Homework #4 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75%
credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points):
  a. Find the electric potential a distance of \( z \) above the center of a flat circular disc of
     radius \( R \) that carries a uniform surface charge density \( \sigma \), using direct integration
     over the charge density.

  b. Compute the \( z \)-component of the electric field from your answer to (a), and verify
     that you recover the solution from Problem 5 on HW #2.

Problem 2 (20 points):
  a. Use Gauss’s law to compute the electric field inside and outside of a long hollow
     cylindrical tube that carries a uniform surface charge density \( \sigma \).

  b. Verify that the change in electric field between the inside and the outside of the
     tube agrees with Eq. 2.33

Problem 3 (20 points): Consider four charges arranged in a square with sides of
length \( a \). If the upper left and lower right charges are \(-q\) and the upper right and
lower left are \(+q\), compute the total work needed to assemble this configuration.

Problem 4 (20 points): Find the electrostatic energy stored in a solid sphere of
radius \( R \) with a uniform volume charge density \( \rho \), and thus a total charge \( Q = \frac{4}{3} \pi R^3 \rho \)
(same as Problems 2&5 on HW #3). Express your answers in terms of \( Q \), not \( \rho \).

  a. First, use Eq. 2.43, with the potential you solved for last week.

  b. Next, use Eq. 2.45.

Problem 5 (20 points): Consider a metal sphere of radius \( R \) carrying charge \( Q \),
surrounded by a thin concentric spherical metal shell with inner radius \( a \) and outer
radius \( b \), carrying no net charge.

  a. Find the surface charge density \( \sigma \) at \( r = R \), \( r = a \), and \( r = b \).

  b. Find the electric potential at the center of the sphere, with respect to infinity.