\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 J \]

**Electricity and Magnetism I: 3811**

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture
Announcements

- I will be absent next Monday & Wednesday
  - Prof. Baalrud will substitute

- There is homework due next Friday
Midterm #1

- Max = 100
- Min = 12
- Mean = 66
- Std. Dev. = 18
- Median = 68
- Mode = 51, 68, 77 (all tied w/ 3)
- Line integrals
- Spherical coordinates
- Gradient theorem
Integrating over a delta function picks out the value of the multiplying function at the location of the delta function.
Gauss’s law: Integral form

Cartesian coordinates

Potential & work
- Gauss’s law: Differential form
- Cylindrical coordinates
- Boundary conditions
General Eqs. for $E, V$

\[
E(\mathbf{r}) = \frac{1}{4\pi \epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} \, d^3r', \quad \text{cumbersome!}
\]

\[
V(\mathbf{r}) = \frac{1}{4\pi \epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, d^3r'
\]

**Gauss's Law:** $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$

⇒ **Poisson's Eq.:** $\nabla^2 V = -\frac{\rho}{\epsilon_0}$

**Charge-Free Region**

⇒ **Laplace's Eq.:** $\nabla^2 V = 0$

**Cartesian:**

\[
\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0
\]
Laplace's Eq. in 1-D

$$d^2V/dx^2 = 0 \Rightarrow V(x) = mx + b$$

![Graph showing $V$ as a linear function of $x$]

### Interesting Fact:

$$V(x) = \frac{1}{2} [V(x-a) + V(x+a)]$$

![Graph illustrating the function $V(x)$ and its midpoint]

$$V(x) = \frac{1}{2} [V(x-a) + V(x+a)]$$

$$= \frac{1}{2} [m(x-a) + b + m(x+a) + b]$$

$$= mx + b$$

**Zero curvature ⇒ No local extrema (except at endpoints)**
Laplace's Eq. In 2-d
\[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \]

\[ V(x, y) = \frac{1}{2\pi R} \oint V \, dl \text{ around a circle centered at } (x, y) \]

\(-V\) has no local extrema except on boundaries
Laplace’s Equation