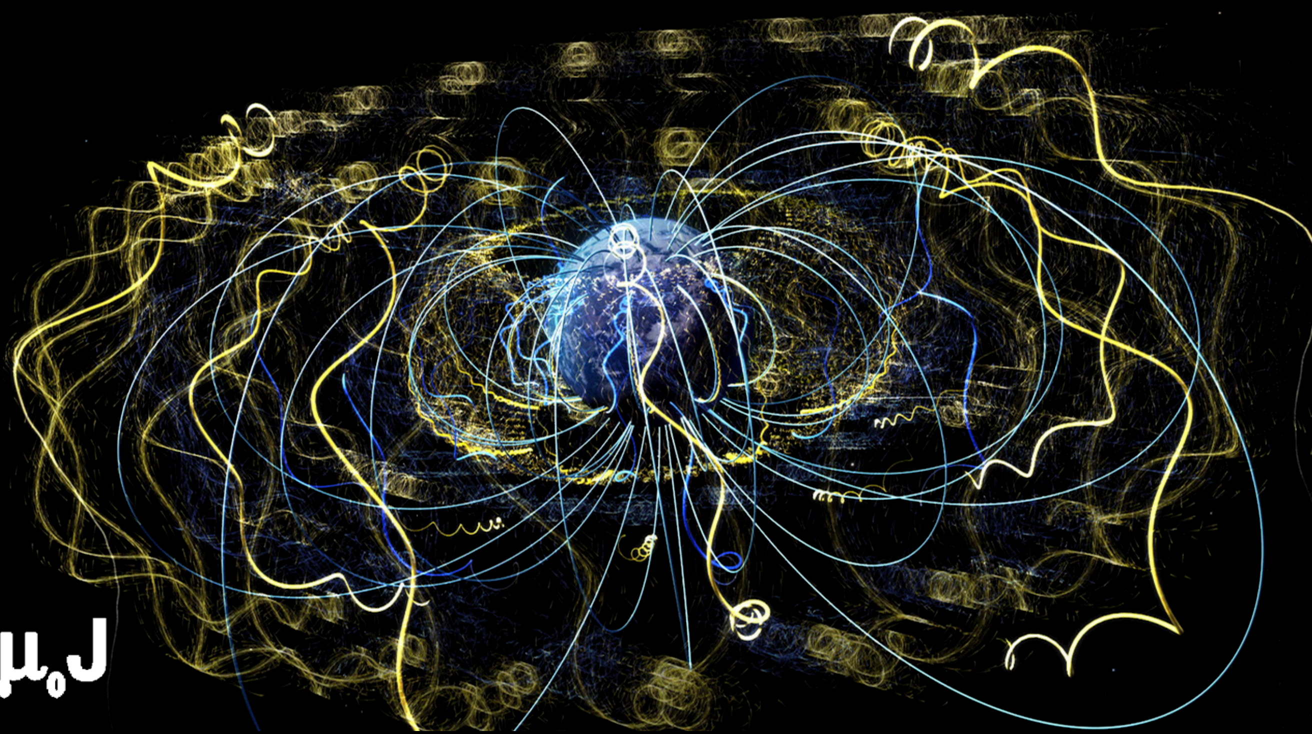
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

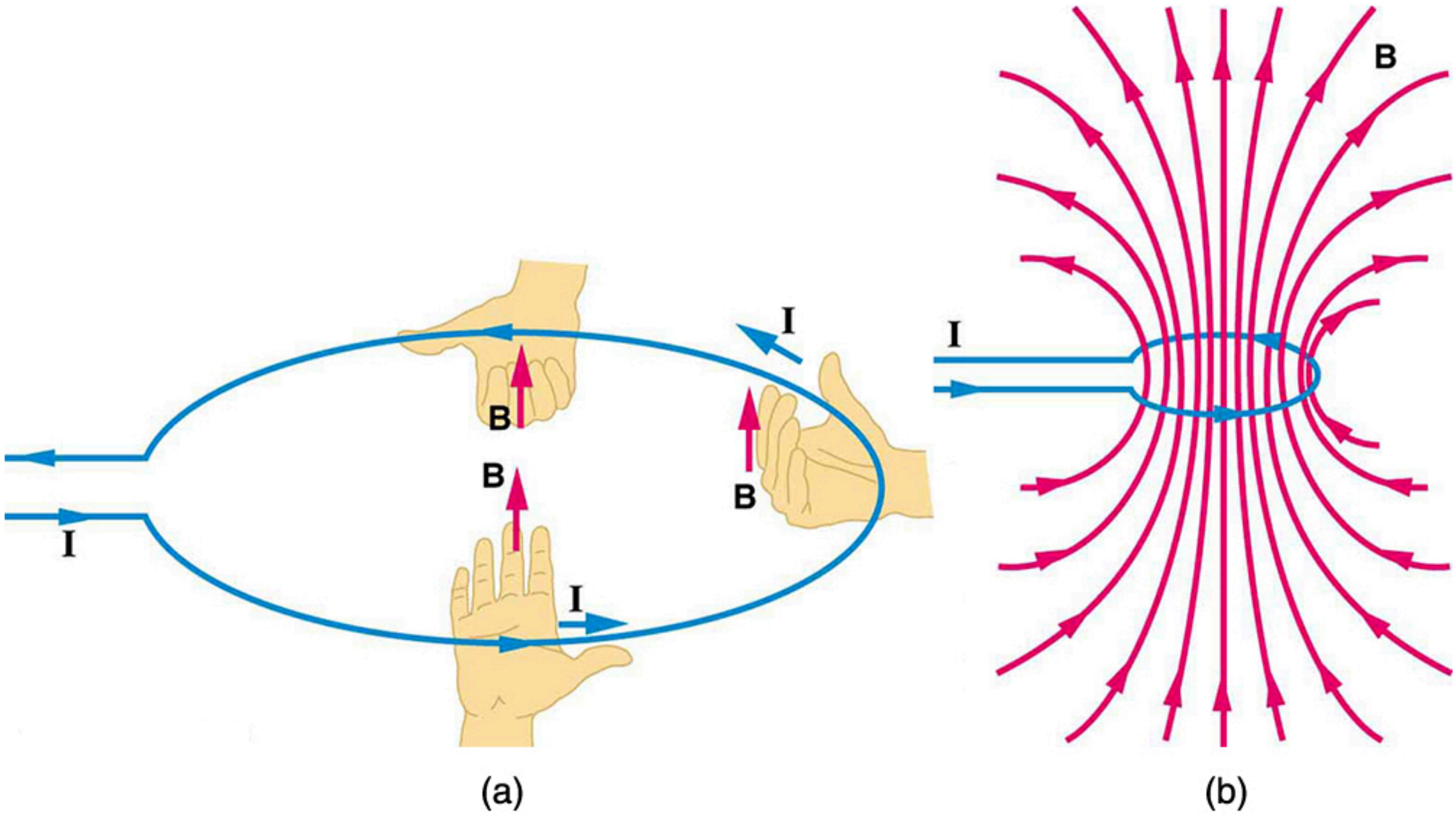
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



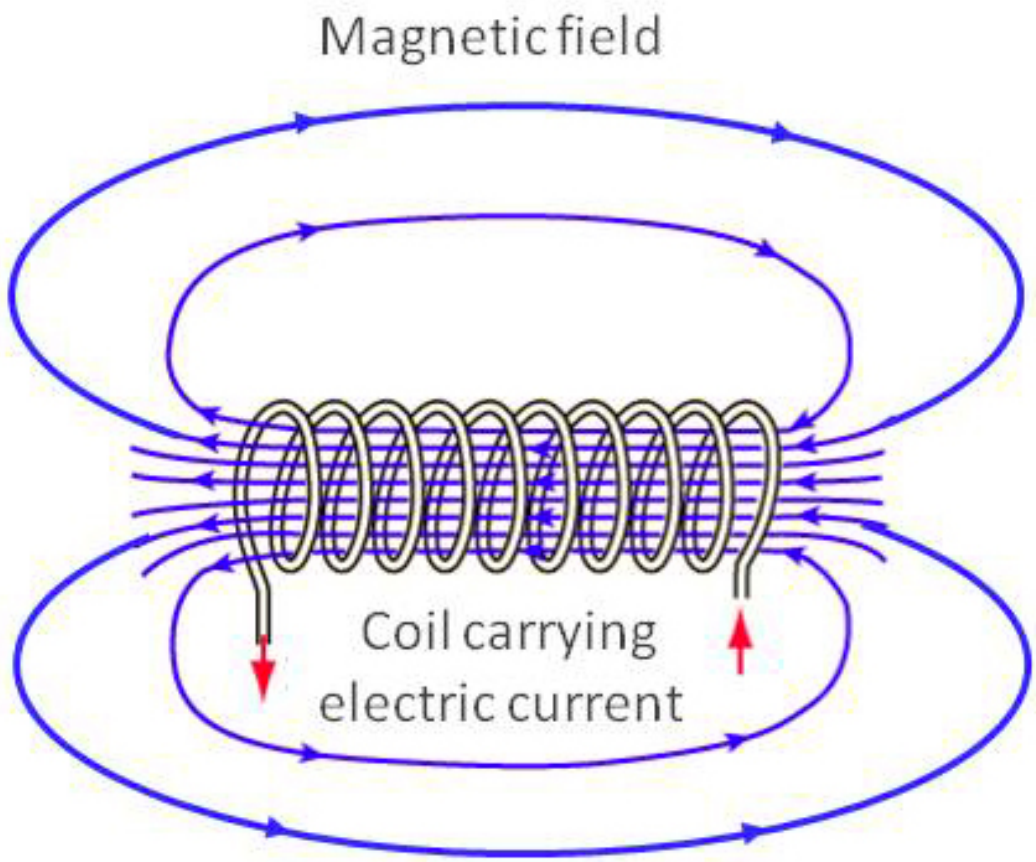
Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Magnetic Field of Current Loop

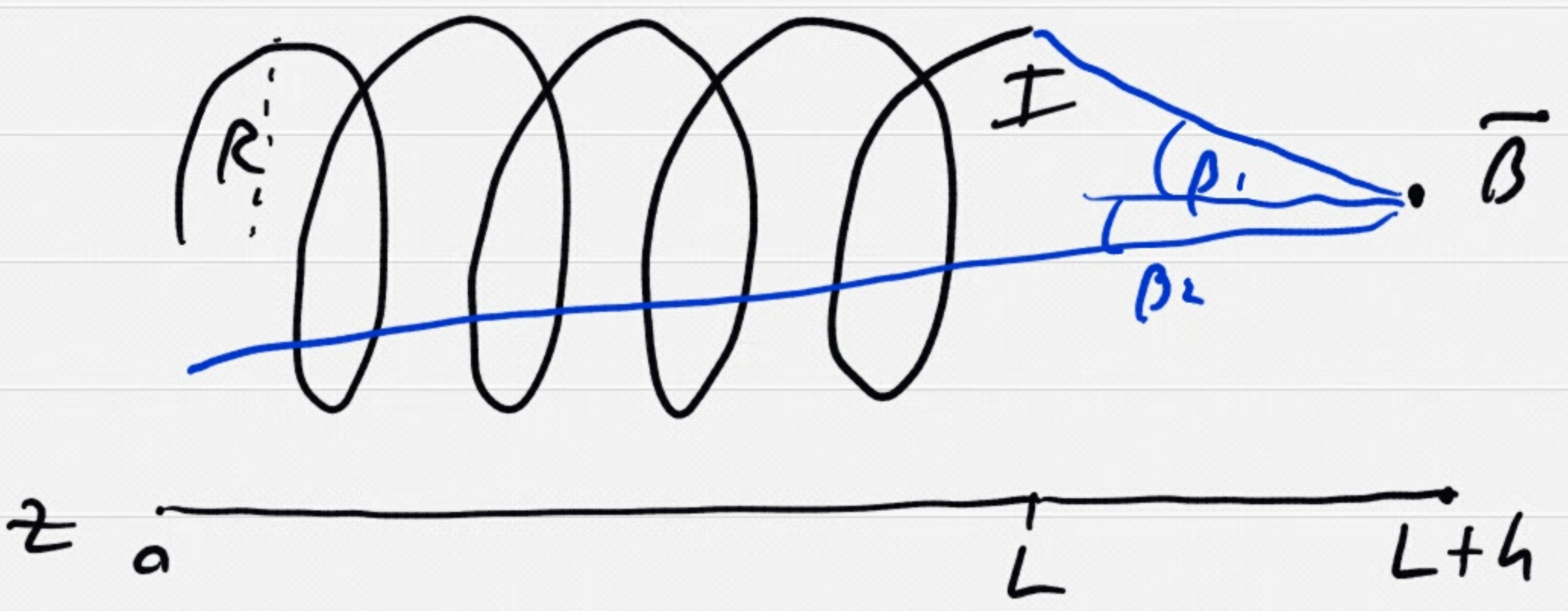


Solenoid



Solenoid

N turns



$$B = \sum \vec{B}_{loop} \sim \int \frac{N \cdot \bar{B}_{loop}}{\text{length}} dz$$

$$= \int_h^{h+L} \frac{N}{L} \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}} dz$$

$$= \frac{N}{L} \frac{\mu_0 I}{2} \frac{z}{\sqrt{R^2+z^2}} \Big|_h^{h+L}$$

$$= \frac{N}{L} \frac{\mu_0 I}{2} \left[\frac{h+L}{\sqrt{R^2+(h+L)^2}} - \frac{h}{\sqrt{R^2+h^2}} \right]$$

$$= \frac{N}{L} \frac{\mu_0 I}{2} \left[\cos\beta_2 - \cos\beta_1 \right]$$

Define $n = N/L$ = turns/length

$$\boxed{B = \frac{\mu_0 n I}{2} \left[\cos\beta_2 - \cos\beta_1 \right]}$$

Infinite Solenoid

$\beta_2 = 0, \quad \beta_1 = \pi$

$$\rightarrow \boxed{B = \mu_0 n I}$$

Electrostatics

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \qquad \nabla \times \vec{E} = 0$$

- Electrostatic field lines stop and start at charges
- Electrostatics are irrotational

Magnetostatics

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times \hat{\Delta r}}{\Delta r^2} \, d\tau'$$

$$\nabla \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \nabla \cdot \left(\vec{J} \times \frac{\hat{\Delta r}}{\Delta r^2} \right) d\tau'$$

$$= \frac{\mu_0}{4\pi} \int \left[\underbrace{\frac{\hat{\Delta r}}{\Delta r^2} \cdot (\nabla \times \vec{J})}_{0 \text{ since } \vec{J}(\vec{r}\,') \text{ not a func. of } \vec{r}} - \vec{J} \cdot \underbrace{\left(\nabla \times \frac{\hat{\Delta r}}{\Delta r^2}\right)}_{0 \text{ since purely radial}} \right] d\tau'$$

$$\Rightarrow \boxed{\nabla \cdot \vec{B} = 0}$$

- Magnetic field lines never stop
- Magnetic fields are solenoidal

$$\nabla \times \vec{B} = \frac{\mu_0}{4\pi} \int \nabla \times \left(\vec{J} \times \frac{\hat{\Delta r}}{\Delta r^2} \right) d\tau'$$

$$= \frac{\mu_0}{4\pi} \int \Big[\left(\frac{\hat{\Delta r}}{\Delta r^2} \cdot \nabla \right) \vec{J} - (\vec{J} \cdot \nabla) \frac{\hat{\Delta r}}{\Delta r^2} + \vec{J} \left(\nabla \cdot \frac{\hat{\Delta r}}{\Delta r^2} \right) - \frac{\hat{\Delta r}}{\Delta r^2} (\nabla \cdot \vec{J}) \Big] d\tau'$$

$\vec{J}(\vec{r}\,')$ doesn't depend on $\vec{r}$

so $$\nabla \times \vec{B} = \frac{\mu_0}{4\pi} \int \Big[-(\vec{J} \cdot \nabla) \frac{\hat{\Delta r}}{\Delta r^2} + \vec{J} \left(\nabla \cdot \frac{\hat{\Delta r}}{\Delta r^2} \right) \Big] d\tau'$$

$$= \frac{\mu_0}{4\pi} \int \Big[(\vec{J} \cdot \nabla') \frac{\hat{\Delta r}}{\Delta r^2} + \vec{J} \cdot 4\pi \delta^3(\Delta r) \Big] d\tau'$$

$$= \frac{\mu_0}{4\pi} \int \Big[\nabla' \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \vec{J} \right) - \frac{\hat{\Delta r}}{\Delta r^2} \nabla' \cdot \vec{J} \Big] d\tau' + \mu_0 \vec{J}(\vec{r})$$

$\nabla' \cdot \vec{J}(\vec{r}\,') = 0$ in magnetostatics

$$\int \nabla' \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \vec{J} \right) d\tau' = \oint \frac{\hat{\Delta r}}{\Delta r^2} \vec{J} \cdot d\vec{a}\,' \to 0$$ for large volume

So $\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$ Ampere's Law

- Magnetostatic fields circulate around currents!

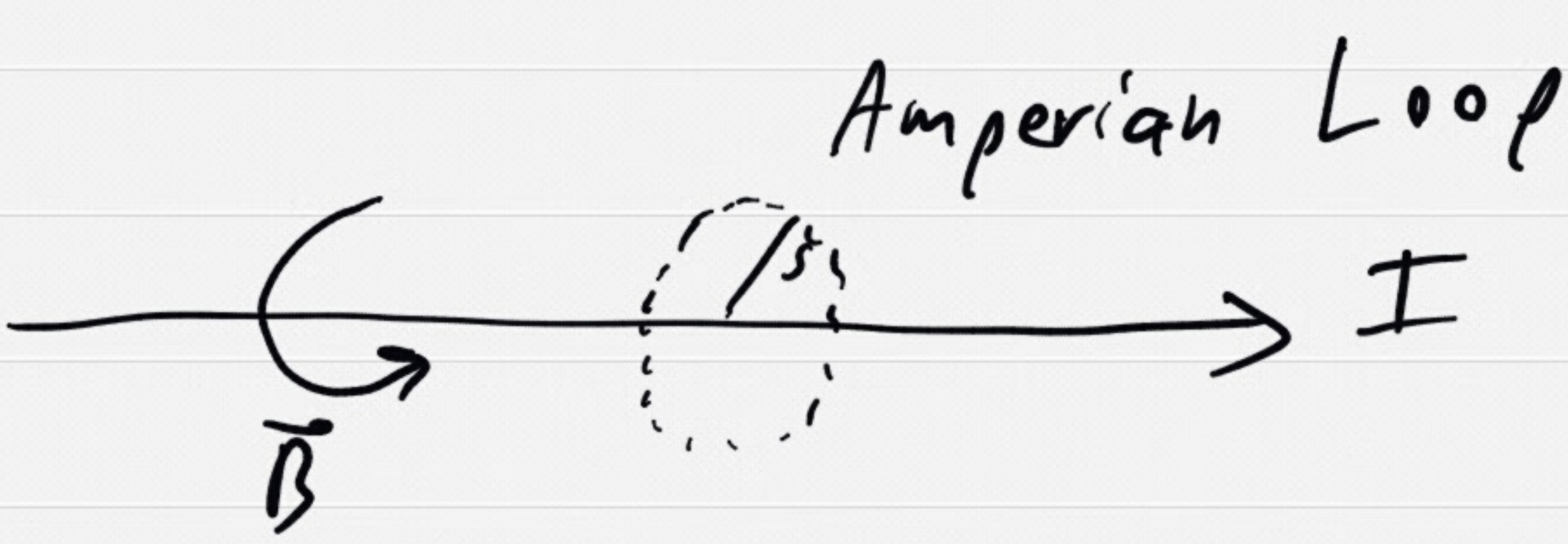
Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\int (\nabla \times \vec{B}) \cdot d\vec{a} = \int \mu_0 \vec{J} \cdot d\vec{a}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Amperes Law: Wire



$$\oint \vec{B} \cdot d\vec{l} = \int B \, dl \quad \text{since } \vec{B} \parallel d\vec{l} \parallel \hat{\phi}$$

$$= 2\pi s B$$

$$= \mu_0 I_{enc}$$

$$= \mu_0 I$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

Way easier than Biot-Savart

Electrostatics vs. Magnetostatics

Gauss	Ampere
$\oint \vec{E} \cdot d\vec{a}$	$\oint \vec{B} \cdot d\vec{l}$
If $\vec{E} \parallel d\vec{a}$	If $\vec{B} \parallel d\vec{l}$
$\rightarrow \oint E\, da$	$\rightarrow \oint B\, dl$
If $\lvert\vec{E}\rvert$ const.	If $\lvert\vec{B}\rvert$ const.
$\rightarrow EA$	$\rightarrow BL$