Electricity and Magnetism 1 [3811] Practice Midterm 2

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

This test is designed to be gender and race neutral.

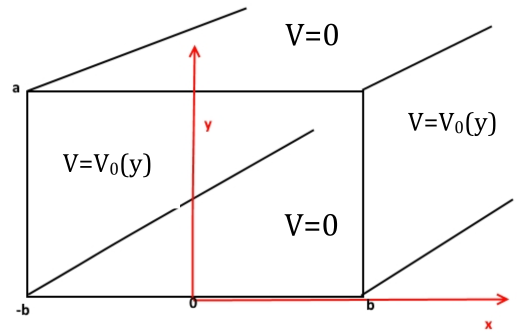
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Question 1 (25 points): Consider a infinitely long rectangular pipe with no charge inside it, formed by grounded plates at $y = 0$ and $y = a$ and plates with a fixed electric potential $V_0(y)$ at $x = \pm b$. If $V_0(y) = \sin(\pi y/a)$, find the solution for $V(x,y)$ in the pipe.



Question 2 (35 points): Consider a conducting sphere of radius *a*, with free charge *Q* on its surface, surrounded by a linear dielectric extending from radius a to radius *2a*. The potential is spherically symmetric, with $V(a) = C_0$ and $V(2a) = C_1$.

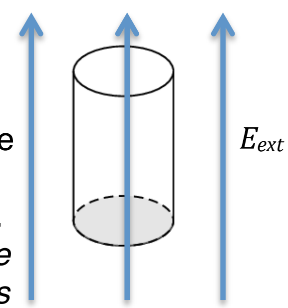
2a (10 points). Explicitly show that *V(r) = 1* and *V(r) = 1/r* satisfy Laplace's equation $\nabla^2 V = 0$ for spherical symmetry (use spherical coordinates). Therefore, the general solution is some combination of these two functions.

2b (15 points). By matching the boundary conditions at *r = a* and *r = 2a*, and requiring a finite potential at *r = 0* and *V = 0* at *r = ∞*, find the potential *V(r)* for $0<r<a$, $a<r<2a$, and $r>2a$.

2c (5 points). Find the electric field $\vec{E}(r)$ in the dielectric ($a<r<2a$).

2d (5 points). By comparing the electric field in the dielectric to what it would be in vacuum, find the dielectric constant ε_r of the dielectric ($a<r<2a$).

Question 3 (10 points): Compute the dipole moment (magnitude and direction) for an arrangement of three charges, $+q/2$ at $(x,y) = (-d/2,d/2)$, $+q/2 = (d/2,d/2)$, and $-q$ at $(0,-d/2)$.



Question 4 (20 points): Consider a cylinder of linear dielectric (with no free charge) immersed in an initially uniform external electric field E_{ext} directed along its axis. Fill in the three diagrams below, showing the lines of polarization P, electric field resulting from the polarization E_{int}, and total electric field $E_{tot} = E_{ext} + E_{int}$ inside and outside of the cylinder. On the polarization diagram, sketch the location of bound charge. *Make sure your sketches are qualitatively accurate, with more lines in regions of stronger field. If you're worried about your drawing skills, feel free to annotate the diagram to make it clear.*