

$$1a. \rho = \epsilon_0 \nabla \cdot \vec{E} \\ = \epsilon_0 \frac{\partial}{\partial x} x = \boxed{\epsilon_0}$$

$$b. \int \vec{E} \cdot d\vec{a} = - \int_{x=0} E_x \cdot dy dz + \int_{x=2} E_x \cdot dy dz \\ = -E_x(0) \cdot 4 + E_x(2) \cdot 4 \\ = \boxed{8}$$

$$c. Q_{enc} = \int \rho d\tau = \boxed{8\epsilon_0} \\ = \epsilon_0 \int \vec{E} \cdot d\vec{a}$$

$$2. \int (r^2 + r \cdot \vec{a}) \int^3 (r - b) d\tau \\ = \boxed{b^2 + b \cdot \vec{a}}$$

$$3a. \int \vec{E} \cdot d\vec{a} = 4\pi r^2 E \\ Q_{enc} = 4\pi \int_0^r \kappa r' \cdot r'^2 dr' \quad r < R \\ = 4\pi \cdot \frac{\kappa r^4}{4} \\ = \pi \kappa r^4 \quad r < R \\ = \pi \kappa R^4 \quad r > R$$

$$\Rightarrow \boxed{\vec{E} = \frac{\kappa r^2}{4\epsilon_0} \hat{r} \quad r < R \\ = \frac{\kappa R^4}{4\epsilon_0 r^2} \hat{r} \quad r > R}$$

$$3b. \Delta V = - \int_{\infty}^R E r \, dr$$

$$= - \int_{\infty}^R \frac{k R^4}{4 \epsilon_0 r^2} \, dr$$

$$= \frac{k R^4}{4 \epsilon_0 r} \Big|_{\infty}^R = \boxed{\frac{k R^3}{4 \epsilon_0}}$$

$$4a. \vec{E} = - \nabla V$$

$$= - \sin \varphi \hat{s} - \frac{1}{s} \cdot s \cos \varphi \hat{\varphi}$$

$$= \boxed{- \sin \varphi \hat{s} - \cos \varphi \hat{\varphi}}$$

$$b. W = \frac{\epsilon_0}{2} \int E^2 \, d\tau$$

$$= \frac{\epsilon_0}{2} \int (\sin^2 \varphi + \cos^2 \varphi) \, d\tau$$

$$= \frac{\epsilon_0}{2} \int d\tau$$

$$= \frac{\epsilon_0}{2} \cdot \pi s^2 \cdot L$$

$$\Rightarrow \boxed{W/L = \frac{\pi \epsilon_0 s^2}{2}}$$