Electricity and Magnetism 1 [3811] Midterm 1
Wednesday October 2nd 2019

Directions:

This exam is closed book. You are allowed a copy of the equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

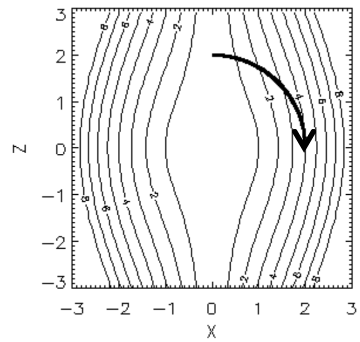
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Question 1 (30 points): Consider the electric potential $V(\vec{r}) = r^2 \sin\theta$ (contours at right).



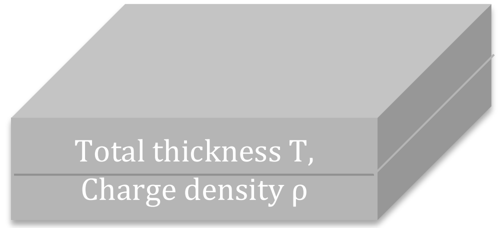
1a (10 points). What is the vector electric field $\vec{E}(\vec{r})$?

1b (15 points). Compute the line integral $\int \vec{E} \cdot \overrightarrow{dl}$ from the point *(r,θ,ϕ)* = (2,0,0) to the point *(r,θ,ϕ)* = (2, $\frac{\pi}{2}$,0), along a path following an arc of constant radius *r* and azimuthal angle ϕ (dark line in figure).

1c (5 points). Use one of the fundamental theorems of vector calculus to compute the answer to question 1b using an alternate method.

Question 2 (10 points): Find the integral $\int (x^2 + y^2)\delta^3(\vec{r} - \vec{a})d\tau$ over all volume, for the vector $\vec{a} = [2,1,1]$.

Question 3 (35 points): A slab of charge with constant volume charge density ρ, with thickness T in the z-direction, and infinite extent in x and y, is centered at the origin (half above and half below the x-y plane).



3a (25 points): Use Gauss's law to compute the vector electric field $\vec{E}(\vec{r})$ as a function of z inside ($|z| < T/2$) and outside ($|z| > T/2$) of the slab. Be explicit about the Gaussian surface(s) you utilize.

3b (10 points): Find the work W that it would take to move a charge $+Q$ from the top of the slab ($z = T/2$) to the center of the slab ($z = 0$).

Question 4 (25 points): An infinitely long cylinder of radius R has a vector electric field equal to $\vec{E}(\vec{r}) = \frac{ks}{2\varepsilon_0}\hat{s}$ inside the cylinder (*s*<*R*), and $\vec{E}(\vec{r}) = \frac{kR^2}{\varepsilon_0 s}\hat{s}$ outside the cylinder (*s*>*R*), with *k* a constant.

4a (15 points): Find the volume charge density $\rho(\vec{r})$ inside and outside of the cylinder.

4c (10 points): Using the boundary conditions on the electric field at $R = s$, calculate the surface charge density σ on the surface of the cylinder.