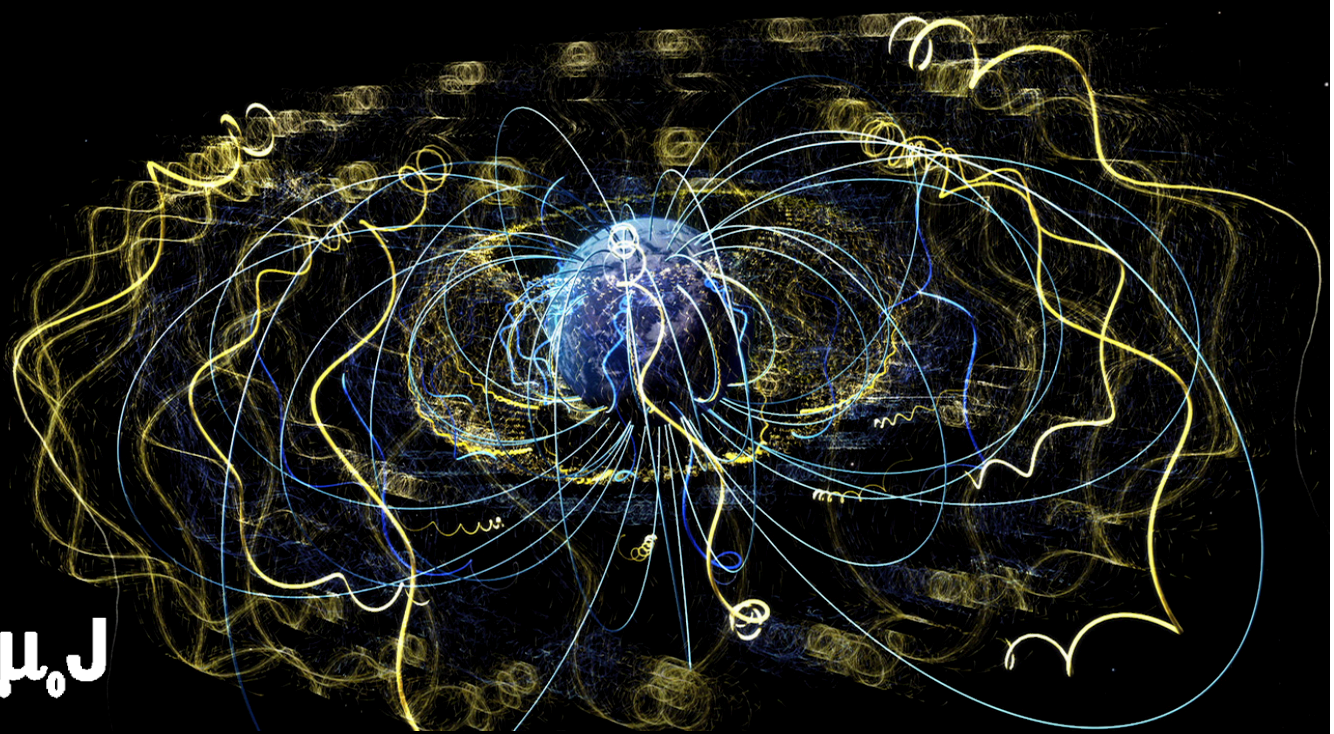
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

$$\boxed{\nabla \cdot \vec{E} = \rho(\vec{r})/\varepsilon_0}$$

What about $\nabla \times \vec{E}$?

$$\nabla \times \vec{E} = \frac{1}{4\pi\varepsilon_0} \nabla \times \int_V \rho(\vec{r}\,') \frac{\hat{\Delta r}}{\Delta r^2} d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \int_V \nabla \times \left(\rho(\vec{r}\,') \frac{\hat{\Delta r}}{\Delta r^2} \right) d\tau'$$

$$\nabla \times \left(\rho(\vec{r}\,') \frac{\hat{\Delta r}}{\Delta r^2} \right)$$

$$= \rho(\vec{r}\,') \nabla \times \left(\frac{\hat{\Delta r}}{\Delta r^2} \right) - \frac{\hat{\Delta r}}{\Delta r^2} \times \nabla \rho(\vec{r}\,')$$

$$= \rho(\vec{r}\,') \nabla \times \left(\frac{\hat{\Delta r}}{\Delta r^2} \right)$$

$$\nabla \times \left(\frac{\hat{\Delta r}}{\Delta r^2} \right) = \nabla \times \left(\frac{\hat{r}}{r^2} \right) = 0$$

$$\Rightarrow \nabla \times \vec{E} = 0$$

Irrotational!
(as long as static)

$$\Rightarrow \oint \vec{E} \cdot d\vec{l} = 0$$

$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$ independent of path

$$\vec{E} = \pm \nabla f$$

Electric Potential

Define $V(\vec{r})$ so that

$$\vec{E} = -\nabla V$$

- Negative sign convention so potential higher by + charge, lower by − charge

- Electric field points "downhill" on map of electric potential

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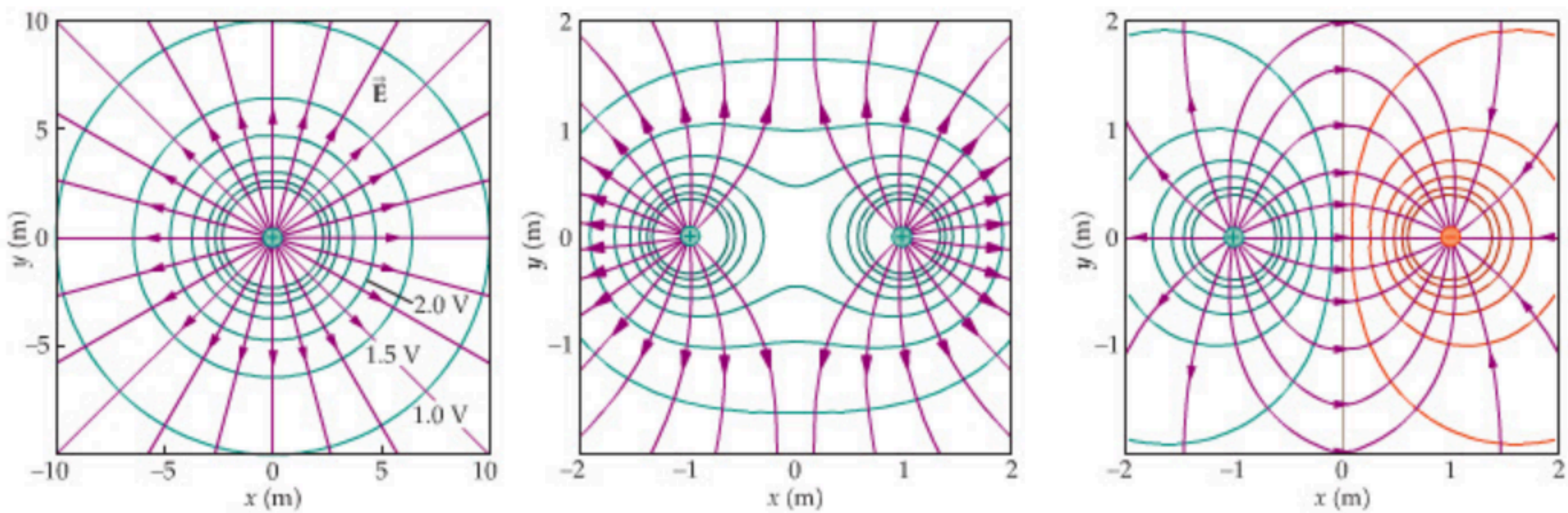
- $V(\vec{r})$ only defined up to a constant $\Rightarrow$ zero of potential arbitrary

$$-\nabla V(\vec{r}) = -\nabla\left(V(\vec{r}) + \text{const.}\right) = \vec{E}(\vec{r})$$

$$\vec{E} = -\nabla V \Rightarrow -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{l} = V(\vec{b}) - V(\vec{a})$$

Independent of path $a \to b$

Electric Field & Electric Potential



Example: $\vec{E} \to V$

- Shell w/ charge Q, radius R
- Gauss's Law

$$\vec{E} = 0 \qquad r < R$$

$$\vec{E} = \frac{Q\,\hat{r}}{4\pi\varepsilon_0 r^2} \qquad r > R$$

$$V(r) = -\int \vec{E} \cdot d\vec{l}$$

pick $V = 0$ @ $r = \infty$

$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$

choose radial path

$$V(r) = -\int_{\infty}^{r} E_r \, dr'$$

$$= -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2} dr' \qquad r > R$$

$$= \frac{Q}{4\pi\varepsilon_0 r'}\Big|_{\infty}^{r} \qquad r > R$$

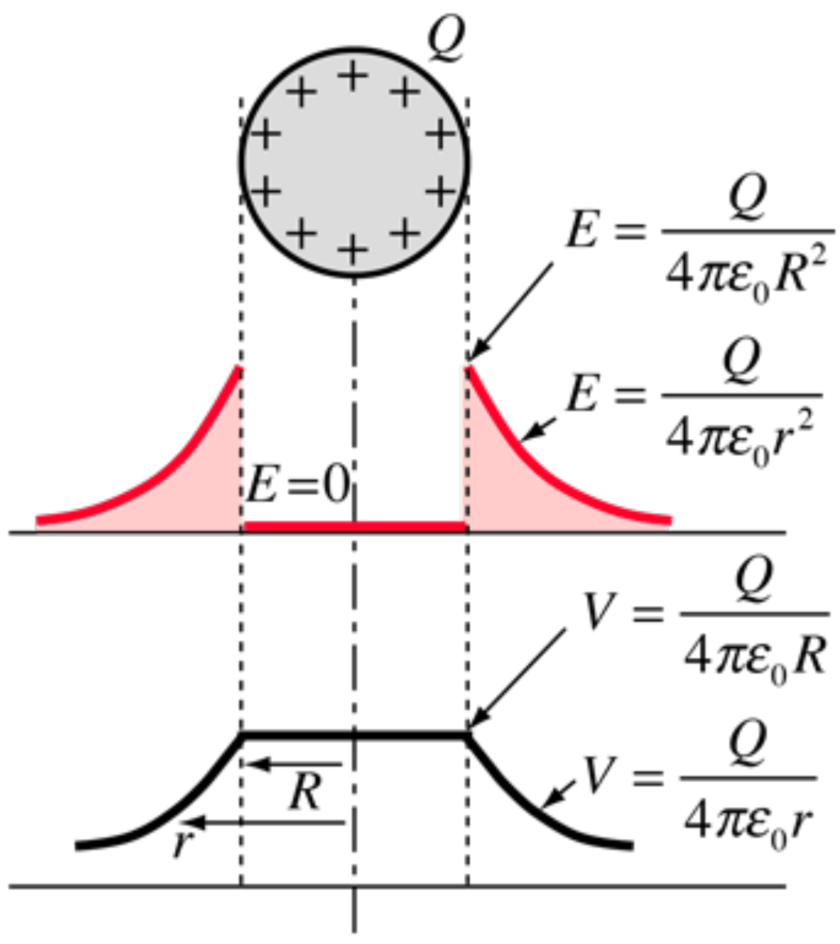
$$= \boxed{Q/(4\pi\varepsilon_0 r) \qquad r > R}$$

$$= V(R) - \int_{R}^{r} E_r \, dr' \qquad r < R$$

$$= Q/4\pi\varepsilon_0 R - \int_{R}^{r} 0 \, dr' \qquad r < R$$

$$= \boxed{Q/(4\pi\varepsilon_0 R) \qquad r < R}$$

Potential of Charged Spherical Shell



Poisson's Equation

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0$$
$$= \nabla \cdot (-\nabla V)$$
$$= -\nabla^2 V$$

$$\Rightarrow \boxed{\nabla^2 V = -\rho/\varepsilon_0}$$

charge density $\leftrightarrow$ curvature of electric potential

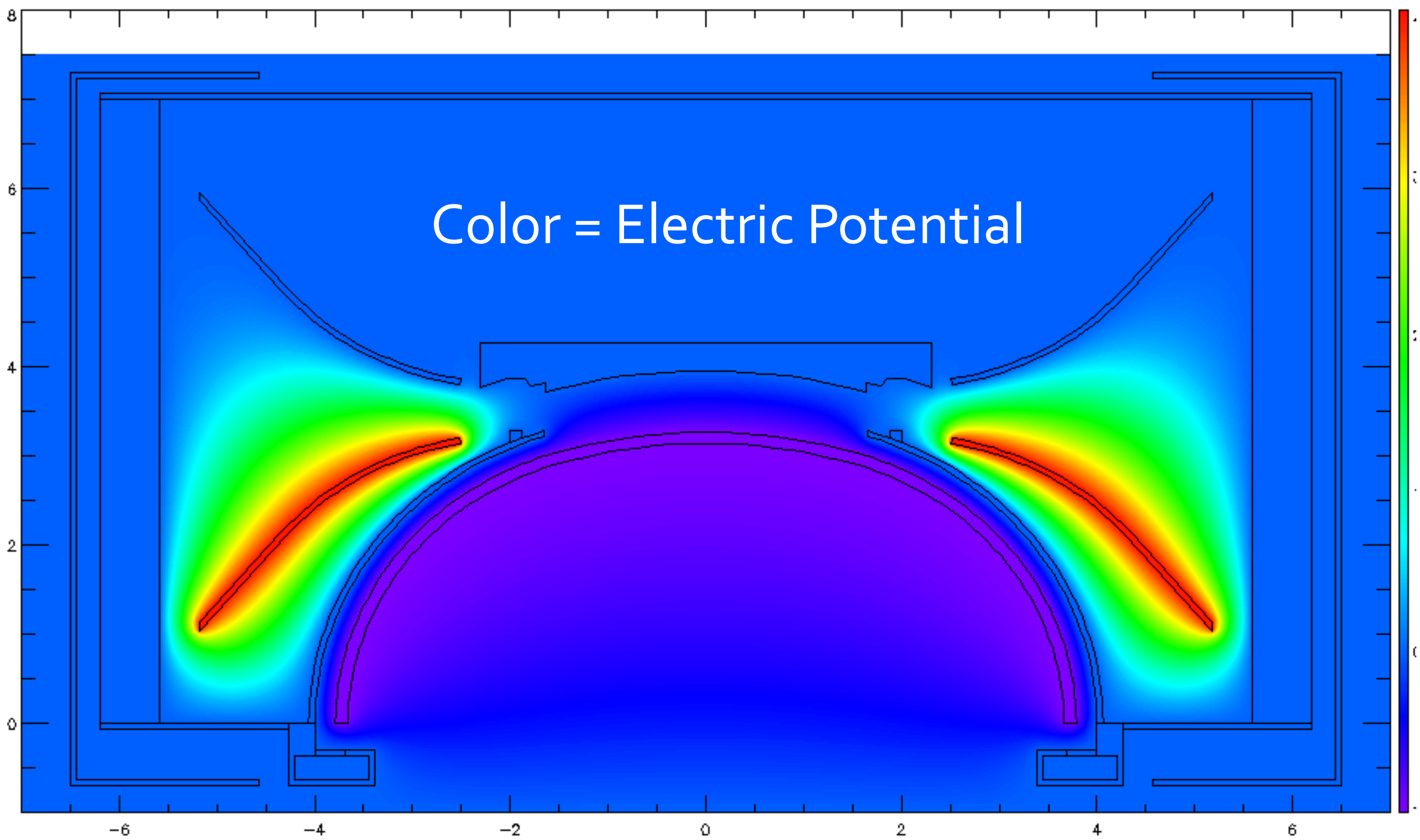
Laplace's Equation

If $\rho(\vec{r}) = 0 \Rightarrow \nabla^2 V = 0$

No charge density $\Rightarrow$ zero curvature

V on boundary of charge-free region
$\Rightarrow$ V everywhere in region

Laplace's Equation: Application



Electric Potential of Arbitrary Charge Distribution

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}\,')}{\Delta r} \, d\tau'$$

Check:

$$\vec{E} = -\nabla V = -\frac{1}{4\pi\varepsilon_0} \nabla \int_V \frac{\rho(\vec{r}\,')}{\Delta r} \, d\tau'$$

$$= -\frac{1}{4\pi\varepsilon_0} \int_V \nabla\left(\frac{\rho(\vec{r}\,')}{\Delta r}\right) d\tau'$$

$$\nabla\left(\frac{\rho(\vec{r}\,')}{\Delta r}\right) = \underbrace{\frac{1}{\Delta r} \nabla \rho(\vec{r}\,')}_{0} + \underbrace{\rho(\vec{r}\,') \nabla\left(\frac{1}{\Delta r}\right)}_{\rho(\vec{r}\,') \cdot -\frac{\hat{\Delta r}}{\Delta r^2}}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \rho(\vec{r}\,') \frac{\hat{\Delta r}}{\Delta r^2} \, d\tau' \quad //$$

Point Charge

$$\rho(\vec{r}\,') = q_i \, \delta^3(\vec{r}\,' - \vec{r}_i)$$

$$V_i(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{q_i \, \delta^3(\vec{r}\,' - \vec{r}_i)}{\Delta r} \, d\tau'$$

$$= \frac{q_i}{4\pi\varepsilon_0 \, \Delta r_i} \quad \text{w/ } \Delta r_i = |\vec{r} - \vec{r}_i|$$

$$V(\vec{r}) = \sum_i V_i = \sum_i \frac{q_i}{4\pi\varepsilon_0 \, \Delta r_i}$$