Homework #2 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points): Check the divergence theorem \( \iiint (\nabla \cdot \vec{A}) \, dV = \iint \vec{A} \cdot d\vec{a} \) for the function \( \vec{A}(r, \theta, \phi) = r^2 \hat{r} \), using as your volume a sphere of radius \( R \) centered at the origin.

Problem 2 (20 points): Evaluate the following volume integrals.

a. \( \iiint (r^2 + \hat{r} \cdot \vec{a} + a^2) \delta^3 (r - \vec{a}) \, d\tau \) over all space (\( \vec{a} \) is a fixed vector of magnitude \( a \))

b. \( \iiint (r^4 + r^2 \hat{r} \cdot \vec{c} + c^4) \delta^3 (r - \vec{c}) \, d\tau \) over a spherical volume with radius 6 centered at the origin, for the vector \( \vec{c} = 5\hat{x} + 3\hat{y} + 2\hat{z} \).

Problem 3 (20 points): Take the vector functions \( \vec{F}_1 = x^2 \hat{z} \) and \( \vec{F}_2 = x\hat{x} + y\hat{y} + z\hat{z} \).

a. Calculate the divergence and curl of each one of these functions.

b. Which one can be written as the gradient of a scalar function? For this one, find an example of a scalar function that has the right gradient.

c. Which one can be written as the curl of a vector function? For this one, find an example of a vector function that has the right curl.

Problem 4 (20 points): Find the vector electric field a distance \( z \) above the center of a circular loop of radius \( R \) that carries a uniform line charge density \( \lambda \).

Problem 5 (20 points): Find the vector electric field a distance \( z \) above the center of a flat circular disk of radius \( R \) that carries a uniform surface charge density \( \sigma \).