\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 J \]

Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture
Physical Dipole

Multipole expansion

\[ V(\vec{r}/\alpha) = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \int_0^1 \sin^n(\alpha \sigma) \rho(\sigma) d\sigma \]

Dipole Term

\[ \vec{V} = \frac{1}{4\pi\varepsilon_0 r^2} \hat{r} \cdot \vec{p} \]

\[ \vec{p} = \int \vec{r} \cdot \left( q \delta^3(\vec{r} - \vec{r}_+ - \vec{r}_- - q \delta^3(\vec{r} - \vec{r}_+ + \vec{r}_-)) \right) d\sigma \]

\[ = q \vec{r}_+ - q \vec{r}_- \]

\[ = q \vec{d} \]

\[ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r^2} \hat{r} \cdot q \vec{d} \]

\[ = \frac{qd \cos \alpha}{4\pi\varepsilon_0 r^2} \]

-same as Taylor expansion result

Note: Physical dipole also has quadrupole, octupole, etc.

- Perfect dipole for \( d \to 0 \)
Pure Vs. Physical Dipole

(a) Field of a "pure" dipole
(b) Field of a "physical" dipole

*Figure 3.32*
Origin of Coordinates

Moments depend on origin

- Monopole \( \int \rho(r) \, dr \) does not

- Dipole \( \vec{D} = \int \vec{r} \, \rho(r) \, dr \)

\[
= \int (\vec{r} - \vec{a}) \, \rho(r) \, dr
\]

\[
\vec{p} = \int \vec{r} \, \rho(r) \, dr - \vec{a} \int \rho(r) \, dr
\]

\[
= \vec{p} - Q \vec{a}
\]

different if \( Q \neq 0 \)
Dipole Electric Field

\[ \mathbf{E}_{\text{dip}}(r, \theta) = \frac{\mathbf{r} \cdot \mathbf{p}}{4 \pi \varepsilon_0 r^2} \]

\[ = \frac{p \cos \theta}{4 \pi \varepsilon_0 r^2} \]

for dipole @ origin

w/ \( \mathbf{p} = p \hat{\mathbf{z}} \)

\[ E_r = -\frac{\partial \mathbf{E}}{\partial r} = \frac{2p \cos \theta}{4 \pi \varepsilon_0 r^3} \]

\[ E_\theta = -\frac{1}{r} \frac{\partial \mathbf{E}}{\partial \theta} = \frac{p \sin \theta}{4 \pi \varepsilon_0 r^3} \]

\[ E_\phi = -\frac{1}{r \sin \theta} \frac{\partial \mathbf{E}}{\partial \phi} = 0 \]

\[ \Rightarrow \mathbf{E}_{\text{dip}}(r, \theta) = \frac{p}{4 \pi \varepsilon_0 r^3} [2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\mathbf{z}}] \]
Conductor

\[ \vec{E} = 0 \]

All charge on surface

Conductor in \( \vec{E} \)

\[ \vec{E} \rightarrow \vec{E} \]

\[ \vec{E} = 0 \]

Insulator

\[ \vec{E} = 0 \]

Only if no applied field or net charge
neutral molecules

random dipoles
field cancels

Insulator in $\vec{E}$

Dipoles align with $\vec{E}$
Induced dipole

\[ \vec{p} \rightarrow \vec{E} \rightarrow \vec{\rho} \]

\[ \vec{p}_{\text{induced}} = \alpha \vec{E} \]

\[ \alpha = \text{"atomic polarizability"} \]

Intrinsic Dipole

\[ \vec{\rho} \]
Induced Dipoles (Polarizability)

Unpolarized

Polarized

Large-scale view of polarized atom
Intrinsic Dipoles (Polar Molecules)

Electronegativity: H 2.1, C 2.5, O 3.5 => electrons closer to O than C, H

(a) No net dipole moment

(b) Net dipole moment
Force on dipole

\[ \vec{F} = \vec{F}_+ + \vec{F}_- \]
\[ = q \vec{E}_+ - q \vec{E}_- \]
\[ = q \Delta \vec{E} \]
\[ = q (\vec{r} - \vec{r}') \times \vec{E} \]
\[ \Rightarrow \vec{F} = (\vec{r} - \vec{r}') \times \vec{E} \]

\[ \vec{r} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- \]
\[ = \frac{1}{2} \vec{r}_+ \times q \vec{E} + -\frac{1}{2} \vec{r}_- \times -q \vec{E} \]
\[ = q \vec{r} \times \vec{E} \]
\[ \Rightarrow \vec{r} = \vec{r} \times \vec{E} \]