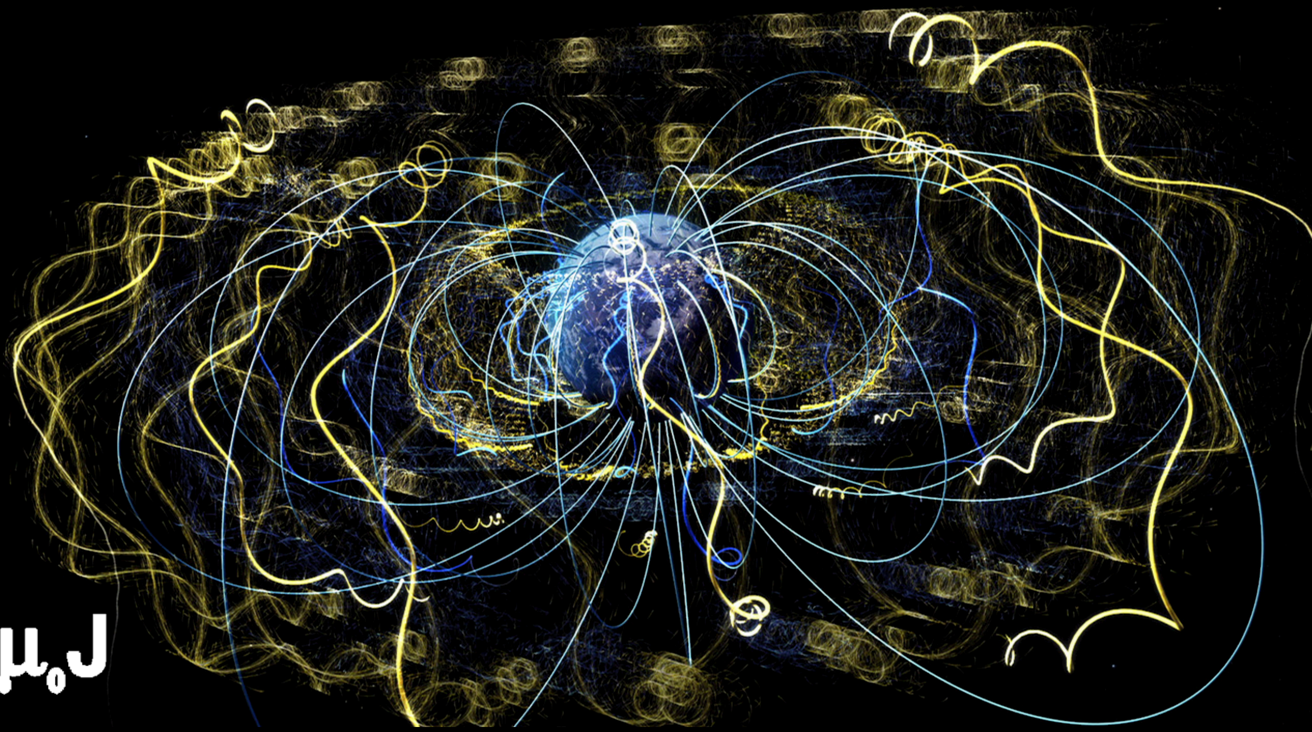
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

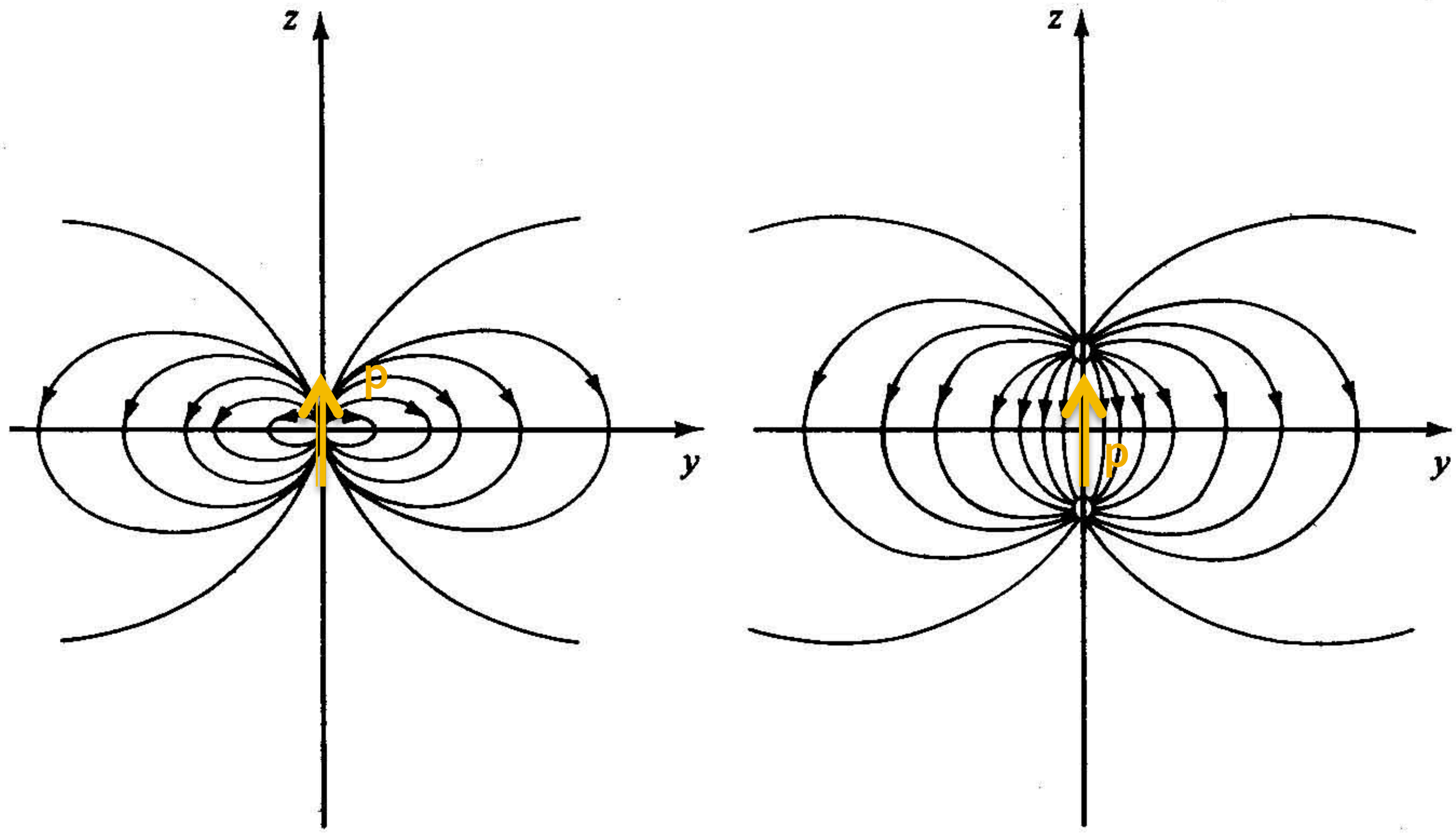
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Pure Vs. Physical Dipole

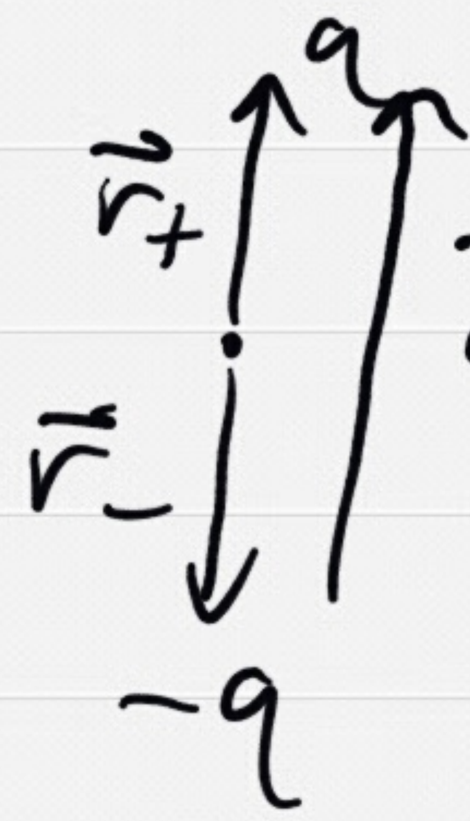


(a) Field of a "pure" dipole

(b) Field of a "physical" dipole

Figure 3.32

Physical Dipole



$$\vec{d} = \vec{r}_+ - \vec{r}_-$$

Monopole moment $Q = \int \rho(\vec{r}')\,d\tau'$
$= 0$

Dipole moment $\vec{p} = \int \vec{r}'\rho(\vec{r}')\,d\tau'$

$$\rho(\vec{r}') = q\,\delta^3(\vec{r}' - \vec{r}_+) - q\,\delta^3(\vec{r}' - \vec{r}_-)$$

$$\Rightarrow \vec{p} = q\int\left(\vec{r}'\delta^3(\vec{r}' - \vec{r}_+) - \vec{r}'\delta^3(\vec{r}' - \vec{r}_-)\right)d\tau'$$

$$= q(\vec{r}_+ - \vec{r}_-)$$

$$= q\vec{d}$$

$$V_{dip} = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2} = \frac{p\cos\alpha}{4\pi\varepsilon_0 r^2}$$

Note: - physical dipole also has non-zero quadrupole, octupole, etc.
- perfect dipole if $d \to 0$

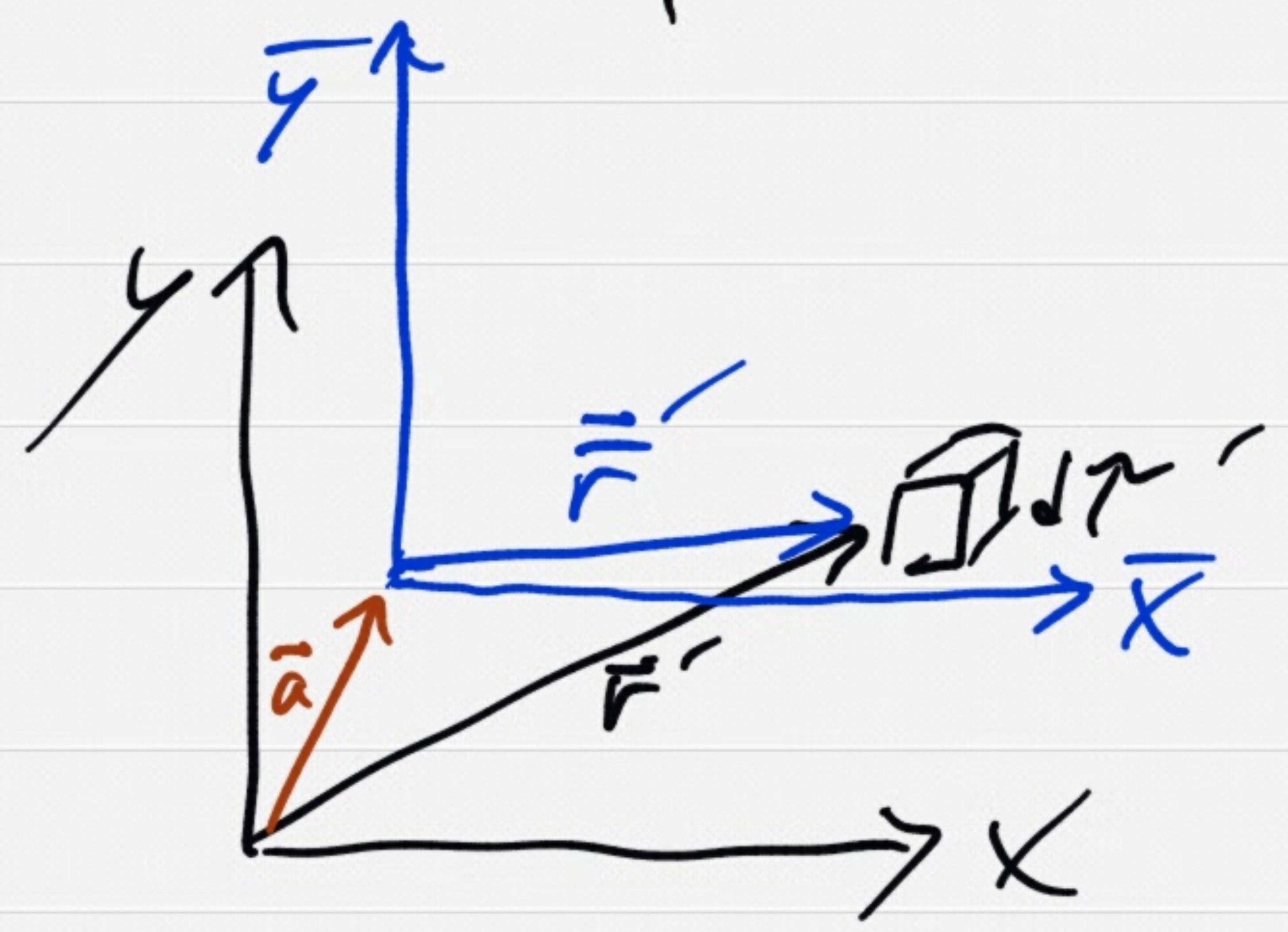
Moments & Origin of Coordinates

- Monopole moment

$$\int \rho(\vec{r}')\,d\tau' = Q \quad \text{ind. of origin}$$

- Dipole moment

$$\vec{p} = \int \vec{r}'\rho(\vec{r}')\,d\tau'$$



$$\bar{\vec{p}} = \int \bar{\vec{r}}'\rho(\bar{\vec{r}}')\,d\tau'$$

$$= \int (\vec{r}'-\vec{a})\,\rho(\vec{r}')\,d\tau'$$

$$= \int \vec{r}'\rho(\vec{r}')\,d\tau' - \int \vec{a}\,\rho(\vec{r}')\,d\tau'$$

$$= \vec{p} - \vec{a}Q$$

$\bar{\vec{p}} \neq \vec{p}$ unless $Q = 0$

Dipole Electric Field

$$V_{dip}(r,\theta) = \frac{\hat{r}\cdot\vec{p}}{4\pi\varepsilon_0 r^2}$$

$$= \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

for dipole @ origin
aligned w/ z-axis

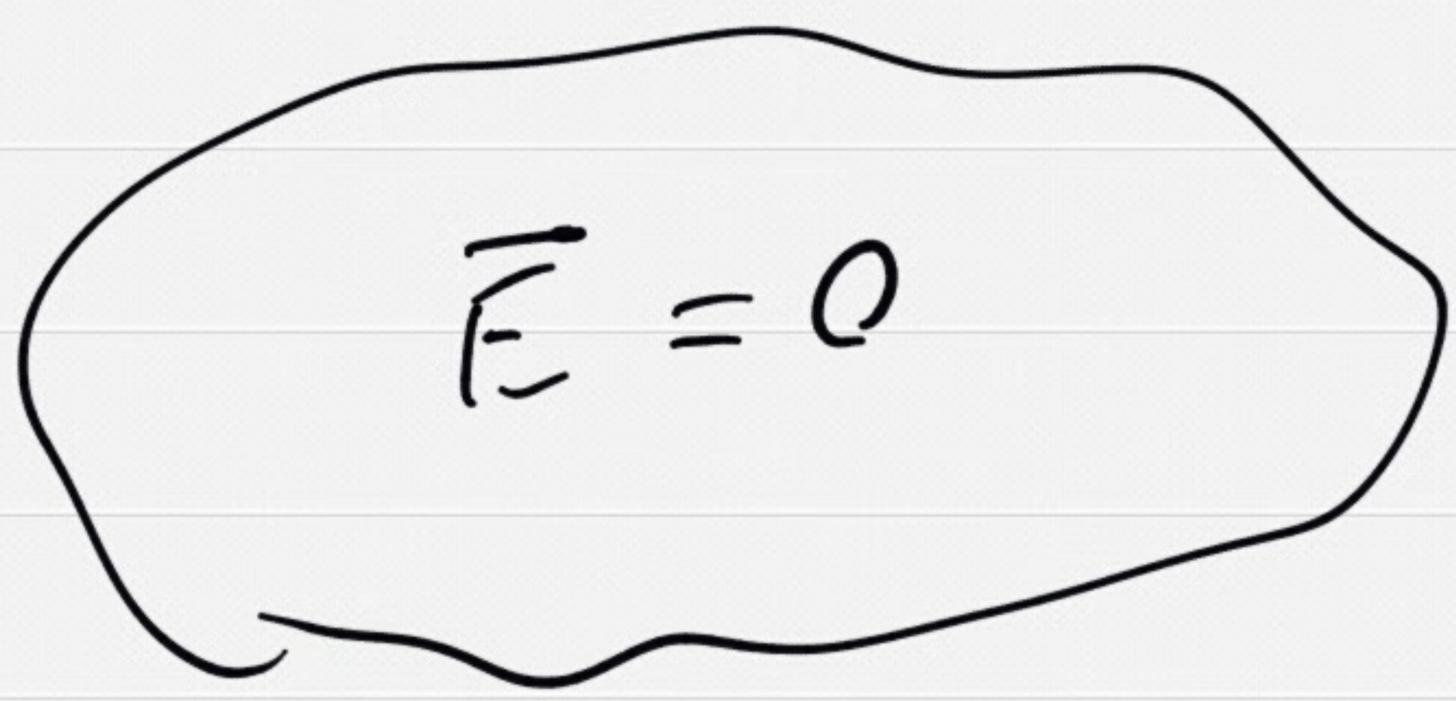
$$E_r = -\partial V/\partial r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$$

$$E_\theta = -\frac{1}{r}\,\partial V/\partial\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

$$E_\varphi = -\frac{1}{r\sin\theta}\,\partial V/\partial\varphi = 0$$

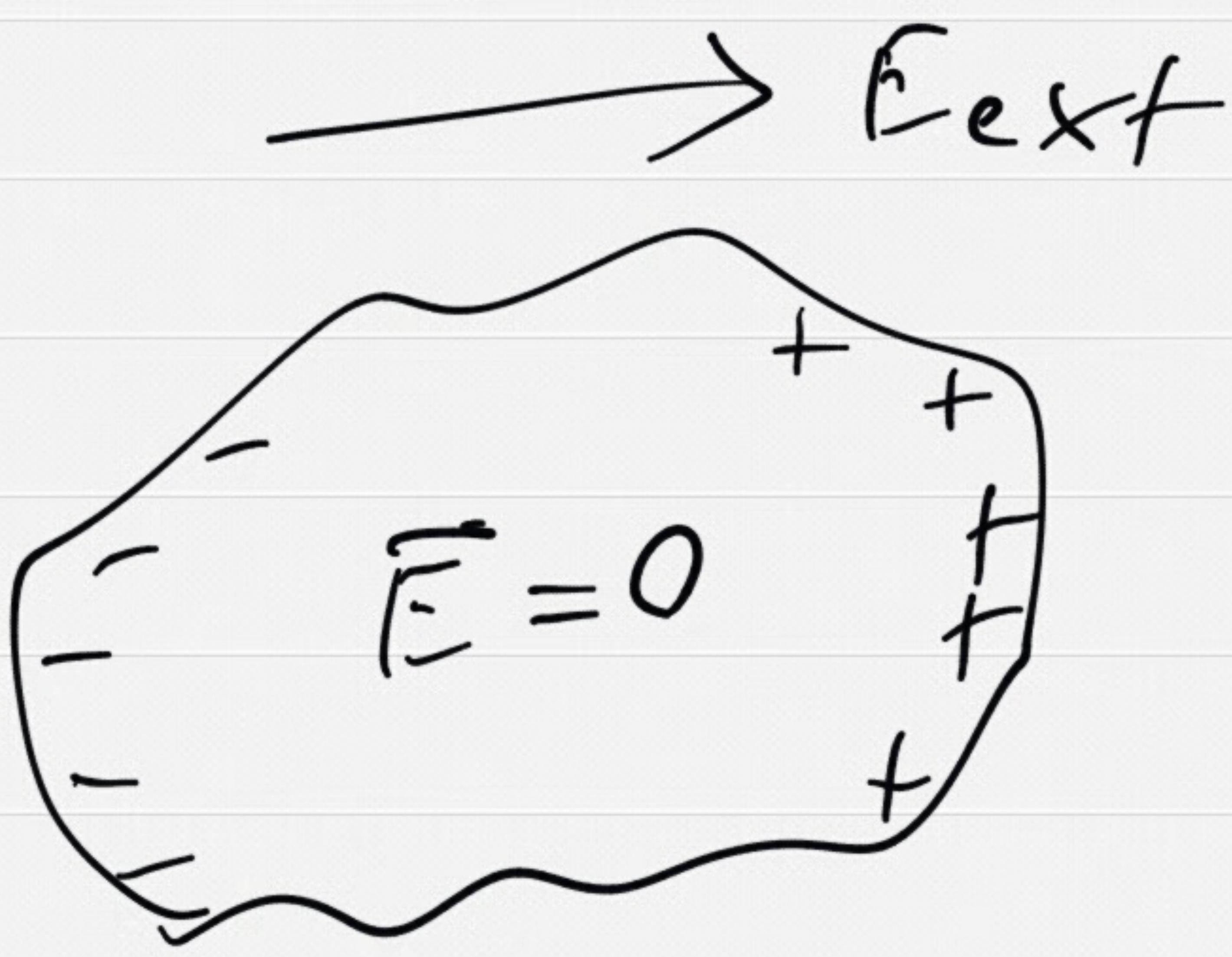
$$\vec{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

Conductor



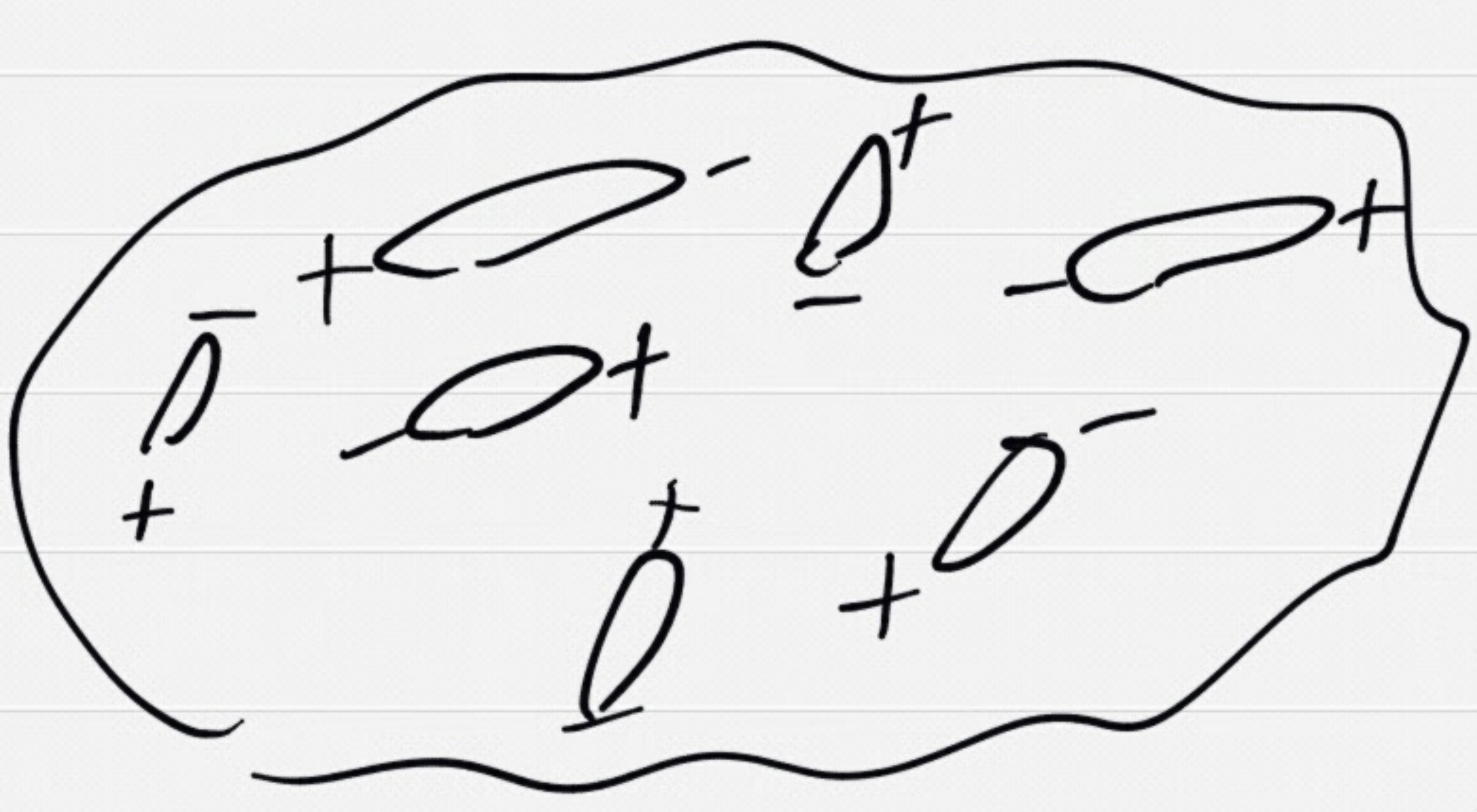
All charge on surface

Conductor in external $\vec{E}$



Charge cancels out $\vec{E}_{ext}$ in conductor

Insulator

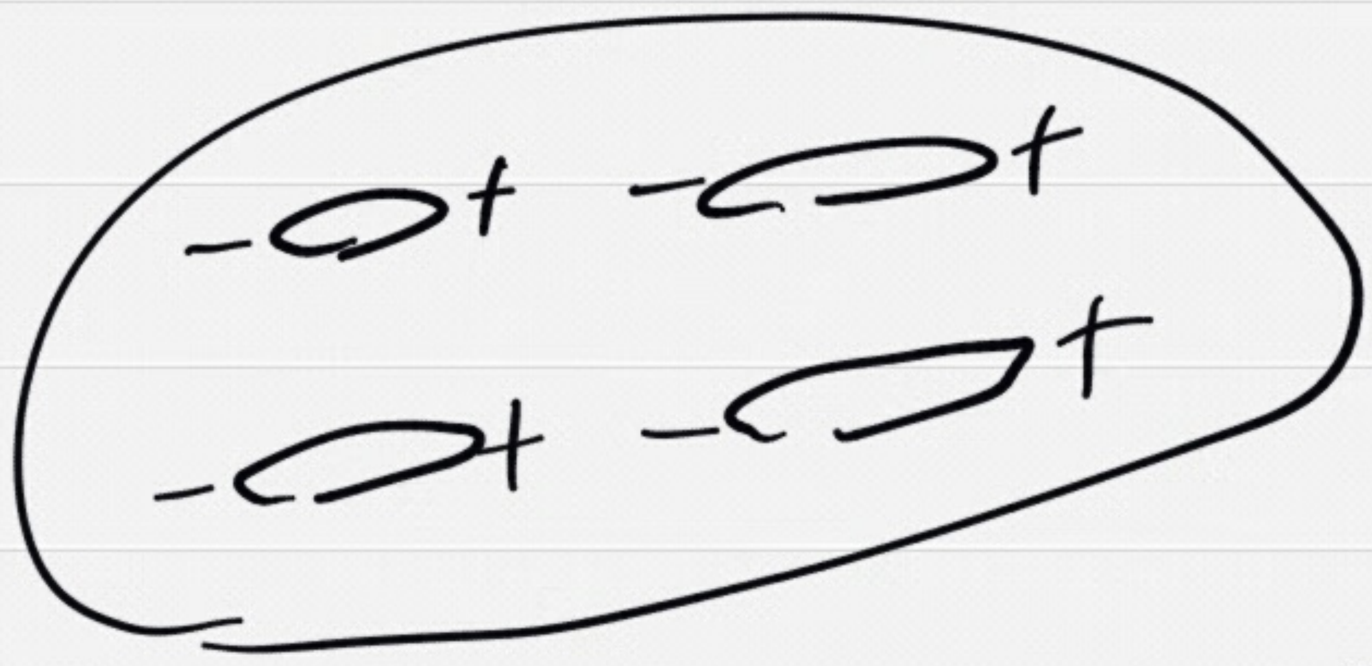


- Made up of dipoles

- Usually randomly oriented so net field zero

Insulator in External $\vec{E}$

$\longrightarrow \vec{E}_{ext}$

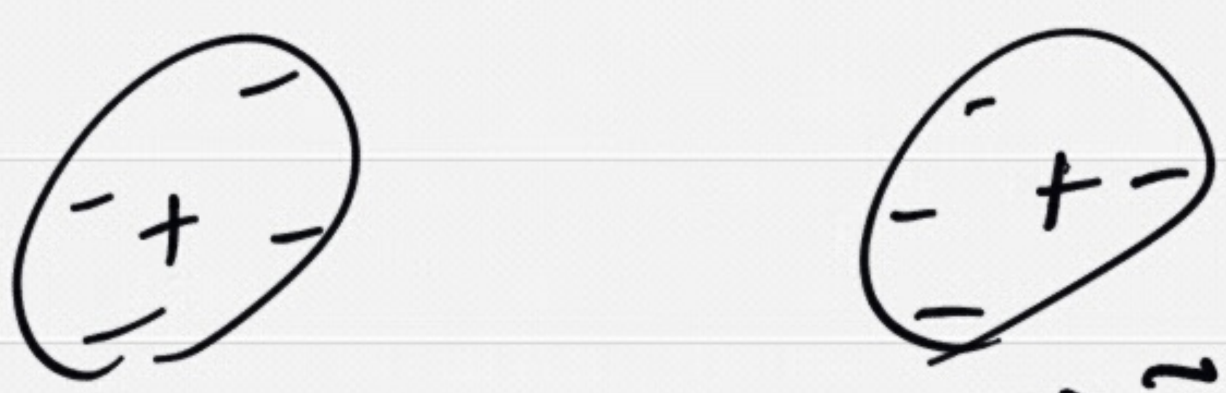


- Dipoles align w/ $\vec{E}_{ext}$
- Weaken but don't cancel out $\vec{E}_{ext}$

Two kinds of dipoles in material

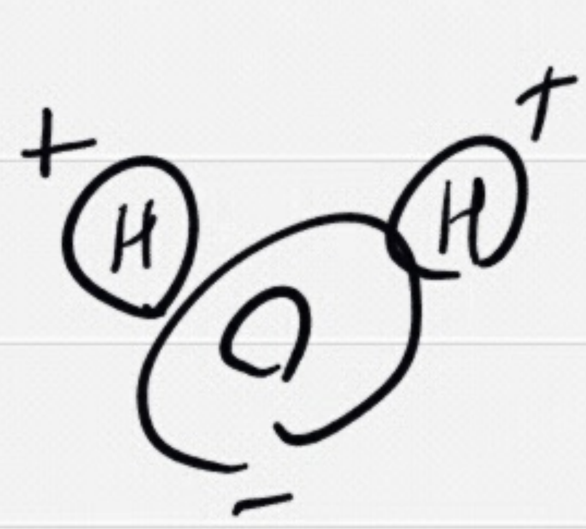
Induced Dipoles

$\rightarrow \vec{E}_{ext}$



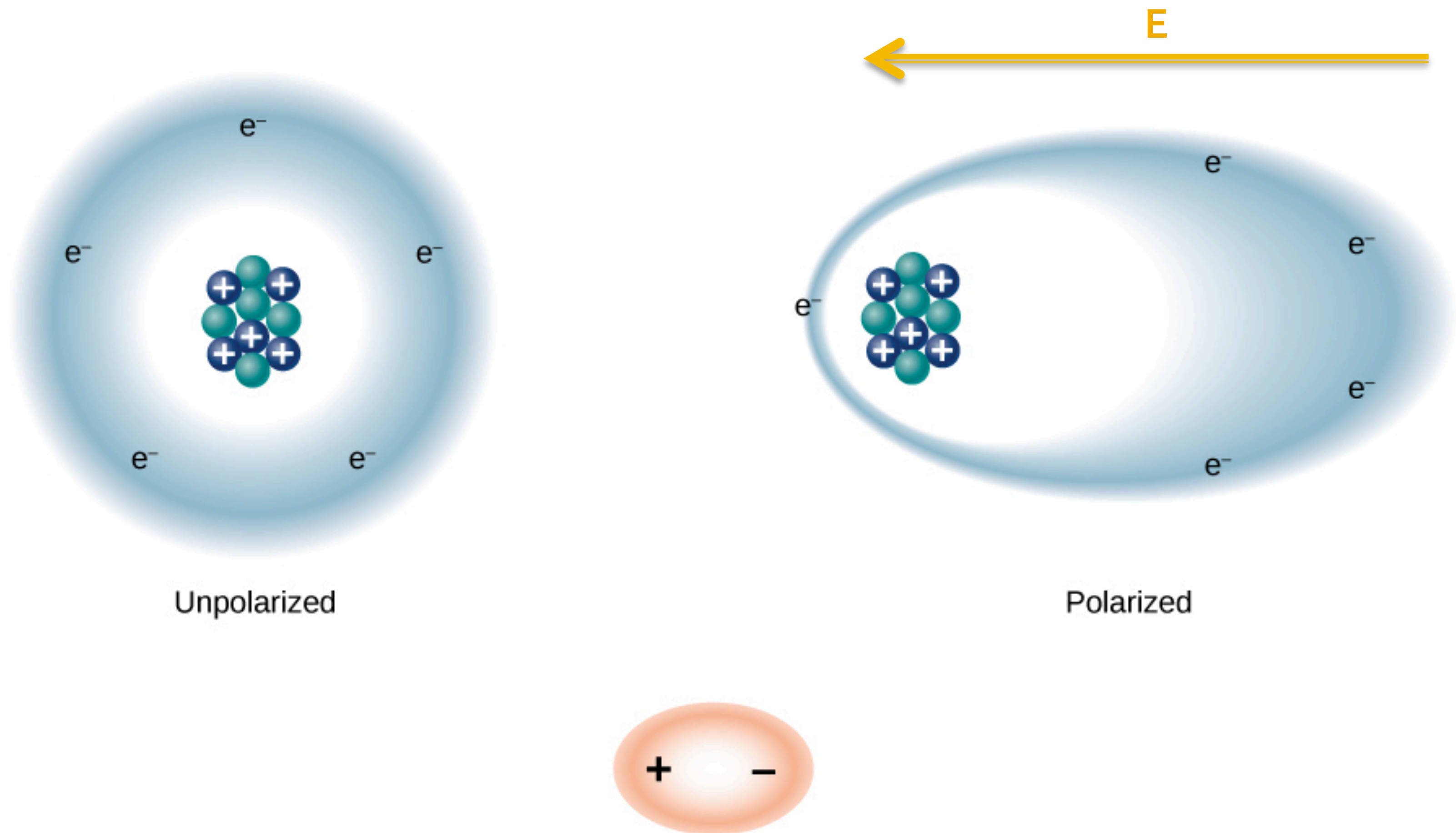
$\longrightarrow \vec{p}$

Intrinsic Dipoles



$\uparrow \vec{p}$

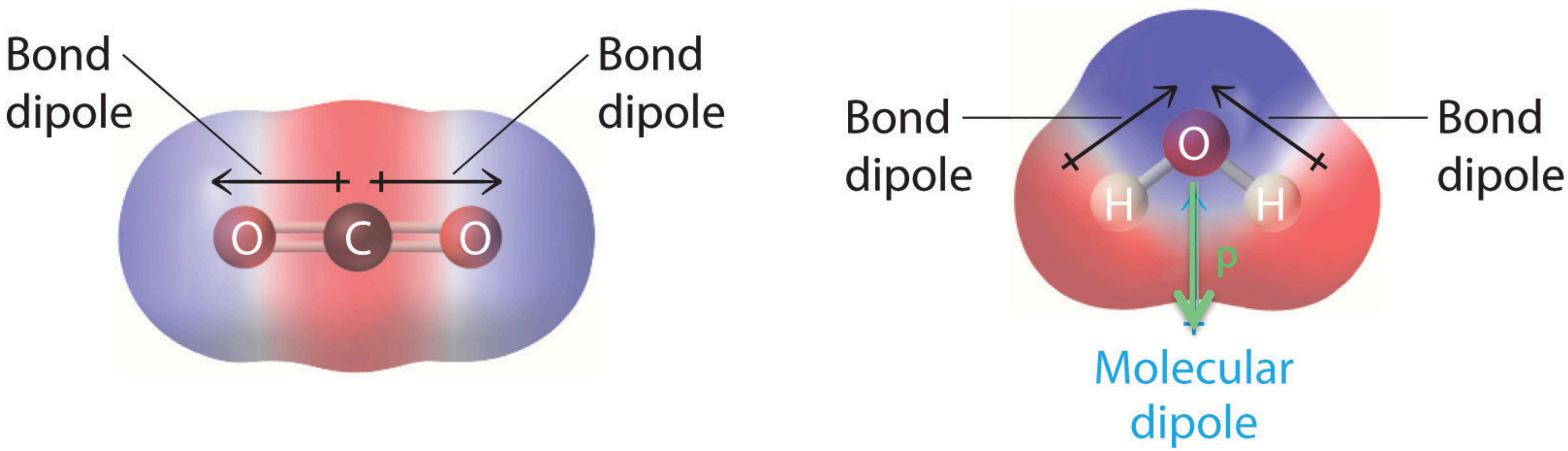
Induced Dipoles (Polarizability)



Large-scale view of polarized atom

Intrinsic Dipoles (Polar Molecules)

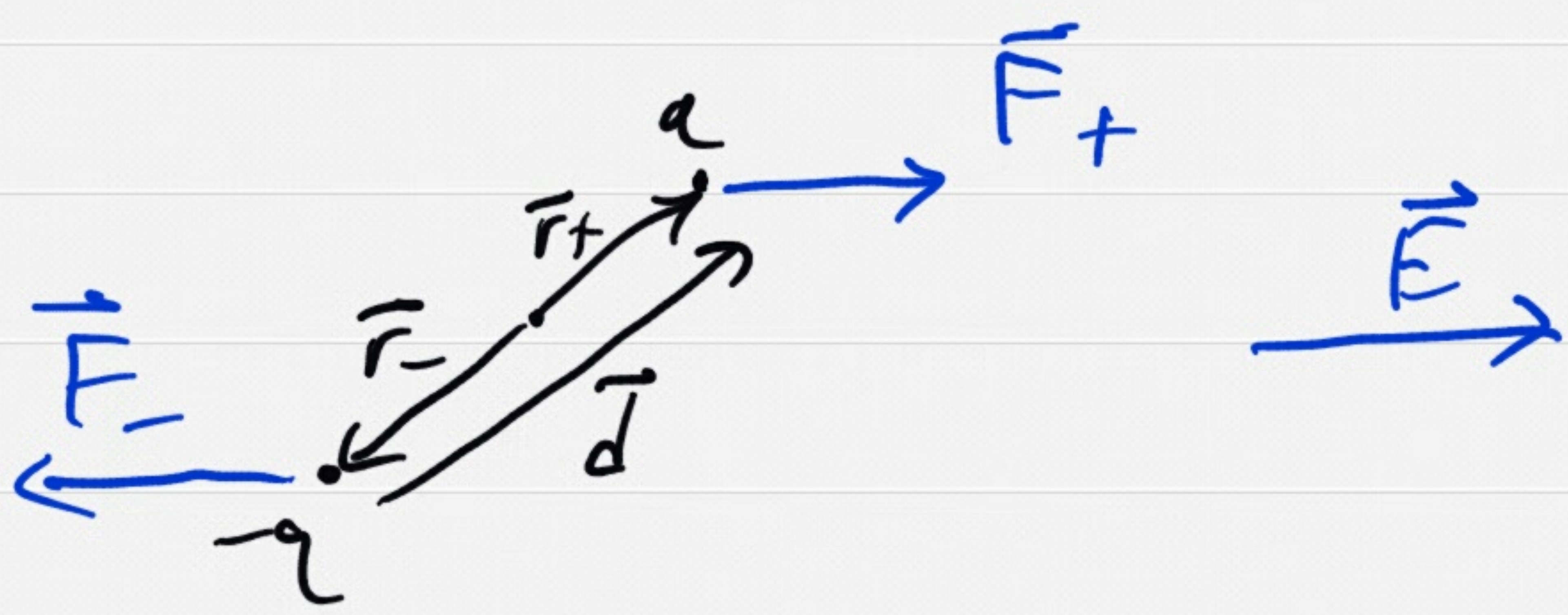
Electronegativity: H 2.1, C 2.5, O 3.5 => electrons closer to O than C, H



(a) No net dipole moment

(b) Net dipole moment

Force on Dipole (Tinker-Toy Model)



$$
\begin{aligned}
\vec{F} &= \vec{F}_+ + \vec{F}_- \\
&= q\vec{E}_+ - q\vec{E}_- \\
&= q\,\Delta\vec{E} \\
&= q\,(\vec{d}\cdot\nabla)\vec{E} \\
&= \boxed{(\vec{p}\cdot\nabla)\vec{E}}
\end{aligned}
$$

$$
\begin{aligned}
\vec{\tau} &= \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- \\
&= \vec{d}/2 \times q\vec{E} + -\vec{d}/2 \times -q\vec{E} \\
&= q\vec{d} \times \vec{E} \\
&= \boxed{\vec{p} \times \vec{E}}
\end{aligned}
$$