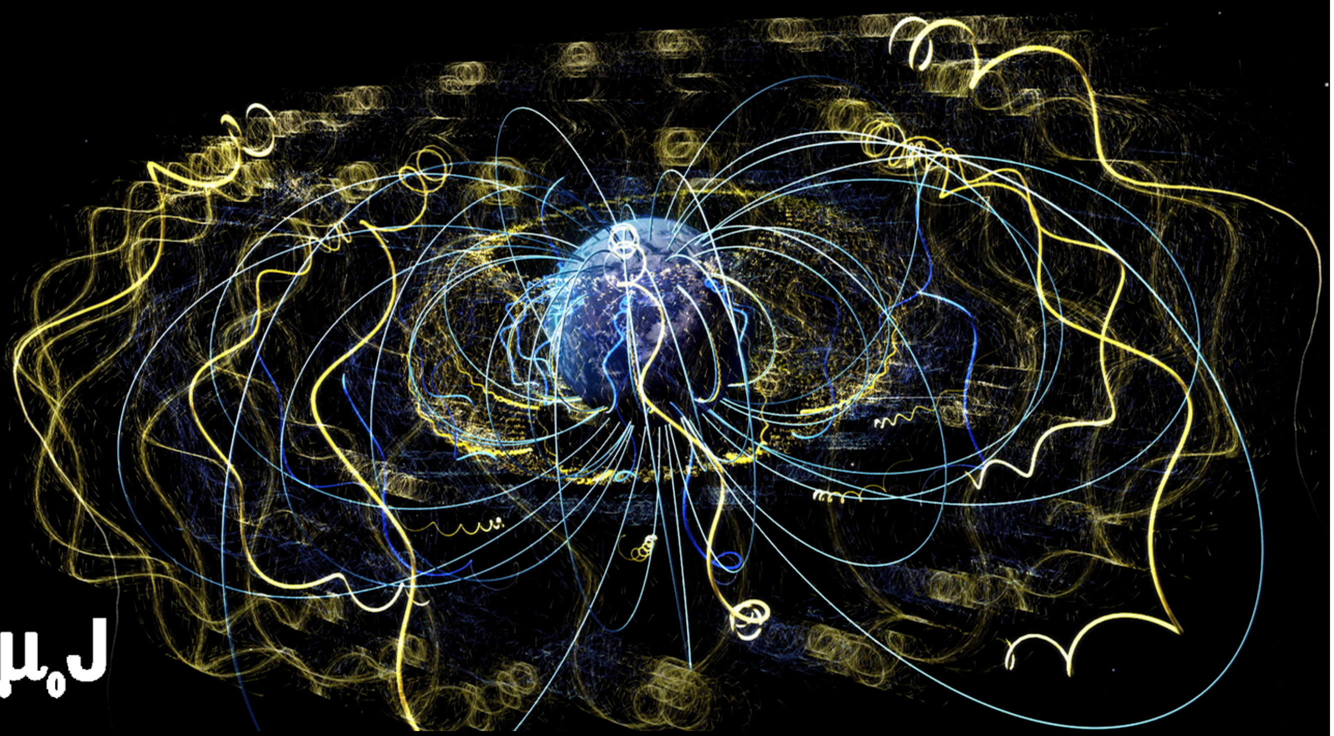
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas

Van Allen 301

MWF 9:30-10:20 Lecture

Continuity

conservation of charge

$$dq_{total}/dt = 0$$

- Flow of charge out of volume

$$I = \oint \vec{J} \cdot d\vec{a}$$
$$= \int_V \nabla \cdot \vec{J} \, d\tau$$
$$= -dq_{enc}/dt$$
$$= -\frac{d}{dt} \int_V \rho \, d\tau$$
$$= -\int_V \partial\rho/\partial t \, d\tau$$

True for any volume

$$\Rightarrow \boxed{\nabla \cdot \vec{J} = -\partial\rho/\partial t}$$

Magnetostatics: $\nabla \cdot \vec{J} = 0$

Biot-Savart Law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}\,') \times \hat{\Delta r}}{\Delta r^2}\, dl'$$

compare to

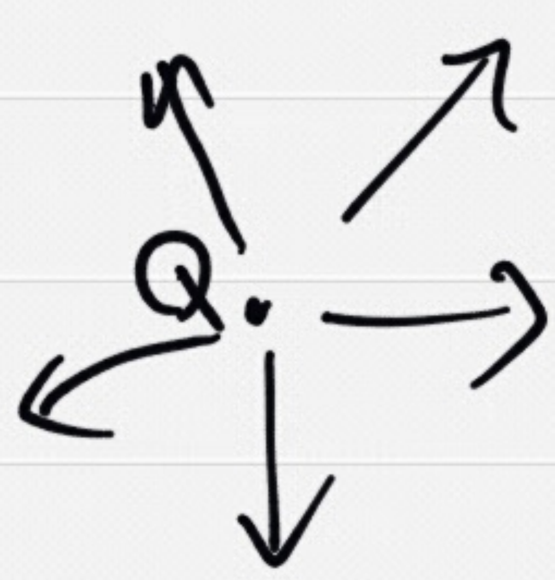
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\vec{r}\,')\, \hat{\Delta r}}{\Delta r^2}\, dl'$$

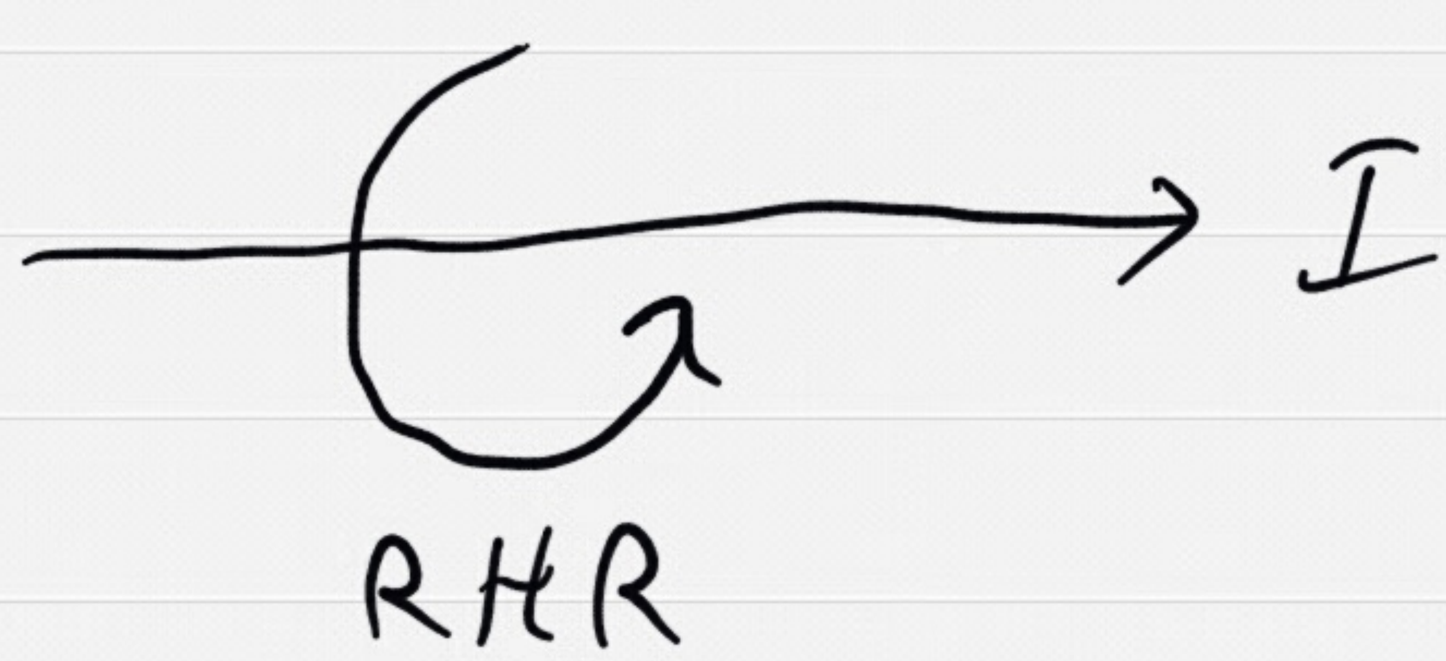
for line charge

$d\vec{E} \parallel \hat{\Delta r}$ radial from source

$d\vec{B} \parallel \vec{I} \times \hat{\Delta r}$ circulating around source

$\vec{E}$ $\qquad\qquad$ $\vec{B}$

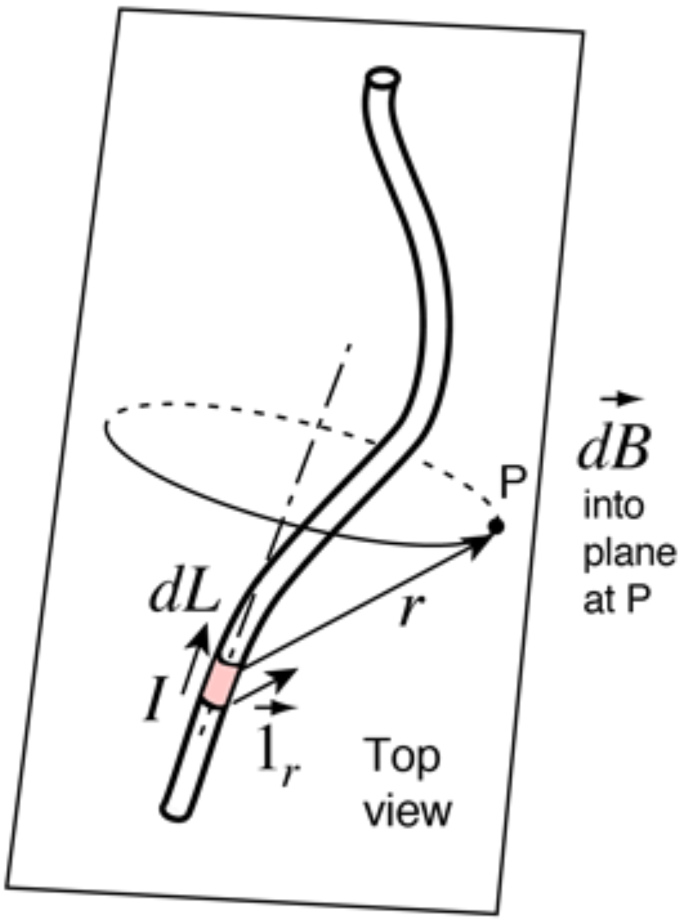




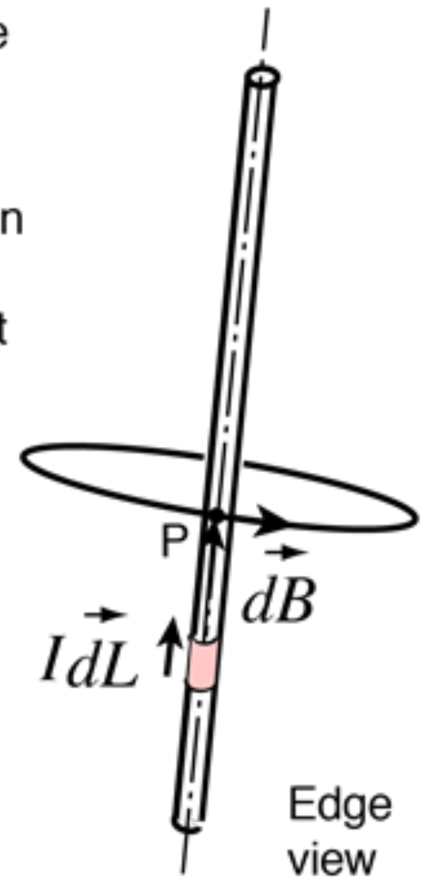
μ_0 = permeability of free space

$= 4\pi \times 10^{-7}$ in SI

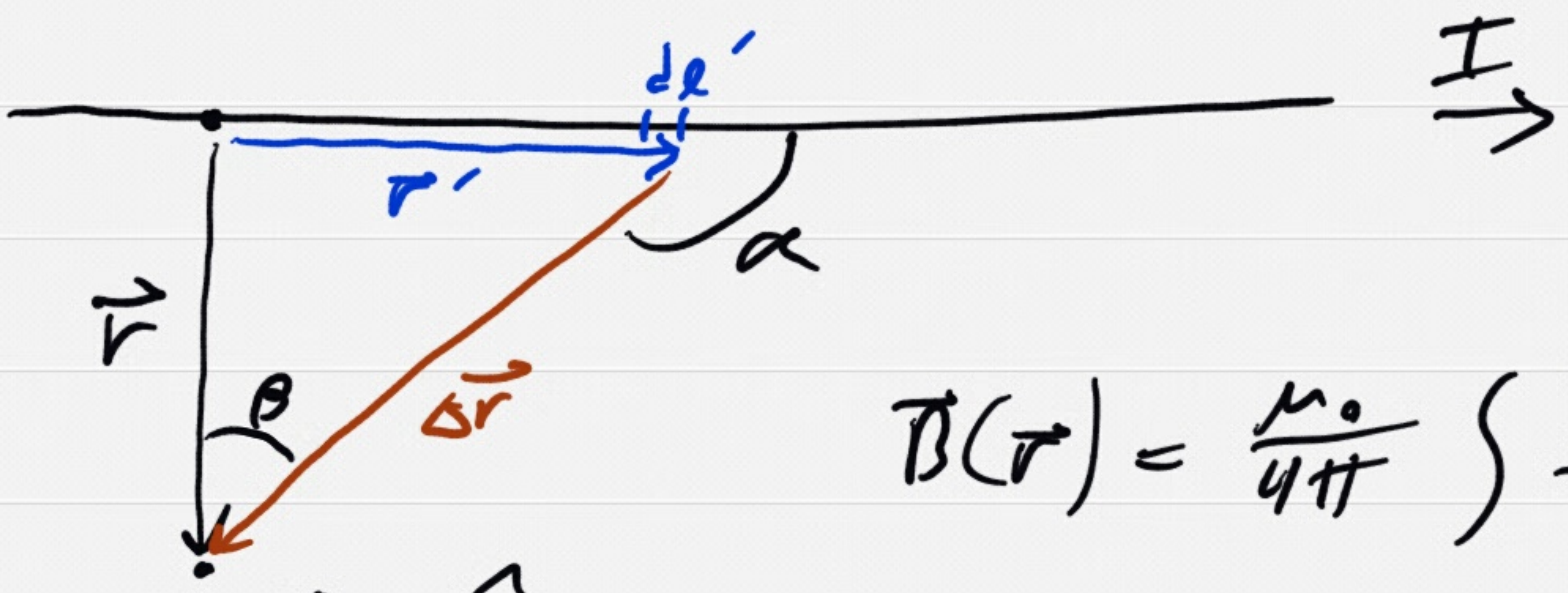
Biot-Savart Law



$\vec{dB}$ is the magnetic field contribution at P from the current element $I\vec{dL}$



Current - Carrying Wire Segment



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I \times \hat{\Delta r}}{\Delta r^2}\, d\ell'$$

$\otimes$ $I \times \hat{\Delta r}$

$$= I \sin\alpha \, \hat{\varphi}$$

$$= I \sin(\pi - (\pi/2 - \beta))\, \hat{\varphi}$$

$$= I \sin(\beta + \pi/2)\, \hat{\varphi}$$

$$= I \cos(\beta)\, \hat{\varphi} = I \, |\vec{r}|/\Delta r \, \hat{\varphi}$$

$\vec{r} = s\hat{s}$, $\vec{r}' = \ell' \hat{z}$, $\Delta r = \sqrt{s^2 + \ell'^2}$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I \hat{\varphi}}{4\pi} \int \frac{s}{(s^2 + \ell'^2)^{3/2}}\, d\ell'$$

$$= \frac{\mu_0 I}{4\pi} \hat{\varphi} \left. \frac{\ell'}{s\sqrt{s^2 + \ell'^2}} \right|_{\ell_1}^{\ell_2}$$

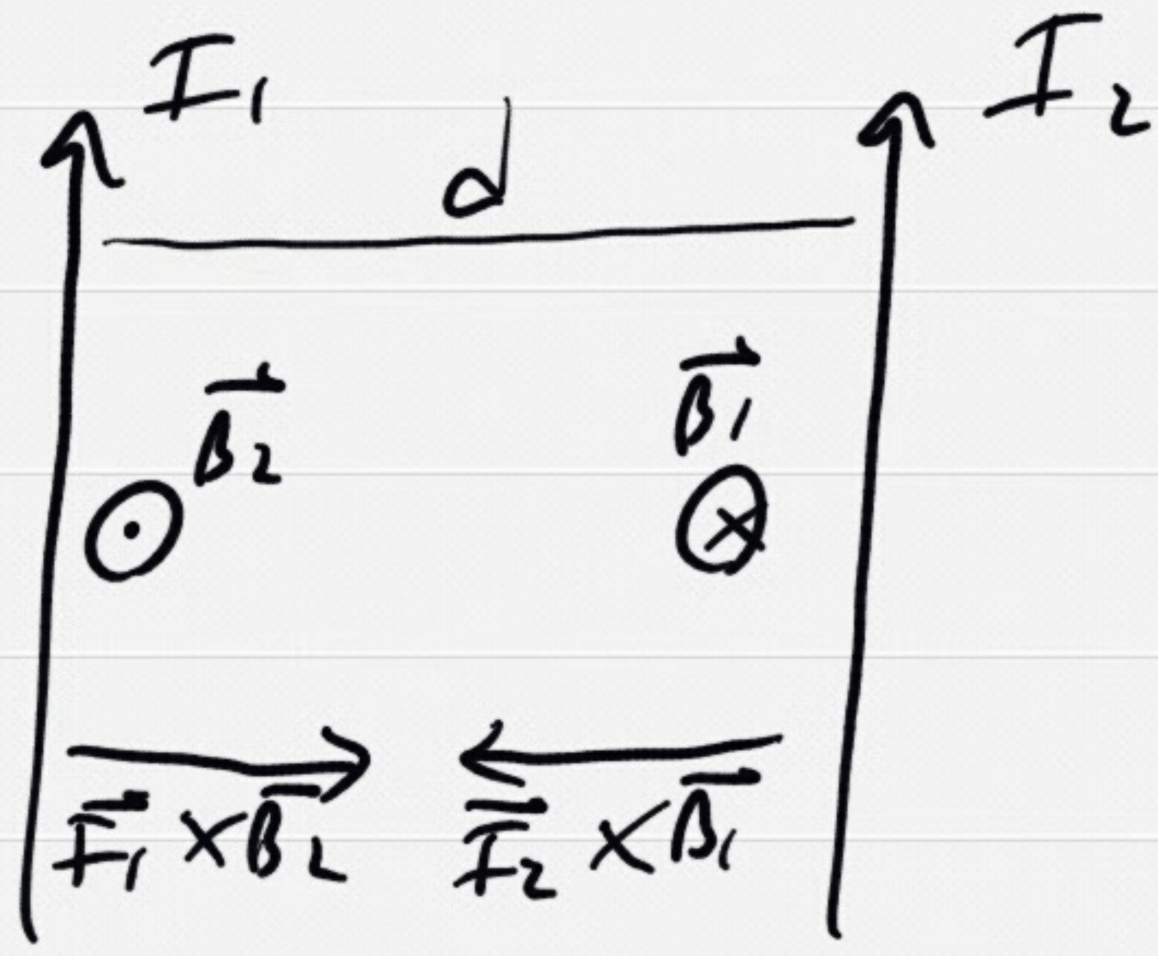
$$= \boxed{\frac{\mu_0 I}{4\pi s} \hat{\varphi} \left[\frac{\ell_2}{\sqrt{s^2 + \ell_2^2}} - \frac{\ell_1}{\sqrt{s^2 + \ell_1^2}} \right]}$$

$$= \boxed{\frac{\mu_0 I}{4\pi s} \hat{\varphi} \left[\sin\beta_2 - \sin\beta_1 \right]}$$

For $\ell_2 \to \infty$, $\ell_1 \to -\infty$

$$\boxed{\vec{B}(\vec{r}) \to \frac{\mu_0 I}{2\pi s} \hat{\varphi}}$$

Force Between Wires



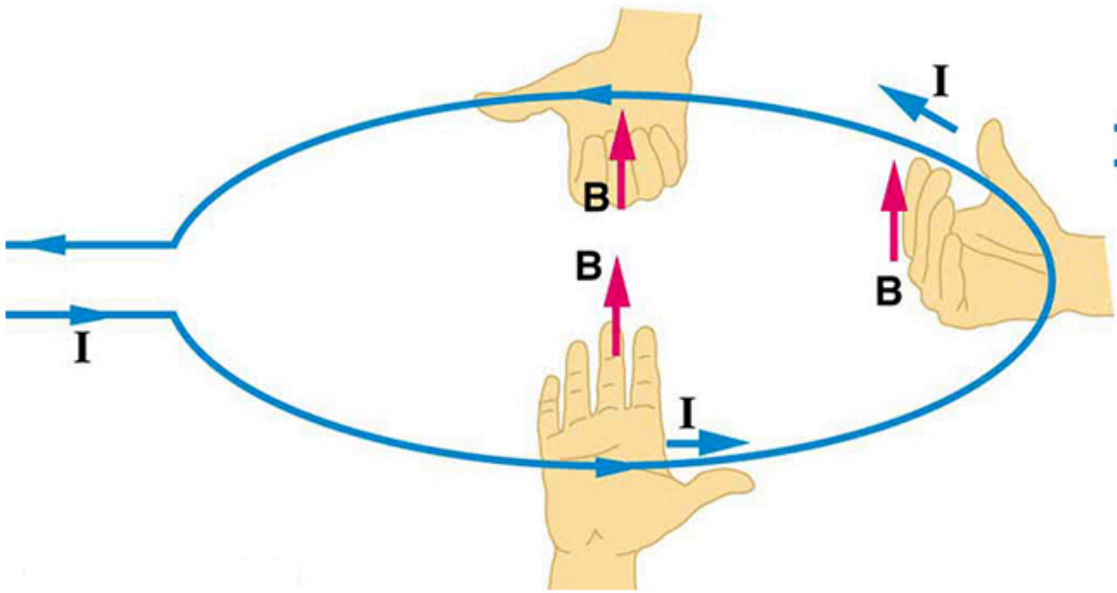
$$\vec{F}_{21} = \int I_2 \times \vec{B}_1 \, dl_2$$
$$= (\vec{I}_2 \times \vec{B}_1) \cdot L$$
$$= -\frac{\mu_0 I_1 I_2}{2\pi d} L \, \hat{x}$$

$$\vec{F}_{21}/L = - \frac{\mu_0 I_1 I_2}{2\pi d} \hat{x}$$

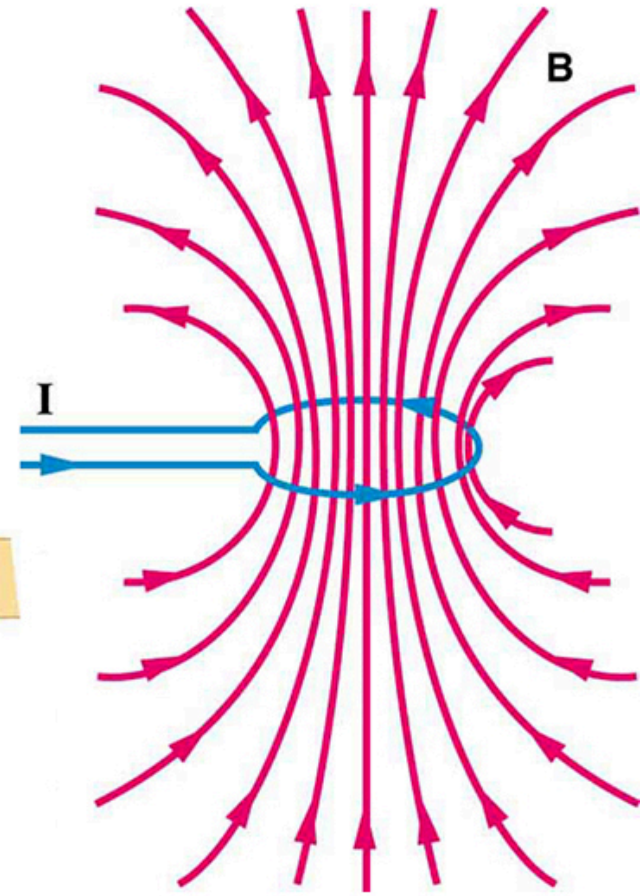
$$\vec{F}_{12}/L = + \frac{\mu_0 I_1 I_2}{2\pi d} \hat{x}$$

- Like currents attract
- Opposite currents repel

Magnetic Field of Current Loop

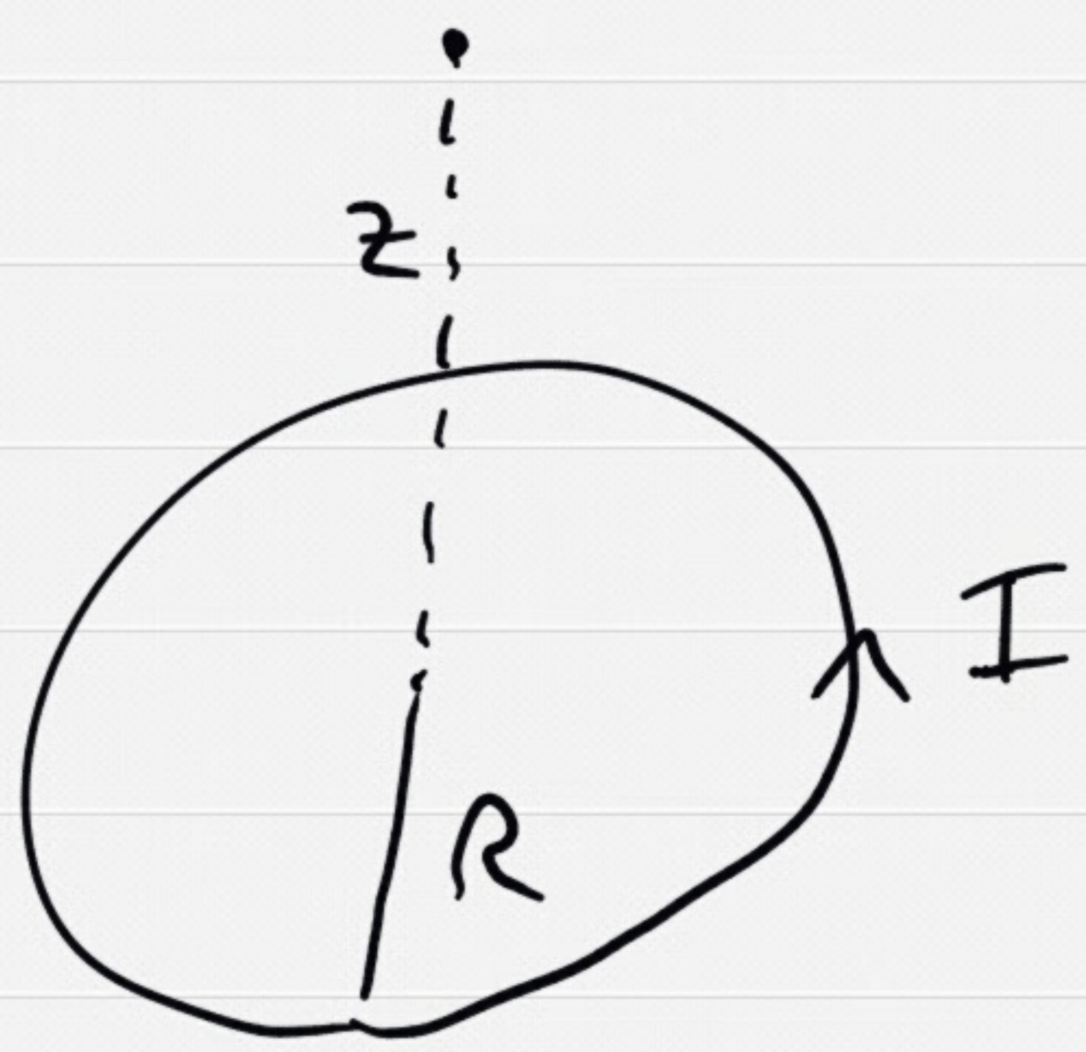


(a)

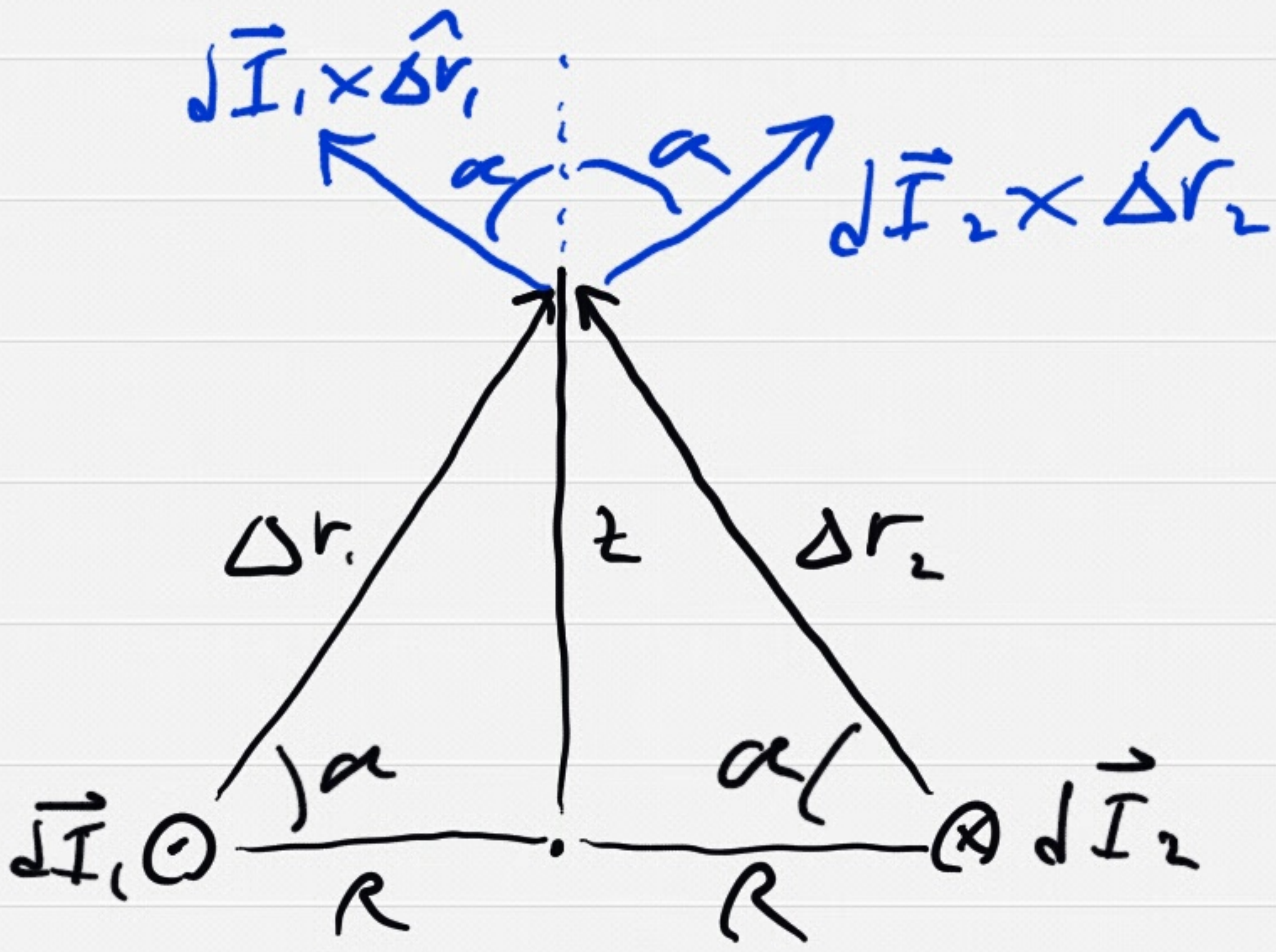


(b)

Current Loop



Side View



Horizontal components cancel
Vertical components add

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{\Delta r}}{\Delta r^2}$$

$$B_z = \frac{\mu_0 I}{4\pi} \cos\alpha \int dl' / \Delta r^2$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{R}{\sqrt{R^2+z^2}} \int dl' / R^2+z^2$$

$$= \frac{\mu_0 I}{4\pi} \frac{R \cdot 2\pi R}{(R^2+z^2)^{3/2}} = \boxed{\frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$

For $z \gg R$

$$B_z \cong \frac{\mu_0 I R^2}{2 z^3}$$

$$= \frac{\mu_0 I \cdot \pi R^2}{2\pi z^3}$$

$$= \frac{\mu_0 \cdot I \cdot \text{Area}}{2\pi z^3}$$

- This is a dipole field
- $I \cdot \text{Area} = m$ = magnetic dipole moment

We'll come back to this!