Homework #9 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points): Given a magnetic field of the form \( \vec{B} = k z \hat{x} \) (with \( k \) a constant), find the force on a square loop with sides of length \( a \) lying in the y-z plane, centered at the origin. The loop has a current \( I \) that flows counterclockwise as seen from a viewpoint looking along the x-axis.

Problem 2 (20 points): Consider a total current \( I \) flowing down a cylindrical wire with a circular cross-section of radius \( a \).

2a (10 points): If the current \( I \) flows entirely on the surface of the wire (uniformly distributed across the surface), what is the surface current density \( K \)?

2b (10 points): If instead the volume current density is inversely proportional to the distance \( s \) from the axis, what is \( J(s) \) in terms of \( I \) and \( a \)?

Problem 3 (30 points): Calculate the magnetic field at the center of a uniformly charged spherical shell of radius \( R \), carrying total charge \( Q \), and spinning around the z-axis with a uniform angular velocity \( \omega \). Hint: Start with the solution derived for the magnetic field above/below the center of a circular loop of current.

Problem 4 (30 points): Consider two infinite straight line charges with linear charge density \( \lambda \), aligned parallel to each other and separated by a distance \( d \). How fast would these two line charges have to move in order for the magnetic attraction between the wires to balance the electrostatic repulsion? Is this possible?