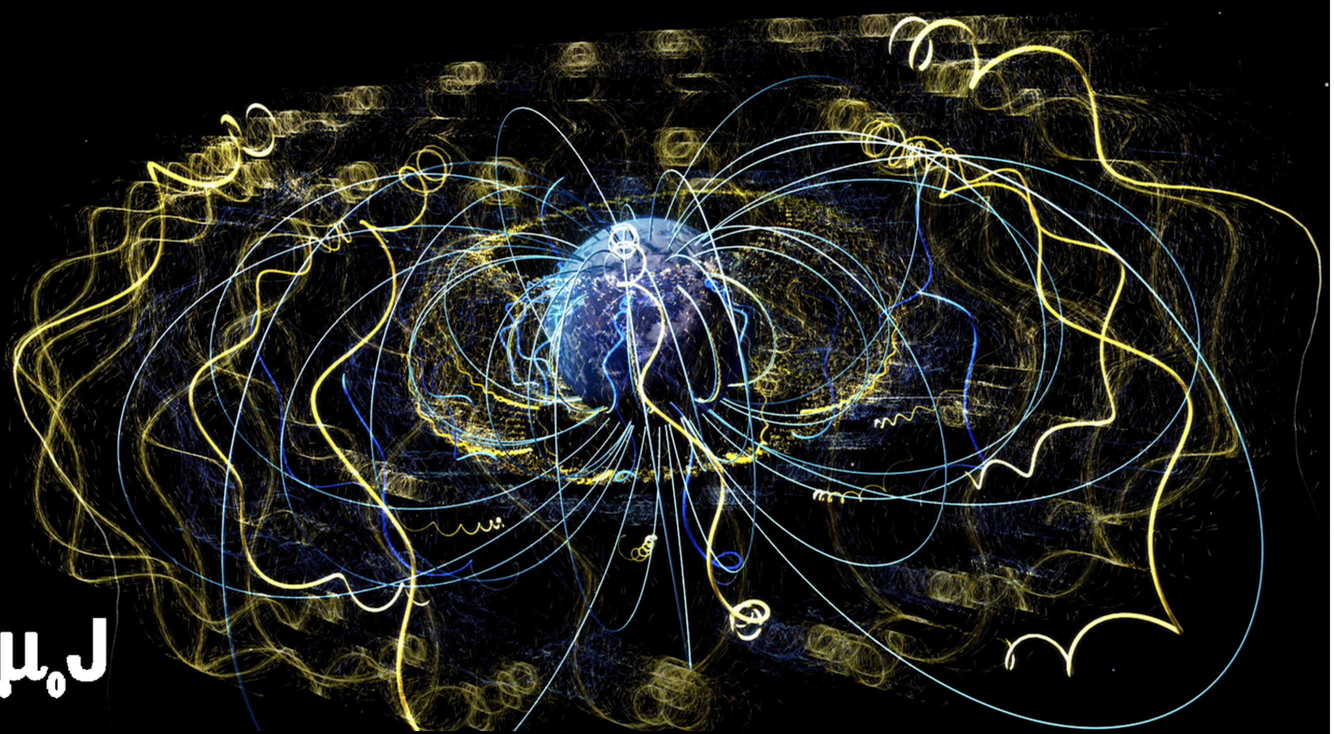
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



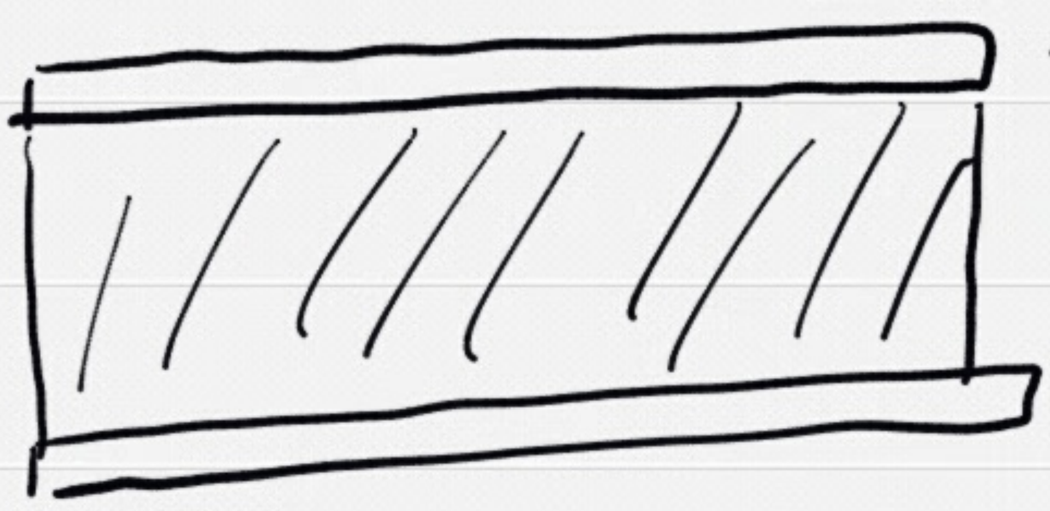
Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Announcements

- Updated equation sheet posted this morning
- Practice midterms will be posted Wednesday
  - Solutions to follow
- Exam two is a week from Wednesday (11/6)

Example : Capacitor



$$\vec{D} = \sigma_f = Q/A$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$
$$= \varepsilon_0 (1+\chi_e) \vec{E}$$
$$= \varepsilon \vec{E}$$

$$\Rightarrow \vec{E} = Q/\varepsilon A$$

$$\Delta V = E \cdot d = Qd/\varepsilon A$$

$$C = Q/\Delta V = \varepsilon A/d$$

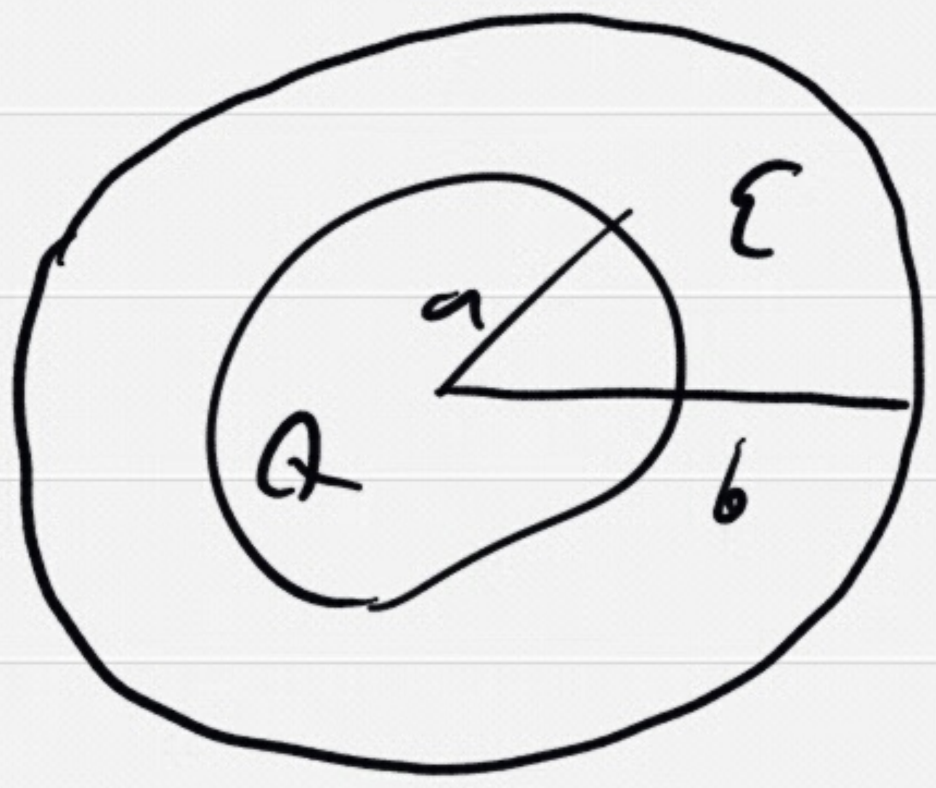
compare to $C_{vacuum} = \varepsilon_0 A/d$

$$C/C_{vac} = \varepsilon/\varepsilon_0 = \varepsilon_r$$

- Less voltage drop for same free charge $\Rightarrow$ higher capacitance

Most capacitors are filled w/ dielectric material

Example



charged conductor surrounded by linear dielectric

$$\oint \vec{D} \cdot d\vec{a} = Q_{fenc}$$

$$\Rightarrow D \cdot 4\pi r^2 = Q \quad r > a$$
$$= 0 \quad r < a$$

$$\Rightarrow \vec{D} = \frac{Q}{4\pi r^2}\hat{r} \quad r > a$$
$$= 0 \quad r < a$$

$$\Rightarrow \quad \vec{E} = 0 \quad r < a$$
$$= \frac{Q}{4\pi \varepsilon r^2} \quad a < r < b$$
$$= \frac{Q}{4\pi \varepsilon_0 r^2} \quad r > b$$

$\vec{E}$ reduced by polarization inside dielectric

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$= \frac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon r^2}\hat{r} \qquad a < r < b$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$= -\frac{1}{r^2}\,\partial/\partial r\,(r^2 P_r)$$

$$= 0$$

$$\sigma_b = \vec{P}\cdot\hat{n}$$

$$= \frac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon b^2} \qquad r = b$$

$$-\frac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon a^2} \qquad r = a$$

$$Q_b = \sigma_{bb}\cdot 4\pi b^2 + \sigma_{ba}\cdot 4\pi a^2$$

$$= \chi_e \frac{\varepsilon_0}{\varepsilon} Q\,[1 - 1] = 0$$

$$V(a) = -\int_\infty^a \vec{E}\cdot d\vec{l}$$

$$= -\int_\infty^b \frac{Q}{4\pi\varepsilon_0 r^2}\,dr - \int_b^a \frac{Q}{4\pi\varepsilon r^2}\,dr - \int_a^a 0\,dr$$

$$= \frac{Q}{4\pi\varepsilon_0 b} + \frac{Q}{4\pi\varepsilon a} - \frac{Q}{4\pi\varepsilon b}$$

reduced from $Q/4\pi\varepsilon_0 a$ w/o dielectric

Boundary Value w/ Linear Dielectrics

In homogeneous linear dielectric
$$\rho_f = 0$$

Boundary conds. $\Delta D_\perp = \sigma_f$
$$\Rightarrow \Delta(\varepsilon E_\perp) = \sigma_f$$

$$\Rightarrow \Delta(\varepsilon \, \partial V/\partial n) = -\sigma_f$$
and $\Delta V = 0$ as always

Example:

$$V_< = \sum_0^\infty A_\ell r^\ell P_\ell(\cos\theta)$$
$$V_> = -E_0 r \cos\theta + \sum_0^\infty \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_<(R) = V_>(R)$$
$$\Rightarrow \sum_0^\infty A_\ell R^\ell P_\ell(\cos\theta) = -E_0 R\cos\theta + \sum_0^\infty \frac{B_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

$$\Rightarrow A_1 R = -E_0 R + \frac{B_1}{R^2}$$

$$A_\ell R^\ell = B_\ell / R^{\ell+1} \quad \ell \neq 1$$

$$\varepsilon \, {\partial V_< }/{\partial r}\Big|_R = \varepsilon_0 \, {\partial V_> }/{\partial r}\Big|_R$$

$$\varepsilon \sum_0^\infty A_\ell \cdot \ell \cdot R^{\ell-1} P_\ell(\cos\theta)$$

$$= -\varepsilon_0 E_0 \cos\theta - \varepsilon_0 \sum_0^\infty B_\ell \cdot (\ell+1)/R^{\ell+2} P_\ell(\cos\theta)$$

$$\Rightarrow \varepsilon A_1 = -\varepsilon_0 E_0 - \varepsilon_0 \cdot B_1 \cdot 2/R^3$$

$$\varepsilon A_\ell \cdot \ell \cdot R^{\ell-1} = -\varepsilon_0 B_\ell \cdot (\ell+1)/R^{\ell+2} \quad \ell \neq 1$$

For $\ell \neq 1$ A_ℓ proportional to both B_ℓ & $-B_\ell$

$$\Rightarrow A_\ell = B_\ell = 0$$

$$A_1 R = -E_0 R + B_1/R^2$$

$$\varepsilon_r A_1 = -E_0 - 2 \, B_1/R^3$$

$$\Rightarrow \varepsilon_r(-E_0 + B_1/R^3) = -E_0 - 2 \, B_1/R^3$$

$$\Rightarrow B_1 = \frac{(\varepsilon_r - 1) E_0}{\varepsilon_r + 2} R^3$$

$$\Rightarrow A_1 = \frac{-3E_0}{\varepsilon_r + 2}$$

So $V_<(r,\theta) = -\dfrac{3E_0}{\varepsilon_r + 2} r\cos\theta$

and $\vec{E}_< = \dfrac{3}{\varepsilon_r + 2} \vec{E}_0$ uniform!