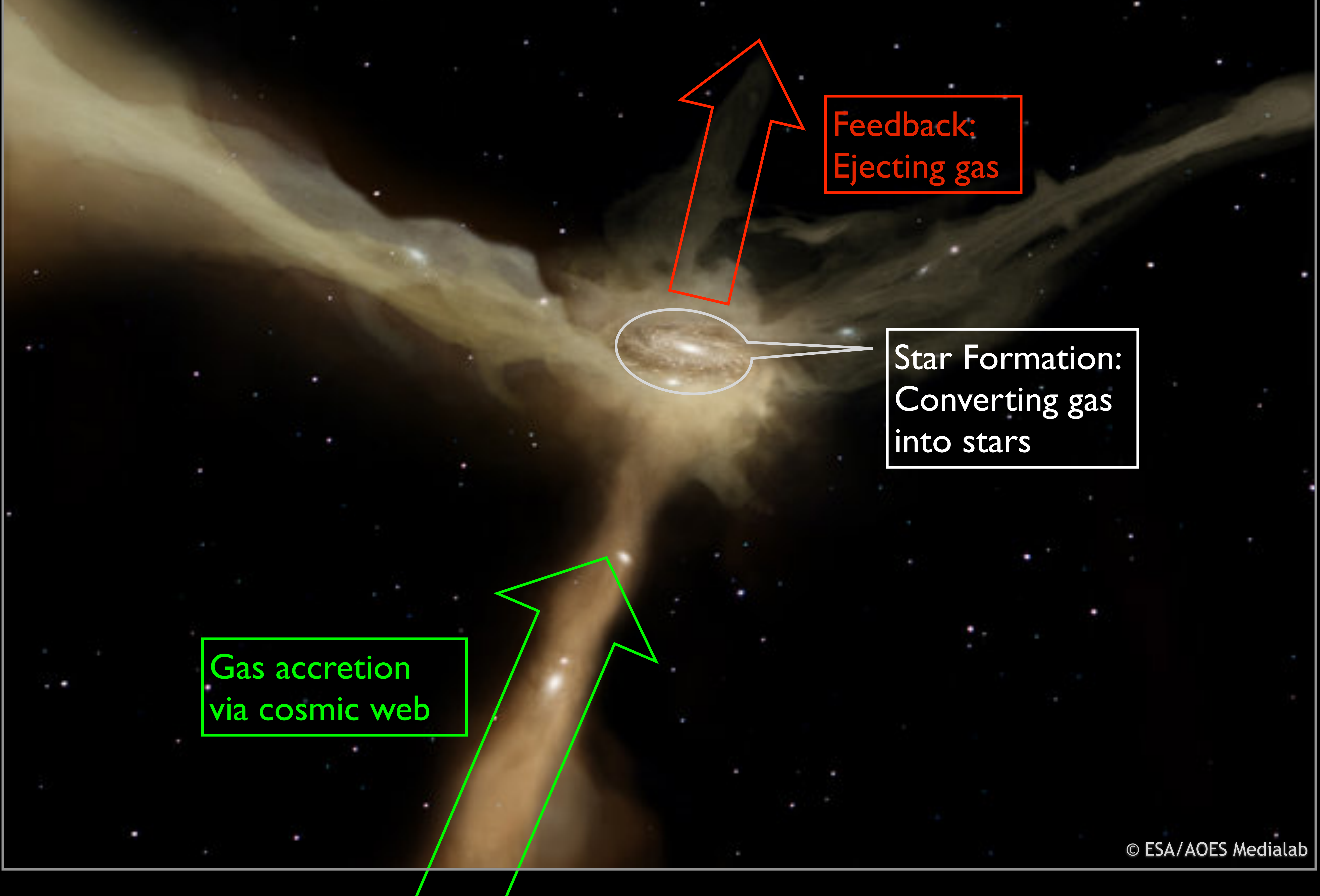


The Cold Gas Accretion Rate

$$\frac{dM_{\rm gas}}{dt} = \epsilon_{\rm cold} f_{\rm baryon} \frac{dM_{\rm halo}}{dt}$$



Baryonic Processes in Galaxy Evolution



The "Bathtub" Model: Accretion-Driven Star Formation

a continuity equation coupled with
a halo growth history and a star formation law

Change in Cold Gas Reservoir | **Accretion Rate** ∝ **Halo Growth Rate** | **Gas Consumption Rate** ∝ **Star Formation Rate**

$$\left\{\begin{array}{l} \dfrac{dM_{\rm gas}}{dt} = \epsilon_{\rm cold} f_{\rm baryon} \dfrac{dM_{\rm halo}}{dt} - (1 - f_{\rm recycle} + f_{\rm outflow}) \dfrac{dM_{\rm star}}{dt} \\ \dfrac{dM_{\rm halo}}{dt} \propto M_{\rm halo}^{1.1} (1+z)^{2.2} \\ \dfrac{dM_{\rm star}}{dt} = {\rm SFR} = \epsilon_{\rm SF} \dfrac{M_{\rm gas}}{\tau_{\rm dyn}} \end{array}\right.$$

⬅ **Halo Growth Rate from EPS** (second equation)

⬅ **Kennicutt-Schmidt Relation** (third equation)

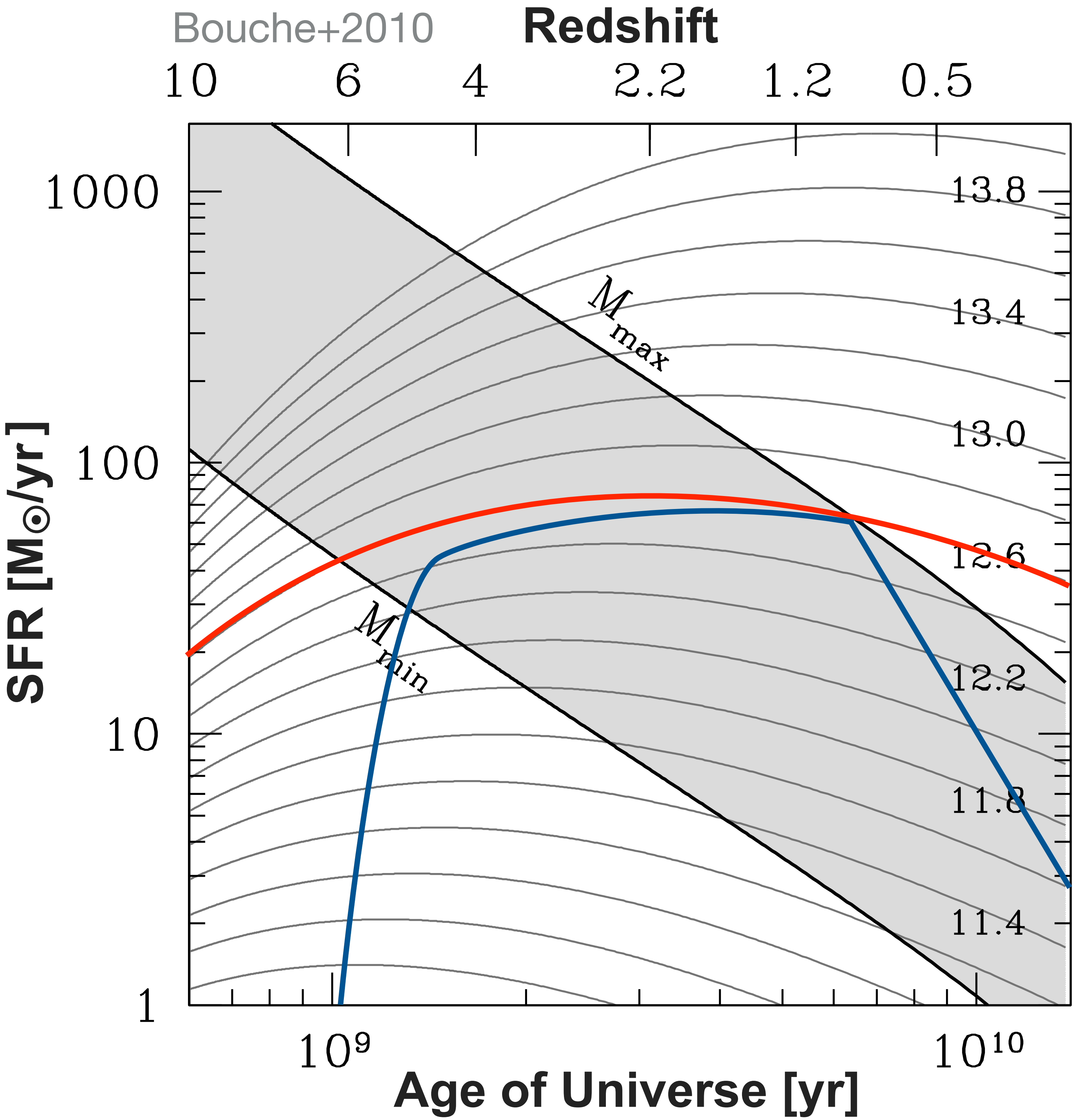
cold gas accretion efficiency:

$$\left\{\begin{array}{l} \epsilon_{\rm cold} = 0.0 \text{ if } M_{\rm halo} < 10^{11} M_\odot \\ \epsilon_{\rm cold} = 0.7 \text{ if } 10^{11} < M_{\rm halo} < 10^{12.3} M_\odot \\ \epsilon_{\rm cold} = 0.0 \text{ if } M_{\rm halo} > 10^{12.3} M_\odot \end{array}\right.$$

recycle & feedback:

$$\left\{\begin{array}{l} f_{\rm recycle} = 0.5 \\ f_{\rm outflow} = 0.6 \end{array}\right.$$

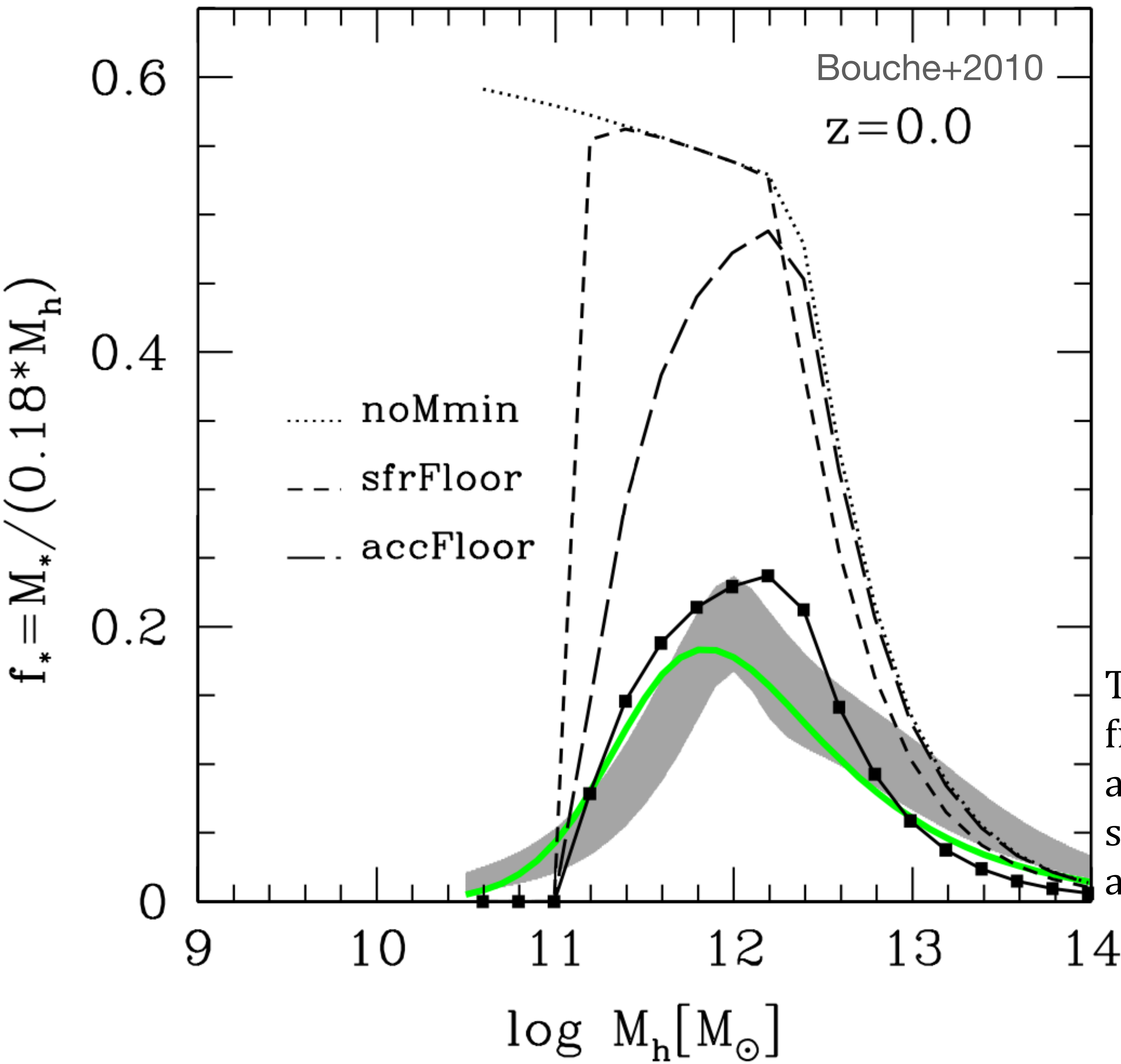
Accretion-Driven Star Formation History



- **Grey region**: efficient cold gas accretion **$10^{11} < M_{Halo} < 1.5 \times 10^{12}\ M_\odot$**
- **Gas accretion history** of a $10^{12.6}\ M_\odot$ halo (mass at $z = 0$)
- **Star formation history** from the continuity equation:

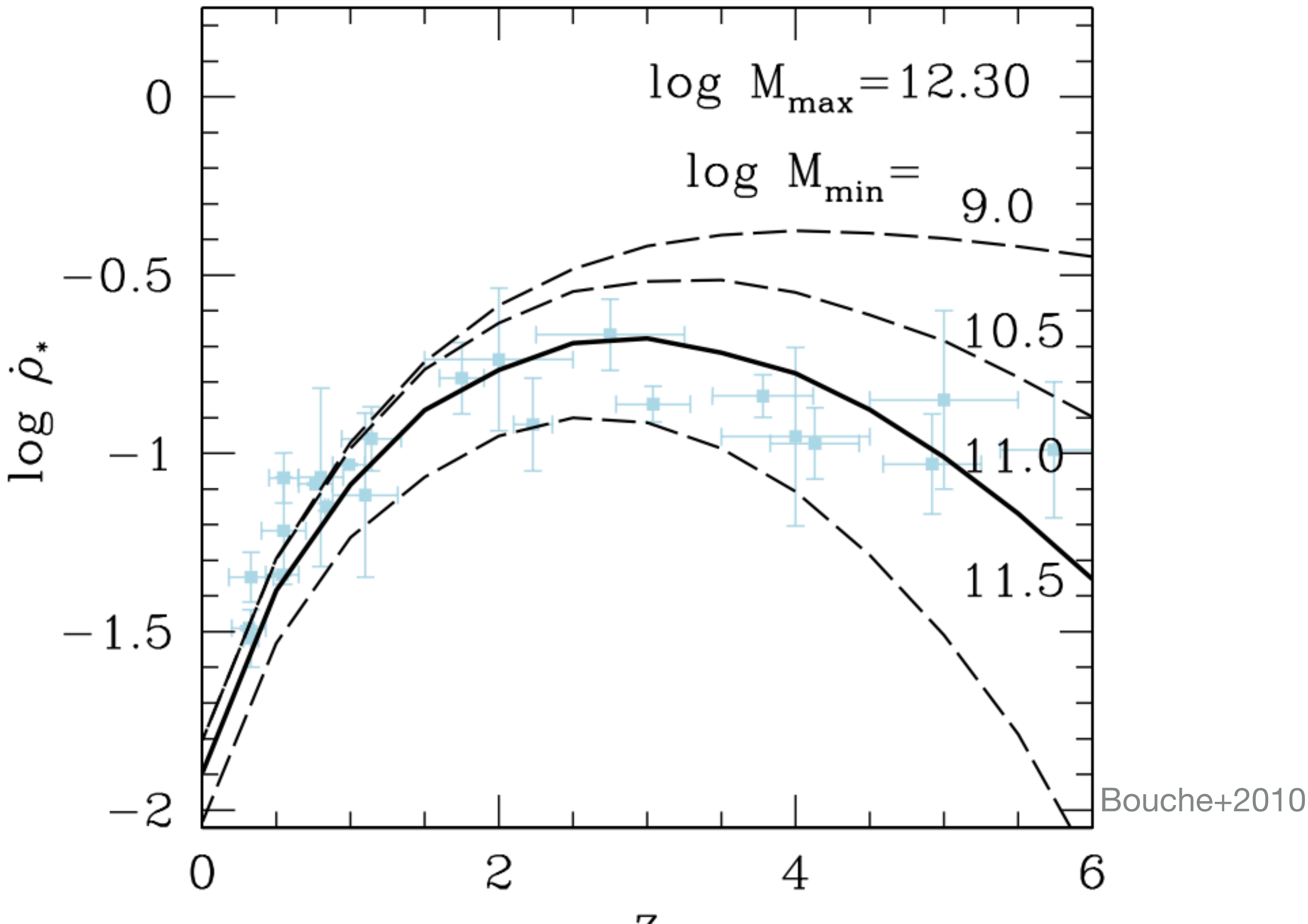
1. Once the halo crosses the minimum mass (**$10^{11}\ M_\odot$**), the SFR rapidly rises to reach a steady state;
2. As the halo mass reaches **$10^{12.3}\ M_\odot$**, cold gas accretion is choked and the SFR starts to decline with an e-folding time of **2-3 Gyr (=2 τ_{dyn}/ϵ_{SF})**.

Stellar mass fraction



The z = 0 stellar fractions from Moster et al. (2010) and Guo et al. (2010) are shown as the shaded area and thick line, respectively.

Stellar mass fraction



Textbook Recommendations for Continued Learning