Homework #5 (10 points) - Show all work on the following problems:

**Problem 1 (2 points):** Find the average potential over a spherical surface of radius \( R \) due to a point charge located inside the sphere (not at the center).

**Problem 2 (2 points):** In 1-d, the functional form of the general solution to Laplace’s equation is \( V(x) = mx + b \).

**2a (1 point):** Find the functional form of the general solution to Laplace’s equation in 3-d spherical coordinates for the case where \( V \) only depends on the radial coordinate \( r \).

**2b (1 point):** Find the functional form of the general solution to Laplace’s equation in 3-d cylindrical coordinates for the case where \( V \) only depends on the radial coordinate \( s \).

**Problem 3 (2 points):** Consider an infinite grounded conducting plane with two charges above the plane: \(-2q\) at height \( d \), and \(+q\) and height \( 3d \). Use image charges to determine the force on the upper charge (+q).

**Problem 4 (4 points):** Consider a point charge \( q \) at a distance \( a \) from the center of a grounded conducting sphere of radius \( R \) (with \( a > R \)), as in Example 3.2 in Griffiths.

**4a (1 point):** Use the law of cosines to show that you can write

\[
V(r, \theta) = \frac{1}{4\pi \varepsilon_0} \left[ \frac{q}{\sqrt{r^2 + a^2 - 2ar \cos \theta}} - \frac{q}{\sqrt{r^2 + \left(\frac{R}{R}ight)^2 - 2r \cos \theta}} \right]
\]

**4b (1 point):** Use the boundary conditions on the electric field (and thus the normal derivative of \( V \)) at the surface of the sphere to find the induced surface charge density \( \sigma \) on the sphere, as a function of \( \theta \).

**4c (1 point):** Integrate the charge density over the surface of the sphere to find the total induced charge.

**4d (1 point):** Calculate the energy of this configuration by determining the energy required to bring the charge \( q \) from infinity.