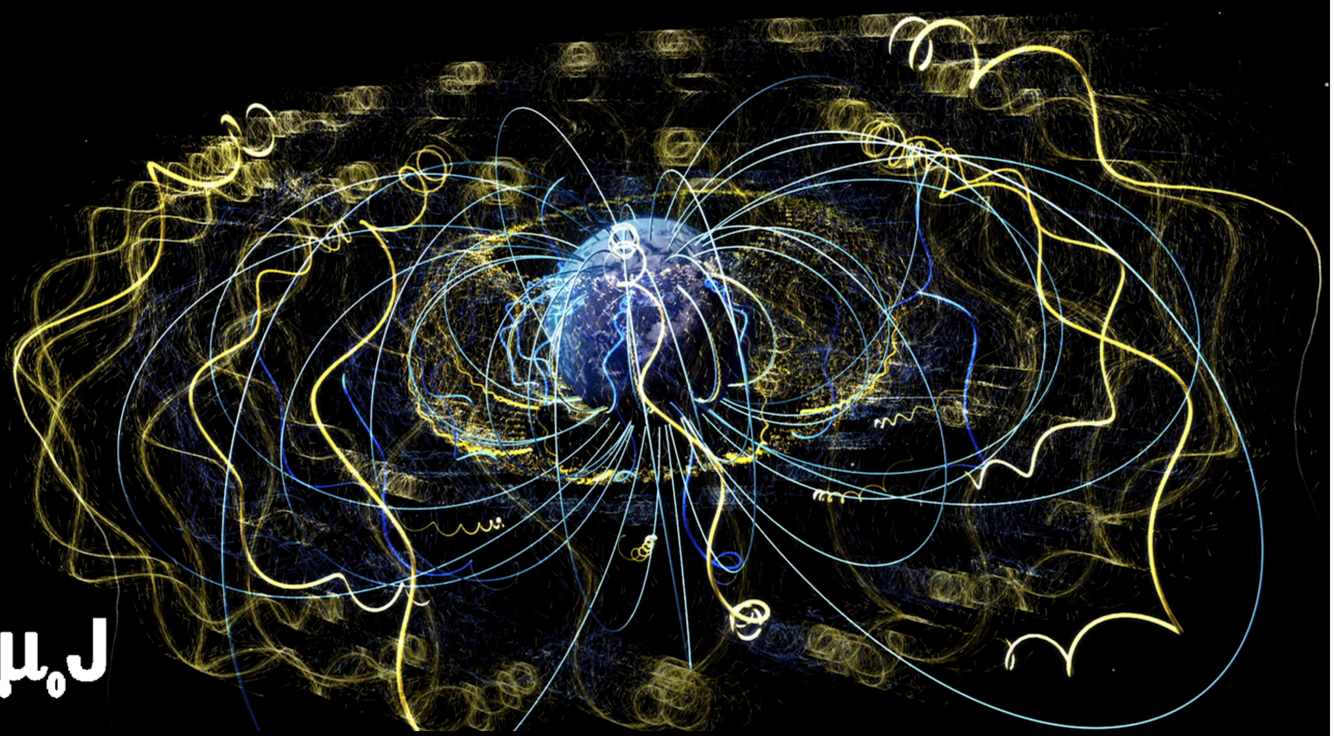
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Announcements

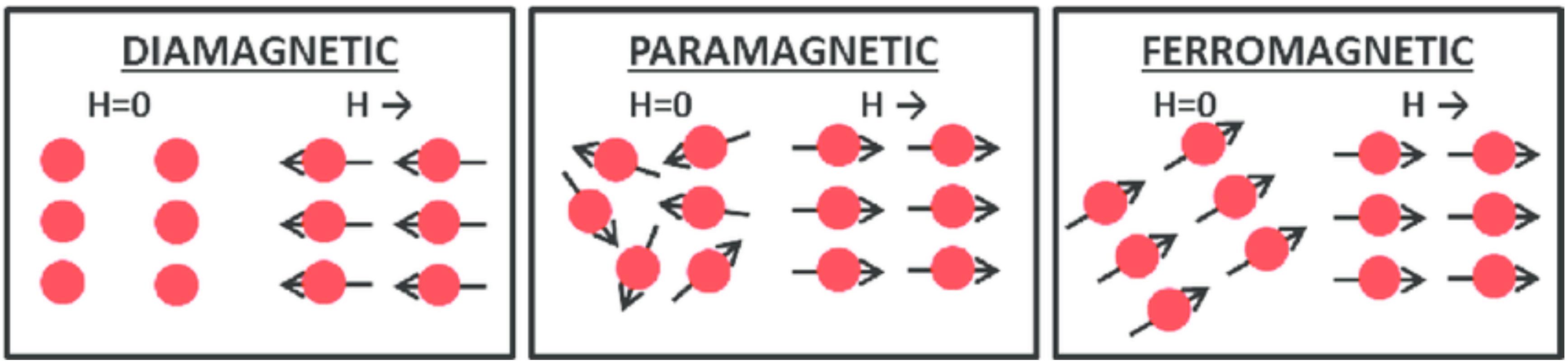
- Equation sheet for final exam posted on course web site
  - Please review

Announcements II

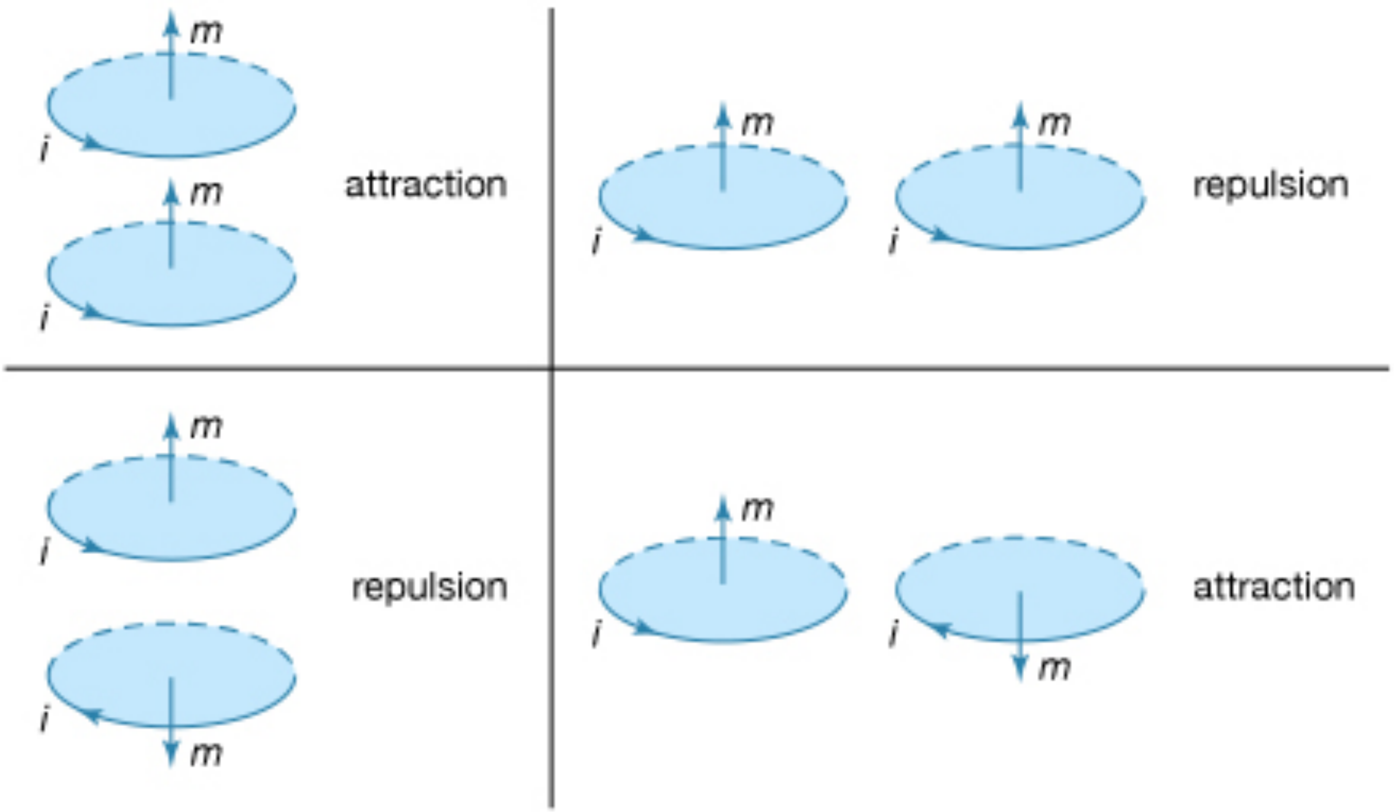
- Course Evaluations are now open
  - Please take a few minutes to fill these out
  - The course evaluations are very valuable for me
  - I read all evaluations carefully, and use them to improve my teaching

  - This is your big chance if you'd like to see anything different next semester!

Magnetism



Force Between Dipoles



Diamagnetic Frogs

- https://www.youtube.com/watch?v=A1vyB-O5i6E

Magnetization

$$\vec{M} = \frac{\vec{m}}{\text{volume}}$$

$$\vec{m} = \int \vec{M}\, d\tau$$

- just like polarization & electric dipole moment

Vector Potential

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \quad \text{for dipole @ origin}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{\Delta r}}{\Delta r^2} \quad \text{for dipole @ } \vec{r}'$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{\Delta r}}{\Delta r^2}\, d\tau'$$

for magnetized material

Bound Currents

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{m}(\vec{r}') \times \hat{\Delta r}}{\Delta r^2}\, d\tau'$$

Use $\nabla'\left(\frac{1}{\Delta r}\right) = \frac{\hat{\Delta r}}{\Delta r^2}$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \left[\vec{m}(\vec{r}') \times \nabla'\left(\frac{1}{\Delta r}\right)\right] d\tau'$$

$$= \frac{\mu_0}{4\pi}\left[\int \frac{1}{\Delta r}\left(\nabla' \times \vec{m}(\vec{r}')\right) d\tau' - \int \left(\nabla' \times \left(\frac{\vec{m}(\vec{r}')}{\Delta r}\right)\right) d\tau'\right]$$

$$= \frac{\mu_0}{4\pi}\int \frac{\nabla' \times \vec{m}(\vec{r}')}{\Delta r}\, d\tau' + \frac{\mu_0}{4\pi}\oint \frac{\vec{m}(\vec{r}')}{\Delta r} \times d\vec{a}'$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{J}_b(\vec{r}')}{\Delta r}\, d\tau' + \frac{\mu_0}{4\pi}\oint \frac{\vec{K}_b(\vec{r}')}{\Delta r}\, da'$$

w/
$$\boxed{\begin{aligned} \vec{J}_b &= \nabla \times \vec{M} \\ \vec{K}_b &= \vec{M} \times \hat{n} \end{aligned}}$$

compare to
$$\rho_b = -\nabla \cdot \vec{P}$$
$$\sigma_b = \vec{P} \cdot \hat{n}$$

Auxiliary Field

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

$$= \frac{1}{\mu_0}(\nabla \times \vec{B})$$

$$= \vec{J}_f + \nabla \times \vec{M}$$

$$\Rightarrow \nabla \times (\vec{B}/\mu_0 - \vec{M}) = \vec{J}_f$$

Define $\boxed{\vec{H} = \vec{B}/\mu_0 - \vec{M}}$

$$\nabla \times \vec{H} = \vec{J}_f$$

Nomenclature:

$\vec{H}$: magnetic field intensity, magnetic field, magnetic field strength, magnetizing field, auxiliary field

$\vec{B}$: magnetic field, magnetic flux density, magnetic induction

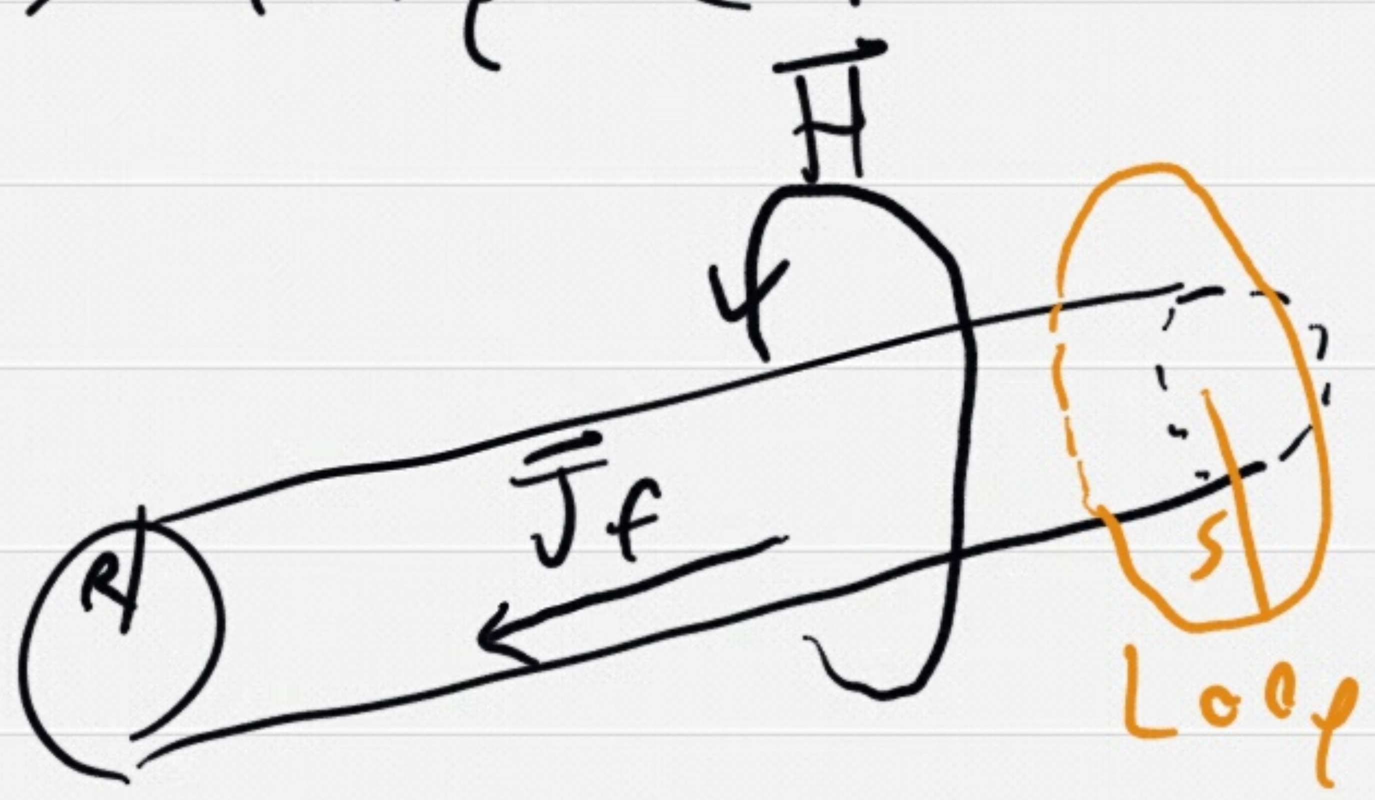
Griffiths: Just call $\vec{H}$ "H"

$$\nabla \times \vec{H} = \vec{J}_f$$

$$\int (\nabla \times \vec{H}) \cdot d\vec{a} = \int \vec{J}_f \cdot d\vec{a}$$

$$\boxed{\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}}$$

Example:



$\vec{J}_f = J \hat{z}$

$\vec{M} = ?$

$$\oint \vec{H} \cdot d\vec{l} = H \cdot 2\pi s$$
$$= I_{f\,enc}$$
$$= J \cdot \pi s^2 \quad s < R$$
$$ J \cdot \pi R^2 \quad s > R$$

$$\Rightarrow \vec{H} = \frac{Js}{2}\hat{\varphi} \quad s < R$$
$$= \frac{J \cdot R^2}{2s}\hat{\varphi} = \frac{I}{2\pi s}\hat{\varphi} \quad s > R$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = ? \quad s < R$$
$$= \frac{\mu_0 I \hat{\varphi}}{2\pi s} \quad s > R$$

Same as unmagnetized cylinder outside

Electrostatics	Magnetostatics
$\vec{E}$	$\vec{B}$
$\vec{P}$	$\vec{M}$
$\sigma_b = \vec{P} \cdot \hat{n}$	$\vec{K}_b = \vec{M} \times \hat{n}$
$\rho_b = -\nabla \cdot \vec{P}$	$\vec{J}_b = \nabla \times \vec{M}$
$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$	$\vec{H} = \vec{B}/\mu_0 - \vec{M}$
$\nabla \cdot \vec{D} = \rho_f$	$\nabla \times \vec{H} = \vec{J}_f$