\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot B = 0 \]

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 \mathbf{J} \]

Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture
Suppose the general solutions of Laplace’s equation for a particular problem in Cartesian coordinates are:

\[ V_n(x,y) = [A_n e^{kx} + B_n e^{-kx}] \times [C_n \cos(n\pi y/a) + D_n \sin(n\pi y/a)] \]

What are the values of k for each n?
Q1: 

$$X_n(x) = A_n e^{kx} + B_n e^{-kx}$$

$$Y_n(y) = C_n \cos \left( \frac{n \pi y}{a} \right) + D_n \sin \left( \frac{n \pi y}{a} \right)$$

$$\frac{d^2 X_n}{dx^2} = k^2 \left( A_n e^{kx} + B_n e^{-kx} \right)$$

$$\frac{d^2 Y_n}{dy^2} = -\frac{n^2 \pi^2}{a^2} \left( C_n \cos \left( \frac{n \pi y}{a} \right) + D_n \sin \left( \frac{n \pi y}{a} \right) \right)$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} = 0$$

$$\Rightarrow k^2 - \frac{n^2 \pi^2}{a^2} = 0$$

$$\Rightarrow k = \frac{n \pi}{a}$$
Suppose the general solutions of Laplace’s equation for a particular problem in Cartesian coordinates are:

\[ V_n(x,y) = [A_n e^{kx} + B_n e^{-kx}] \times [C_n \cos(n\pi y/a) + D_n \sin(n\pi y/a)] \]

If the boundary conditions are such that \( V = 0 \) at \( y = \pm a/2 \), \( V = \cos(\pi y/a) \) at \( x = 0 \), and \( V \to 0 \) as \( x \to \infty \) and, what are \( A_n, B_n, C_n, D_n \) for all \( n \)?
Q.2: \[ V(x, y) = \sum_{n} V_n(x, y) \]

if \( V(0, y) = \cos\left(\frac{\pi y}{a}\right) \)

\[ \Rightarrow \text{only } n = 1 \text{ contributes} \]
And only \( c_1 \neq 0 \)

\[ e^{\infty} \rightarrow \infty \Leftrightarrow x = \infty \]
so only \( b_1 \neq 0 \)

\[ \Rightarrow V(x, y) = e^{-\frac{\pi x}{a}} \cos\left(\frac{\pi y}{a}\right) \]
The general solutions of Laplace’s equation in spherical coordinates are:

- $V_l(r, \theta) = (A_l r^l + B_l/r^{l+1})P_l(\cos \theta)$

If you want to find a solution valid from $r = R$ to $r = \infty$, and you know $V(R, \theta) = P_3(\cos \theta)$, what is the full solution $V(r, \theta)$?
Q 3: \( V(r, \theta) = \sum \frac{A e r^2 + \theta}{r^{2+l}} p_l(\cos \theta) \)

For \( r > R \), \( A e = 0 \)

\( V_r(V_r, \theta) = \sum \frac{\theta}{r^{2+l}} p_l(\cos \theta) \)

but \( V(R, \theta) = p_3(\cos \theta) \)

\[ \Rightarrow \text{only } p_3 \neq 0 \]

\( V_r(R, \theta) = \frac{\theta}{R^4} p_3(\cos \theta) \)

\[ \Rightarrow p_3 = R^4 \]

\[ V_r(V_r, \theta) = \frac{R^4}{r^4} p_3(\cos \theta) \]
• Consider a dielectric shell surrounding a conductor with charge +Q on its surface.
• Draw three diagrams, showing the lines of polarization, electric field, and electric displacement. Be qualitatively accurate, with more lines for stronger fields.
Q4: $\vec{p}$
Consider a dielectric sphere embedded in an initially uniform external field. Sketch the resulting lines of polarization, electric field, and electric displacement.
Dielectric Sphere Electric Field

$E_{\text{ext}}$

$E_{\text{int}}$

$E_{\text{tot}}$
Q5: \[ \vec{P} \quad \vec{E} \quad \sigma_b \]
In the problem with the dielectric sphere, there is no free charge anywhere.

\[ \int \mathbf{D} \cdot d\mathbf{a} = Q_{f\,enc} \]

- But \( D \neq 0 \)
- Why not?
Q \bar{b} : \oint \overline{\overrightarrow{D}} \cdot \overrightarrow{d\alpha} = Q_{\text{fence}} = 0

But \overline{\overrightarrow{D}} \neq 0

\overline{\overrightarrow{D} \cdot \overrightarrow{n}} > 0 \text{ over half}
\overline{\overrightarrow{D} \cdot \overrightarrow{n}} < 0 \text{ over half}

cancels out in \oint \overline{\overrightarrow{D} \cdot d\overrightarrow{a}}