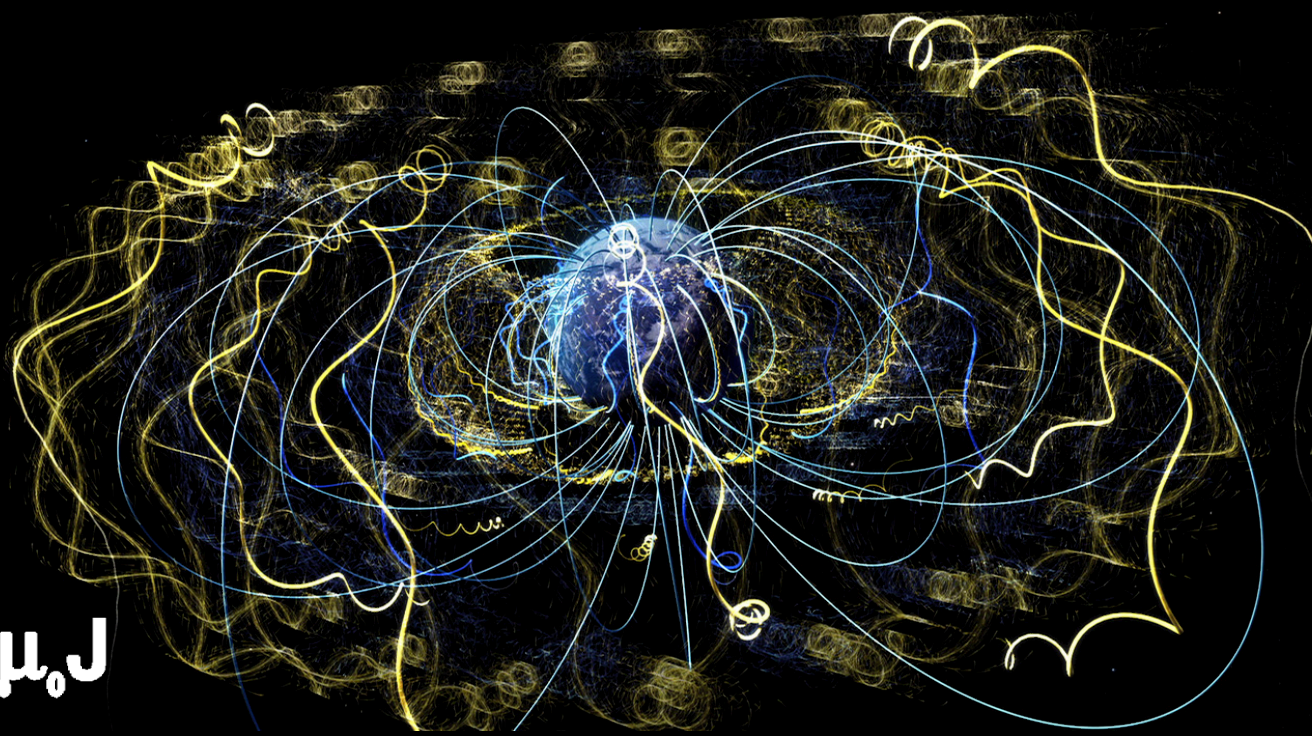
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



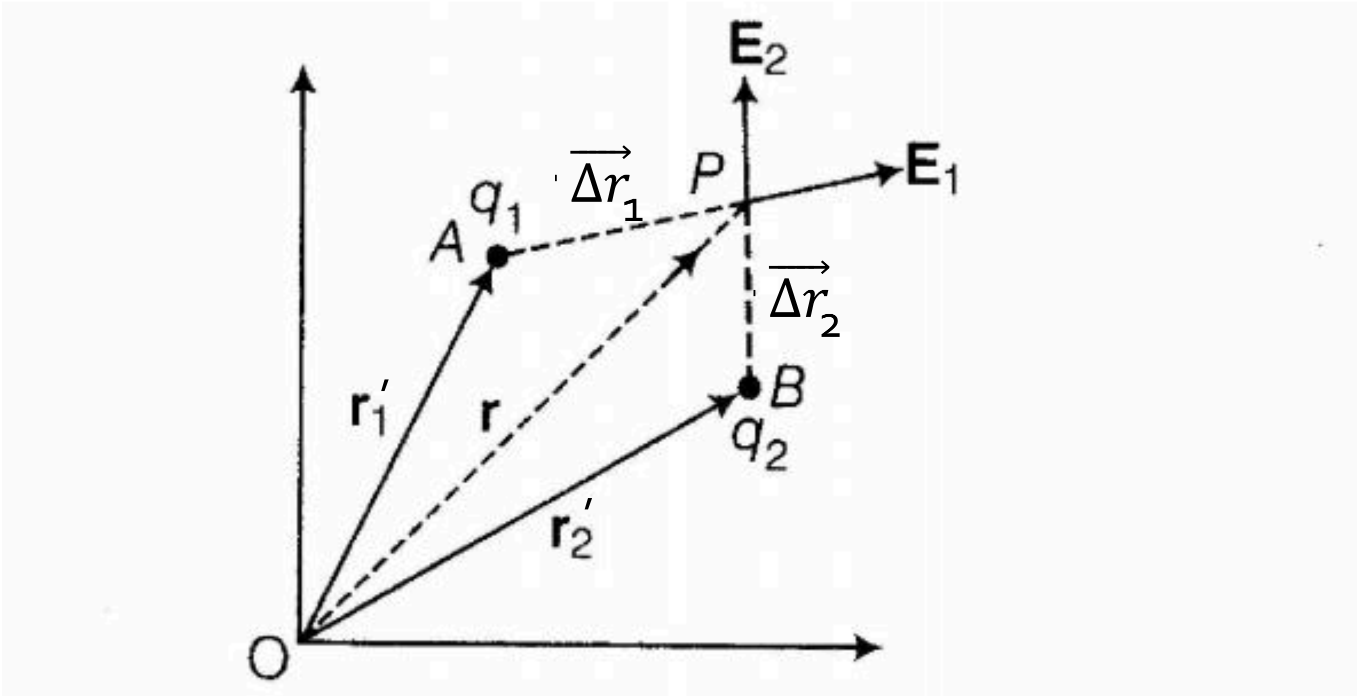
Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Midterm Details

- First midterm exam is Wednesday (Oct. 2) during normal class hours
  - Exam covers Griffiths Ch. 1-2 & lectures through Friday 9/27
  - Equation sheet posted
    - You are responsible for printing this, annotating it if you desire, and bringing it to the test
  - Two sample midterms (with solutions) posted
    - Format of this year's exam will be very similar
    - Problems will include spherical, cylindrical, and Cartesian geometry

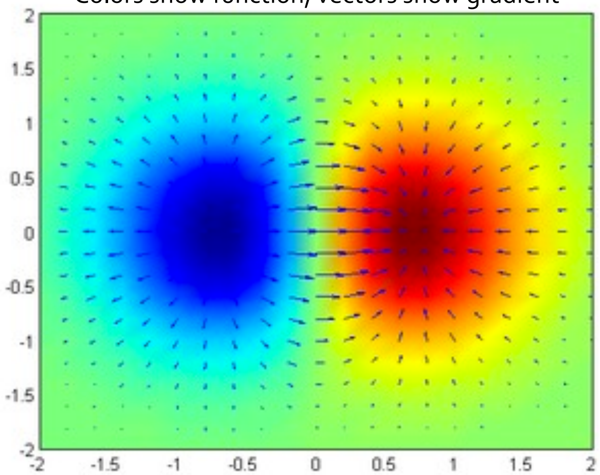
Vector Calculus: Position & Source Vectors



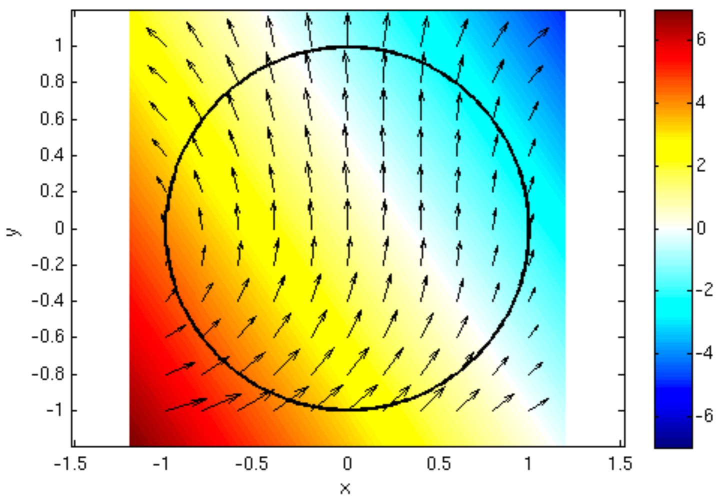
Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Vector Calculus: Derivatives

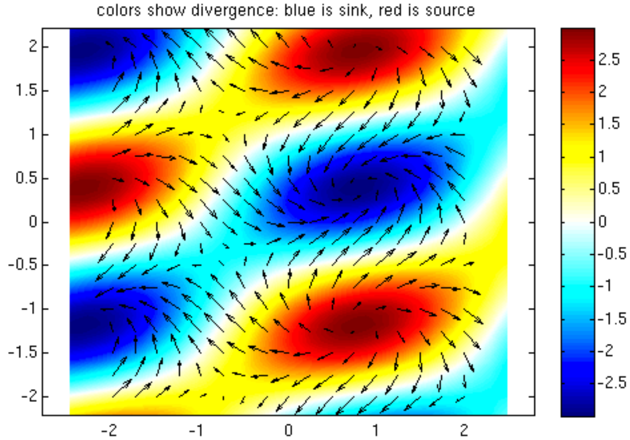
Colors show function, vectors show gradient



colors show curl: blue is clockwise, red is counterclockwise

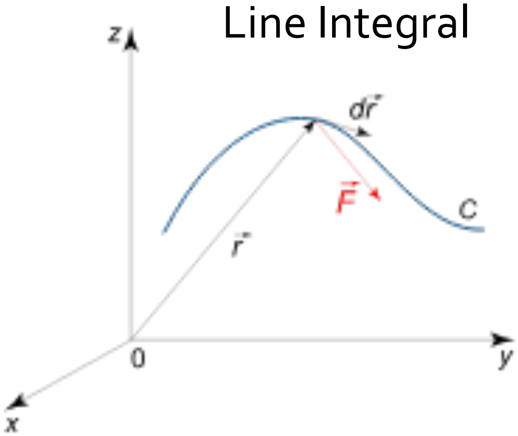


colors show divergence: blue is sink, red is source

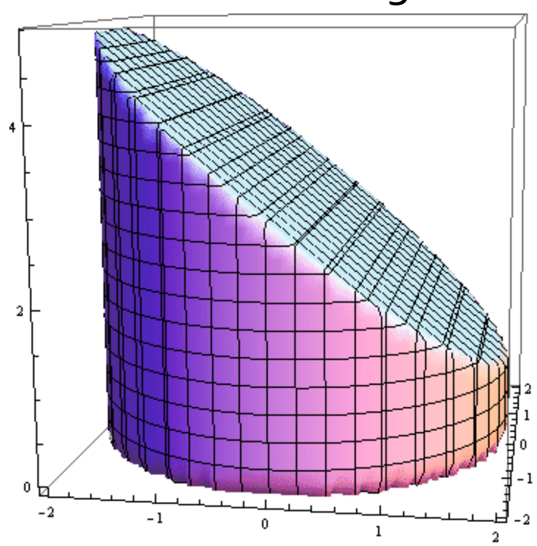


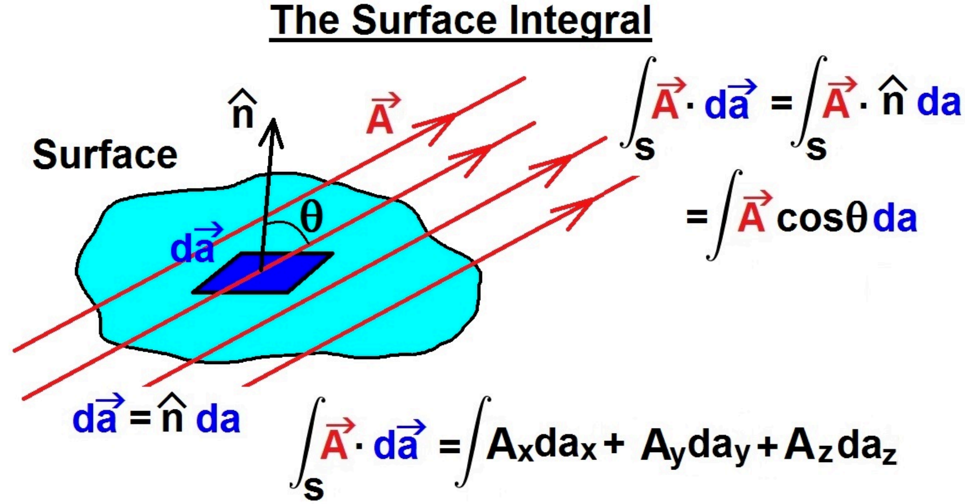
Vector Calculus: Integrals

Line Integral



Volume Integral



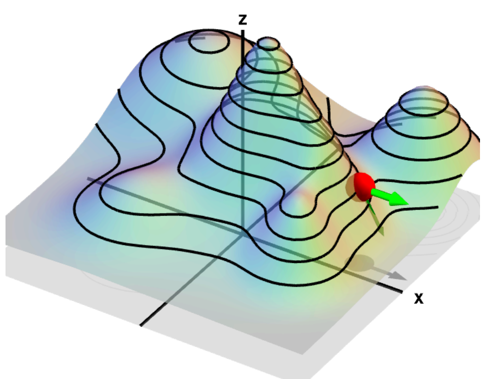


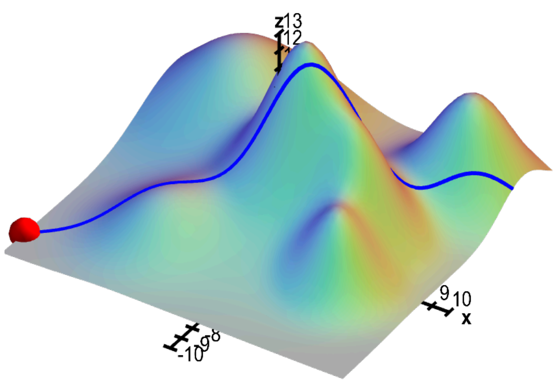
Fundamental Theorem(s)

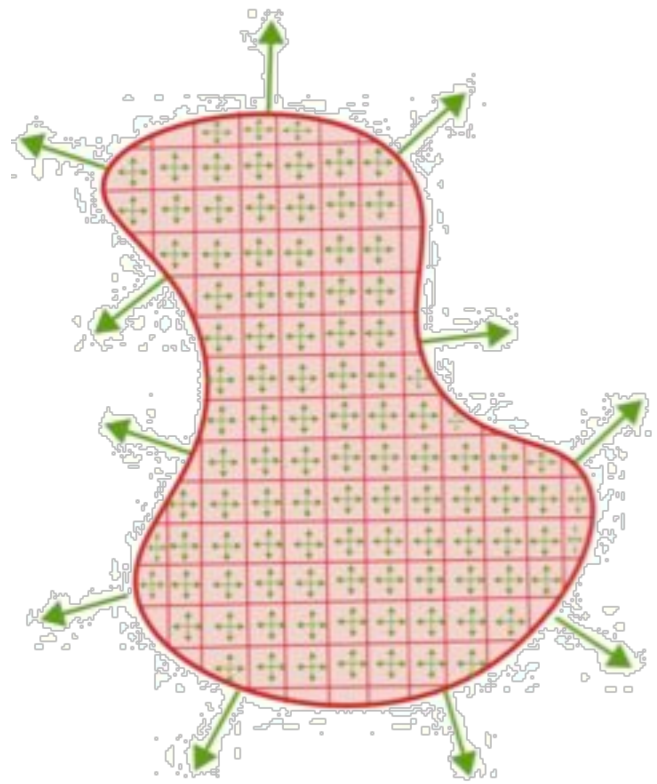
$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a})$

$\int \nabla \cdot \vec{A} \, d\tau = \oint \vec{A} \cdot \overrightarrow{da}$

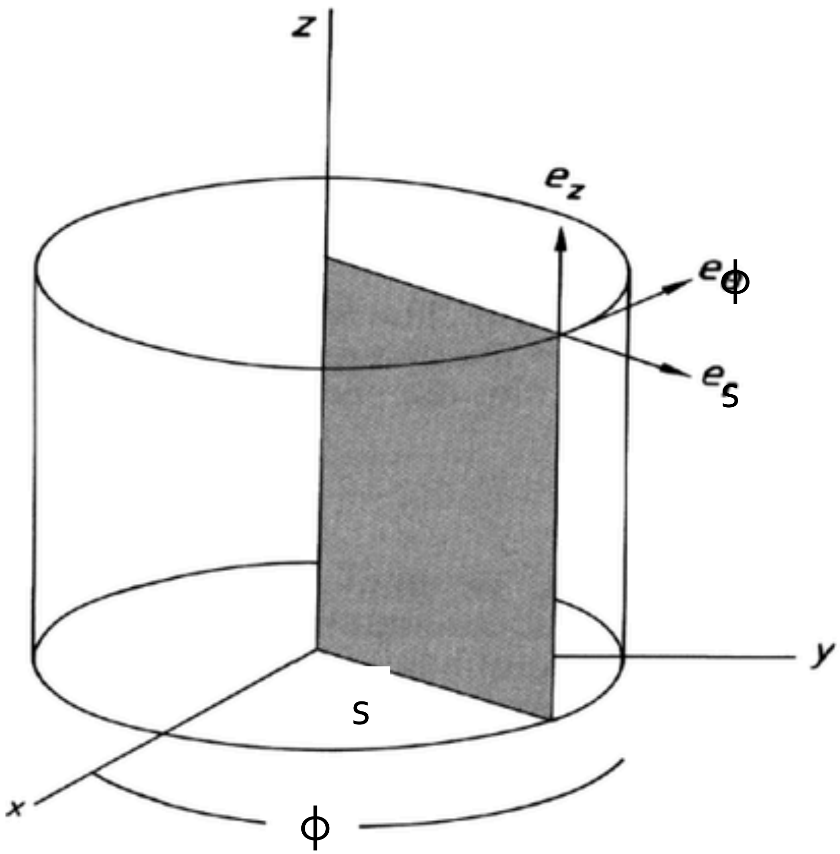
$\int (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$



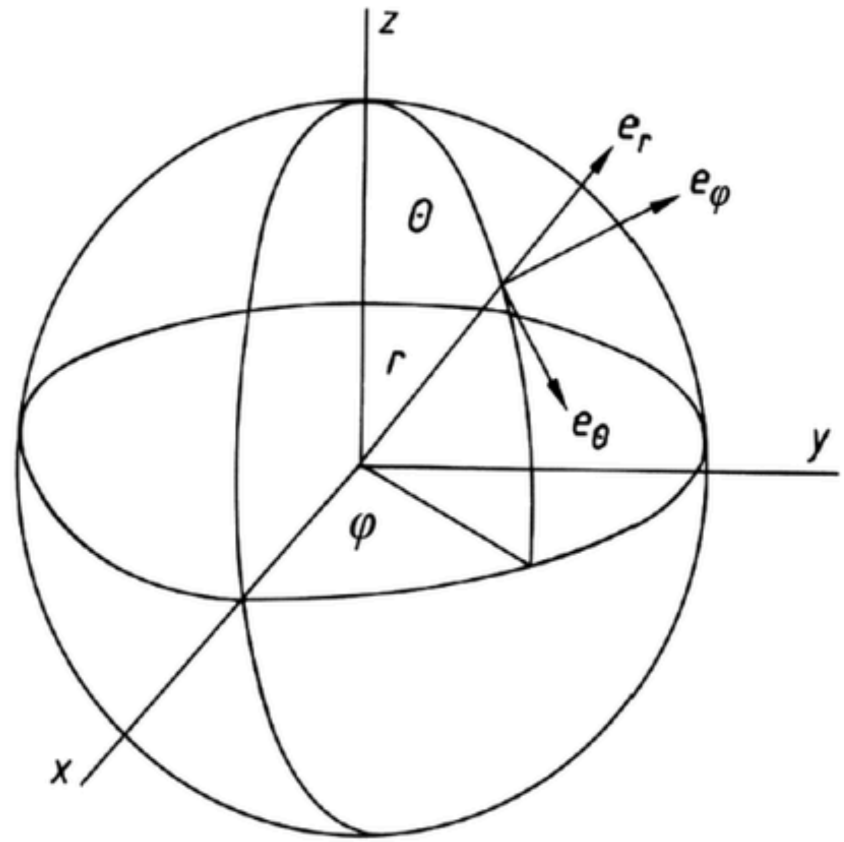




Coordinate Systems



(a)



(b)

Vector Calculus in Different Coordinate Systems

Cartesian Coordinates: $\vec{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ $\quad d\tau = dx\, dy\, dz$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

Spherical Coordinates: $x = r\, sin\theta\, cos\phi,\ y = r\, sin\theta\, sin\phi,\ z = r\, cos\theta$

$$\vec{dl} = dr\hat{r} + r\, d\theta\hat{\theta} + r\, sin\theta\, d\phi\hat{\phi} \qquad d\tau = r^2 sin\theta\, dr\, d\theta\, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\, sin\theta}\frac{\partial (sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\, sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r\, sin\theta}\left(\frac{\partial (sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r\, A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\phi}$$

Cylindrical Coordinates: $x = s\, cos\phi,\ y = s\, sin\phi,\ z = z$

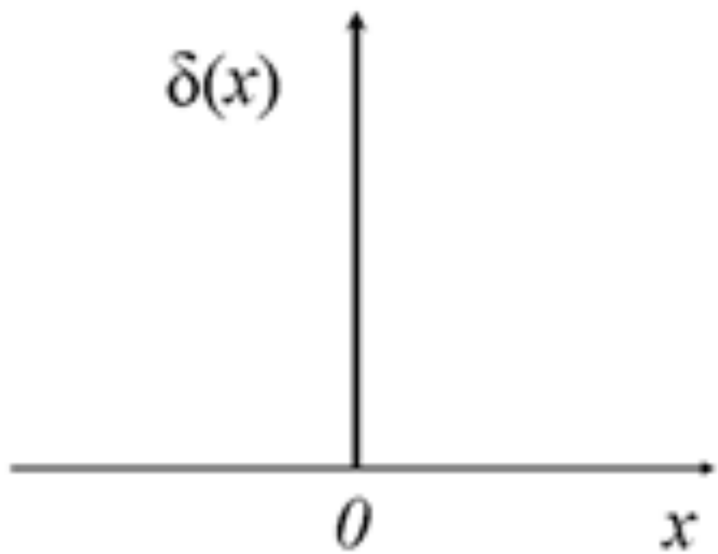
$$\vec{dl} = ds\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z} \qquad d\tau = s\, ds\, d\phi\, dz$$

$$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial (s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$$

Dirac Delta Function

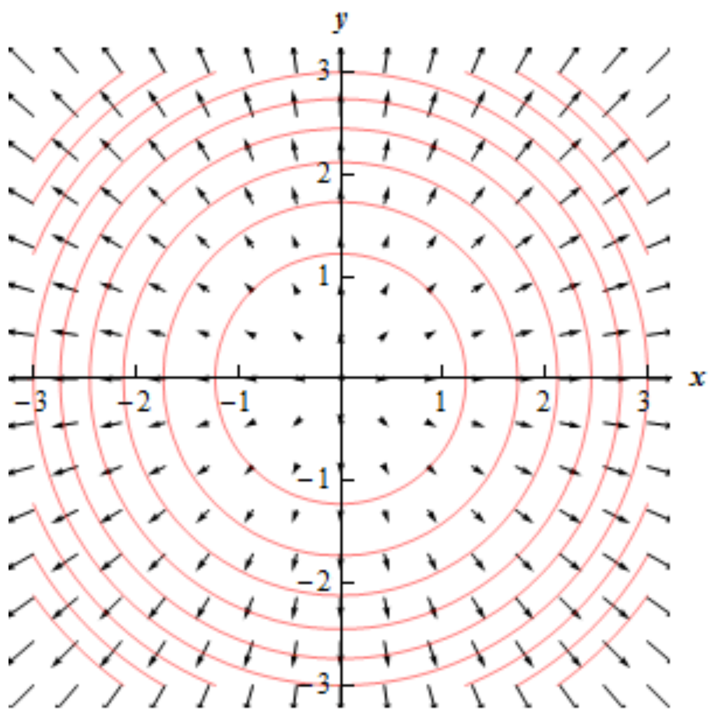
Dirac Delta Function: $\int \delta^3(\vec{r} - \vec{a})\, d\tau = 1$ if $\vec{a}$ contained in volume, $\delta^3(\overrightarrow{\Delta r}) = \frac{1}{4\pi} \nabla \cdot \left(\frac{\widehat{\Delta r}}{\Delta r^2}\right)$

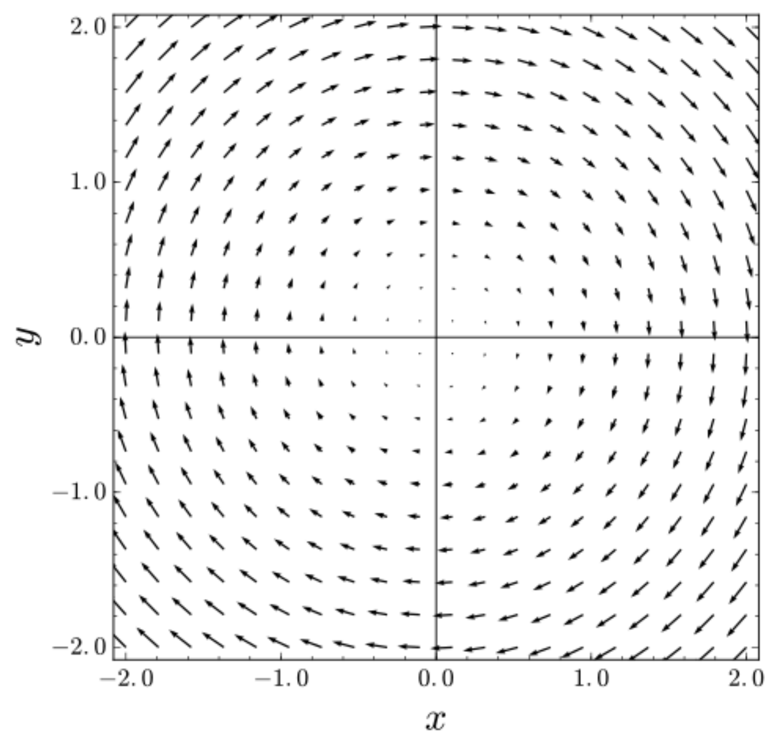


Special Vector Functions

$$\nabla \times \vec{A} = 0 \qquad \vec{A} = \nabla f \qquad \oint \vec{A} \cdot \overrightarrow{dl} = 0$$

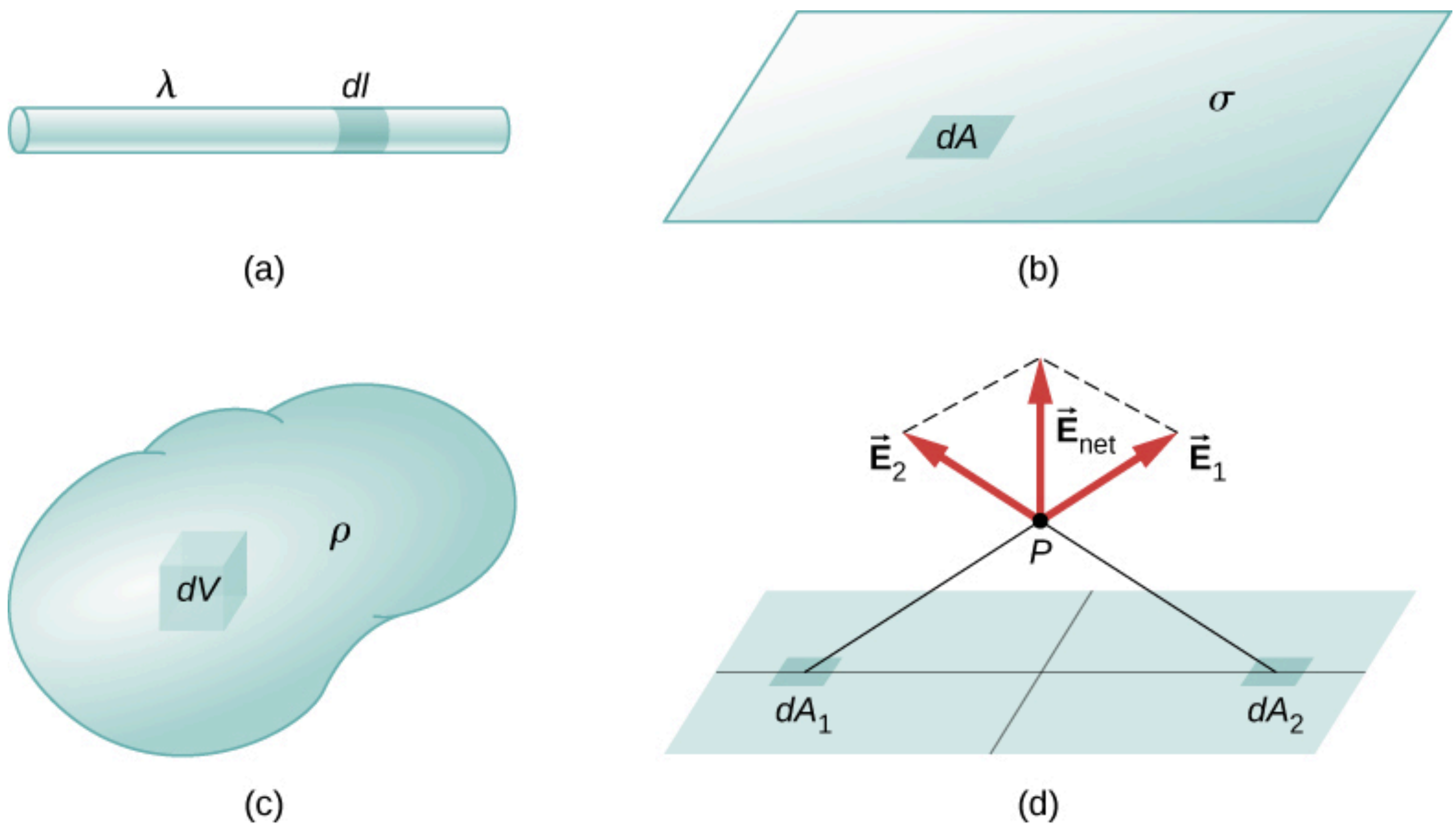
$$\nabla \cdot \vec{F} = 0 \qquad \vec{F} = \nabla \times \vec{A} \qquad \oint \vec{F} \cdot \overrightarrow{da} = 0$$





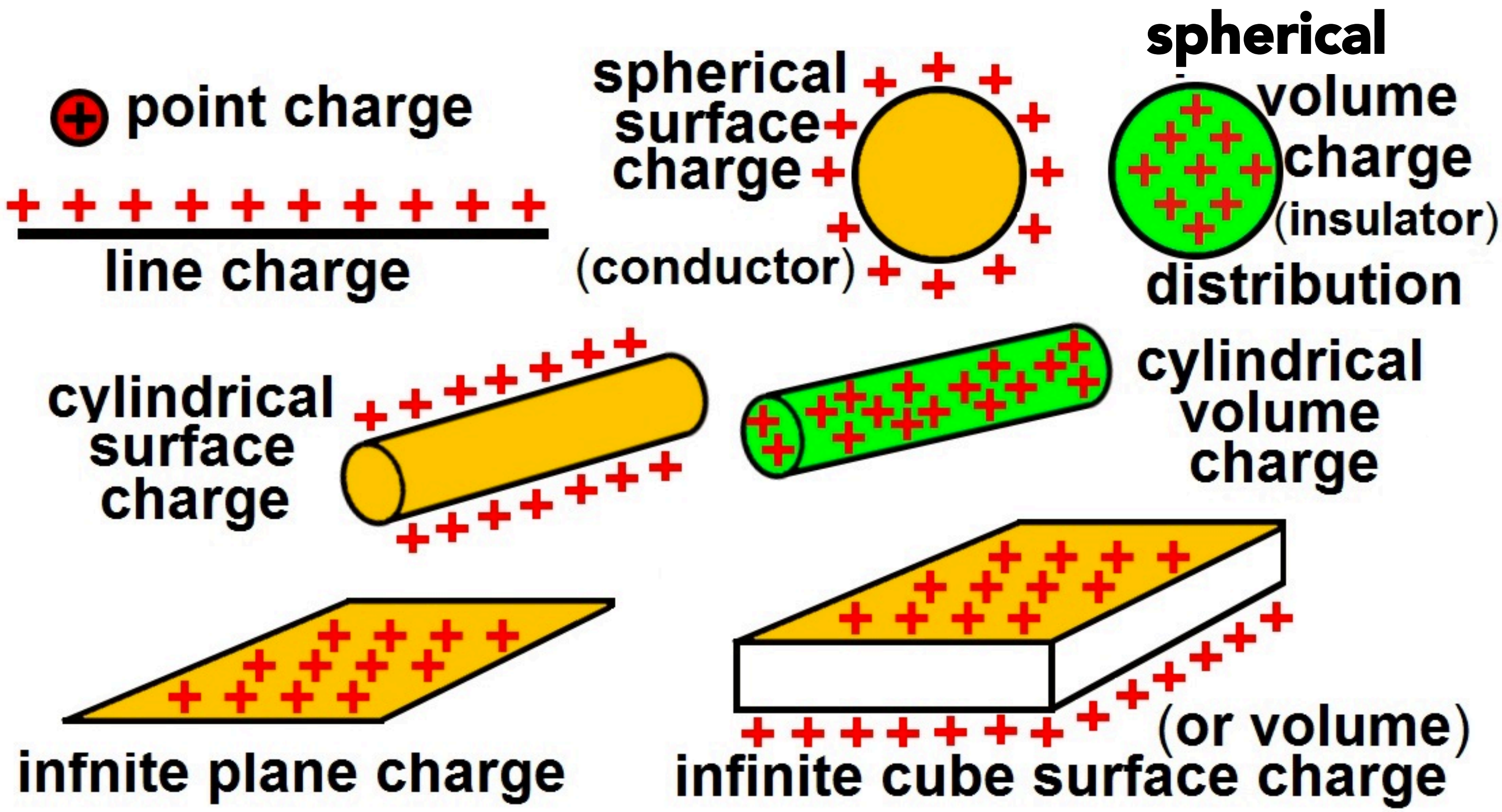
Electric Field

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\, d\tau', \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\, da', \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r\prime})}{\Delta r^2}\widehat{\Delta r}\, dl'$$



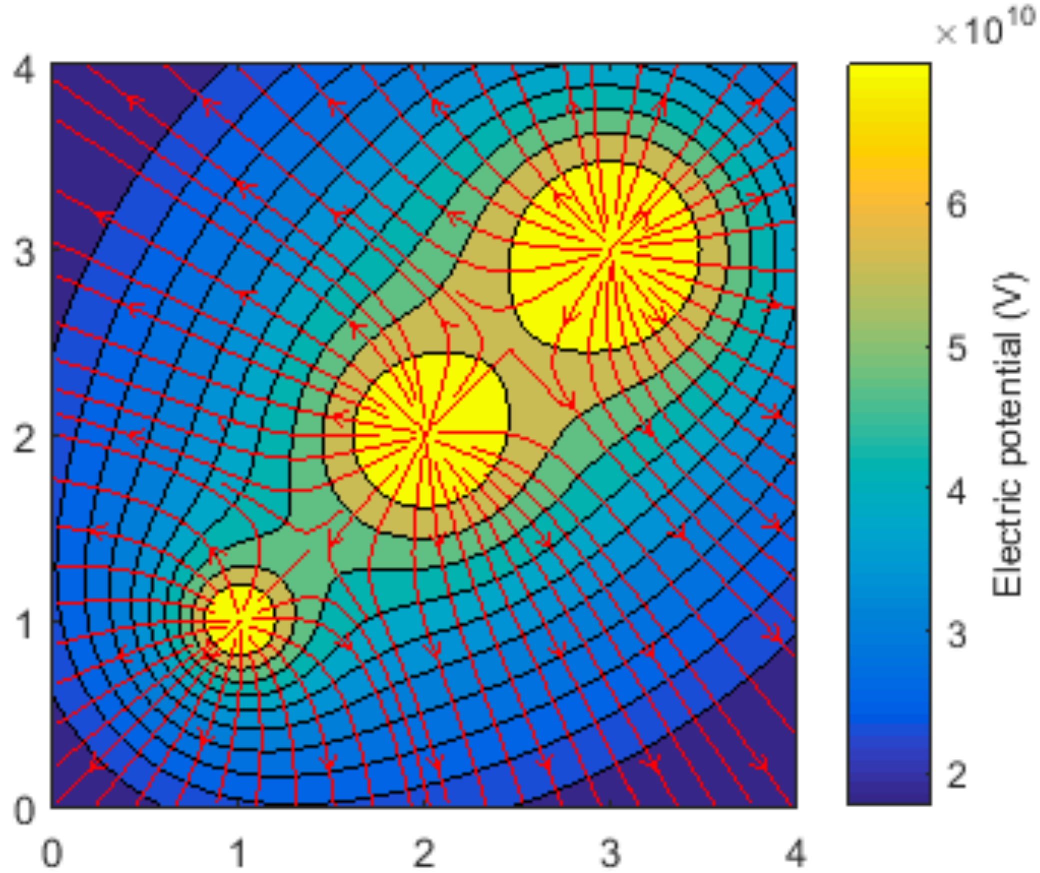
Gauss's Law

$$\oint \vec{E} \cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0, \quad \nabla \cdot \vec{E} = \rho/\varepsilon_0$$



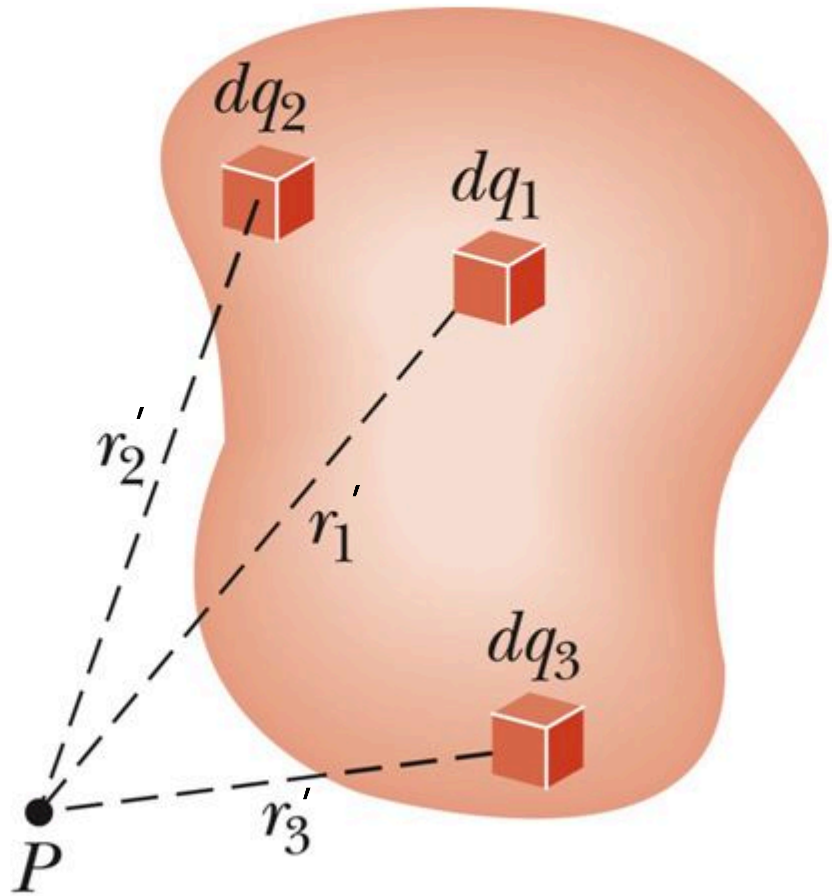
Electric Potential

$$\vec{E} = -\nabla V, \quad V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot \overrightarrow{dl}, \quad \nabla^2 V = -\rho/\varepsilon_0$$

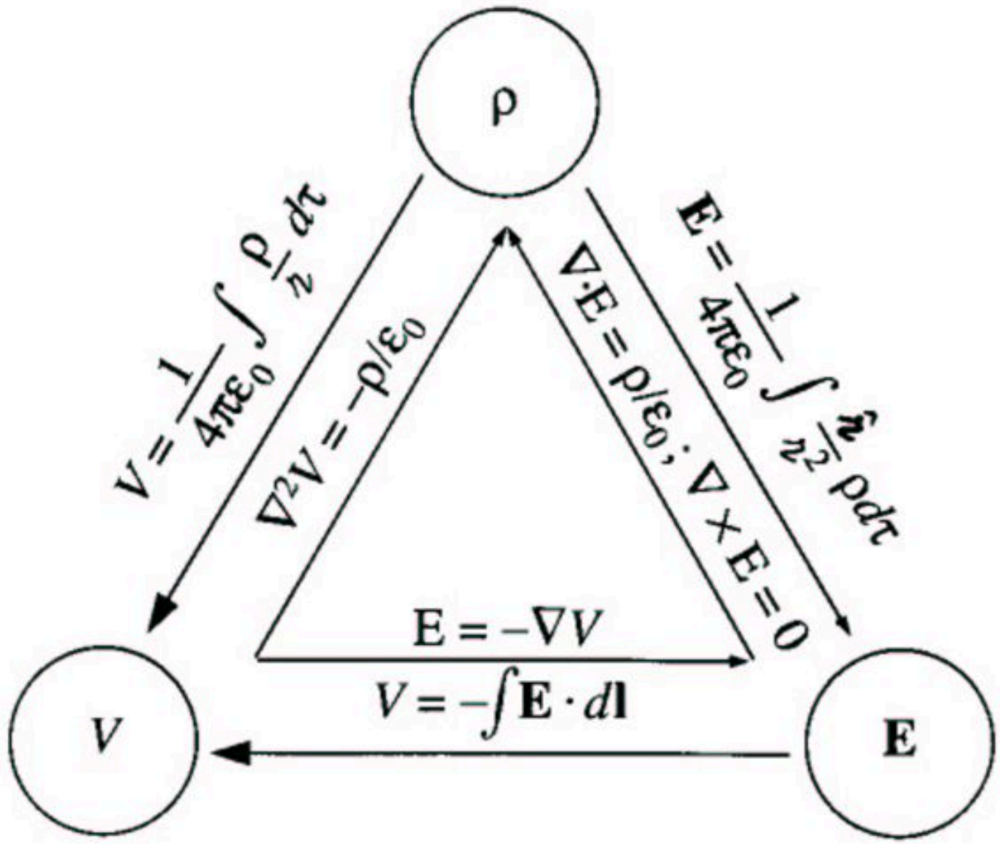


Electric Potential

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\overrightarrow{r\prime})}{\Delta r}d\tau', \; V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(\overrightarrow{r\prime})}{\Delta r}da', \; V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(\overrightarrow{r\prime})}{\Delta r}dl'$$



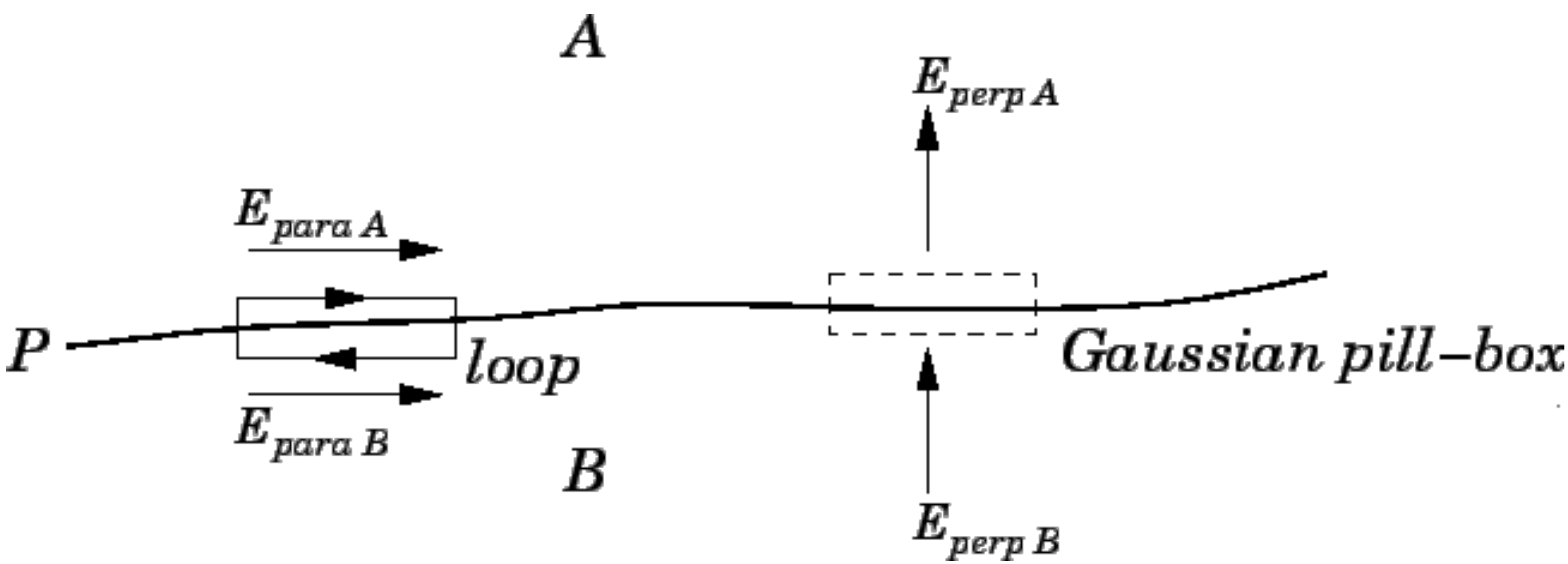
Charge <-> Potential <-> Field



Note:
$\mathscr{r} = \Delta r$

Boundary Conditions

$$\Delta\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n},\ \Delta V = 0$$



Electrostatic Energy

$$W = Q\Delta V, \quad W = \frac{\varepsilon_0}{2}\int E^2 d\tau$$