1. a. $\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}$

$= -2r\sin\theta\,\hat{r} - r\cos\theta\,\hat{\theta}$

b. $\int \vec{E}\cdot d\vec{l} = \int E_r\,dr + \int E_\theta \cdot r\,d\theta$

$+ \int E_\varphi \cdot r\sin\theta\,d\varphi$

Along arc, only θ changes

$\Rightarrow \int \vec{E}\cdot d\vec{l} = \int E_\theta \cdot r\,d\theta$

$= \int_0^{\pi/2} -r^2\cos\theta\,d\theta$

$= -r^2\sin\theta \Big|_0^{\pi/2}$

$= -r^2 = -4$

c. $\int \vec{E}\cdot d\vec{l} = -\Delta V$

$= -[V(2,\pi/2,0) - V(2,0,0)]$

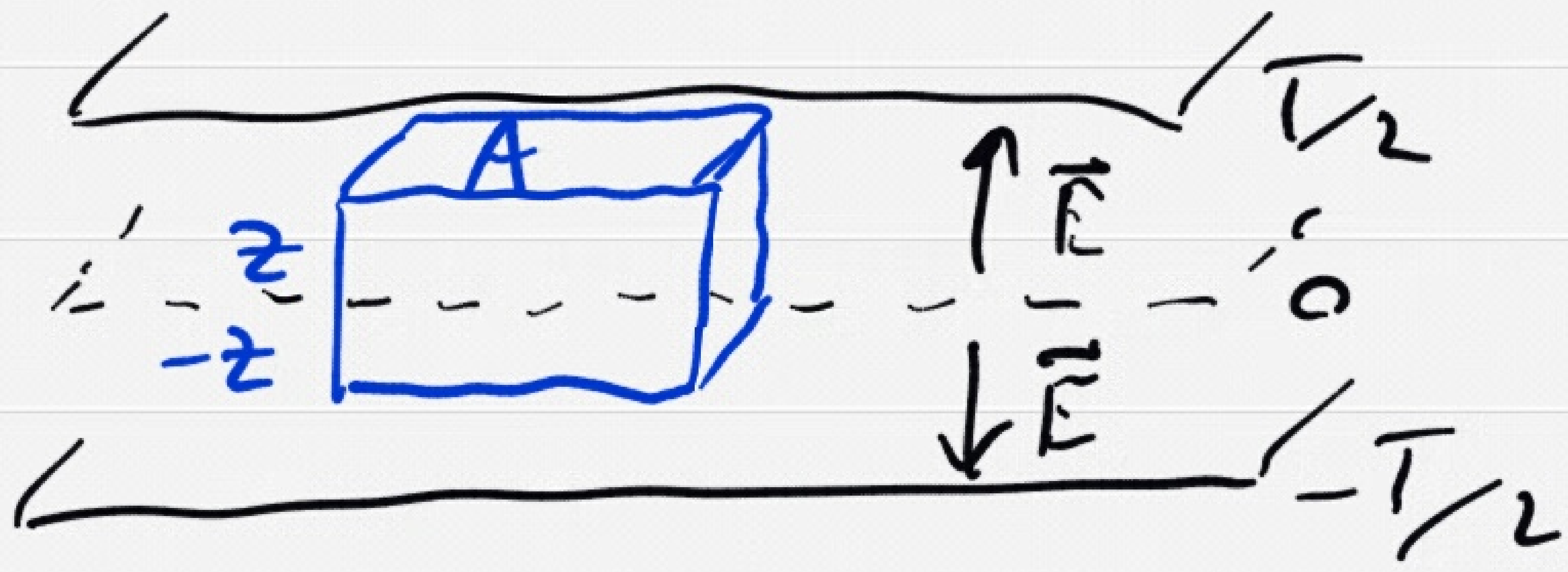
$= -4$ //

2. $\int f(\vec{r})\,\delta^3(\vec{r}-\vec{a}) = f(\vec{a})$

$= 2^2 + 1^2$

$= 5$

3. a.



$\vec{E}$ symmetric in z

$$\oint \vec{E} \cdot d\vec{a} = E_z(z) A - E_z(-z) A = 2E_z(z) \cdot A$$

$$Q_{enc} = 2\rho A z \quad |z| < T/2$$

$$= \rho A T \quad |z| > T/2$$

$$\Rightarrow \boxed{\begin{aligned} E_z &= \rho z/\varepsilon_0 & |z| < T/2 \\ E_z &= \rho T/2\varepsilon_0 \, \hat{n} & |z| > T/2 \end{aligned}}$$

b. $W = Q \Delta V$

$$= Q \cdot -\int \vec{E} \cdot d\vec{l}$$

$$= Q \cdot -\int_{T/2}^{0} \rho z/\varepsilon_0 \, dz$$

$$= Q \cdot -\rho z^2/2\varepsilon_0 \Big|_{T/2}^{0}$$

$$= \boxed{Q \cdot \rho T^2/8\varepsilon_0}$$

4. a. $\rho = \varepsilon_0 \nabla \cdot \vec{E}$

$= \varepsilon_0 \cdot \frac{1}{s} \partial/\partial s \left(s E_s \right)$

$= \varepsilon_0 \cdot \frac{1}{s} \partial/\partial s \left(\frac{k s^2}{2 \varepsilon_0} \right) \qquad s < R$

$= \boxed{k \quad s < R}$

$= \varepsilon_0 \cdot \frac{1}{s} \partial/\partial s \left(\frac{k R^2}{\varepsilon_0} \right) \qquad s > R$

$= \boxed{0 \quad s > R}$

b. $\Delta \vec{E} = \sigma/\varepsilon_0 \, \hat{s}$

$= \vec{E}_{out} - \vec{E}_{in}$

$= \frac{kR^2}{\varepsilon_0 R} \hat{s} - \frac{kR}{2\varepsilon_0} \hat{s}$

$= \frac{kR}{2\varepsilon_0} \hat{s}$

$\Rightarrow \boxed{\sigma = kR/2}$