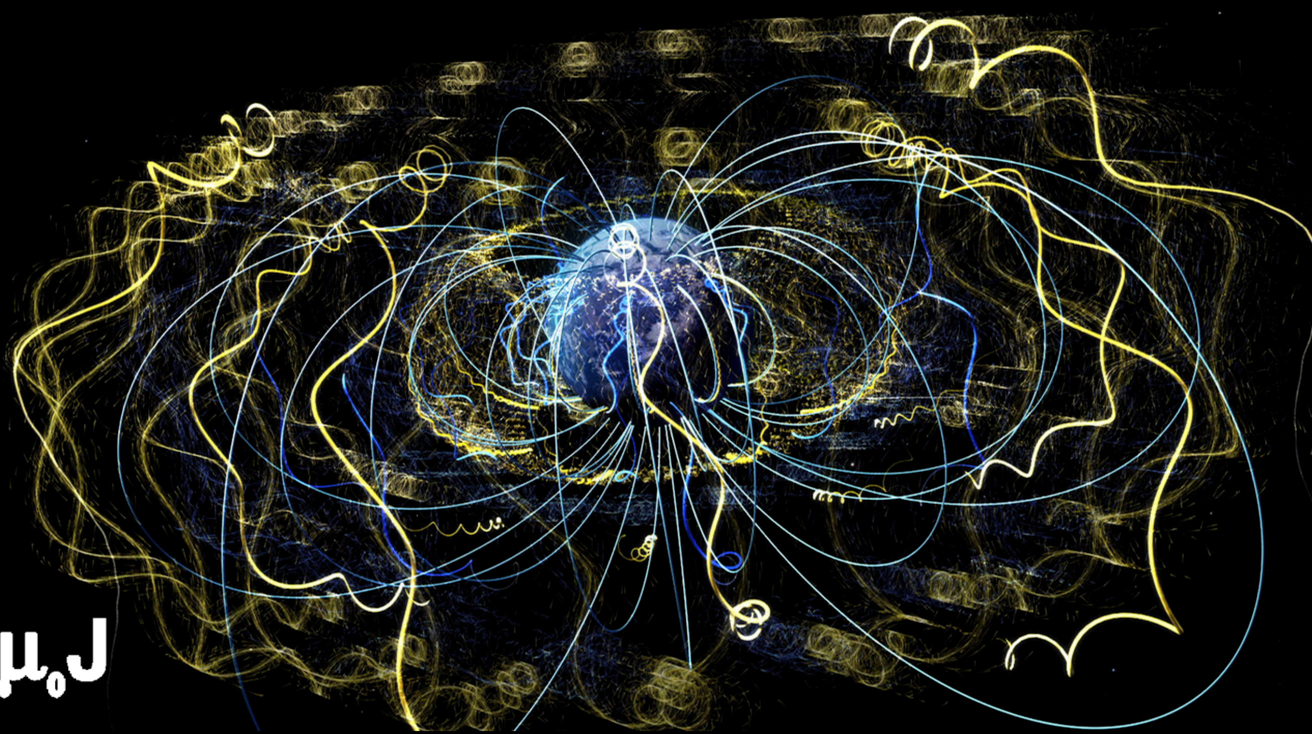
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

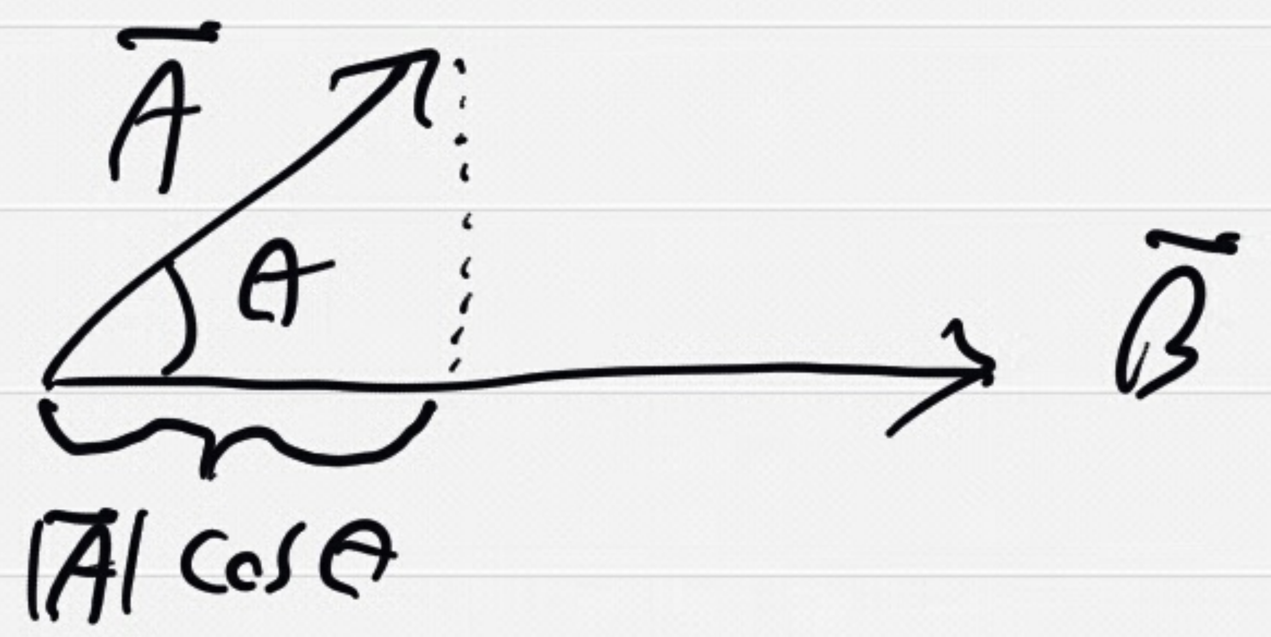
Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Vector Multiplication

I. Dot Product (Scalar Product)

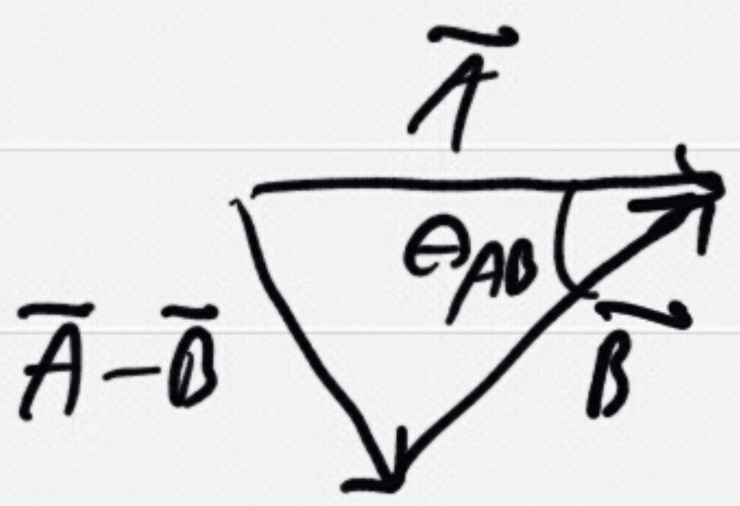
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= |\vec{A}|\,|\vec{B}| \cos\theta$$

Projection



$$\vec{A} \cdot \vec{A} = |\vec{A}|^2 = A^2$$
$$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1 \;, \quad \hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{x} \cdot \hat{z} = 0$$

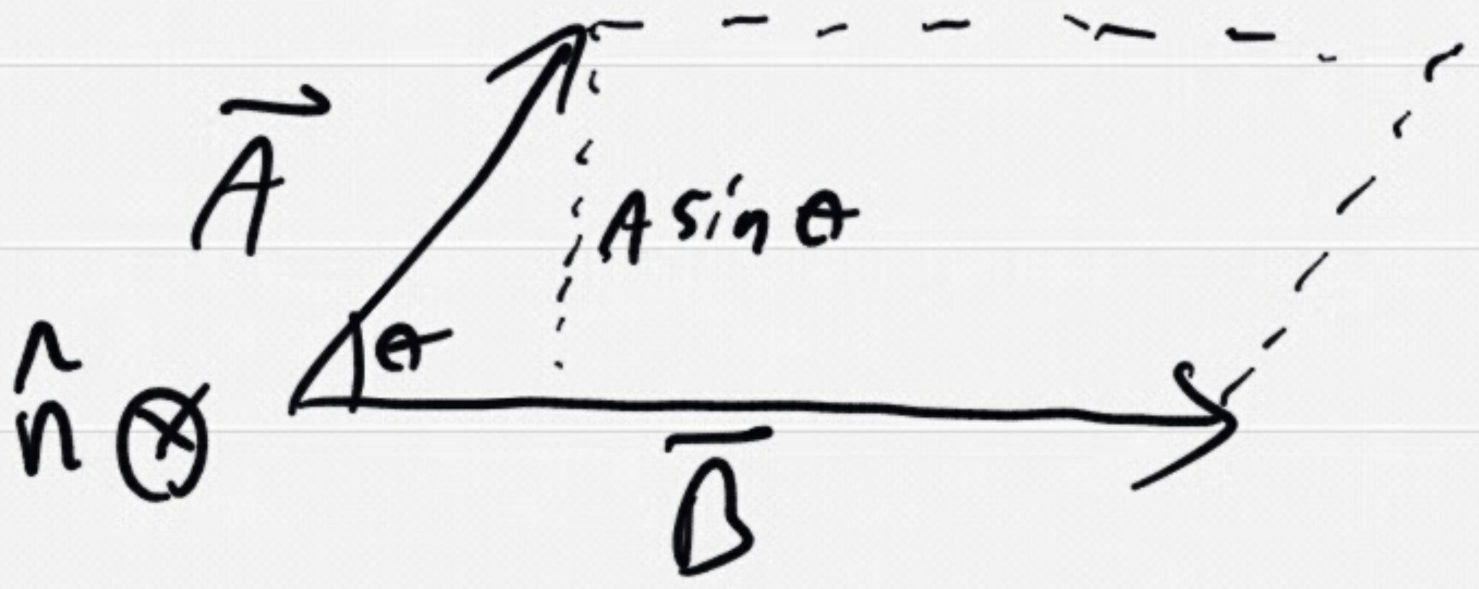
$$(\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) = A^2 + B^2 - 2AB\cos\theta$$



" law of cosines

II. Cross Product (vector product)

$$\vec{A} \times \vec{B} = AB \sin\theta_{AB}\, \hat{n}$$
w/ $\hat{n}$ from right-hand-rule



$AB \sin\theta$ = area of parallelogram

$$\vec{A} \times \vec{A} = 0$$

$$\hat{x} \times \hat{y} = \hat{z} \qquad \hat{y} \times \hat{x} = -\hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x} \qquad \hat{z} \times \hat{y} = -\hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y} \qquad \hat{x} \times \hat{z} = -\hat{y}$$

$$\Rightarrow \quad \vec{A} \times \vec{B} =$$

$$[A_x, A_y, A_z] \times [B_x, B_y, B_z]$$

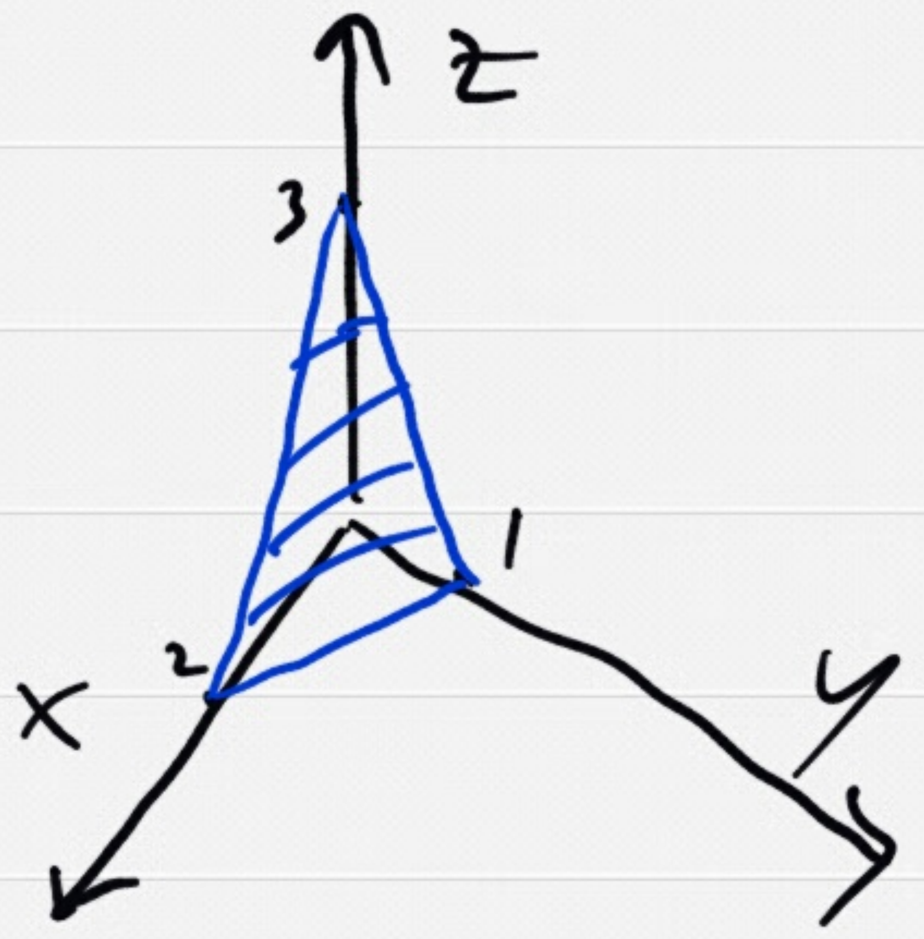
$$\begin{aligned} = \; & A_x B_x \cdot 0 + A_x B_y \hat{z} - A_x B_z \hat{y} \\ & - A_y B_x \cdot \hat{z} + A_y B_y \cdot 0 + A_y B_z \hat{x} \\ & + A_z B_x \hat{y} - A_z B_y \hat{x} + A_z B_z \cdot 0 \end{aligned}$$

$$\begin{aligned} = \; & (A_y B_z - A_z B_y) \hat{x} \\ & + (A_z B_x - A_x B_z) \hat{y} \\ & + (A_x B_y - A_y B_x) \hat{z} \end{aligned}$$

Trick: $\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

Sample Problem

Find normal to plane



Pick $\vec{A}$ from
$[2,0,0]$ to $[0,1,0]$

$\Rightarrow \vec{A} = [-2,1,0]$

Pick $\vec{B}$ from
$[2,0,0]$ to $[0,0,3]$

$\Rightarrow \vec{B} = [-2,0,3]$

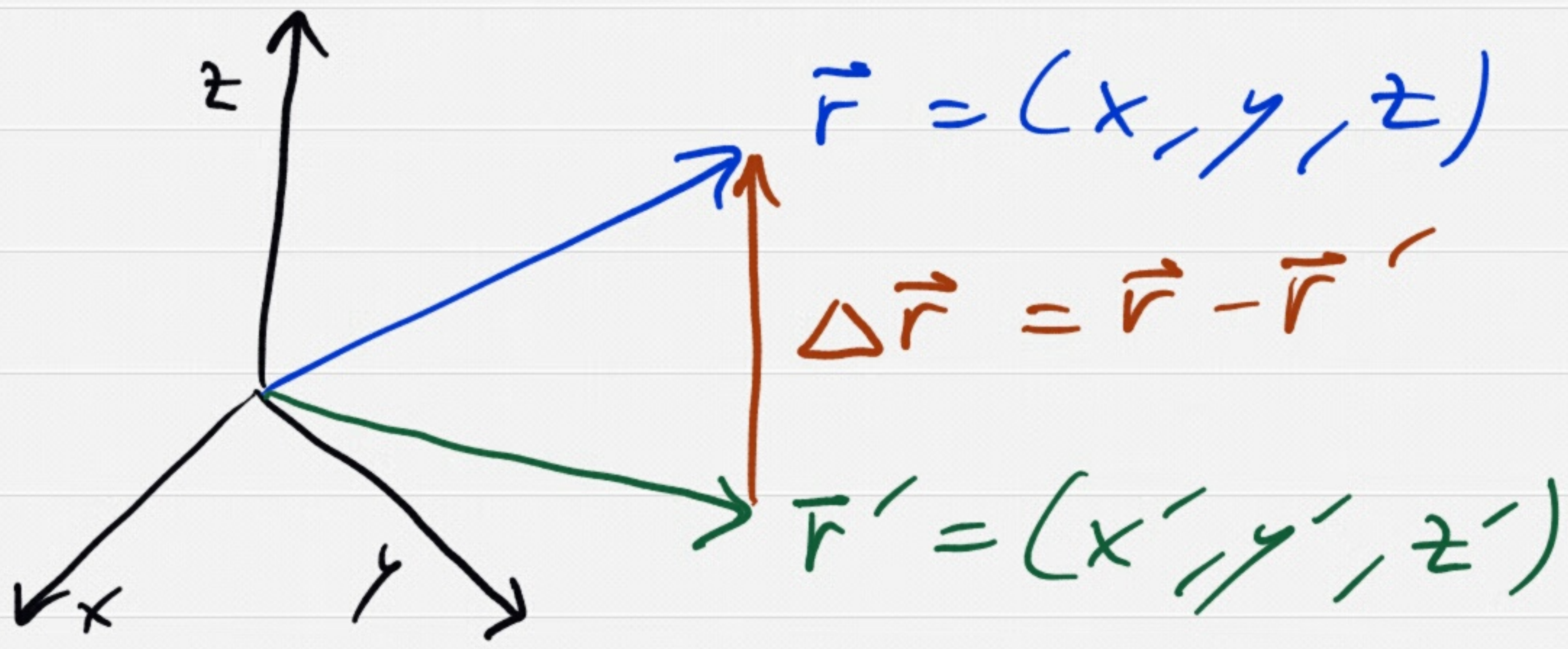
$$\vec{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -2 & 1 & 0 \\ -2 & 0 & 3 \end{vmatrix}$$

$$= [3, 6, 2]$$

$$\hat{n} = \vec{n}/|\vec{n}| = [3,6,2]/\sqrt{9+36+4}$$

$$= [3/7, 6/7, 2/7]$$

Position Vectors



$\vec{r}$ = position vector

$\vec{r}\,'$ = source vector

$\Delta\vec{r}$ = separation vector

Note: Book uses $\vec{\mathscr{r}}$ for $\Delta\vec{r}$

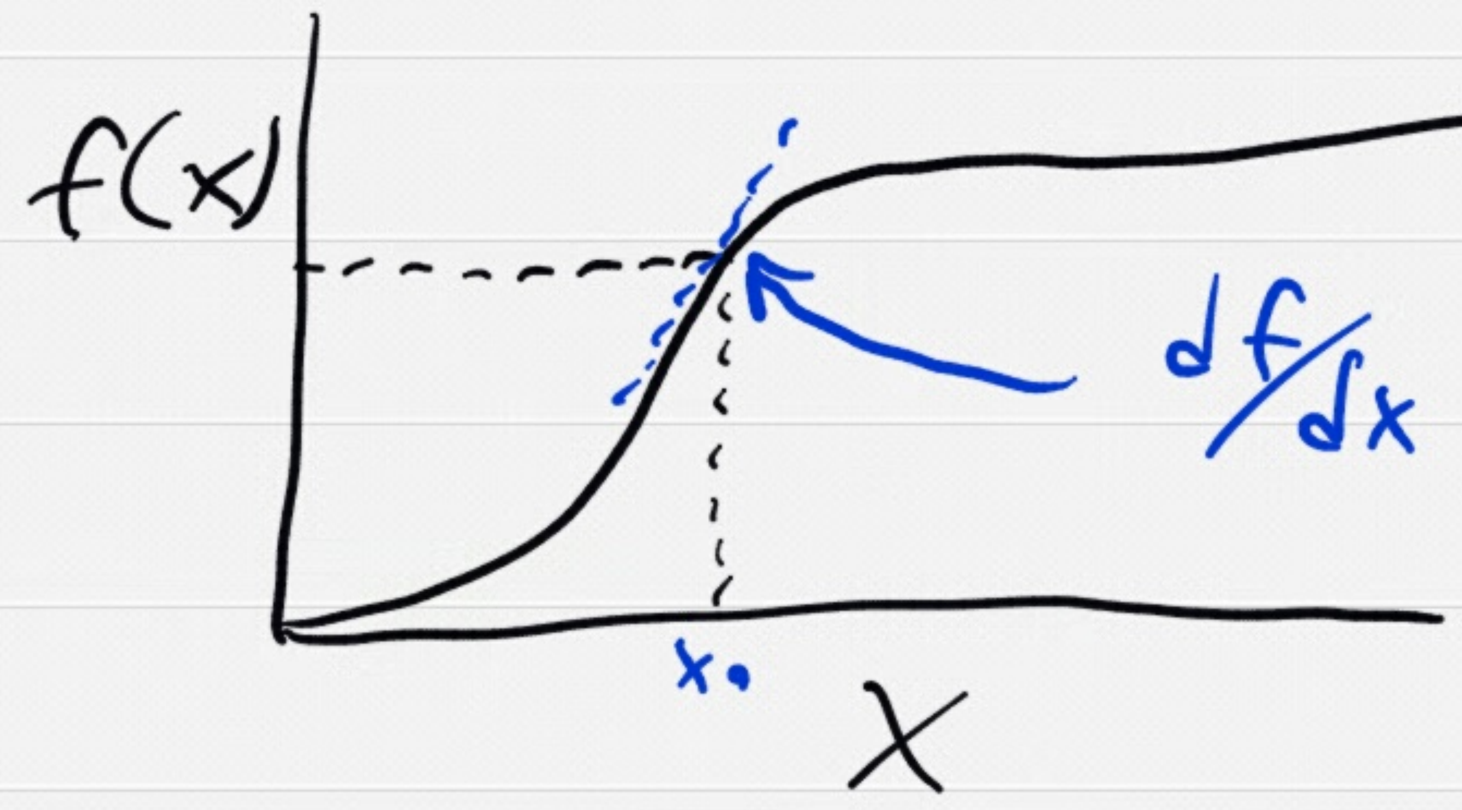
Example:

Electric field at $\vec{r}$ from charge q at $\vec{r}\,'$

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \frac{\hat{\Delta r}}{|\Delta r|^2}$$

- Field radially out from $\vec{r}\,'$
- Falls off w/ inverse square of distance from $\vec{r}\,'$

1-d derivative



$\left.\frac{df}{dx}\right|_{x_0}$ = slope of $f(x)$ @ $x = x_0$

$$df = \left(\frac{df}{dx}\right) dx$$

change in f = $\frac{df}{dx}$ × change in x

Gradient

$f(x,y,z)$ is function of position

$$df = \left(\frac{df}{dx}\right) dx + \left(\frac{df}{dy}\right) dy + \left(\frac{\partial f}{\partial z}\right) dz$$

$$= \left[\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}\right] \cdot [dx, dy, dz]$$

$$= \nabla f \cdot d\vec{l}$$

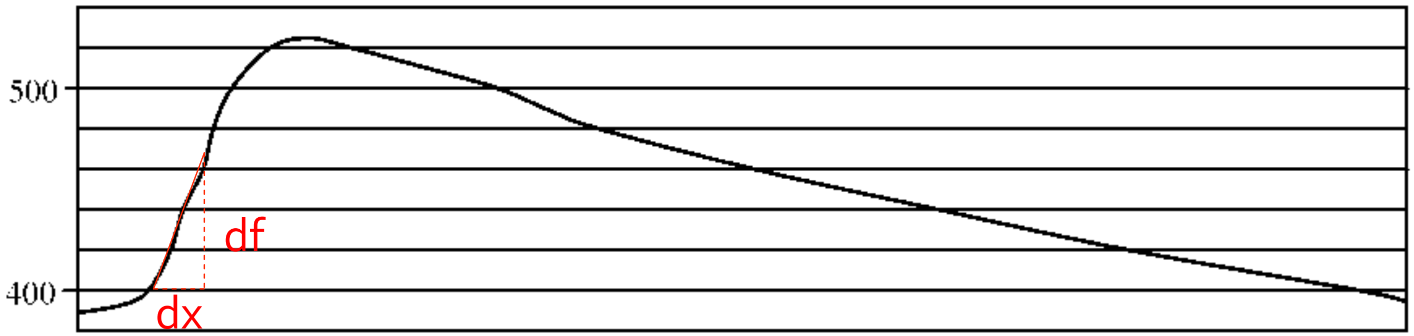
$df = 0$ along contour of const. f

$\Rightarrow \nabla f \perp$ constant f line

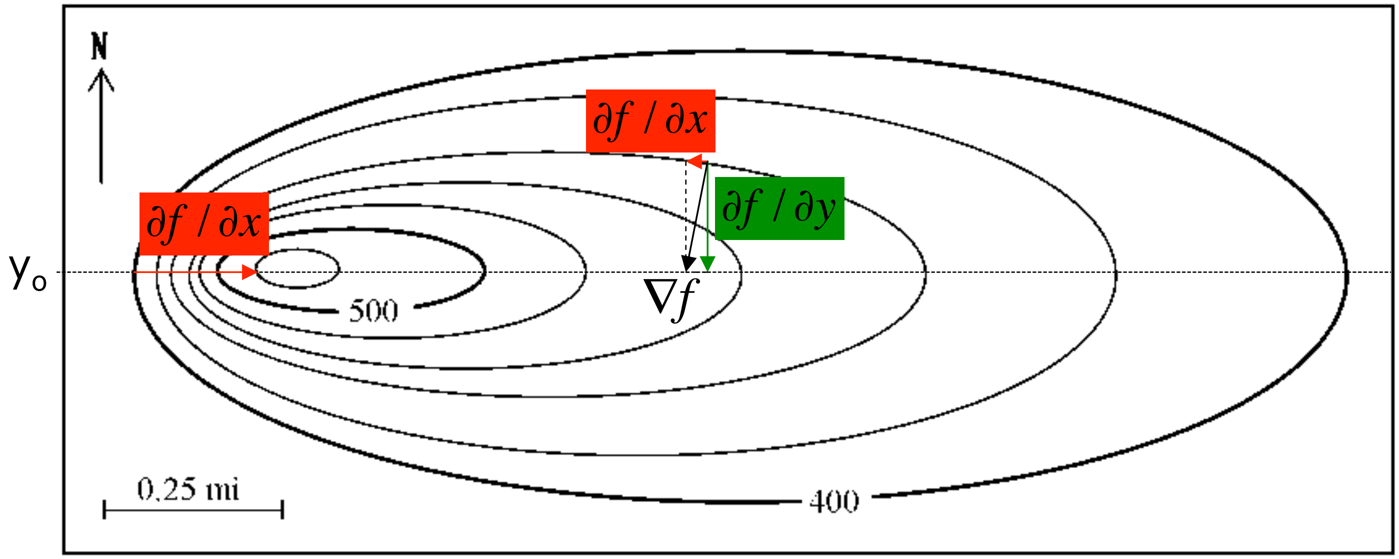
df max for $\nabla f \parallel d\vec{l}$

$\Rightarrow \nabla f$ points in direction of steepest ascent

Gradients



$f(x,y=y_o)$



$f(x,y)$