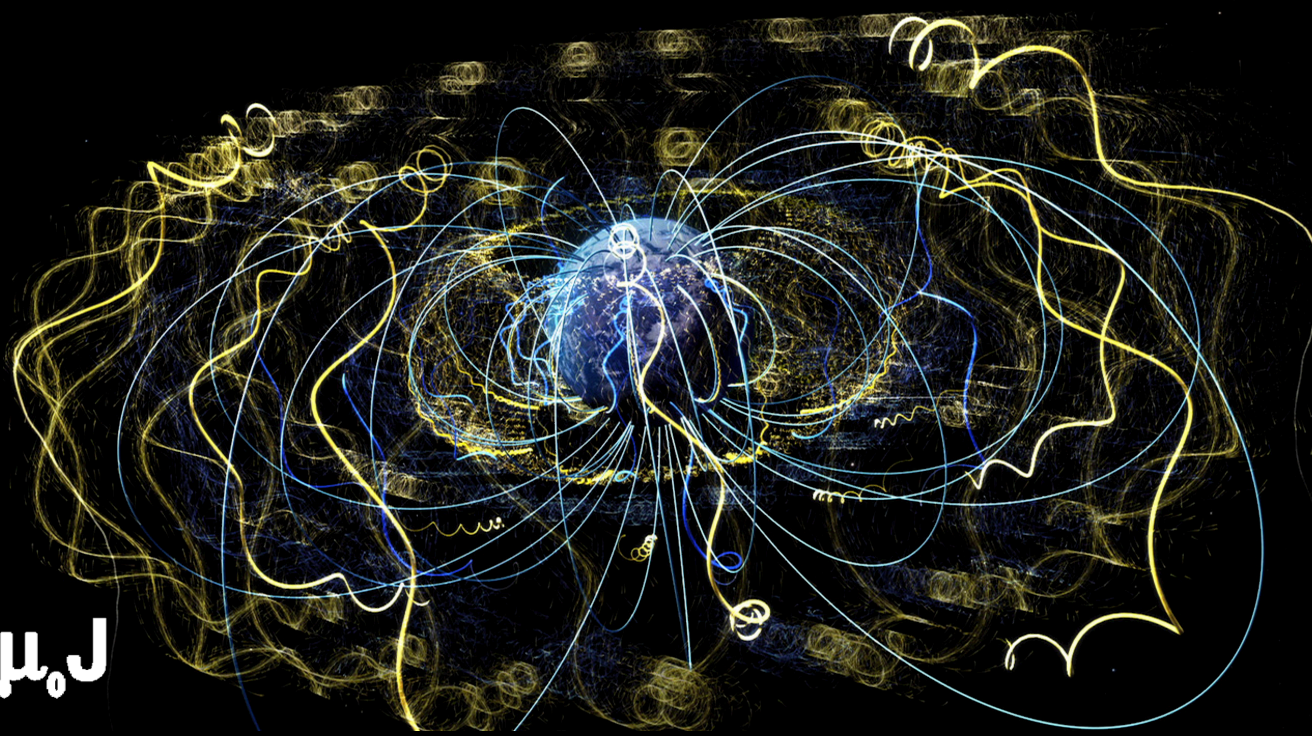
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Announcements

- Updated equation sheet posted Monday
- Practice midterms posted today
  - Note: Last year's exam 2 covered one more week of material, so each practice test is one problem short of a full exam
  - Solutions to follow
- Exam 2 is a week from today (11/6)
  - Covers Chapters 3 & 4 in Griffiths - all lecture material since the first exam

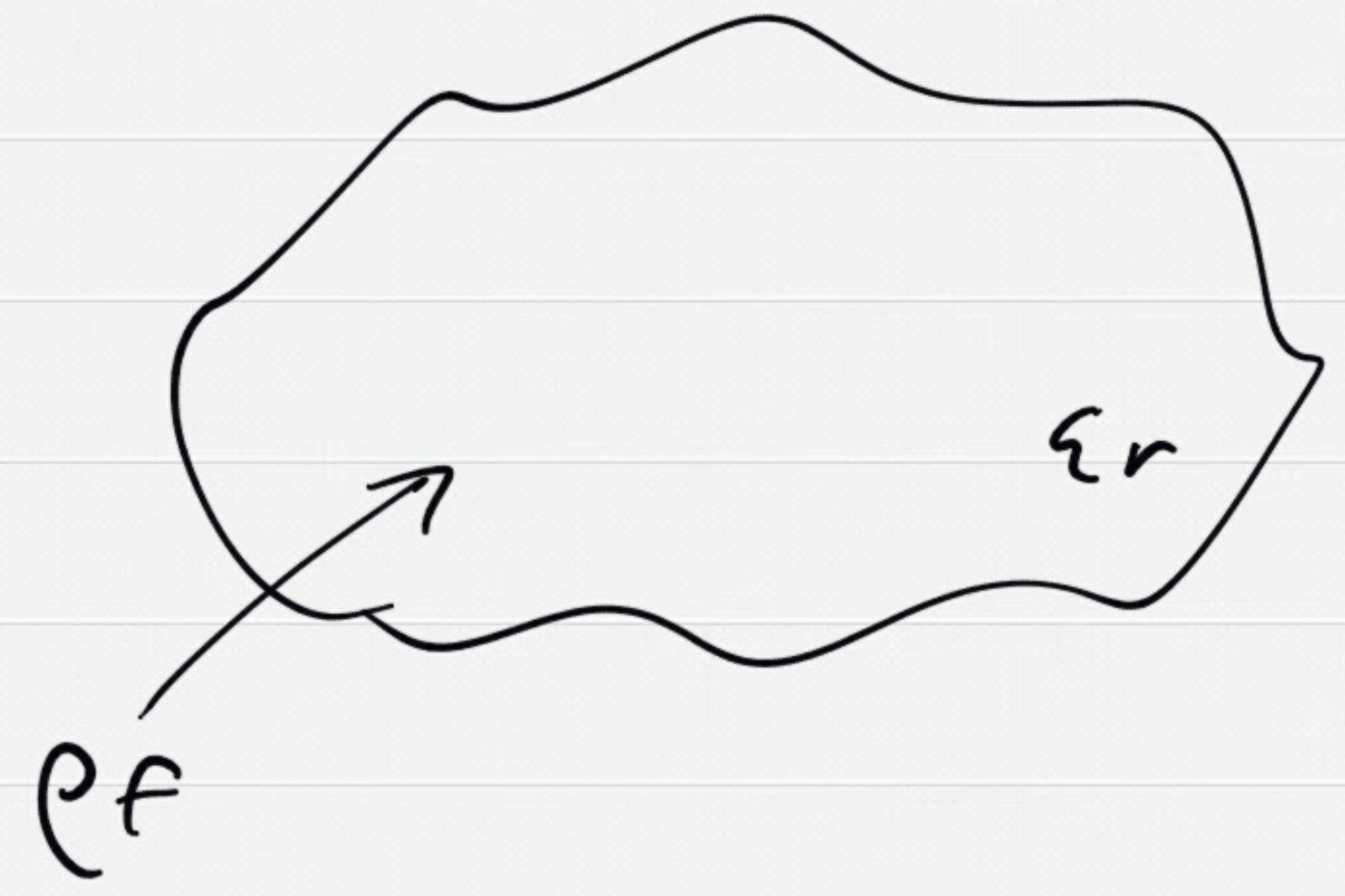
Energy in Dielectrics

Capacitor $W = \frac{1}{2} C V^2$

$= \frac{1}{2} \varepsilon_r C_{vac} V^2$

so more work to charge capacitor w/ dielectric

Generally



- Assemble free charge
- Free charge polarizes dielectric

Work increment

$$\Delta W = \int \Delta\rho_f \cdot V \, d\tau$$

$$= \int (\nabla \cdot \Delta\vec{D}) V \, d\tau$$

$$(\nabla \cdot \Delta\vec{D}) V = \nabla \cdot (\Delta\vec{D} V) - \nabla V \cdot \Delta\vec{D}$$

$$= \nabla \cdot (\Delta\vec{D} V) + \vec{E} \cdot \Delta\vec{D}$$

$$\Rightarrow \Delta W = \int \nabla \cdot (\Delta \vec{D} V)\, d\tau + \int \vec{E} \cdot \Delta \vec{D}\, d\tau$$

$$= \oint (\Delta \vec{D} \cdot V) \cdot d\vec{a} + \int \vec{E} \cdot \Delta \vec{D}\, d\tau$$

zero for large volume

$$\text{so } \Delta W = \int_{\text{all space}} \vec{E} \cdot \Delta \vec{D}\, d\tau$$

For linear dielectric

$$\vec{D} = \varepsilon \vec{E}$$

$$\Delta (\vec{D} \cdot \vec{E}) = \Delta (\varepsilon \vec{E} \cdot \vec{E})$$
$$= \varepsilon \vec{E} \cdot \Delta \vec{E} + \varepsilon \Delta \vec{E} \cdot \vec{E}$$
$$= 2 \varepsilon \vec{E} \cdot \Delta \vec{E}$$
$$= 2 \vec{E} \cdot \Delta \vec{D}$$

$$\Rightarrow \Delta W = \Delta \left(\frac{1}{2} \int \vec{D} \cdot \vec{E}\, d\tau \right)$$

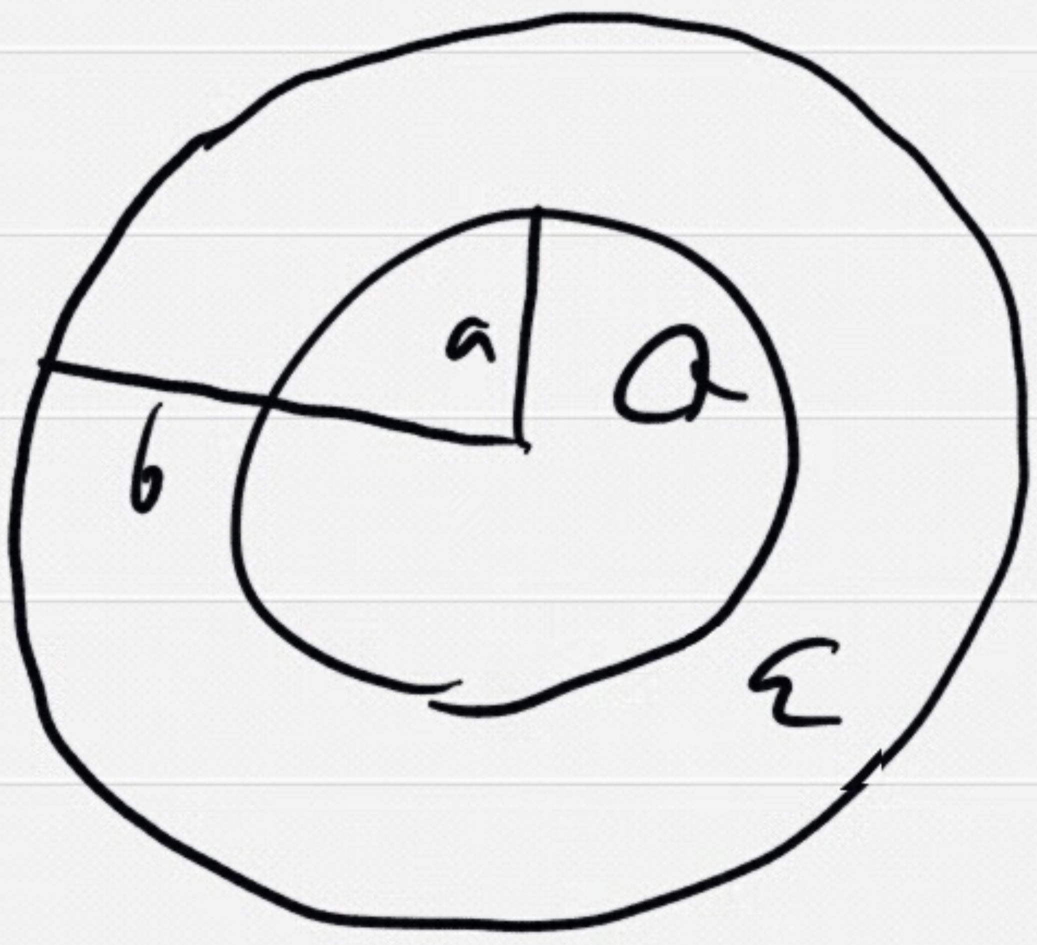
$$\Rightarrow W = \frac{1}{2} \int \vec{D} \cdot \vec{E}\, d\tau$$

$$= \frac{1}{2} \varepsilon \int E^2\, d\tau$$

bigger by $\varepsilon_r = \varepsilon / \varepsilon_0$

Only valid for linear dielectric

Example



$$\vec{D} = \begin{cases} 0 & r < a \\ Q/4\pi r^2 \,\hat{r} & r > a \end{cases}$$

$$\vec{E} = \begin{cases} 0 & r < a \\ Q/4\pi\varepsilon r^2 \,\hat{r} & a < r < b \\ Q/4\pi\varepsilon_0 r^2 \,\hat{r} & r > b \end{cases}$$

$$W_{total} = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau$$

$$= \frac{1}{2}\int_a^b \frac{Q^2}{(4\pi)^2 \varepsilon r^4} d\tau \quad + \frac{1}{2}\int_b^\infty \frac{Q^2}{(4\pi)^2 \varepsilon_0 r^4} d\tau$$

$$\int_a^b \frac{1}{r^4} d\tau = \int_0^{2\pi}\int_0^{\pi}\int_a^b \frac{1}{r^4}\cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 4\pi \cdot -\frac{1}{r}\Big|_a^b$$

$$\text{so } W_{total} = \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q^2}{8\pi\varepsilon_0}\frac{1}{b}$$

$$= \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b} + \varepsilon_r/b\right)$$

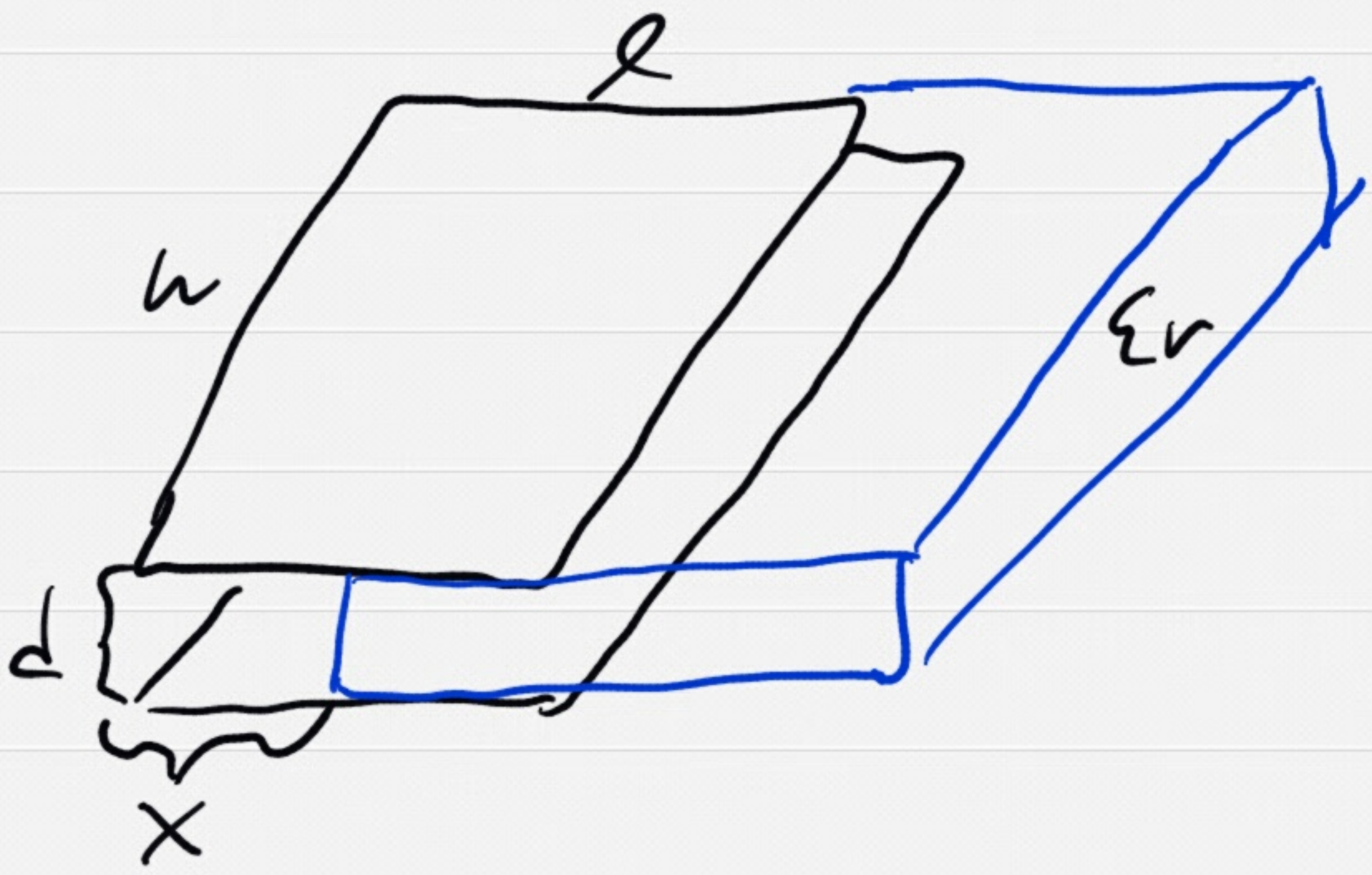
$$= \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b} + \frac{1+\chi_e}{b}\right)$$

$$= \boxed{\frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{a} + \frac{\chi_e}{b}\right)}$$

$$W_{elec} = \frac{1}{2}\varepsilon_0\int E^2 d\tau = \frac{Q^2\varepsilon_0}{8\pi\varepsilon^2}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q^2}{8\pi\varepsilon_0}\frac{1}{b}$$

This term smaller by $\varepsilon_0/\varepsilon$

Force on Dielectrics



Exert $\vec{F}_{me}$ to pull dielectric

$$dW = F_{me}\, dx$$

so $$F_e = -F_{me} = -dW/dx$$

w/ $$W = \tfrac{1}{2} C V^2 = \tfrac{1}{2} Q^2/C$$

$$C = \frac{\varepsilon_0 x w}{d} + \frac{\varepsilon (l-x) w}{d}$$

$$F_e = -\tfrac{d}{dx}\left(\tfrac{1}{2} Q^2/C\right)$$

$$= \tfrac{1}{2} Q^2/C^2 \; dC/dx$$

$$= \tfrac{1}{2} V^2 \, dC/dx$$

$$= \tfrac{1}{2} V^2 \, \frac{(\varepsilon_0 - \varepsilon) w}{d}$$

$\varepsilon > \varepsilon_0$ so $F < 0$

pulls dielectric into capacitor