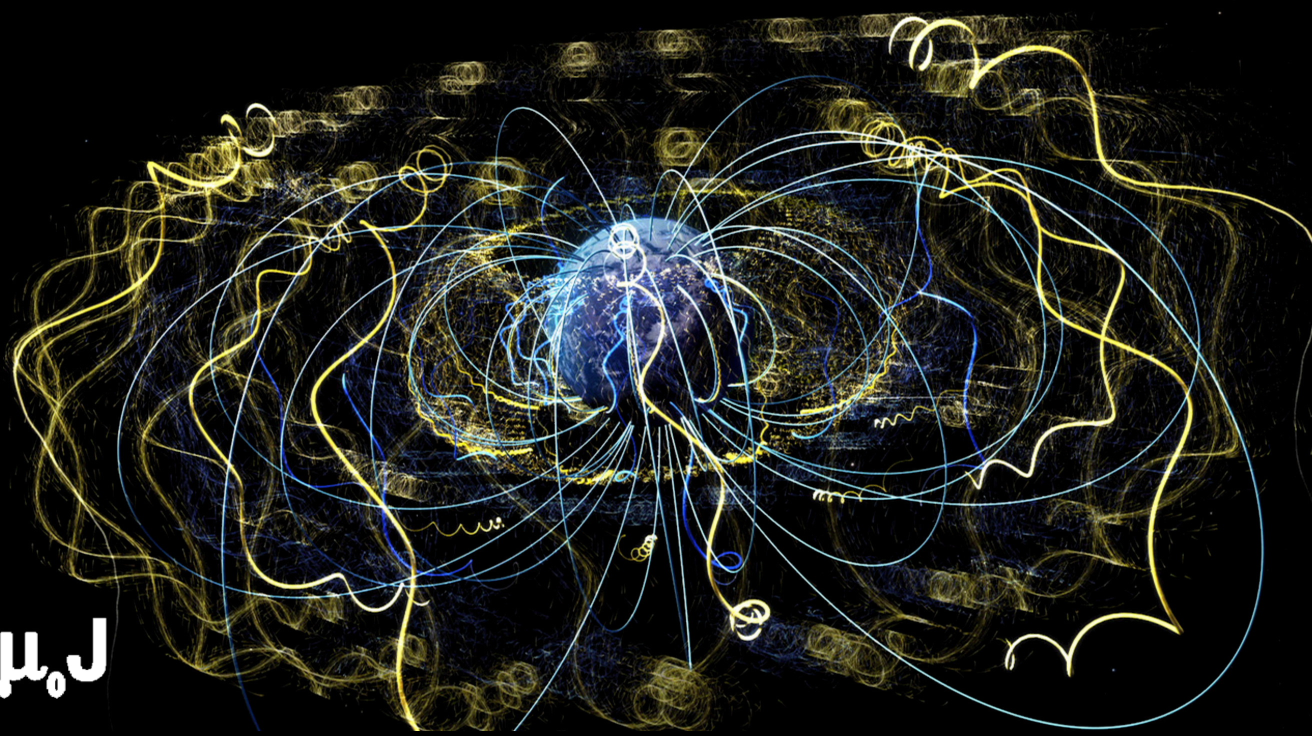
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

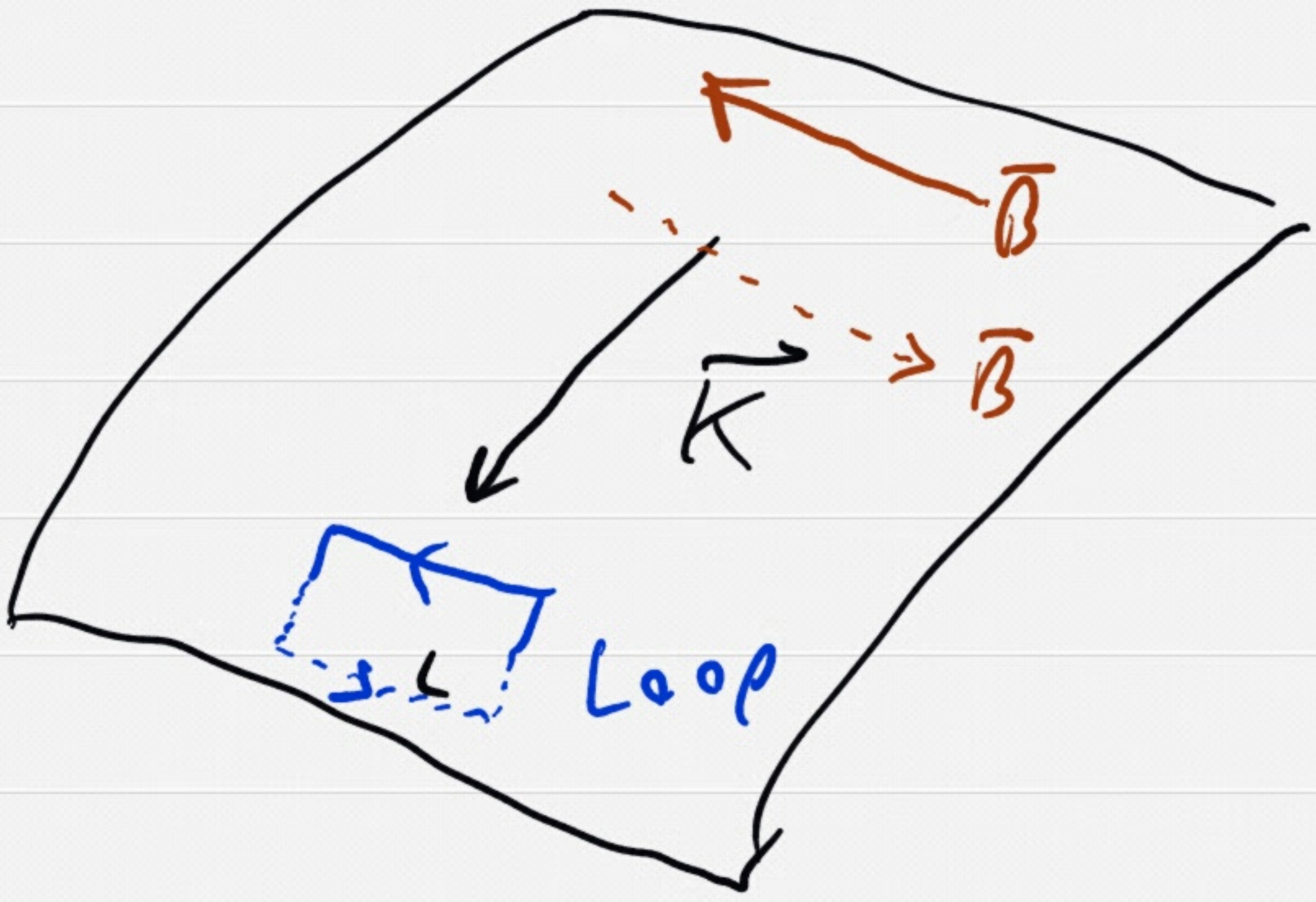
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Infinite Sheet of Current

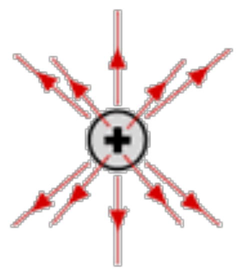
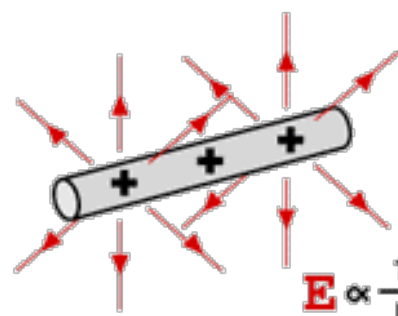
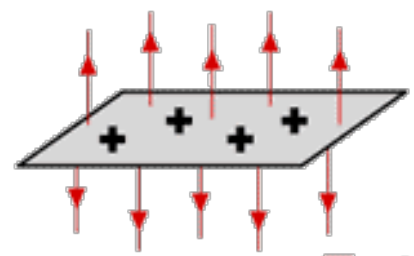
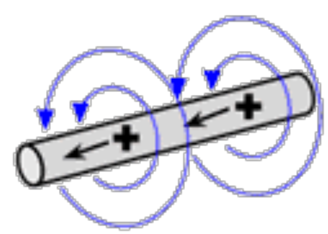
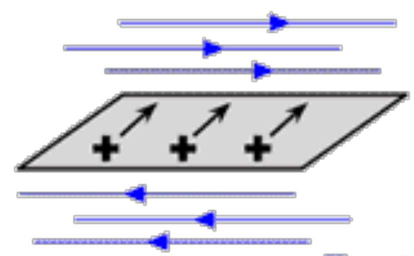


$$\oint \vec{B}\cdot d\vec{l} = 2BL$$
$$= \mu_0 I_{enc}$$
$$= \mu_0 K L$$

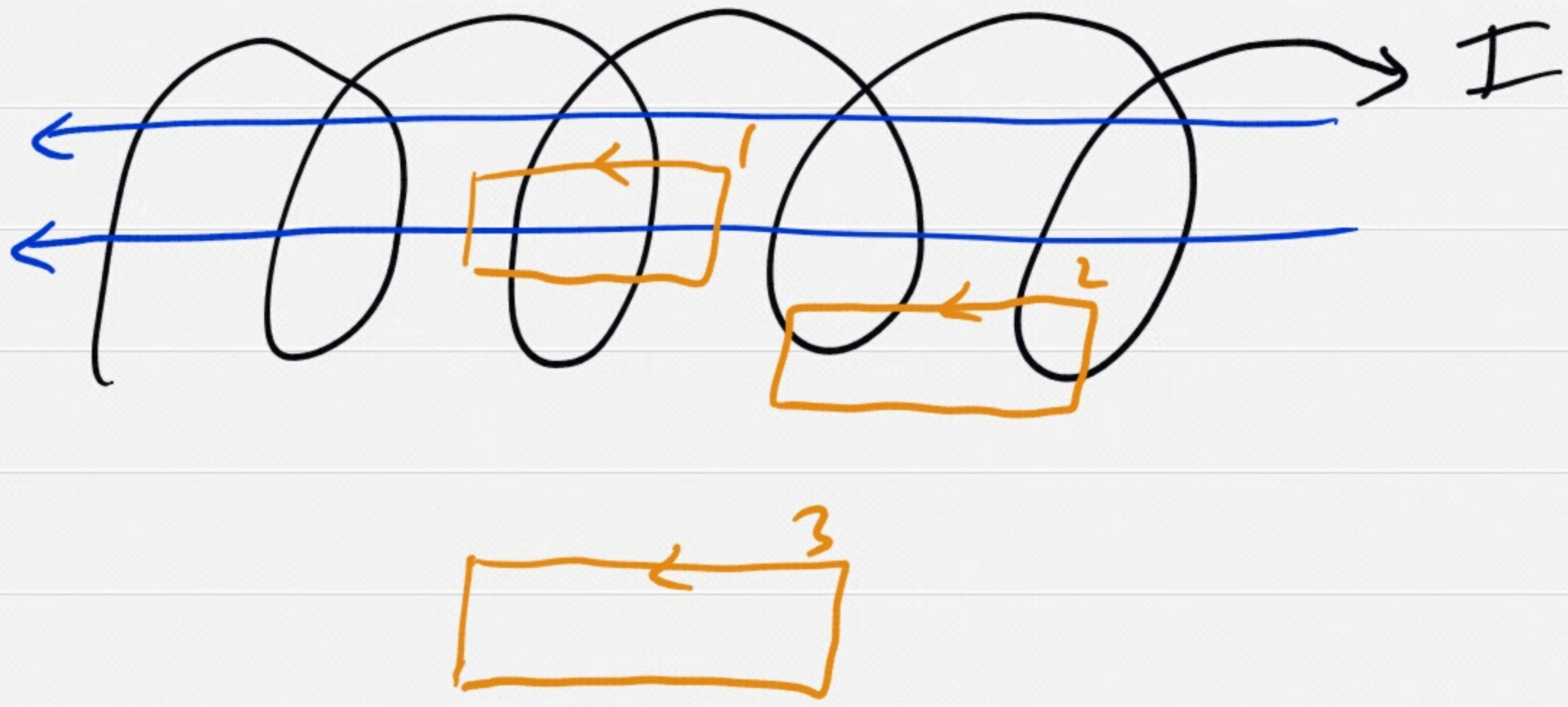
$$\Rightarrow \boxed{B = \frac{\mu_0 K}{2}}$$

Compare to $E = \sigma/2\varepsilon_0$
for infinite sheet of charge

Basic Geometries: E&B

	point charge	infinite line of charge	infinite plane of charge
electric field $\vec{E}$ units: N/C	 $E \propto \frac{1}{r^2}$	 $E \propto \frac{1}{r}$	 $E \propto 1$
magnetic field $\vec{B}$ units: Tesla (T)	(no magnetic monopoles)	 $B \propto \frac{1}{r}$	 $B \propto 1$

Infinite Solenoid



Loop 1 : $\oint \vec{B} \cdot d\vec{\ell} = (B_{top} - B_{bottom}) \cdot L$

$= 0$

$\Rightarrow B =$ const. inside

Loop 3 : $\oint \vec{B} \cdot d\vec{\ell} = (B_{top} - B_{bottom}) \cdot L$

$= 0$

$\Rightarrow B =$ const. outside

$B \to 0$ @ ∞

so $B = 0$ outside

Loop 2 : $\oint \vec{B} \cdot d\vec{\ell} = B_{in} \cdot L$

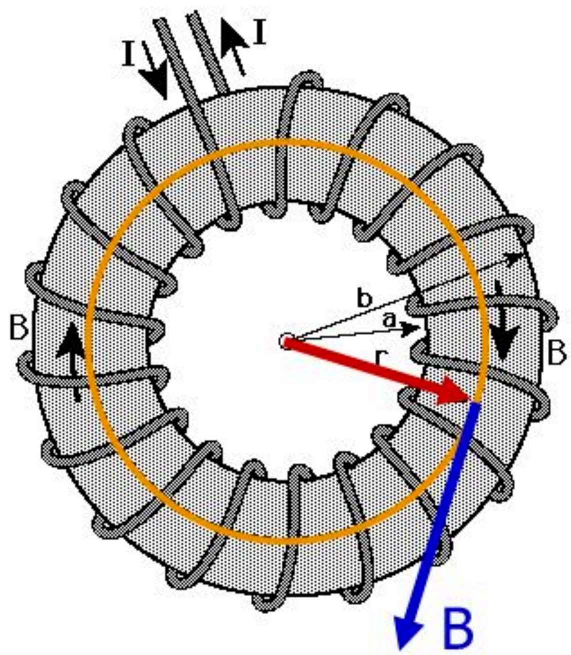
$= \mu_0 I_{enc}$

$= \mu_0 N I$

$$\Rightarrow B_{in} = \mu_0 I \frac{N}{L} = \boxed{\mu_0 n I}$$

Toroid

A toroid* is just a solenoid "hooked up" to itself.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed} = \mu_0 N I$$

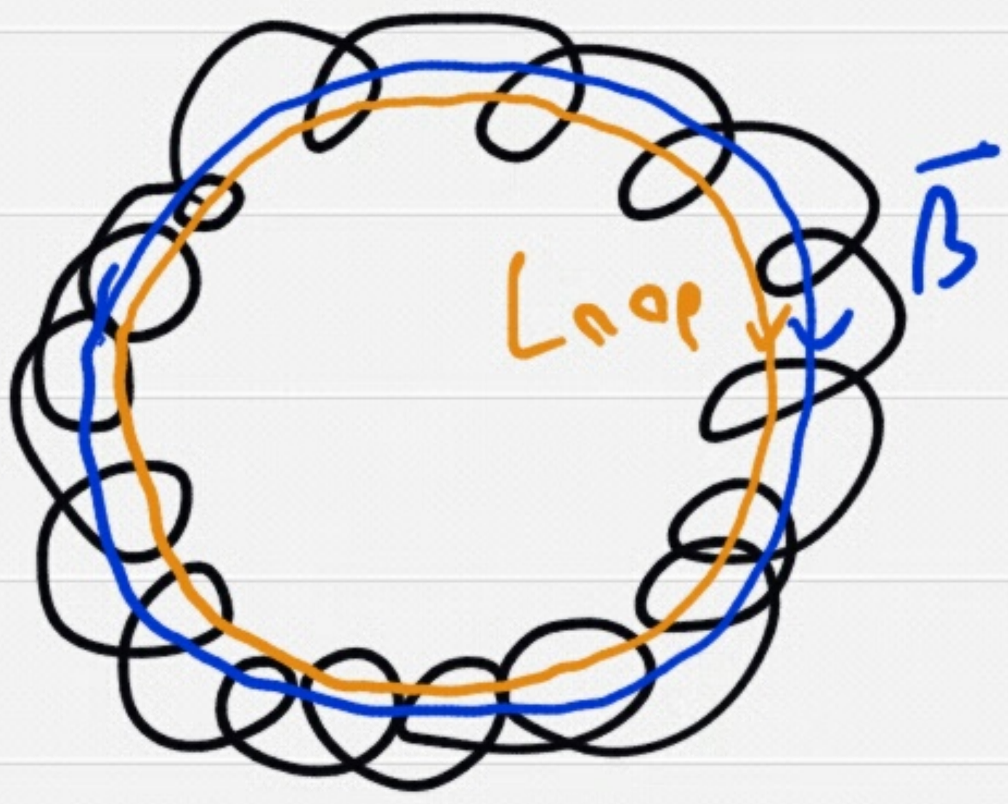
$$\oint \vec{B} \cdot d\vec{s} = B \int ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Magnetic field inside a toroid of N loops, current I.

Toroid



$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r$$

$$= \mu_0 \, I_{enc}$$

$$= \mu_0 \, N I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Special Vector Functions

Irrotational

$$\nabla \times \vec{E} = 0$$

$$\Updownarrow$$

$$\vec{E} = -\nabla V$$

$$\Updownarrow$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Solenoidal

$$\nabla \cdot \vec{B} = 0$$

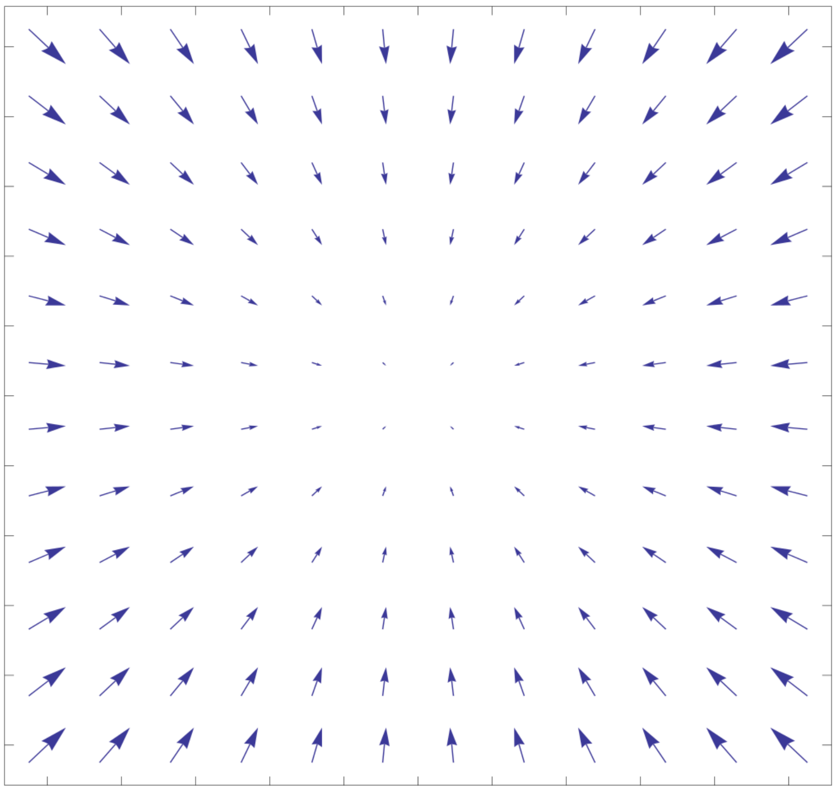
$$\Updownarrow$$

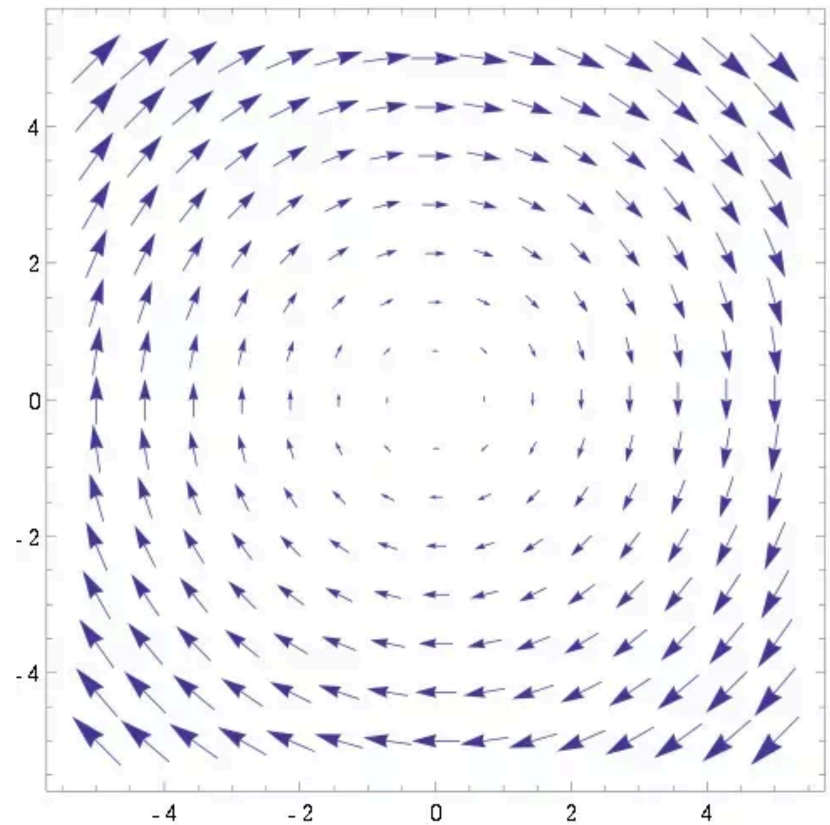
$$\vec{B} = \nabla \times \vec{A}$$

$$\Updownarrow$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

Irrotational vs. Solenoidal Fields





Magnetic Vector Potential

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A})$$
$$= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$
$$= \mu_0 \vec{J}$$

- Can pick a gauge such that $\nabla \cdot \vec{A} = 0$

If $\nabla \cdot \vec{A}_0 \neq 0$

Add $\nabla \lambda$ (w/ λ a scalar function) to $\vec{A}_0$

$$\nabla \cdot \vec{A} = \nabla \cdot \vec{A}_0 + \nabla \cdot (\nabla \lambda)$$
$$= 0 \text{ if}$$
$$\nabla^2 \lambda = -\nabla \cdot \vec{A}_0$$

- This is Poisson's Eq. w/ $\nabla \cdot \vec{A}_0$ in place of ρ/ε_0
    - Has a solution, so we can pick $\nabla \cdot \vec{A} = 0$

- The condition $\nabla \cdot \vec{A} = 0$ is called the Coulomb gauge

- Putting $\vec{A}$ in this gauge is allowed because it doesn't change $\vec{B}$
  - Just like adding a constant to electric potential

$$\nabla \times \vec{A} = \nabla \times (\vec{A}_0 + \nabla \lambda)$$
$$= \nabla \times \vec{A}_0$$
$$= \vec{B}$$

In Coulomb Gauge:

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

Each component like Poisson's Eq.

$$\Rightarrow \boxed{\begin{aligned} \vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\Delta r} d\tau' \\ &\text{or } \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}')}{\Delta r} da' \\ &\text{or } \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{\Delta r} dl' \end{aligned}}$$