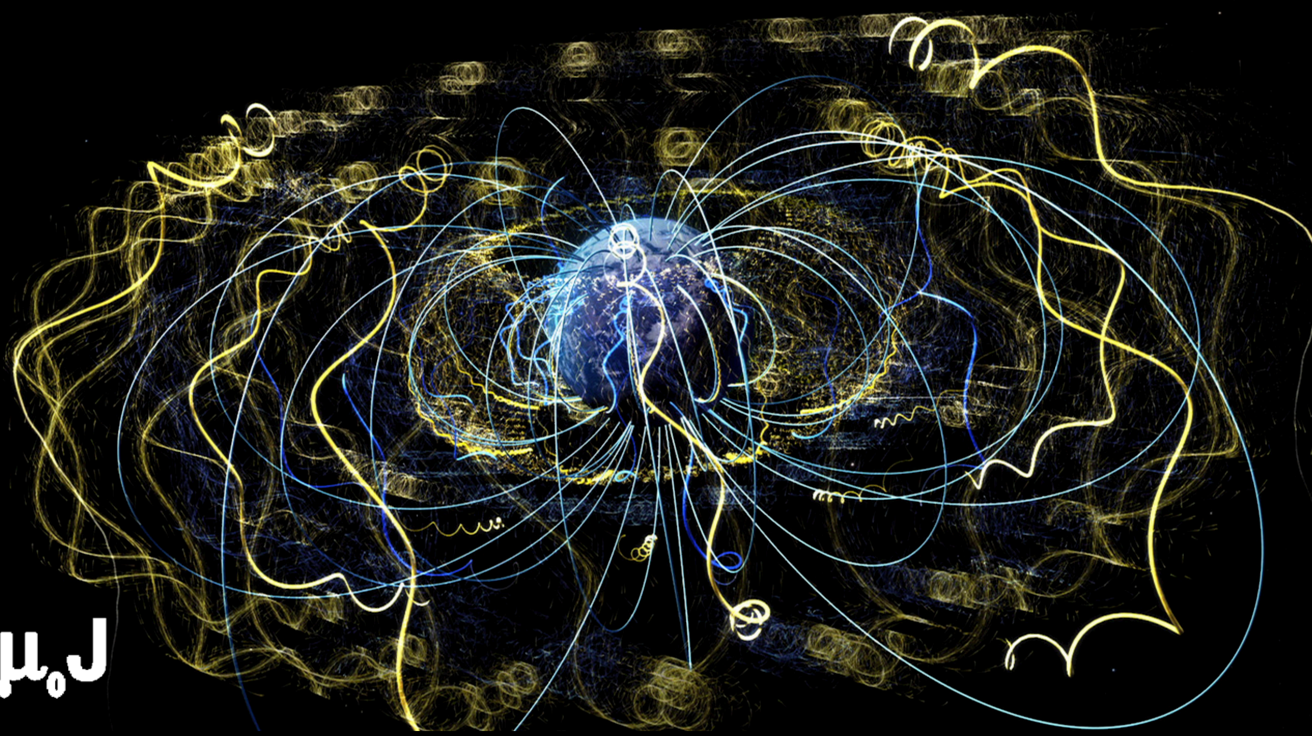
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Continuous Charge Distribution

$\frac{1}{2} \sum_i q_i V(\vec{r}_i) \rightarrow \boxed{W = \frac{1}{2} \int \rho V \, d\tau}$

In terms of $\vec{E}$:

$$\rho = \varepsilon_0 \nabla \cdot \vec{E}$$

$$W = \frac{\varepsilon_0}{2} \int (\nabla \cdot \vec{E}) V \, d\tau$$

Integrate by parts

$$W = \frac{\varepsilon_0}{2} \left[-\int \vec{E} \cdot \nabla V \, d\tau + \oint V \vec{E} \cdot d\vec{a} \right]$$

$$= \frac{\varepsilon_0}{2} \int \vec{E} \cdot \vec{E} \, d\tau + \frac{\varepsilon_0}{2} \oint V \vec{E} \cdot d\vec{a}$$

For all space $\rightarrow 0$

$\Rightarrow \boxed{W = \frac{1}{2} \varepsilon_0 \int E^2 \, d\tau}$

- Electric field contains energy!

Example

- Shell w/ charge Q / radius R

$$\vec{E} = \begin{cases} Q/(4\pi\varepsilon_0 r^2)\,\hat{r} & r > R \\ 0 & r < R \end{cases}$$

$$V = \begin{cases} Q/(4\pi\varepsilon_0 r) & r > R \\ Q/(4\pi\varepsilon_0 R) & r < R \end{cases}$$

$$W = \frac{1}{2}\int \rho V \, d\tau = \frac{1}{2}\int \sigma V \, da$$

$$= \frac{1}{2}\int \frac{Q}{4\pi R^2}\,\frac{Q}{4\pi\varepsilon_0 R}\cdot da$$

$$= \frac{1}{2}\,\frac{Q}{4\pi R^2}\cdot\frac{Q}{4\pi\varepsilon_0 R}\cdot 4\pi R^2$$

$$= \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$W = \frac{1}{2}\varepsilon_0 \int E^2 \, d\tau$$

$$\frac{1}{2}\varepsilon_0 \int_0^{2\pi}\int_0^{\pi}\int_R^{\infty} \frac{Q^2}{(4\pi\varepsilon_0 r^2)^2}\cdot r^2 dr \sin\theta \, d\theta \, d\varphi$$

$$= \frac{Q^2}{32\pi^2\varepsilon_0}\cdot\int_R^{\infty}\frac{1}{r^2}\,dr \int_0^{2\pi}\int_0^{\pi}\sin\theta \, d\theta \, d\varphi$$

$$= \frac{Q^2}{32\pi^2\varepsilon_0}\cdot\frac{1}{R}\cdot 4\pi = \frac{Q^2}{8\pi\varepsilon_0 R} \;//$$

Conductors

- Materials w/ many unbound "free electrons" which can quickly and easily move in response to any electric field

Steady-State Conductors

1. $\vec{E} = 0$ inside

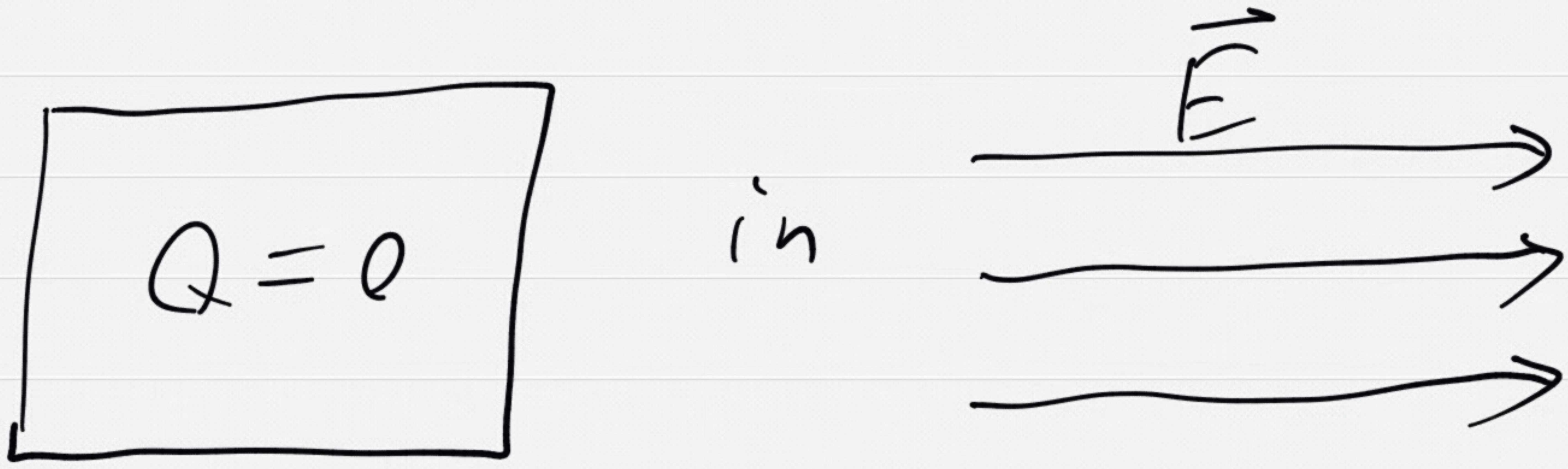
2. $\rho = 0$ inside
   $(\nabla \cdot \vec{E} = 0)$

3. Net charge only on surface

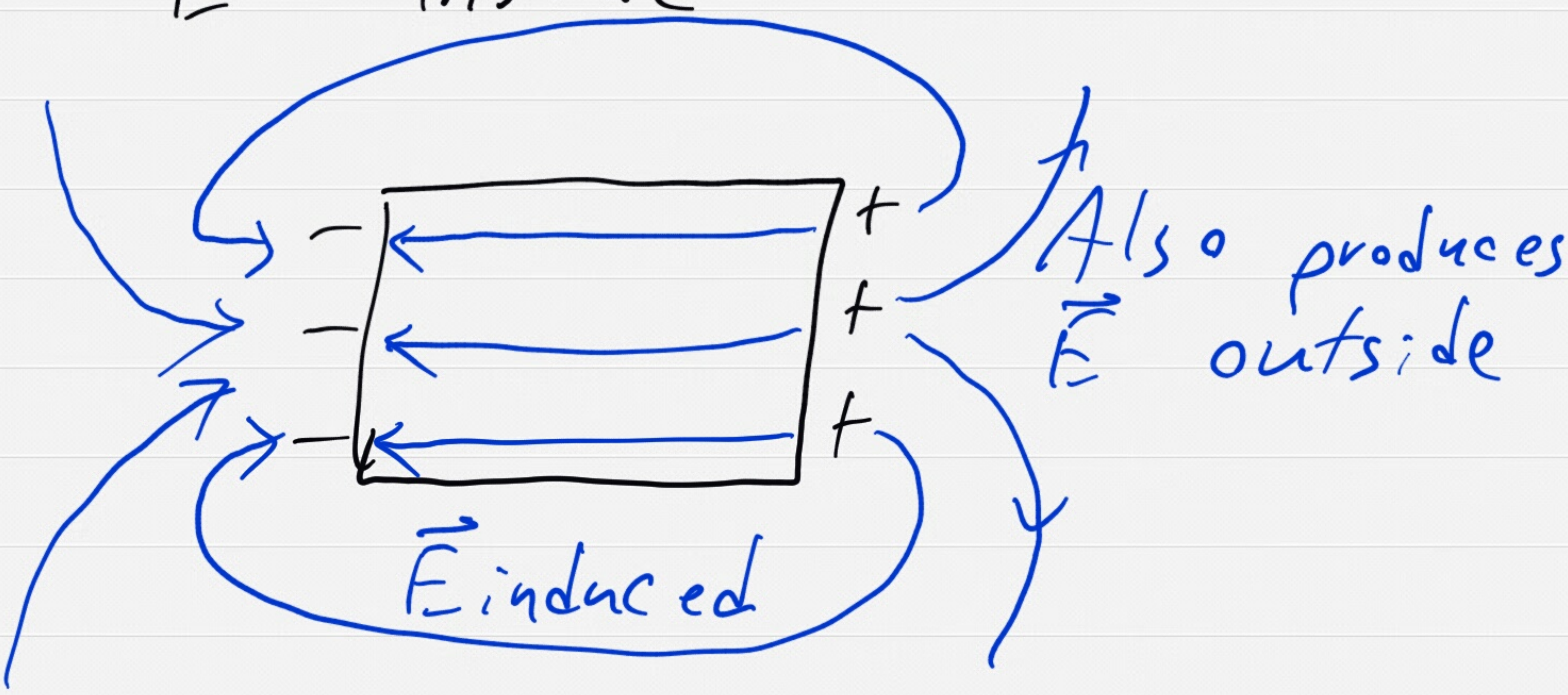
4. Conductor is equipotential
   $\Delta V = -\int \vec{E} \cdot d\vec{l} = 0$

5. $\vec{E}$ normal to surface
   $\Delta \vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n} = \vec{E}_{outside}$

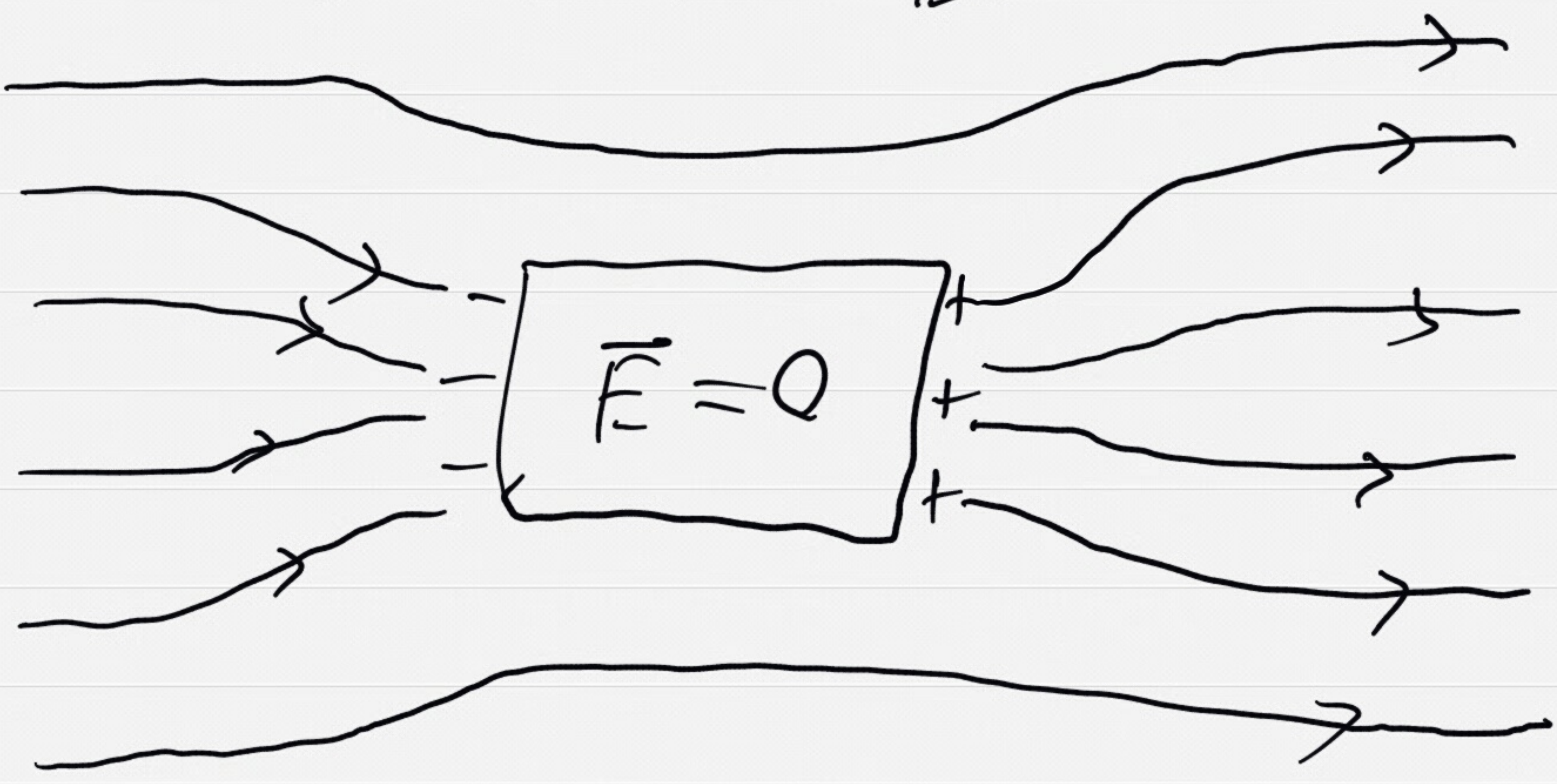
Induced Charge and Field



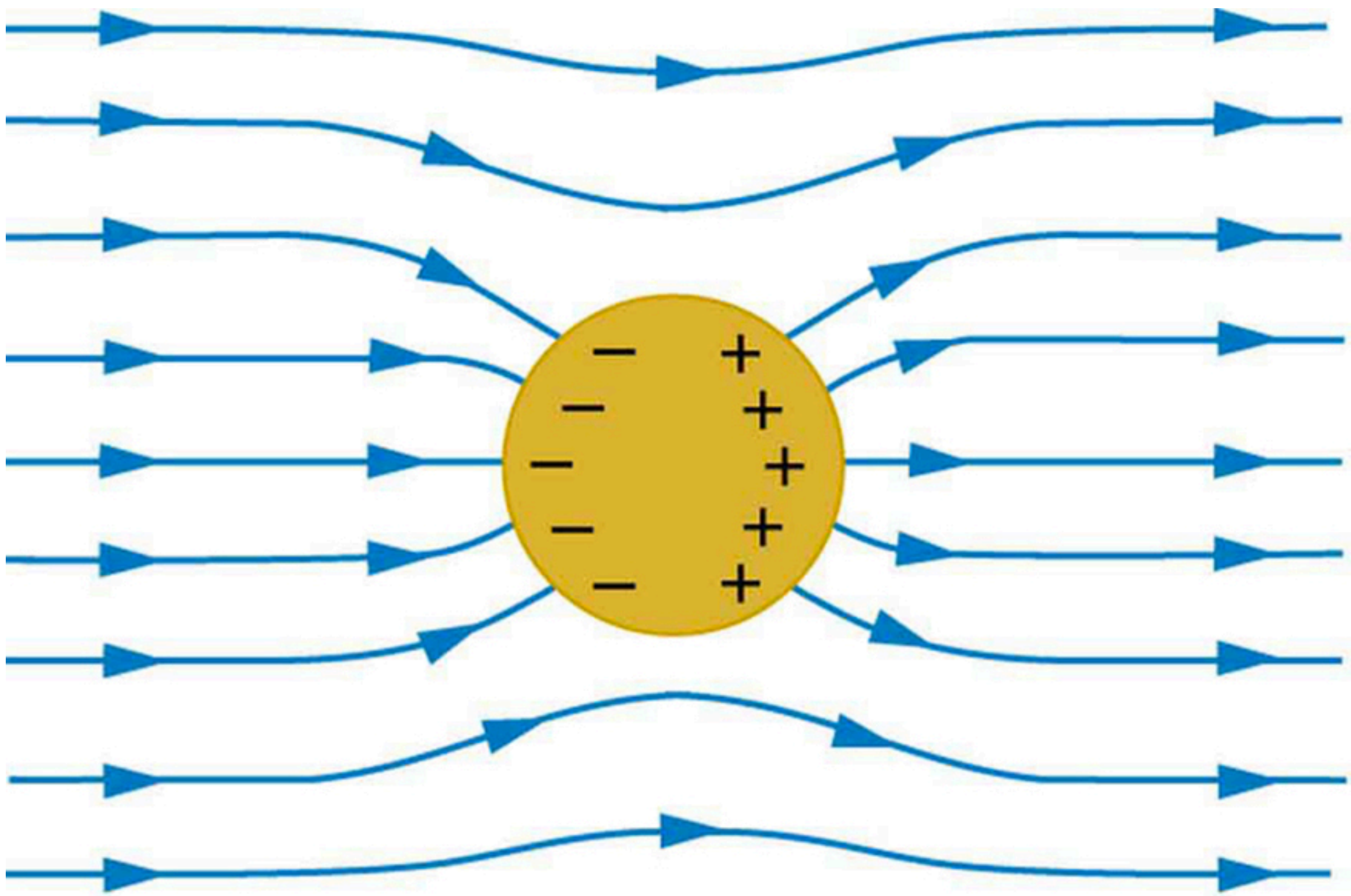
Charge flows to cancel $\vec{E}$ inside



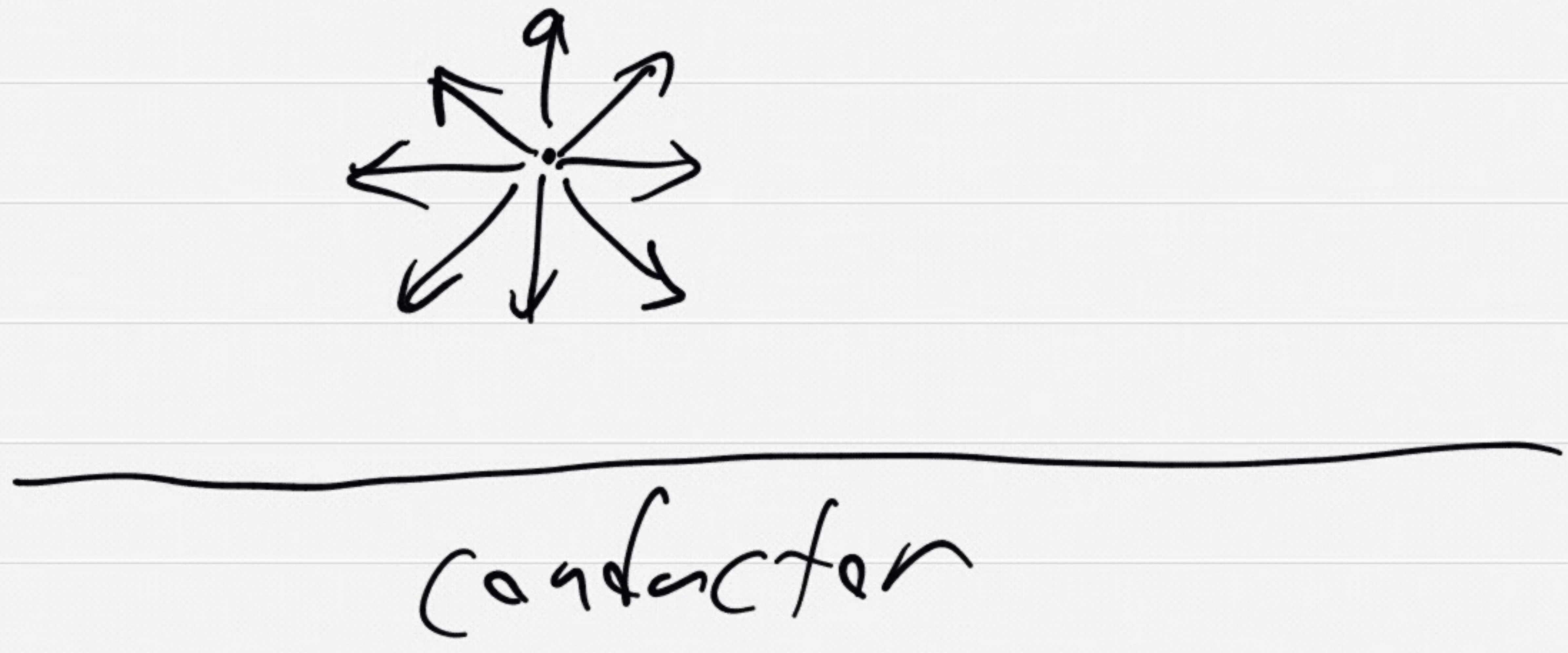
Total $\vec{E}$



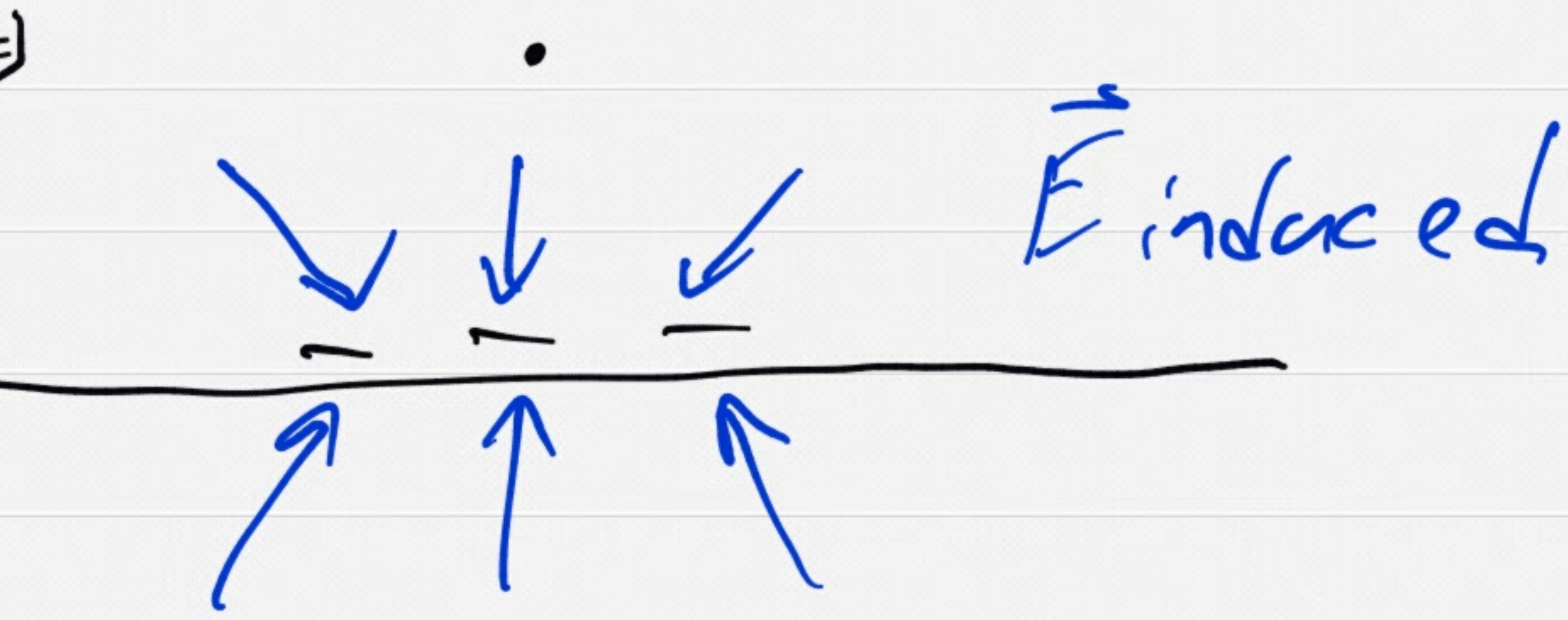
Conductors



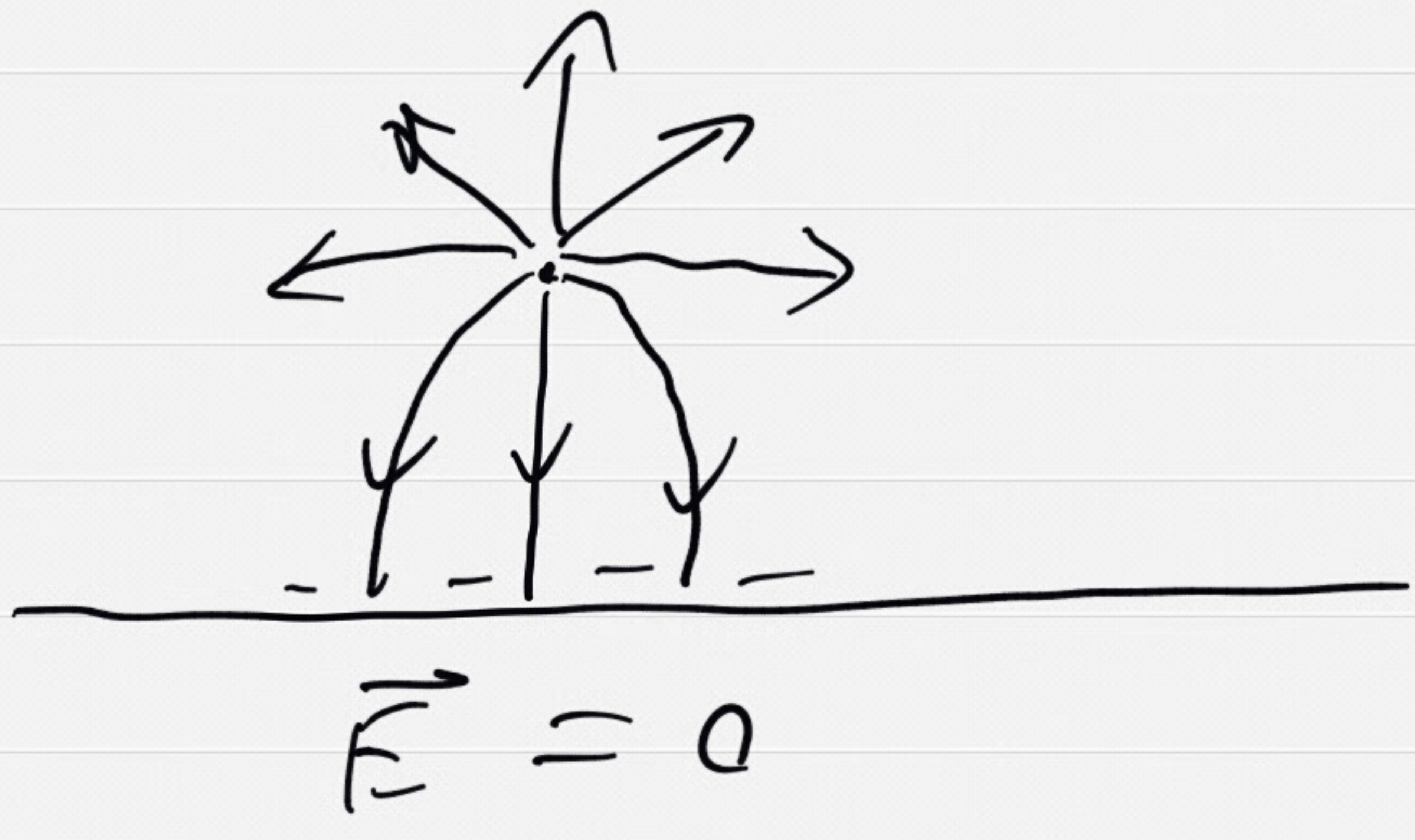
Charge Near Conductor



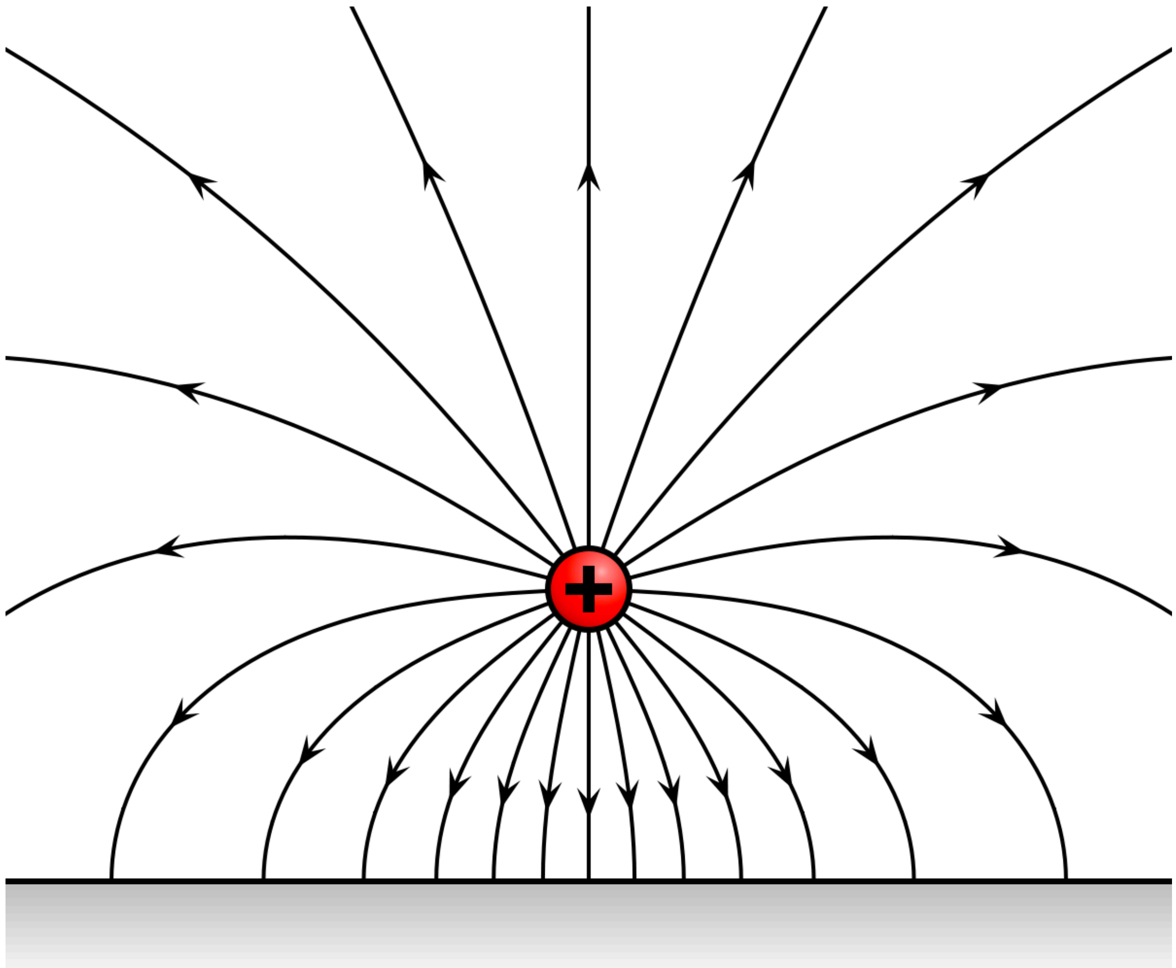
$\Rightarrow$



Total Field



Charge Near Conductor



Conductors