1. - Use separation of variables
- solution for x-direction is $X(x) = Ae^{-kx} + Be^{kx}$
- must go to zero @ $x = \infty$

$$\Rightarrow X(x) = Ae^{-kx}$$

$$\frac{1}{X} \frac{d^2X}{dx^2} = k^2 = C_1$$

$$\frac{1}{Y} \frac{d^2Y}{dy^2} = -\left(\frac{3\pi}{a}\right)^2 = C_2$$

$$C_1 + C_2 = 0 \Rightarrow k = 3\pi/a$$

$$\boxed{V(x,y) = \sin\left(\frac{3\pi y}{a}\right) e^{-3\pi x/a}}$$

($e^0 = 1$ so already normalized)

2. a. General solution

$$V(r,\theta) = \sum \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$$

$$\Rightarrow \boxed{\begin{aligned} V_{in}(r,\theta) &= V_0 \frac{r}{R} \cos\theta \\ V_{out}(r,\theta) &= \frac{V_0 R^2}{r^2} \cos\theta \end{aligned}}$$

b. $\vec{E} = -\nabla V$

$$\boxed{\begin{aligned} \vec{E}_{in}(r,\theta) &= -\frac{V_0}{R}\cos\theta\,\hat{r} + \frac{V_0}{R}\sin\theta\,\hat{\theta} \\ \vec{E}_{out}(r,\theta) &= \frac{2V_0R^2}{r^3}\cos\theta\,\hat{r} + \frac{V_0R^2}{r^3}\sin\theta\,\hat{\theta} \end{aligned}}$$

2c. $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$$\vec{D}_{in} = \left(P - \frac{\varepsilon_0 V_0}{R}\right)\cos\theta\,\hat{r} + \left(P + \frac{\varepsilon_0 V_0}{R}\right)\sin\theta\,\hat{\theta}$$

$$\vec{D}_{out} = \frac{2\varepsilon_0 V_0 R^2}{r^3}\cos\theta\,\hat{r} + \frac{\varepsilon_0 V_0 R^2}{r^3}\sin\theta\,\hat{\theta}$$

d. $D_{in\_r}|_R = D_{out\_r}|_R$

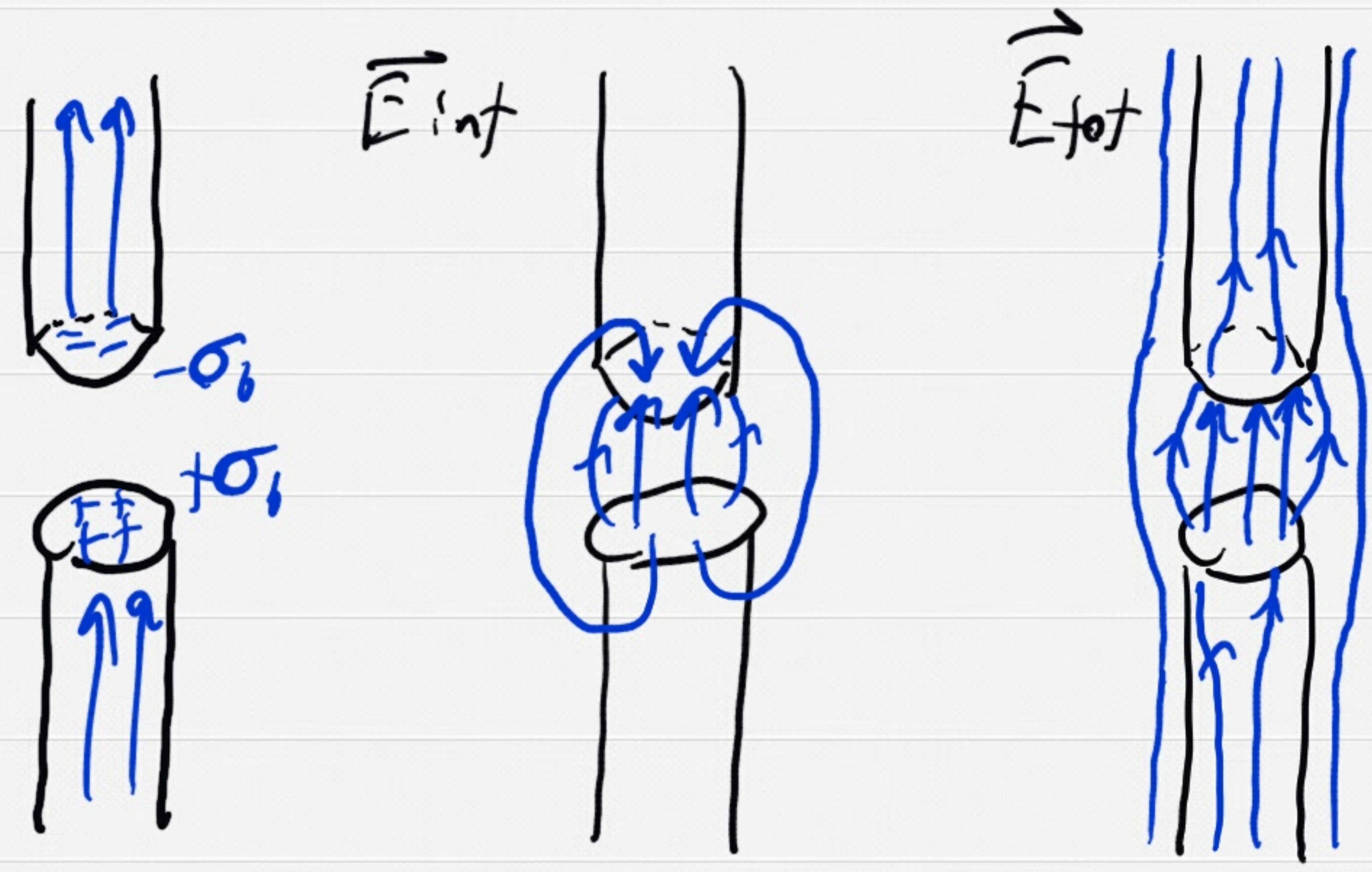
$$\Rightarrow P - \frac{\varepsilon_0 V_0}{R} = \frac{2\varepsilon_0 V_0}{R}$$

$$\Rightarrow P = \frac{3\varepsilon_0 V_0}{R}$$

3. $$\vec{p} = \int \vec{r}'\rho(\vec{r}')\,d\tau'$$
$$= 3q\,[d/2, 0] + q\,[-d/2, 0]$$
$$= qd\,\hat{x}$$

4.

$\vec{P}$



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