\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J} \]

**Electricity and Magnetism I: 3811**

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture
Announcements

- Midterm 2 coming up on Wednesday 11/7
- Updated Eq. Sheet posted on “Notes” page
  - Please look over and make sure it makes sense
Magnetostatics

- So far, we've studied electrostatics, w/ no moving charges
  - This situation implies no magnetic fields, it turns out

- Now we'll transition to magnetostatics, w/ steadily moving charges (currents), but no changing currents
  - This implies only steady magnetic fields
    \[ \Rightarrow \text{no radiation} \]
**Lorentz Force Law**

Force on moving charge

\[ \vec{F} = q (\vec{E} + \vec{v} \times \vec{B}) \]

- Electric force on any charge, along \( \vec{E} \)
- Magnetic force on only moving charges, perpendicular to \( \vec{B} \)

**Example: Cyclotron Motion**

\[ \vec{B} = -B \hat{z} \]
\[ \vec{v}_o = x \hat{x} \]
\[ \vec{j}_o = v \hat{y} \]

\( \vec{F} \perp \vec{v} \)

\[ |\vec{v}| = \text{const.} = v \]
\[ \Rightarrow \text{circular motion} \]

\[ qvB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB} \]

\[ w = \frac{v}{R} = \frac{qB}{m} \]
If \( \mathbf{v} \) has a component along \( \mathbf{B} \), separate into

\[
\frac{v_{\parallel}}{v_{\perp}}
\]

\( v_{\parallel} = \text{const.} \)

\( |v_{\perp}| = \text{const.} \)

\[
q v_{\perp} \mathbf{B} = \frac{m v_{\perp}^2}{R} \Rightarrow R = \frac{m v_{\perp}}{q B}
\]

\( \text{helical path} \)

- Left-handed for \( +Q \)
- Right-handed for \( -Q \)
**Gyro-Motion**

**Figure 29.18** A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

Ions = Left-Handed Gyration
Electrons = Right-Handed Gyration
Magnetic Forces & Work

\[ \vec{F}_{mag} = q (\vec{v} \times \vec{B}) \]

\[ \vec{F}_{mag} \cdot \vec{\nabla} = 0 \]

\[ w = \int \vec{F}_{mag} \cdot d\vec{r} \]

\[ = \int \vec{F}_{mag} \cdot d\vec{r} / dt \cdot dt \]

\[ = \int \vec{F}_{mag} \cdot \vec{v} \, dt \]

\[ = 0 \]

- Magnetic Forces Do No Work!

- In specific examples, very hard to find what does do the work, but it is never static magnetic fields
Cycloidal Motion

\[ \mathbf{F} = q (E \mathbf{\hat{E}} + \mathbf{v} \times \mathbf{B}) \]

\[ \mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{\hat{x}} & \mathbf{\hat{y}} & \mathbf{\hat{z}} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} \]

\[ = v_y B \mathbf{\hat{x}} - v_x B \mathbf{\hat{y}} \]

\[ \mathbf{F} = q E \mathbf{\hat{y}} + q v_y B \mathbf{\hat{x}} - q v_x B \mathbf{\hat{y}} \]

\[ = m \mathbf{\ddot{u}} = m \left( \frac{d}{dt} \left( \frac{q B}{m} v_y \mathbf{\hat{x}} \right) + \frac{d}{dt} \left( \frac{q B}{m} v_x \mathbf{\hat{y}} \right) \right) \]

\[ \Rightarrow \frac{dv_x}{dt} = \frac{q B}{m} v_y = \omega v_y \]

\[ \frac{dv_y}{dt} = \frac{q E}{m} - \frac{q B}{m} v_x = \omega \left( \frac{E}{B} - v_x \right) \]

\[ \frac{d^2v_y}{dt^2} = -\omega \frac{dv_x}{dt} = -\omega^2 v_y \]

\[ \Rightarrow v_y = V_0 \sin(\omega t) \]
\[ w V_0 \cos(wt) = w \left( \frac{E}{B} - V_x \right) \]

\[ \Rightarrow V_x = \left( \frac{E}{B} - V_0 \cos(wt) \right) \]

\[ V_x(0) = 0 \Rightarrow V_0 = \frac{E}{B} \]

\[ \Rightarrow V_x = \frac{E}{B} (1 - \cos(wt)) \]

\[ V_y = \frac{E}{B} \sin(wt) \]

- Circular motion + constant drift in x-direction
- Drift not along \( \vec{E} \) but along \( \vec{E} \times \vec{B} \)
Cycloidal Motion
Cycloid Motion