1. $X_n(x) = A\cos(k_x x) + B\sin(k_x x)$

$= B\sin\left(\frac{n\pi x}{a}\right)$ by boundary cond.

$Y(y) = Ce^{k_y y} + De^{-k_y y}$

even so $C = D \Rightarrow Y(y) = 2C\cosh(k_y y)$

$C_1 + C_2 = k_y^2 - \frac{n^2\pi^2}{a} = 0$

$\Rightarrow k_y = \frac{n\pi}{a}$

$$\boxed{V_n(x) = C_n \sin\left(\frac{n\pi x}{a}\right)\cosh\left(\frac{n\pi y}{a}\right)}$$

2. $V_<(R,\theta) = V_>(R,\theta)$

$\Rightarrow A_\ell R^\ell = B_\ell / R^{\ell+1} \Rightarrow B_\ell = A_\ell R^{2\ell+1}$

$\Delta E_\perp = \Delta\left(-\frac{\partial V}{\partial n}\right)_R = P_2(\cos\theta)/\varepsilon_0$

$\Rightarrow \sum_{\ell=0}^{\infty}\left[\frac{B_\ell\cdot(\ell+1)}{R^{\ell+2}} + A_\ell\cdot\ell\cdot R^{\ell-1}\right]P_\ell(\cos\theta) = \frac{P_2(\cos\theta)}{\varepsilon_0}$

$\Rightarrow \sum_{\ell=0}^{\infty} A_\ell\cdot(2\ell+1)\cdot R^{\ell-1} P_\ell(\cos\theta) = P_2(\cos\theta)/\varepsilon_0$

$\Rightarrow A_2\cdot 5\cdot R = 1/\varepsilon_0,\ A_{\ell\neq 2} = 0$

$$\Rightarrow \boxed{\begin{array}{ll} A_2 = \frac{1}{5\varepsilon_0 R} & A_{\ell\neq 2} = 0 \\ B_2 = \frac{R^4}{5\varepsilon_0} & B_{\ell\neq 2} = 0 \end{array}}$$

3. a. $\rho_b(r) = -\nabla \cdot \vec{P}$

$$= -\frac{1}{r^2}\partial/\partial r(P_0 r^3)$$

$$= \boxed{-3P_0}$$

$$\sigma_b = \vec{P}\cdot\hat{n} = \boxed{P_0 R}$$

b. $\boxed{\vec{D} = 0}$ everywhere
since $Q_{f\,enc} = 0$
& spherically symmetric

$$\vec{E} = (\vec{D} - \vec{P})/\varepsilon_0$$

$$\boxed{\begin{aligned} &= 0 && r > R \\ &= -\frac{P_0 r}{\varepsilon_0}\hat{r} && r < R \end{aligned}}$$

4. $\vec{p} = \int \vec{r}'\lambda(\vec{r}')\,dl'$

$$= \int_{-d/2}^{d/2} z'\hat{z}\cdot Q/z'\,dz'$$

$$= \boxed{Qd\,\hat{z}}$$

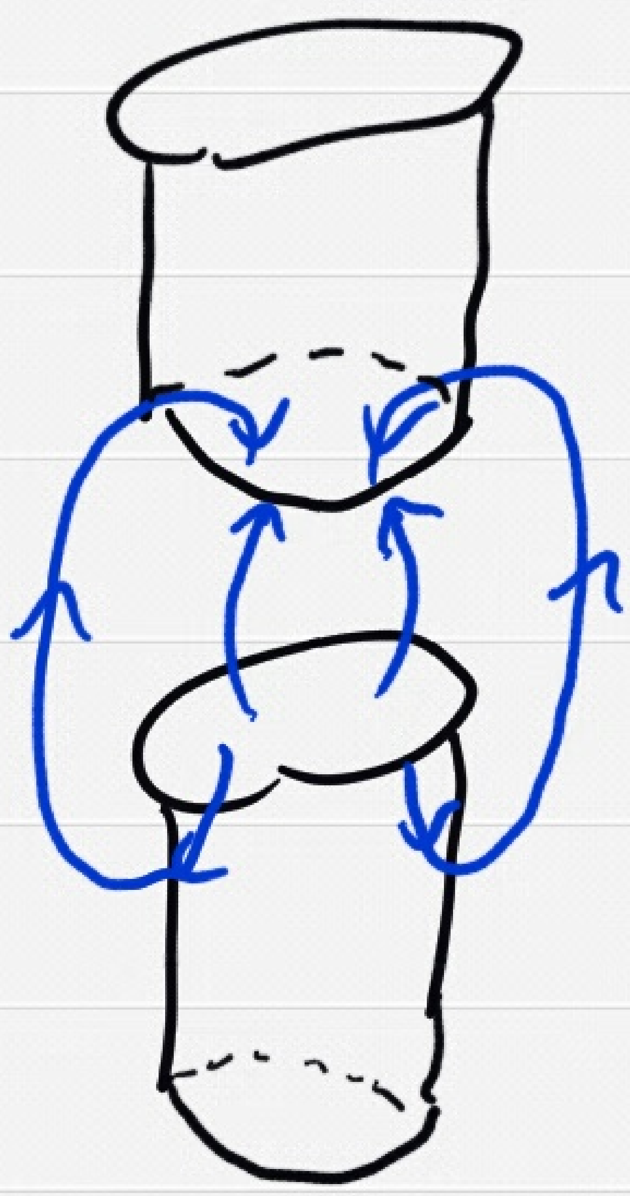
5.

σ_b

$\vec{P}$

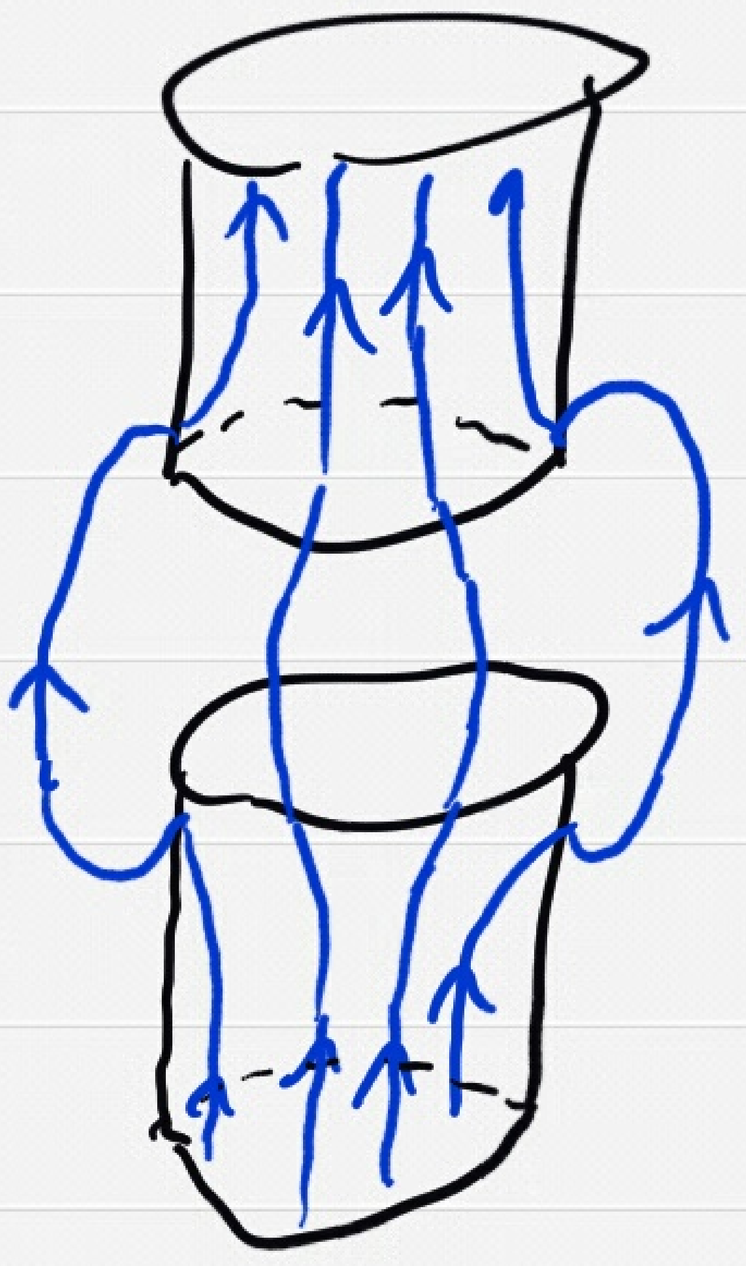
$\sigma_b = \vec{P}\cdot\hat{n}$

$\vec{E}$



$\vec{E}$ out of $+\sigma_b$ in to $-\sigma_b$

$\vec{D}$



$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

- normal component continuous
- tangential component same discontinuity as $\vec{P}$
- $\vec{D} \parallel \vec{E}$ if $\vec{P} = 0$