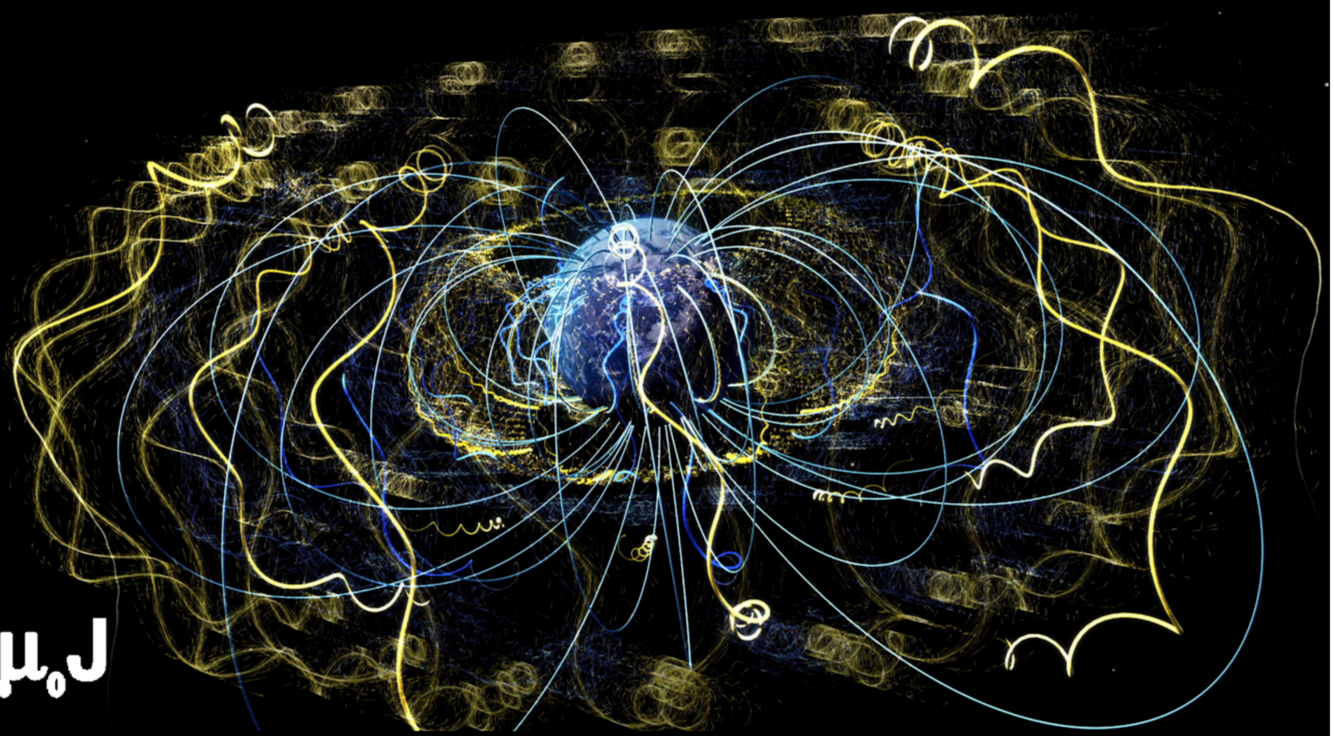
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



# Electricity and Magnetism I: 3811

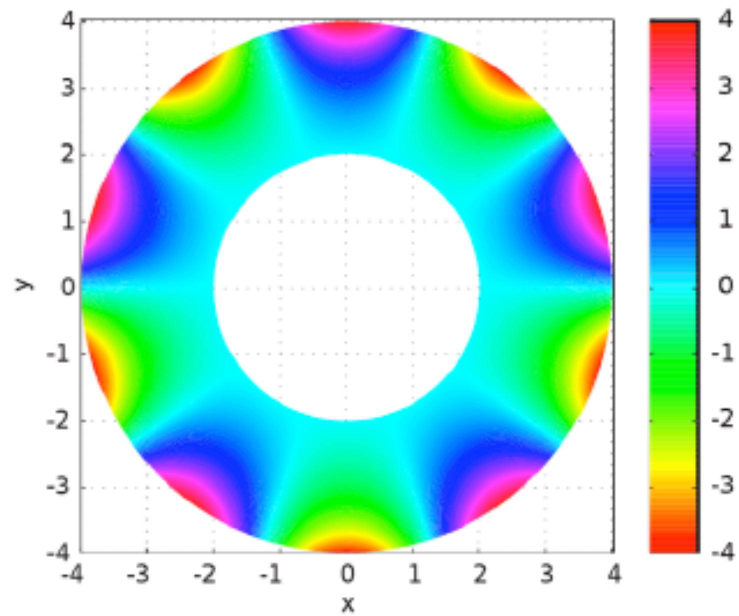
Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

# Midterm II

- Exam 2 on Wednesday 11/6
  - In class, same rules as Exam 1
  - Covers Ch. 3-4 in Griffiths = lecture material since last exam
  - Exam 2 Equation sheet posted
    - Bring yours to the exam, annotated as desired
  - Practice midterms and solutions posted

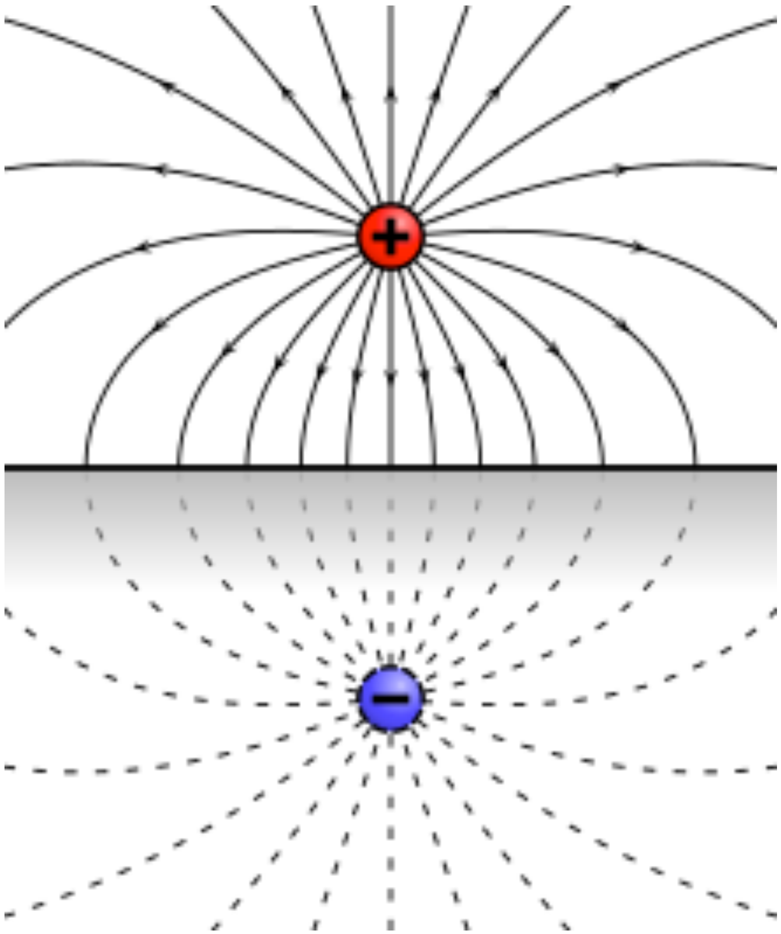
# Laplace's Equation

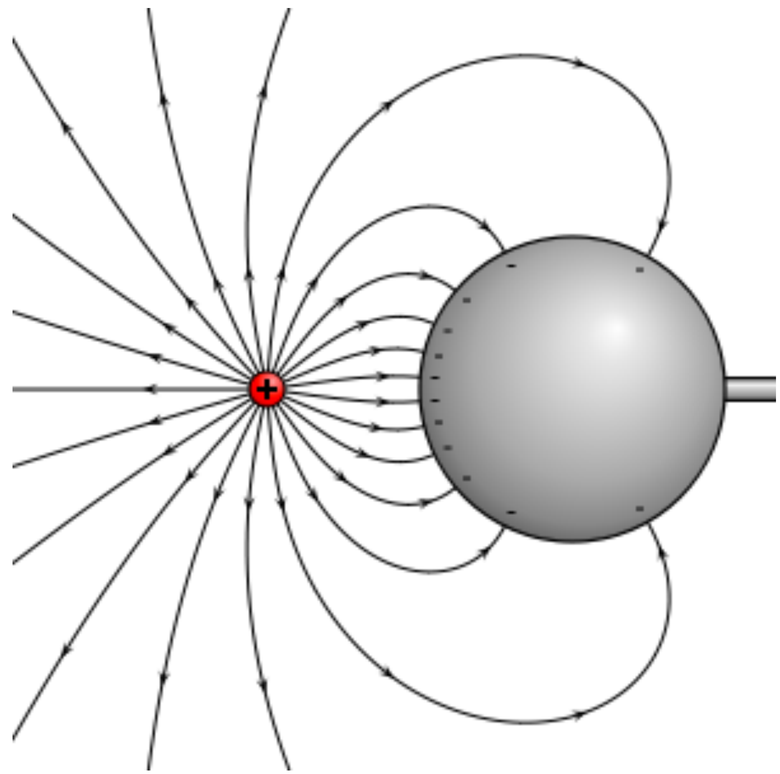
- Solutions have zero curvature
- Value at any point is average of "local neighborhood"
- Local extrema only on boundaries
- If you know V on the boundary, you can find a unique solution




**Laplace's Equation:** $\nabla^2 V = 0 \; if \; \rho = 0$

# Image charges: A rarely useful trick





# The Laplacian Operator

Cartesian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

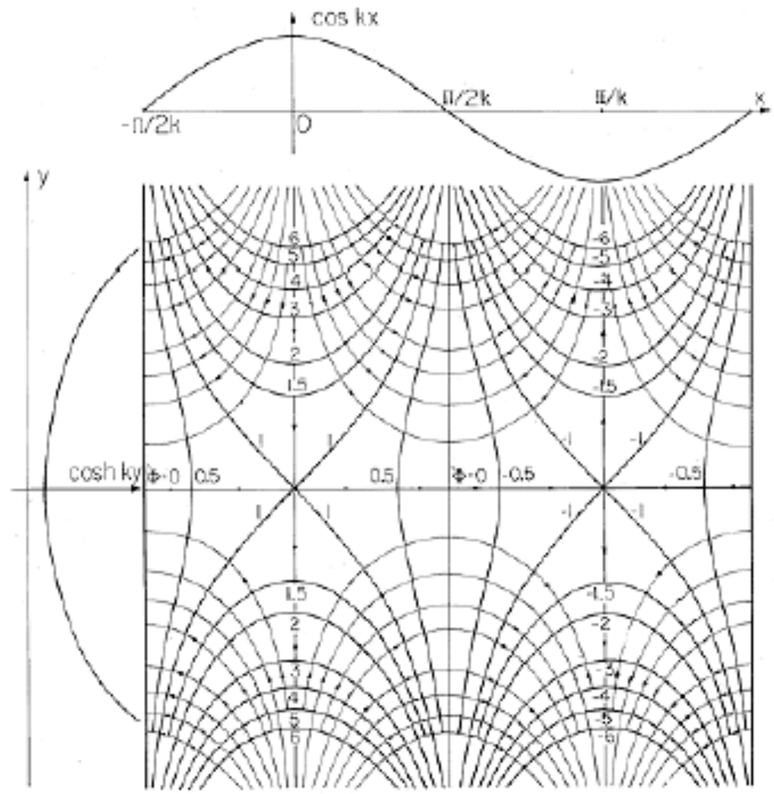
Spherical

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Cylindrical

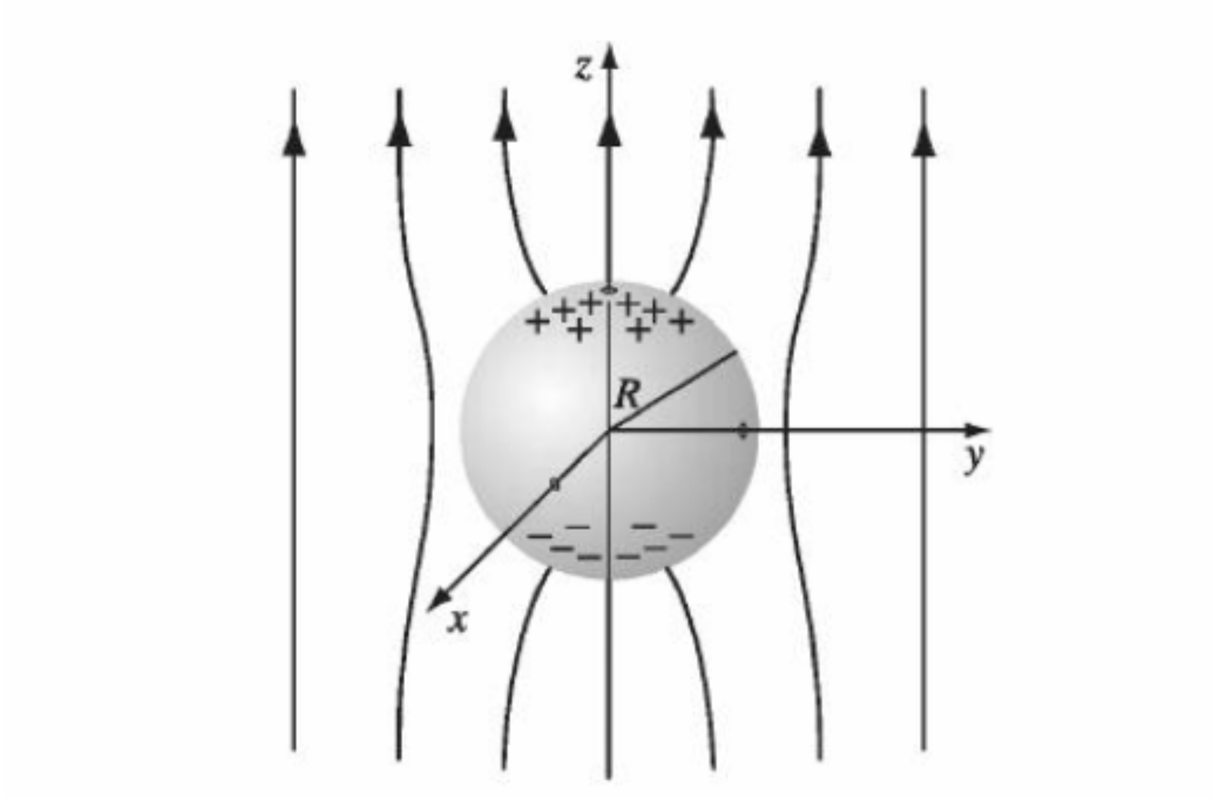
$$\nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

# Separation of Variables: Cartesian




**Separation of Variables:** $\frac{d^2X}{dx^2} = C_1X, \frac{d^2Y}{dy^2} = C_2Y, \frac{d^2Z}{dz^2} = C_3Z$, $C_1+C_2+C_3=0$, $V = X(x)Y(y)Z(z)$

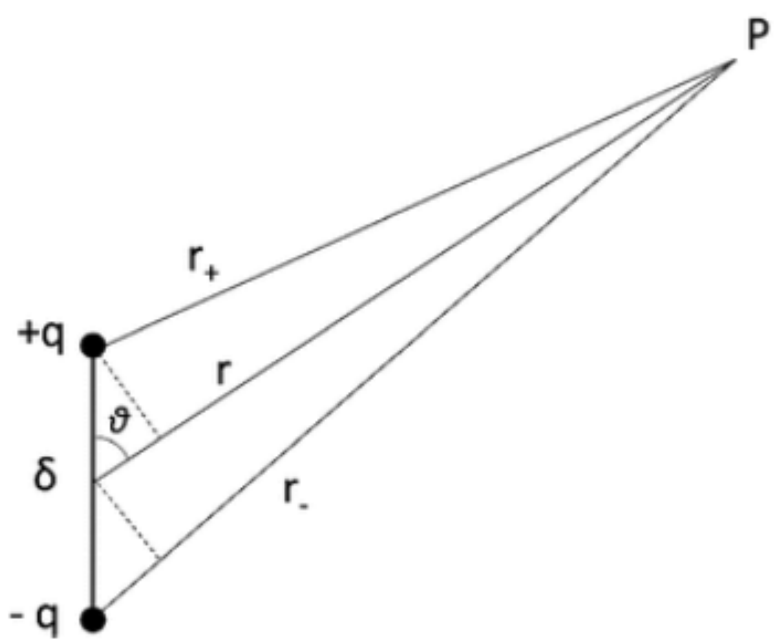
# Separation of Variables: Spherical

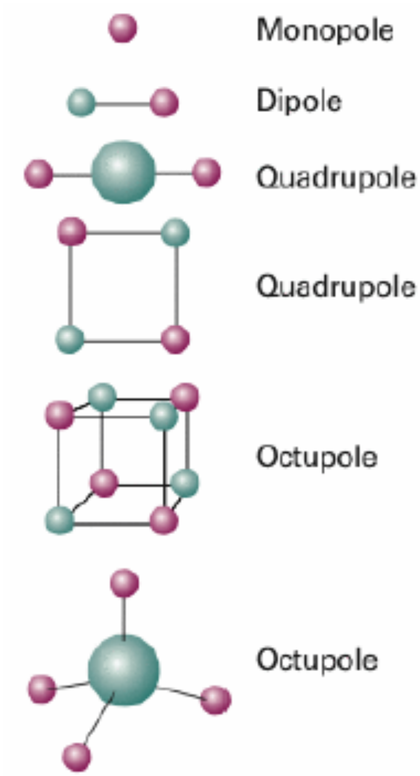



**Separation of Variables (Spherical):** $V(r,\theta) = \sum_0^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(cos\theta)$

$$P_0(cos\ \theta) = 1, P_1(\cos\theta) = \cos\theta\ , P_2(\cos\theta) = \frac{3cos^2\theta - 1}{2}$$

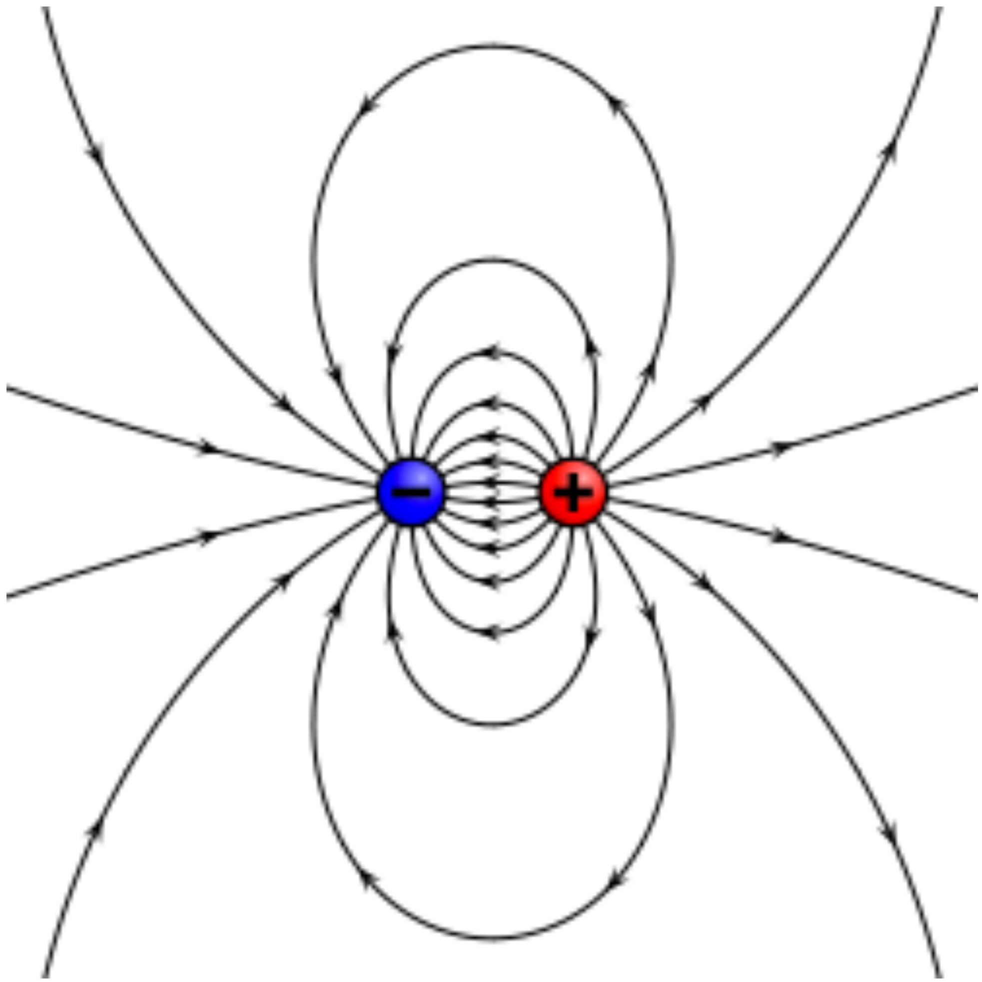
# Multipole Expansion



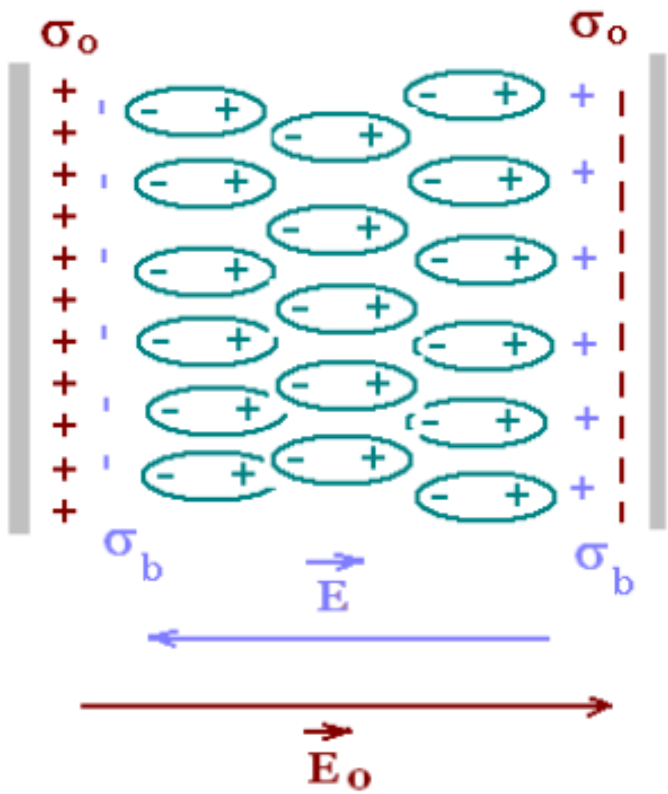

**Multipole Expansion:** $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_0^\infty \frac{1}{r^{n+1}}\int (r')^n P_n(cos\alpha)\rho(\vec{r}')d\tau'$

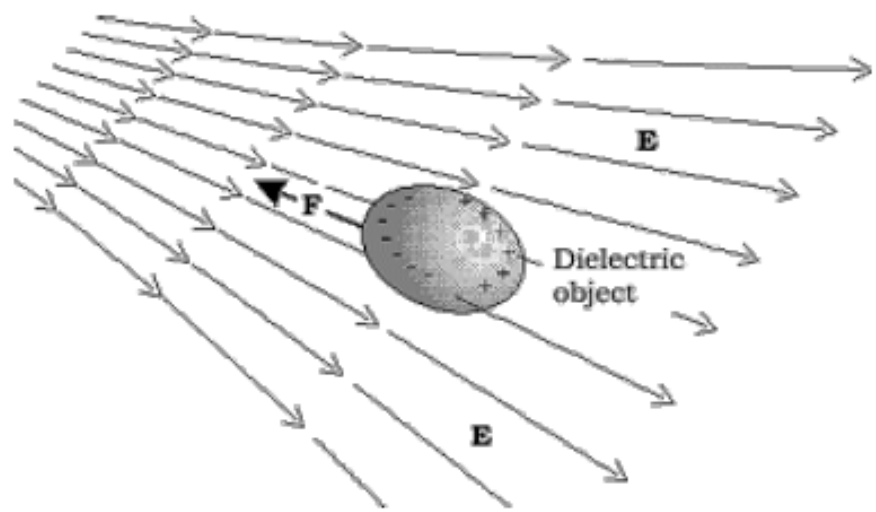
# Dipole Moment



**Dipoles:** $\vec{p} = \int \vec{r}' \rho(\vec{r}') d\tau', \;\; \vec{p} = \int \vec{r}' \sigma(\vec{r}') da', \;\;\; \vec{p} = \int \vec{r}' \lambda(\vec{r}') dl'$

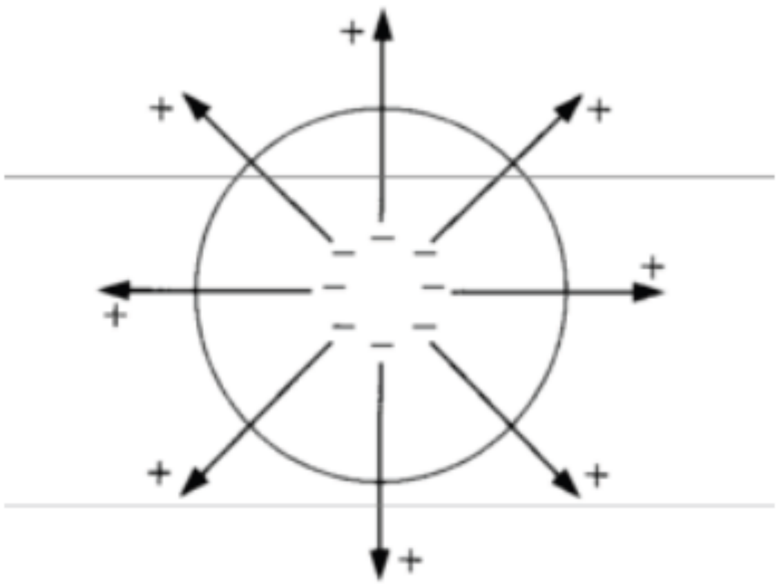
# Dipoles and Polarization

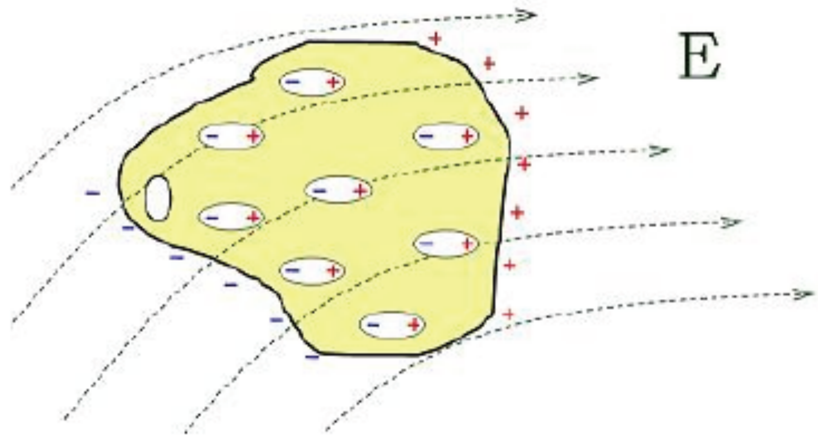



$$V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}, \quad \vec{\tau} = \vec{p}\times\vec{E}, \ \vec{F} = (\vec{p}\cdot\nabla)\vec{E}, \quad \vec{P} = \vec{p}/volume$$

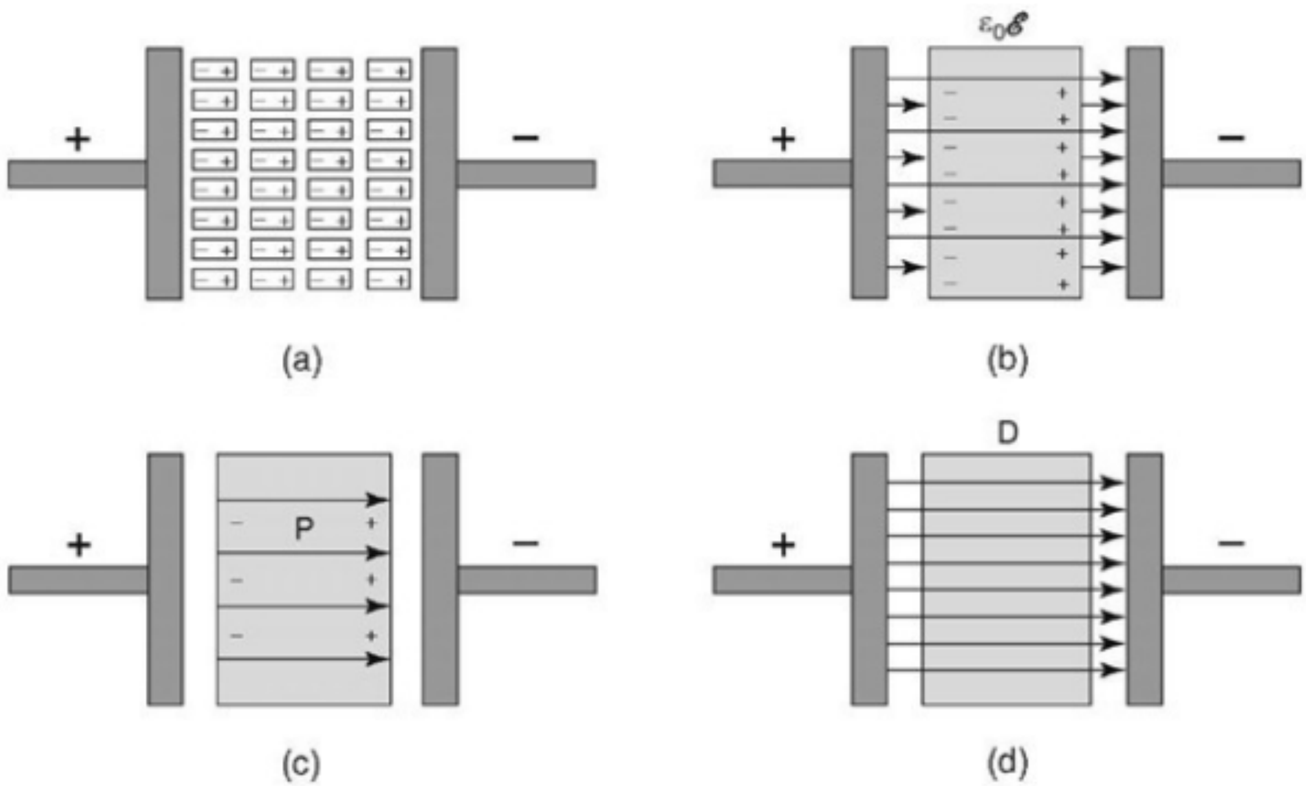
# Bound Charge





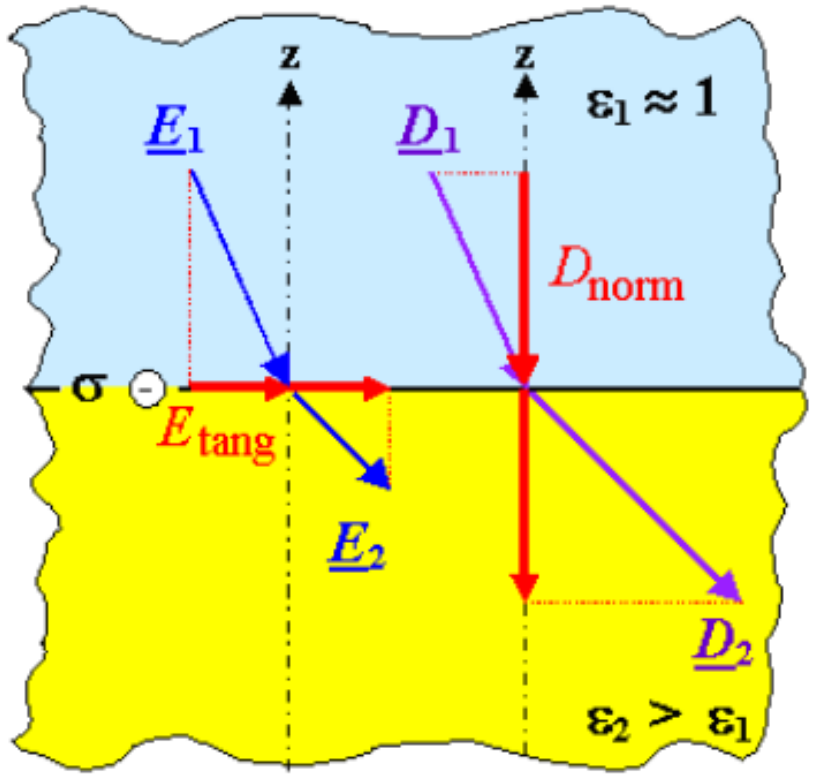

$$\sigma_b = \vec{P} \cdot \hat{n},\ \rho_b = -\nabla \cdot \vec{P}$$

# Electric Displacement



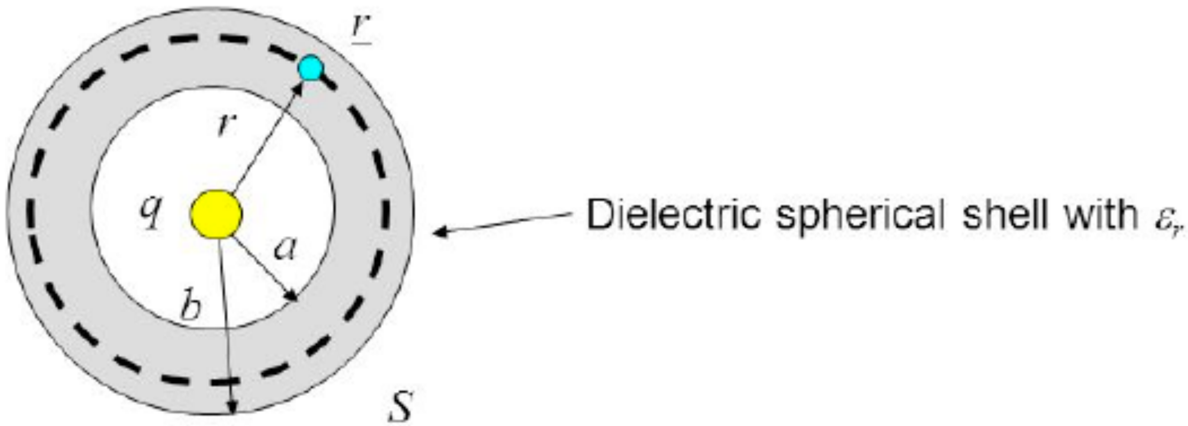

**Dielectrics:** $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ $\oint \vec{D}\cdot\overrightarrow{da} = Q_{f\_enc}$, $\nabla\cdot\vec{D} = \rho_{free}$

# Boundary Conditions




**Boundary Conditions:** $\Delta V = 0,\ \Delta\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n},\quad \Delta D_\perp = \sigma_f,\quad \Delta\vec{D}_{||} = \Delta\vec{P}_{||}$

# Linear Dielectrics




**Linear Dielectrics:** $\vec{P} = \varepsilon_0 \chi_e \vec{E},\ \ \vec{D} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_0 \vec{E} = (1+\chi_e)\varepsilon_0 \vec{E}$

$$W_{elec+polarization} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,d\tau$$