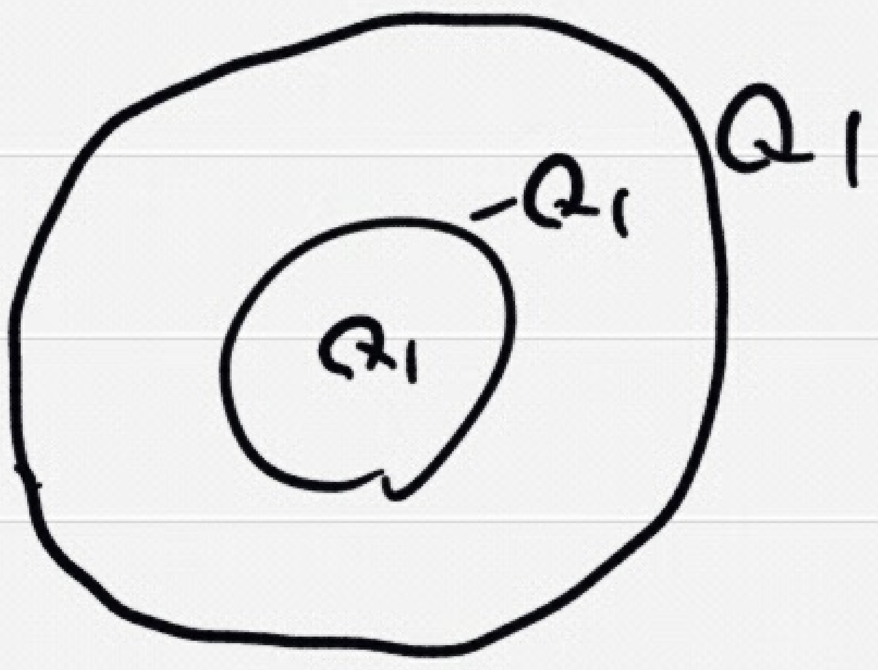
1\. a.



$$\boxed{\begin{aligned} \sigma_a &= -Q_1/4\pi a^2 \\ \sigma_b &= +Q_1/4\pi b^2 \end{aligned}}$$

b. $\oint \vec{E} \cdot d\vec{a} = Q_{enc}/\varepsilon_0$

$$\Rightarrow \boxed{\vec{E} = \begin{cases} \dfrac{Q_1}{4\pi\varepsilon_0 r^2}\hat{r} & r < a \\ 0 & a < r < b \\ Q_1/4\pi\varepsilon_0 r^2\,\hat{r} & r > b \end{cases}}$$

c. $\Delta E_\perp = \sigma/\varepsilon_0$

$$\boxed{\begin{aligned} \Delta E_a &= -Q_1/4\pi\varepsilon_0 a^2 = \sigma_a/\varepsilon_0 \\ \Delta E_b &= +Q_1/4\pi\varepsilon_0 b^2 = \sigma_b/\varepsilon_0 \end{aligned}}$$

d.
$$\begin{aligned} W &= Q_2 \Delta V \\ &= Q_2 \cdot -\int_\infty^a E\,dr \\ &= Q_2 \cdot \left[-\int_\infty^b \frac{Q_1}{4\pi\varepsilon_0 r^2}\,dr - \int_b^a 0\,dr\right] \\ &= \boxed{\frac{Q_2 Q_1}{4\pi\varepsilon_0 b}} \end{aligned}$$

2. a. $\vec{E} = -\nabla V$

$= -\partial V/\partial r \, \hat{r} - \frac{1}{r} \partial V/\partial\theta \, \hat{\theta}$

$= \boxed{-\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}}$

b. $\rho = \varepsilon_0 \nabla \cdot \vec{E}$

$= \varepsilon_0 \left[\frac{1}{r^2} \partial/\partial r (r^2 \cdot -\cos\theta) + \frac{1}{r\sin\theta} \partial/\partial\theta (\sin^2\theta) \right]$

$= \varepsilon_0 \left[-\frac{2\cos\theta}{r} + \frac{2\sin\theta\cos\theta}{r\sin\theta} \right]$

$= \boxed{0}$

3. $\oint \vec{E} \cdot d\vec{a} = Q_{enc}/\varepsilon_0 = \int \rho \, d\tau / \varepsilon_0$

$E \cdot 2\pi s \cdot L = \int C/s' \cdot s' ds' d\phi dz / \varepsilon_0$

$= 2\pi C s L/\varepsilon_0 \quad s < R$

$2\pi C R L/\varepsilon_0 \quad s > R$

$\Rightarrow \boxed{\vec{E} = \begin{array}{ll} C/\varepsilon_0 \, \hat{s} & s < R \\ CR/\varepsilon_0 s \, \hat{s} & s > R \end{array}}$

4. $\nabla^2 V = \partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2$

$= -K_x^2 V - K_y^2 V + \partial^2 V/\partial z^2$

$= 0$

$\Rightarrow \frac{1}{V} \partial^2 V/\partial z^2 = K_x^2 + K_y^2$

$$\Rightarrow \boxed{V(x,y,z) = V_0 \sin(K_x x)\sin(K_y y) e^{-\sqrt{K_x^2 + K_y^2}\cdot z}}$$

Note: No $e^{+\sqrt{K_x^2+K_y^2}\, z}$
since this blows up
for $z > 0$

5. a. $\nabla \cdot \vec{B} = 0 \Rightarrow \Delta B_\perp = 0$
so this must be H

b. $\Delta \vec{H}_\parallel = 0 \Rightarrow \vec{K}_f = 0$
only $\vec{K}_b$

c. If $\sigma_f = 0$ $\Delta D_\perp = 0$
so this must be E

6\. a. $\oint \vec{D} \cdot d\vec{a} = Q_{f\,enc}$

$$D \cdot 2\pi s L = \lambda \cdot L$$

$$\Rightarrow \boxed{\vec{D} = \frac{\lambda}{2\pi s} \hat{s}}$$

$$\vec{D} = (1+\chi_e) \varepsilon_0 \vec{E}$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{1+\chi_e} \frac{\lambda}{2\pi \varepsilon_0 s} \hat{s}}$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$\Rightarrow \boxed{\vec{P} = \frac{\chi_e}{1+\chi_e} \frac{\lambda}{2\pi s} \hat{s}}$$

b. $\sigma_b = \vec{P} \cdot \hat{n}$

$$\boxed{\begin{aligned} \sigma_{ab} &= -\frac{\chi_e}{1+\chi_e} \frac{\lambda}{2\pi a} \\ \sigma_{bb} &= \frac{\chi_e}{1+\chi_e} \frac{\lambda}{2\pi b} \end{aligned}}$$

7.a. $I = \int \vec{J} \cdot d\vec{a}$

$= \int C \cdot s \cdot s \, ds \, d\varphi$

$= 2\pi C s^3/3 \Big|_0^R$

$= 2\pi C R^3/3$

$\Rightarrow C = 3I/2\pi R^3$

$\Rightarrow \boxed{J(s) = \frac{3Is}{2\pi R^3}}$

b. $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$B \cdot 2\pi s = \mu_0 \int \vec{J} \cdot d\vec{a}$

$= \mu_0 \cdot \frac{3I}{2\pi R^3} \cdot \frac{2\pi s^3}{3}$

$= \mu_0 I \, s^3/R^3 \quad s < R$

$\mu_0 I \quad s > R$

$$\Rightarrow \boxed{\begin{aligned} \vec{B} &= \frac{\mu_0 I s^2}{2\pi R^3} \hat{\varphi} \quad s < R \\ &= \frac{\mu_0 I}{2\pi s} \hat{\varphi} \quad s > R \end{aligned}}$$

8. a. $\vec{J}_b = \nabla \times \vec{M}$

$$= \frac{\partial}{\partial z}(M_0 z)\,\hat{y}$$

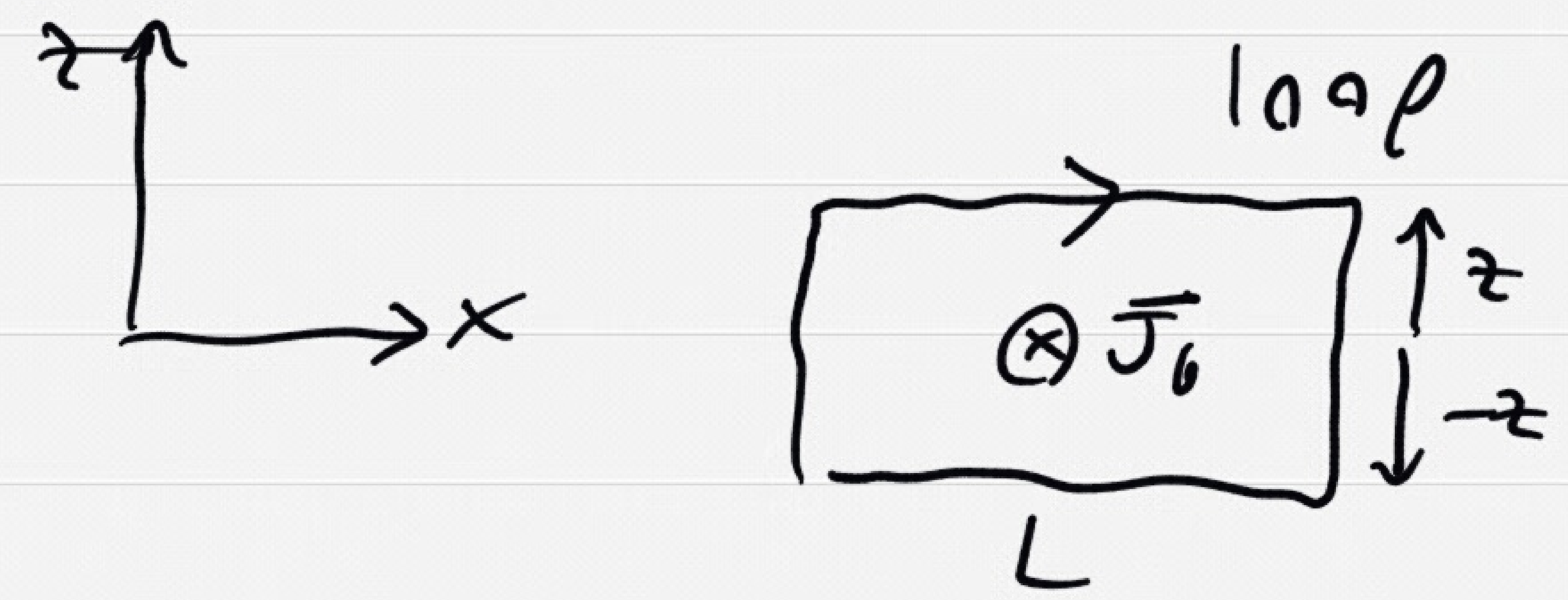
$$= \boxed{M_0\,\hat{y}}$$

$$\vec{K}_b = \vec{M} \times \hat{n} = M_0 z\,\hat{x} \times \pm\hat{z}$$

$$= M_0 \cdot \pm T/2 \cdot \mp\hat{y}$$

$$= \boxed{-\frac{M_0 T}{2}\,\hat{y}}$$ same top and bottom

b. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$



$$B \cdot 2L = \mu_0 M_0 \cdot 2z \cdot L \qquad z < T/2$$

$$= \mu_0\left(M_0 \cdot 2\,T/2 \cdot L - 2 \cdot \frac{M_0 T}{2} \cdot L\right) \qquad z > T/2$$

$$\Rightarrow \boxed{\begin{aligned} \vec{B} &= \mu_0 M_0 z\,\hat{x} & |z| < T/2 \\ &= 0 & |z| > T/2 \end{aligned}}$$

c. $\vec{H} = \vec{B}/\mu_0 - \vec{M}$

$$\boxed{\vec{H} = 0 \text{ everywhere (and continuous)}}$$