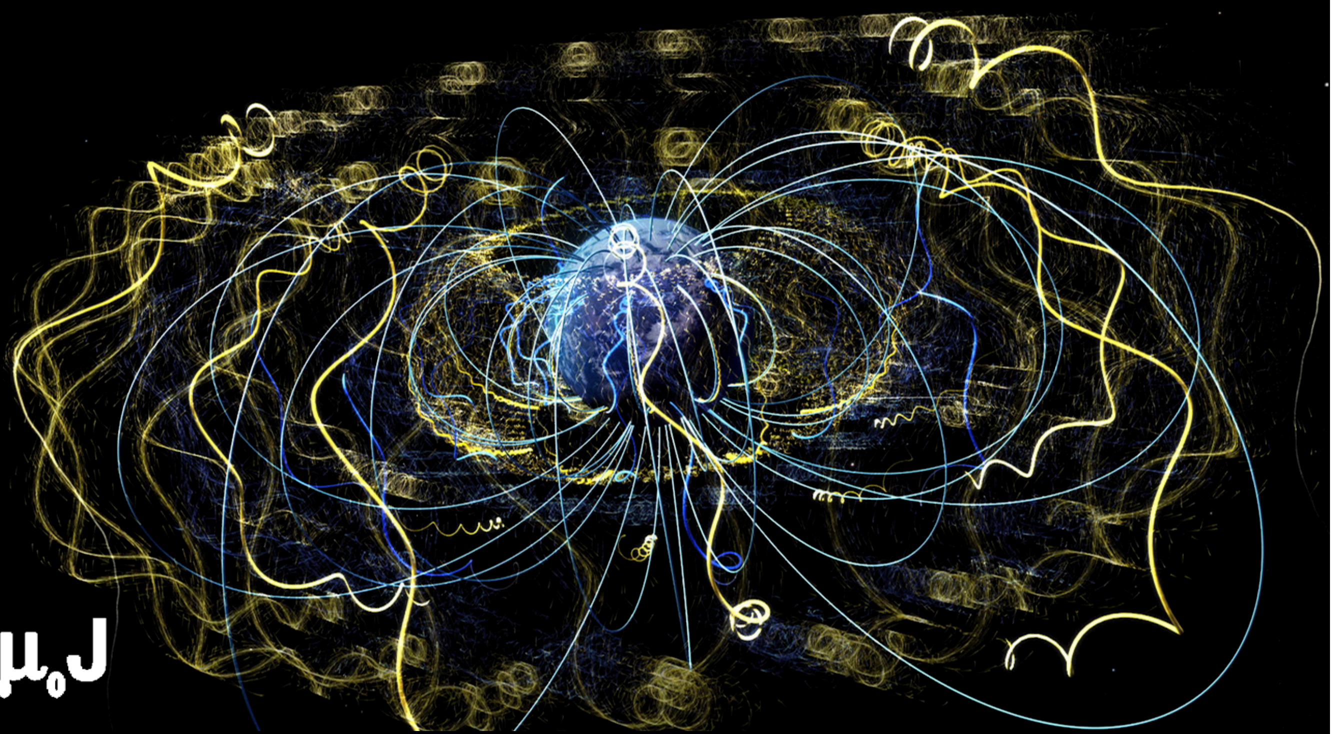
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

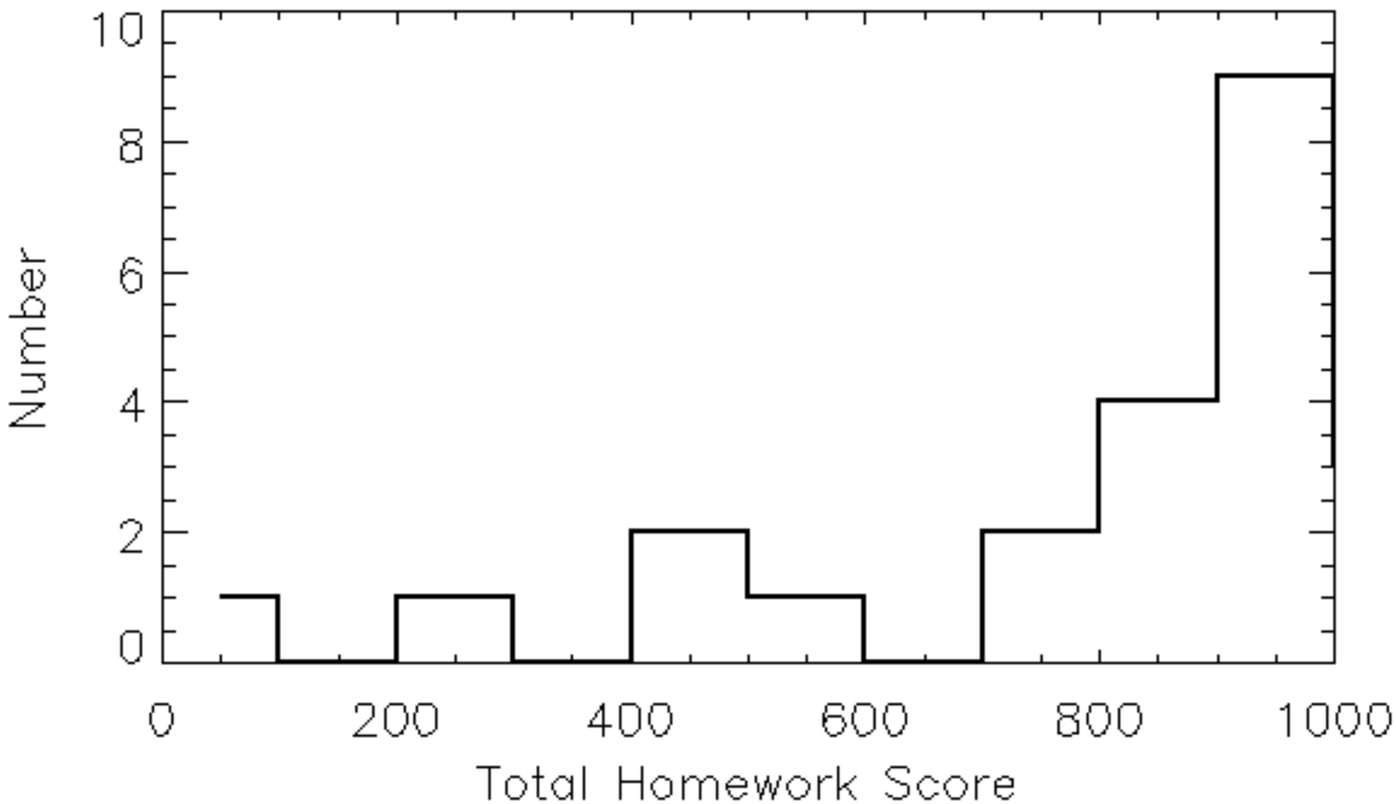


Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Announcements

- All homework scores uploaded and total homework score calculated



Announcements

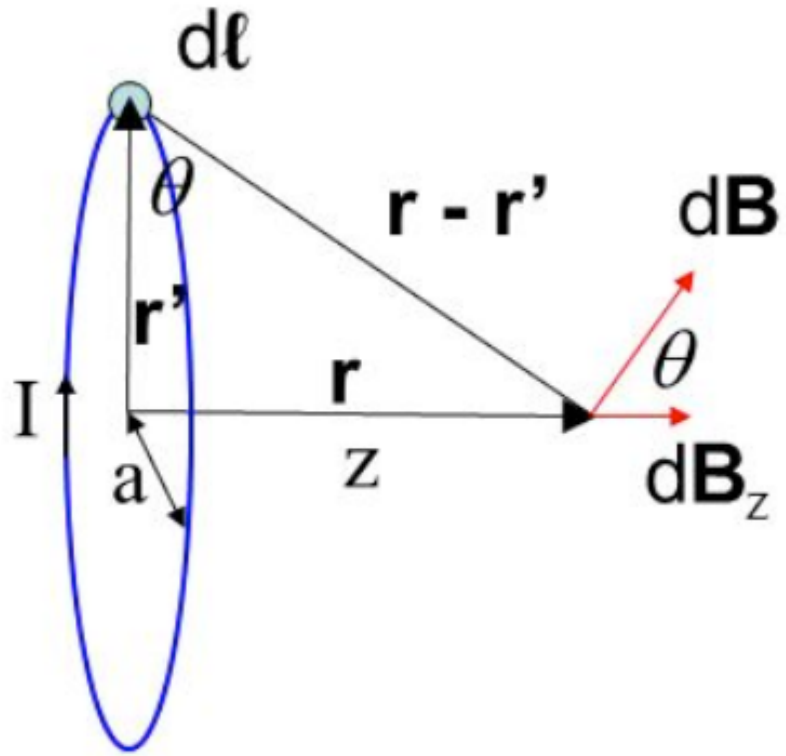
- Final Wednesday 12/18 12:30-2:30 in Van 301
  - The eternal question: Donuts or cookies?
  - Final Equation Sheet posted
    - Annotate as desired and bring to exam
  - Final exam from last year (and solutions) posted
- Please fill out course evaluations
  - Response rate so far: 10/23 - not bad but still time for more!
- Extra Office Hours Next Week
  - Monday 9am-12pm, Tuesday 12pm-2pm

Final Exam Details

- Final exam will have 8 questions covering material from Ch. 1-6 of Griffiths
  - No problems specifically on Ch. 1 – but most problems require the use of concepts or mathematical techniques from Ch. 1
  - Approximate distribution of points:
    - 62 points electrostatics
    - 38 points magnetostatics

Position and Source Coordinates

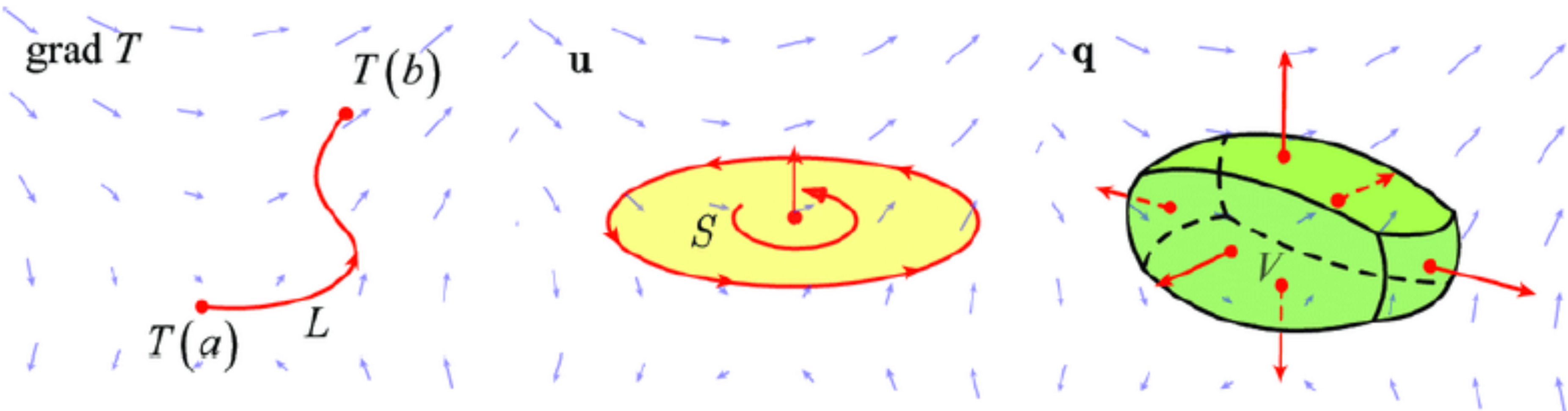
Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$



Fundamental Theorems of Vector Calculus

Fundamental Theorems of Vector Calculus:

$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a})$ $\int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da}$ $\int (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$



The Right Coordinates for the Job

Cartesian Coordinates: $\vec{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ $\quad d\tau = dx\,dy\,dz$

$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$ $\quad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$ $\quad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical Coordinates: $x = r\,sin\theta\,cos\phi,\ y = r\,sin\theta\,sin\phi,\ z = r\,cos\theta$

$\vec{dl} = dr\hat{r} + r\,d\theta\hat{\theta} + r\,sin\theta\,d\phi\hat{\phi}$ $\quad d\tau = r^2 sin\theta\,dr\,d\theta\,d\phi$

$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\,sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$ $\quad \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\,sin\theta}\frac{\partial (sin\theta\,A_\theta)}{\partial \theta} + \frac{1}{r\,sin\theta}\frac{\partial A_\phi}{\partial \phi}$

$\nabla \times \vec{A} = \frac{1}{r\,sin\theta}\left(\frac{\partial (sin\theta\,A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r\,A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r\,A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\phi}$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$

Cylindrical Coordinates: $x = s\,cos\phi,\ y = s\,sin\phi,\ z = z$

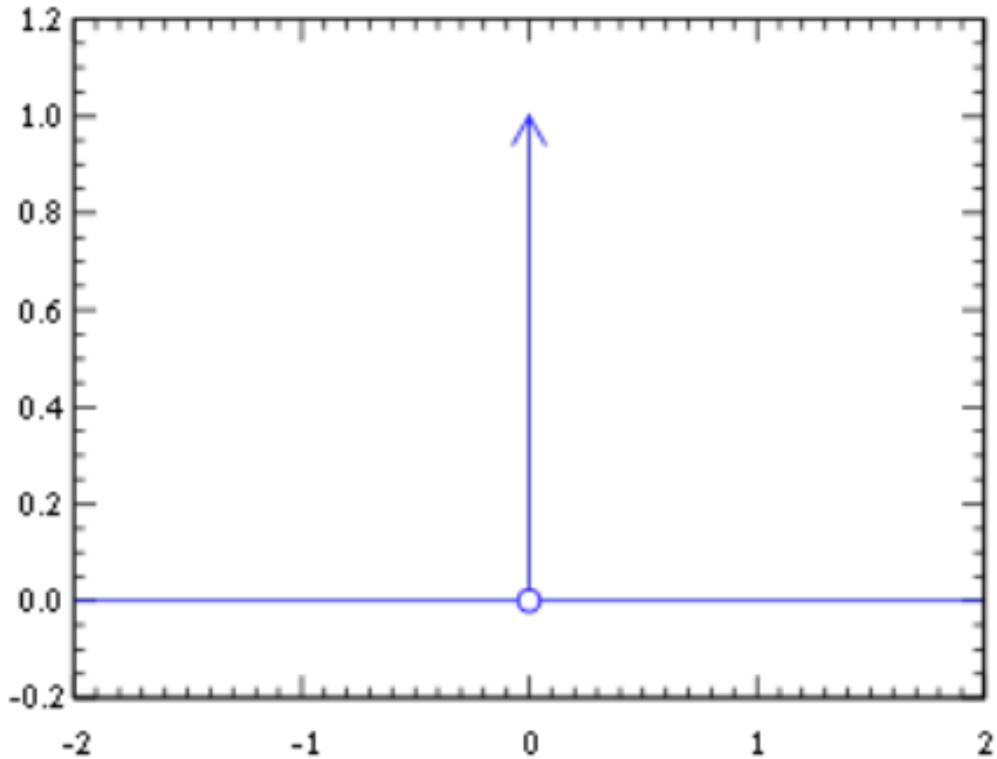
$\vec{dl} = ds\hat{s} + s\,d\phi\hat{\phi} + dz\hat{z}$ $\quad d\tau = s\,ds\,d\phi\,dz$

$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$ $\quad \nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial (s\,A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

$\nabla \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s\,A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$ $\quad \nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

Dirac Delta Function

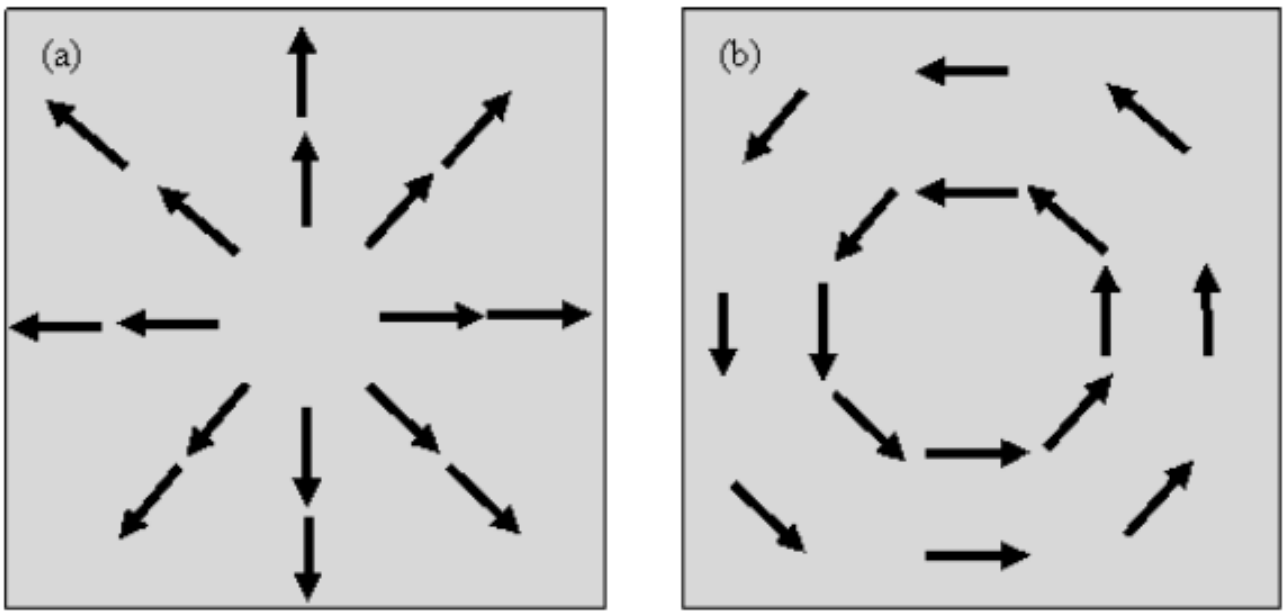
Dirac Delta Function: $\int \delta^3(\vec{r}-\vec{a})\, d\tau = 1$ if $\vec{a}$ contained in volume, $\delta^3\left(\overrightarrow{\Delta r}\right) = \frac{1}{4\pi}\nabla\cdot\left(\frac{\widehat{\Delta r}}{\Delta r^2}\right)$



Special Functions

Irrotational Function (e.g. Electrostatic Field): $\nabla\times\vec{E} = 0 \qquad \vec{E} = -\nabla V \qquad \oint \vec{E}\cdot\vec{dl} = 0$

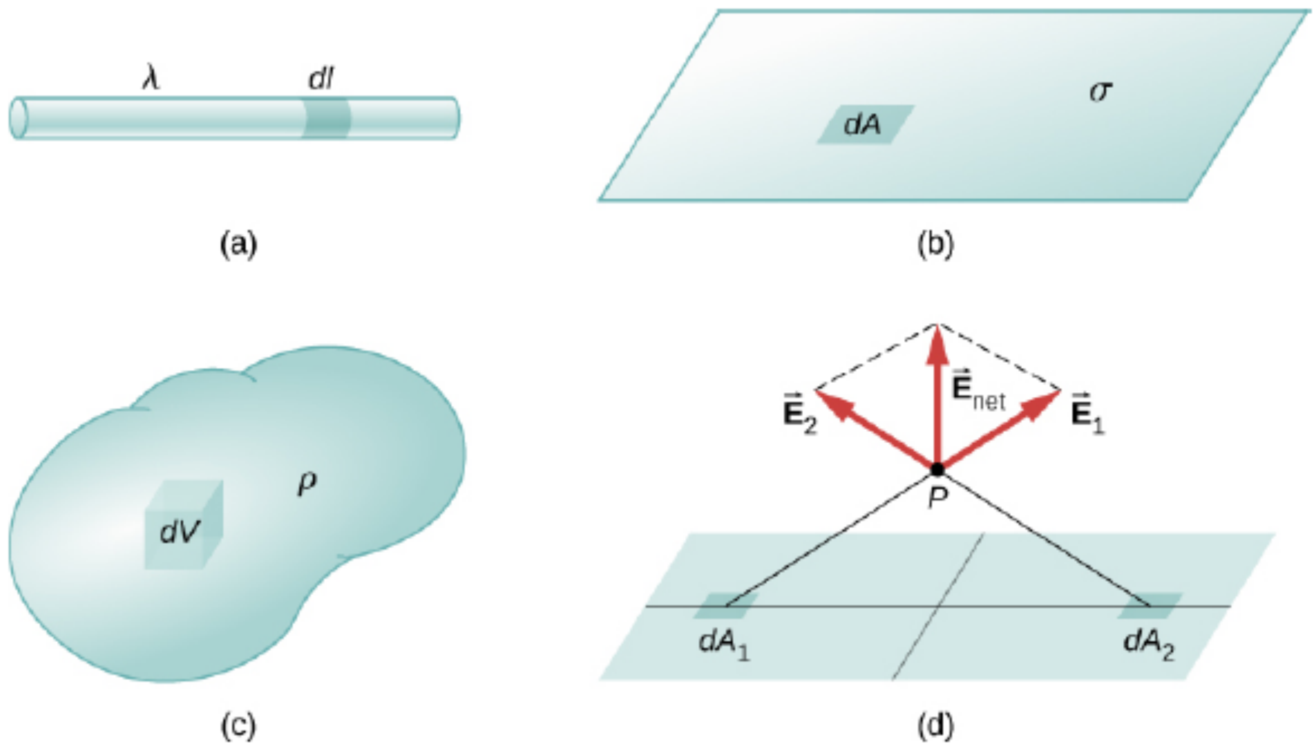
Solenoidal Function (e.g. Magnetostatic Field): $\nabla\cdot\vec{B} = 0 \qquad \vec{B} = \nabla\times\vec{A} \qquad \oint \vec{B}\cdot\vec{da} = 0$



Fields from Direct Integration

Electric Field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{\Delta r^2}\widehat{\Delta r}\, d\tau', \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}')}{\Delta r^2}\widehat{\Delta r}\, da', \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r}')}{\Delta r^2}\widehat{\Delta r}\, dl'$

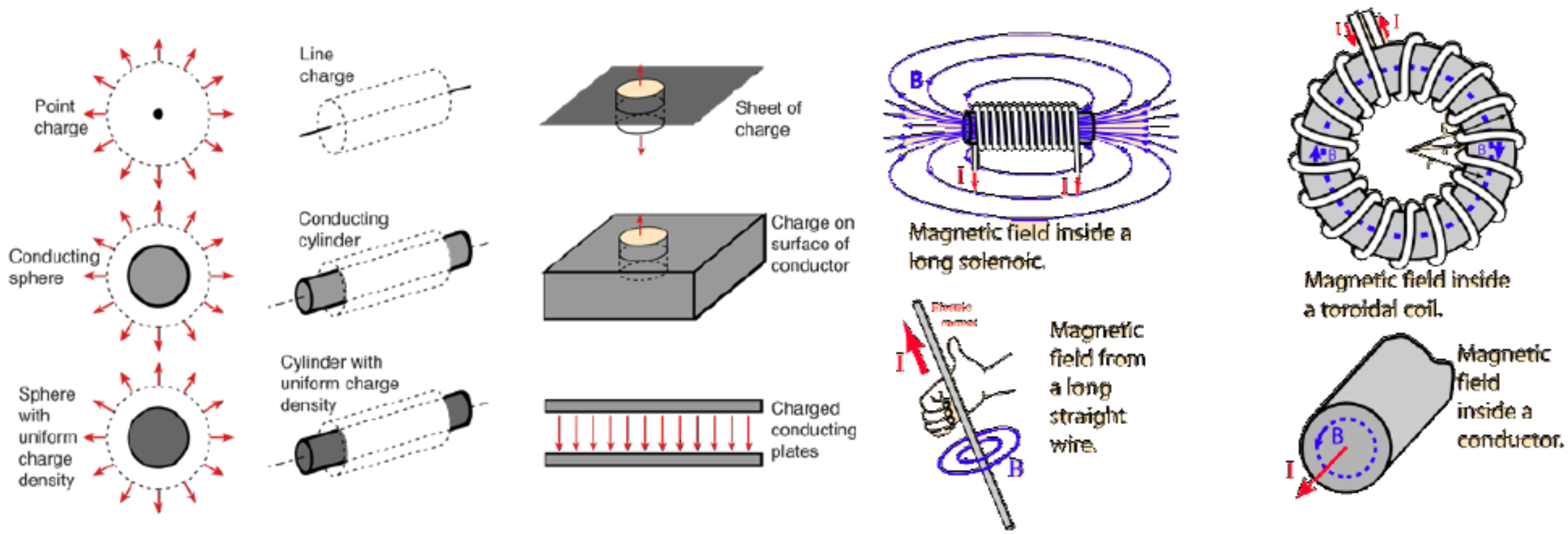
Magnetic Field: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\times\widehat{\Delta r}}{\Delta r^2} d\tau',\ \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}')\times\widehat{\Delta r}}{\Delta r^2} da', \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r}')\times\widehat{\Delta r}}{\Delta r^2} dl'$



Exploiting Symmetry

Gauss's Law: $\oint \vec{E} \cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0$, $\nabla \cdot \vec{E} = \rho/\varepsilon_0$, $\oint \vec{D} \cdot \overrightarrow{da} = Q_{f\_enc}$, $\nabla \cdot \vec{D} = \rho_{free}$

Ampere's Law: $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc}$, $\nabla \times \vec{B} = \mu_0 \vec{J}$, $\oint \vec{H} \cdot \overrightarrow{dl} = I_{free\_enc}$, $\nabla \times \vec{H} = \vec{J}_{free}$



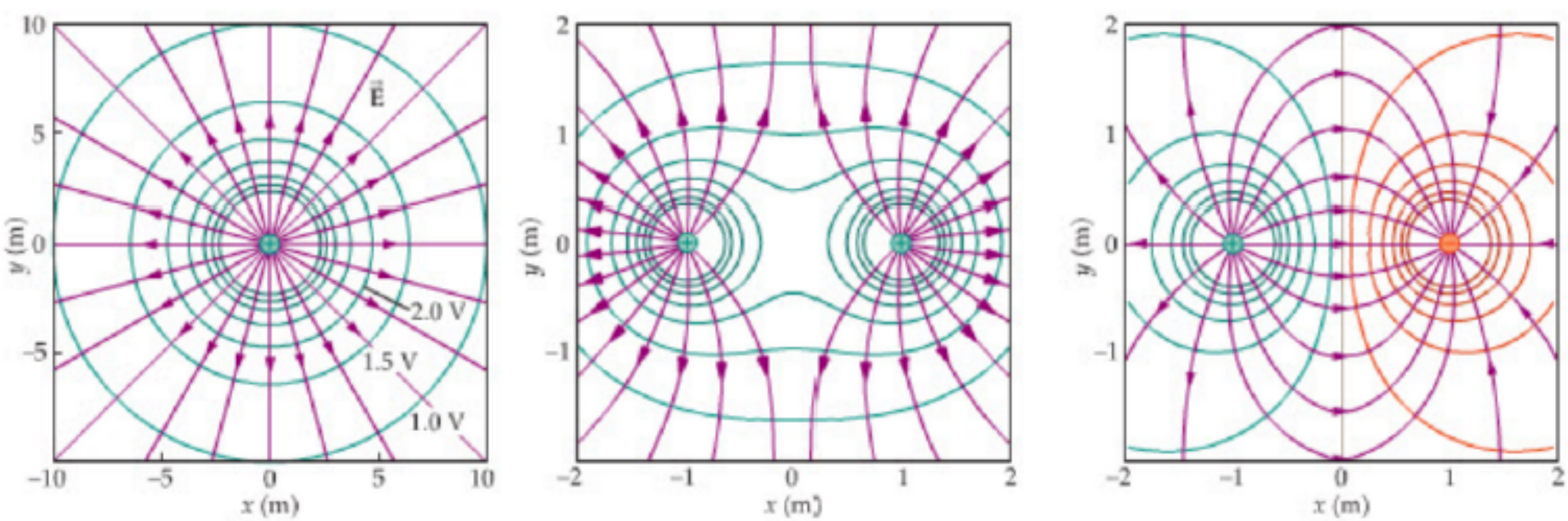
Potential Theory

Electric Potential: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r'})}{\Delta r} d\tau'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r'})}{\Delta r} da'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r'})}{\Delta r} dl'$

$\vec{E} = -\nabla V$, $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot \vec{dl}$, $\nabla^2 V = -\rho/\varepsilon_0$

Magnetic Vector Potential: $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r'})}{\Delta r} d\tau'$, $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r'})}{\Delta r} da'$, $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r'})}{\Delta r} dl'$

$\vec{B} = \nabla \times \vec{A}$, $\nabla \cdot \vec{A} = 0$ & $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ (Coulomb gauge)



Laplace's Equation

Laplace's Equation: $\nabla^2 V = 0 \; if \; \rho = 0, \;\; \nabla^2 \vec{A} = 0 \; if \; \vec{J} = 0$

Separation of Variables: $\frac{d^2X}{dx^2} = C_1 X, \frac{d^2Y}{dy^2} = C_2 Y, \frac{d^2Z}{dz^2} = C_3 Z, C_1 + C_2 + C_3 = 0, V(x,y,z) = X(x)Y(y)Z(z)$

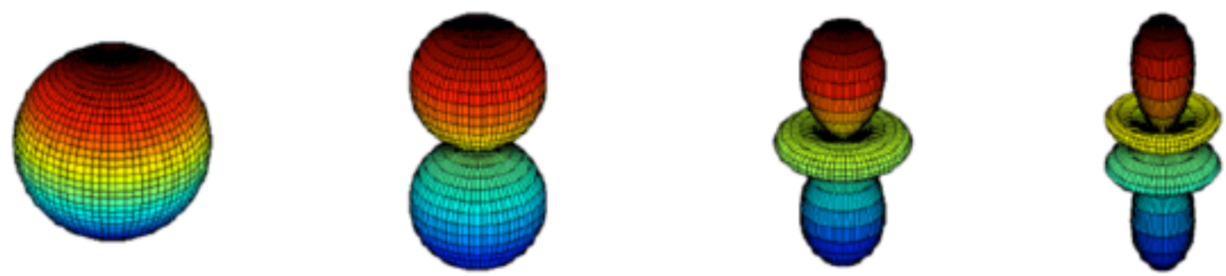
Separation of Variables (Spherical): $V(r,\theta) = \sum_0^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(cos\theta)$

$P_0(\cos\Theta) = 1$

$P_1(\cos\Theta) = \cos\Theta$

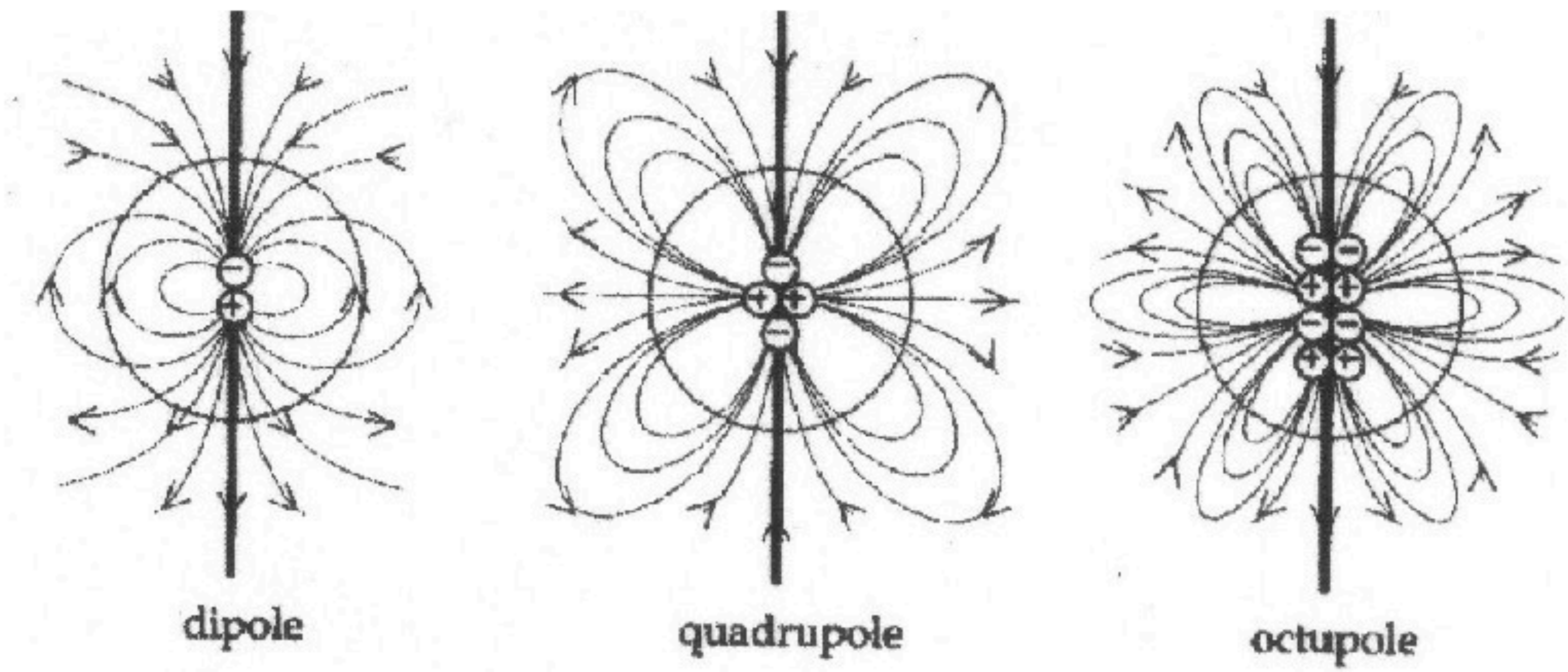
$P_2(\cos\Theta) = \frac{1}{2}(3\cos^2\Theta - 1)$

$P_3(\cos\Theta) = \frac{1}{2}(5\cos^3\Theta - 3\cos\Theta)$



Multipole Expansion

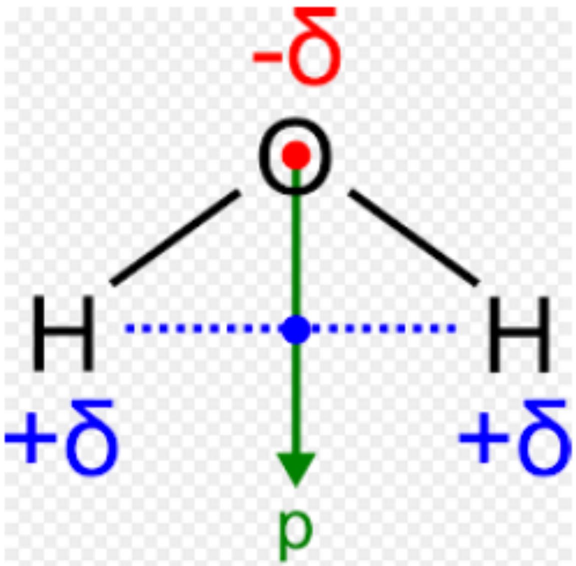
Multipoles: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_0^\infty \frac{1}{r^{n+1}}\int (r')^n P_n(cos\alpha)\rho(\vec{r}')d\tau'$, $\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\sum_0^\infty \frac{1}{r^{n+1}}\int (r')^n P_n(cos\alpha)\overrightarrow{dl}'$

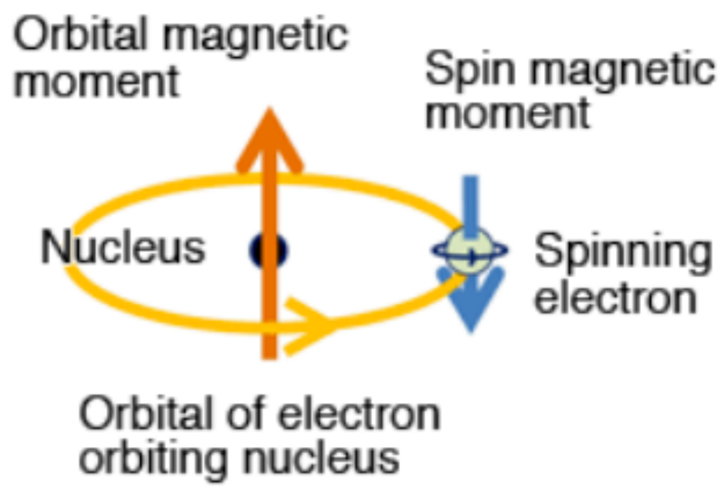


Dipoles

Dipoles: $\vec{p} = \int \vec{r}'\rho(\vec{r}')d\tau'$, $V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$, $\vec{\tau} = \vec{p}\times\vec{E}$, $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$, $\vec{P} = \vec{p}/volume$

$\vec{m} = I\vec{a}$, $\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$, $\vec{\tau} = \vec{m}\times\vec{B}$, $\vec{F} = \nabla(\vec{m}\cdot\vec{B})$, $\vec{M} = \vec{m}/volume$

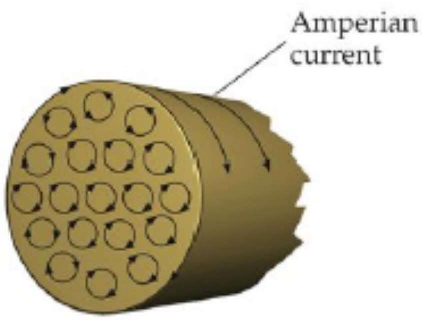


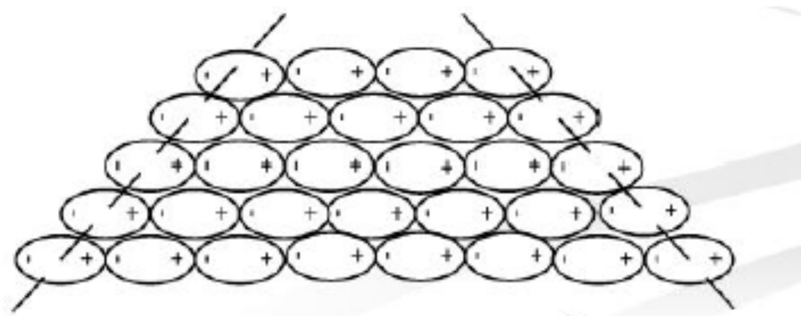


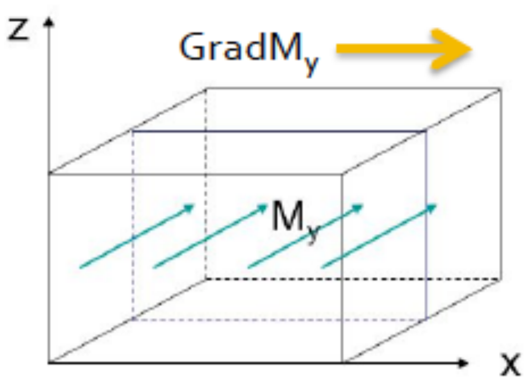
Fields in Matter

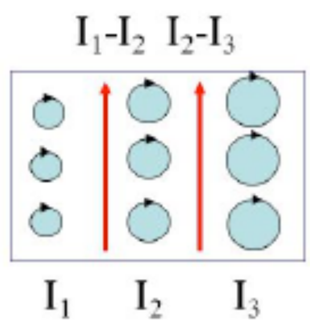
Fields in Matter: $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$, $\sigma_b = \vec{P}\cdot\hat{n}$, $\rho_b = -\nabla\cdot\vec{P}$

$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$, $\vec{K}_b = \vec{M}\times\hat{n}$, $\vec{J}_b = \nabla\times\vec{M}$





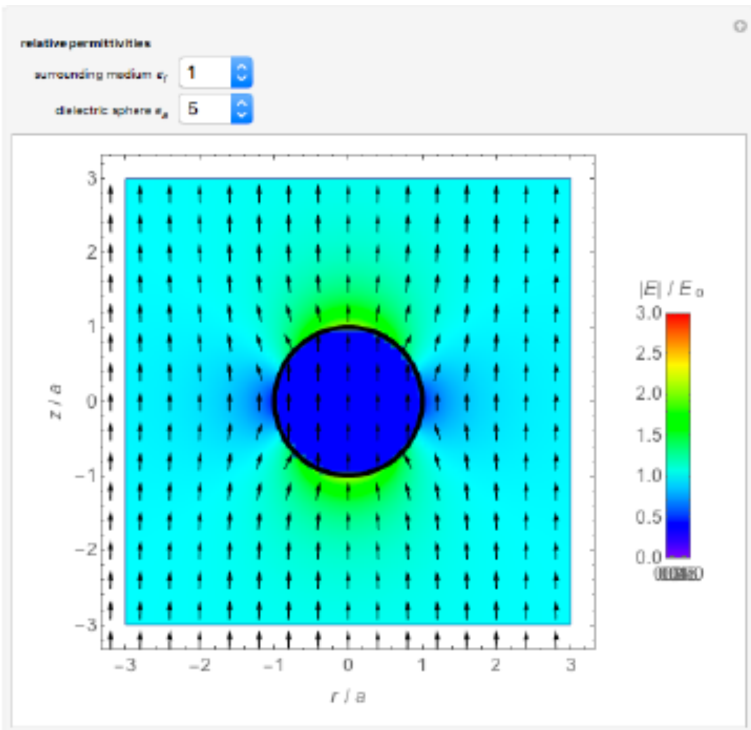


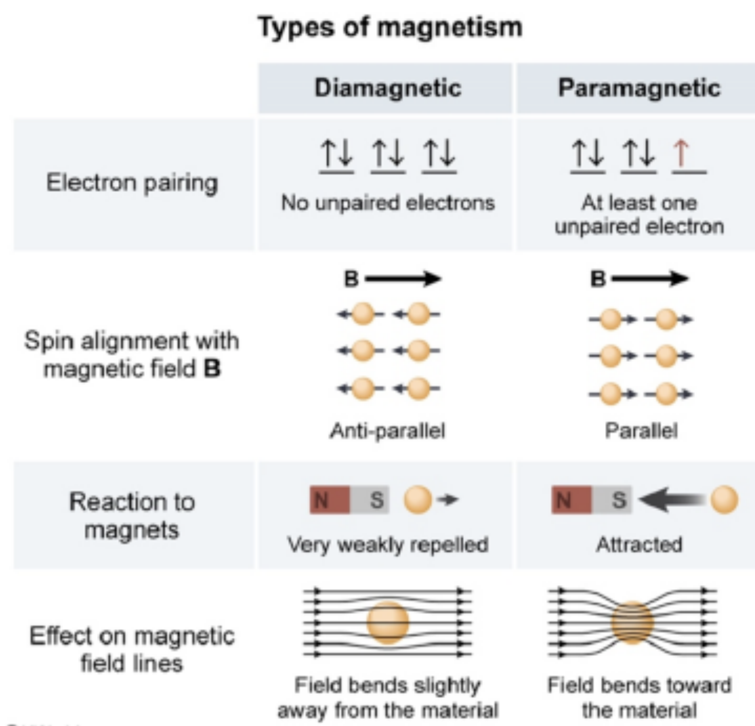


Linear Media

Linear Materials: $\vec{P} = \varepsilon_0 \chi_e \vec{E}, \quad \vec{D} = \varepsilon \vec{E} = (1+\chi_e)\varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$

$$\vec{M} = \chi_m \vec{H}, \quad \vec{B} = \mu \vec{H} = (1+\chi_m)\mu_0 \vec{H}$$

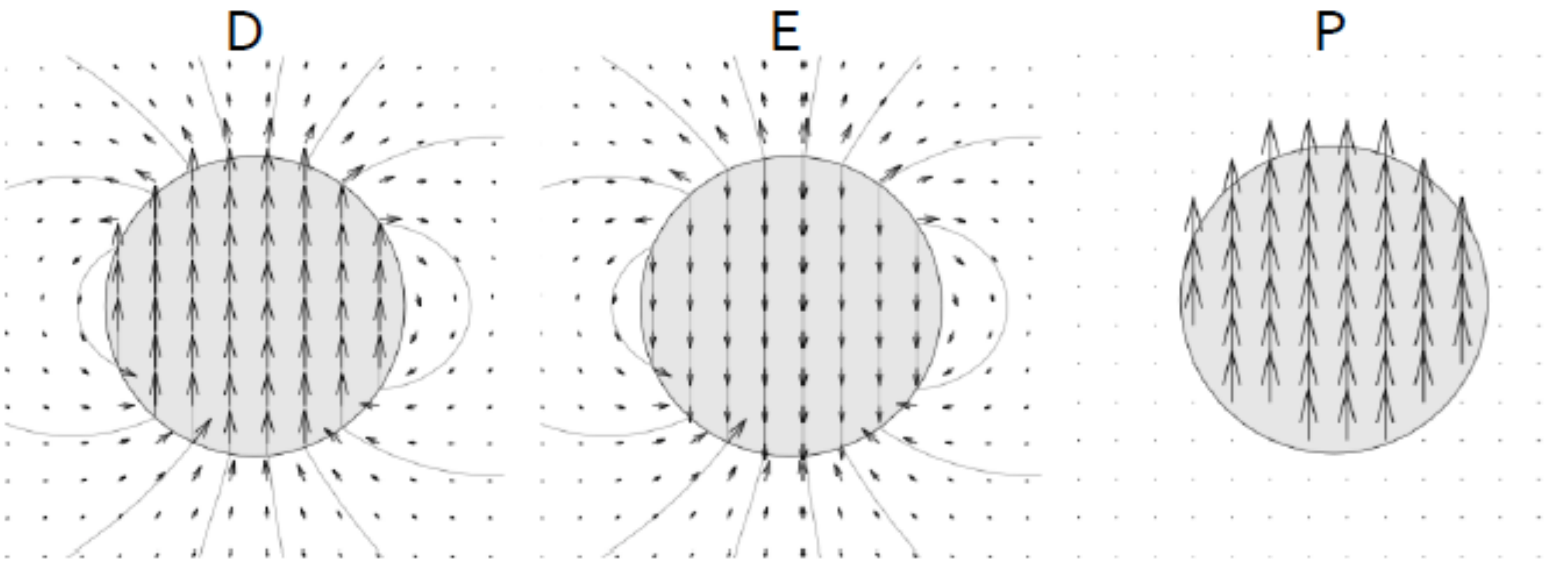




Boundary Conditions

Boundary Conditions: $\Delta V = 0,\ \Delta\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} = -\Delta\left(\frac{\partial V}{\partial n}\right)\hat{n},\quad \Delta D_\perp = \sigma_f,\qquad \Delta\vec{D}_{||} = \Delta\vec{P}_{||}$

$\Delta\vec{A} = 0,\ \Delta\vec{B} = \mu_0(\vec{K}\times\hat{n}),\ \Delta\left(\frac{\partial\vec{A}}{\partial n}\right) = -\mu_0\vec{K},\ \Delta\vec{H}_{||} = \vec{K}_{free}\times\hat{n},\ \Delta H_\perp = -\Delta M_\perp$



(a) **B**-field (b) **H**-field (c) **M**-field

Miscellaneous

Continuity: $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ (=0 in electrostatics/magnetostatics)

Lorentz Force: $\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$, On Wire: $\vec{F}_{mag} = \int I(\overrightarrow{dl}\times\vec{B})$

Work and Energy: $W = Q\Delta V$, $W_{electrostatic} = \frac{\varepsilon_0}{2}\int E^2 d\tau$, $W_{elec+polarization} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\, d\tau$