

$$1a. \vec{E} = -\nabla V$$

$$= -2xy \hat{x} - (x^2+1) \hat{y}$$

$$b. \int \vec{E} \cdot d\vec{r} = \int -2xy dx - (x^2+1) dy$$

but $x=y$, $dx=dy$ on path

$$\int \vec{E} \cdot d\vec{r} = \int_0^2 (-2x^2 - (x^2+1)) dx$$

$$= -3x^3/3 - x \Big|_0^2$$

$$= \boxed{-10}$$

$$c. V(0,0,0) = 0$$

$$V(2,2,2) = 10$$

$$\Delta V = 10 = -\int \vec{E} \cdot d\vec{r}$$

as expected

$$2. \rho(\vec{r}) = q \delta^3(\vec{r} - \vec{r}_1) - q \delta^3(\vec{r} - \vec{r}_2)$$

$$3a. \int \vec{E} \cdot d\vec{a} = E \cdot 2\pi s \cdot L$$

$$Q_{enc} = \pi s^2 L K \quad s < R$$

$$\pi R^2 L K \quad s > R$$

$$\vec{E} = \frac{Ks}{2\epsilon_0} \hat{s} \quad s < R$$

$$\frac{KR^2}{2\epsilon_0 s} \hat{s} \quad s > R$$

$$\begin{aligned}
 3b. \quad \Delta V &= - \int_0^R \vec{E} \cdot d\vec{l} \\
 &= - \int_0^R \frac{\kappa s}{2\epsilon_0} ds \\
 &= - \left. \frac{\kappa s^2}{4\epsilon_0} \right|_0^R \\
 &= \boxed{-\frac{\kappa R^2}{4\epsilon_0}}
 \end{aligned}$$

$$\begin{aligned}
 4a. \quad \rho &= \epsilon_0 \nabla \cdot \vec{E} \\
 &= \epsilon_0 \cdot \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot r \sin^2 \theta) \\
 &= \epsilon_0 \cdot \frac{1}{r^2} \cdot 3r^2 \sin^2 \theta \\
 &= \boxed{3\epsilon_0 \sin^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 b. \quad Q &= \int \rho(\vec{r}) d\tau \\
 &= \int 3\epsilon_0 \sin^2 \theta \cdot r^2 \sin \theta d\theta d\phi dr \\
 &= 3\epsilon_0 \int_0^{2\pi} d\phi \int_0^R r^2 dr \int_0^\pi \sin^3 \theta d\theta \\
 &= 6\pi \epsilon_0 \cdot \frac{R^3}{3} \cdot \frac{\pi}{2} \\
 &= \boxed{\pi^2 \epsilon_0 R^3}
 \end{aligned}$$

$$\begin{aligned}
 Q_a &= -Q \\
 Q_b &= +Q
 \end{aligned}$$