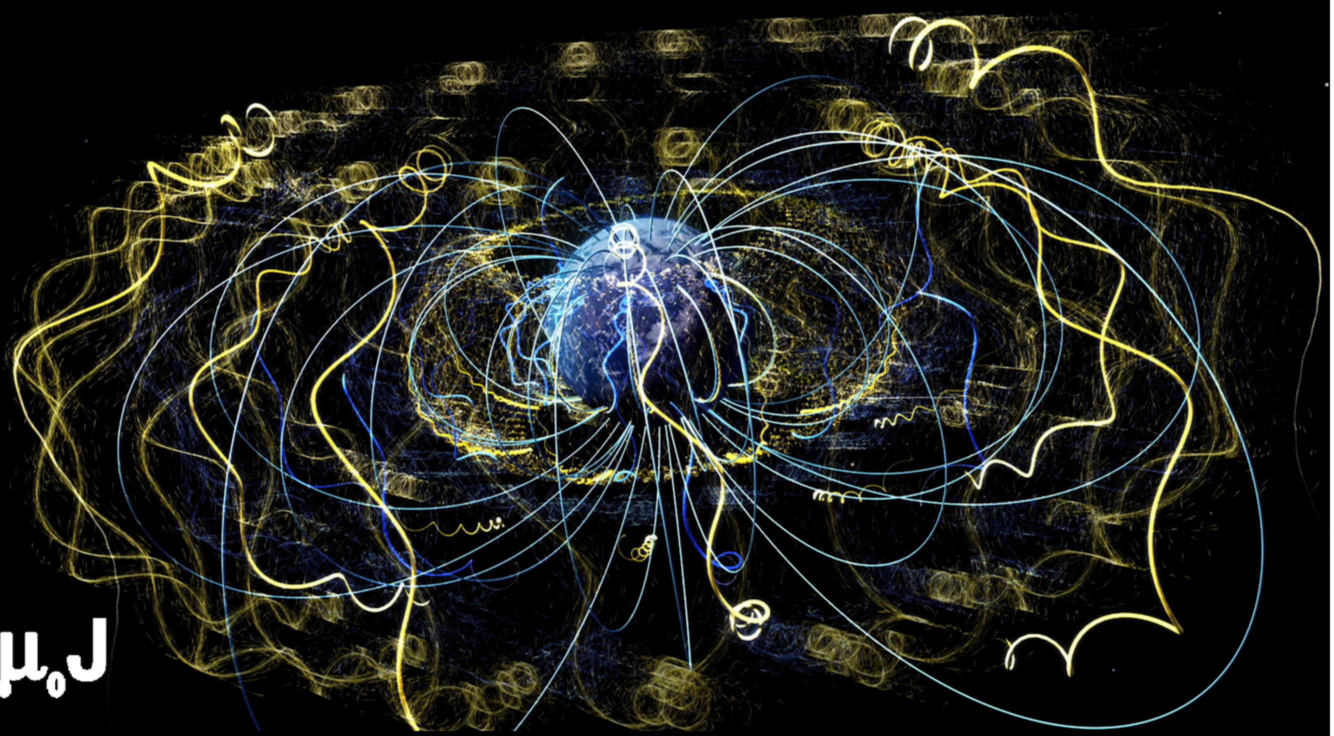
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas

Van Allen 301

MWF 9:30-10:20 Lecture

Polarization

$$\vec{P} = \frac{\text{dipole moment}}{\text{volume}}$$

$$\vec{p} = \int \vec{P}(\vec{r}\,')\, d\tau'$$

Potential of polarized object

$$V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$ for dipole at origin

$$V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\Delta\hat{r}}{\Delta r^2}$$ for dipole at $\vec{r}\,'$

$$\Rightarrow V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\vec{P}(\vec{r}\,')\cdot\Delta\hat{r}}{\Delta r^2}\, d\tau'$$

for polarized object

Uniform Polarization & Bound Charge

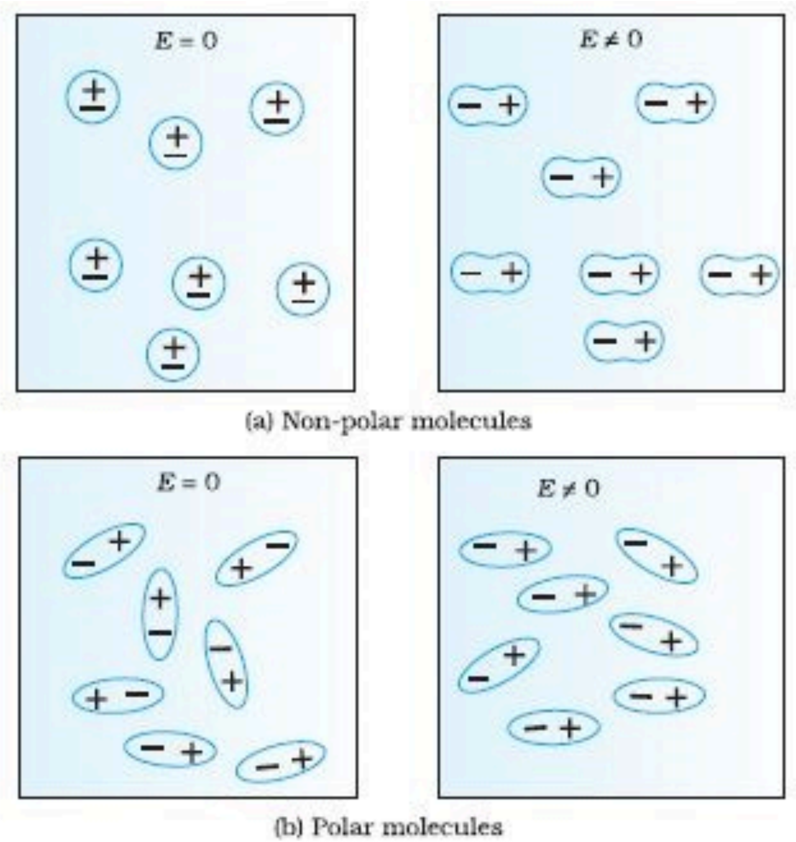


FIGURE A dielectric develops a net dipole moment in an external electric field. (a) Non-polar molecules, (b) Polar molecules.

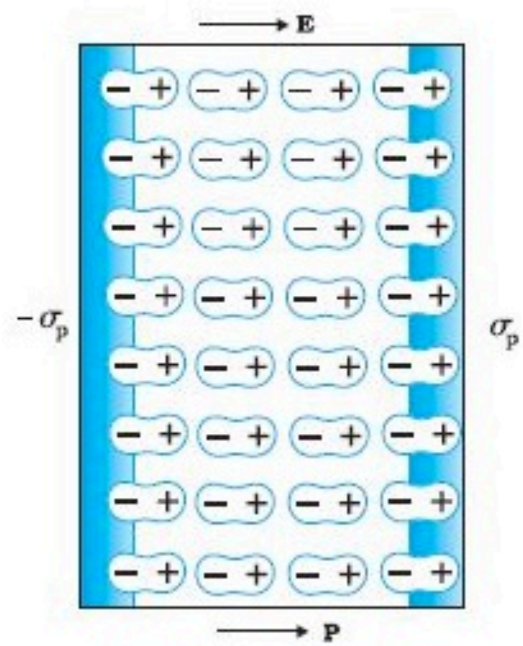
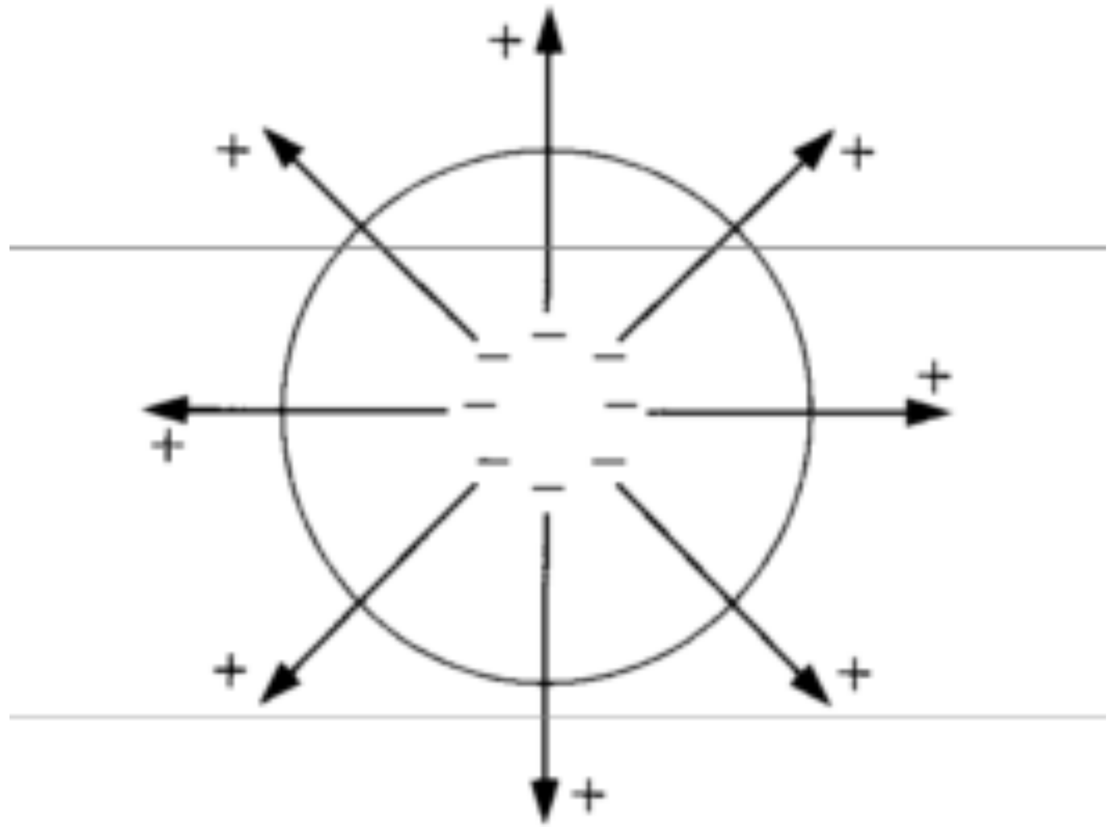


FIGURE A uniformly polarised dielectric amounts to induced surface charge density, but no volume charge density.

Non-Uniform Polarization and Bound Charge



Identity $\nabla\left(\frac{1}{\Delta r}\right) = -\frac{\Delta\hat{r}}{\Delta r^2}$

Change of variables

$$\Delta\vec{r} = \vec{r} - \vec{r}\,'$$

$$\Rightarrow \quad \nabla'\left(\frac{1}{\Delta r}\right) = +\frac{\Delta\hat{r}}{\Delta r^2}$$

w/ ∇' gradient w/ respect to $\vec{r}\,'$

So: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \vec{P}(\vec{r}\,')\cdot\nabla'\left(\frac{1}{\Delta r}\right)d\tau'$

$$= \frac{1}{4\pi\varepsilon_0}\int\left[\nabla'\cdot\left(\frac{\vec{P}}{\Delta r}\right) - \frac{1}{\Delta r}\nabla'\cdot\vec{P}\right]d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\oint\frac{\vec{P}\cdot d\vec{a}\,'}{\Delta r} - \frac{1}{4\pi\varepsilon_0}\int\frac{\nabla\cdot\vec{P}}{\Delta r}d\tau'$$

Looks like:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint\frac{\sigma_b}{\Delta r}da' + \frac{1}{4\pi\varepsilon_0}\int\frac{\rho_b}{\Delta r}d\tau'$$

surface charge + volume charge

$$\boxed{\sigma_b = \vec{P}\cdot\hat{n}, \quad \rho_b = -\nabla\cdot\vec{P}}$$

Bound Charge

$$Q_{bs} = \oint \sigma_b \, da = \oint \vec{P} \cdot \hat{n} \, da$$
$$= \oint \vec{P} \cdot d\vec{a}$$
$$= \int \nabla \cdot \vec{P} \, d\tau$$

- If $\vec{P}$ uniform, $Q_{bs} = 0$
- If $\vec{P}$ non-uniform

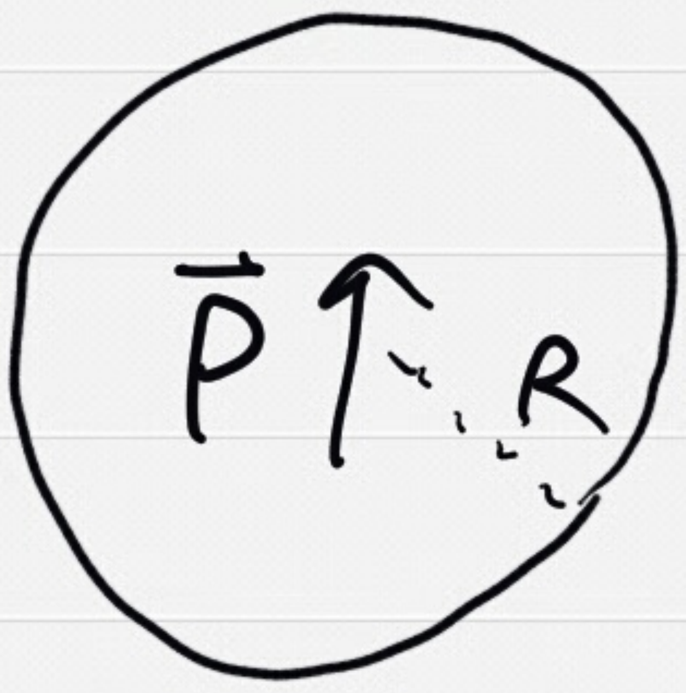
$$Q_{bs} = \int \nabla \cdot \vec{P} \, d\tau$$

$$Q_{bv} = \int \rho_b \, d\tau$$
$$= \int -\nabla \cdot \vec{P} \, d\tau$$
$$= -Q_{bs}$$

$$Q_b = Q_{bs} + Q_{bv} = 0$$

- Net bound charge is always zero
  - Insulators are net neutral
  - Could be added free charge

Example: Polarized Sphere



$$\bar{P} = P\hat{z}$$

$$\sigma_b = \bar{P}\cdot\hat{n} = P\,\hat{z}\cdot\hat{r}$$
$$= P\cos\theta$$

$$V(r,\theta) = \sum\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$$

only $\ell = 1$ since $\cos\theta = P_1(\cos\theta)$

$$\Rightarrow V_<(r,\theta) = A_1 r\cos\theta \qquad r<R$$
$$V_>(r,\theta) = \frac{B_1}{r^2}\cos\theta \qquad r>R$$

$$V_<(R,\theta) = V_>(R,\theta) \Rightarrow B_1 = R^3 A_1$$

$$\Delta E_r = -\partial V_>/\partial r - \left(-\partial V_</\partial r\right)\Big|_{r=R}$$
$$= \frac{2B_1}{R^3}\cos\theta + A_1\cos\theta$$
$$= 3A_1\cos\theta = \sigma/\varepsilon_0 = P\cos\theta/\varepsilon_0$$
$$\Rightarrow A_1 = P/3\varepsilon_0$$

$$\Rightarrow V(r,\theta) = \frac{P}{3\varepsilon_0}\,r\cos\theta \qquad r<R$$
$$= \frac{P}{3\varepsilon_0}\frac{R^3}{r^2}\cos\theta \qquad r>R$$

Inside: $V(r,\theta) = \frac{P}{3\varepsilon_0} z$

$$\vec{E} = -\nabla V = -\frac{P}{3\varepsilon_0}\hat{z}$$

$$= -\vec{P}/3\varepsilon_0$$

(opposite $\vec{P}$ as expected)

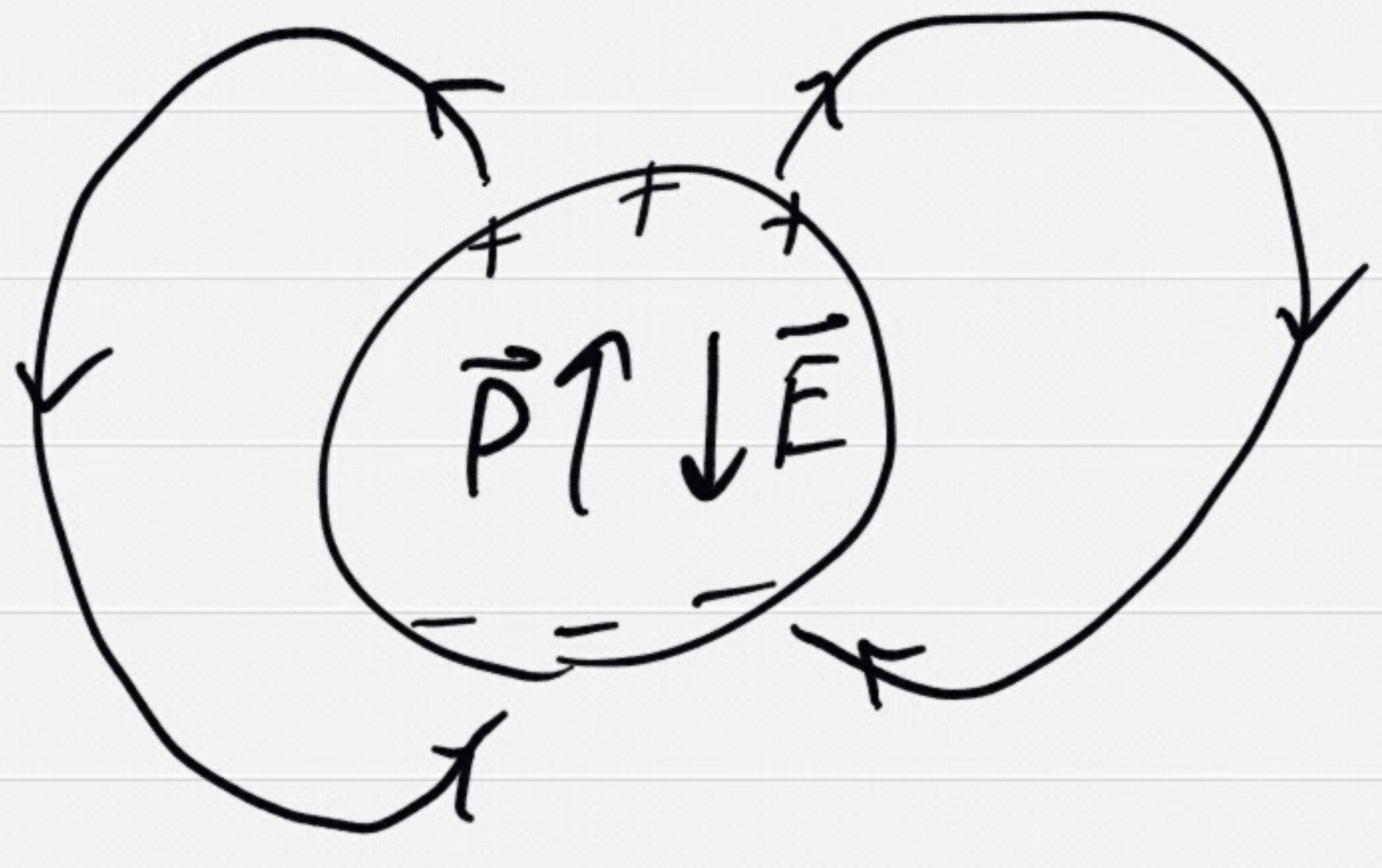
Outside: $V(r,\theta) = \frac{\vec{P}\cdot\hat{r}}{3\varepsilon_0}\frac{R^3}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

w/ $\vec{p} = \vec{P}\cdot 4/3\,\pi R^3$

$$= \vec{P}\cdot\text{Volume}$$

(as expected for uniform $\vec{P}$)



Polarized Sphere