1. Use separation of variables

solution is $X(x) = Ae^{\kappa x} + Be^{-\kappa x}$

even, so $X(x) = A(e^{\kappa x} + e^{-\kappa x})$

$$d^2X/dx^2 = \kappa^2 X$$
$$d^2Y/dy^2 = -(\pi/a)^2$$
$$\kappa^2 - (\pi/a)^2 = 0 \Rightarrow \kappa = \pi/a$$

$$\Rightarrow V(x,y) = A\sin\left(\frac{\pi y}{a}\right)\left(e^{\pi x/a} + e^{-\pi x/a}\right)$$

Normalize to match boundary condition

$$\Rightarrow \boxed{V(x,y) = \sin\left(\frac{\pi y}{a}\right)\frac{\left(e^{\pi x/a} + e^{-\pi x/a}\right)}{e^{\pi b/a} + e^{-\pi b/a}} = \sin\left(\frac{\pi y}{a}\right)\cosh(\pi x/a)/\cosh(\pi b/a)}$$

2.a. $\nabla^2 V = \frac{1}{r^2}\partial/\partial r\left(r^2 \partial V/\partial r\right)$

$\partial/\partial r(1) = 0 \Rightarrow 1$ is solution

$\partial/\partial r(r^2 \partial/\partial r \, 1/r) = \partial/\partial r(-1) = 0 \Rightarrow 1/r$ is solution

b. $r < a$: $V = A \cdot 1 = C_0$

$r > 2a$: $V = B/r = C_1 \cdot 2a/r$

$a < r < 2a$: $V = A' + B'/r$

$$V(a) = C_0 = A' + B'/a$$
$$V(2a) = C_1 = A' + B'/2a$$
$$B'(1/a - 1/2a) = C_0 - C_1$$
$$\Rightarrow B' = 2a(C_0 - C_1)$$
$$A' = C_0 - 2(C_0 - C_1) = 2C_1 - C_0$$

$$V(r) = 2C_1 - C_0 + 2a(C_0 - C_1)/r$$

2c. $\vec{E} = -\nabla V$

$= 2a(c_0 - c_1)/r^2 \, \hat{r}$

2d. $\vec{E}_{vac} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

$\dfrac{\varepsilon}{\varepsilon_0} = \vec{E}_{vac}/\vec{E} = \varepsilon_r = \dfrac{Q}{4\pi\varepsilon_0 \cdot 2a(c_0 - c_1)}$

3. $\vec{p}(\vec{r}) = \int \vec{r}\,'\rho(\vec{r}\,')\,d\tau'$

$= \Sigma q\vec{r}\,'$ for point charges

$= q/2\,[-d/2, d/2] + q/2\,[d/2, d/2]$
$\quad - q\,[0, -d/2]$

$= q d\,\hat{y}$

4. $\vec{p}$ $\qquad$ $\vec{E}_{int}$ $\qquad$ $\vec{E}_{total}$