Electricity and Magnetism 1 [3811] Final Exam
Wednesday December 18th, 2019

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If a question asks for a vector quantity, give both the magnitude and direction.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

This test is designed to be gender and race neutral.

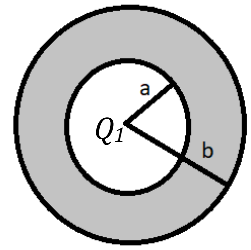
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Problem 1 (16 points): A perfectly conducting spherical shell with no net charge has inner and outer radii $r = a$ and $r = b$. A point charge Q_1 is located at the center ($r = 0$).



1a (4 points): Find the surface charge density on the inner and outer surfaces of the shell (σ_a at $r = a$, σ_b at $r = b$).

1b (4 points): Find the electric field $\vec{E}(r)$ as a function of radius inside ($r < a$), within ($a < r < b$), and outside ($r > b$) of the conducting shell.

1c (4 points): Show explicitly that the electric fields at the inner and outer surfaces of the shell ($r = a$, $r = b$) satisfy the electrostatic boundary conditions.

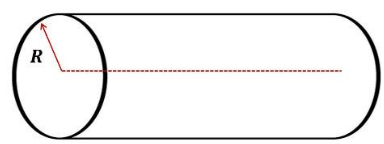
1c (4 points): How much energy does it take to move a second point charge Q_2 from infinity to the inner surface of the shell ($r = a$)?

Problem 2 (8 points): The electric potential in a region of space is $V(\vec{r}) = r\,cos(\theta)$.

2a (4 points): Find the corresponding electric field $\vec{E}(\vec{r})$, in spherical coordinates.

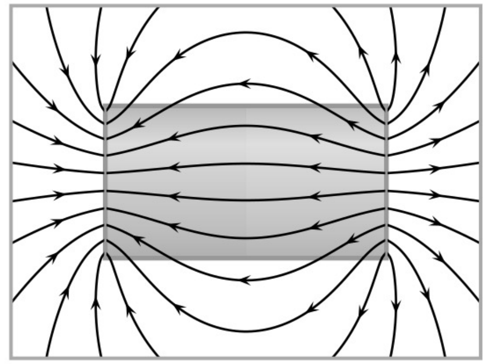
2b (4 points): Find the corresponding volume charge density $\rho(\vec{r})$.

Problem 3 (10 points): An infinitely long cylinder of radius R has a volume charge density *ρ(s) = C/s* that varies inversely with radius. Find the electric field $\vec{E}(s)$ as a function of radius inside (*s* < *R*) and outside (*s* > *R*) of the cylinder.



Problem 4 (12 points): A flat electrode in the *x-y* plane is biased to create a fixed electric potential distribution $V(x, y, z = 0) = V_0 \sin(k_x x) \sin(k_y y)$. The space above the electrode is completely free of charge. What is the full solution for the electric potential $V(x, y, z)$ valid in the entire half-space above the biased electrode (*z > 0*)?

Problem 5 (12 points): Consider the field lines around the rectangular object (shaded grey) in the image at right.



5a (4 points): If this is a magnetostatic problem, are these lines of B or lines of H? *Explain why, either in a sentence or using equations.*

5b (4 points): Given your answer to 5a, is there any free surface current in the picture, or is there only bound surface current? *Explain why, either in a sentence or using equations.*

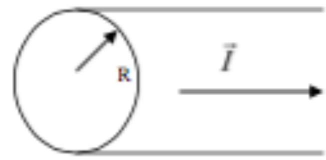
5c (4 points): If this is instead an electrostatic problem, and assuming there is no free charge present, are these lines of D or lines of E? *Explain why, either in a sentence or using equations.*

Problem 6 (12 points): An infinitely long charged cylinder of radius *a* holds a constant linear charge density λ. The charged cylinder is surrounded by a linear dielectric material with electric susceptibility χ_e that extends from $s = a$ to $s = b$.

6a (8 points): Find $\vec{D}$, $\vec{E}$, and $\vec{P}$ within the dielectric material ($a < s < b$).

6b (4 points): Find the bound surface charge density on the inner and outer surfaces of the dielectric (σ_{ab} at $s = a$, σ_{bb} at $s = b$).

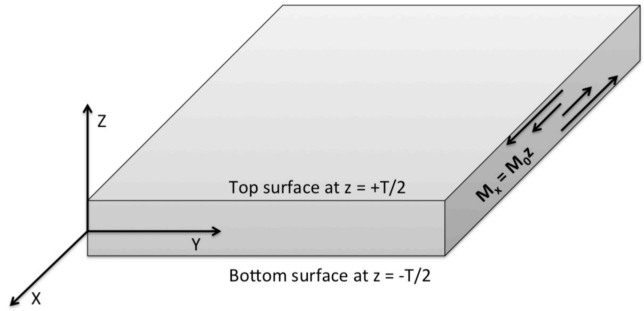
Problem 7 (14 points): An infinitely long cylinder with radius *R* has a total current *I* flowing along the cylinder.



7a (6 points): If the volume current density *J* is directly proportional to the distance *s* from the axis of the cylinder (i.e. *J* = constant × *s*), what is *J(s)* in terms of *s*, *I,* and *R*? *In other words, what is the constant?*

7b (8 points): What is the magnetic field $\vec{B}(s)$ as a function of the distance *s* from the axis of the cylinder inside (*s* < *R*) and outside (*s* > *R*) of the cylinder?

Problem 8 (16 points): An infinite magnetized slab with thickness *T* lies in the x-y plane, extending from *z = -T/2* to *z = T/2*. The magnetization of the slab $\vec{M} = M_0 z\,\hat{x}$ is proportional to z, and thus points in the *+x* direction for *+z* and the *–x* direction for *–z*, as shown. *There is no free current anywhere.*



8a (6 points): Find the magnitude and direction of the bound volume current density J_b within the slab and the surface current density K_b at the top and bottom of the slab.

8b (6 points): Find the magnetic field $\vec{B}(z)$ as a function of *z* below ($z < -T/2$), within ($-T/2 < z < T/2$), and above ($z > T/2$) the slab.

8c (4 points): Find $\vec{H}(z)$ as a function of *z* below ($z < -T/2$), within ($-T/2 < z < T/2$), and above ($z > T/2$) the slab, and verify that it is continuous.