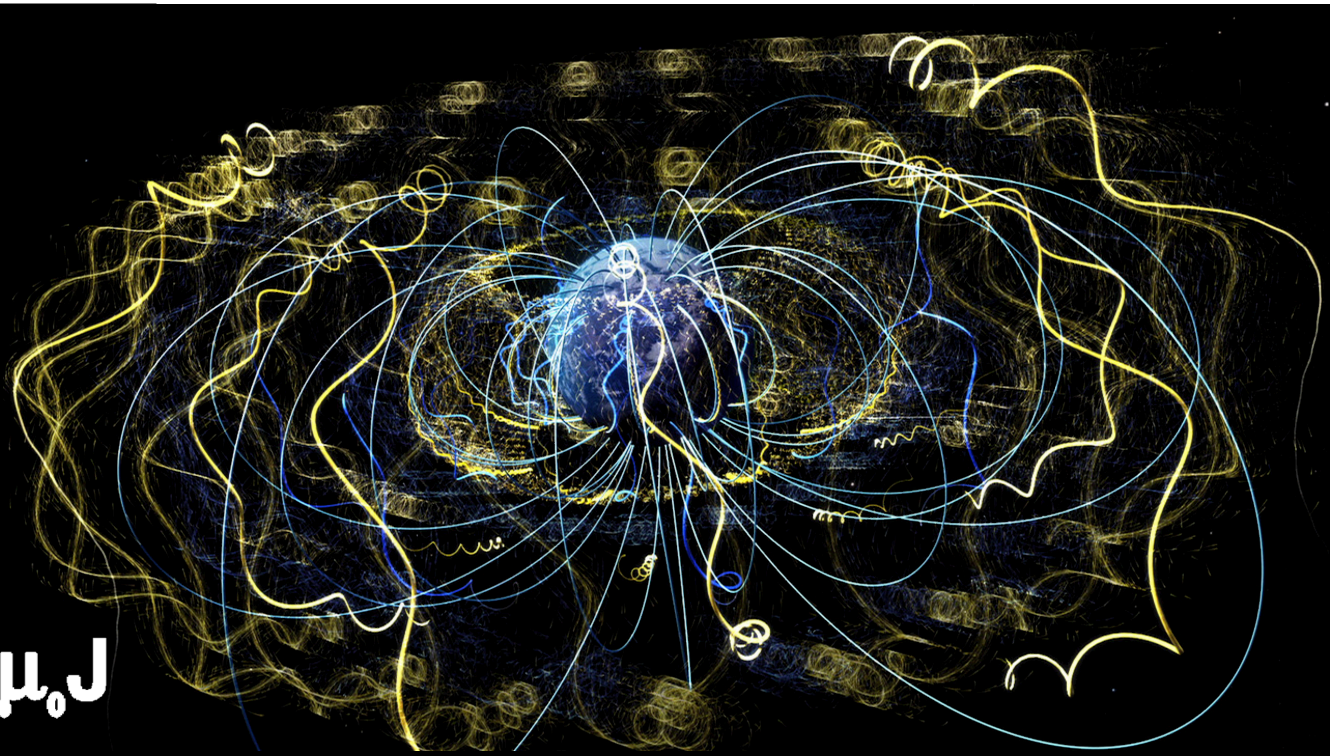
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



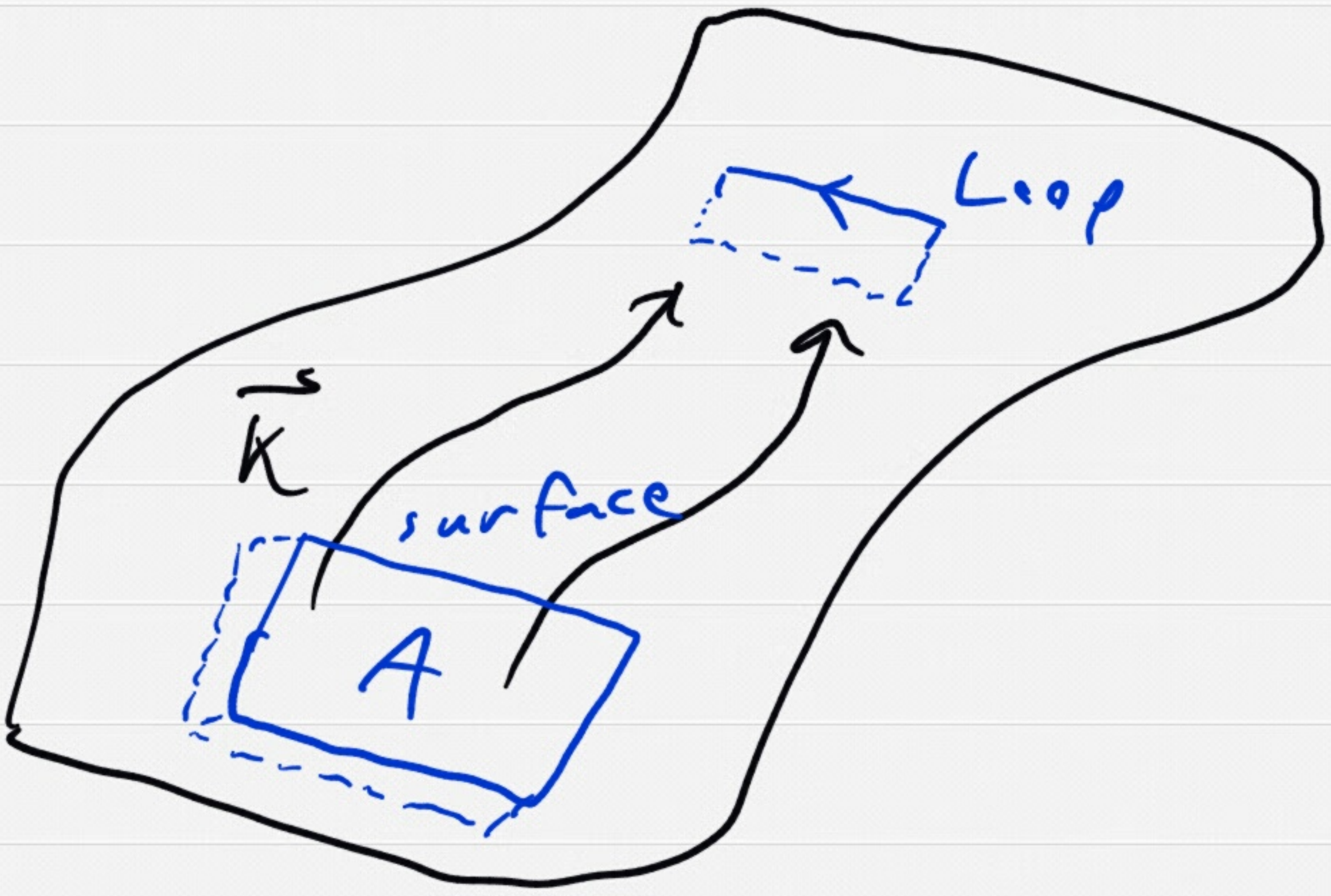
Electricity and Magnetism I: 3811

Professor Jasper Halekas

Van Allen 301

MWF 9:30-10:20 Lecture

Boundary Conditions



Surface: $\oint \vec{B} \cdot d\vec{a} = 0$

$$= B_{\perp\,\text{above}} \cdot A - B_{\perp\,\text{below}} \cdot A$$

$$\Rightarrow \boxed{\Delta B_{\perp} = 0}$$

Loop: $\oint \vec{B} \cdot d\vec{l} = B_{\parallel\,\text{above}} \cdot L - B_{\parallel\,\text{below}} \cdot L$

$$= \mu_0 I_{enc}$$

$$= \mu_0 K \cdot L$$

$$\Rightarrow \boxed{\Delta B_{\parallel} = \mu_0 K}$$

Combined vector form

$$\boxed{\Delta \vec{B} = \mu_0 \vec{K} \times \hat{n}}$$

Complete B.C.

$$\Delta \vec{E} = \sigma/\varepsilon_0 \, \hat{n} \quad \text{or} \quad \Delta\left(\frac{\partial V}{\partial n}\right) = -\sigma/\varepsilon_0$$

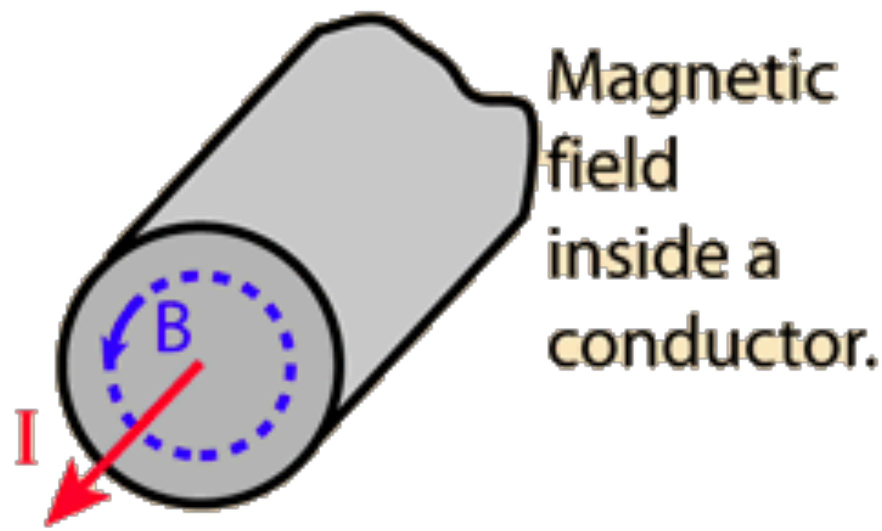
$$\Delta V = 0$$

$$\Delta \vec{B} = \mu_0 \vec{K} \times \hat{n} \quad \text{or} \quad \Delta\left(\partial \vec{A}/\partial n\right) = -\mu_0 \vec{K}$$

$$\Delta \vec{A} = 0$$

Check Your Understanding #1

- A cylindrical conductor of radius R carries a total current I.
- If the current is evenly distributed across the conductor, what is the volume current density J?



Q1:

$$\vec{J} = dI/da_{\perp} = \text{const.}$$

$$\int \vec{J} \cdot d\vec{a} = |\vec{J}| \cdot A$$

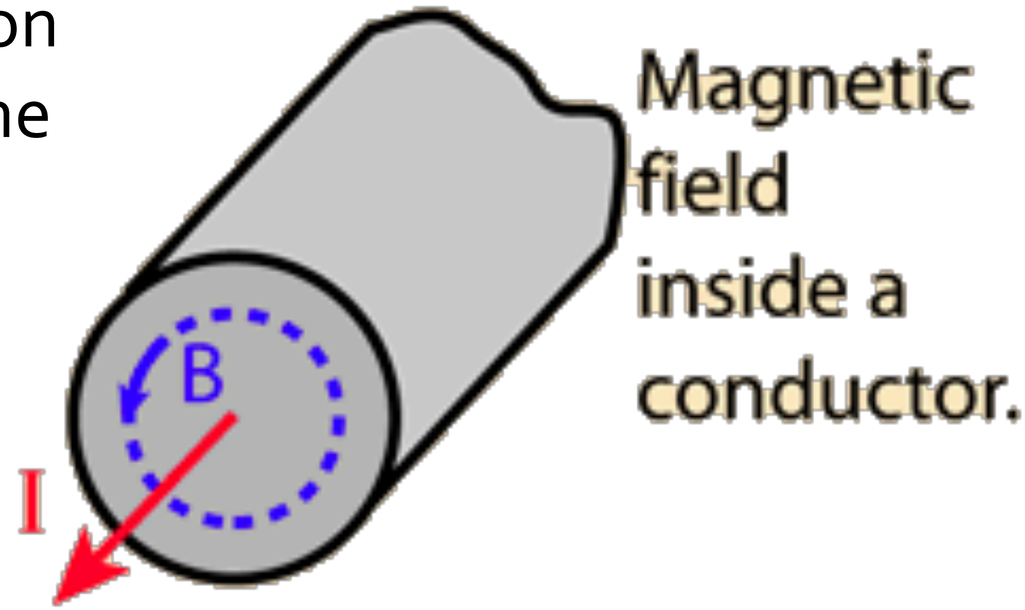
$$= |\vec{J}| \cdot \pi R^2$$

$$= I$$

$$\Rightarrow \boxed{|\vec{J}| = I/\pi R^2}$$

Check Your Understanding #2

- What is the magnetic field B(s) as a function of distance s from the axis of the cylinder?



Q2: $\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi s$

$= \mu_0 I_{enc}$

$= \mu_0 \int \vec{J} \cdot d\vec{a}$

$= \mu_0 J \cdot \pi s^2 \quad s < R$

$= \mu_0 J \cdot \pi R^2 \quad s > R$

$$\Rightarrow \boxed{\begin{aligned} B(s) &= \frac{\mu_0 J s}{2} & s < R \\ &= \frac{\mu_0 J R^2}{2s} & s > R \end{aligned}}$$

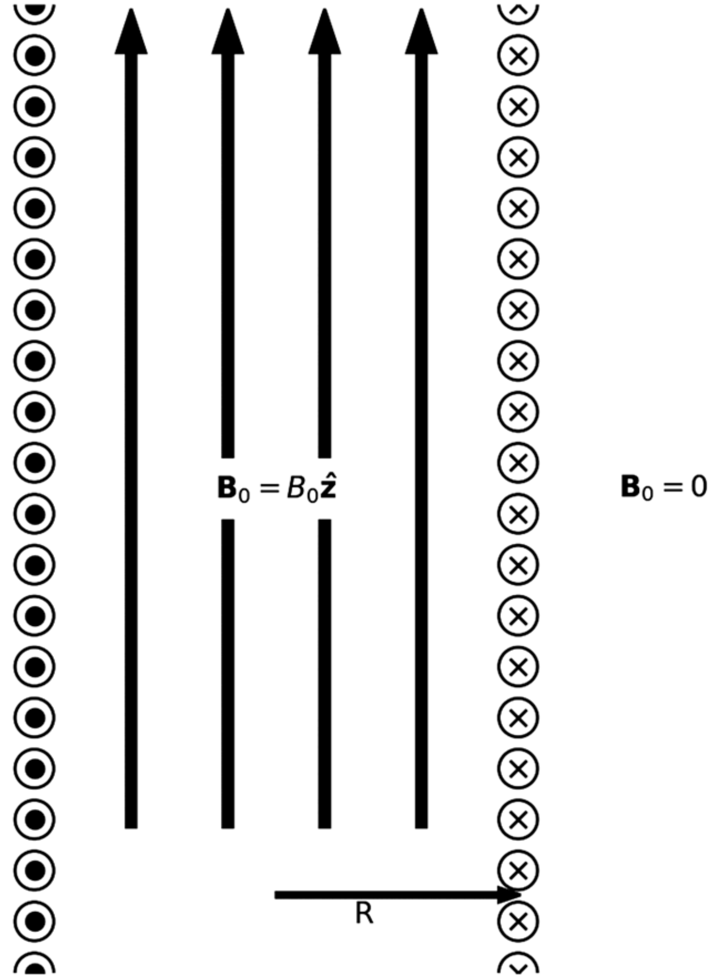
or in terms of I

$$\boxed{\begin{aligned} B(s) &= \frac{\mu_0 I}{2\pi s} \cdot \frac{\pi s^2}{\pi R^2} & s < R \\ &= \frac{\mu_0 I}{2\pi s} & s > R \end{aligned}}$$

Check Your Understanding 3

- An infinite solenoid is aligned with the Z axis as shown at right

- What direction is the magnetic vector potential A inside the solenoid?



Q3: $\bar{A}$ of single loop purely azimuthal

so, $\vec{A} = A\hat{\varphi}$

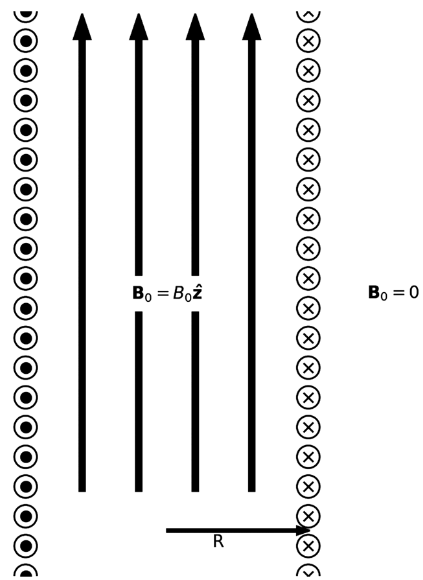
- Also $\vec{B} = \nabla \times \vec{A} = B_0\hat{z}$
  so $\bar{A}$ must be azimuthal or radial
- If it was radial it would have to depend on φ but by symmetry that doesn't make sense

Check Your Understanding 4

- Given the following formula for the curl in cylindrical coordinates

$$\nabla\times\vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s\,A_\phi)}{\partial s} - \frac{\partial A_s}{\partial\phi}\right)\hat{z}$$

- What is the magnitude of the magnetic vector potential A(s) as a function of distance s from the axis of the solenoid?



Q4i $\vec{B} = \nabla \times \vec{A}$

$$= \frac{1}{s} \partial(s A_\varphi)/\partial s \; \hat{z}$$

$$= B_0 \hat{z}$$

$$\Rightarrow B_0 s = \partial/\partial s (s A_\varphi)$$

$$\Rightarrow s A_\varphi = \frac{B_0 s^2}{2} + \text{const.}$$

$$\Rightarrow \boxed{A_\varphi = \frac{B_0 s}{2} + \text{const.}/s}$$