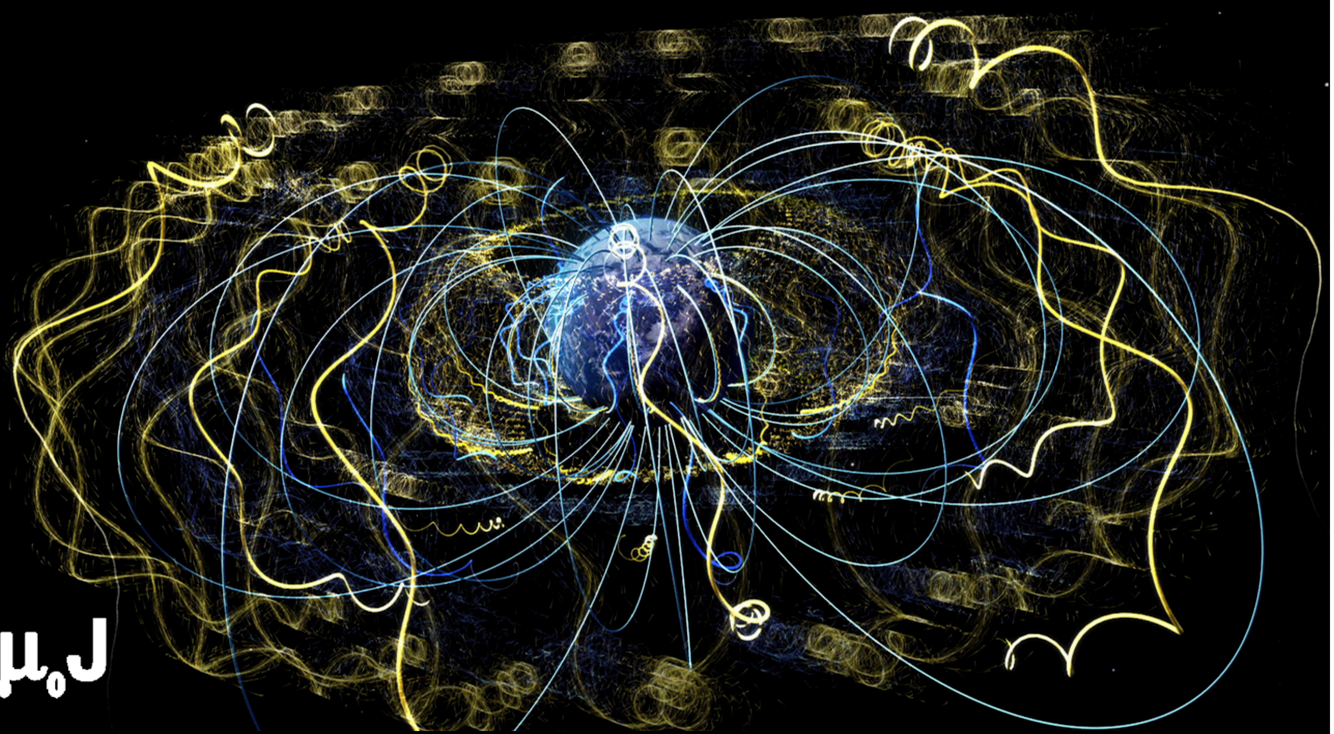
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

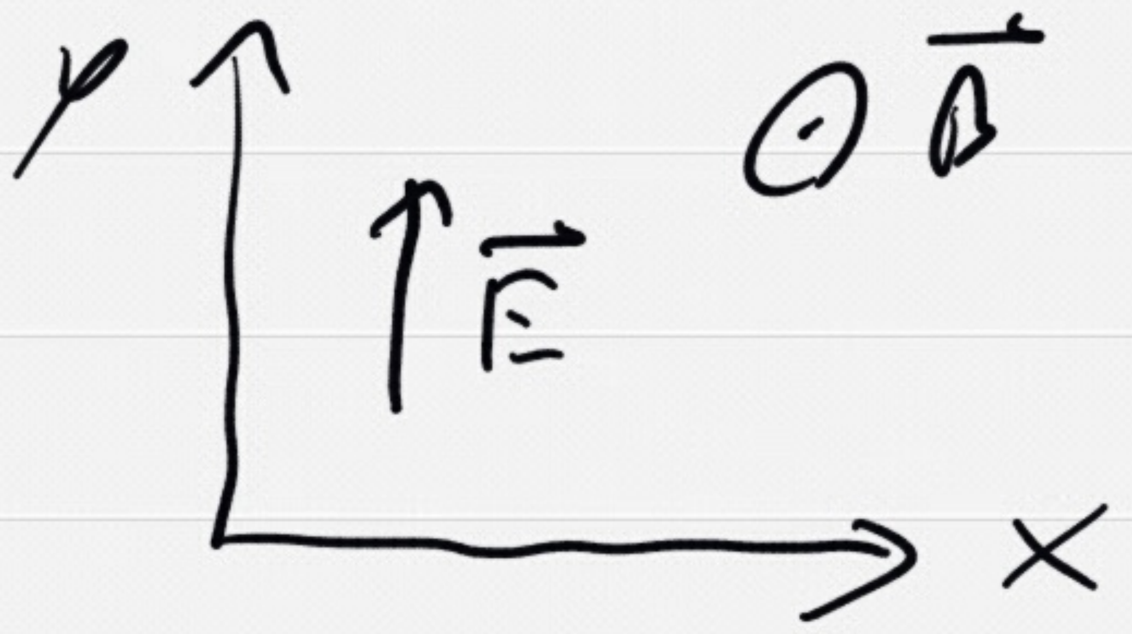
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Cycloid Motion: $\vec{E}\times\vec{B}$ Drift



$\vec{r}_0 = 0$

$\vec{v}_0 = 0$

$$\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$$

$$= QE\hat{y} + Qv_yB\hat{x} - Qv_xB\hat{y}$$

$$= m\vec{a} = m\left(dv_x/dt\,\hat{x} + dv_y/dt\,\hat{y}\right)$$

$$\Rightarrow \quad dv_x/dt = \frac{QB}{m}v_y = \omega v_y$$

$$dv_y/dt = \frac{QE}{m} - \frac{QB}{m}v_x = \omega\left(\frac{E}{B} - v_x\right)$$

$$d^2v_y/dt^2 = -\omega\, dv_x/dt = -\omega^2 v_y$$

$\Rightarrow v_y = v_0\sin(\omega t)$ since $v_y(0)=0$

$$dv_x/dt = \omega v_0 \sin(\omega t)$$

$\Rightarrow v_x =$ const. $- v_0\cos(\omega t)$

$= v_0(1-\cos(\omega t))$ since $v_x(0)=0$

$$v_0\omega\cos(\omega t) = \omega\left(\frac{E}{B} - v_0 + v_0\cos(\omega t)\right)$$

$$\Rightarrow v_0 = E/B$$

$V_x = E/B\,(1 - \cos(\omega t))$
$V_y = E/B\,\sin(\omega t)$

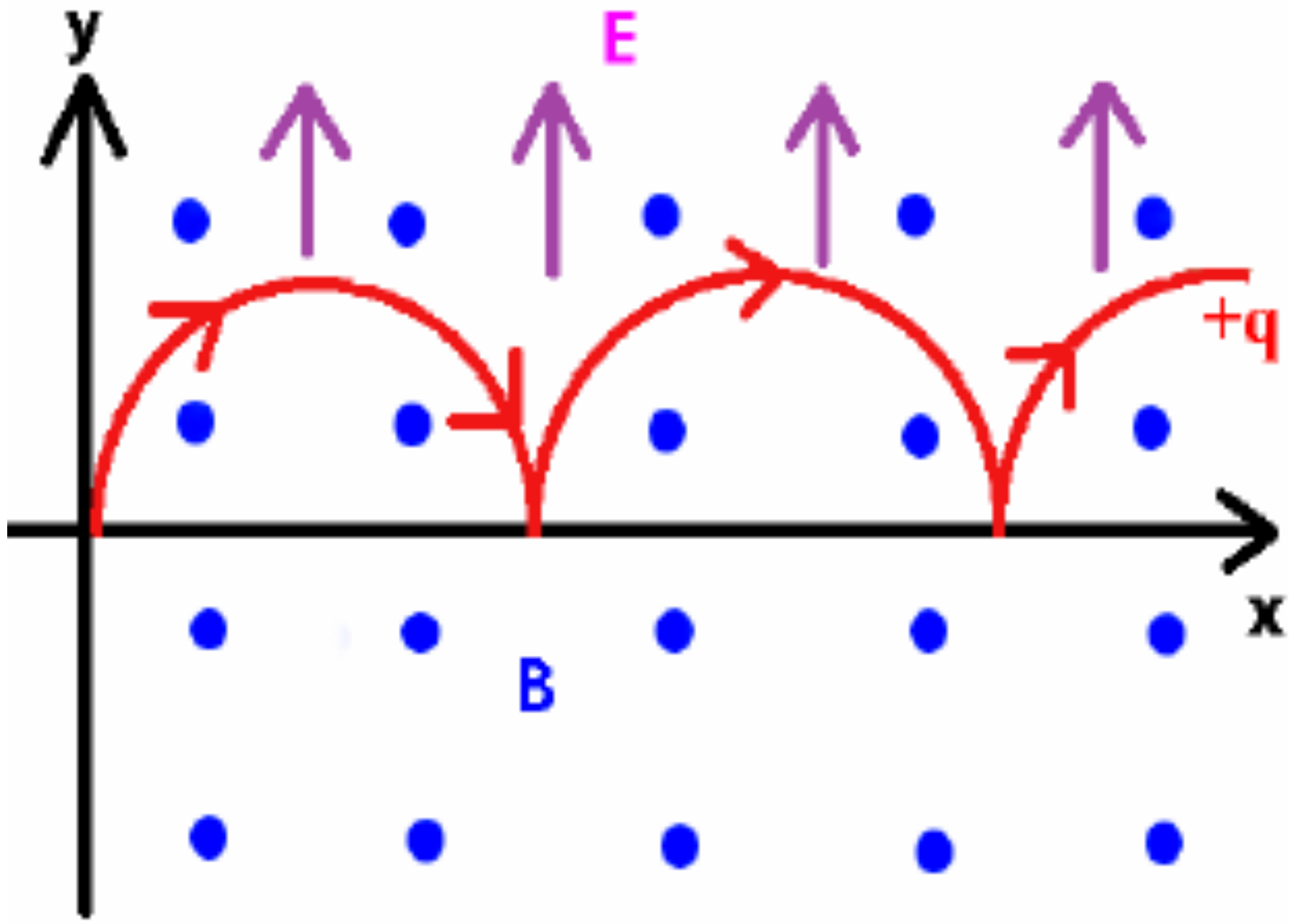
circular motion
+ constant $V_x = E/B$

drift is along $\vec{E} \times \vec{B}$
direction even though
average force is along $\vec{E}$

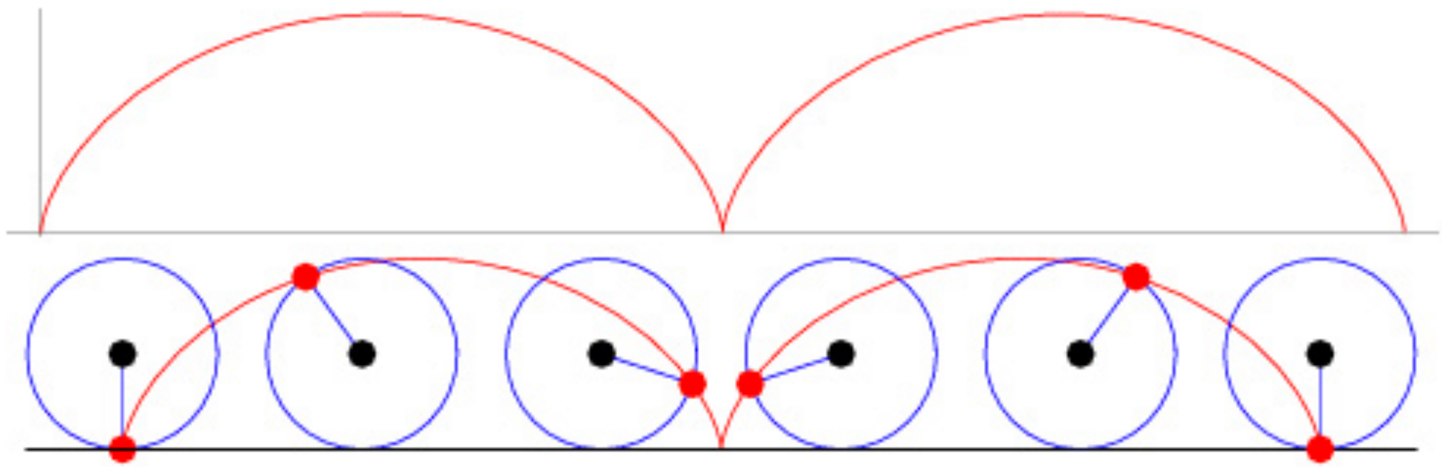
average velocity $= E/B$

generally $\vec{V}_{drift} = \dfrac{\vec{E} \times \vec{B}}{B^2}$

Cycloid Motion

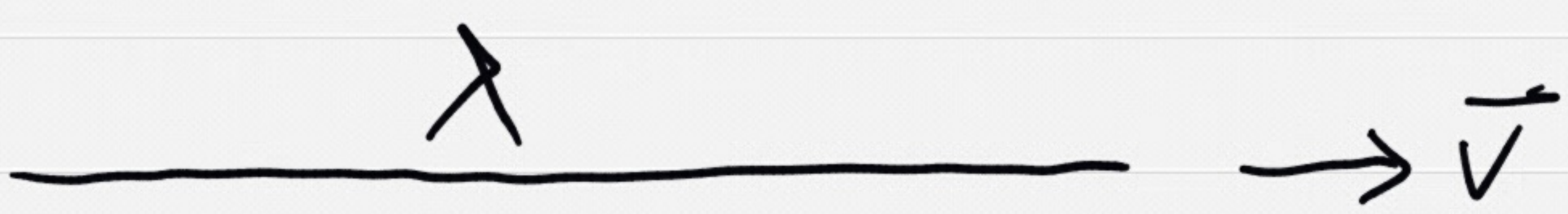


Cycloid Motion



Currents

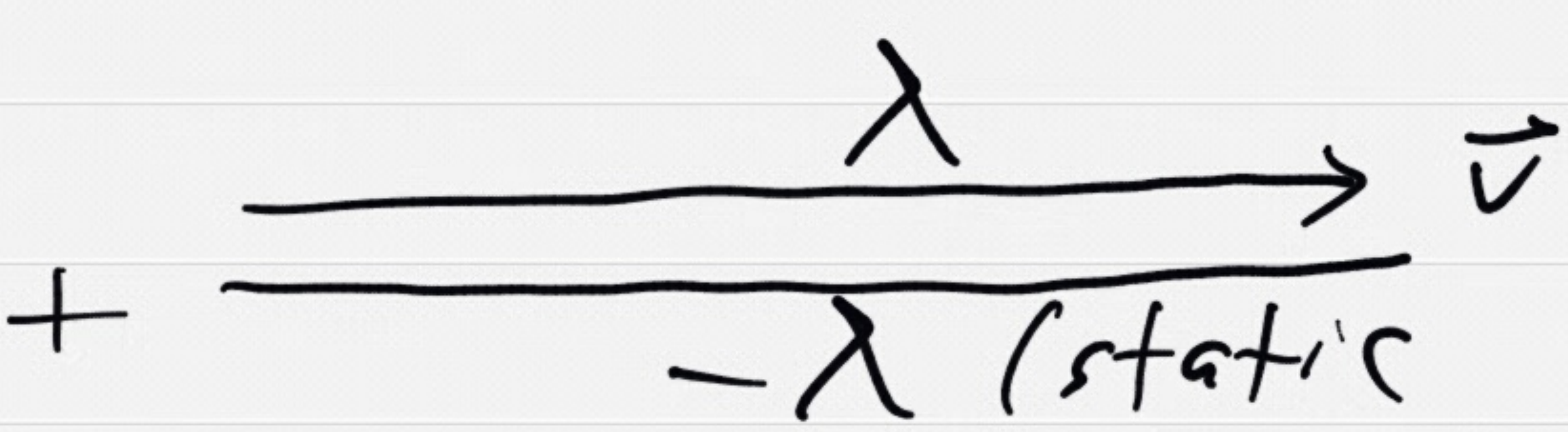
current = charge passing per unit time



$$\vec{I} = \lambda \vec{v} = \frac{\text{charge}}{\text{length}} \cdot \frac{\text{length}}{\text{time}}$$

$$= \text{charge}/\text{time}$$

Note:



- Has no net charge, but still has a current

Like a neutral wire w/ electrons carrying current

Force on Currents

$$\vec{F}_b = \int (\vec{v} \times \vec{B})\, dq$$

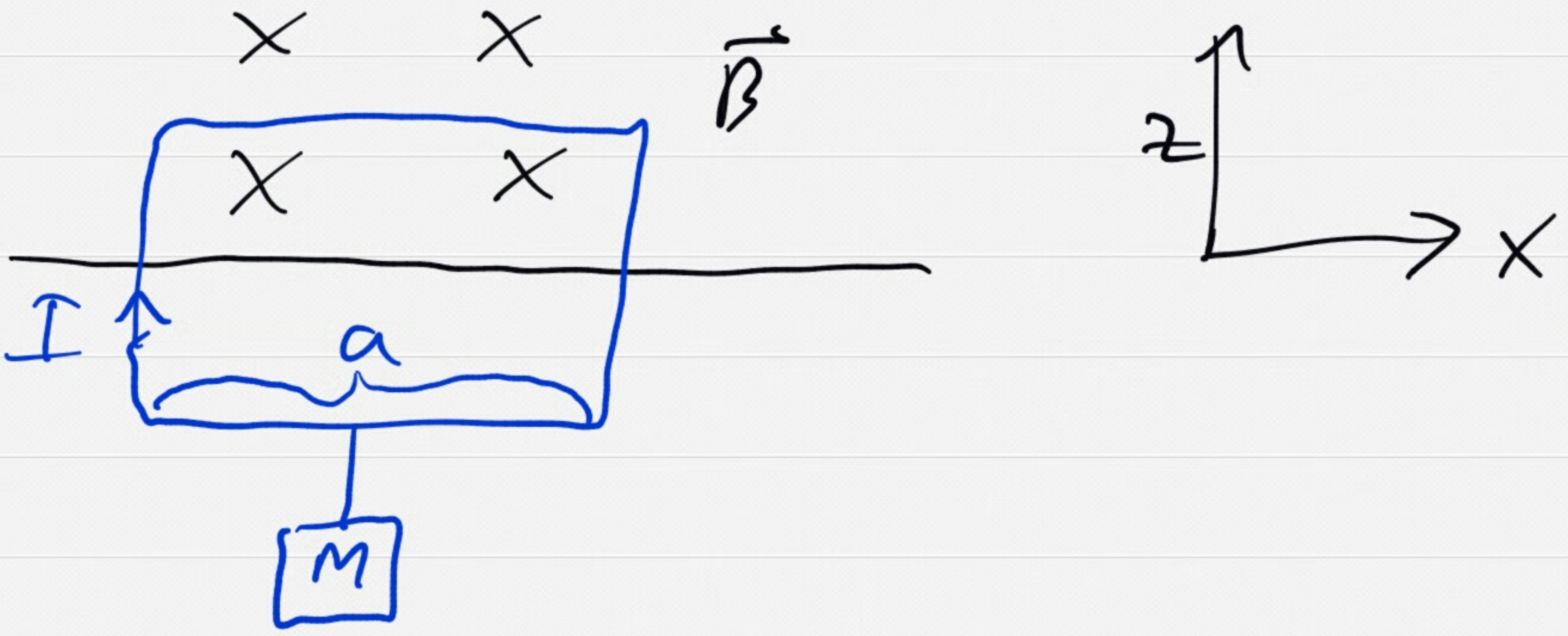
$$= \int (\vec{v} \times \vec{B})\, \lambda\, dl$$

$$= \int (\vec{I} \times \vec{B})\, dl$$

$$= \int I\, d\vec{l} \times \vec{B}$$

if $I = \text{const.}$

Example



$$\vec{F}_g = -mg\,\hat{z}$$

$$\vec{F}_b = \int I\, d\vec{\ell} \times \vec{B}$$

$$= IaB\,\hat{z}$$

$IaB = mg$ balances

What if $IaB > mg$

$\Rightarrow$ upward velocity u_z

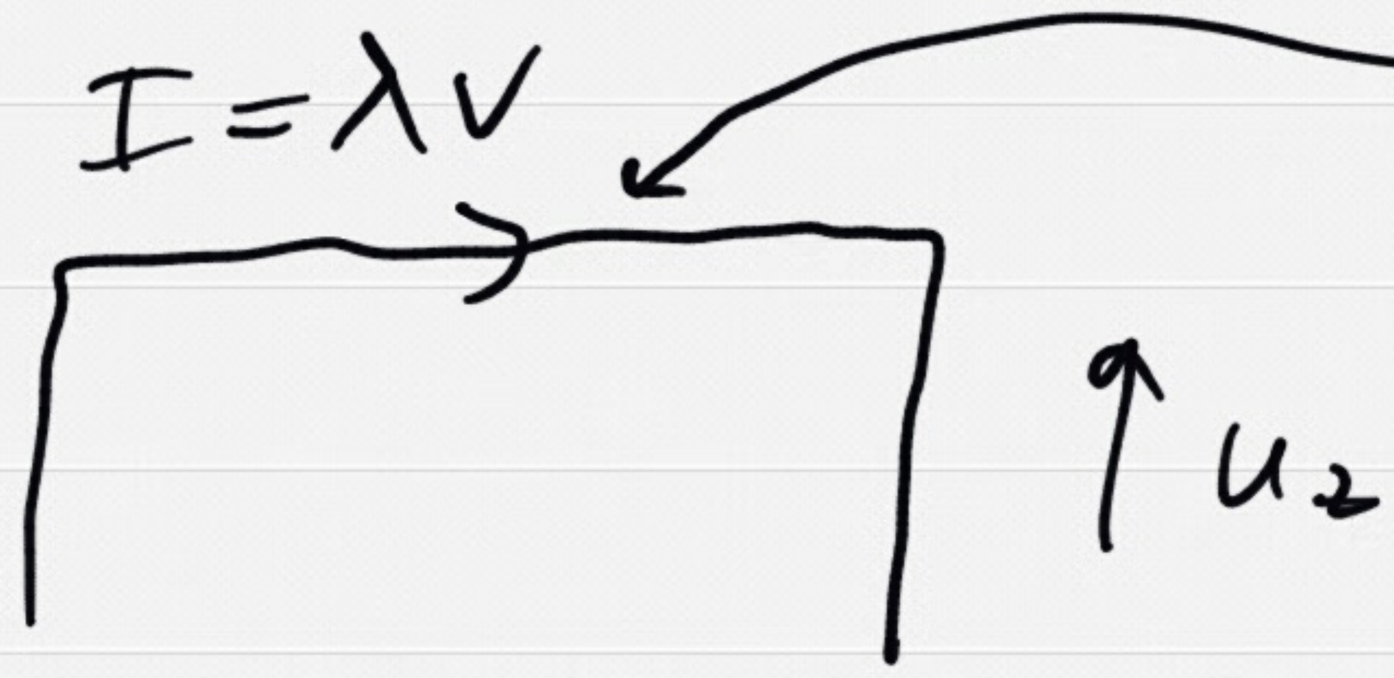
$$W = \int F dz = \int (IaB - mg)\, dz$$

$$= IaBh - mgh$$

$$> 0$$

What does work??

Consider charge here



$$\bar{F}_1 = I a B \hat{z} - \lambda a u_z B \hat{x}$$

tries to slow down charge

To maintain I, battery must overcome

$$\lambda a u_z B \hat{x}$$

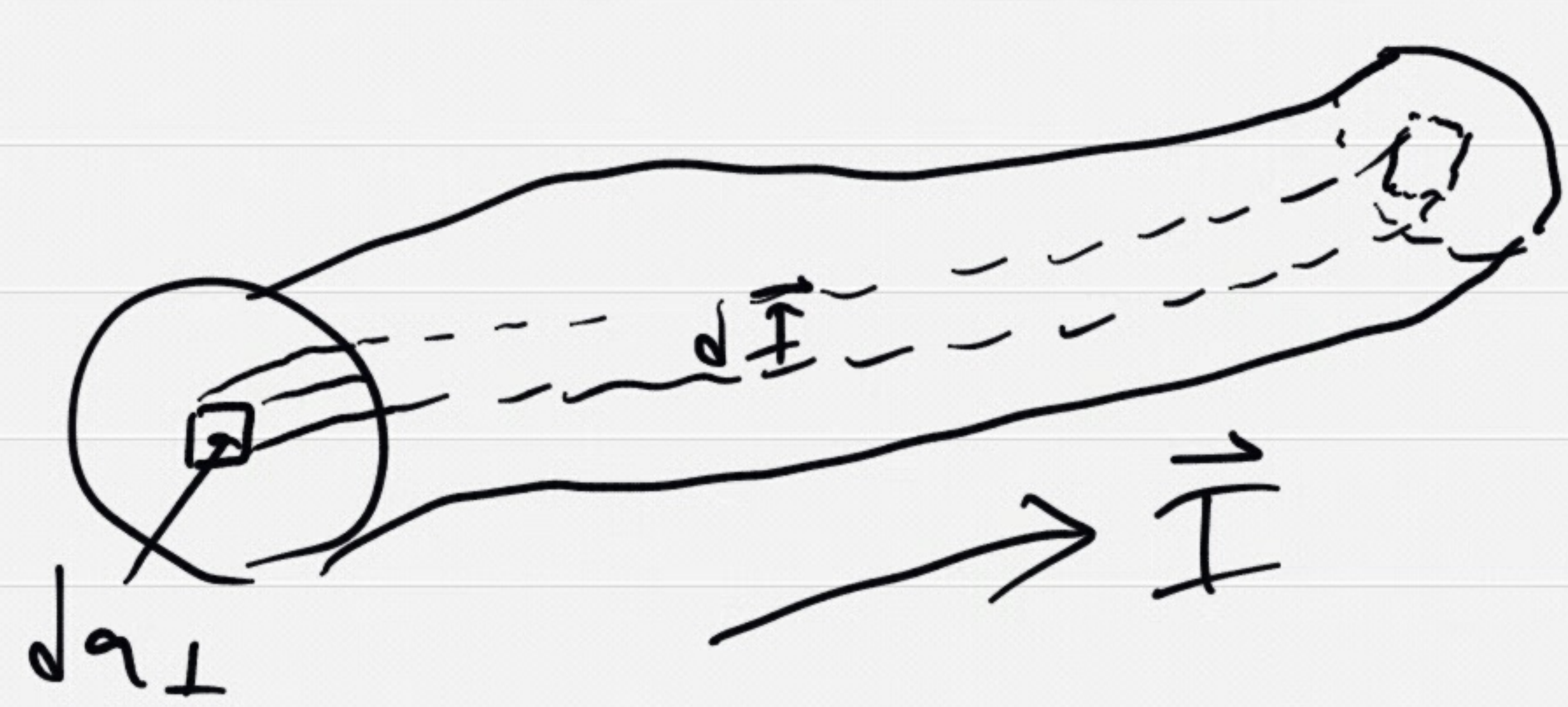
$$
\begin{aligned}
W_{battery} &= \int \lambda a u_z B \, dx \\
&= \int \lambda a u_z B \frac{dx}{dt} \, dt \\
&= \int (\lambda v) a u_z B \, dt \\
&= \int I a u_z B \, dt \\
&= I a B h
\end{aligned}
$$

Supplies all positive work

Surface & Volume Current density

$$\vec{K} = d\vec{I}/dl_\perp$$

$$= \sigma \vec{v} \quad (A/m)$$



$$\vec{J} = d\vec{I}/da_\perp = \rho \vec{v} \quad (A/m^2)$$