

$$\begin{aligned}
 \text{1. a. } \vec{E} &= -\nabla V = -\frac{\partial V}{\partial r} \hat{r} - \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} \\
 &= \boxed{-2r \sin \theta \hat{r} - r \cos \theta \hat{\theta}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \int \vec{E} \cdot d\vec{l} &= \int E_r dr + \int E_\theta \cdot r d\theta \\
 &\quad + \int E_\phi \cdot r \sin \theta d\phi
 \end{aligned}$$

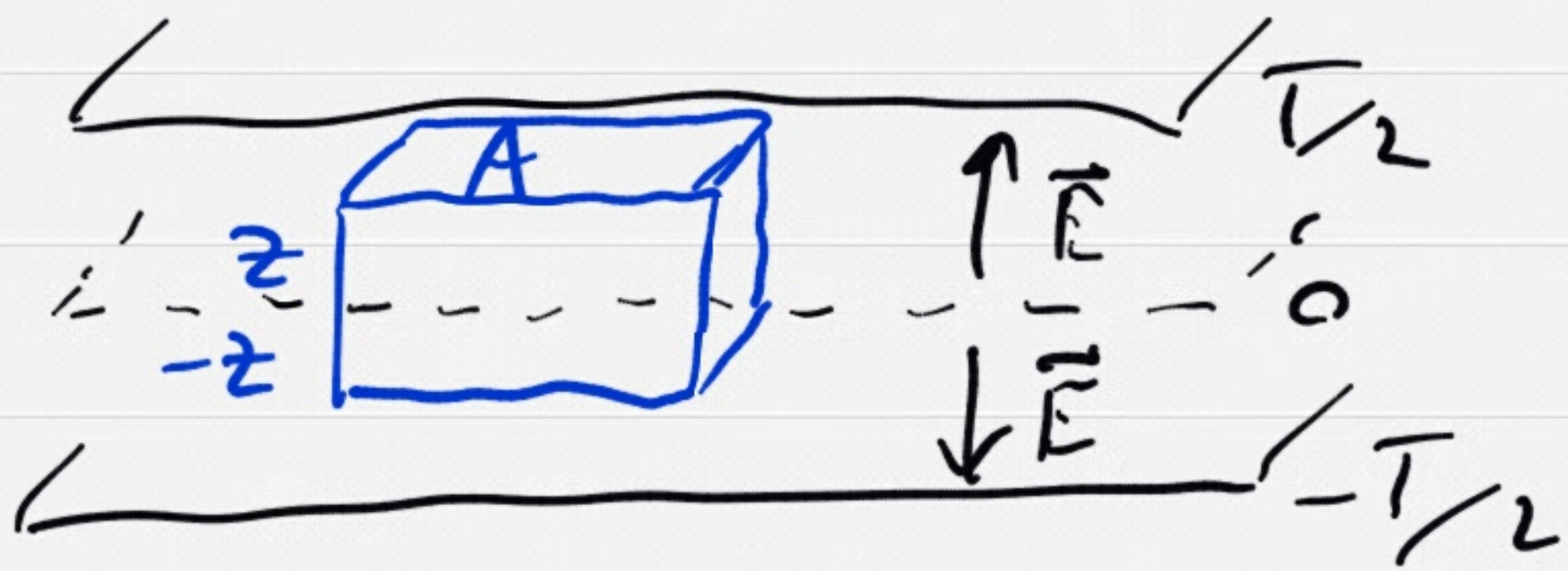
Along arc, only θ changes

$$\begin{aligned}
 \Rightarrow \int \vec{E} \cdot d\vec{l} &= \int E_\theta \cdot r d\theta \\
 &= \int_0^{\pi/2} -r^2 \cos \theta d\theta \\
 &= -r^2 \sin \theta \Big|_0^{\pi/2} \\
 &= -r^2 = \boxed{-4}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \int \vec{E} \cdot d\vec{l} &= -\Delta V \\
 &= -[V(2, \pi/2, 0) - V(2, 0, 0)] \\
 &= \boxed{-4} //
 \end{aligned}$$

$$\begin{aligned}
 \text{2. } \int f(\vec{r}) \delta^3(\vec{r} - \vec{a}) &= f(\vec{a}) \\
 &= 2^2 + 1^2 \\
 &= \boxed{5}
 \end{aligned}$$

3. a.



\vec{E} symmetric
in z

$$\int \vec{E} \cdot d\vec{a} = E_z(z) \cdot A - E_z(-z) \cdot A = 2E_z(z) \cdot A$$

$$Q_{enc} = 2\rho A z \quad |z| < T/2$$

$$= \rho A T \quad |z| > T/2$$

$$\Rightarrow \begin{cases} E_z = \rho z / \epsilon_0 & |z| < T/2 \\ E_z = \rho T / 2\epsilon_0 \hat{n} & |z| > T/2 \end{cases}$$

6. $W = Q \Delta V$

$$= Q \cdot - \int \vec{E} \cdot d\vec{r}$$

$$= Q \cdot - \int_{T/2}^0 \frac{\rho z}{\epsilon_0} \sqrt{z} dz$$

$$= Q \cdot - \left. \frac{\rho z^2}{2\epsilon_0} \right|_{T/2}^0$$

$$= \boxed{Q \cdot \frac{\rho T^2}{8\epsilon_0}}$$

$$\begin{aligned}
 4. a. \quad \rho &= \epsilon_0 \nabla \cdot \vec{E} \\
 &= \epsilon_0 \cdot \frac{1}{s} \frac{\partial}{\partial s} (s E_s) \\
 &= \epsilon_0 \cdot \frac{1}{s} \frac{\partial}{\partial s} \left(\frac{\kappa s^2}{2 \epsilon_0} \right) \quad s < R \\
 &= \boxed{\kappa \quad s < R} \\
 &= \epsilon_0 \cdot \frac{1}{s} \frac{\partial}{\partial s} \left(\frac{\kappa R^2}{\epsilon_0} \right) \quad s > R \\
 &= \boxed{0 \quad s > R}
 \end{aligned}$$

$$\begin{aligned}
 b. \quad \Delta \vec{E} &= \frac{\sigma}{\epsilon_0} \hat{s} \\
 &= \vec{E}_{out} - \vec{E}_{in} \\
 &= \frac{\kappa R^2}{\epsilon_0 R} \hat{s} - \frac{\kappa R}{2 \epsilon_0} \hat{s} \\
 &= \frac{\kappa R}{2 \epsilon_0} \hat{s} \\
 \Rightarrow \boxed{\sigma = \kappa R / 2}
 \end{aligned}$$