Electricity and Magnetism 1 [3811] Midterm 2
Wednesday November 6th, 2019

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

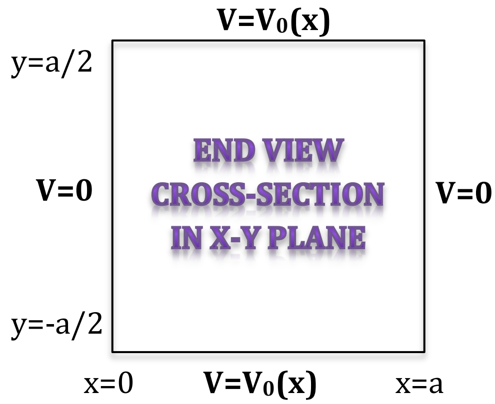
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Question 1 (20 points): A long square pipe has equal thickness and height *a*. The pipe is infinite in the z-direction, and extends from $x = 0$ to $x = a$, and from $y = -a/2$ to $y = a/2$. The walls at $y = -a/2$ and $y = a/2$ have the same specified potential $V_0(x)$, and the walls at $x = 0$ and $x = a$ are grounded ($V = 0$).



Find a valid set of solutions of Laplace's equation $V_n(x,y) = X(x)Y(y)$ for this problem.

Be sure to evaluate the constants C_1 and C_2 (or k_x, k_y) and the relationships between them for each solution, to match the boundary conditions. However, you do not need to find the coefficients C_n or an equation for the coefficients – just the set of solutions that would make up a general solution.

Question 2 (25 points): A hollow shell of radius R, with vacuum inside and out, has a charge density $\sigma(\theta) = P_2(\cos\theta)$. Use the continuity of the electric potential and the boundary conditions on the electric field at $r = R$ to find the coefficients A_l for the potential inside the sphere $V_<(r,\theta) = \sum_0^\infty (A_l r^l) P_l(\cos\theta)$ and the coefficients B_l for the potential outside the sphere $V_>(r,\theta) = \sum_0^\infty \left(\frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$.

Question 3 (20 points): A dielectric sphere of radius *R* has no free charge, but has a purely radial polarization that increases with radius as $\vec{P} = P_0 r \hat{r}$.

Don't assume the dielectric is linear.

3a (10 points): Find the volume charge density $\rho_b(r)$ within the sphere and the surface charge density σ_b at the surface of the sphere.

3b (10 points): Find the electric displacement $\vec{D}(r)$ and the electric field $\vec{E}(r)$ inside and outside of the sphere.

Question 4 (15 points): Find the dipole moment $\vec{p}$ of a charged wire lying along the z-axis, extending from *z = -d/2* to *z = d/2*, with a linear charge density that varies as $\lambda(z) = Q/z$.

Question 5 (20 points): A very long uniformly polarized cylinder has a short segment removed. In the vicinity of the resulting gap (inside the grey dashed ellipses), sketch the bound charge density σ_b, the electric field $\vec{E}$, and the electric displacement $\vec{D}$. Make sure your sketches are qualitatively accurate and obey the boundary conditions on $\vec{E}$ and $\vec{D}$. Feel free to annotate the diagrams to make them clear.