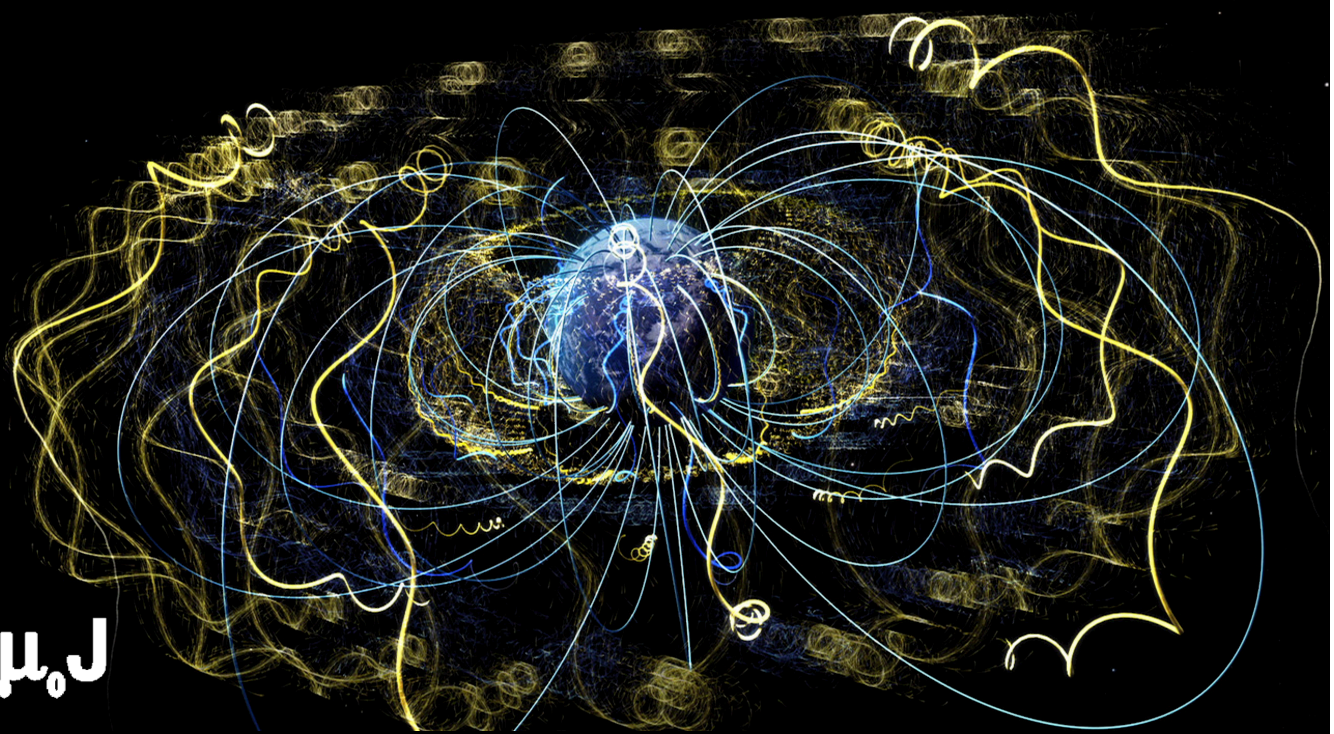
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas

Van Allen 301

MWF 9:30-10:20 Lecture

Check Your Understanding #1

- Suppose the general solutions of Laplace's equation for a particular problem in Cartesian coordinates are:
  - $V_n(x,y) = [A_n e^{kx} + B_n e^{-kx}]*[C_n \cos(n\pi y/a) + D_n \sin(n\pi y/a)]$
- What are the values of k for each n?

Q1:

$$X_n(x) = A_n e^{kx} + B_n e^{-kx}$$

$$Y_n(y) = C_n \cos\left(\frac{n\pi y}{a}\right) + D_n \sin\left(\frac{n\pi y}{a}\right)$$

$$d^2X_n/dx^2 = k^2\left(A_n e^{kx} + B_n e^{-kx}\right)$$

$$d^2Y_n/dy^2 = -\frac{n^2\pi^2}{a^2}\left(C_n \cos\left(\frac{n\pi y}{a}\right) + D_n \sin\left(\frac{n\pi y}{a}\right)\right)$$

$$\frac{1}{X} d^2X/dx^2 + \frac{1}{Y} d^2Y/dy^2 = 0$$

$$\Rightarrow k^2 - \frac{n^2\pi^2}{a^2} = 0$$

$$\Rightarrow k = \frac{n\pi}{a}$$

Check Your Understanding #2

- Suppose the general solutions of Laplace's equation for a particular problem in Cartesian coordinates are:
  - $V_n(x,y) = [A_n e^{kx} + B_n e^{-kx}]*[C_n \cos(n\pi y/a) + D_n \sin(n\pi y/a)]$
- If the boundary conditions are such that V= 0 at $y = \pm a/2$, $V = \cos(\pi y/a)$ at $x = 0$, and V -> 0 as x -> ∞ and, what are A_n, B_n, C_n, D_n for all n?

Q2: $V(x,y) = \sum_n V_n(x,y)$

if $V(0,y) = \cos\left(\frac{\pi y}{a}\right)$

$\Rightarrow$ only $n=1$ contributes

And only $C_1 \neq 0$

$e^{kx} \to \infty$ @ $x = \infty$

so only $B_1 \neq 0$

$$\Rightarrow V(x,y) = e^{-\frac{\pi x}{a}} \cos\left(\frac{\pi y}{a}\right)$$

Check Your Understanding #3

- The general solutions of Laplace's equation in spherical coordinates are:
  - $V_l(r,\theta) = (A_l r^l + B_l/r^{l+1})P_l(\cos\theta)$
- If you want to find a solution valid from $r = R$ to $r = \infty$, and you know $V(R,\theta) = P_3(\cos\theta)$, what is the full solution $V(r,\theta)$?

Q 3: $V(r,\theta) = \sum_l \left(A_l r^l + B_l/r^{l+1}\right) P_l(\cos\theta)$

for $r > R$ $A_l = 0$

$$V_>(r,\theta) = \sum_l \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

but $V(R,\theta) = P_3(\cos\theta)$

$\Rightarrow$ only $B_3 \neq 0$

$$V_>(R,\theta) = \frac{B_3}{R^4} P_3(\cos\theta)$$

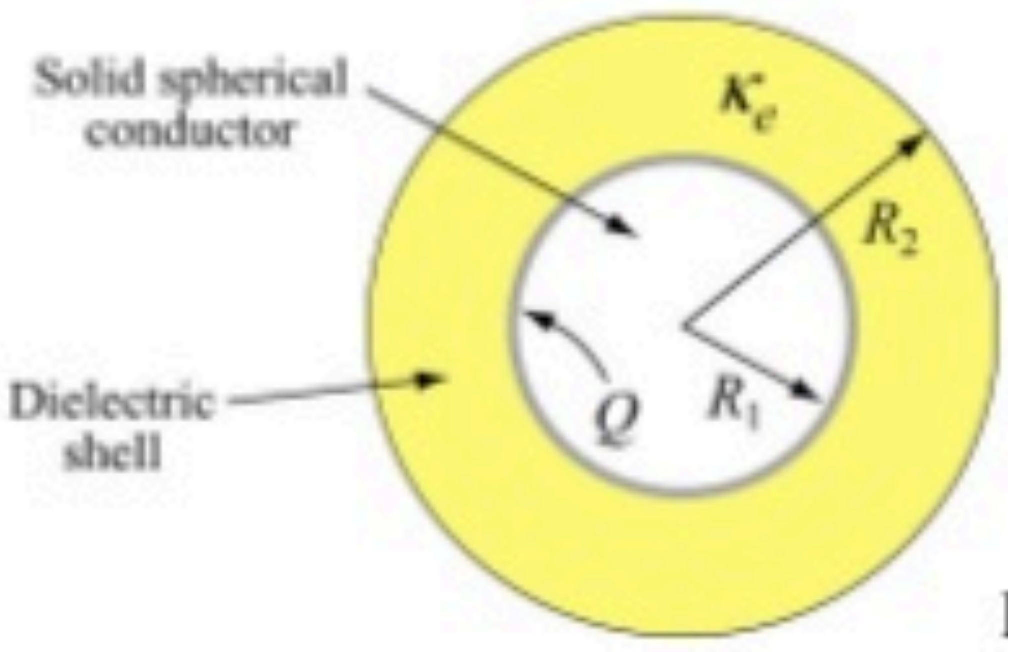
$$= P_3(\cos\theta)$$

$$\Rightarrow B_3 = R^4$$

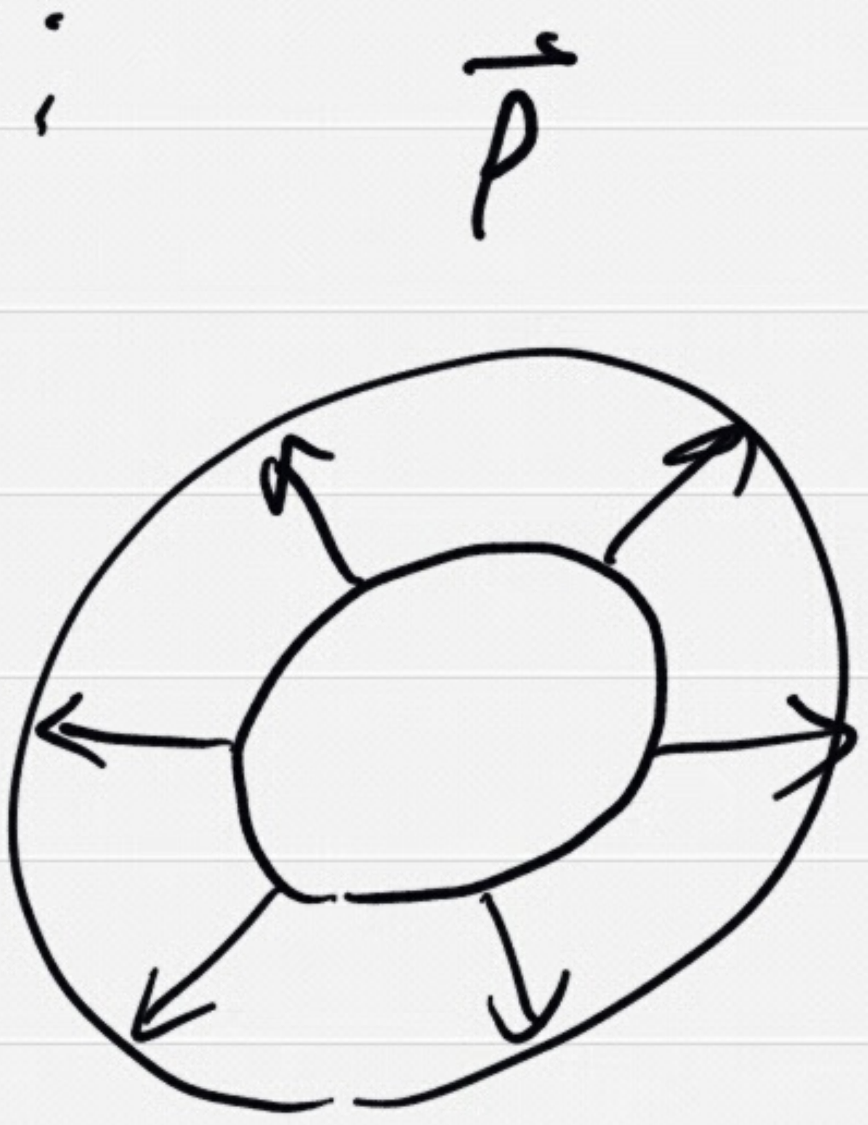
$$\boxed{V_>(r,\theta) = R^4/r^4 \, P_3(\cos\theta)}$$

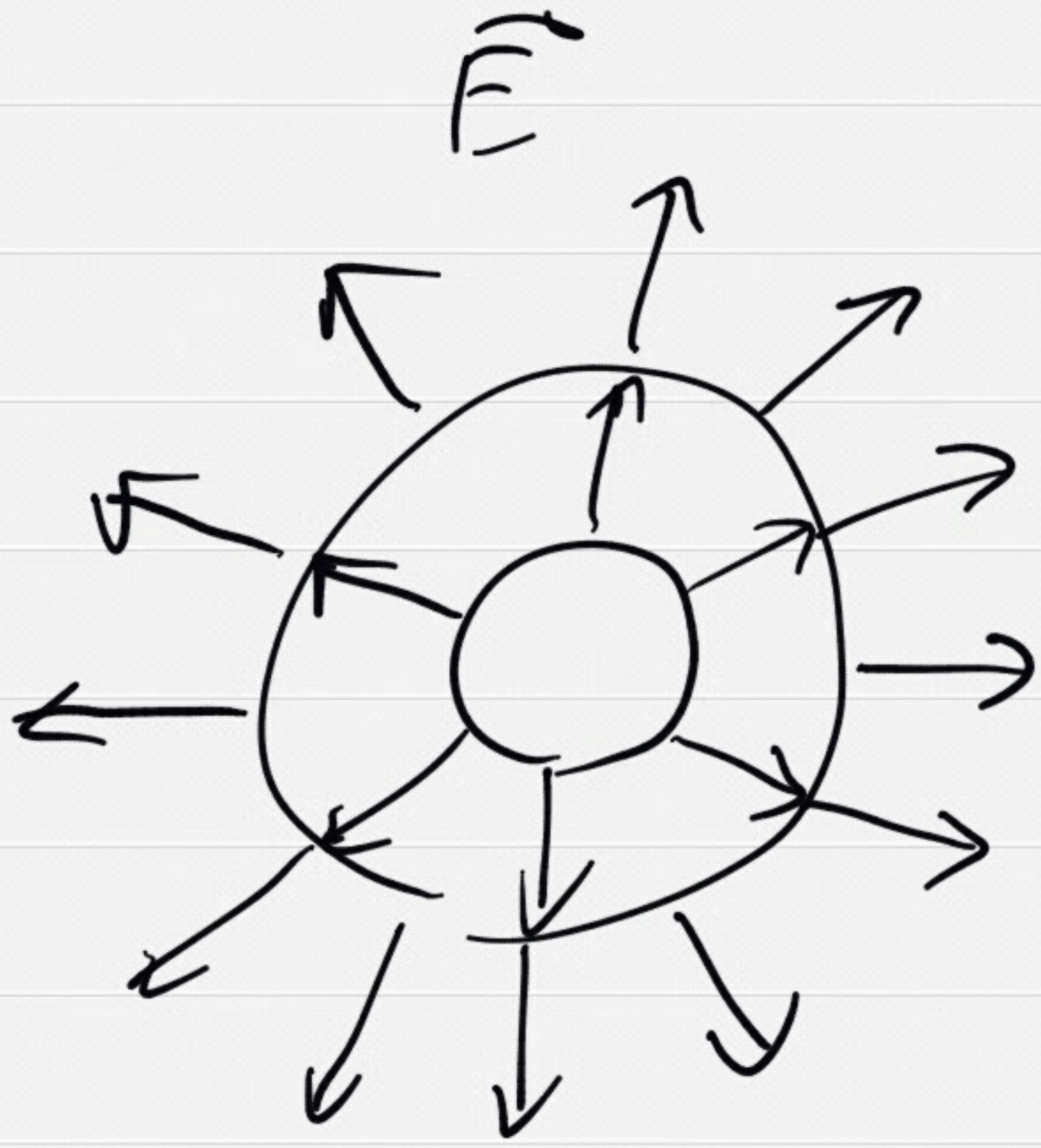
Check Your Understanding #4

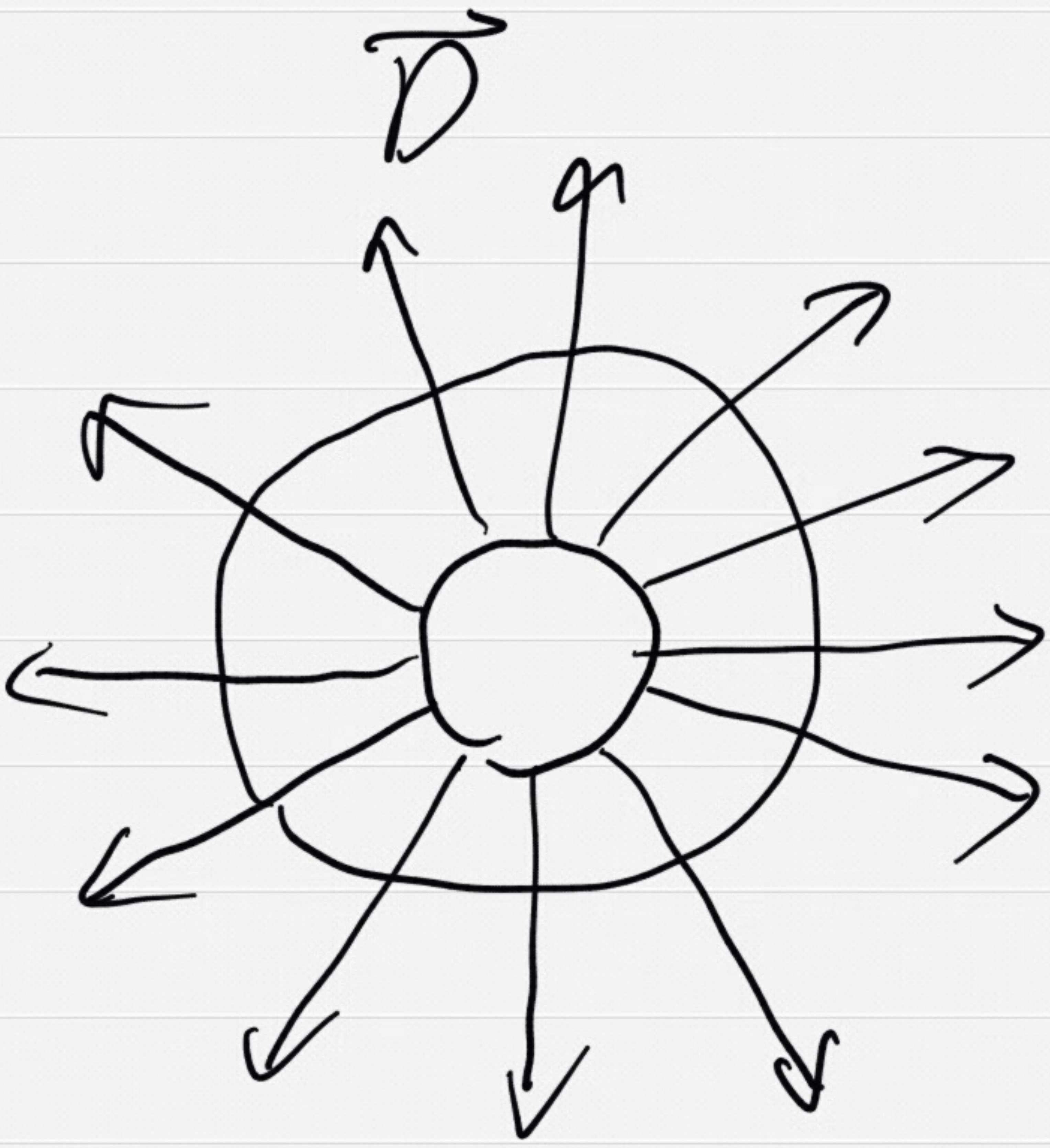
- Consider a dielectric shell surrounding a conductor with charge +Q on its surface.
- Draw three diagrams, showing the lines of polarization, electric field, and electric displacement. Be qualitatively accurate, with more lines for stronger fields.

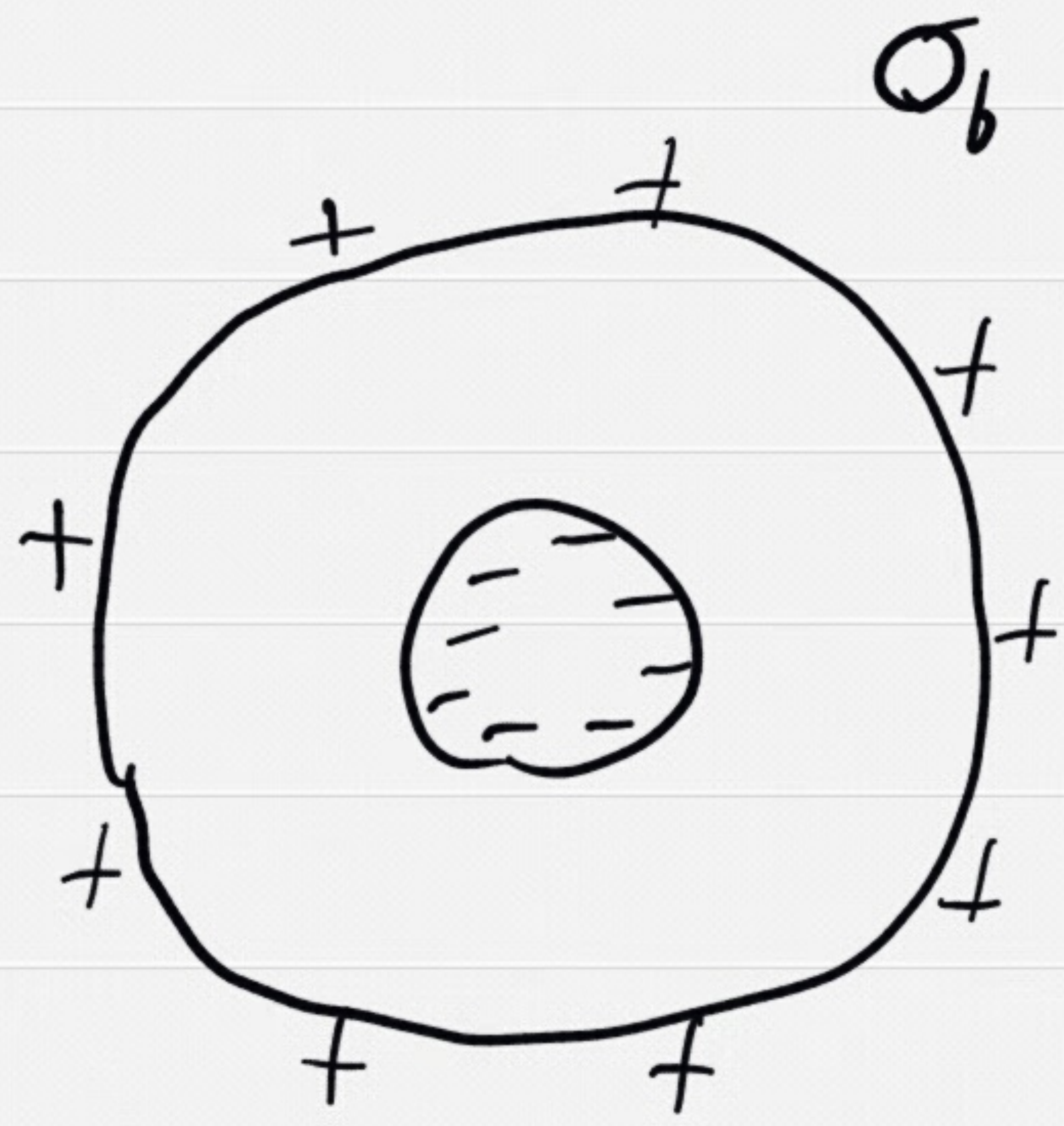


Q4:



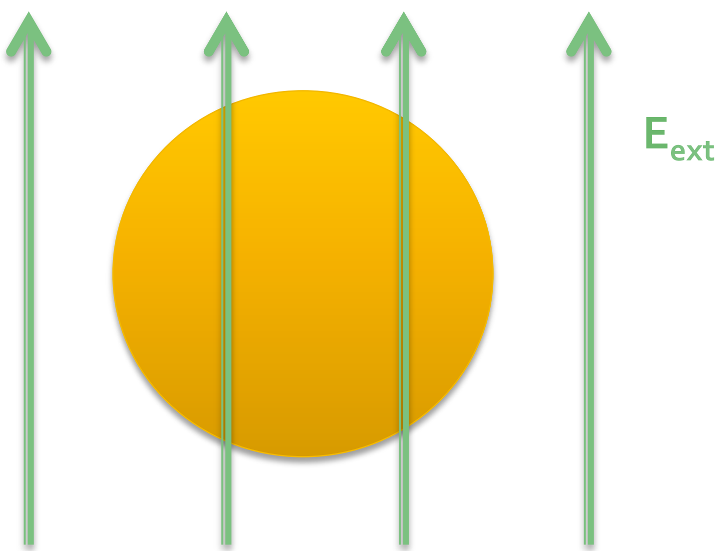






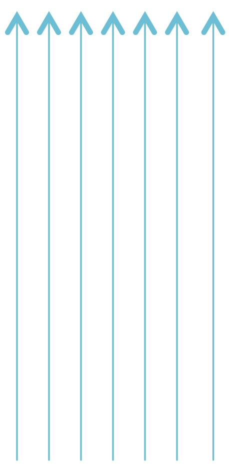
Check Your Understanding #5

- Consider a dielectric sphere embedded in an initially uniform external field. Sketch the resulting lines of polarization, electric field, and electric displacement.

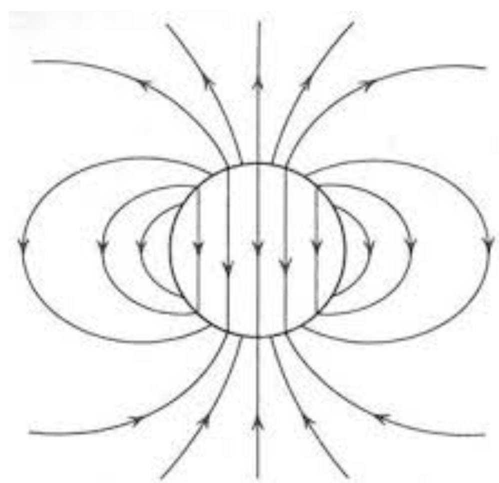


Dielectric Sphere Electric Field

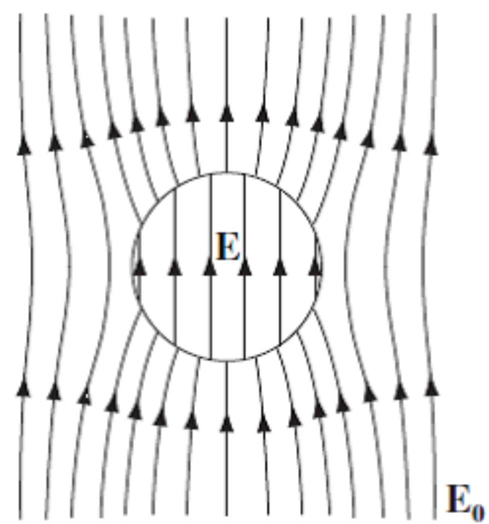
E_{ext}



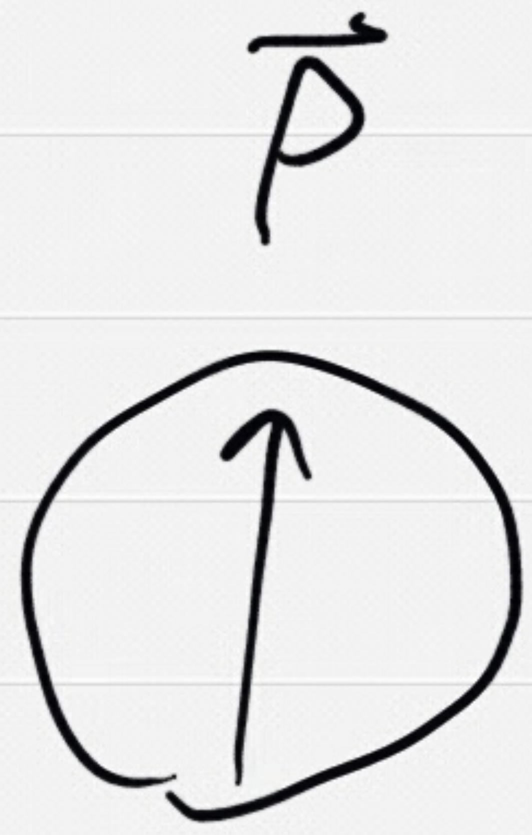
E_{int}

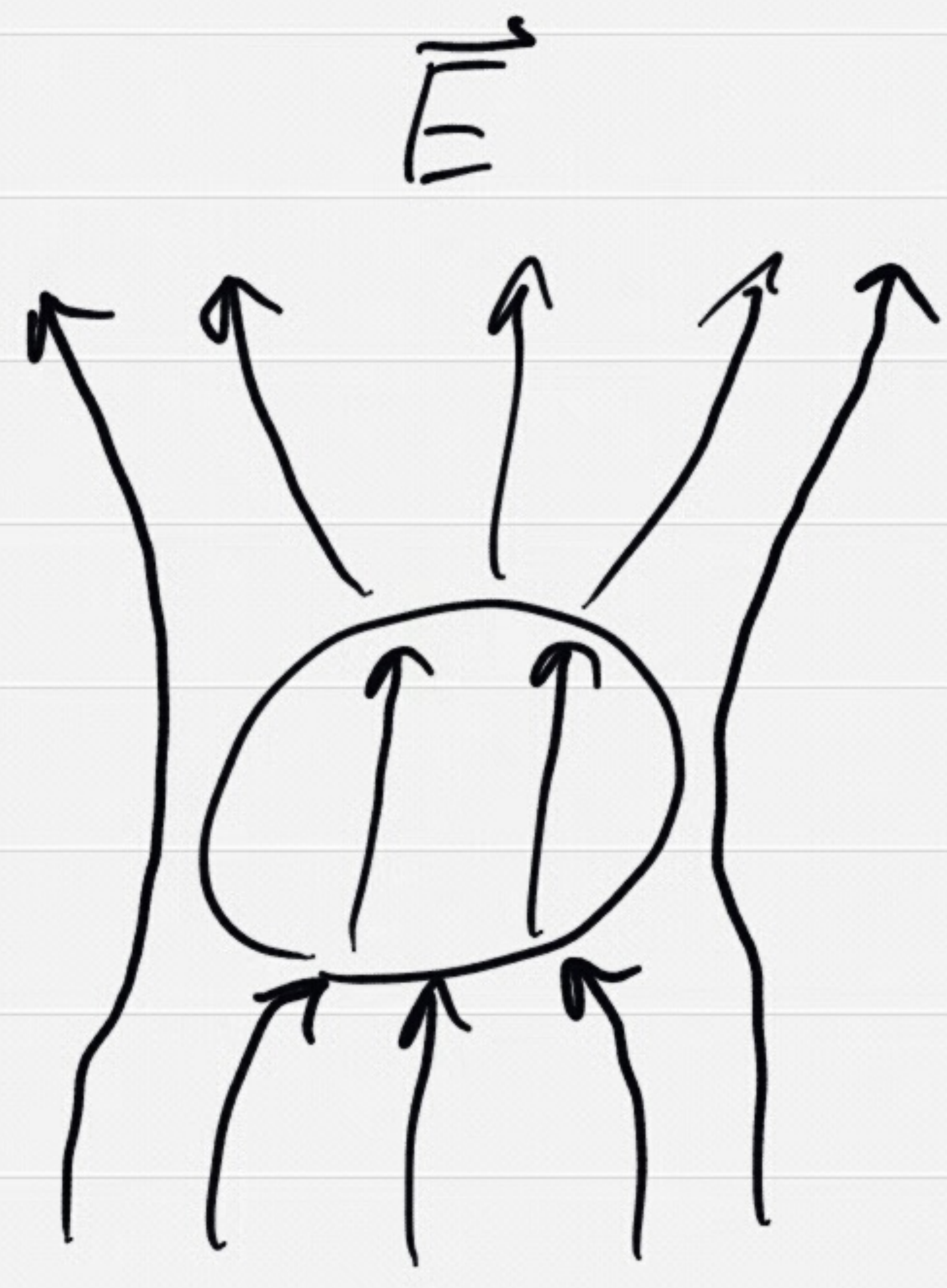


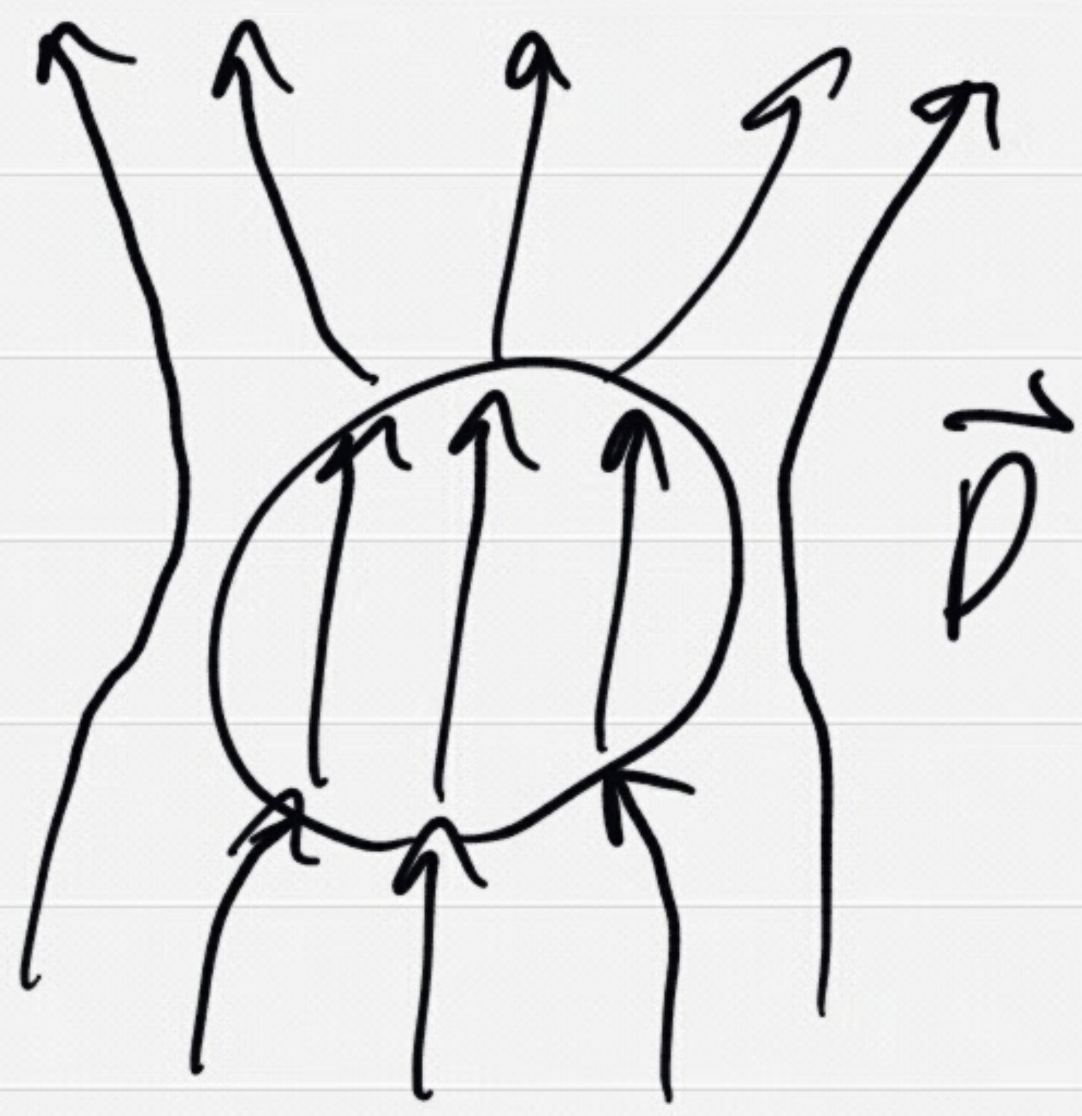
E_{tot}

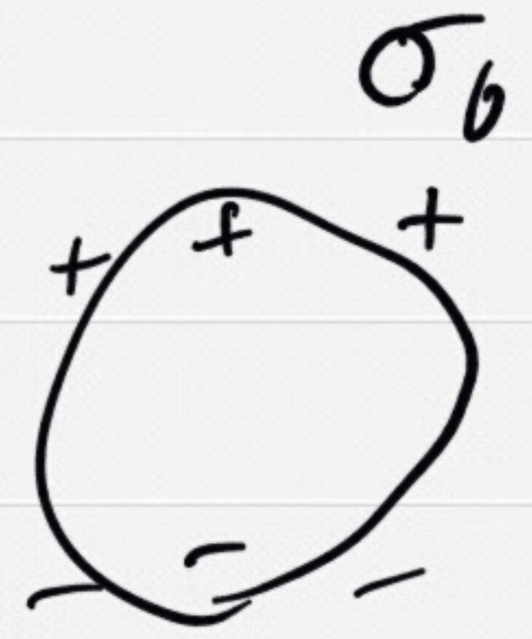


Q5:









Check Your Understanding #6

- In the problem with the dielectric sphere, there is no free charge anywhere

$$\oint \mathbf{D} \cdot da = Q_{f\ enc}$$

- But D ≠ 0

- Why not?

Q6:

$$\oint \vec{D} \cdot d\vec{a} = Q_{f\,enc}$$
$$= 0$$

But $\vec{D} \neq 0$

$\vec{D} \cdot \hat{n} > 0$ over half

$\vec{D} \cdot \hat{n} < 0$ over half

cancels out in $\oint \vec{D} \cdot d\vec{a}$